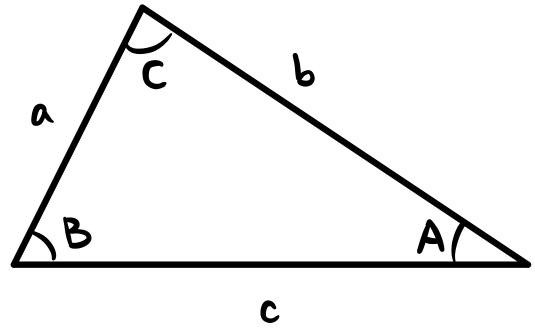
# 9: TRIGONOMETRY

## 9.1 & 9.2: Sine & Cosine Rule

### Sine Rule

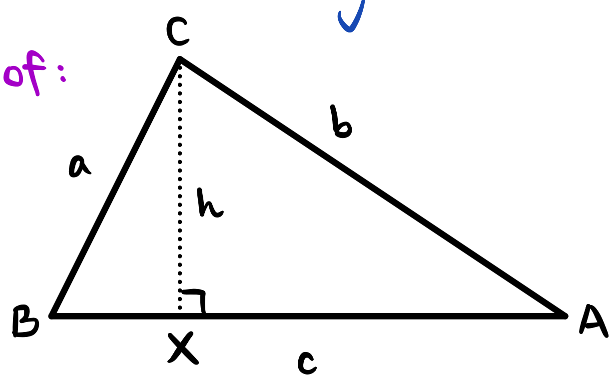
when you know 1 angle and 2 sides
or
2 angles and 1 side

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$




ratios between side & opposite angle are same in triangle

Proof:




$\sin B = \frac{h}{a}$

$h = a\sin B$

$\sin A = \frac{h}{b}$

$h = b\sin A$

Therefore,

$a\sin B = b\sin A$

$$\underline{\frac{\sin A}{a} = \frac{\sin B}{b}}$$

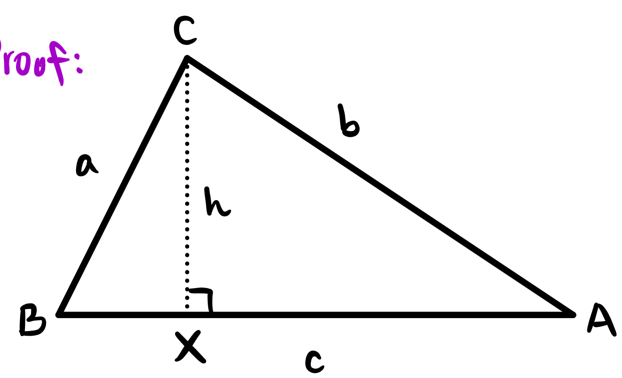
### Cosine Rule

when you know 3 sides

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$

These rules can be applied to <u>ALL</u> triangles

Proof:




$\cos A = \frac{AX}{b}$

$AX = b\cos A$

$BX = c - AX$

$= c - b\cos A$

$\sin A = \frac{h}{b}$

$h = b\sin A$

$a^2 = (BX)^2 + h^2$

$a^2 = c^2 - 2bc\cos A + b^2\cos^2 A + b^2\sin^2 A$

$a^2 = b^2(\sin^2 A + \cos^2 A) + c^2 - 2bc\cos A$

$$\underline{a^2 = b^2 + c^2 - 2bc\cos A}$$

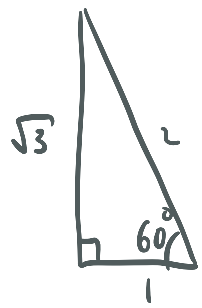
# EX 9A (p. 177)

① a) $(AB)^2 = 6.5^2 + 8.4^2 - 2(6.5)(8.4)\cos 20°$

$AB = \sqrt{42.25 + 70.56 - 109.2\cos 20°}$

$= 3.2$

b) $(BC)^2 = 2^2 + 1^2 + 4\cos 60$

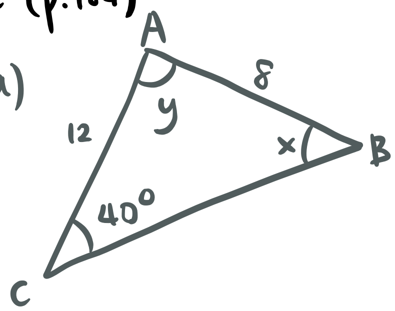



$BC = \sqrt{4+5+2}$

$= 3.3$

# EX 9C (p. 184)

② a)




