

Claudio Mirabello

Intro to Neural Networks and Deep Learning







Resources

- MIT lectures on Deep Learning (http://introtodeeplearning.com/)
- TensorFlow Playground (https://playground.tensorflow.org)
- Keras Docs (https://keras.io)



Quick notes

- ► All the code is in python, sorry R users! (you can still do everything we teach here in R as well)
- You must be tired of online lectures, so we will be taking plenty of breaks
- Interrupt at any time if you have questions, keep in mind that I might not see the chat
- Labs usually not about bioinformatics, not easy to build interesting datasets that can be processed in a seconds/minutes
- But everything you learn will be very much relevant to your research

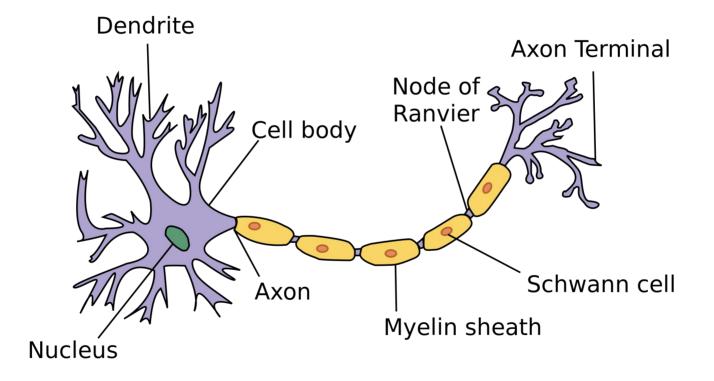


Quick notes

- We're going to cover mostly classification because it is easy to visualize and the most common task
- Regression in the end is not that different, as we will see it is mostly about changing one line of code

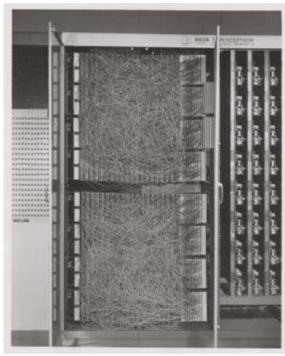


This is a neuron

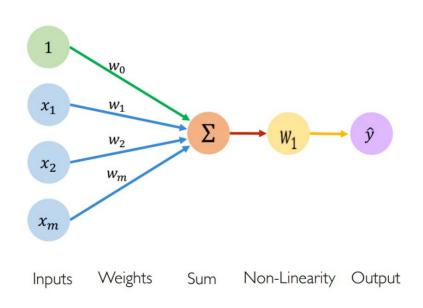


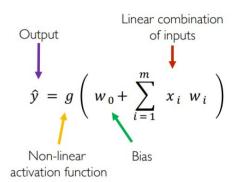


This is a perceptron (1958)



Mark I Perceptron machine wikipedia.org





MIT "Intro to Deep Learning"



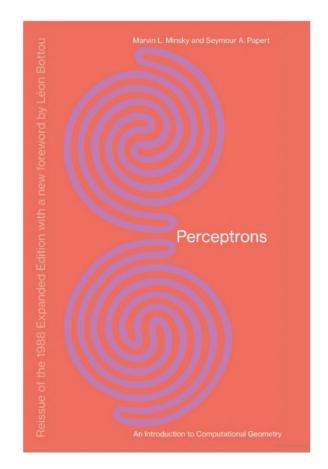
A lot was expected of perceptrons

"the embryo of an electronic computer that [the Navy] expects will be able to walk, talk, see, write, reproduce itself and be conscious of its existence."

The New York Times

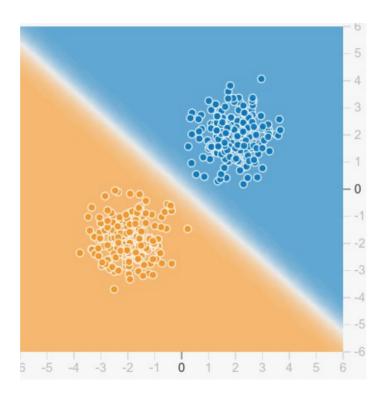


Perceptrons can only learn linearly separable classes



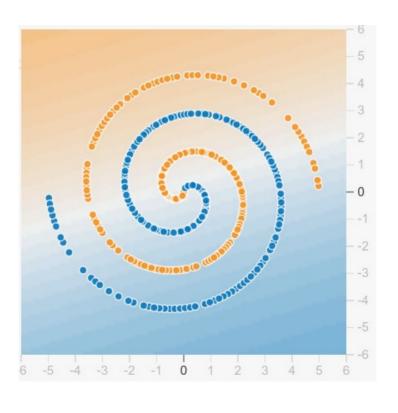


Perceptrons can only learn linearly separable classes





But sometimes you want to model non-linear functions

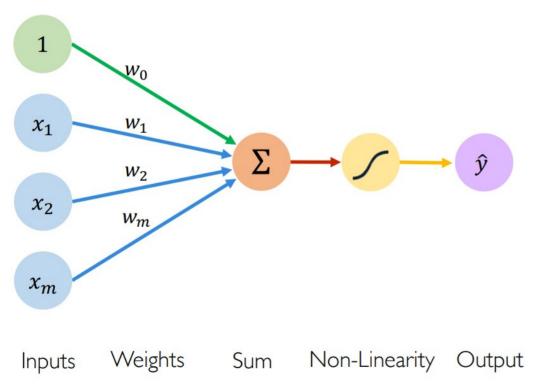




How do we make this non-linear then? Two ingredients to add



Non-linear activation functions

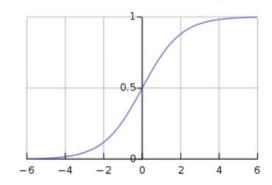


Activation Functions

$$\hat{y} = g(w_0 + X^T W)$$

• Example: sigmoid function

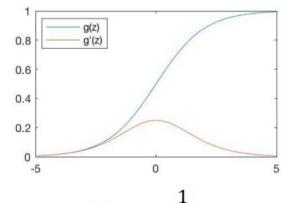
$$g(z) = \sigma(z) = \frac{1}{1 + e^{-z}}$$





Common activation functions

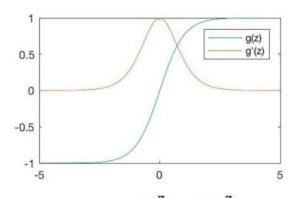
Sigmoid Function



$$g(z) = \frac{1}{1 + e^{-z}}$$

$$g'(z) = g(z)(1 - g(z))$$

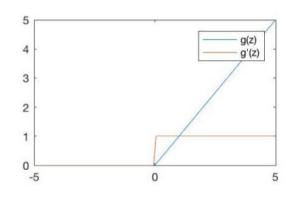
Hyperbolic Tangent



$$g(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

$$g'(z) = 1 - g(z)^2$$

Rectified Linear Unit (ReLU)



$$g(z) = \max(0, z)$$

$$g'(z) = \begin{cases} 1, & z > 0 \\ 0, & \text{otherwise} \end{cases}$$



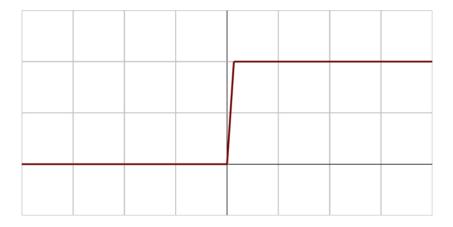
Special case: softmax

- Used in classification problems
- Given k classes, it decides which one is more likely
- One output per class, each output is assigned a probability from 0 to 1
- The sum of probabilities for all outputs is 1

$$g(z)_j = \frac{e^{z_j}}{\sum_{k=1}^k e^{z_k}}$$
 for $j = 1, ..., k$



Wait a second, the perceptron already has a non-linear (step) activation function!



