Arithmetic Series

$$S_n = a + (a+d) + (a+2d) + \ldots + (a+(n-1)d)$$

where $u_n = n^{th}$ term $= a+(n-1)d$
$= l$ (last term)

or...

$$S_n = a + (a+d) + (a+2d) + \ldots + (l-d) + l$$

Sum Formulas

$$S_n = \frac{n}{2}(a+l)$$

or

$$S_n = \frac{n}{2}\left(2a+(n-1)d\right)$$

Geometric Series

$$S_n = a + ar + ar^2 + ar^3 + \ldots ar^{n-1}$$

n^{th} term $= u_n = ar^{n-1}$

Sum Formulas

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}$$

We can consider an infinite geometric series if $|r|<1$ or $-1<r<1$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{a(1-r^n)}{1-r}$$

$$= \frac{a\left(1-\lim_{n\to\infty} r^n\right)}{1-r}$$

Since $|r|<1$, $\lim_{n\to\infty} r^n = 0$

$$= \frac{a}{1-r} = S_\infty$$

Ex 3I (p. 84)

1(a) $4000 + 9\times 200 = 4000 + 1800 = £5800$ at start of 10^{th} month.

(b) $4000 + 200(m-1)$

3(a) $1+2+3+\ldots+42$

$S_n = \frac{n}{2}(a+l) = \frac{42}{2}(1+42) = 21\times 43 = 903p = £9.03$

(b) Find n such that $S_n = 10000$

$S_n = \frac{n}{2}(2a+(n-1)d)$

$10000 = \frac{n}{2}(2+n-1)$

$n^2+n-20{,}000 = 0$ Round up the +ve root.

6(a) $v(1-0.15)^3 = 11054.25$

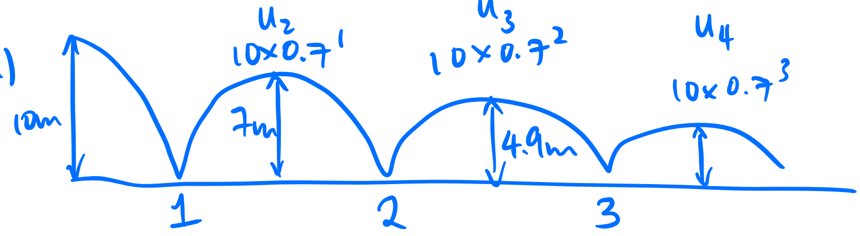
$v = 11054.25/(1-0.15)^3 = £18000$

(b) $18000(1-0.15)^n = 5000$

$n = \log_{0.85}\left(\frac{5000}{18000}\right) = 7.88$

$= 7$ years 11 months

14(a)



4 bounces: $10\times0.7^4 = 2.401\text{m}$

(b) Total vertical dist. travelled up to 6^{th} time hitting floor

$$= 2\times\frac{10(1-0.7^6)}{1-0.7} - 10 = 48.8234$$

Ex 3I

2. $a = £20000$ Reaches max of £25000 after 10 years.
$d = £500$

a) $S_{10} = \frac{10}{2}(2(20000) + (10-1)(500)) = £222500$ ✓

b) $222500 + 5(25000) = £347500$ ✓

c) This model does not account for inflation. ✗ Unlikely her salary will rise the same amount each year

5. Gear 1 : $40 \text{ kmh}^{-1} \longrightarrow a$ or u_1 $40r^{4-1} = 120$

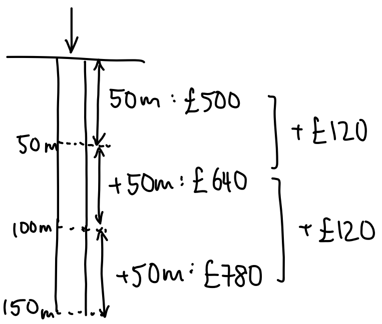
Gear 4 : $120 \text{ kmh}^{-1} \longrightarrow u_4$ $r^3 = 3$

$r = \sqrt[3]{3} \approx 1.44$

$\therefore$ Gear 2 : $40 \times \sqrt[3]{3} = 57.7 \text{ kmh}^{-1}$ ✓

Gear 3 : $40 \times (\sqrt[3]{3})^2 = 83.2 \text{ kmh}^{-1}$ ✓

8. Drilling



$a = u_1 = 500$ $d = 120$
$u_2 = 640$
$u_3 = 780$

$\frac{10}{2}(2(500) + (10-1)(140))$ 140

a) 500m : $\frac{500}{50} = 10$ ✓ $\therefore$ Cost $= S_{10} = \frac{10}{2}(2(500) + (10-1)(120)) = £10400$ ✗ £11300

b) Cost $= S_n = \frac{n}{2}(1000 + (n-1)(120)) < 76000$

140 NOT 120

$1000n + 120n^2 - 120n < 152000$

$120n^2 + 880n - 152000 < 0$

$3n^2 + 22n - 3800 < 0$

If $3n^2 + 22n - 3800 = 0$,

$n = \frac{-22 \pm \sqrt{22^2 + 4(3)(3800)}}{6} = 32.1, -39.4$ $\therefore -39.4 < n < 32.1$

To the nearest 50m : $32 \times 50 = 1600\text{m}$ ✗

1500m

13. 1^{st} square : 1
2^{nd} square : 2
3^{rd} square : $2^2 = 4$
4^{th} square : $2^3 = 8$

Geometric Sequence
$a = 1 \quad r = 2$

$\therefore$ Total grains of corn $= S_{64} = \dfrac{1(1-2^{64})}{1-2} = 2^{64} - 1 = 1.84 \times 10^{19}$

15. $a = 10$ miles $\quad r = 1.1$ (Increase by 10%)

a) $S_n = \dfrac{10(1.1^n - 1)}{1.1 - 1} = 100(1.1^n - 1) = 1000$

$$1.1^n - 1 = 10$$
$$1.1^n = 11$$
$$n = \log_{1.1} 11 = 25.16$$

$\therefore$ 26 days.

b) On the 25^{th} day: $U_{25} = 10 \times 1.1^{24} = 98.5$ miles

On the 26^{th} day: $10000 - S_{25} = 1000 - \dfrac{10(1.1^{25} - 1)}{1.1 - 1} = 16.5$ miles

$\therefore$ Greatest number of miles in 1 day = 98.5 miles on day 25.