$\frac{\sin x}{12} = \frac{\sin 40}{8}$

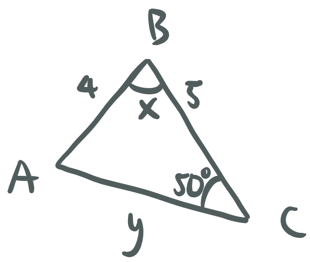
$x = \sin^{-1}(12\sin 40/8)$

$= 74.6°$ or $105.4°$

$\Downarrow$ $\Downarrow$

$y = 65.4°$ or $34.6°$

c)




$\frac{\sin \angle BAC}{5} = \frac{\sin 50}{4}$

$\angle BAC = \sin^{-1}(\frac{5}{4}\sin 50)$

$= 73.2°$ or $106.8°$

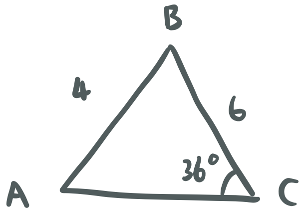
$\Downarrow$

$x = 56.8°$ or $23.2°$

$\frac{y}{\sin 56.8°} = \frac{4}{\sin 50°}$

$y = \frac{4\sin 56.8°}{\sin 50°}$ or $y = \frac{4\sin 23.2°}{\sin 50°}$

(4)




$$\frac{\sin \angle BAC}{6} = \frac{\sin 36}{4}$$

$$\angle BAC = \sin^{-1}\left(\frac{3}{2}\sin 36\right)$$

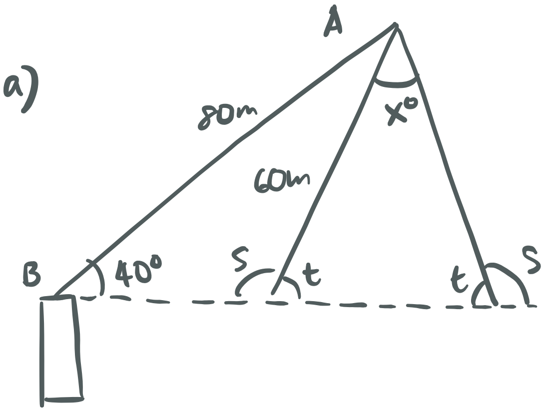
$$= 61.8^\circ \text{ or } 118.2$$


$$\angle ABC = 180 - 36 - 61.8 = 82.2^\circ$$

or

$$= 180 - 36 - 118.2 = 25.8^\circ$$

(6) a)




t is acute, s is obtuse

$$\frac{\sin 40}{60} = \frac{\sin s}{80}$$

$$s = \sin^{-1}\left(\frac{4}{3}\sin 40\right)$$

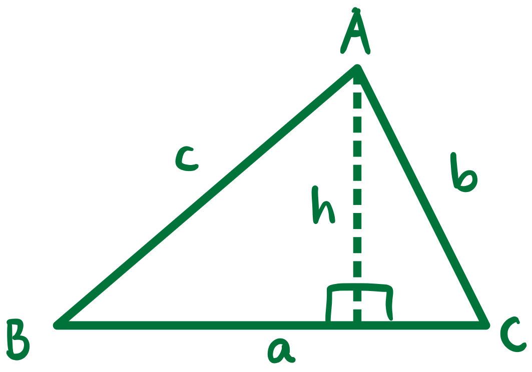
$$= 59.0^\circ \text{ or } 121.0^\circ$$

$$t = 180 - 121 = 59.0^\circ$$

$$x = 180 - 2(59.0) = 62$$

b) we assumed the triangle is isoceles (the swing is perfectly symmetric)