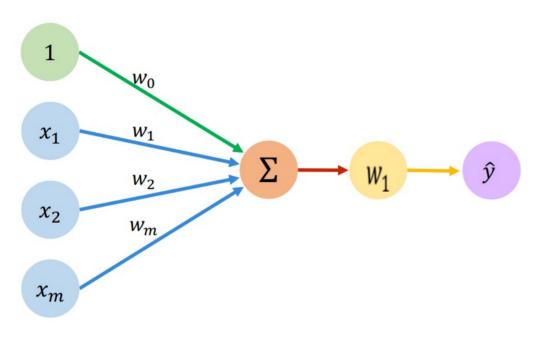


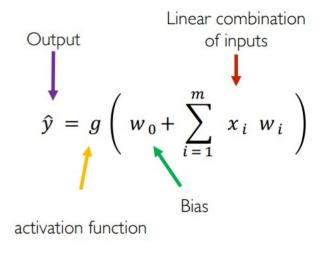
That was in the 50s

- Perceptron can't deal with many types of more complex patterns (XOR)
- Now we have huge, complex datasets (Big Data)
- Faster processors, more memory
- And a better understanding of learning processes



This is a perceptron

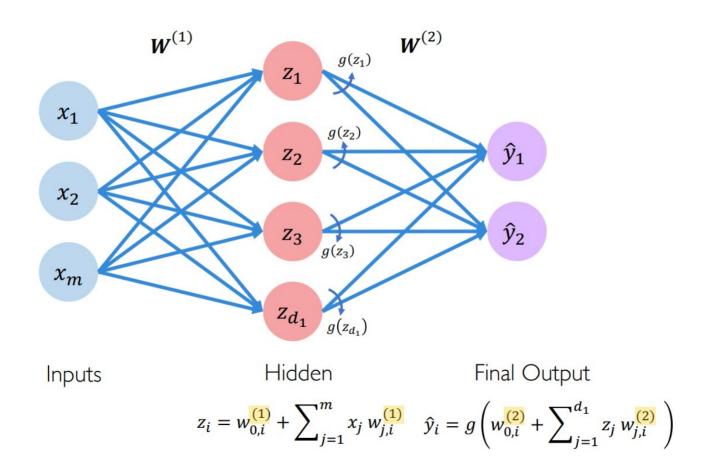




Inputs Weights Sum Non-Linearity Output

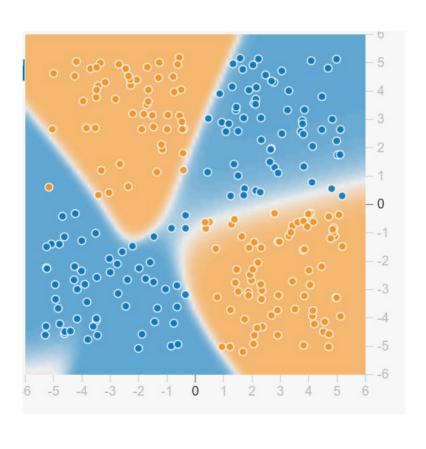


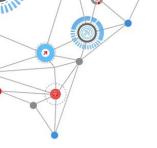
Multi-layer Perceptron (1986)



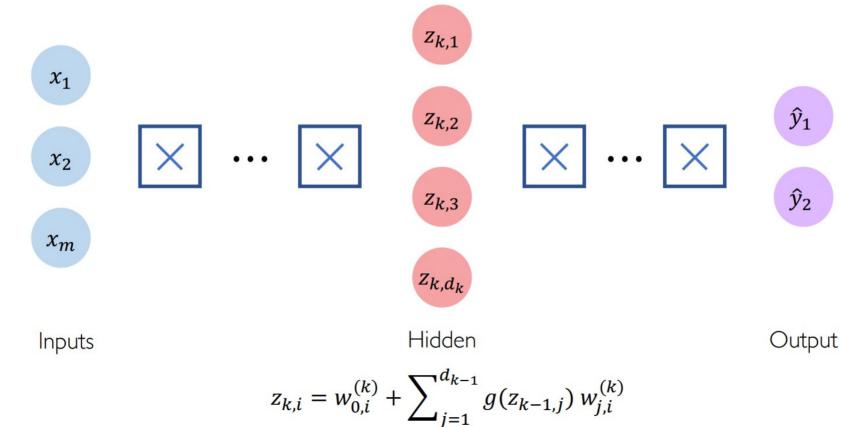


Now we're getting somewhere



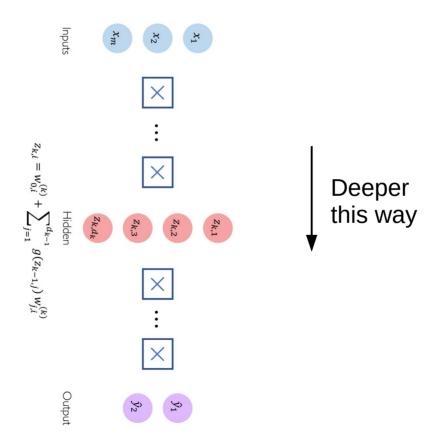


Why stop at one hidden layer?





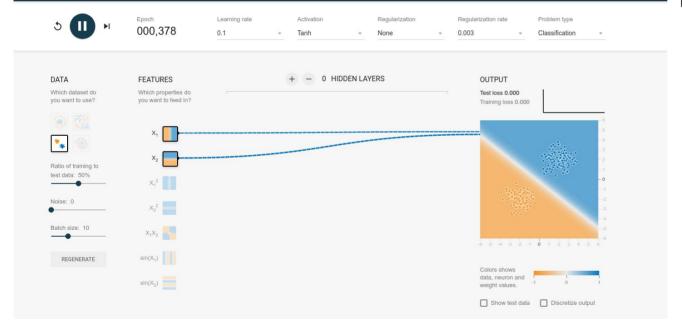
Deep Networks are simply NNs with multiple hidden layers





https://playground.tensorflow.org

Tinker With a **Neural Network** Right Here in Your Browser. Don't Worry, You Can't Break It. We Promise.



Let's review:

- Perceptron
- XOR problem
- Activations
- Multi-layer perceptron



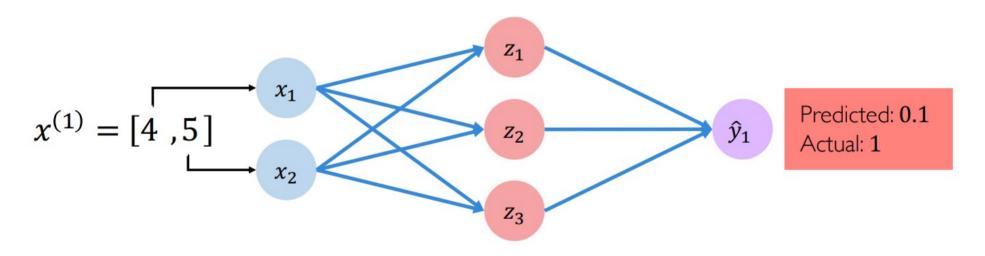
How do we decide which weights are optimal?

- A linear regressor's weights (coefficients) are calculated in closed form
- This can't be done if you have hidden layers and non-linear activations



How do we decide which weights are optimal?

The **loss** of our network measures the cost incurred from incorrect predictions



$$\mathcal{L}\left(f\left(x^{(i)}; \boldsymbol{W}\right), y^{(i)}\right)$$
Predicted Actual

How do we decide which weights are optimal?

The **empirical loss** measures the total loss over our entire dataset

$$\mathbf{X} = \begin{bmatrix} 4, & 5 \\ 2, & 1 \\ 5, & 8 \\ \vdots & \vdots \end{bmatrix} \qquad \begin{array}{c} \mathbf{x_1} \\ \mathbf{x_2} \\ \end{array} \qquad \begin{array}{c} \mathbf{f}(\mathbf{x}) \\ \mathbf{y} \\ \end{array} \qquad \begin{array}{c} \mathbf{f}(\mathbf{x}) \\ \begin{bmatrix} 0.1 \\ 0.8 \\ 0.6 \\ \vdots \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 0 \\ 1 \\ \vdots \end{bmatrix}$$

Also known as:

- Objective function
- Cost function
- Empirical Risk

 $-J(\mathbf{W}) = \frac{1}{n} \sum_{i=1}^{n} \mathcal{L}(f(x^{(i)}; \mathbf{W}), y^{(i)})$

