Mechanics

Base units:

candella	[cd]	
Kelvin	[K]	Temperature
metre	[m]	Distance
Kilogram	[kg]	Mass
Ampere	[A]	Current
Second	[s]	Time
Mole	[mol]	Amount

Prefixes

P	peta	10^{15}
T	tera	10^{12}
G	giga	10^{9}
M	mega	10^{6}
k	kilo	10^{3}
h	hecto	10^{2}
da	deca	10^{1}
d	deci	10^{-1}
c	centi	10^{-2}
m	milli	10^{-3}
μ	micro	10^{-6}
n	nano	10^{-9}
p	pico	10^{-12}
f	femto	10^{-15}

Base units for: Force $[N] = [kgms^{-2}]$

Energy $[J] = [kgm^2s^{-2}]$

Volt $[V] = [kgm^2s^{-3}A^{-1}]$

Pressure $[Pa] = [kgm^{-1}s^{-2}]$

Power $[W] = [kgm^2s^{-3}]$

Charge $[C] = As$

Converting

1. $60\ kmh^{-1} = \frac{60 \times 1000}{60 \times 60}\ ms^{-1}$

   $= 16.67\ ms^{-1}$

2. $12m^2 = 12 \times 100^2\ cm^2$

   $= 120000 = 1.2 \times 10^5\ cm^2$

3. $1\ kWh = 1000\ Wh = 1000\ \frac{J}{s} \times 3600\ s$

   $= 3.6 \times 10^6\ J$

4. $1\ cm^3 = \frac{1}{100^3}\ m^3$

   $= 1 \times 10^{-6}\ m^3$

Types of measurement

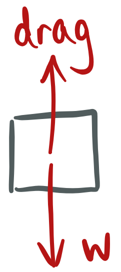
SCALARS: have magnitude ONLY, no direction

VECTORS: have magnitude AND direction

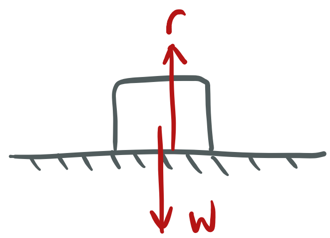
SCALARS	VECTORS
speed	velocity
distance	displacement
mass	force
energy	acceleration
(time)	momentum
density	
temperature	

Forces

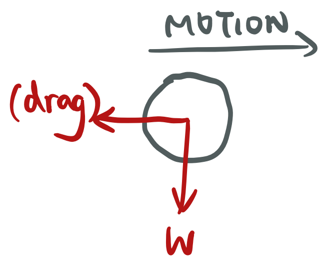
falling object



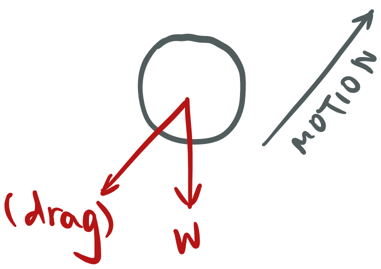
object on ground



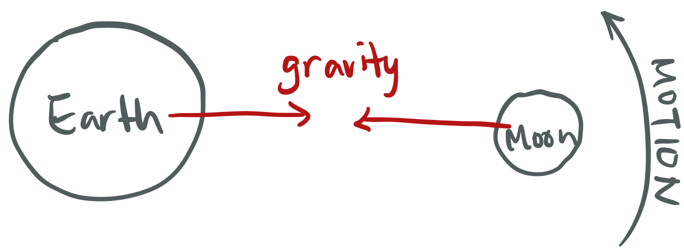
ball travelling horizontally



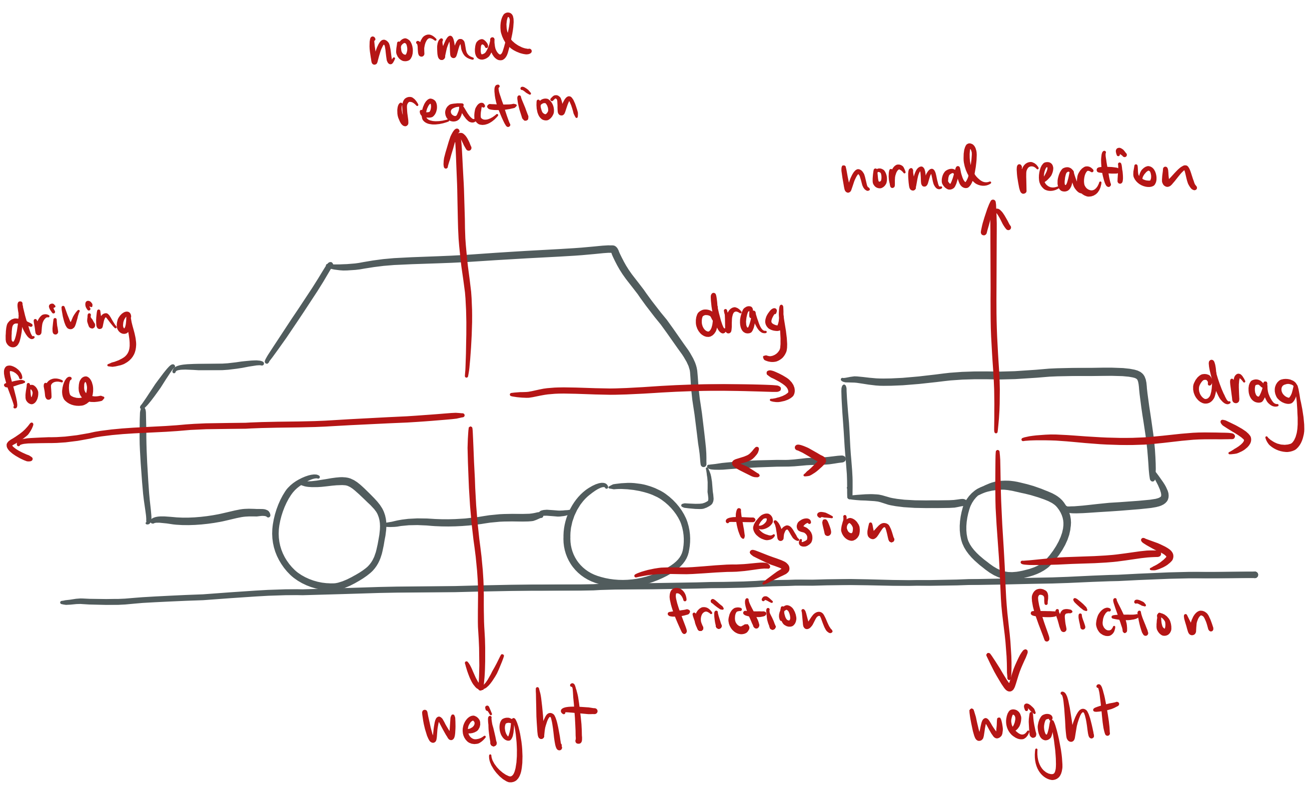
ball flying up at an angele



earth-moon system



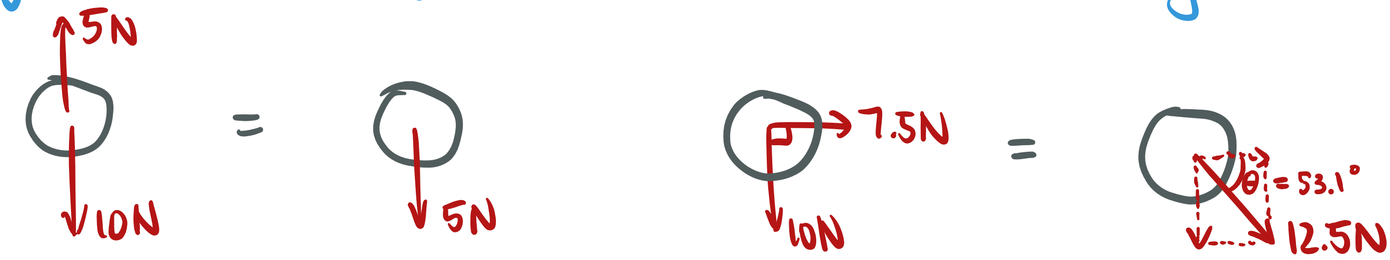
vehicle towing a trailer

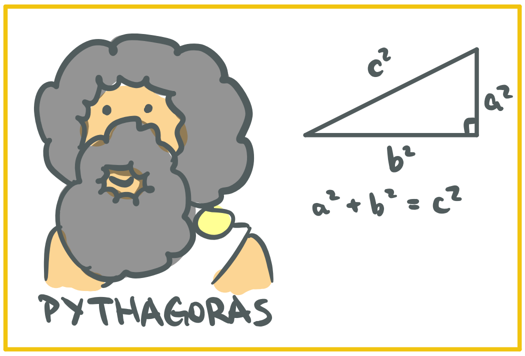


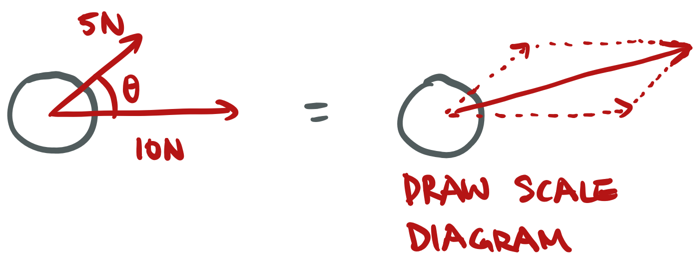
COMBINING FORCES

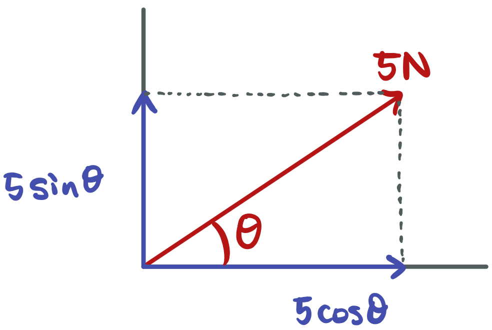
resolving forces to create a resultant force

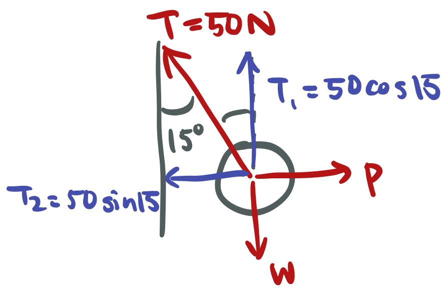
a single force which is equal to the sum of all the forces acting on an object



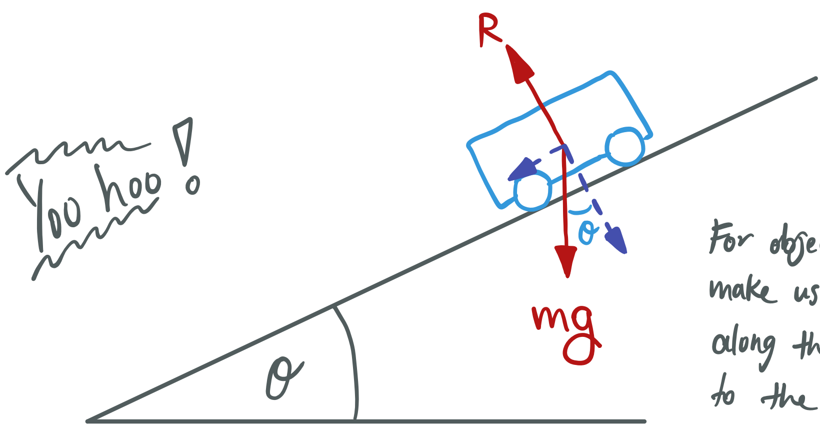








Forces on an inclined plane (ie ramp or slope)



Force down a plane is: $mg\sin\theta$!

For objects on inclined planes, make use of the force components along the slope and perpendicular to the slope.