# AREA OF TRIANGLES



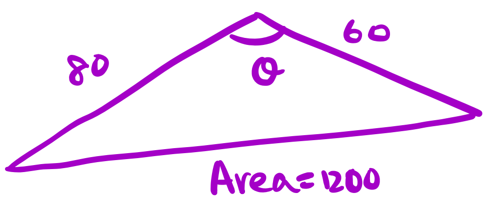

$\text{Area} = \frac{1}{2}ah$ , $\sin C = \frac{h}{b}$

$h = b\sin C$

$$\boxed{\text{Area} = \frac{1}{2}ab\sin C}$$

## Ex 9D (p.186)

3




$\text{Area} = \frac{1}{2}ab\sin C$

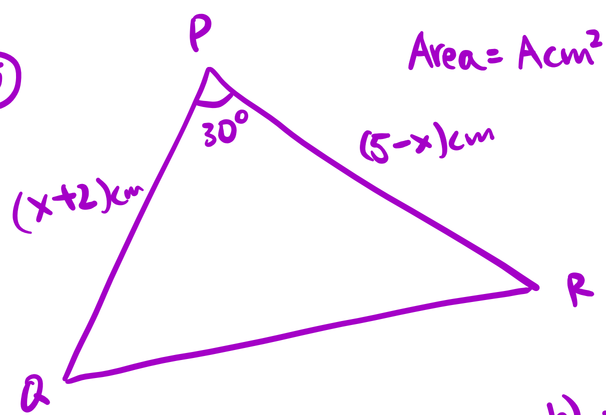
$1200 = \frac{1}{2}(80)(60)\sin\theta$

$\theta = \sin^{-1}\left(\frac{1200}{40\times 60}\right)$

$= 30^\circ$ or $150^\circ$

$c = \sqrt{80^2+60^2-2(80)(60)\cos 150} = 135\text{m}$

5




a) $\text{Area} = \frac{1}{2}ab\sin C$

$A = -\frac{1}{2}(x+2)(x-5)\sin 30$

$= -\frac{1}{4}(x^2-3x-10)$

$= \frac{1}{4}(10+3x-x^2)$

b) $-\frac{1}{4}x^2 + \frac{3}{4}x + \frac{5}{2} = 0$

$-x^2+3x+10=0$

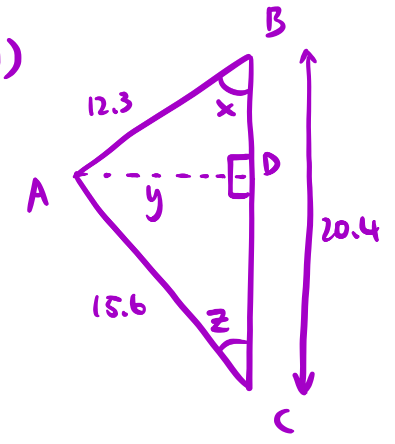
$-(x^2-\frac{3}{2})^2+10+\frac{9}{4}=0$

$(x^2-\frac{3}{2})^2+\frac{49}{4}=0$

$\text{Area} = \frac{49}{4}$ at $x = \frac{3}{2}$

# EX 9E (p.189)

① i)