Predicted

Actual

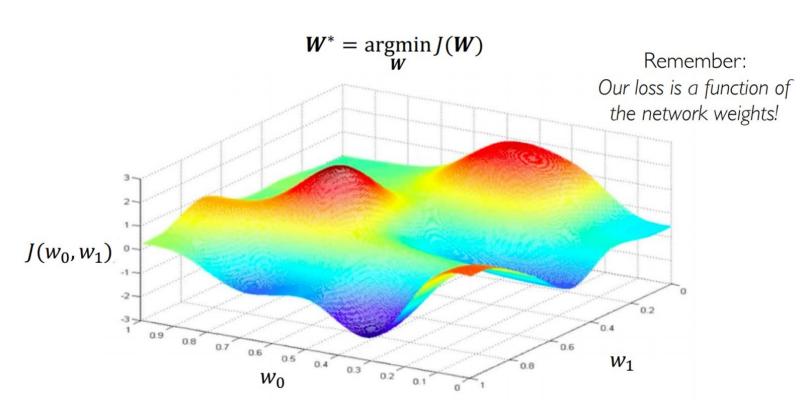


Lower loss => better predictions

We want to find the network weights that achieve the lowest loss

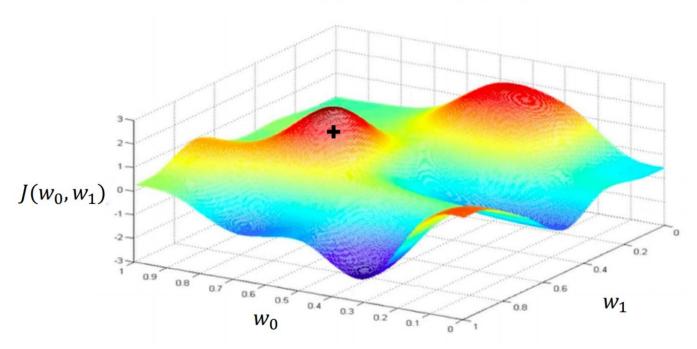
$$W^* = \underset{W}{\operatorname{argmin}} \frac{1}{n} \sum_{i=1}^{n} \mathcal{L}(f(x^{(i)}; W), y^{(i)})$$
$$W^* = \underset{W}{\operatorname{argmin}} J(W)$$





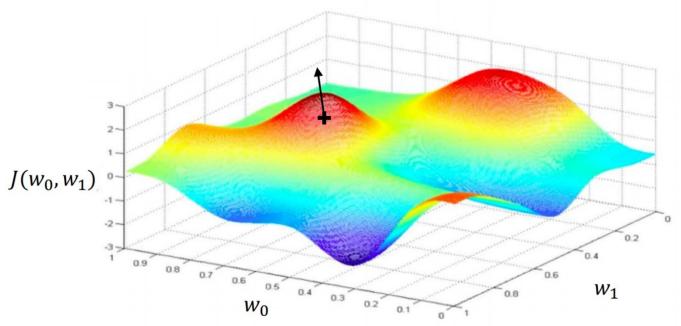


Randomly pick an initial (w_0, w_1)



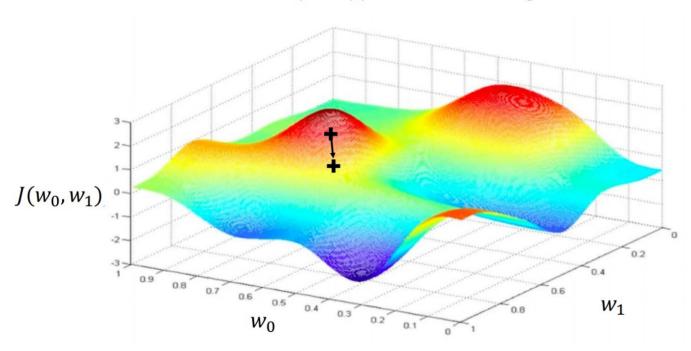






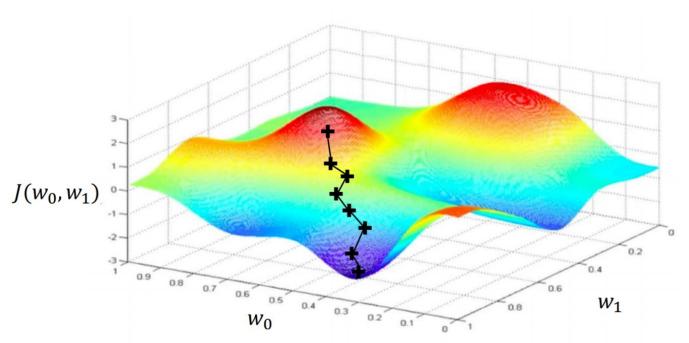


Take small step in opposite direction of gradient











Gradient descent

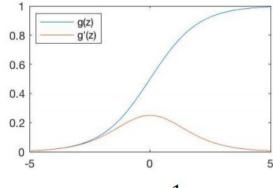
Algorithm

- Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
- 2. Loop until convergence:
- Compute gradient, $\frac{\partial J(W)}{\partial W}$ Update weights, $W \leftarrow W \eta \frac{\partial J(W)}{\partial W}$
- 5. Return weights



Activation functions have to be differentiable!

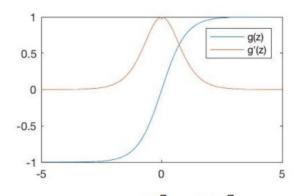
Sigmoid Function



$$g(z) = \frac{1}{1 + e^{-z}}$$

$$g'(z) = g(z)(1 - g(z))$$

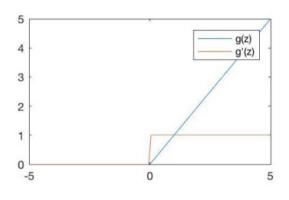
Hyperbolic Tangent



$$g(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

$$g'(z) = 1 - g(z)^2$$

Rectified Linear Unit (ReLU)



$$g(z) = \max(0, z)$$

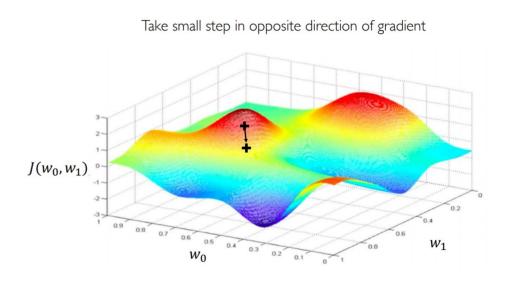
$$g'(z) = \begin{cases} 1, & z > 0 \\ 0, & \text{otherwise} \end{cases}$$



The learning rate η

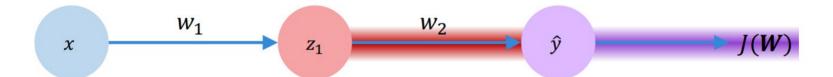
Algorithm

- 1. Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
- 2. Loop until convergence:
- 3. Compute gradient, $\frac{\partial J(W)}{\partial W}$
- 4. Update weights, $\mathbf{W} \leftarrow \mathbf{W} \eta \frac{\partial J(\mathbf{W})}{\partial \mathbf{W}}$
- 5. Return weights





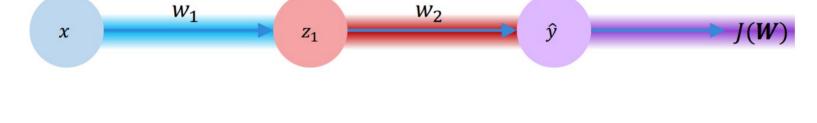
Backpropagation



$$\frac{\partial J(\boldsymbol{W})}{\partial w_2} = \frac{\partial J(\boldsymbol{W})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial w_2}$$



Backpropagation



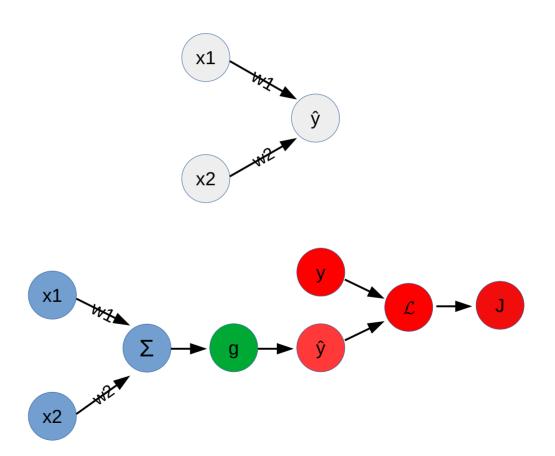
$$\frac{\partial J(\mathbf{W})}{\partial w_1} = \frac{\partial J(\mathbf{W})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial z_1} * \frac{\partial z_1}{\partial w_1}$$

