Algebraic expressions

① Simplifying e.g. $3x^2(2x+1) - 5x^2(3x-4)$

$= 6x^3 + 3x^2 - 15x^3 + 20x^2$

$= -9x^3 + 23x^2$

② Expanding e.g. $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

③ Factorising e.g. $4x^4 - 13x^2 + 9 = (4x^2 - 9)(x^2 - 1)$

$= (2x+3)(2x-3)(x+1)(x-1)$

④ Negative & Fractional Indices

$\mathbb{Z}$ Integer : whole numbers (not fractions/decimals)

Can be +ve of -ve

$\mathbb{Q}$ Rational number : a number that can be expressed as a fraction ($\frac{p}{q}$ where p & q are integers)

⑤ Surds ‼ They are IRRATIONAL

* Any multiple of $\sqrt{n}$, where n is an integer but not square #

i $\frac{3-2\sqrt{5}}{\sqrt{5}-1} \Rightarrow p+q\sqrt{5}$ $\quad \frac{3\sqrt{5}-10+3-2\sqrt{5}}{5-1} = -\frac{7}{4} + \frac{1}{4}\sqrt{5}$

ii a) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$

$= a - b$

b) $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \dots + \frac{1}{\sqrt{24}+\sqrt{25}}$

$= \frac{\sqrt{1}-\sqrt{2}+\sqrt{2}-\sqrt{3}+\dots+\sqrt{24}-\sqrt{25}}{-1}$

$= \frac{1-5}{-1} = 4$

P15-17 MIXED EX 1

1. a) y^8 b) $6x^7$ c) $32x$ d) $12b^9$

2. a) $x^2-8x-15$ b) $6x^2+2x-21x-7$
$= 6x^2-19x-7$ c) $6x^2-2xy+4x+15x-5y+10$
$= 6x^2+19x-2xy-5y+10$

3 a) $x(x^2+3x-4)$
$= x^3+3x^2-4x$ b) $(x^2-x-6)(x+7)$
$= x^3+7x^2-x^2-7x-6x-42$
$= x^3+6x^2-13x-42$

c) $(6x^2-2x+9x-3)(x-2) = (6x^2+7x-3)(x-2)$
$= 6x^3-12x^2+7x^2-14x-3x+6 = 6x^3-5x^2-17x+6$

4 a) $15y+12$ b) $15x^2-25x^3+10x^4$ c) $10x^2+15x-2x+6x^2$
$= 16x^2+13x$

d) $3x^2+9x^2-6x^2+4x$
$= 9x^3-3x^2+4x$

5 a) $x(3x+4)$ b) $2y(2y+5)$ c) $x(x+y+y^2)$ d) $2xy(4y+5x)$

6 a) $(x+2)(x+1)$ b) $3x(x+2)$ c) $(x-7)(x+5)$

d) $2x-3$, $x+1$
$(2x-3)(x+1)$

e) $5x+2$, $x-3$
$(5x+2)(x-3)$

f) $6-x$, $1+x$
$(6-x)(1+x)$

7 a) $2x(x^2+3)$ b) $x(x^2-36)$ c) $x(2x^2+7x-15)$

8 a) $3x^6$ b) $4^{1/2} = 2$ c) $6x^2$ d) $\frac{1}{2}x^{-1/3}$

9 a) $(2/3)^2 = \frac{4}{9}$ b) $\left(\sqrt{255}/\sqrt{289}\right)^3 = (15/17)^3 = \frac{3375}{4913}$

10 a) $\frac{3\sqrt{63}}{63} = \frac{\sqrt{63}}{21}$ b) $2\sqrt{5} + 6\sqrt{5} - 4\sqrt{5} = 4\sqrt{5}$

11 a) $35(25)^2 + 2(25) - 48 = 21877$

b) $5x + 6$ $(5x+6)(7x-8)$

$7x - 8$ sub $x=25$: $21877 = (131)(167)$

12 a) $3\sqrt{2} + \sqrt{10}$ b $10 + 2\sqrt{3} - 5\sqrt{5} - \sqrt{15}$ c) $24 - 6\sqrt{7} - 4\sqrt{2} + \sqrt{14}$

13 a) $\frac{\sqrt{3}}{3}$ b) $\frac{\sqrt{2}+1}{1} = \sqrt{2}+1$ c) $\frac{3(\sqrt{3}+2)}{-1} = -3\sqrt{3} - 6$

d) $\frac{23 - 2\sqrt{851} + 37}{23 - 37} = -\frac{60 - 2\sqrt{851}}{14} = -\frac{30 - \sqrt{851}}{7}$

e) $\frac{1}{4 + 4\sqrt{3} + 3} = \frac{1}{7 + 4\sqrt{3}} = \frac{7 - 4\sqrt{3}}{49 - 48} = 7 - 4\sqrt{3}$

f) $\frac{1}{16 - 8\sqrt{7} + 7} = \frac{1}{23 - 8\sqrt{7}} = \frac{23 - 8\sqrt{7}}{81}$

14 a) $(x+3)(x^2 - 4x - 5)$ $\therefore b = -4,\ c = -5$

b) $(x+3)(x-5)(x+1)$

15 a) $y^{1/3} = \frac{1}{4}x$ b) $4y^{-1} = 4/y = 4(64)/x^3 = \frac{256}{x^3}$

16 $\frac{5(\sqrt{75} + \sqrt{50})}{75 - 50} = \frac{5\sqrt{15} + 5\sqrt{10}}{25} = \frac{\sqrt{15} + \sqrt{10}}{5}$

17 $(\sqrt{11} - 5)(5 - \sqrt{11}) = 5\sqrt{11} - 11 - 25 + 5\sqrt{11} = 10\sqrt{11} - 36$

18 $x(1 - 64x^2) = x(1 - 8x)(1 + 8x)$

19 $(3^3)^{2x+1} = 3^{6x+3}$ $\therefore y = 6x+3$

20 $8\sqrt{3} + 6x = 8x$

$2x = 8\sqrt{3}$

$x = 4\sqrt{3}$

21 $x(1+\sqrt{3}) = \sqrt{12}$

$x = \frac{2\sqrt{3}-6}{-2} = -\sqrt{3}+3 = 3-\sqrt{3}$

22 $\frac{4-4\sqrt{x}+x}{\sqrt{x}} = 4x^{-1/2} - 4 + x^{1/2}$

23 a) $3^{11/2}$ $a = \frac{11}{2}$ b) $x+3y = \frac{11}{2}$

$y = \frac{11}{6} - \frac{1}{3}x$

24 $4x^{5/2} + x^2$ $a = \frac{5}{2}$ $b = 2$