Sprinngggssss and Strings

Springs & strings both follow hooke's law

Mass pulling down, tension holding up.

$T \propto x$ $\quad$ T = tension $\quad$ x = extension of spring

$T = kx$

upgrade!

Hooke's law

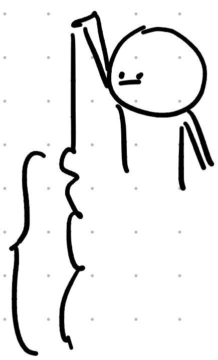
$T = \frac{\lambda x}{l}$ $\quad$ $\left(k = \frac{\lambda}{l}\right)$ $\quad$ $k\ [Nm^{-1}]$

λ = modulus of elasticity [N]

l = original length of spring [m]

high modulus
↓
stiff spring

low modulus
↓
soft spring



Hooke's law holds true for BOTH extension AND compression of springs.

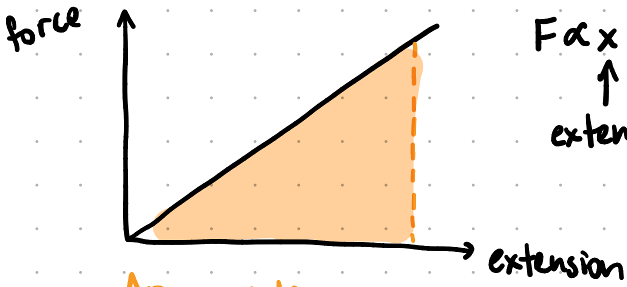
Strings <u>cannot</u> compress

compressed spring

compressed string

Elastic Potential Energy, EPE

when extended/compressed, springs store EPE



Area under curve
= energy stored.

$F \propto x$

↑

extension is also distance

$F \times d$ = Work to extend spring

= EPE stored.

What is the area?

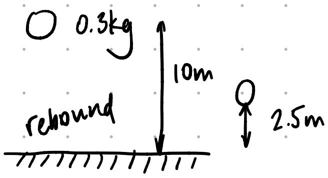
By triangle: $\frac{1}{2}bh = \frac{1}{2}Fx = \frac{1}{2}\left(\frac{\lambda x}{l}\right)x = \frac{\lambda x^2}{2l}$

By Integration: $\int_0^x \frac{\lambda x}{l}\,dx = \left[\frac{\lambda x^2}{2l}\right]_0^x = \frac{\lambda x^2}{2l}$

$EPE = \frac{\lambda x^2}{2l}$

for extensions AND compressions

Review Exercise 1

1)


($\uparrow$ is positive)

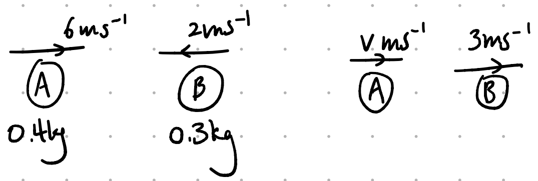
$v^2 = u^2 + 2as = 0^2 + 2(9.8)(10) = 196$

$v = \sqrt{196} = 14ms^{-1}$ downwards

$v^2 = u^2 + 2as = u^2 + 2(-9.8)(2.5) = 0$

$u = \sqrt{5 \times 9.8} = 7ms^{-1}$ upwards

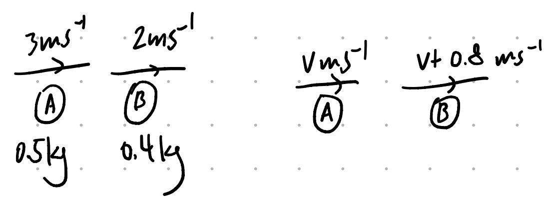
Impulse $= mv - mu = 0.3(7) - 0.3(-14) = 6.3Ns$ ✓