MIXED EX 3

20. 2012 population: 25000

$\uparrow$ by 2% each year $\quad \therefore a = 2500 \quad r = 1.02$

a) 2 years later: $25000 \times 1.02^2 = 26010$

b) After n years $\longrightarrow$ population $= 25000 \times 1.02^n > 50000$

$$1.02^n > 2$$
$$n > \log_{1.02} 2 = \frac{\log 2}{\log 1.02}$$

c) $\dfrac{\log 2}{\log 1.02} = 35.003 \quad \therefore$ 35 years from 2012 $\longrightarrow$ During 2047

25000

d) End of 2019: 8 years from start of 2012 $\quad \therefore S_8 = \dfrac{2500(1.02^8 - 1)}{1.02 - 1} = 21457$ appointments.

214574

e) People turn 18 in the middle of the year & people die

Some people are healthy & don't go to the doctor.

22. a) Geometric series $|r|<1$

2^{nd} term $= U_2 = -3 \quad \therefore \quad a = U_1 = \frac{-3}{r}$

$S_\infty = \frac{a}{1-r} = \frac{\frac{-3}{r}}{1-r} = \frac{-3}{r-r^2} = 6.75$

$-3 = 6.75r - 6.75r^2$

$6.75r^2 - 6.75r - 3 = 0$

$27r^2 - 27r - 12 = 0$

b) Solving $27r^2 - 27r - 12 = 0$,

$3r \quad -4$

$3r \quad +1$

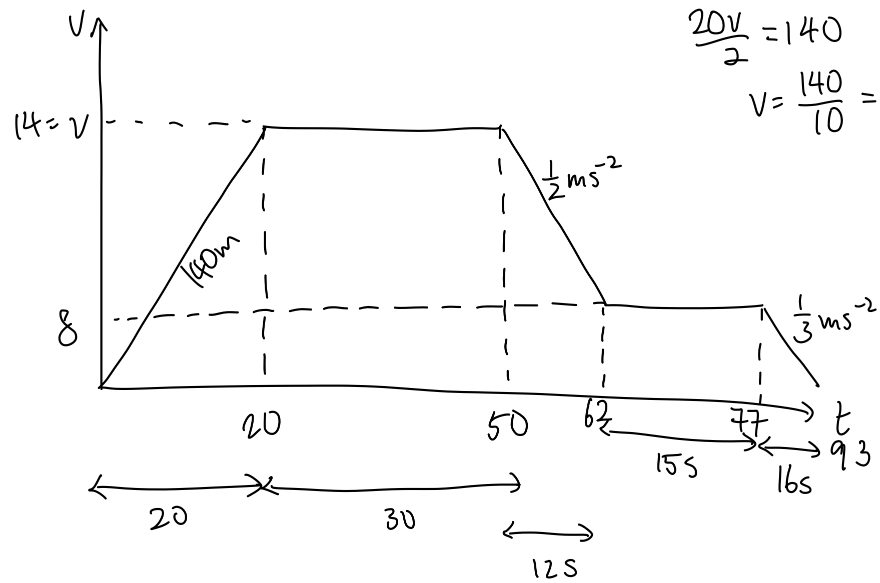
$9r^2 - 9r^2 - 4 = 0$

$(3r-4)(3r+1) = 0$

$r = -\frac{1}{3}, \frac{4}{3}$ rej. since $|\frac{4}{3}| > 1$

$\therefore r = -\frac{1}{3}$

c) $a = \frac{-3}{-\frac{1}{3}} = 9 \qquad S_5 = \frac{9(1-(-\frac{1}{3})^5)}{1-(-\frac{1}{3})} = 6.78$



$\frac{20v}{2} = 140$

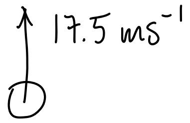
$v = \frac{140}{10} = 14ms^{-1}$

$v = u + at$

$8 = 14 + (-\frac{1}{2})t$

$t = 12s$

$0 = 8 - \frac{1}{2}t$



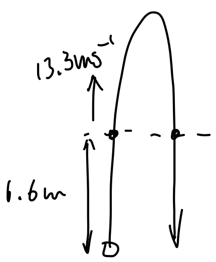
max height?

$v^2 = u^2 + 2as$

$0 = 17.5^2 + 2(-9.81)\,s$

$s = \dfrac{17.5^2}{2 \times 9.81} = 15.60907238 \text{ m}$

velocity at 6.6 m



$v = \sqrt{u^2 + 2as} = \sqrt{17.5^2 + 2(-9.81)(6.6)}$

$= 13.29503667 \text{ ms}^{-1}$

$s = ut + \frac{1}{2}at^2$

$0 = 13.295\ldots t + \frac{1}{2}(-9.81)t^2$

$t\left(\frac{1}{2}(-9.81)t + 13.295\ldots\right) = 0$

$t = 2.7s$