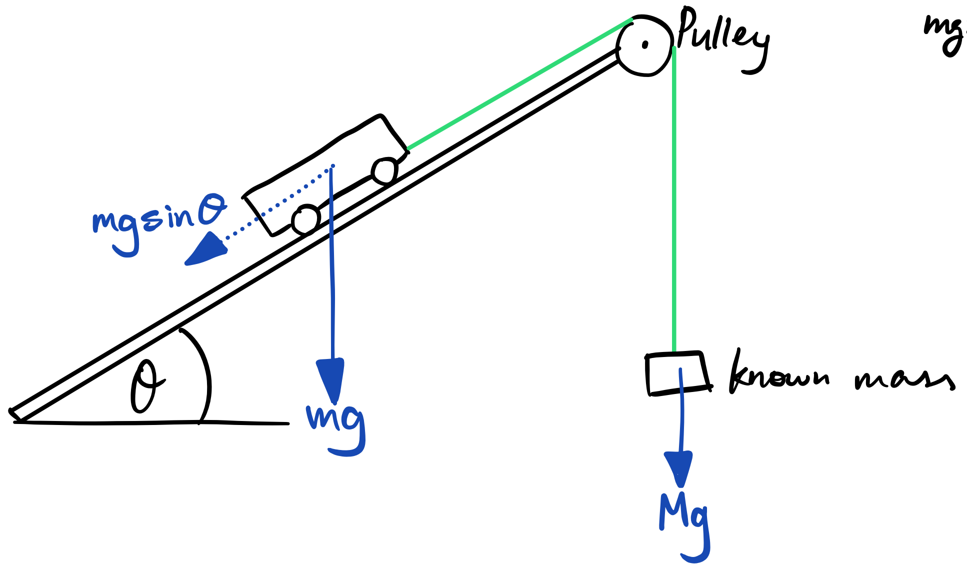
Component of weight along slope = $mg\sin\theta$ the more useful one!

Component of weight perpendicular to the slope = $mg\cos\theta$ = R

Measurements on an Inclined Plane

when in equilibrium,

$mg\sin\theta = Mg$



For different values of M, adjust the ramp and measure θ when the trolley is balanced.

RESULTS

measure using sinθ: measure length of plane & height

INDEPENDENT VARIABLE	DEPENDENT VARIABLE					
KNOWN MASS / g	LENGTH / m	HEIGHT / m 1	2	3	MEAN	sin θ
0.100	0.969 0.968 0.969 ⇓ 0.969	0.201	0.198	0.196		0.204
0.150		0.295	0.290	0.288		0.300
0.200		0.382	0.378	0.376		0.391
0.250		0.494	0.491	0.489		0.507
0.300						

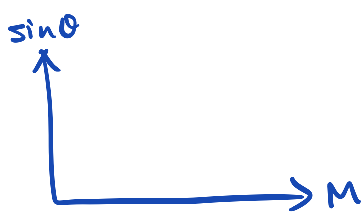
$y = mx + c$

$mg\sin\theta = Mg$

$m\sin\theta = M \longrightarrow \sin\theta = \frac{M}{m}$

$\sin\theta = \frac{1}{m}M$

$y = m\ x$



gradient $= \frac{1}{m}$

$M = m\sin\theta$

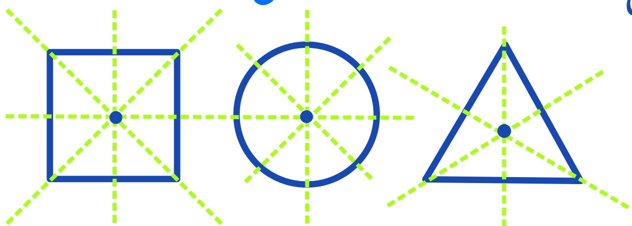
$y = m\ x$

gradient = m

CENTER OF MASS AND MOMENTS

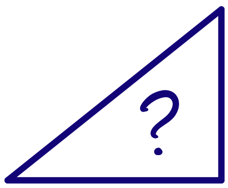
[Center of Mass (CoM): A single point through which we assume all the weight acts]
(AKA Center of Gravity)

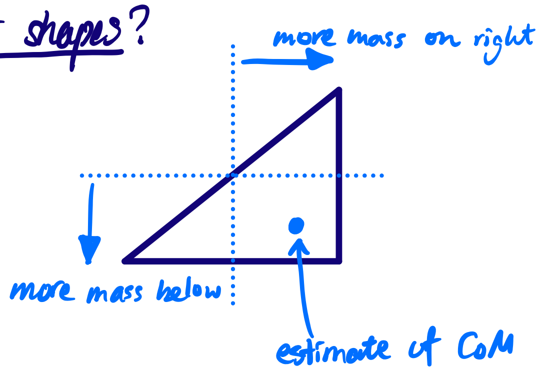
e.g.

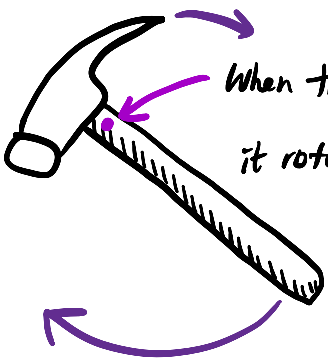


Symmetrical objects have easy CoMs.

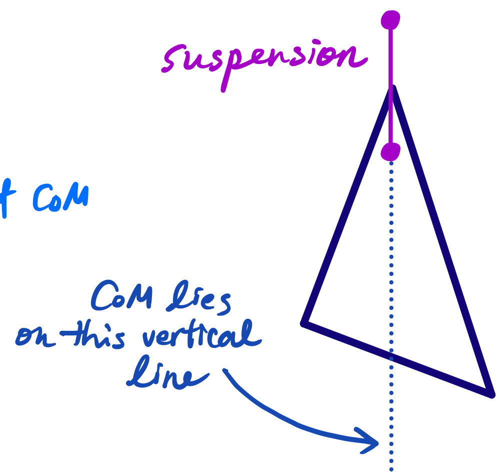
What about irregular shapes?



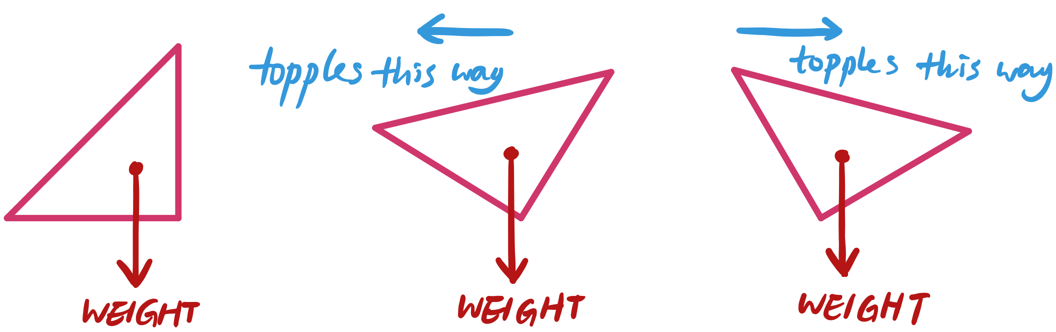




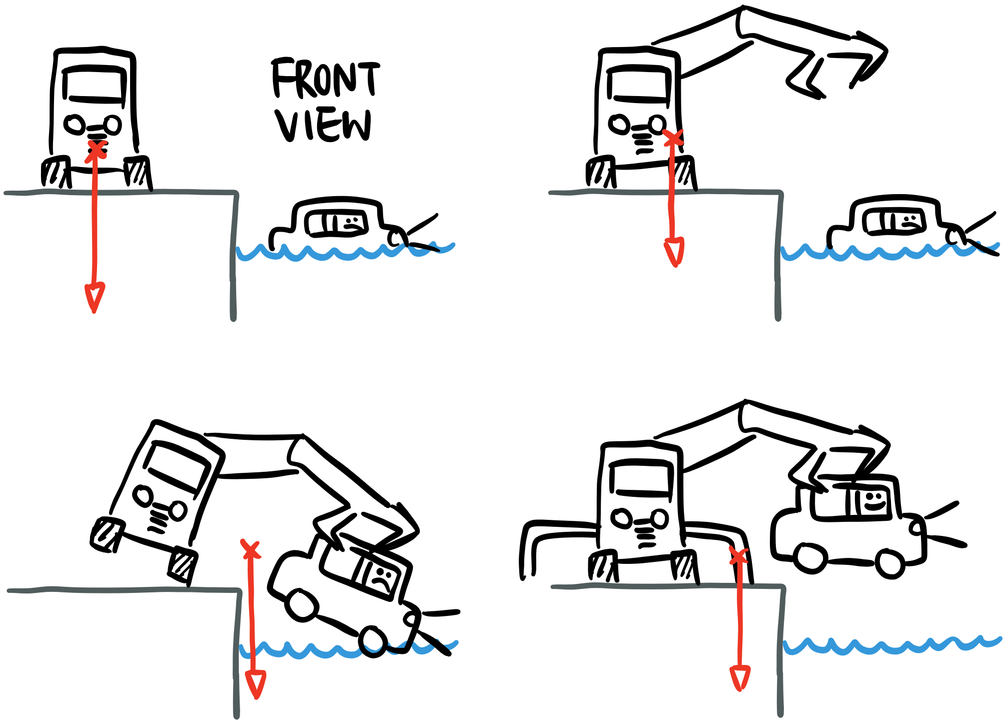
When throwing a hammer, it rotates around its CoM



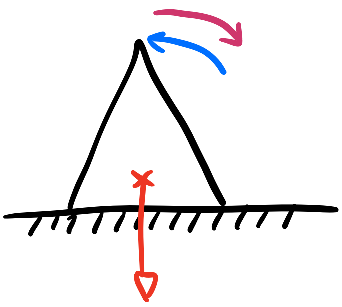
STABILITY (and how CoM relates)



CRANE & CAR

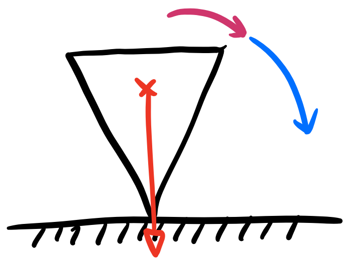


TYPES OF EQUILIBRIUM



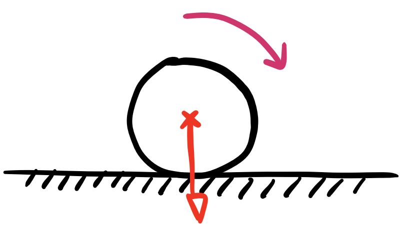
STABLE

Given a small displacement, the object returns to its original position



UNSTABLE

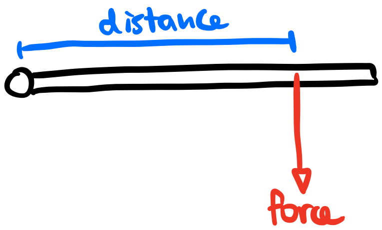
Given a small displacement, the object continues to move away from its original position



NEUTRAL

Given a small displacement, the object stays in its new position

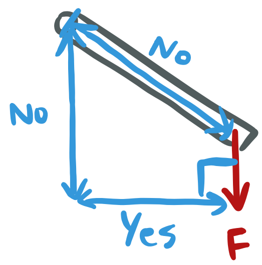
MOMENTS



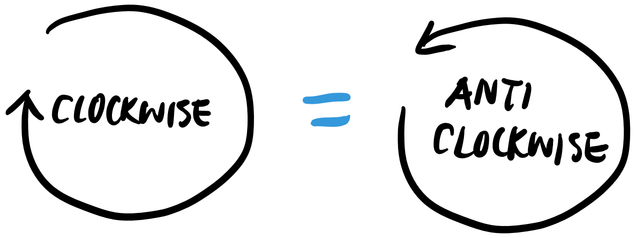
MOMENT = FORCE × DISTANCE

[Nm] = [N] × [m]

Distance is PERPENDICULAR
(perpendicular to the force)



LAW/PRINCIPLE OF MOMENTS

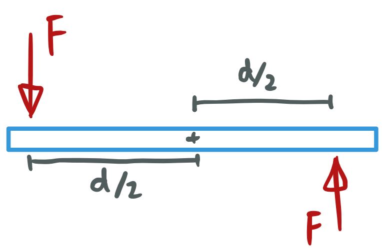


WHEN IN EQUILIBRIUM

You can move the fulcrum and the net moment won't change

WHEN EQUILIBRIUM, Net force = 0, Net moment = 0

Couple:



The forces: same magnitude
same object
opposite directions
different lines of action

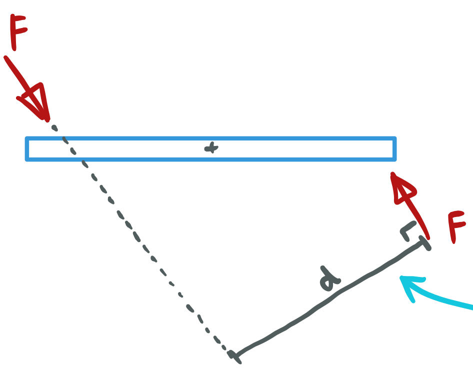
turning effect by couple is called TORQUE

turning effect $= F \times d/2 + F \times d/2$

$= F(d/2 + d/2)$

$= F\,d$

Magnitude of ONE force.
DO NOT SUM THEM!