3)


a) PCLM: $0.4 \times 6 - 0.3 \times 2 = 0.4v + 0.3 \times 3$

$v = (2.4 - 0.6 - 0.9)/0.4 = 2.25ms^{-1}$ to the right, no change ✓

b) Impulse on B $= mv - mu = 0.3 \times 3 - 0.3 \times (-2) = 0.3 \times 5 = 1.5$ Ns ✓

5)


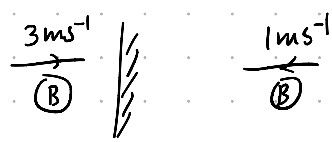
a) PCLM: $0.5 \times 3 + 0.4 \times 2 = 0.5v + 0.4(v + 0.8)$

$0.9v + 0.32 = 1.5 + 0.8$

$v = (1.5 + 0.8 - 0.32)/0.9 = 2.2\ ms^{-1}$ (A) ✓

$v + 0.8 = 3\ ms^{-1}$ (B) ✓

b) Impulse on A $= mv - mu = 0.5 \times 2.2 - 0.5 \times 3 = 1.1 - 1.5 = -0.4$ Ns ✓

c)


Impulse $= mv - mu = 0.4 \times (-1) - 0.4 \times 3 = -0.4 - 1.2 = -1.6$ Ns
(1.6 Ns to the left) ✓

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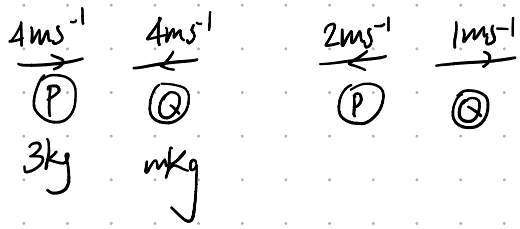

a) PCOM: $3\times4-4m=-3\times2+m$

$5m=12+6$

$m=18/5=3.6\,kg$

b) Impulse on Q $=mv-mu=3.6\times1-3.6\times(-4)=3.6+14.4=18\,Ns$

9 a) $v=\begin{pmatrix}t^2+2\\-6t\end{pmatrix}$ $\quad a=\frac{dv}{dt}=\begin{pmatrix}2t\\-6\end{pmatrix}$ $\quad F=ma=0.75\begin{pmatrix}2t\\-6\end{pmatrix}=\begin{pmatrix}1.5t\\-4.5\end{pmatrix}N$

$t=4$: $F=\begin{pmatrix}6\\-4.5\end{pmatrix}N$ $\quad |F|=\sqrt{6^2+4.5^2}=7.5N$

b) $t=5$, $u=\begin{pmatrix}25+2\\-30\end{pmatrix}=\begin{pmatrix}27\\-30\end{pmatrix}$ $\quad$ Impulse $=\lambda\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}\lambda\\-\lambda\end{pmatrix}$ $\quad \left|\begin{matrix}\lambda\\-\lambda\end{matrix}\right|=9\sqrt{2}=\lambda\sqrt{2}$

$\lambda=9$

$\therefore$ Impulse $=mv-mu=0.75v-0.75\begin{pmatrix}27\\-30\end{pmatrix}=\begin{pmatrix}9\\-9\end{pmatrix}$

$0.75v=\begin{pmatrix}9+20.25\\-9-22.5\end{pmatrix}$ $\quad v=\begin{pmatrix}39\\42\end{pmatrix}ms^{-1}$

11 a) $v=\int\begin{pmatrix}2\\t\end{pmatrix}dt=\begin{pmatrix}2t\\\frac{1}{2}t^2\end{pmatrix}+\begin{pmatrix}2\\-4\end{pmatrix}=\begin{pmatrix}2t+2\\\frac{1}{2}t^2-4\end{pmatrix}ms^{-1}$

b) $t=2$: $u=\begin{pmatrix}4+2\\2-4\end{pmatrix}=\begin{pmatrix}6\\-2\end{pmatrix}ms^{-1}$ $\quad$ Impulse $=mv-mu=0.5v-0.5\begin{pmatrix}6\\-2\end{pmatrix}=\begin{pmatrix}3\\-1.5\end{pmatrix}$

$v=2\left(\begin{pmatrix}3\\1.5\end{pmatrix}+\begin{pmatrix}3\\-1\end{pmatrix}\right)=2\begin{pmatrix}6\\0.5\end{pmatrix}=\begin{pmatrix}12\\1\end{pmatrix}ms^{-1}$?

