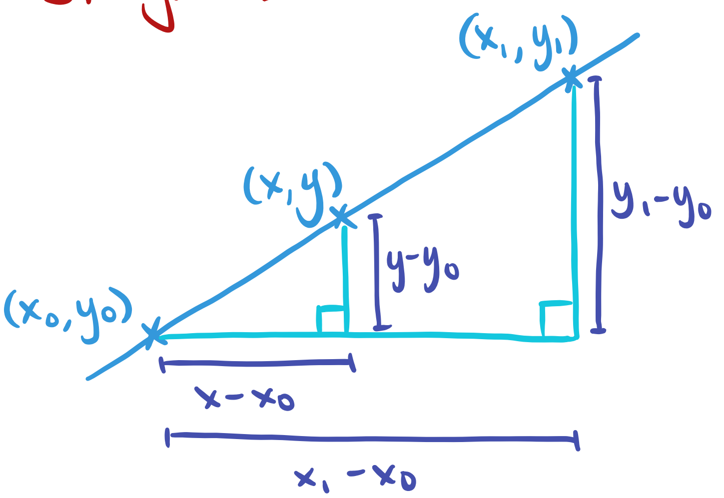
# Straight Lines




$(x_i, y_i)$ are fixed points

$(x, y)$ is a general point.

**Deriving $y=mx+c$**

$$y-y_0 = \frac{y_1-y_0}{x_1-x_0}(x-x_0)$$

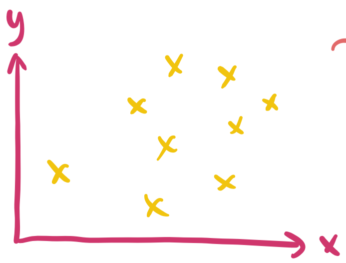
let $m = \dfrac{y_1-y_0}{x_1-x_0}$

$$y-y_0 = m(x-x_0) \longrightarrow y-y_0 = mx - mx_0$$

$$y = mx + y_0 - mx_0$$

let $c = y_0 - mx_0$

$$y = mx + c$$




} Scatter plot.

data set $= (x_i, y_i)$

$i = 1, \dots, n$

AIM: objectively find the line of best fit.

$\hat{y}_i = \alpha + \beta x_i$ where $\hat{y}_i$ is the estimate of $y_i$

A line is a best fit when: $Q = \underbrace{\sum_{i=1}^{n} (y_i - (\alpha + \beta x_i))^2}_{\text{sum of square of difference between } y_i \text{ and } \hat{y}_i}$ is minimized.

↳ achieved by minimizing $Q$ with respect to $\alpha$ and $\beta$

Now the math: ($x_i$, $y_i$ and $\sum_{i=1}^{n}$ have been simplified to $x$, $y$ and $\Sigma$)

Step 1: Differentiate & Simplify

$$\frac{\delta Q}{\delta \alpha} = -2\Sigma(y - (\alpha + \beta x)) = 0 \quad \text{(chain rule)}$$

$$\Sigma y - \Sigma(\alpha + \beta x) = 0$$

$$\Sigma y - n\alpha - \beta \Sigma x = 0 \quad \text{— (1)}$$

$$\frac{\delta Q}{\delta \beta} = -2x\Sigma(y - (\alpha + \beta x)) = 0 \quad \text{(chain rule)}$$

$$\Sigma xy - \Sigma(x\alpha + \beta x^2) = 0$$

$$\Sigma xy - \alpha \Sigma x - \beta \Sigma x^2 = 0 \quad \text{— (2)}$$

Step 2: Eliminate $\alpha$

$① \times \Sigma x$

$$\Sigma x \Sigma y - n\alpha \Sigma x - \beta(\Sigma x)^2 = 0 \quad — ①'$$

$② \times n$

$$n\Sigma xy - n\alpha\Sigma x - n\beta\Sigma x^2 = 0 \quad — ②'$$

Step 3: Solve for $\beta$

$①' - ②'$

$$\Sigma x \Sigma y - n\Sigma xy - \beta(\Sigma x)^2 + n\beta\Sigma x^2 = 0$$

$$\beta\left(n\Sigma x^2 - (\Sigma x)^2\right) = n\Sigma xy - \Sigma x \Sigma y$$

$$\boxed{\beta = \frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma x^2 - (\Sigma x)^2}}$$