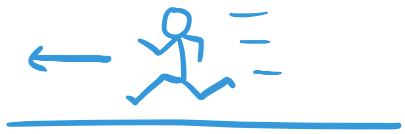


Perpendicular distance between the 2 forces
(NOT THROUGH PIVOT)

Linear Motion

ie motion in a straight line

DISTANCE: SCALAR
TOTAL LENGTH OF TRAVEL



DISPLACEMENT: VECTOR
(s) STRAIGHT LINE DISTANCE BETWEEN 2 POINTS

SPEED = $\frac{\text{DISTANCE}}{\text{TIME}}$ = RATE OF CHANGE OF DISTANCE

VELOCITY (v) = $\frac{\text{DISPLACEMENT}}{\text{TIME}}$ = RATE OF CHANGE OF DISPLACEMENT

ACCELERATION (a) = $\frac{\text{CHANGE OF VELOCITY}}{\text{TIME}}$ = RATE OF CHANGE OF VELOCITY

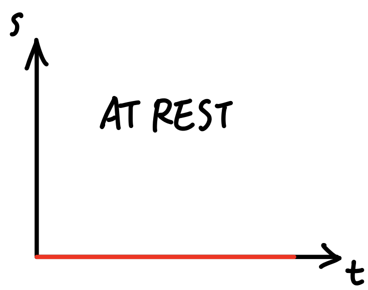
⇓ something over time

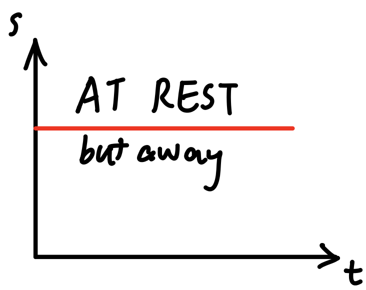
$$\boxed{a = \frac{\Delta v}{t} = \frac{v-u}{t}}$$

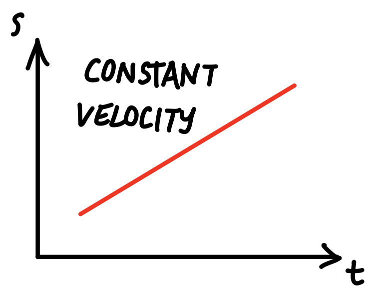
Summary		
s =	displacement	[m]
t =	time	[s]
u =	initial velocity	[ms^{-1}]
v =	final velocity	[ms^{-1}]
a =	acceleration	[ms^{-2}]

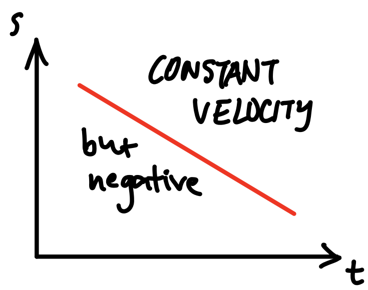
GRAPHS OF MOTION

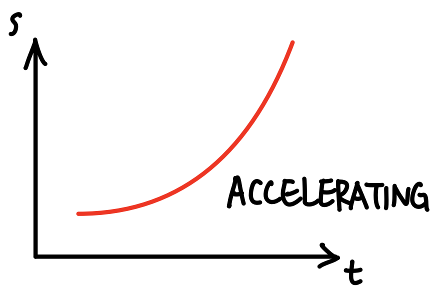
DISPLACEMENT-time graphs (s-t graphs)



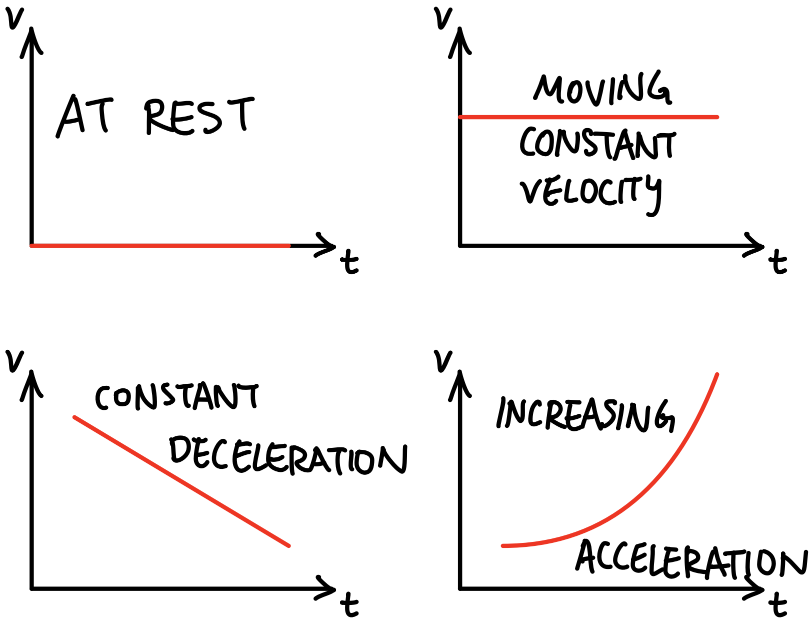




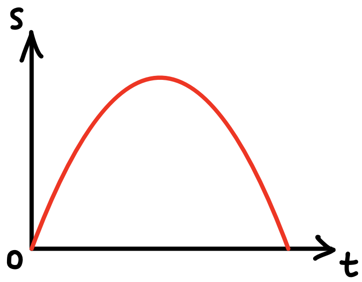


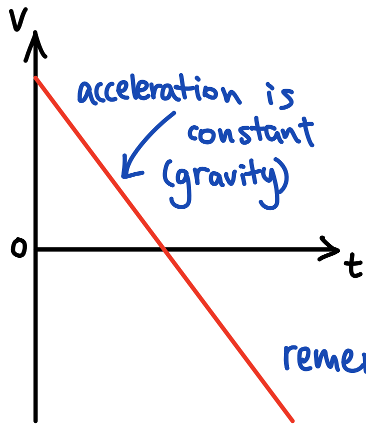


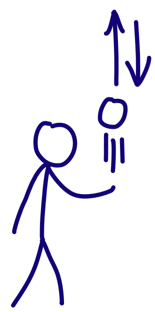
VELOCITY - time graphs (v-t graphs)



BALL: UP & DOWN

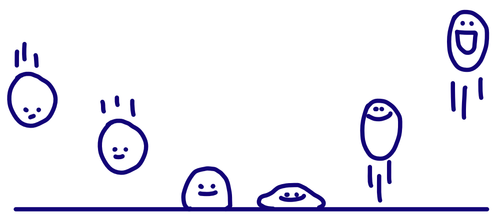


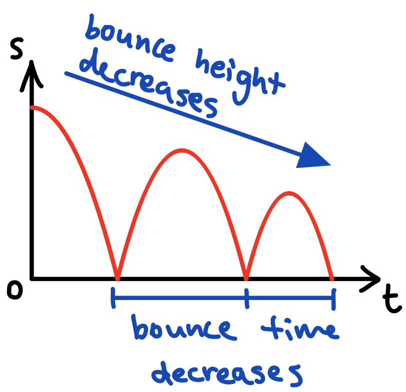


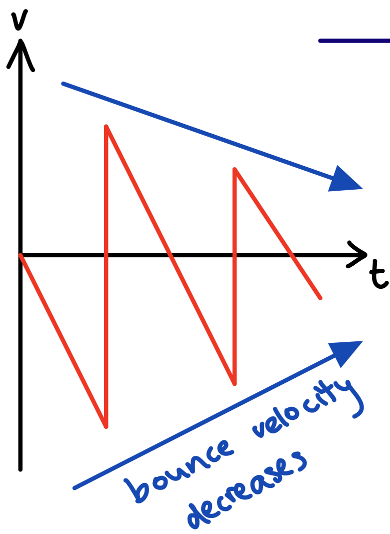


remember: velocity is a VECTOR!

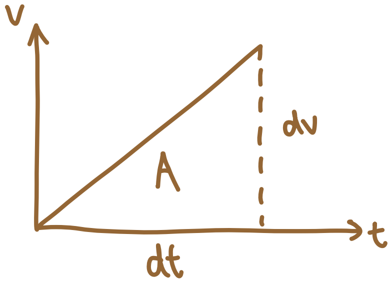
BALL: DROP & BOUNCE 3 TIMES







More on v-t graphs:



slope $= \frac{dv}{dt} =$ acceleration

area $= A = \int v\,dt =$ displacement

SUVAT

s	displacement	[m]
u	initial velocity	[ms^{-1}]
v	final velocity	[ms^{-1}]
a	acceleration	[ms^{-2}]
t	time	[s]

ONLY VALID WHEN CONSTANT ACCELERATION

($a = 0$ is ok!)

① $a = \frac{\Delta v}{t} = \frac{v-u}{t}$

rearranging, $\boxed{v = u + at}$

② Average velocity:

$\frac{v+u}{2} = \frac{s}{t}$

rearranging, $\boxed{s = \frac{1}{2}(u+v)t}$

③ Substitute eq^n 1 into 2

$s = \frac{1}{2}(2u+at)t$

$\boxed{s = ut + \frac{1}{2}at^2}$

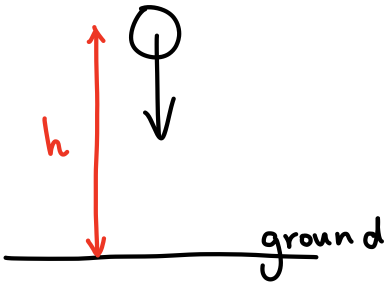
④ eq^n 1: $t = \frac{v-u}{a}$

substitute eq^n 1 into 2

$s = \frac{1}{2}(u+v)\frac{(v-u)}{a}$

rearranging, $\boxed{v^2 = u^2 + 2as}$

Calculating g ($9.81 ms^{-1}$)

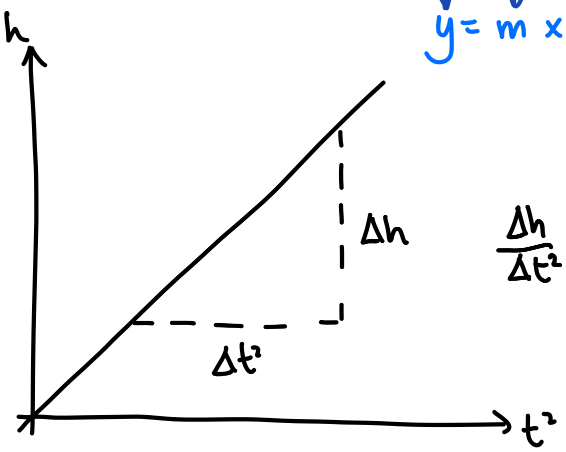


$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}at^2$ (ball dropped with $u=0$)

$h = \frac{1}{2}gt^2$

$y = mx + c$



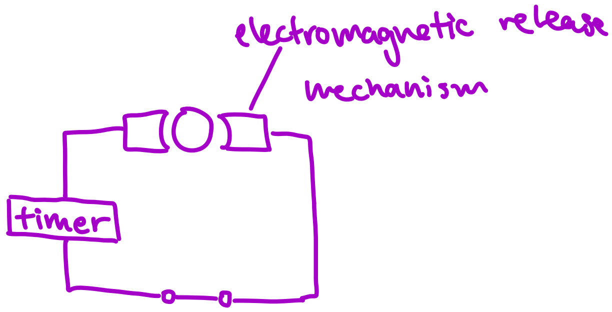
$\frac{\Delta h}{\Delta t^2} = \frac{1}{2}g$

however, human reaction time is too SLOW (0.2s), large % uncertainty

Improvements

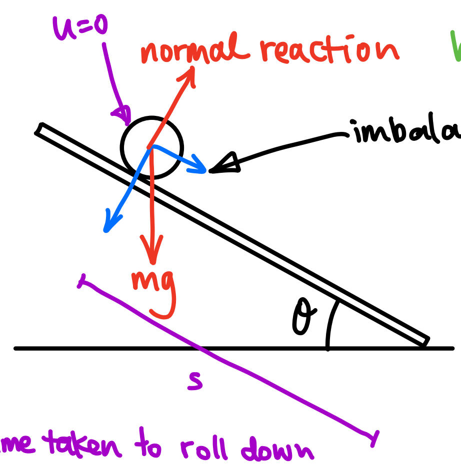
- ↑ drop height
- Computer-controlled drop & time
- slo-mo video on phone
- roll ball down slope