$15.6^2 = 12.3^2 + 20.4^2 - 2(12.3)(20.4)\cos x$

$x = \cos^{-1}\left(\frac{12.3^2 + 20.4^2 - 15.6^2}{2(12.3)(20.4)}\right)$

$= 49.8^\circ$

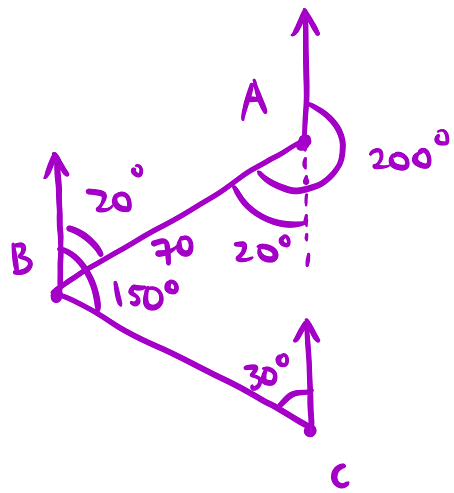
$\sin x = \frac{y}{12.3}$

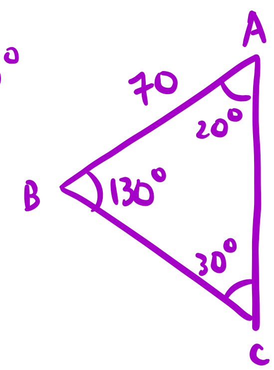
$y = 12.3 \sin 49.8 = 9.39$

$\frac{\sin z}{y} = \frac{\sin 90}{15.6}$

$z = \sin^{-1}\left(\frac{9.39}{15.6}\right) = 37^\circ$

④



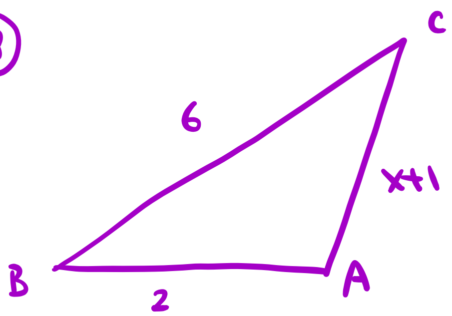




$\frac{AC}{\sin 130} = \frac{70}{\sin 30}$

$AC = \frac{70 \sin 130}{\sin 30}$

$= 107 \text{ km}$

⑧




$\cos \angle ABC = 5/8$

a) $(x+1)^2 = 6^2 + 2^2 - 24(5/8)$

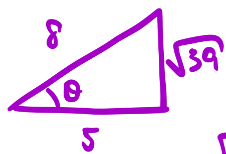
$x^2 + 2x + 1 = 36 + 4 - 15$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x = 4$ (rej. $-6$)

b)

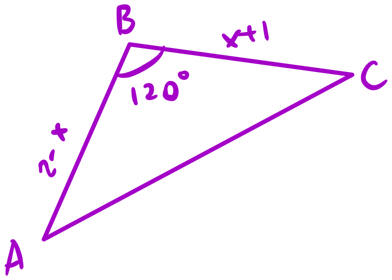



$\Rightarrow \sin \angle ABC = \frac{\sqrt{39}}{8}$

$\text{Area} = \frac{1}{2}(6)(2)\frac{\sqrt{39}}{8}$

$= \frac{3\sqrt{39}}{4}$

(10)




a) $|AC|^2 = 4-4x+x^2+x^2+2x+1+2(x+1)(x-2)\cos 120$

$= 2x^2-2x+5+(2x^2-2x-4)\cos 120$

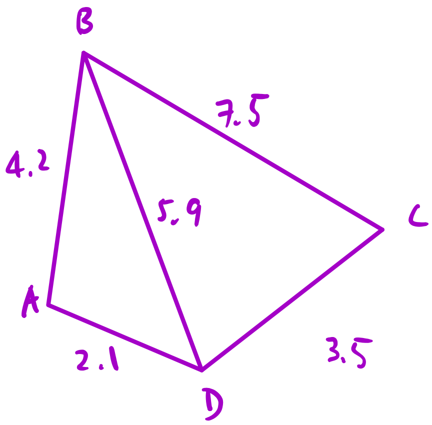
$= 2x^2-2x+5-x^2+x+2$

$= x^2-x+7$

b) $x^2-x+7 = (x-\frac{1}{2})^2+7-\frac{1}{4} = (x-\frac{1}{2})+\frac{29}{4}$

$\therefore$ minimised when $x = \frac{1}{2}$

(14)




