Hypothesis Testing

Poisson Distribution:

$H_0 : \lambda = \lambda_0$ $\quad \lambda_0 \in \mathbb{R}^+$ $\quad$ Observe $X = X^*$ (test statistic)

$H_1 : \lambda < \lambda_0$ $\quad X \sim Po(\lambda_0)$ $\quad$ p-value $= P(X \leq X^*)$ $\quad$ sign direction must match H_1

(null distribution)

If we observe $X \in \{X \leq X^*\}$, this casts doubt on the truth of H_0 ← If p-value is "small", then under the assumption of H_0, observing $X \leq X^*$ is highly unlikely.

Ex (4A)

(1) $H_0 : \lambda = 8$ $\quad X \sim Po(8)$ — null dist$^{\underline{n}}$ $\quad$ significance of 5%

$H_1 : \lambda < 8$ $\quad$ We observe $X = 3$ $\quad$ p-value $= P(X \leq 3) = 0.0424 < 0.05$

$\therefore$ Sufficient evidence to suggest that H_0 is false

(7) Coffee machine seizes 0.2 times a week.

After new coffee beans, the machine seizes 3 times in 5 weeks.

$H_0 : \lambda = 1$ $\quad X \sim Po(1)$ - null dist$^{\underline{n}}$ $\quad$ p-value $= P(X \geq 3)$

$H_1 : \lambda > 1$ $\quad X$ observed to be 3 $\quad = P(X \leq 2)$

(X = # of seizes per 5 weeks) $\quad = 0.0803 \nless 0.05$

$\therefore$ Fail to reject H_0

(14) 1000 components, 1% defective $\quad X \sim B(1000, 0.01)$

n large, p small $\longrightarrow X \dot{\sim} Po(1000 \times 0.01 = 10)$

c) $H_0 : \lambda = 10$ $\quad X$ observed to be 5

$H_1 : \lambda < 10$ $\quad$ p-value $= P(X \leq 5) = 0.0671 \nless 0.05$

$\therefore$ Insufficient evidence to reject H_0

Finding the CRITICAL REGION of the Poisson Distribution

Ex (4B)

(2) A fisherman catchs fish at 5 per hour (records fish in a 2-hour period)

Does new equipment improve fish-catching?

$H_0: \lambda = 10$ Null $dist^{n}$ $X \sim Po(10)$

$H_1: \lambda > 10$ Find smallest a s.t. $P(X \geq a) < 0.05$

find a to satisfy the inequality. The critical region will be $\{X \geq a\}$

$1 - P(X \leq a-1) < 0.05$

$P(X \leq a-1) > 0.95$

Use calculator

Poisson CD → "list"

$\lambda = 10$

x	P
10	
11	
12	
13	
⋮	⋮

$P(X \leq 14) = 0.9165$ $P(X \leq 15) = 0.9512$

$a - 1 = 15$

$a = 16$ ∴ critical region is $\{X \geq 16\}$

(7) c) $H_0: \lambda = 9.5$ Significance level of 5%

$H_1: \lambda \neq 9.5$ each tail as close as possible to 2.5%

"as close as possible"

which one is closer?

0.025

$P(X \leq L)$ as close to 0.025 as possible

$P(X \leq 3) = 0.0148$ $P(X \leq 4) = 0.0402$

closer to 0.025

$L = 3$

$P(X \leq U-1)$ as close to 0.975 as possible

$P(X \leq 15) = 0.9665$ $P(X \leq 16) = 0.9822$

closer to 0.025

$U = 17$

Critical region is

$\{X \leq 3\} \cup \{X \geq 17\}$

Actual significance level $= \underbrace{0.0148 + (1 - 0.9822)}_{\text{just add probabilities of each tail.}} = 0.0326$

Hypothesis testing for Geometric Distribution.

$H_0: p = p_0 \longrightarrow H_0: E[X] = \frac{1}{p} = \frac{1}{p_0}$ $\quad X \sim Geo(p_0)$ Null dist$^{\underline{n}}$

$H_1: p < p_0 \longrightarrow H_1: E[X] = \frac{1}{p} > \frac{1}{p_0}$

Observe $X = X^*$ $\quad$ p-value $= P(X \geq X^*) = (1-p_0)^{X^*-1}$ $\rightarrow$ compare with significance.

matching direction (opposite direction of p)

Ex (4C)

(1) $H_0: p = 0.25 \longrightarrow H_0: E[X] = 4$ $\quad X \sim Geo(0.25)$ Null dist$^{\underline{n}}$

$H_1: p < 0.25 \qquad H_1: E[X] > 4$ $\quad$ X observed to be 9

p-value $= P(X \geq 9) = 0.75^8 = 0.1001 \nless 0.05$

$\therefore$ fail to reject H_0 at significance level of 5%

(5) $H_0: p = 0.02 \longrightarrow H_0: E[X] = 50$ $\quad X \sim Geo(0.02)$ Null dist$^{\underline{n}}$

$H_1: p > 0.02 \qquad H_1: E[X] < 50$ $\quad$ X observed to be 2

$P(X \leq 2) = 1 - P(X \geq 3) = 1 - (1-0.02)^2 = 0.0396 < 0.05$

$\therefore$ Enough evidence to reject H_0 at significance level of 5%

(11) Shoot penalties until scored $\quad$ attempts $= X \sim Geo(0.3)$ Null dist$^{\underline{n}}$

$H_0: p = 0.3 \longrightarrow H_0: E[X] = 10/3 \simeq 3.33$ $\quad$ Takes 10 tries to score

$H_1: p < 0.3 \qquad H_0: E[X] > 10/3 \simeq 3.33$ $\quad \hookrightarrow$ X observed to be 10

p-value $= P(X \geq 10) = (1-0.3)^9 = 0.0404 < 0.05$

$\therefore$ Enough evidence to reject H_0 at significance level of 5%

Critical Regions of Geometric Dist^{n}

Ex (4D)

(1) $H_0: p = 0.3$ $X \sim Geo(p=0.3)$ Critical region: $P(X \geq a) = (1-0.3)^{a-1} < 0.05$

$H_1: p < 0.3$

$(a-1)\log 0.7 < \log 0.05$

reverse sign when dividing both sides by $\log 0.7$ since $\log 0.7 < 0$

$$a > 1 + \frac{\log 0.05}{\log 0.7} = 9.4$$

$\therefore$ Critical region is $\{X \geq 10\}$

Actual significance = $P(X \geq 10) = 0.7^9 = 0.0404$ or 4.04%

(6) Medical condition: 60% to tremor every day.

Medicine will lower the chance to tremor.

$H_0: p = 0.6$ X = days until 1st tremor $X \sim Geo(p=0.6)$ Null dist^{n}

$H_1: p < 0.6$ Critical region: $P(X \geq a) = 0.4^{a-1} < 0.05$

$(a-1)\log 0.4 < \log 0.05$ $\therefore$ Critical region = $\{X \geq 5\}$

$a > 1 + \dfrac{\log 0.05}{\log 0.4} = 4.27$ (If 5 days tremor-free, medicine works)