Reduce error



ball drop → start timer

ball drops on switch to stop timer



When θ is small, t is large, % error is small

imbalanced force accelerates ball

$F = ma = mg\sin\theta$

$a = g\sin\theta$

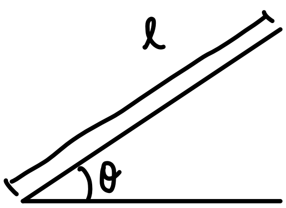
eqⁿ: $s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}at^2$

$s = \frac{1}{2}gt^2\sin\theta$

Results of Experiment
(calculating the value of g)

s (m)	t_1 (s)	t_2 (s)	t (s)	t^2 (s^2)
0.900	1.60	1.59	1.60	2.54
0.800	1.62	1.49	1.60	2.42
0.700	1.33	1.65	1.49	2.22
0.600	1.40	1.42	1.41	1.99
0.500	1.50	1.50	1.50	2.25
0.400	1.23	1.30	1.27	1.60
0.300	1.05	1.02	1.04	1.07
0.200	0.85	0.95	0.90	0.81



$\sin\theta = \frac{h}{s}$

$h = 0.123m$

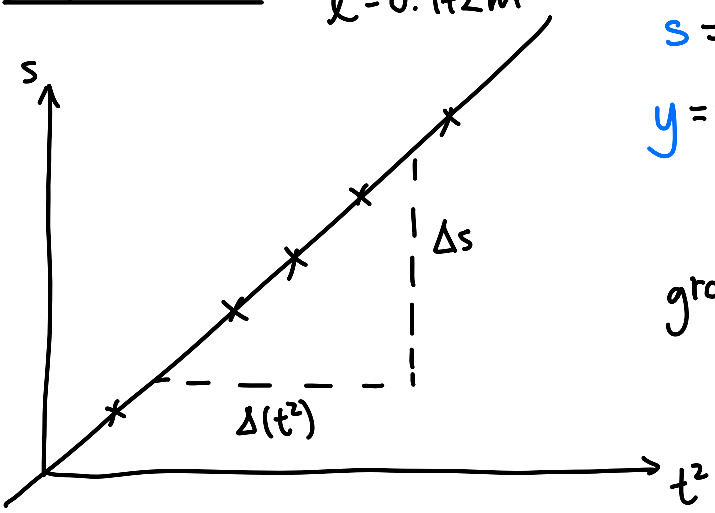
$l = 0.972m$

$$s = \frac{1}{2} g t^2 \sin\theta$$

$$s = \frac{1}{2} g \sin\theta \, t^2$$

$$s = \frac{1}{2} g \sin\theta \, t^2 + 0$$

$$y = mx + c$$

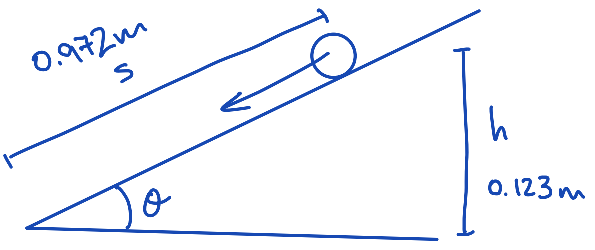


$$\text{gradient} = \frac{\Delta s}{\Delta(t^2)} = \frac{1}{2} g \sin\theta$$

My result: $g = 5.42\ ms^{-2}$

Ramp Practical Analysis

Tuesday, October 20, 2020 2:56 PM



Analysis:

1. Is this g by freefall?

2 a) What is the uncertainty in your measurements of:

h: ±0.001 m

l ~~x~~: ±0.001m

b) Hence, calculate the percentage uncertainty in your calculation of $\sin\theta$ ~~x~~?

$\sin\theta = \frac{h}{l}$

$\% h: \frac{0.001}{0.123} = \pm 0.813\%$ uncertainty

$\% l: \frac{0.001}{0.972} = \pm 0.103\%$ uncertainty

$\% \sin\theta: \%h + \%l = 0.916\%$ uncertainty

$= 0.92\%$ uncertainty

3 a) What is the uncertainty in your measurement of:

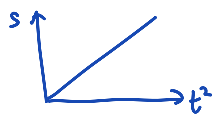
your smallest value of *s*: ±0.001 m

your corresponding the value of *t*: $\frac{1}{2}(0.95 - 0.85) = \pm 0.05s$

b)

Hence, using these uncertainties, calculate the percentage uncertainty in your calculation of the gradient of your graph.

$\text{gradient} = \frac{\Delta s}{\Delta(t^2)}$



$\% s = \frac{0.001}{0.200} = \pm 0.50\%$ uncertainty

$\% t = \frac{0.05}{0.90} = \pm 5.56\%$ uncertainty

$\%\text{gradient} = \%s + 2(\%t) = 0.5 + 2(0.556) = 11.6\%$

$= 12\%$ uncertainty

4.

The actual value of *g* is *9.81 ms*$^{-2}$. Calculate the percentage difference between this value and your calculated value of g (from question 4).

5. List the possible sources of error that may exist in this experiment and what type of error they are (e.g. systematic, zero, random, …)

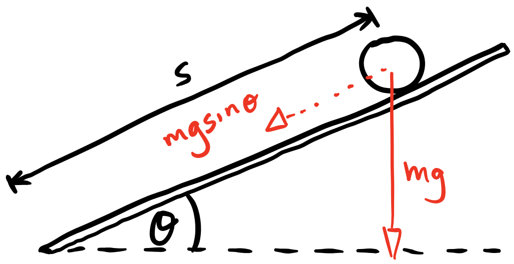
human reaction time (random)
friction is present (random)
ball follows different path (random)

6. How did you make your measurement of *D* as accurate as possible?

7. Comment on the reliability of your experimental results:

8. If you were to use a similar set of apparatus, what would you do differently to improve the accuracy of your measurements? Explain how the changes would improve the accuracy.

Re-Do: The value of g



Measure time taken for ball to roll from rest, down the ramp, from different distances.

$F = ma$

$\Downarrow$

$mg\sin\theta = ma$

$g\sin\theta = a$

$s = ut + \frac{1}{2}at^2$

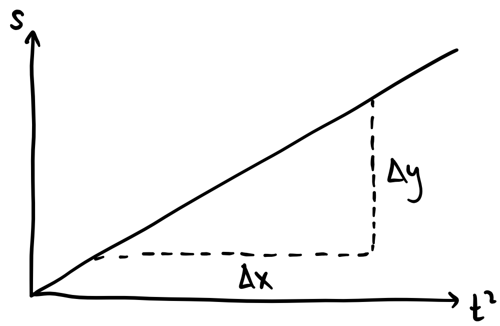
$\Downarrow$

$s = \frac{1}{2}at^2$

$a = \frac{2s}{t^2}$

$g\sin\theta = \frac{2s}{t^2}$

$s = \left(\frac{1}{2}g\sin\theta\right)t^2$



$\text{gradient} = \frac{\Delta y}{\Delta x} = \frac{1}{2}g\sin\theta$

$\therefore g = \frac{2 \times \text{gradient}}{\sin\theta}$

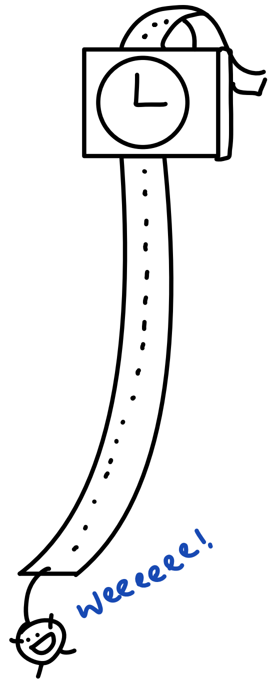
Table of Results

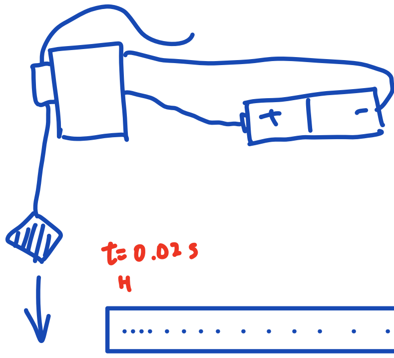
$\theta =$

$\sin\theta =$

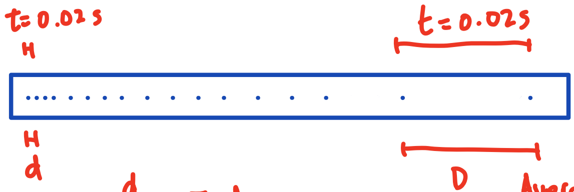
s / m	t_1 / s	t_2 / s	t_3 / s	$\bar{t}$ / s	$(\bar{t})^2$ / s^2

Alternative: Ticker tape timer! (Ticker timer & Ticker tape)





usually 50 Hz (depending on mains)