Define $S_{xy} = \Sigma(x-\bar{x})(y-\bar{y})$

then $S_{xx} = \Sigma(x-\bar{x})(x-\bar{x}) = \Sigma(x-\bar{x})^2$

Expand $S_{xy} = \Sigma xy - \Sigma\bar{x}y - \Sigma x\bar{y} + \Sigma\bar{x}\bar{y}$

$= \Sigma xy - n\bar{x}\bar{y} - n\bar{x}\bar{y} + n\bar{x}\bar{y}$

$= \Sigma xy - n\bar{x}\bar{y}$

Therefore $S_{xx} = \Sigma x^2 - n\bar{x}^2$

and $S_{yy} = \Sigma y^2 - n\bar{y}^2$

PEARSON'S COEFFICIENT OF CORRELATION

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{xy}}} = \frac{S_{xy}}{\sqrt{(\Sigma x^2 - n\bar{x}^2)(\Sigma y^2 - n\bar{y}^2)}}$$

ALL THIS IS OUT OF SYLLABUS.

# Lines of Best Fit (this time IN syllabus)

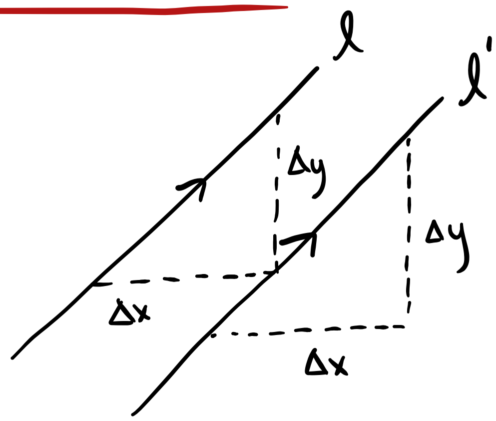

Solve for m & c for $y=mx+c$

m is the change in y per unit change in x
(e.g. -0.06°C when going up 1m)

c is the y value when x is 0
(e.g. 20°C when altitude = 0m)

Interpreting the meaning of m & c

# Parallel lines



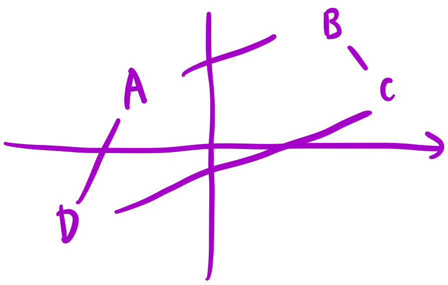

$m_l = \frac{\Delta y}{\Delta x}$

$m_{l'} = \frac{\Delta y}{\Delta x}$

$\therefore m_l = m_{l'}$

## Ex 5E (p.97)

(3) Show ABCD is a trapezium (hint: trapeziums have 1 set of parallel sides)




Either AB//CD or ~~BC//DA~~ Not possible from graph

$m_{AB} = \frac{6}{10} = \frac{3}{5}$ $\quad m_{CD} = \frac{9}{15} = \frac{3}{5}$