Backpropagation example, step by step: https://mattmazur.com/2015/03/17/a-step-by-step-backpropagation-example/



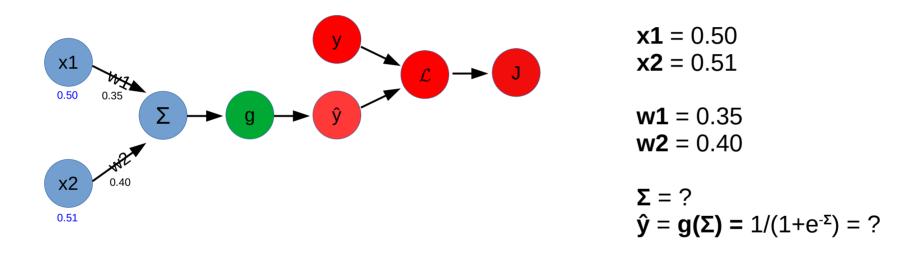
Practical example

- Simple network:
 - ► Two inputs [x1, x2]
 - ► Two weights [w1, w2]
 - ▶ No bias
 - Activation function g()
 - ► One output ŷ
 - ► One label y
 - \triangleright Loss function $\mathcal{L}()$
 - Weight-dependent error J(W)



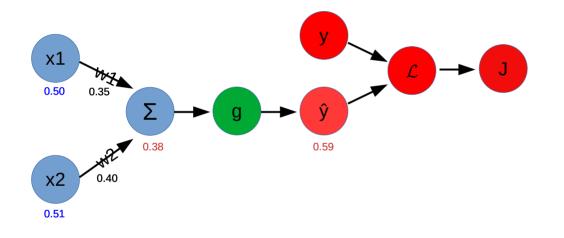


1. Forward pass





1. Forward pass



$$x1 = 0.50$$

 $x2 = 0.51$

$$w1 = 0.35$$

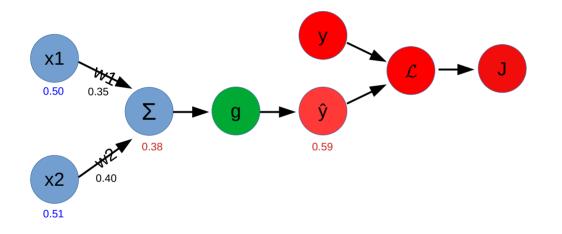
 $w2 = 0.40$

$$\Sigma = 0.35 * x1 + 0.4 * w2 = 0.38$$

 $\hat{y} = g(\Sigma) = 1/(1+e^{-\Sigma}) = 0.59$



2. Calculate Loss



$$x1 = 0.50$$

 $x2 = 0.51$

$$w1 = 0.35$$

 $w2 = 0.40$

$$\Sigma = 0.35 * x1 + 0.4 * w2 = 0.38$$

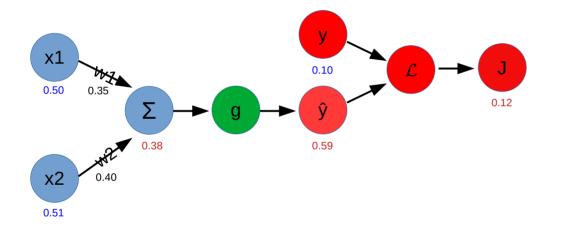
 $\hat{y} = g(\Sigma) = 1/(1+e^{-\Sigma}) = 0.59$

$$Y = 0.10$$

 $J(W) = \mathcal{L}(y, \hat{y}) = \frac{1}{2} * (y - \hat{y})^2 = ?$



2. Calculate Loss



$$x1 = 0.50$$

 $x2 = 0.51$

$$w1 = 0.35$$

 $w2 = 0.40$

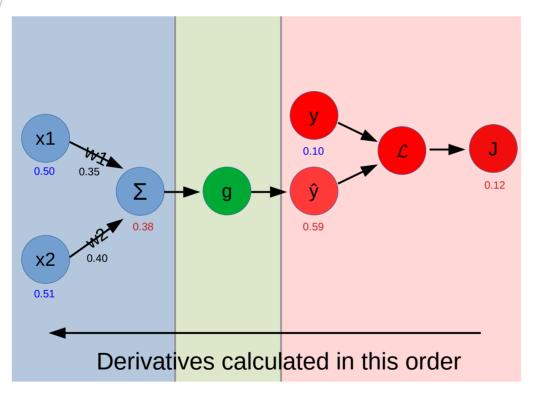
$$\Sigma = 0.35 * x1 + 0.4 * w2 = 0.38$$

 $\hat{y} = g(\Sigma) = 1/(1+e^{-\Sigma}) = 0.59$

$$Y = 0.10$$

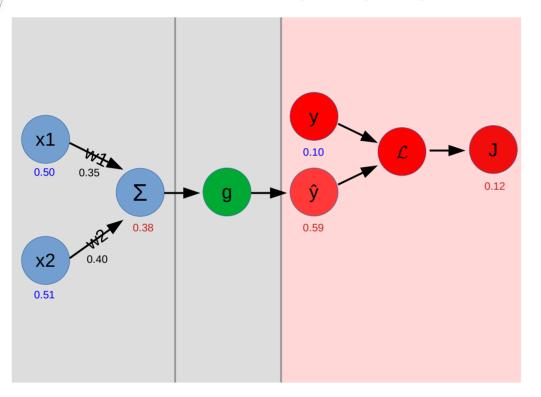
 $J(W) = \mathcal{L}(y, \hat{y}) = \frac{1}{2} * (y - \hat{y})^2 = 0.12$

3. Backpropagate the error



```
J(W) = \mathcal{L}(g(X * W))
\partial J(W)/\partial w1 = \partial J(W) / \partial \hat{y} \text{ (loss)}
* \partial \hat{y} / \Sigma \text{ (activation)}
* \partial \Sigma / \partial w1 \text{ (weight)}
```

3. Backpropagate the error: loss



$$J(W) = \mathcal{L}(g(X * W))$$

$$\partial J(W)/\partial w1 = \partial J(W) / \partial \hat{y} \text{ (loss)}$$

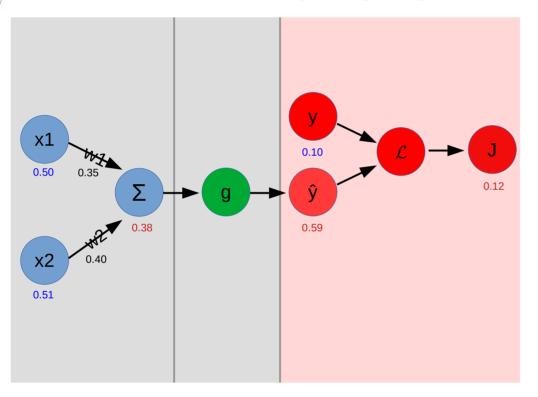
$$* \partial \hat{y} / \Sigma \text{ (activation)}$$

$$* \partial \Sigma / \partial w1 \text{ (weight)}$$

$$J(W) = \mathcal{L}(y, \hat{y}) = \frac{1}{2} * (y - \hat{y})^2$$

$$\partial J(W) / \partial \hat{y} = \partial \mathcal{L}(y, \hat{y}) / \partial \hat{y} = ?$$