Average $u = \frac{d}{0.02} = 50d$

Average $v = \frac{D}{0.02} = 50D$

$D = 72\ mm = 0.072\ m$

$V = 3.6\ ms^{-1}$

$d = 4mm = 0.004m$

$V = 0.2\ ms^{-1}$

23 dots → 0.46s

$a = \frac{3.4}{0.46} = 7.39\ ms^{-2}$

$s = \overbrace{ut}^{0} + \frac{1}{2}at^2$

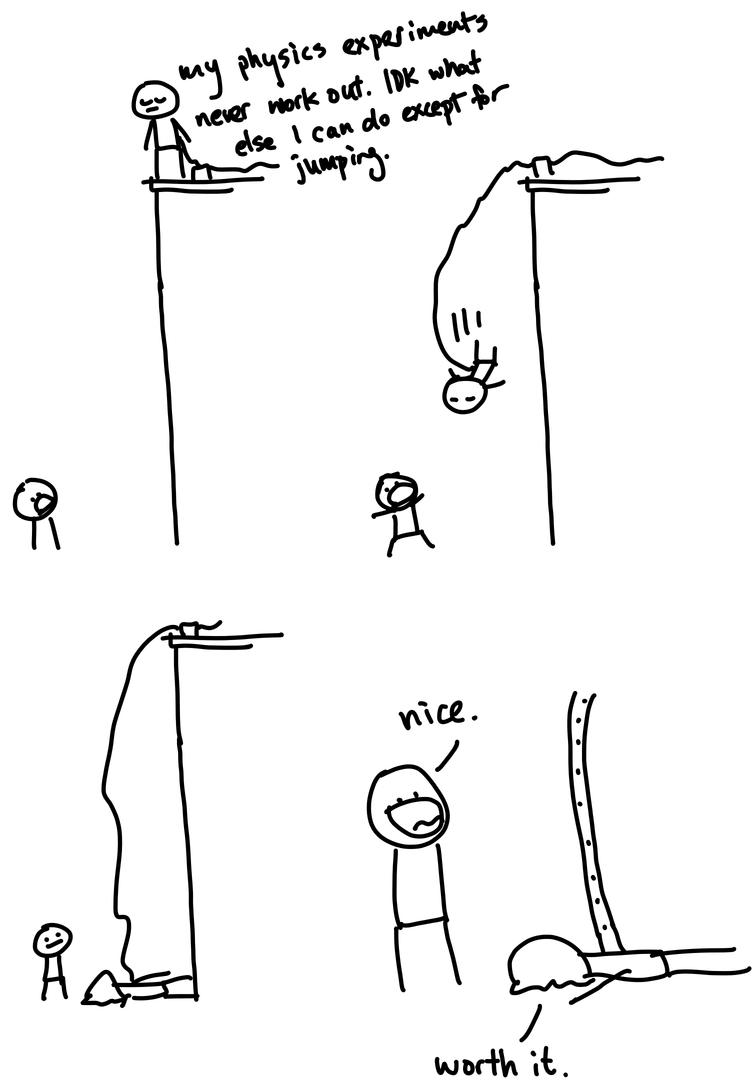
$s = \frac{1}{2}at^2$

$a = \frac{2s}{t^2}$

$s = 940\ mm = 0.94m$

$t = 0.46\ s$

$a = \frac{2 \times 0.94}{0.46^2} = 8.88\ ms^{-2}$

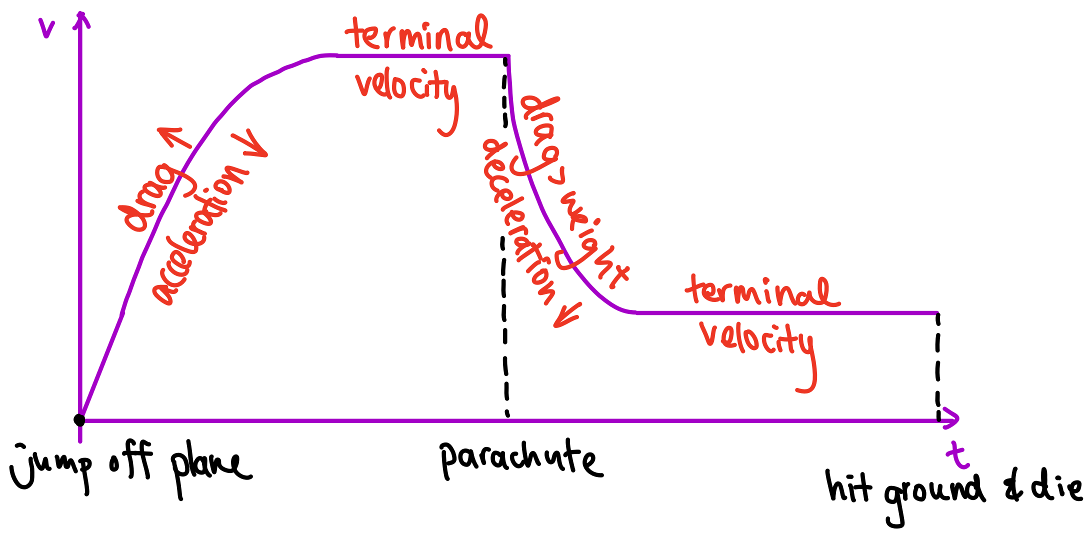


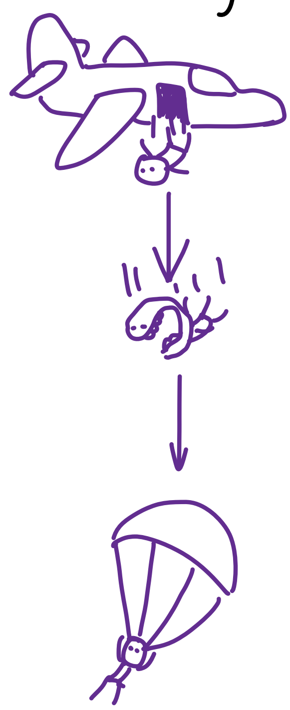
Terminal Velocity

Terminal velocity : weight = air resistance

No net force → no acceleration → constant velocity

The v-t graph





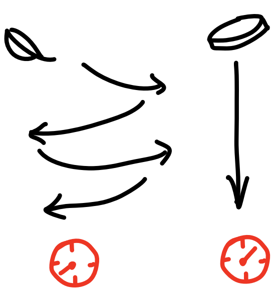
Guinea & Feather Experiment

not the pig!

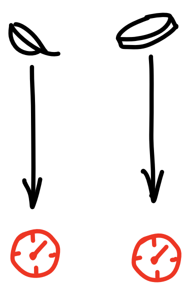
(a guinea is an old coin)

Feather falls slower because of air resistance

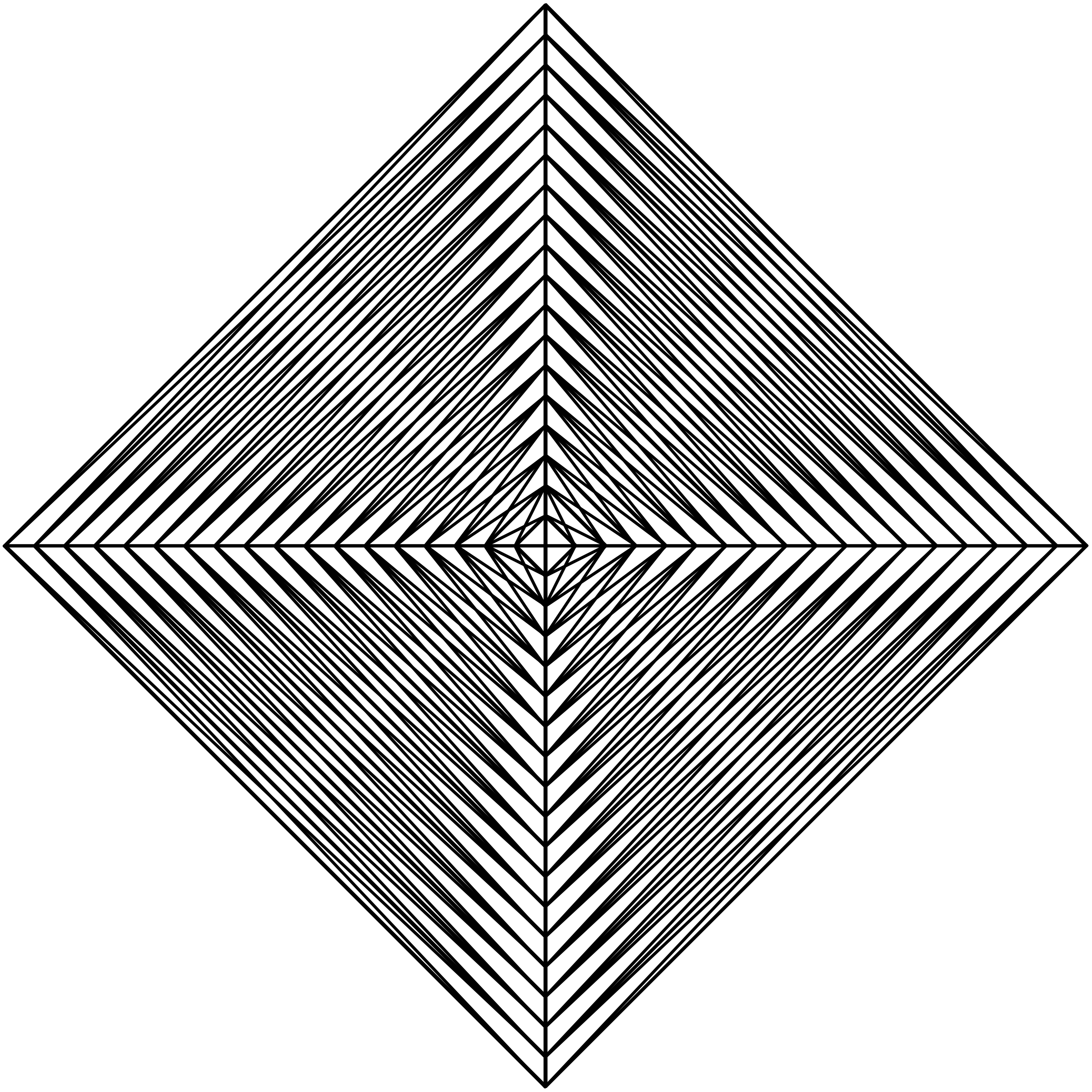
In atmosphere



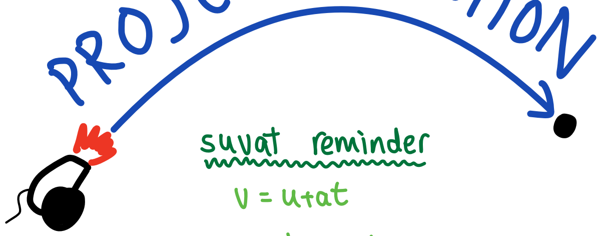
In vacuum



⚠ do not assume air resistance unless told to!



PROJECTILE MOTION



suvat reminder

$v = u + at$

$s = \frac{1}{2}(u+v)t$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

Vertical: Constant acceleration downwards

$9.81\ ms^{-2}$

SUVAT!

Horizontal: No acceleration / deceleration

no driving force

no air resistance

Constant velocity $v = \frac{s}{t}$!

do not affect each other (independent)

Vertical fall (time taken)

$5\,m \Rightarrow s = ut + \frac{1}{2}at^2 \quad (u=0)$

$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{10}{9.81}} = 1.01\,s\ (3sf)$

$10\,m \Rightarrow t = \sqrt{\frac{20}{9.81}} = 1.43\,s\ (3sf)$

$15\,m \Rightarrow t = \sqrt{\frac{30}{9.81}} = 1.75\,s\ (3sf)$

NOT DIRECTLY PROPORTIONAL!

Results for investigation below:

INVESTIGATION 1							
Initial Velocity (m/s)	0	5	10	15	20	25	30
Time taken (s)	1.11	1.11	1.11	1.11	1.11	1.11	1.11
Range (m)	0	5.53	11.06	16.59	22.12	27.65	33.18
INVESTIGATION 2							
Height (m)	5	10	15				
Time taken (s)	1.01	1.43	1.75				
Range (m)	20.19	28.56	34.97				

time of flight depends on height, not horizontal velocity

horizontal velocity ∝ range

Projectile Investigation using Simulation

02 November 2020 10:37

Online, search for "**PhET Projectile Motion**"

Click on the result: Projectile Motion - Kinematics | Air Resistance ... - PhET

Run the simulation by selecting the '**Intro**' option.

Familiarise yourself with the simulation then work through the investigations below…

Do not write your answers/measurements on this sheet – there is absolutely not enough space!

Investigation 1:

Make sure Air Resistance is turned off (not ticked)

Adjust the height of the cannon to 6m.

Keep the cannon in the horizontal position.

Fire a projectile at various speeds (from 0 to 30 m/s).

For each speed, use the measurement tool to record the time taken to hit the ground and the range (the horizontal distance achieved). Tabulate your results.

Comment on your measurements – e.g. what do they demonstrate?

Investigation 2:

Make sure Air Resistance is turned off (not ticked)

Keep the cannon in the horizontal position.

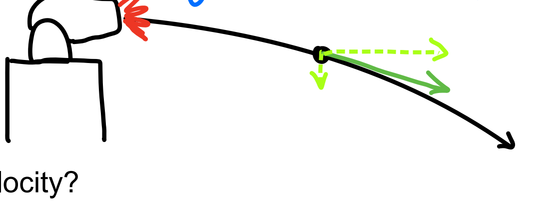
Set the initial speed of the projectile to 20 m/s.

For the heights of 5m, 10m, 15m

a. **Calculate** the time taken for an object to fall vertically from rest from this height.
b. Use the simulation to measure the time taken to hit the ground and the range.

Comment on your results – what do they demonstrate?

velocity tangential to path



Investigation 3:

For a horizontal cannon at height 13m and initial speed 16 m/s

Turn on the Velocity Vectors – Components (by ticking the box)

What do you observe about the horizontal and vertical vectors of velocity?

Investigation 4:

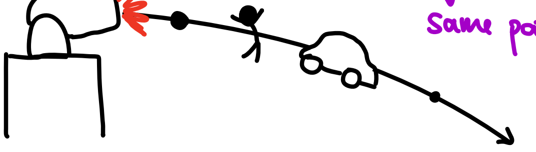
Set a cannon height and initial projectile speed of your choice.

Fire different projectiles from the cannon.

No air resistance

↓

same path



Comment on what you observe.

Projectiles step-by-step

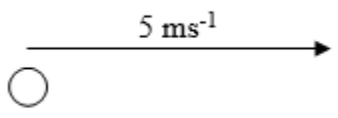
04 November 2020 08:23

this worksheet guides you step-by-step through the process!

Projectiles: step-by-step

[Use the value of g = 9.81 ms^{-2}]

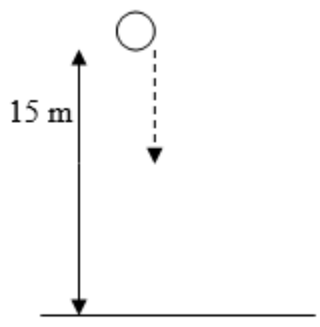
1.



