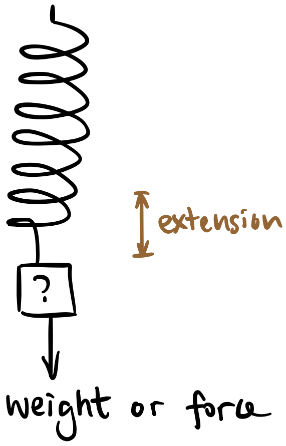
Hooke's Law

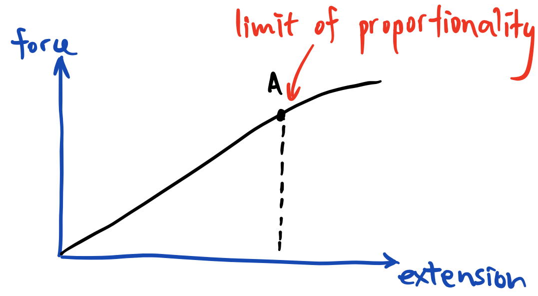
$F \propto x$

Force $\propto$ extension

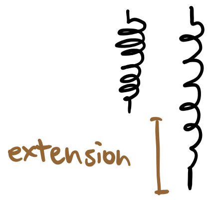
$F = kx$

stiffness constant of spring

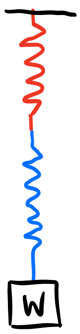




⚠ The extension does not consider the length of the spring!

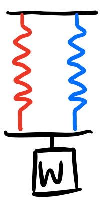


Spring Combinations



"Series"

Combo A



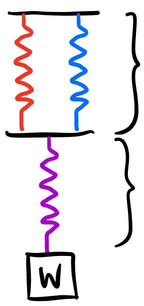
"Parallel"

Combo B

If each spring had stiffness k,

Combo A : $\frac{1}{K_T} = \frac{1}{k} + \frac{1}{k} \Rightarrow K_T = \frac{k}{2}$

Combo B : $K_T = k + k = 2k$



Parallel 2K

Series K

The formulas are opposite of electrical resistance

$\frac{1}{K_T} = \frac{1}{2K} + \frac{1}{K}$

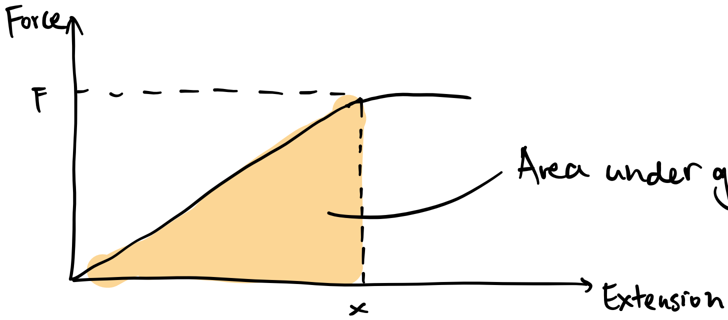
$K_T = \frac{2K}{3} = \frac{2}{3}K$

Plastic behaviour

When out of the elastic limit, force will deform the material permanently.

Stretching a spring

Work = force × distance



Area under graph = Work done = $\frac{1}{2}Fx$

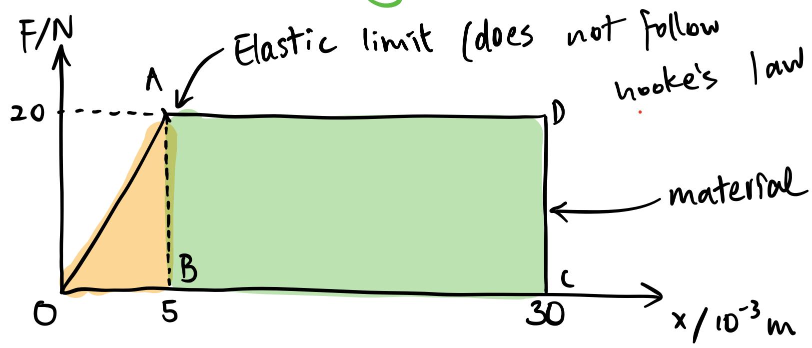
Not the full force, but an average half the force

Elastic Potential Energy (EPE) (AKA STRAIN ENERGY)

EPE = work done to stretch it = $\frac{1}{2}Fx = \frac{1}{2}kx^2$

$F = kx$

Question: Stretching Metal



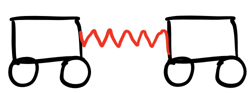
Elastic limit (does not follow hooke's law anymore)

material breaks.

Work done to reach elastic limit = $(5\times10^{-3}\times20)/2 = 0.05J$

Work done to break metal = $0.05 + 20\times25\times10^{-3} = 0.55J$

Question: Compressed Spring Explosion

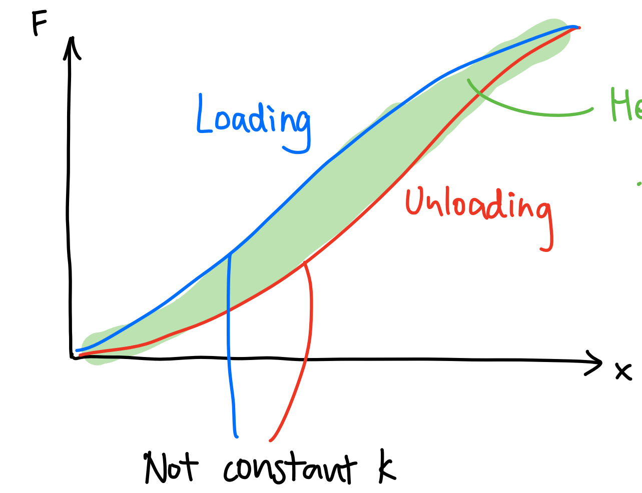


Spring properties: $k = 80Nm^{-1}$ (Compressed by 0.06m)