3. Backpropagate the error: loss



$$J(W) = \mathcal{L}(g(X * W))$$

$$\partial J(W)/\partial w1 = \partial J(W) / \partial \hat{y} \text{ (loss)}$$

$$* \partial \hat{y} / \Sigma \text{ (activation)}$$

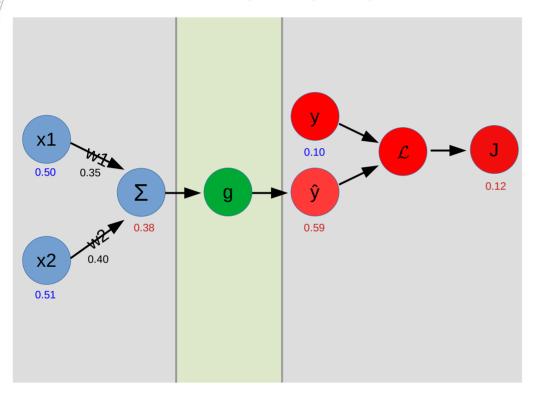
$$* \partial \Sigma / \partial w1 \text{ (weight)}$$

$$J(W) = \mathcal{L}(y, \hat{y}) = \frac{1}{2} * (y - \hat{y})^2$$

$$\partial J(W) / \partial \hat{y} = 2 * \frac{1}{2} * (y - \hat{y}) * -1$$

$$= -y + \hat{y} = -0.1 + 0.59 = 0.49$$

3. Backpropagate the error: activation



$$J(W) = \mathcal{L}(g(X * W))$$

$$\partial J(W)/\partial w1 = 0.49 \qquad \text{(loss)}$$

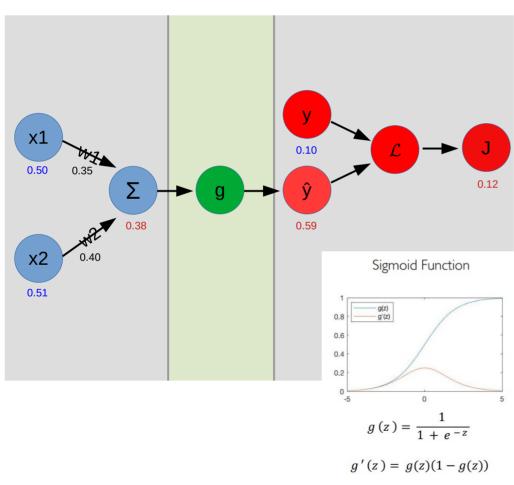
$$* \partial \hat{y} / \partial \Sigma \qquad \text{(activation)}$$

$$* \partial \Sigma / \partial w1 \qquad \text{(weight)}$$

$$\hat{\mathbf{y}} = g(\Sigma) = 1/(1 + e^{-\Sigma})$$

$$\partial \hat{y} / \partial \Sigma = ?$$

3. Backpropagate the error: activation



$$J(W) = \mathcal{L}(g(X * W))$$

$$\partial J(W)/\partial w1 = 0.49 \qquad \text{(loss)}$$

$$* \partial \hat{y} / \partial \Sigma \qquad \text{(activation)}$$

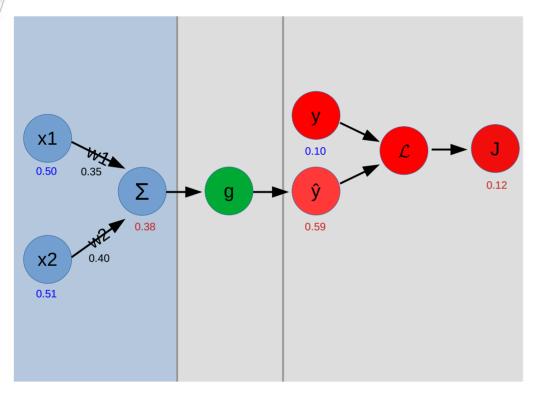
$$* \partial \Sigma / \partial w1 \qquad \text{(weight)}$$

$$\hat{\mathbf{y}} = g(\Sigma) = 1/(1 + e^{-\Sigma})$$

$$\partial \hat{y} / \partial \Sigma = 1/(1 + e^{-\Sigma}) * (1 - 1/(1 + e^{-\Sigma}))$$

$$= 0.24$$

3. Backpropagate the error: weight



$$J(W) = \mathcal{L}(g(X * W))$$

$$\partial J(W)/\partial w1 = 0.49 \qquad \text{(loss)}$$

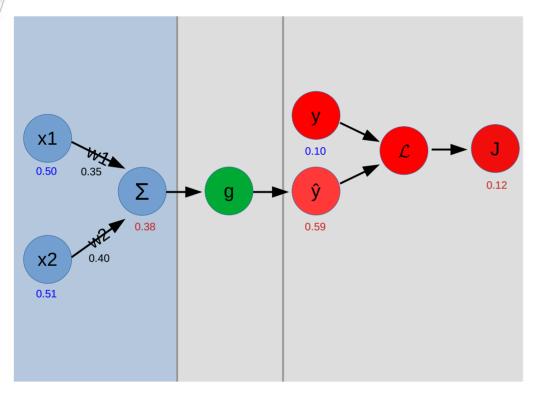
$$* 0.24 \qquad \text{(activation)}$$

$$* \partial \Sigma / \partial w1 \qquad \text{(weight)}$$

$$\Sigma = X * W = x1 * w1 + x2 * w2$$

 $\partial \Sigma / \partial w 1 = ?$

3. Backpropagate the error: weight



$$J(W) = \mathcal{L}(g(X * W))$$

$$\partial J(W)/\partial w1 = 0.49 \qquad \text{(loss)}$$

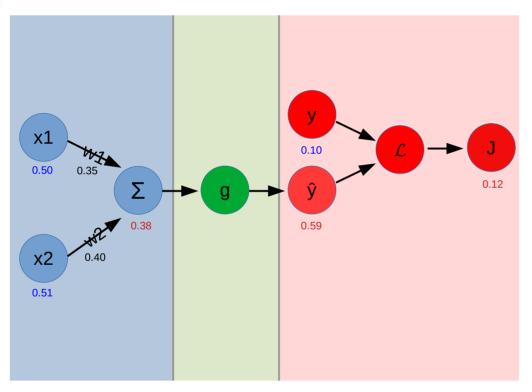
$$* 0.24 \qquad \text{(activation)}$$

$$* \partial \Sigma / \partial w1 \qquad \text{(weight)}$$

$$\Sigma = X * W = x1 * w1 + x2 * w2$$

 $\partial \Sigma / \partial w 1 = x1 + 0 = 0.5$

4. Weight update



$$J(W) = \mathcal{L}(g(X * W))$$

$$\partial J(W)/\partial w1 = 0.49 \qquad \text{(loss)}$$

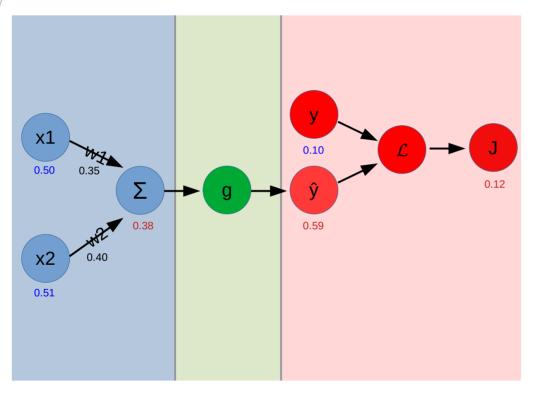
$$* 0.24 \qquad \text{(activation)}$$

$$* 0.5 \qquad \text{(weight)}$$

$$= 0.06 \qquad \text{(gradient)}$$

$$w1' = ?$$

4. Weight update



```
J(W) = \mathcal{L}(g(X * W))

\partial J(W)/\partial w1 = 0.49 (loss)

* 0.24 (activation)

* 0.5 (weight)

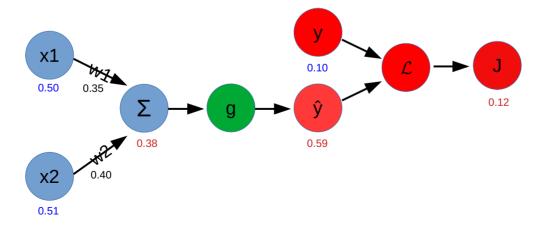
= 0.06 (gradient)

w1' = w1 - \eta*0.06 = 0.35 - 0.06 = 0.29
```



Exercise

- Can you calculate the weight update for w2? How many new gradients do you need to calculate?
- What is the new predicted output? Has the error gone down?
- What if I had another layer before this one?