$v = \frac{s}{t} \Rightarrow s = vt = 5 \times 2 = 10\,m$

Calculate the distance travelled in 2 seconds.

2.



$s = ut + \frac{1}{2}at^2$

$\Downarrow$

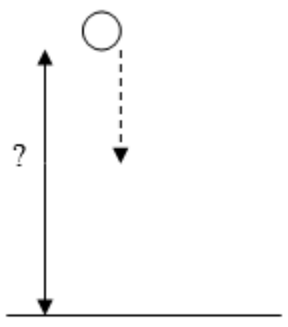
$s = \frac{1}{2}at^2$

$\Downarrow$

$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{30}{9.81}} = 1.75\,s$

Calculate the time taken for the ball to hit the ground, if released from rest.

3.



$s = ut + \frac{1}{2}at^2$

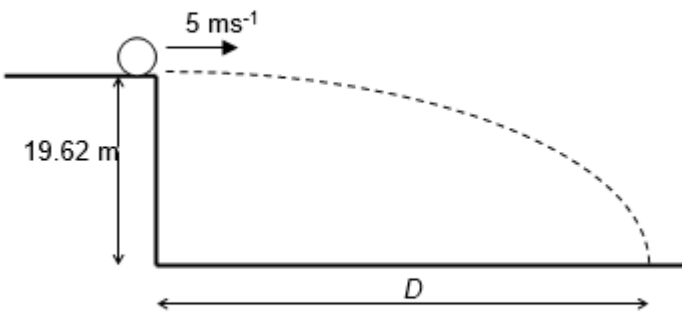
$\Downarrow$

$s = \frac{1}{2}at^2$

$= \frac{1}{2}(9.81)(2)^2 = 19.62\,m$

If the time of flight is 2s, calculate the height it has dropped through (from rest).

4.

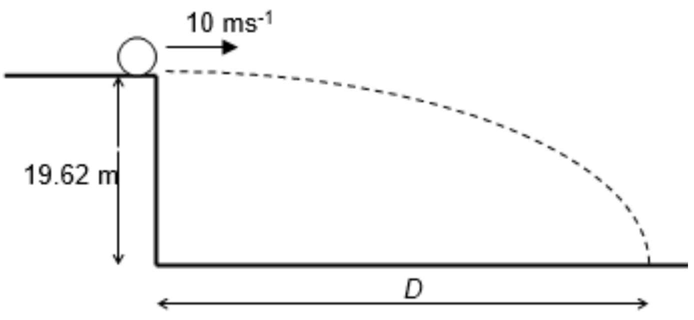


Calculate the horizontal distance, *D*.

time of flight = 2 s

$v = \frac{s}{t} \Rightarrow s = vt = 5 \times 2 = 10\,m$

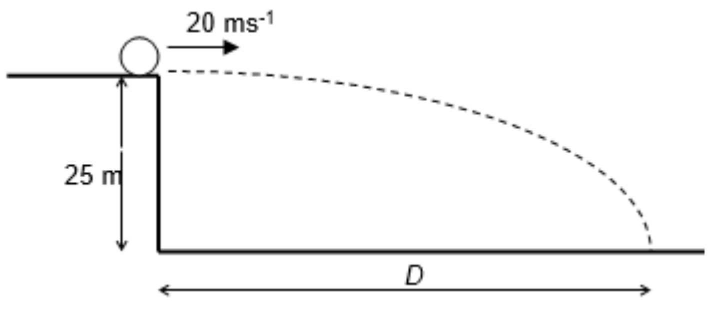
5.



Calculate the horizontal distance, *D*.

$v = \frac{s}{t} \Rightarrow s = vt = 10 \times 2 = 20\,m$

6.



a) Calculate the time of flight.

$s = \frac{1}{2}at^2$

$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{50}{9.81}} = 2.26\,s$

b) Calculate the horizontal distance, *D*.

$s = vt = 20 \times 2.26 = 45.2\,m$

c) Calculate the *vertical* velocity when it hits the ground

$v = u + at = 9.81 \times 2.26 = 22.1\,ms^{-1}$

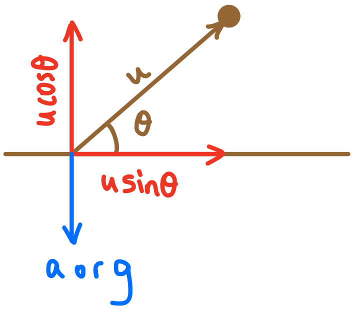
d) What is the *horizontal* velocity when it hits the ground?

$20\,ms^{-1}$

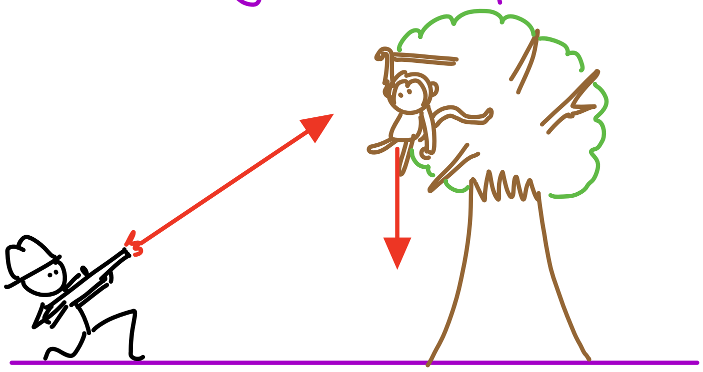
e) Calculate the *Magnitude* and *Direction* of the *resultant velocity* when it hits the ground.

Magnitude $= \sqrt{20^2 + 22.1^2} = 29.8\,ms^{-1}$ Direction $= \tan^{-1}\left(\frac{22.1}{20}\right) = 47.9°$ to the horizontal

Projectiles fired from an angle
(angled launches)

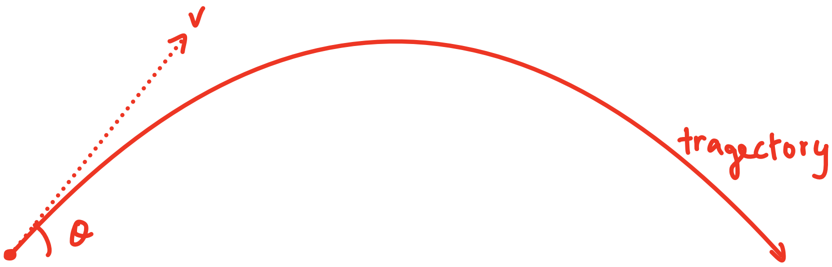


The monkey & hunter problem



monkey drops & hunter fires.
does monkey go bye-bye?

YES!



max height?
$v^2 = u^2 + 2as$
and
solve!

$v=0$
u
$a = -9.81$

how long does it fly?

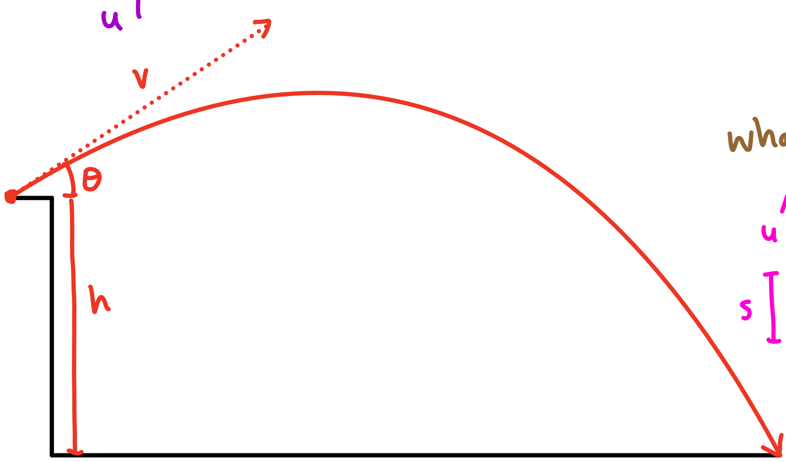
Method 1: use $a = \frac{v-u}{t}$ or $v = u + at$
for $\underbrace{\text{ascent \& descent}}_{\text{both have same time}}$

$v=0$
u
$a = -9.81$

Method 2: use $a = \frac{v-u}{t}$ or $v = u + at$
for entire thing

attention to signs!

$a = -9.81$
u
$v = -u$



what about now?

u
s
v

$s = ut + \frac{1}{2}at^2$

substitute, then solve
quadratic for t

(pay attention to signs!)

Projectiles: Further Questions

05 November 2020 12:01

Example 1:

A coin is flipped into the air. Initial velocity = 8.0ms^{-1}. Using g=10ms^{-2} and ignoring air resistance;

a) What is the max height that the coin reaches?

b) What is the velocity of the coin when returning to my hand?

c) How long is the coin in the air for?

a) $v^2 = u^2 + 2as$

$s = \frac{v^2 - u^2}{2a} = \frac{0-64}{-20} = 3.2m$

b) $-8\,ms^{-1}$ ($8ms^{-1}$ downwards)

c) $t = \frac{-8-8}{-10} = 1.6\,s$

Example 2:

A stone is thrown over the edge of a cliff with horizontal velocity 15 ms^{-1}. The cliff is 150m high. Ignoring air resistance and using g=10ms^{-2};

a) How long does it take the stone to reach the ground?

b) How far does it land from the foot of the cliff?

c) The stone's velocity when it hits the ground?

a) $s = ut + \frac{1}{2}at^2$

$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2(150)}{10}} = \sqrt{30} = 5.48\,s$

b) $s = vt = 15 \times 5.48 = 82.2\,m$

c) $v = u + at = at = 10 \times 5.48 = 54.8\,ms^{-1}$ (vertical)

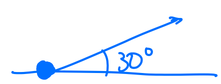
velocity $= \sqrt{15^2 + 54.8^2} = 56.8\,ms^{-1}$

Example 3:

A golfer hits a ball so it moves off with a velocity of 26ms^{-1} at 30^{o} to the horizontal. Ignoring air resistance and using g=10ms^{-2};

a) How long is the ball in the air?

b) The horizontal distance travelled by the ball?



a) vertical $u: 26 \sin 30 = 13\,ms^{-1}$

$\uparrow -13ms^{-1}$ } -ve

$\downarrow 10ms^{-2}$ } +ve

$s = ut + \frac{1}{2}at^2$

$0 = -13t + 5t^2$

$5t^2 - 13t = 0$

$t = 13/5 = 2.6s$

b) horizontal $u = 26 \cos 30 = 22.5\,ms^{-1}$

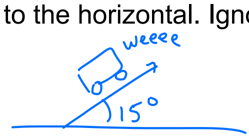
$s = vt = 2.6 \times 22.5 = 58.5m$

Example 4:

A bus travelling at 50mph (~ 20 ms^{-1}) takes off from a ramp at 15^{o} to the horizontal. Ignoring air resistance and using g=10ms^{-2};

a) How long is the bus in the air?

b) The horizontal distance travelled by the bus?



a) vertical $u = 20 \sin 15 = 5.18\,ms^{-1}$

$s = ut + \frac{1}{2}at^2$

$0 = 5.18t + 5t^2 = t(5t + 5.18)$

$t = \frac{5.18}{5} = 1.04\,s$

b) horizontal $u = 20 \cos 15$

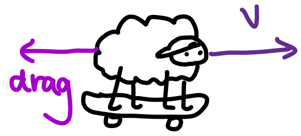
$s = vt = 1.04 \times 20 \cos 15 = 20m$

No calculations with air resistance needed

But we need to know the effects of air resistance
ie drag

↑ velocity, ↑ drag

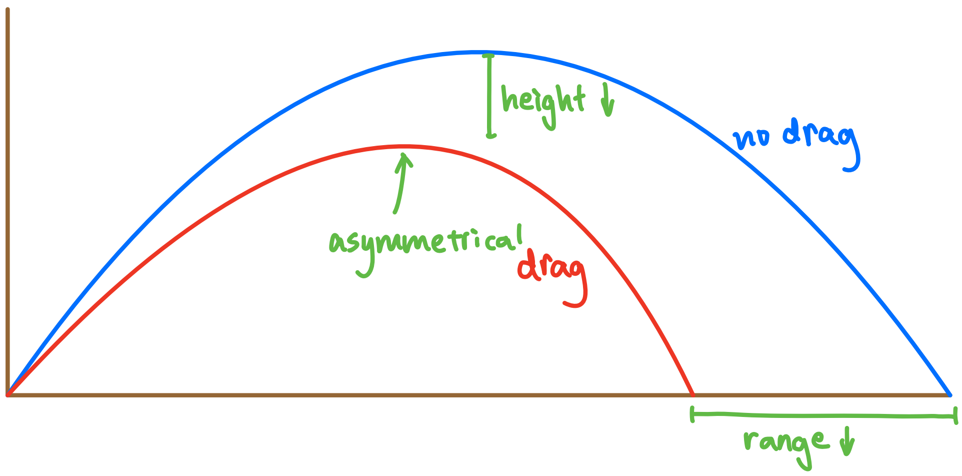
acts in opposite direction of velocity



Range ↓ when drag

Max Height ↓ when drag

asymmetrical flight path / trajectory



Newton's laws of motion

① Body at rest / at constant velocity unless acted on by an external resultant force

