QUADRATICS

from the latin word "quadratus", meaning "square"

Consider : $f(x) = ax^2 + bx + c$

how do the coefficients a, b and c influence the shape of the quadratic curve?

$\boxed{a}$ the dominant term in $f(x)$

$\Downarrow$

when x is large, ax^2 matters most

a determines U/∩ shape & how "closed" the curve is

$\boxed{b}$

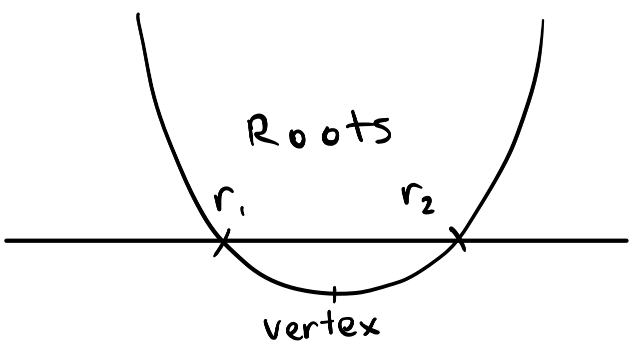
$$ax^2+bx+c = a\left(x^2+\frac{b}{a}x\right)+c$$
$$= a\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a^2}$$

minimum when $x = -\frac{b}{2a}$

$\therefore$ a & b determines vertex position

$\boxed{c}$ c = y-intercept

c determines the y intercept and translates the graph up & down



$$f(x) = a(x-r_1)(x-r_2)$$

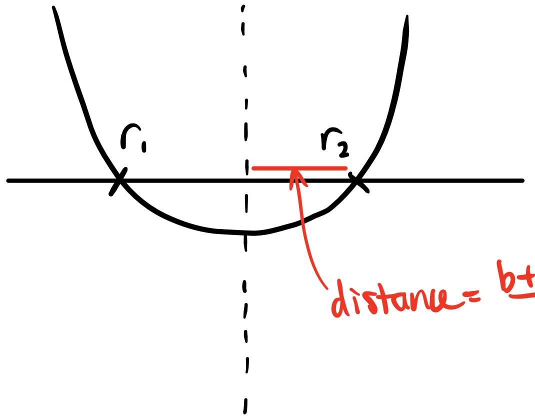
$$\text{vertex} = \left(\frac{r_1+r_2}{2}, f\left(\frac{r_1+r_2}{2}\right)\right)$$

Finding the quadratic formula

$$ax^2+bx+c=0$$
$$a\left(x^2+\frac{b}{a}x\right)+c=0$$
$$a\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a}=0$$
$$a\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a}-\frac{4a^2c}{4a^2}$$
$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{4ac}{4a^2}$$

$$\rightarrow \quad x+\frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$



distance $= \frac{b+\sqrt{b^2-4ac}}{2a} - \frac{b}{2a} = \frac{\sqrt{b^2-4ac}}{2a}$

when sketching a quadratic graph, you HAVE to label the vertex & intercepts

6. $f(x) = x^2 - 2x + 2$

a) $f(x) = (x-1)^2 + 1$

b) vertex $= (1, 1)$

$\because a > 0$, $\therefore$ upwards curve

$\therefore$ curve never crosses x-axis

OR

$\Delta = b^2 - 4ac = 4 - 8 = -4 < 0$

7. a) $(x^3+8)(x^3+1) = 0$

$x = -2, -1$

b) $(x^2-8)(x^2-4) = 0$

$x = \pm 2, \pm 2\sqrt{2}$

c) $(27x^3-1)(x^3+1) = 0$

$x = \frac{1}{3}, -1$

THE DISCRIMINANT (Δ)!

from $ax^2+bx+c=0$,

$$x = \underbrace{-\frac{b}{2a}}_{\text{line of symmetry}} \pm \underbrace{\frac{\sqrt{b^2+4ac}}{2a}}_{\text{distance from line of symmetry to the roots}}$$

b^2+4ac determines # of roots.

if $\Delta > 0$, 2 roots

if $\Delta = 0$, 1 root

if $\Delta < 0$, no roots

P.32 EX2G

2 $\Delta = b^2 - 4ac = 36 - 4k > 0$

$k < 9$

5 $\Delta = b^2 - 4ac = 16 - 12k < 0$

$k > \frac{4}{3}$

7 a) $\Delta = k^2 + 8k + 16 - 8k$

$= k^2 + 16$

b) Since $k^2 \geq 0$,

$k^2 + 16$ ie $\Delta \geq 16 > 0$

CHALLENGE (p.32 Ex2G)

a) $\Delta = b^2 - 4ac > 0$

$b^2 > 4ac$

CASE 1: same sign ($a, c > 0$ or $a, c < 0$)

Choose b such that $b^2 > 4ac$

CASE 2: different sign ($a > 0, c < 0$ or $a < 0, c > 0$)

$b^2 > 4ac$ will always be true because $b^2 \geq 0 > 4ac$

b) Yes. $\Delta = b^2 + 4ac = 0$

$b^2 = 4ac$

$\underbrace{b = \pm 2\sqrt{ac}}_{\text{only if } ac \text{ is +ve}}$

p.34 EX2H

1 a) height of the bridge from the water level

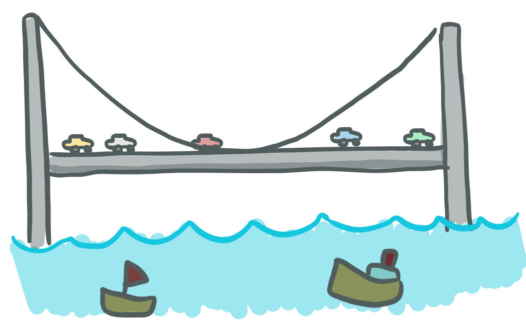
b) $0.00012x^2 + 200 = 346$

$0.00012x^2 = 146$

$x^2 \approx 1216667$

$x \approx 1103$ m

c) length $= 1103 \times 2 = 2206$ m



4 a) when $t = 10000$, $p = 30$

$10000 = M - 30000$

$M = 40000$

b) $r = -1000p^2 + 40000p$

$= -1000(p-20)^2 + 400000$

c) maximum when $p - 20 = 0$

$\therefore p = 20$

£20

CHALLENGE (p.35 EX2H)

a) $d(20) = 400a + 20b + c = 6$ —— ①

$d(30) = 900a + 30b + c = 14$ —— ②

$d(40) = 1600a + 40b + c = 24$ —— ③

② − ①: $500a + 10b = 8$ —— ④

③ − ②: $700a + 10b = 10$ —— ⑤

⑤ − ④: $200a = 2$

$$\begin{cases} a = \frac{1}{100} \text{ or } 0.01 \\ b = \frac{3}{10} \text{ or } 0.3 \\ c = -4 \end{cases}$$

b) $0.01s^2 + 0.3s - 4 = 20$

$s^2 + 30s - 2400 = 0$

$(s+15)^2 - 2400 - 225 = 0$

$s + 15 = 5\sqrt{105}$

$s = -15 + 5\sqrt{105}$

≈ 36 mph