Gradient vanishing

What happens if we backpropagate on a network with many (N > k > 1) hidden layers?

$$z_{k,1}$$
 $z_{k,2}$
 $z_{k,2}$
 $z_{k,3}$
 $z_{k,3}$
 z_{k,d_k}

Inputs

Hidden

 $z_{k,i} = w_{0,i}^{(k)} + \sum_{j=1}^{d_{k-1}} g(z_{k-1,j}) \, w_{j,i}^{(k)}$

Output

$$\partial E/\partial W1 = \partial J/\partial \hat{Y} * \partial \hat{Y}/\partial \Sigma_{N} * \partial \Sigma_{N}/\partial W_{N} * \dots * \partial Z_{k}/\partial \Sigma_{k} * \partial \Sigma_{k}/\partial W_{k} * \dots \partial Z_{1}/\partial \Sigma_{1} * \partial \Sigma_{1}/\partial X_{1}$$



Gradient vanishing

- These are all "zero-point-somethings" multiplied by each other
- So the gradient becomes smaller by orders of magnitudes as we go back more and more layers until it's so small that the network is stuck

$$\partial E/\partial W1 = \partial J/\partial \hat{Y} * \partial \hat{Y}/\partial \Sigma_{N} * \partial \Sigma_{N}/\partial W_{N} * ... * \partial Z_{k}/\partial \Sigma_{k} * \partial \Sigma_{k}/\partial W_{k} * ... \partial Z_{1}/\partial \Sigma_{1} * \partial \Sigma_{1}/\partial X_{1} = O(10^{-N})$$

initial w1 = 0.5optimal w1 = -0.25-layer gradient ~ 0.00001

How many iterations do we need to get from 0.5 to -0.2?



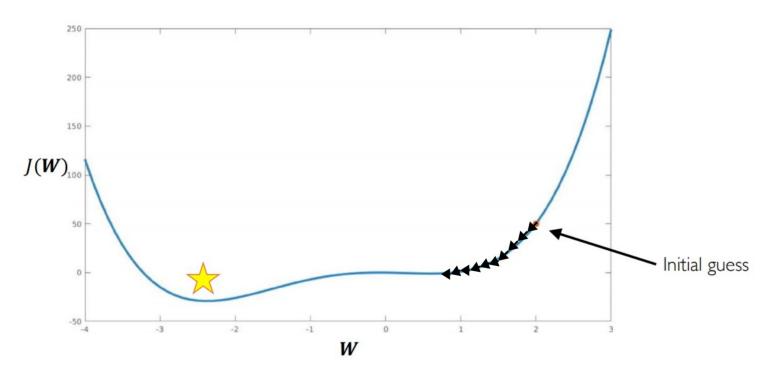
Other issues

Finding the right learning rate (η, eta) can be tricky

Computing the loss over the full set at every iteration is slow

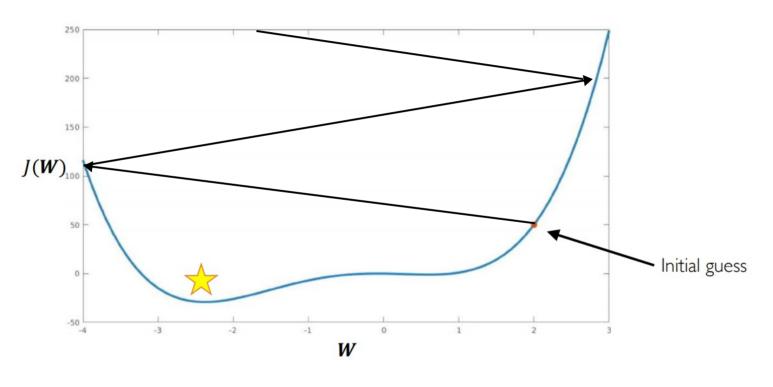


Too small: slow, gets stuck



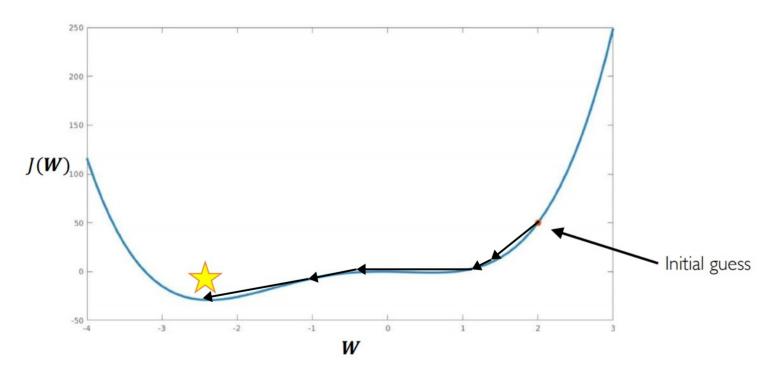


Too large: fast, might explode





Adaptive learning rate

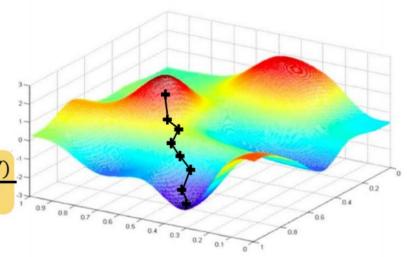




Stochastic gradient descent

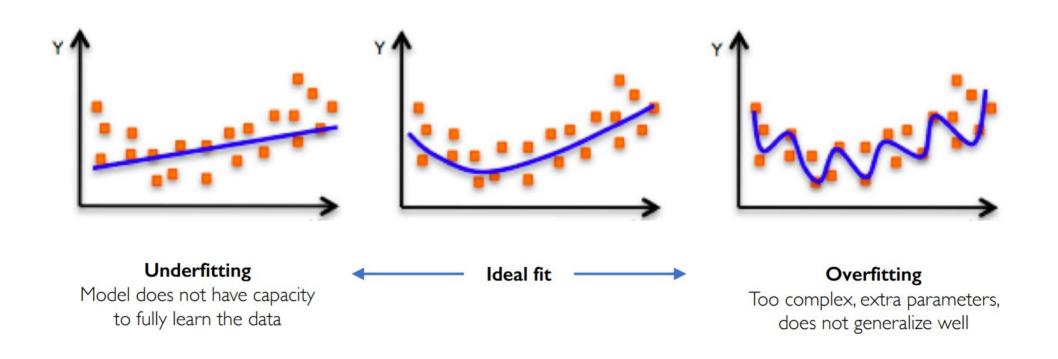
Algorithm

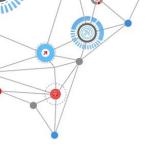
- 1. Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
- 2. Loop until convergence:
- 3. Pick batch of *B* data points
- 4. Compute gradient, $\frac{\partial J(W)}{\partial W} = \frac{1}{B} \sum_{k=1}^{B} \frac{\partial J_k(W)}{\partial W}$
- 5. Update weights, $\mathbf{W} \leftarrow \mathbf{W} \eta \frac{\partial J(\mathbf{W})}{\partial \mathbf{W}}$
- 6. Return weights





Other issues: under/overfitting





Regularization to limit overfitting

L1 (Lasso):

$$L(x,y) \equiv \sum_{i=1}^{n} (y_i - h_{\theta}(x_i))^2 + \lambda \sum_{i=1}^{n} |\theta_i|$$

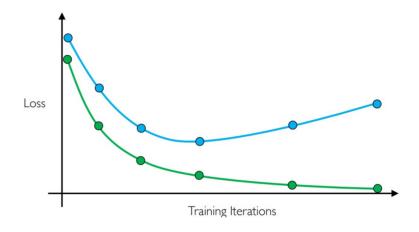
L2 (Ridge):

$$L(x,y) \equiv \sum_{i=1}^{n} (y_i - h_{\theta}(x_i))^2 + \lambda \sum_{i=1}^{n} \theta_i^2$$

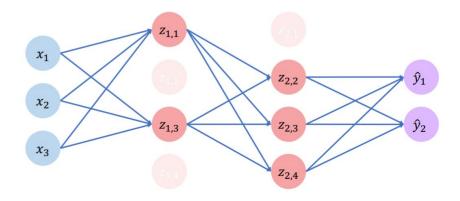


Regularization to limit overfitting

Early stopping



Dropout





Before we continue

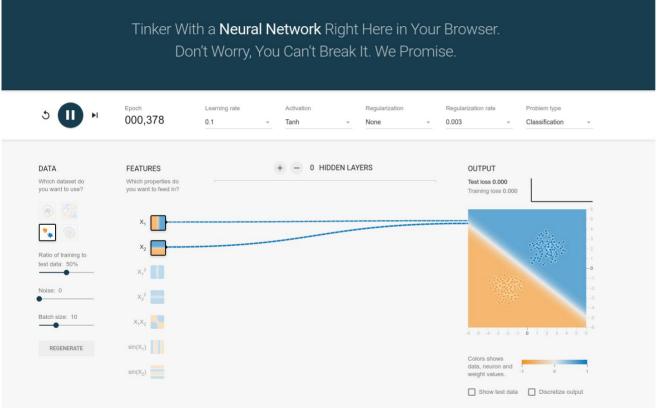
- ► Go to: https://bitbucket.org/clami66/deep_learning_course
- ➤ Clone the repository with git