0.40kg 0.40kg

1. Energy stored = $\frac{1}{2}kx^2 = \frac{1}{2}\times80\times0.06^2 = 0.144J$
2. Trolleys fly apart at equal speeds because they have same mass
3. Speed? $E_K = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{0.144}{0.4}} = \frac{3}{5} = 0.6\ ms^{-1}$

Rubber Bands

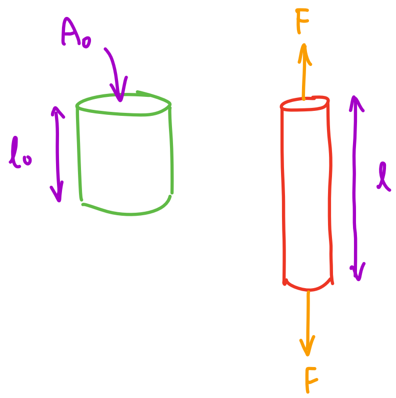


Heat loss (rubber band heats up)
∴ Work done ↓

"Elastic hysteresis"

Young's Modulus.

$$\text{Stress} = \frac{\text{Force}}{\text{Cross Sectional Area}} \qquad \sigma = \frac{F}{A} \qquad [Pa] = \frac{[N]}{[m^2]}$$



No units

$$\text{Strain} = \frac{\text{Extension}}{\text{Original length}} \qquad \varepsilon = \frac{\Delta L}{L} \qquad [\ ] = \frac{[m]}{[m]}$$

$$\text{Young modulus} = \frac{\text{Stress}}{\text{Strain}} \qquad E = \frac{\sigma}{\varepsilon} \qquad [Pa] = \frac{[Pa]}{[\ ]}$$

$$E = \frac{F/A}{\Delta L/L} = \frac{FL}{\Delta E A}$$

Rearranging $E = \frac{FL}{\Delta L A}$:

$\Delta L = \frac{FL}{EA} \longrightarrow \Delta L \propto F \quad \Delta L \propto L \quad \underbrace{\Delta L \propto \frac{1}{E}} \quad \Delta L \propto \frac{1}{A}$

When E↑, material is more stiff

Steel wire

200 cm long

Cross sectional area $0.5\,mm^2$ $\Rightarrow E = \frac{FL}{\Delta L A} = \frac{50 \times 2}{0.001 \times 0.5 \times 10^{-6}} = 2 \times 10^{11}$

Stretched 50N

New length = 200.1 cm

Copper wire

$E = 130\,GPa = 130{,}000{,}000{,}000 = 1.3 \times 10^{11}\,Pa$

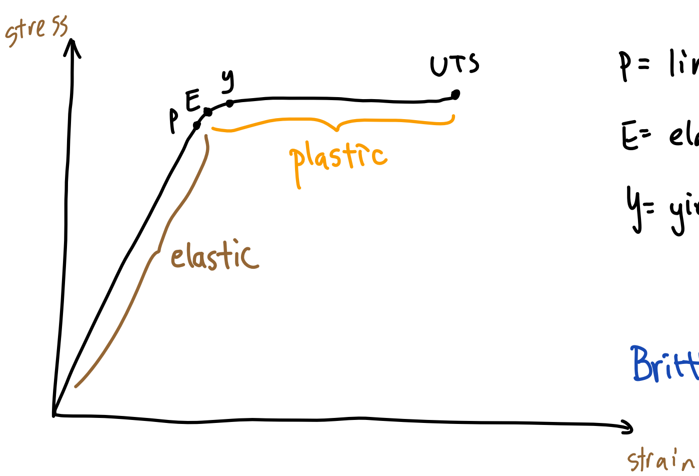
Length = 1m

Diameter = 1mm   Cross sectional area $= \pi r^2 = \pi \left(\frac{0.001}{2}\right)^2 = 2.5\pi \times 10^{-7}\,m^2$

Force = 10N

$\Delta L = \frac{FL}{EA} = \frac{10 \times 1}{1.3 \times 10^{11} \times 2.5\pi \times 10^{-7}} = 9.79 \times 10^{-5}\,m$

Interpreting Stress-Strain Graphs



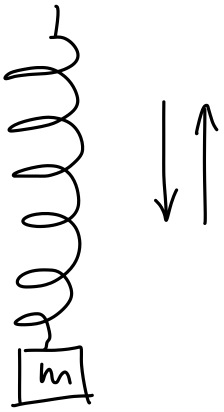
P = limit of proportionality

E = elastic limit

Y = yield point

Brittle : little plastic behaviour

Stress-strain graphs are similar to force-extension graphs



When the mass bounces up and down,

$\boxed{m}$ At top: No velocity, no extension

$\therefore$ Only GPE

$\boxed{m}$ At middle: High velocity, little extension

$\therefore$ GPE, E_k and EPE

$\boxed{m}$ At bottom: No velocity, high extension

$\therefore$ Only EPE

Strain energy $= \frac{1}{2}F\Delta L$

Volume of wire $= AL$

$$\frac{\text{Strain energy}}{\text{volume}} = \frac{\frac{1}{2}F\Delta L}{AL} = \frac{1}{2}\left(\frac{F}{A}\right)\left(\frac{\Delta L}{L}\right) = \frac{1}{2}\sigma\varepsilon = \text{Energy Density}$$

Energy per unit volume

[J m^{-3}]

Area under stress strain graph