$\because m_{AB} = m_{CD}$ $\quad \therefore AB // CD$

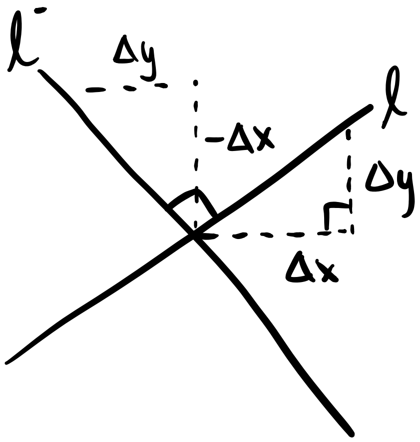
$\therefore$ ABCD is trapezium

(5) $y = -\frac{2}{5}x - 4$

$5y = -2x - 20$

$2x + 5y + 20 = 0$

# Perpendicular lines ($\perp$)




$m_l = \frac{\Delta y}{\Delta x}$

$m_{l'} = \frac{-\Delta x}{\Delta y}$

Therefore,

$m_l \cdot m_{l'} = -1$

## EX 5F (p.99)

③ $l \perp 3x+8y-11=0$ and $(0,-8)$

$8y = -3x+11$

$y = -\frac{3}{8}x + 11$

$l: y = \frac{8}{3}x - 8$

⑪ $l_1: 5x+11y-7=0$

crosses x-axis at A

$l_2 \perp l_1$ and crosses through A

a) $A(\frac{7}{5}, 0)$

b) $l_1: y = -\frac{5}{11}x + \frac{7}{11}$

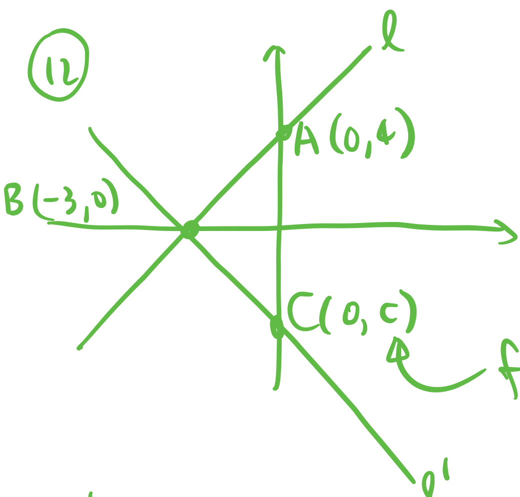
$l_2: 0 = \frac{11}{5}(\frac{7}{5}) + c$

$c = -\frac{77}{25}$

$\therefore l_2: y = \frac{11}{5}x - \frac{77}{25}$

$y - \frac{11}{5}x + \frac{77}{25} = 0$

⑫




find c

$m_l = \frac{4}{3}$

$\therefore m_{l'} = -\frac{3}{4}$

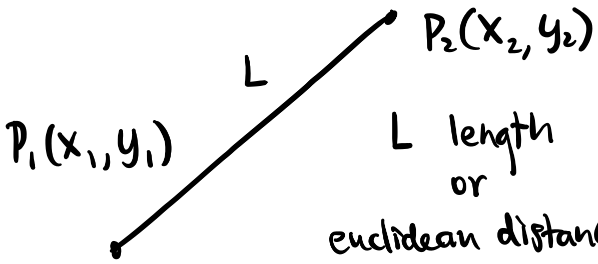
$l': 0 = (-3)(-\frac{3}{4}) + c$

$c = -\frac{9}{4}$

$\therefore l': y = -\frac{3}{4}x - \frac{9}{4}$

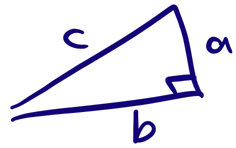
and $C(0, -9/4)$

# § 5.4 : Length & Area




L length
or
euclidean distance

Pythagoras!




$$= \underbrace{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}_{\text{Pythagoras!}}$$

$L_2$ in $\mathbb{R}^2$

There are many ways to define distance

Suppose $\underline{x}, \underline{y} \in \mathbb{R}^n$

if $\underline{x},\underline{y} \in \mathbb{R}^2 \Rightarrow \underline{x} = (x_1, x_2)$, $\underline{y} = (y_1, y_2)$ } 2 dimensions

if $\underline{x},\underline{y} \in \mathbb{R}^n \Rightarrow \underline{x} = (x_1, x_2, \dots, x_n)$, $\underline{y} = (y_1, y_2, \dots, y_n)$ } n dimensions

$$\underbrace{d_m(\underline{x},\underline{y})}_{\text{distance between } \underline{x} \text{ and } \underline{y}} = \sqrt[m]{(x_1-y_1)^m + \dots + (x_n-y_n)^m}$$

where $m \in \mathbb{Z}$, $m > 1$

The above distance would be defined as $L_m$ by a mathematician.

$$\text{In } \mathbb{R}^2, \quad d(P_1,P_2) = \underbrace{|x_1-x_2| + |y_1-y_2|}_{\text{AKA. Manhattan Distance}}$$

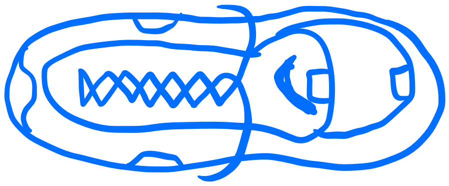
# AREAS

triangle: $\frac{1}{2}bh$ trapezium: $\frac{h(a+b)}{2}$ etc....