 (git clone git@bitbucket.org:clami66/deep_learning_course.git

 git clone https://clami66@bitbucket.org/clami66/deep_learning_course.git)
- Or use the "Download Repository" option
 (left menu → Downloads → Download Repository)
- You will get a Jupyter Notebook as well as the latest version of the slides



https://playground.tensorflow.org



Let's review:

- Learning rate
- Minibatches
- Regularization
- Under/Overfitting



Visual cortex studies ('50s)

- Hubel & Wiesel (1959) hypothesized that the visual cortex is made of two types of cells:
 - Simple cells recognize fixed patterns such as edges
 - Complex cells can do the same task, but in a spatially invariant fashion
 - A model was proposed where simplex and complex cells where combined in a cascade



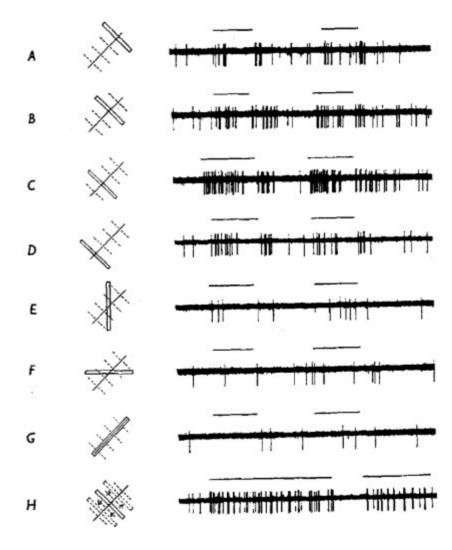


Fig. 4. Responses of a cell with a complex field to stimulation of the left (contralateral) eye with a slit %×2%. Receptive field was in the area centralis and was about 2×3° in size. A–D, % wide slit oriented parallel to receptive field axis. E–G, slit oriented at 45 and 90° to receptive-field axis. H, slit oriented as in A–D, is on throughout the record and is moved rapidly from side to side where indicated by upper beam. Responses from left eye slightly more marked than those from right (Group 3, see Part II). Time 1 sec.



Neocognitron (1979)

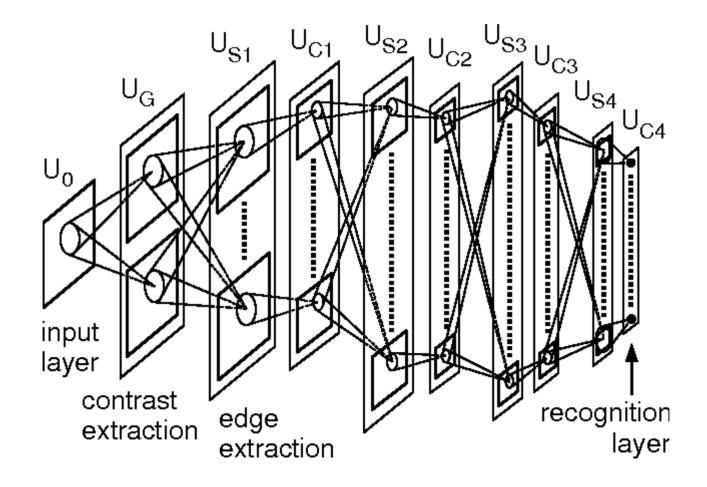




Image processing: kernels

Input image



Convolution Kernel

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Feature map





Image processing: kernels

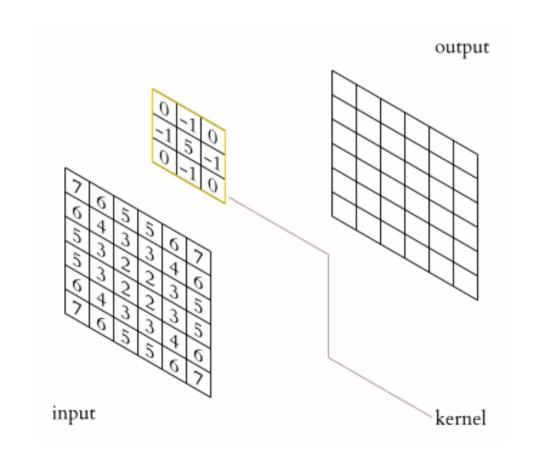
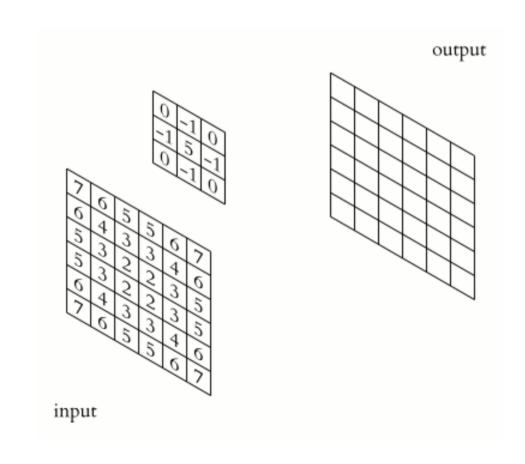


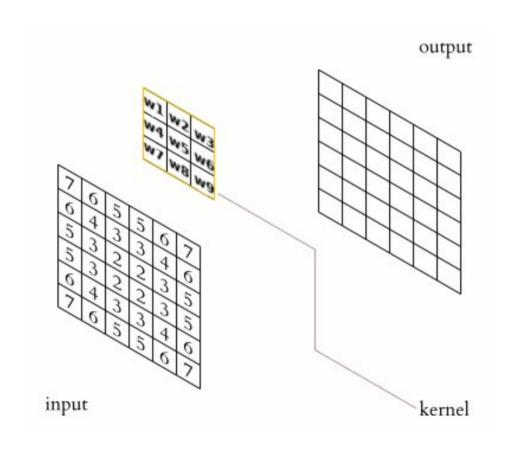


Image processing: kernels



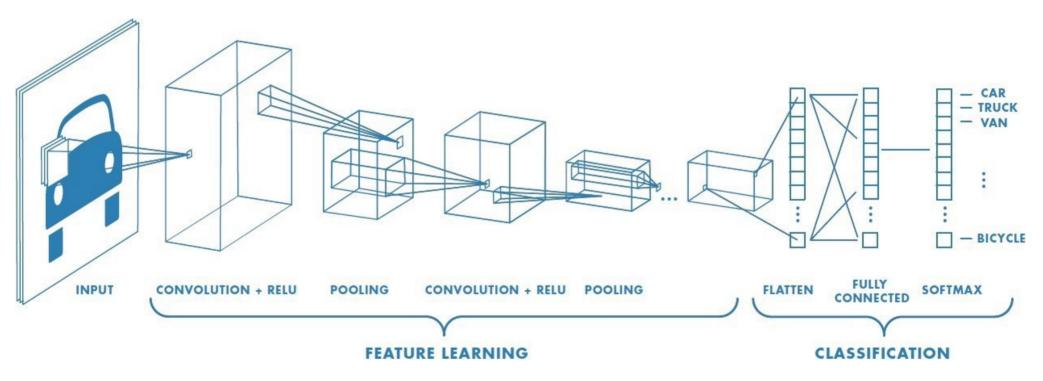


Convnets apply adaptive filters to the inputs



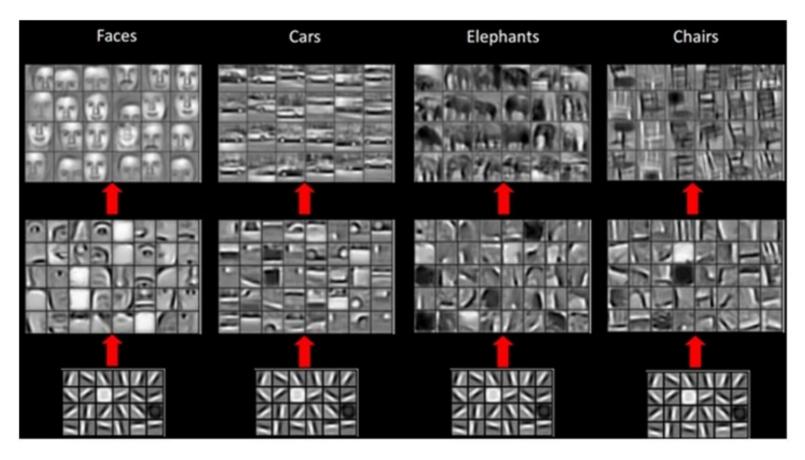


Convolutional Neural Networks (1995)