② Force = rate of change of momentum ($F = \frac{dmv}{dt} = m\frac{dv}{dt} = ma$)

ie

Force causes a change in momentum

③ For every action there is an equal and opposite reaction

If A exerts a force on B, B exerts an equal but opposite force on A

they don't cancel because they're on different objects

N3L

The 2 forces ⇒ "Newton pair"

Properties of a Newton Pair

- Equal magnitude
- Opposite direction
- Act along the same line of action
- Act on different objects
- Act for the same time

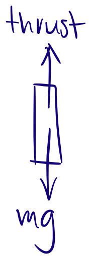
Newton's Law Qs 1

Friday, November 20, 2020 10:35 AM

Example 1: A rocket of mass 1000kg carries 800kg of fuel at take off. The thrust from the engines is 22 000N. Assume g = 10ms^{-2}.

a) What is the initial acceleration?

b) What is the acceleration just as all the fuel is used up, assuming the thrust remained constant.



a) $mg = (1000+800)(10) = -18000$ N

thrust $= 22000$ N

$F_{net} = ma$

$22000 - 18000 = (1000+800)a$

$a = \frac{4000}{1800} = 2.2\,ms^{-2}$

b) $mg = (1000)(10) = -10000$ N

thrust $= 22000$ N

$F_{net} = ma$

$22000 - 10000 = 1000a$

$a = 12\,ms^{-2}$

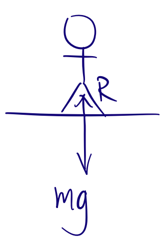
Example 2: A person (mass 70kg) is in a lift. Assuming g= 10 N kg^{-1}, calculate the contact force, R when:

a) The lift is at rest.

b) Lift is accelerating upwards at 1.0ms^{-2}.

c) Lift is accelerating downwards at 2.0ms^{-2}.

d) Lift is ascending at a steady speed.



a) $R = mg = 70 \times 10 = 700$N

b) $F_{net} = ma = 70 \times 1 = 70$N upwards

$R = 700 + 70 = 770$N

c) $F_{net} = ma = 70 \times 2 = 140$ N downwards

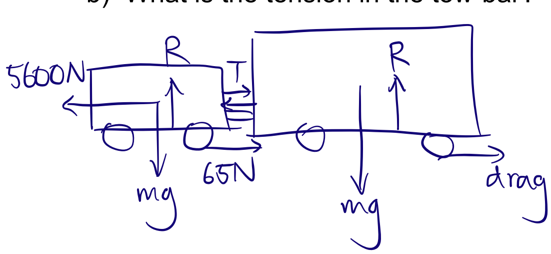
$R = 700 - 140 = 560$N

d) 700N

Example 3: A car of mass 1200kg tows a caravan of mass 1800kg. They are linked by a rigid tow-bar. The car accelerates at 1.80ms^{-2}, driven by 5600N. The resistance on the car is 65N.

a) What is the resistive force of the caravan?

b) What is the tension in the tow-bar?



a) Horizontal: $F_{net} = ma = 3000(1.8) = 5400$

$F_{net} = 5600 - 65 - F = 5400$

$F = 135$N

b) Tension causes caravan to accelerate at 1.80 ms^{-2}

$\therefore$ Tension $-$ drag $= ma$

Tension $= (1800)(1.8) + 135$

$= 3375$ N

Linear Momentum   this is a vector.

how easy/difficult to stop something (not the official definition)

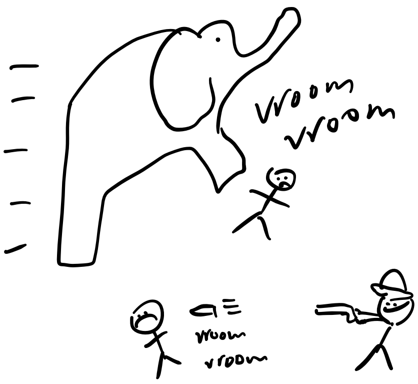
Momentum = mass × velocity

$$p = mv$$

$$[kg\,ms^{-1}] = [kg][ms^{-1}]$$

or

$$[Ns]$$



Newton's 2nd Law: $F = \frac{mv - mu}{t}$ (force = rate of change of momentum)

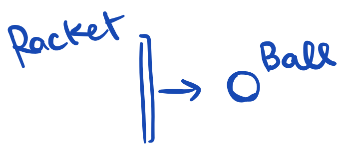
$$= m\left(\frac{v-u}{t}\right) = ma$$

Impulse: Change in momentum (NOT over time)

$$= \Delta p = mv - mu = Ft$$

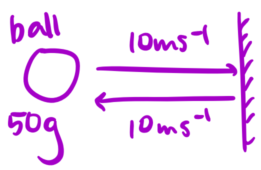
Δp: change in momentum; Ft: force × time

Hitting balls (tennis, football, golf etc.)



Follow through → $t\uparrow$, impulse $\uparrow$, $v\uparrow$

E.g.



Ball bounce: impulse?

$$\Delta p = mv - mu = 0.05(10) - 0.05(-10) = 1\,Ns$$

If ball bounce duration = 0.1s,

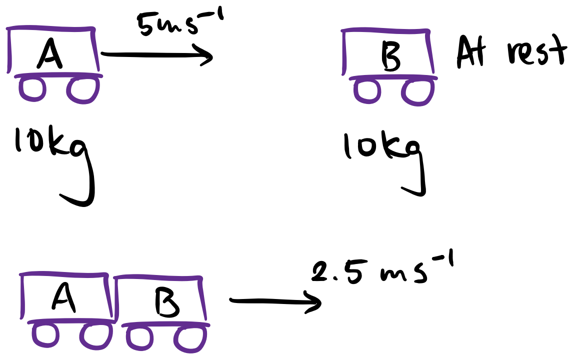
$$\Delta p = Ft$$

$$F = \frac{\Delta p}{t} = \frac{1}{0.1} = 10\,N$$

Conservation Of Momentum

During a collision the total momentum of the system is conserved so long as no external force is applied

Example:



Before

After

Inelastic collision (The carts stick together)

MOMENTUM

1. In a football match, a player kicks a stationary football of mass 0.44 kg and gives it a speed of 32 m s^{-1}.

(a) (i) Calculate the change of momentum of the football.

$mv - mu = 0.44 \times 32 = 14.08\ kgms^{-1}$

(ii) The contact time between the football and the footballer's boot was 9.2 m s. Calculate the average force of impact on the football.

Impulse = Ft

$14.08 = 0.0092F$

$F = 1530\ N$

(3)

(b) A video recording showed that the toe of the boot was moving on a circular arc of radius 0.62 m centred on the knee joint when the football was struck. The force of the impact slowed the boot down from a speed of 24 m s^{-1} to a speed of 15 m s^{-1}.

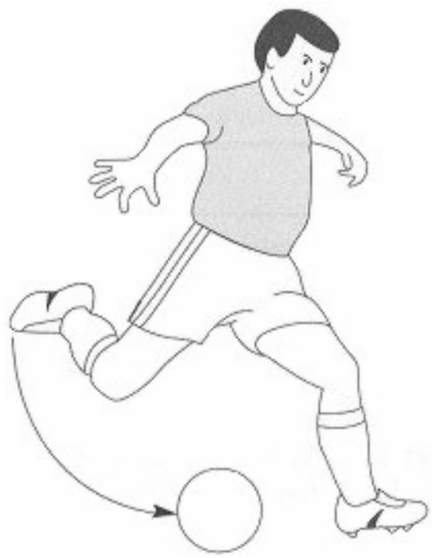


Figure 1

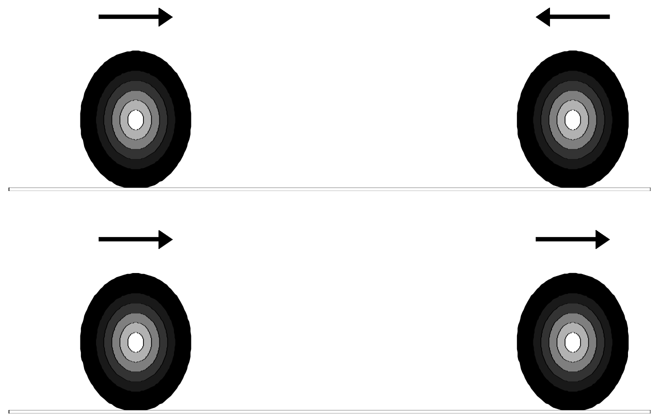
(i) Calculate the deceleration of the boot along the line of the impact force when it struck the football.

$a = \frac{v-u}{t} = \frac{15-24}{0.0092} = -\frac{9}{0.0092} = -978\ ms^{-2}$

(1)
(Total 4 marks)

2. (a) (i) Give an equation showing how the principle of conservation of momentum applies to the colliding snooker balls shown in the diagram.

Before



After

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

(ii) State the condition under which the principle of conservation of momentum applies.

No external forces acting on the system

(3)

(b) A trolley, A, of mass 0.25 kg and a second trolley, B, of mass 0.50 kg are held in contact on a smooth horizontal surface. A compressed spring inside one of the trolleys is released and they then move apart. The speed of A is 2.2 m s^{-1}.

(i) Calculate the speed of B.

$m_1v_1 + m_2v_2 = 0$

$0.25 \times 2.2 + 0.5v_2 = 0$

$v_2 = -0.55/0.5 = -1.1\ ms^{-1}$

speed $= |v_2| = 1.1 ms^{-1}$

(ii) Calculate a minimum value for the energy stored in the spring when compressed.

Energy $= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(0.25)(2.2)^2 + \frac{1}{2}(0.5)(1.1)^2$

$= 0.9075$ J

(4)

(c) The rotor blades of a helicopter sweep out a cross-sectional area, *A*. The motion of the blades helps the helicopter to hover by giving a downward velocity, υ, to a cylinder of air, density ρ. The cylinder of air has the same cross-sectional area as that swept out by the rotor blades.

Explaining your reasoning,

(i) derive an expression for the mass of air flowing downwards per second, and

$m = \rho V$

$\frac{m}{t} = \rho \frac{V}{t} = \rho A v$

(ii) derive an expression for the momentum given per second to this air.

$p = mv$

$\frac{p}{t} = \frac{m}{t} v = \rho A v^2$

(iii) Hence explain why the motion of the air results in an upward force, *F*, on the helicopter given by

$F = \rho A \upsilon^2$.

Momentum given per second = rate of change of momentum = force (from N2L)

$\therefore F = \frac{p}{t} = \rho A v^2$

If air is forced downward, there is an equal but opposite force upwards on the helicopter

(5)

(d) A loaded helicopter has a mass of 2500 kg. The area swept out by its rotor blades is 180 m^2. If the downward flow of air supports 50% of the weight of the helicopter, what speed must be given to the air by the motion of the rotor blades when the helicopter is hovering? Take the density of air to be 1.3 kg m^{-3}.