**shoelace method:** Any shape with points $P_1(x_1,y_1), P_2(x_2,y_2), \dots P_n(x_n,y_n)$

product →

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & \dots & x_n & x_1 \\ y_1 & y_2 & y_3 & \dots & y_n & y_1 \end{array}$$



$$\text{Area} = \frac{1}{2} \cdot \left| (x_1y_2 + x_2y_3 + \dots + x_ny_1) - (x_2y_1 + x_3y_2 + \dots + x_1y_n) \right|$$

$$= \frac{1}{2} \left| (\text{sum of red products}) - (\text{sum of green products}) \right|$$

⚠ Don't forget to multiply by $\frac{1}{2}$!

⚠ Remember to put the points IN ORDER!

EX 5G (p.102)

(5) Distance b/w $(2,y)$ and $(5,7)$ is $3\sqrt{10}$

$\sqrt{(y-7)^2+(2-5)^2} = 3\sqrt{10}$

$y^2-14y+49+9 = 90$

$y^2-14y-32=0$ $\quad \begin{pmatrix} y & -16 \\ y & +2 \end{pmatrix}$ $\quad y=-2, 16$

(8) $A(2,7)$, $B(5,-6)$, $C(8,-6)$

a) show that $\Delta ABC$ is scalene (ie. all sides different lengths)

$AB = \sqrt{9+169} = \sqrt{178}$

$BC = \sqrt{9+0} = 3$

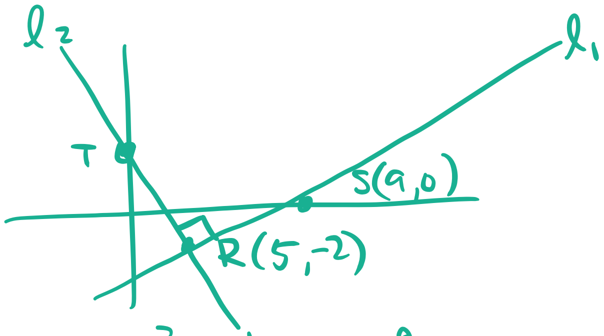
$AC = \sqrt{36+169} = \sqrt{205}$

b) find area of $\Delta ABC$ using shoelace method: $\frac{1}{2}\left|\begin{array}{cccc} 2 & 5 & 8 & 2 \\ 7 & -6 & -6 & 7 \end{array}\right|$

$= \frac{1}{2}\left| \left(2(-6)+5(-6)+8(7)\right) - \left(7(5)+8(-6)+2(-6)\right) \right|$

$= \frac{1}{2}(14+25) = 39/2$ units squared

11




a) $m_{l_1} = \frac{2}{4} = \frac{1}{2}$ $\quad 0 = \frac{9}{2} + c$

$c = -\frac{9}{2}$

$\therefore l_1 : y = \frac{1}{2}x - \frac{9}{2}$

b) $m_{l_2} = -2$ $\quad -2 = -10 + c$

$c = 8$

$\therefore l_2 : y = -2x + 8$

c) $T(0, 8)$

d) $|RS| = \sqrt{16 + 4} = 2\sqrt{5}$

$|TR| = \sqrt{25 + 100} = 5\sqrt{5}$

e) AREA $\Delta RST = \frac{1}{2} \times |RS| \times |TR|$

$= \frac{50}{2}$

$= 25$ units squared

## Extension question

shortest distance from origin to $3x + 4y = 25$

$\Downarrow$

$\perp$ line through origin intersection with line

$y = -\frac{3}{4}y + \frac{25}{4}$ $\quad m = -\frac{3}{4}$ $\quad y = \frac{4}{3}x$ is new line

$m_\perp = \frac{4}{3}$ $\quad 3x + \frac{16}{3} = 25$

$3x = \frac{59}{3}$

$x = \frac{59}{9}$

$y = \frac{236}{27}$

$(59/9, 236/27)$ is intersection

distance $= \sqrt{(59/9)^2 + (236/27)^2}$

$\approx 10.9$

EX 5H Q5-9 (p.106)