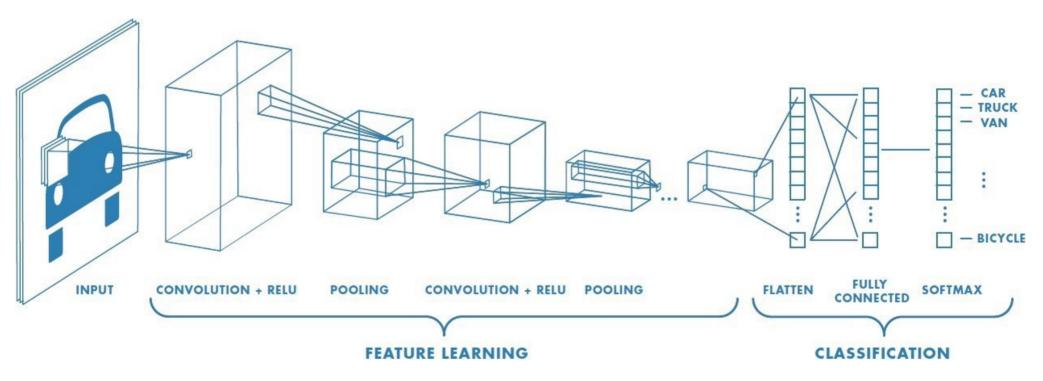


Convnets apply adaptive filters to inputs



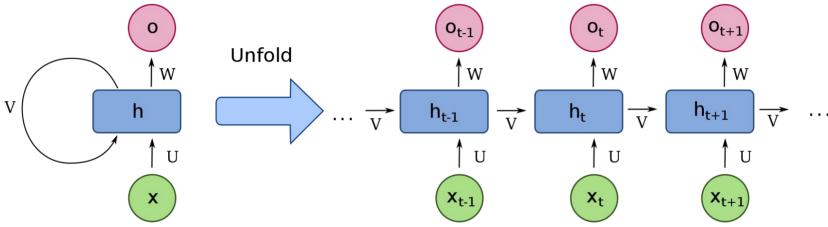


Convolutional Neural Networks (1995)





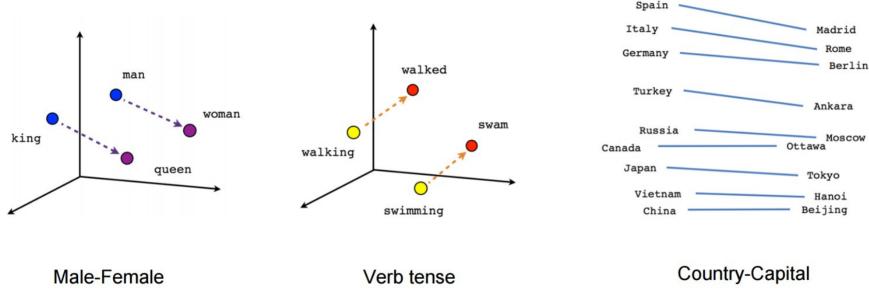
Recurrent Neural Networks



- Connections between nodes form a directed graph along a temporal sequence
- Actually always the same neurons repeated over and over (weight sharing)
- RNNs work very well when your data is like a time series (audio, text, DNA bases...)



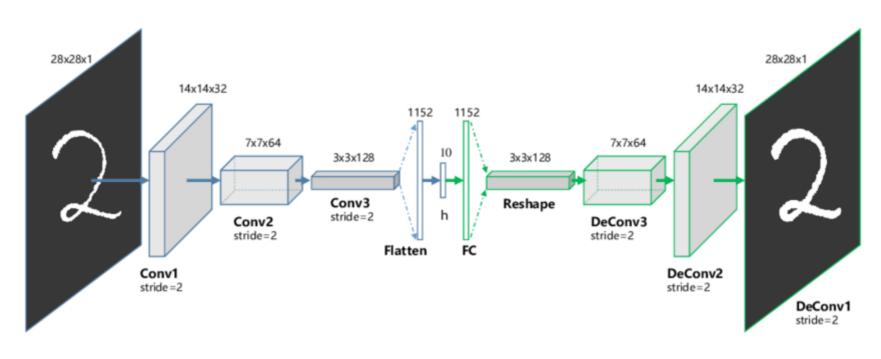
Word embeddings (word2vec)



- Simple two-layer NN that assigns to categorical inputs a floating point vector
- · Inputs that are closely associated will be close in the output space
- It can also find analogies

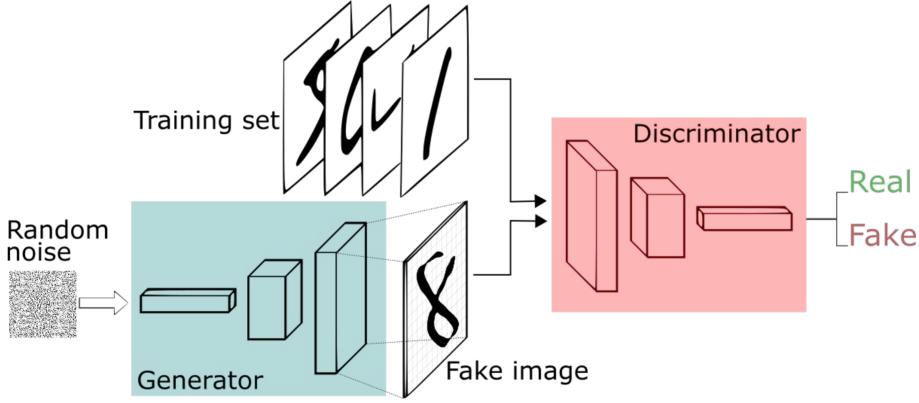


Autoencoders





Generative Adversarial Network (GAN)





Fantastic beasts and where to find them (2016)





Is this the real life? (2019)



NVIDIA, 2019

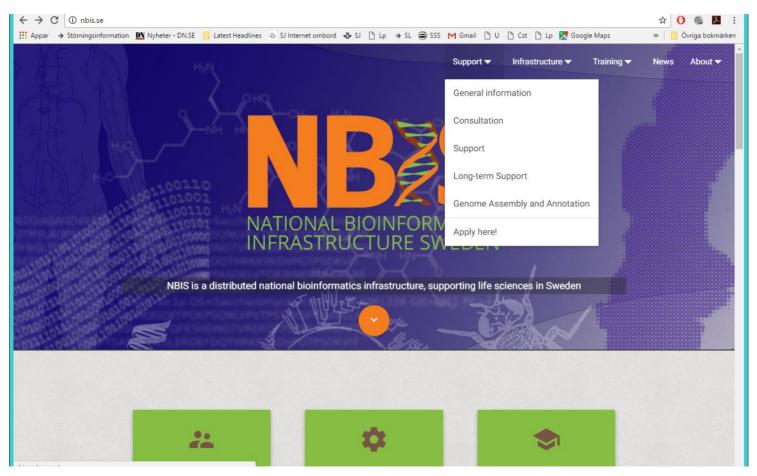
National Bioinformatics Infrastructure Sweden

- Drop-in sessions on Zoom every Tuesday at 14
- Project support (user fee 800 kr/h)
- Long-term support (free; selection by peer review)
- Partner projects (cost: part-time employment)
- PhD advisory program (one call per year)
- Numerous workshops for PhD students and scientists (Neural Networks and Deep learning workshop: Jan 2022)











Expertise

Genomics

Systems biology

Proteomics

Machine Learning

Multi-omics integration

Metabolomics

Phylogenomics

Biostatistics

Computational method development