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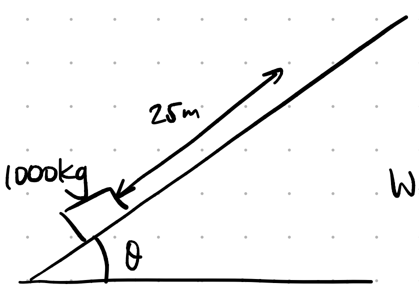


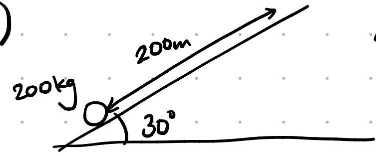
Work done $=\Delta GPE=mg\Delta h=1000(9.8)(25\sin\theta)=19600$

$\sin\theta=\frac{19600}{245000}$

$\theta=\arcsin\left(\frac{19600}{245000}\right)=\arcsin\frac{2}{25}$

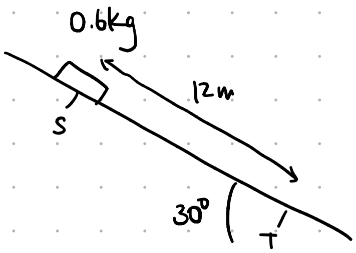
(15) a) $\Delta KE = \frac{1}{2}m(v^2-u^2) = \frac{1}{2}(200)(1.5^2-2^2) = -175\text{ J}$ ✓

b)



$\Delta GPE = mg\Delta h = 200 \times 9.8 \times 200\sin 30° = 196000\text{ J}$ or 196 kJ ✓

(17)



a) $\Delta GPE = mg\Delta h = 0.6 \times 9.8 \times (-12\sin 30°) = -35.28\text{ J}$

$\Delta KE = \frac{1}{2}m(v^2-u^2) = \frac{1}{2}(0.6)(9^2-10^2) = -5.7\text{ J}$

total energy loss $= 35.28 + 5.7 = 40.98\text{ J}$ ✓

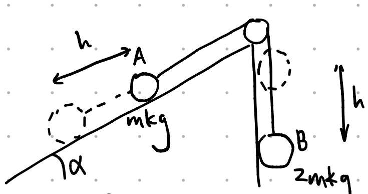
b) energy loss = W.d. against friction $= F \times d = \mu R \times d$

$R = mg\cos\theta = 0.6 \times 9.8 \times \cos 30° = 5.09223\text{ N}$

$40.98 = 5.09223\mu \times 12$

$\mu = \frac{40.98}{12 \times 5.09223} = 0.671$ (3sf)

(19)



$\tan\alpha = \frac{3}{4}$

a) $\Delta GPE_A = mg\Delta h = mgh\sin\alpha = \frac{3}{5}mgh$

$\Delta GPE_B = mg\Delta h = -2mgh$

total $\Delta GPE = \frac{3}{5}mgh - 2mgh = -\frac{7}{5}mgh$ ✓

b) $\mu = \frac{5}{8}$ $\quad F = \mu R = \frac{5}{8}mg\cos\alpha = \frac{1}{2}mg$ $\quad$ W.d. against friction $= \frac{1}{2}mgh$

$KE_A = \frac{1}{2}mv^2$ $\quad KE_B = mv^2$ $\quad$ total KE gain $= \frac{3}{2}mv^2$

KE gain = GPE loss − W.d. against friction

$\frac{3}{2}mv^2 = \frac{7}{5}mgh - \frac{1}{2}mgh$

$v^2 = \frac{9}{10}gh \times \frac{2}{3} = \frac{3}{5}gh$ ✓