Indefinite Integration: find y given $\frac{dy}{dx}$ or $f(x)$ given $f'(x)$

opposite of differentiation

Definite Integrals: find area under graph

find area between curves

$f(x)$ $\quad$ $f'(x)$

$5x^3$ → multiply by power → reduce power by 1 → $15x^2$

divide by new power ← increase power by 1

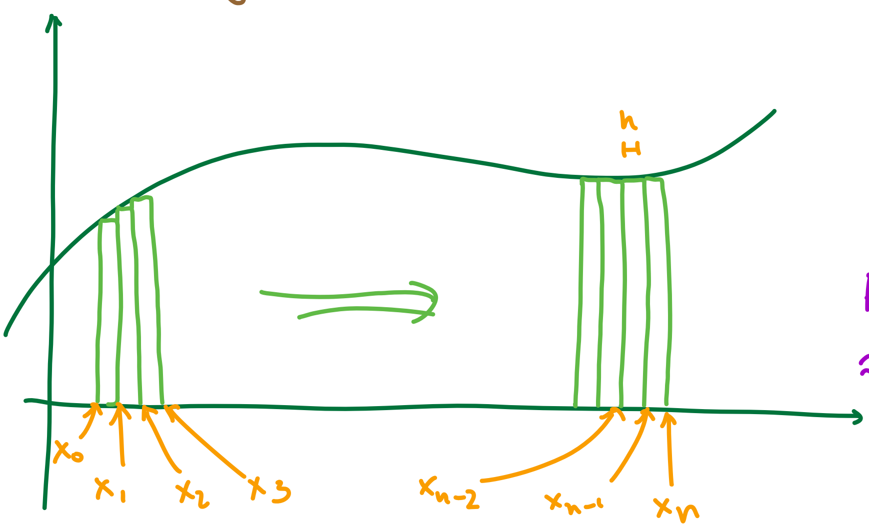
⚠ When integrating, you need to $+c$! (Constants disappear when differentiating)

Ex 13A (p.289)

1 a) $\int x^5\,dx = \frac{1}{6}x^6 + C$ $\quad$ c) $\int -x^{-2}\,dx = x^{-1} + C = \frac{1}{x} + C$ $\quad$ e) $\int x^{\frac{2}{3}}\,dx = \frac{3}{5}x^{\frac{5}{3}} + C$

g) $\int -2x^6\,dx = -\frac{2}{7}x^7 + C$ $\quad$ i) $\int 5x^{-\frac{3}{2}}\,dx = -10x^{-\frac{1}{2}} + C$ $\quad$ k) $\int 36x^{11}\,dx = 3x^{12} + c$

Definite integrals



$h = x_i - x_{i-1}$ where $i = 1, 2, 3, \ldots, n$

ie width

Area under curve

$\approx f(x_0)(x_1 - x_0) + f(x_1)(x_2 - x_1) + \ldots$

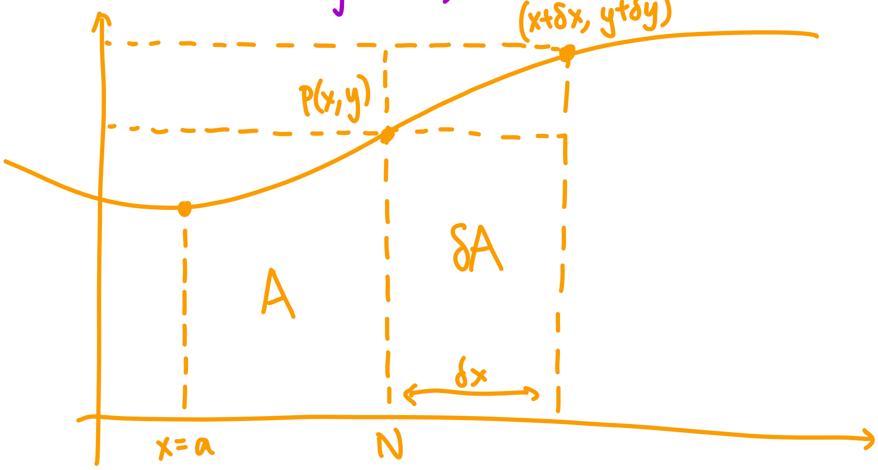
$= \sum_{i=1}^{n-1} f(x_i)(x_i - x_{i-1})$