$F = \rho A v^2 = \frac{mg}{2}$

$1.3 \times 180 \times v^2 = \frac{2500 \times 9.81}{2}$

$v^2 = \left(\frac{2500 \times 9.81}{2}\right) / (1.3 \times 180)$

$v = \sqrt{\frac{2725}{52}} = 7.24 \text{ ms}^{-1}$

(3)
(Total 15 marks)

Elastic & Inelastic Collisions

If there is no external force:

Momentum is **ALWAYS CONSERVED**

but **NOT KINETIC ENERGY**

Elastic collision: E_K is conserved

ie E_K before = E_K after

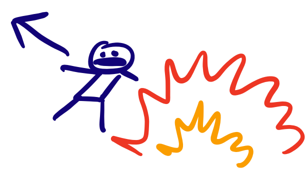
(Reminder: $E_K = \frac{1}{2}mv^2$)

Inelastic collision: E_K is lost

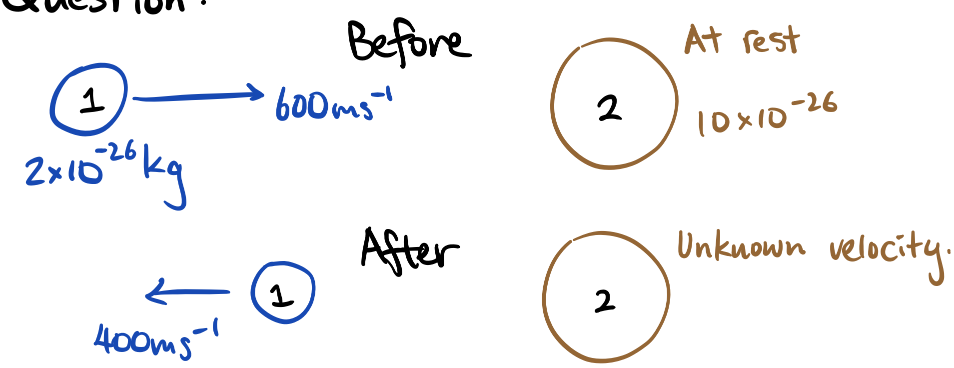
ie E_K before > E_K after e.g.

Super elastic collision: E_K is gained

ie E_K before < E_K after e.g. explosion



Question:



Step 1: find v_2

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$2 \times 10^{-26} \times 600$

$= -400 \times 2 \times 10^{-26} + 10 \times 10^{-26} \times v_2$

$v_2 = 140 \text{ ms}^{-1}$

Step 2: find E_K

E_K before: $\frac{1}{2}mv^2 = \frac{1}{2}(2 \times 10^{-26})(600)^2 = 3.6 \times 10^{-21}$ J

E_K after: $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{1}{2}(2 \times 10^{-26})(400)^2 + \frac{1}{2}(10 \times 10^{-26})(140)^2$

$= 2.58 \times 10^{-21}$ J

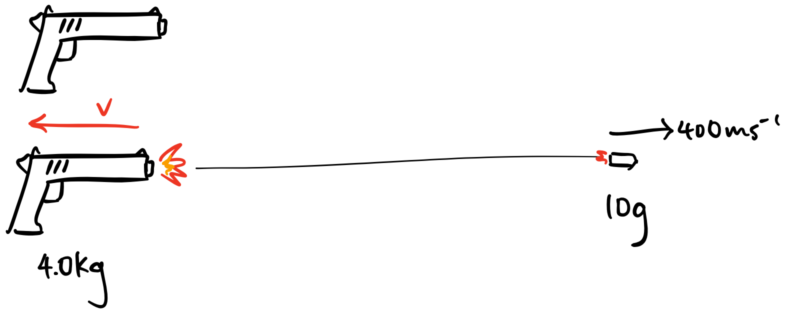
Since E_K before > E_K after,

collision is inelastic.

In real life, most collisions are inelastic

except for the collision of gas molecules, which are elastic ($\because$ gases don't cool down to liquids)

Total momentum = 0



$m_1v_1 + m_2v_2 = 0$

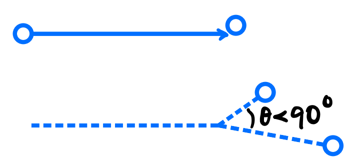
$4v + 0.01 \times 400 = 0$

$v = \frac{-4}{4} = -1\,ms^{-1}$ ie $1\,ms^{-1}$ back

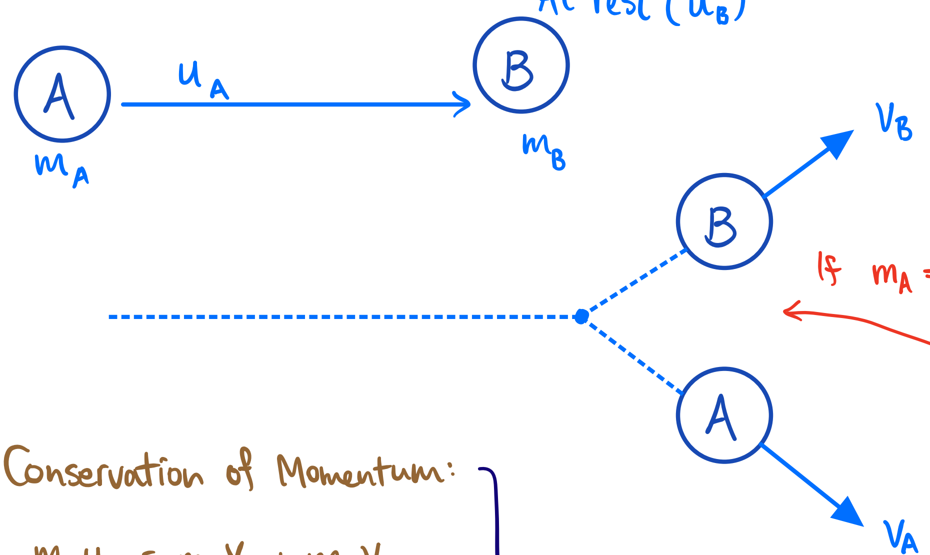
Colliding At An Angle

If $m_A > m_B$, separation angle < 90°

If $m_B > m_A$, separation angle > 90°



At rest (u_B)



If $m_A = m_B$ and the collision is elastic, the angle = 90°

Conservation of Momentum:

$m_A u_A = m_A V_A + m_B V_B$

(u_A, V_A & V_B are vectors)

If elastic:

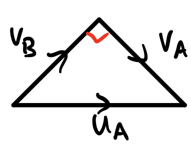
$\frac{1}{2}m_A u_A^2 = \frac{1}{2}m_A V_A^2 + \frac{1}{2}m_B V_B^2$

let's say $m_A = m_B$

↓

$u_A = V_A + V_B$ & $\overbrace{u_A^2 = V_A^2 + V_B^2}^{\text{Is it him? Again?}}$

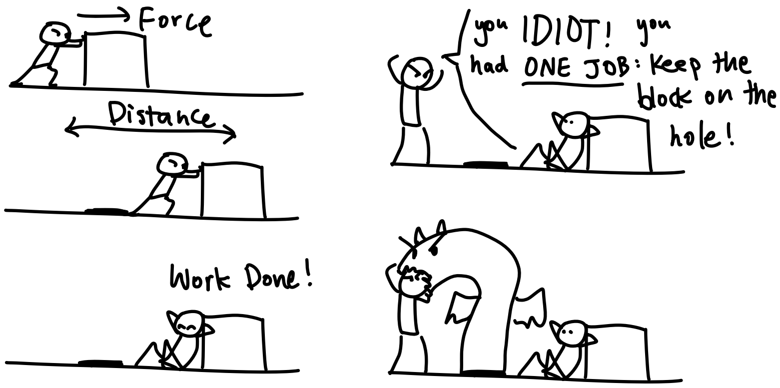
The vector triangle



Pythagoras!

($\because$ The right angle)

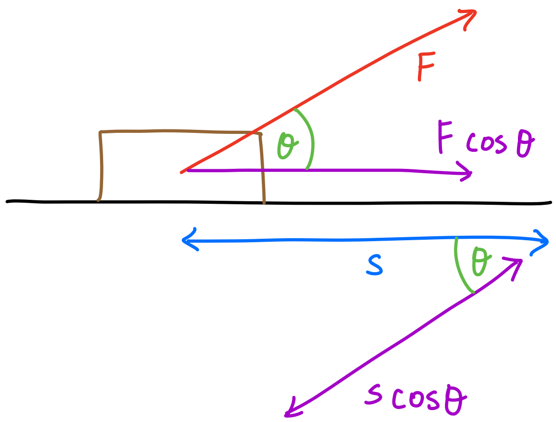
WORK DONE, ENERGY & POWER



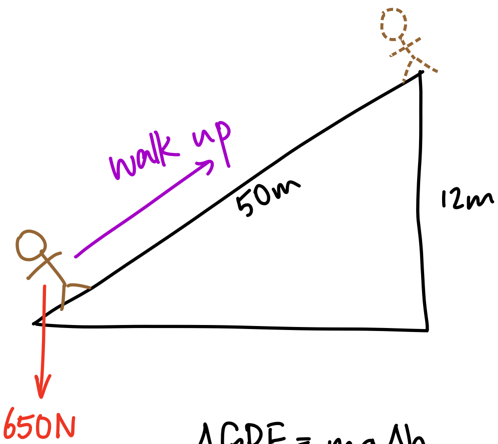
Work Done = Force × Displacement

$W = F \times s$ $\qquad [J] = [N] \times [m]$

Distance over which the force is applied.



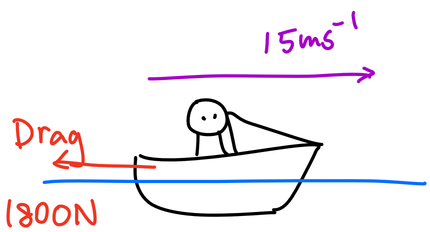
$$\left.\begin{aligned} W &= Fs\cos\theta \\ &= Fs\cos\theta \end{aligned}\right\}$$ As long as the components are parallel



$\underbrace{\text{Work} = 650 \times 12 = 7800\text{J}}_{\text{That's it!}}$

Move up 12m by counteracting weight

$\Delta GPE = \underbrace{mg}_{\text{weight}}\underbrace{\Delta h}_{\text{height travelled}}$



Steady speed

↓

No resultant force

Work Done in 10s:

$W = f \times s = f \times v \times t = 1800 \times 15 \times 10 = 270000\text{ J}$

∴ Power of motor:

$270000/10 = 27000\text{ W}$

TYPES OF ENERGY

Heat	Kinetic	GPE
Light		Elastic PE
Sound		Electrical
		Chemical

Kinetic Energy

$W = fs$

$= mas$

$= m\frac{v^2}{2s}s$

$= \frac{1}{2}mv^2 = E_k$

Gravitational Potential Energy

$W = fs$

$= (\text{Weight})(h)$

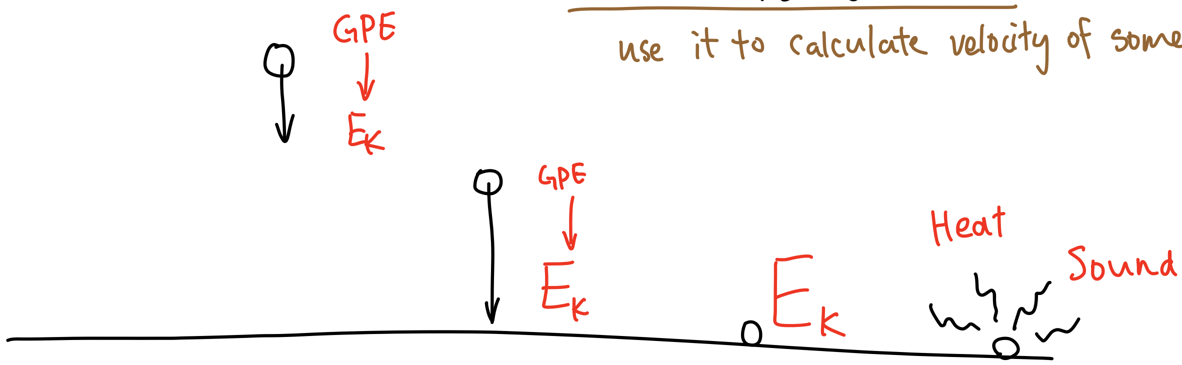
$= mgh = GPE$

Interchange between GPE and E_k

○ GPE

Loss in GPE = Gain in E_k

use it to calculate velocity of something falling



Conservation of Energy

Energy cannot be created nor destroyed, only transferred between different types.

Energy is NOT "LOST". It is DISSIPATED or WASTED.

Heat or sound usually

GPE lost = E_k gained + Work done to overcome friction

Power

force × velocity (Only when velocity is constant)
(If not, we get average power)

$$P = \frac{E}{t} = \frac{W}{t} = \frac{Fs}{t} = Fv$$

energy over time

$$[W] = \frac{[J]}{[s]}$$

Efficiency

A measure of the proportion of energy/power that is transferred into useful forms