(5) a) $a = \frac{9550-7100}{13-6} = 350$  $\therefore C = 350d + 5000$

$b = 7100 - 350(6) = 5000$

b) a = how much more it costs per day

b = how much it would cost if it took 0 days

c) $13400 = 350d + 5000$

$d = \frac{13400-5000}{350} = 24$

(6) a) $a = \frac{68-48.2}{20-9} = 1.8$  $\therefore F = 1.8C + 32$

$b = 68 - 1.8(20) = 32$

b) a = temp. increase in °C when temp. goes up by 1°F

b = when it is 0°C, it is 32°F

c) $101.3 = 1.8C + 32$

$C = \frac{101.3-32}{1.8} = 38.5$

d) $F = 1.8F + 32$

$0.8F = -32$

$F = -40$

$\therefore -40°F = -40°C$

(7) a) $n = 750t + 17500$  b) the growth of n is linear

(8) a) because the ratio between foot length and height is very similar among people

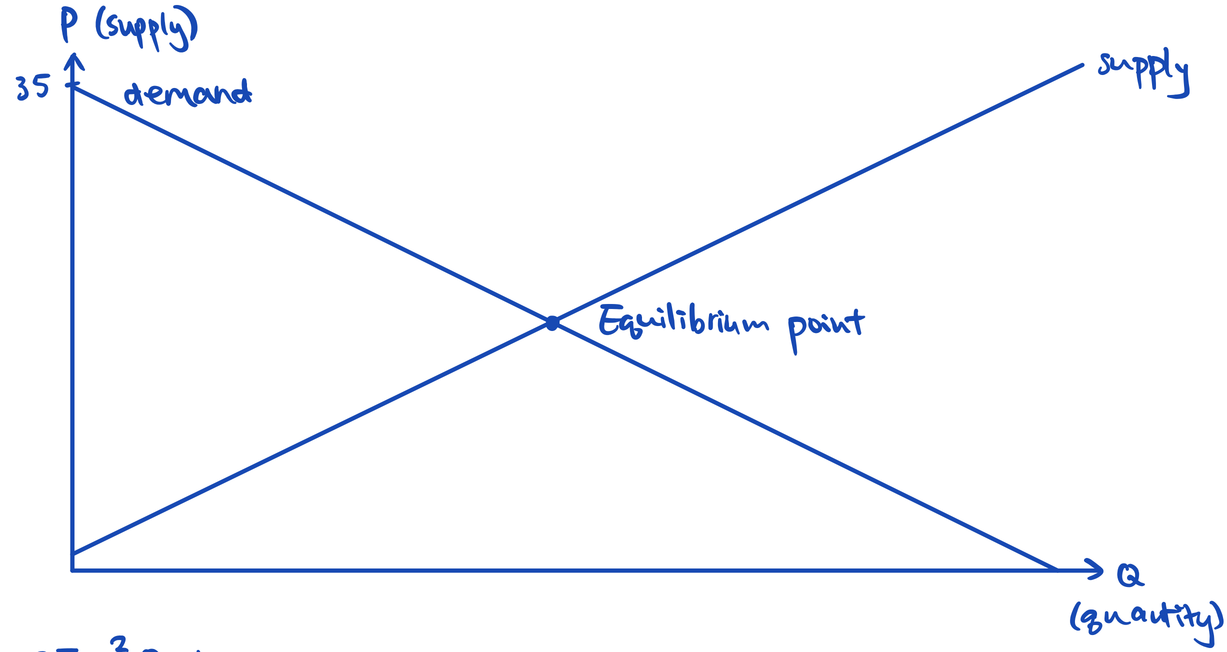
b) $h = af + b$

$a = \frac{177-165}{27-24} = \frac{12}{3} = 4$  $\therefore h = 4f + 69$

$b = 177 - 4(27) = 69$

c) $h = 4(26.5) + 69 = 175\text{cm}$

9 a)




b) $-\frac{3}{4}Q+35=\frac{2}{3}Q+1$

$\frac{17}{4}Q=34$

$Q=8$

$P=\frac{21}{2}=10.5$