Notation: $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n-1} f(x_i)(x_i - x_{i-1})$ or $\lim_{n\to\infty} \sum_{i=1}^{n-1} f(x_i)\cdot h$

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

no constant of integration needed

More! (Out of syllabus)



$$y\delta x \le \delta A \le \delta x(y+\delta y)$$

$$y \le \frac{\delta A}{\delta x} \le y+\delta y$$

Recall: $\frac{\delta f}{\delta x} = \lim_{\delta \to 0} \frac{f(x+\delta)-f(x)}{(x+\delta)-x}$

$= \lim_{\delta x \to 0} \frac{\delta f}{\delta x}$

$$\lim_{\delta x \to 0} y \le \lim_{\delta x \to 0} \frac{\delta A}{\delta x} \le \lim_{\delta x \to 0} y+\delta y$$

limit does nothing $\rightarrow$ $y \le \frac{\delta A}{\delta x} \le y$

$\frac{\delta f}{\delta x} = \lim_{\delta x \to 0} \frac{\delta f}{\delta x}$

when $\delta x \to 0$, $\delta y \to 0$

$$\frac{\delta A}{\delta x} = y$$

$$\int \frac{\delta A}{\delta x} = \int y$$

$$\int \delta A = \int y \, dx$$

$$A = \int y \, dx$$

Challenge (p.295)

① A set of curves, where all curves pass through origin

$\uparrow$

$f_1(x), f_2(x), f_3(x), \ldots$ where $f_n'(x) = f_{n-1}(x)$ and $f_1(x) = x^2$

a) $f_2(x) = \int f_1(x)dx = \int x^2 dx = \frac{1}{3}x^3 + C = \frac{1}{3}x^3$ (C=0 when pass through origin)

$f_3(x) = \int f_2(x)\,dx = \int \frac{1}{3}x^3\,dx = \frac{1}{12}x^4 + C = \frac{1}{12}x^4$

b) $f_n(x) = \frac{(n+1)!}{2} x^{n+1}$

② A set of curves, where all curves pass through (0,1)

$\uparrow$

$f_1(x), f_2(x), f_3(x), \ldots$ where $f_n'(x) = f_{n-1}(x)$ and $f_1(x) = 1$

$f_2(x) = \int f_1(x)dx = \int 1\,dx = x + C = x+1$

$f_3(x) = \int x+1\,dx = \frac{1}{2}x^2 + x + 1$

$f_4(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + 1$

$f_n(x) = \sum_{i=0}^{n-1} \frac{1}{i!}x^i$

More on Definite Integrals (§ 13.4)

Definite integrals produce a value

Indefinite integrals produce a function

Steps: $\int 3x^2\,dx = x^3 + C$

$\Downarrow \qquad \downarrow$

difference of the 2 values

$\int_1^2 3x^2\,dx = [x^3]_1^2 = (2)^3 - (1)^3 = 7$

① Evaluate indefinite integral without C into $[\ldots]_a^b$

② Evaluate $f(b) - f(a)$

Ex13D (p.297)

① a) $\int_2^5 x^3\,dx = \left[\frac{1}{4}x^4\right]_2^5 = \frac{625}{4} - 4 = 152.25$ or $\frac{609}{4}$

b) $\int_1^3 x^4\,dx = \left[\frac{1}{5}x^5\right]_1^3 = \frac{243}{5} - \frac{1}{5} = \frac{242}{5}$

} Simple!

⑧ train velocity $= v = 20 + 5t$, $0 \le t \le 10$

distance $= s = \int_0^{10} (20 + 5t)\,dt = \left[20t + \frac{5}{2}t^2\right]_0^{10} = 200 + 250 = 450$ m

} Modelling $\int v\,dt = s$

Challenge (p.297)

$$\int_k^{3k} \frac{3x+2}{8}\,dx = \frac{1}{8}\int_k^{3k} 3x+2\,dx = \frac{1}{8}\left[\frac{3}{2}x^2 + 2x\right]_k^{3k} = \frac{1}{8}\left(\frac{27}{2}k^2 + 6k - \frac{3}{2}k^2 - 2k\right)$$

$$= \frac{1}{8}(12k^2 + 4k) = \frac{1}{2}(3k^2 + k) = 7$$

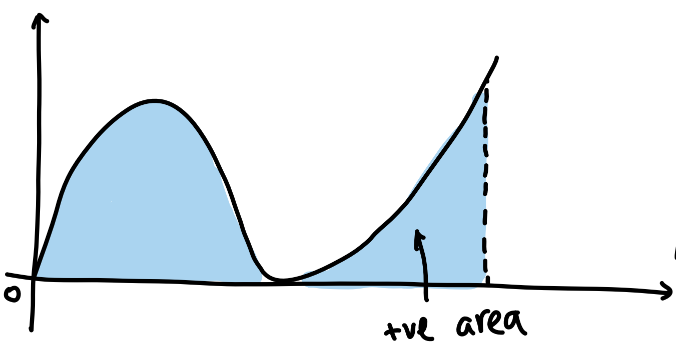
$$3k^2 + k - 14 = 0$$

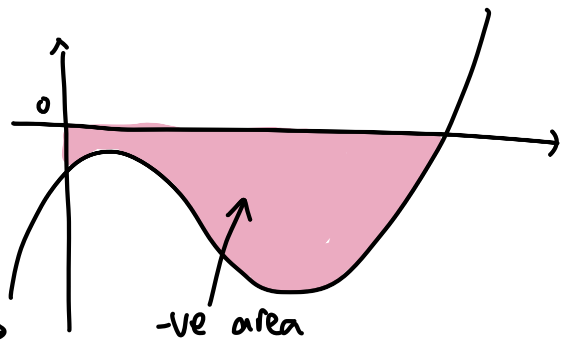
$$\begin{pmatrix} 3k & +7 \\ k & -2 \end{pmatrix}$$

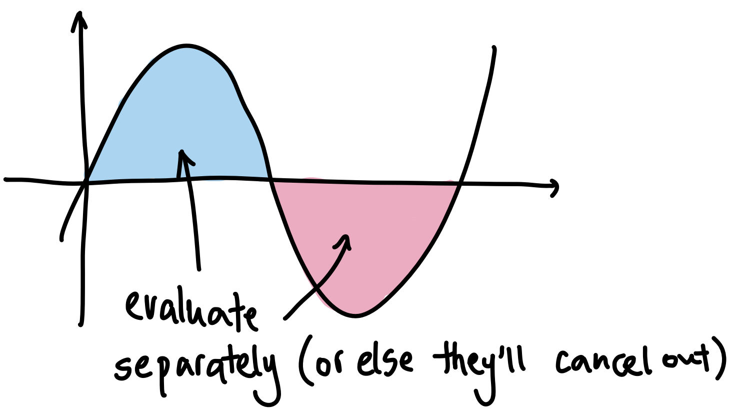
$k = 2, -\frac{7}{3}$ ← rej. (given $k > 0$)

$= 2$

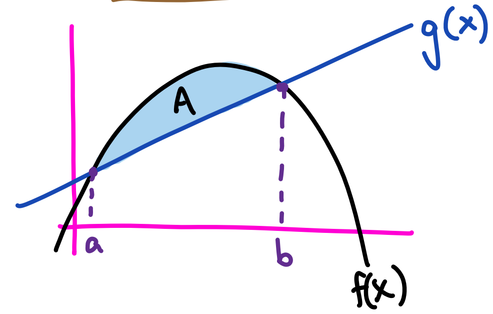
Find area between curve & x-axis:







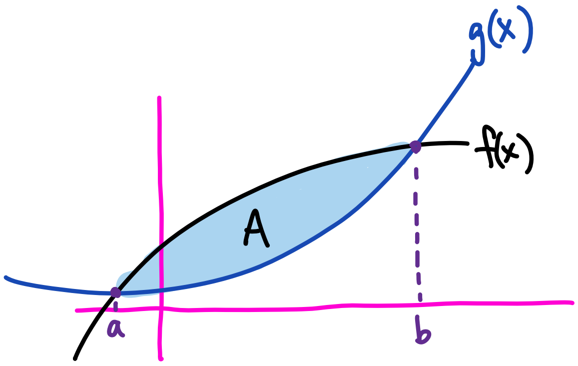
Area between curves



$$\text{Area } A = \int_a^b f(x)\,dx - \text{trapezium}$$

$$\underbrace{= \int_a^b f(x)\,dx - \frac{(f(a)+f(b))(b-a)}{2}}_{\text{only if } g(x) \text{ is a st. line}}$$

OR



$$\text{Area } A = \overbrace{\int_a^b f(x)\,dx - \int_a^b g(x)\,dx}^{\text{difference in area}}$$

$$= \int_a^b f(x) - g(x)\,dx$$