a) $\angle DAB$ $\quad 5.9^2 = 4.2^2+2.1^2-2(4.2)(2.1)\cos(\angle DAB)$

$4.2^2\cos(\angle DAB) = 4.2^2+2.1^2-5.9^2$

$\angle DAB = \cos^{-1}\left(\frac{4.2^2+2.1^2-5.9^2}{4.2^2}\right)$

$= 136.3^\circ$

$\angle BCD$ $\quad 5.9^2 = 3.5^2+7.5^2-2(3.5)(7.5)\cos(\angle BCD)$

$\cos(\angle BCD) = \frac{3.5^2+7.5^2-5.9^2}{2(3.5)(7.5)}$

$\angle BCD = \cos^{-1}\left(\frac{3.5^2+7.5^2-5.9^2}{2(3.5)(7.5)}\right)$

$= 50.1^\circ$

b) $\Delta ABD$ Area $= \frac{1}{2}(4.2)(2.1)\sin 136.3$

$= 3.05 m^2$

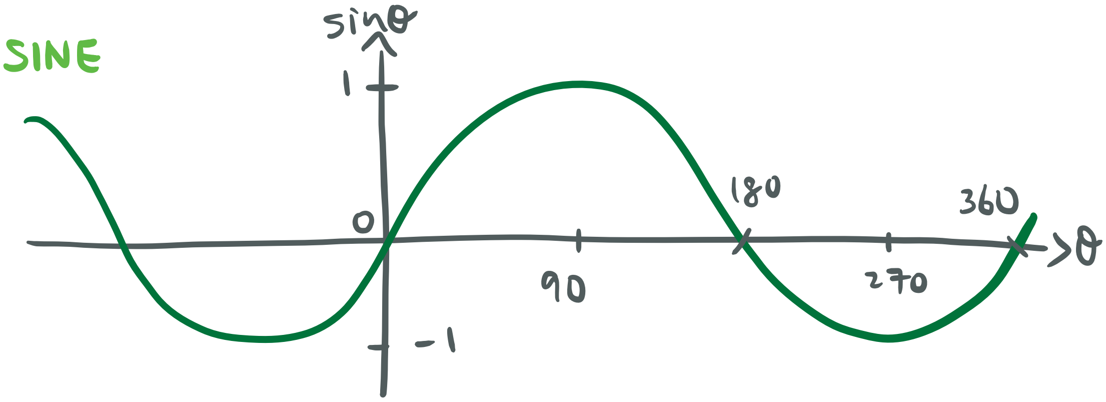
$\Delta BDC$ Area $= \frac{1}{2}(7.5)(3.5)\sin 50.1$

$= 10.07 m^2$

Area $= 10.07+3.05 = 13.12 m^2$

# Trigonometric Graphs

## • SINE




$\sin\theta$ max $= 1$

min $= -1$

period $= 360$ ie $\sin\theta = \sin(\theta + 360k)$, $k \in \mathbb{Z}$

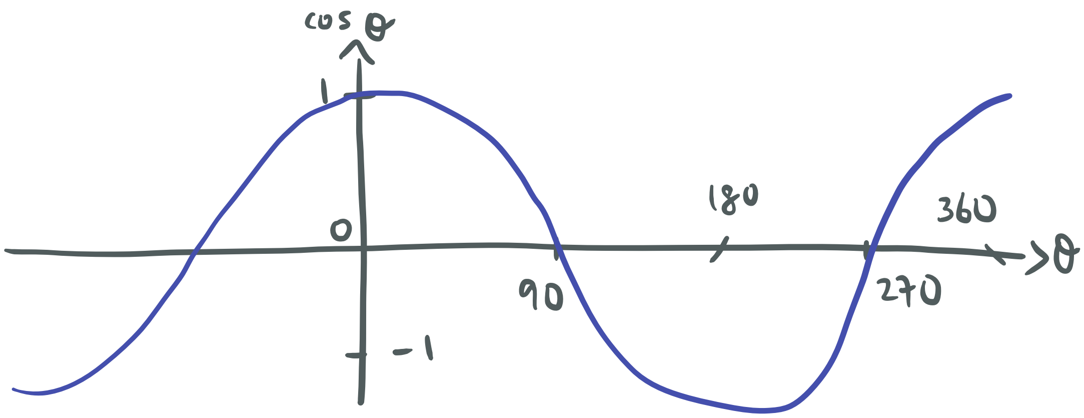
$\sin\theta$ is an odd function $(f(-x) = -f(x))$

$\sin(180-\theta) = \sin\theta$ $\quad$ $\sin(90-\theta) = \cos\theta$

$\sin(360-\theta) = -\sin\theta$ $\quad$ $\sin(90+\theta) = \cos\theta$

use this to find all solutions to $\sin^{-1}$

## • COSINE




$\cos\theta$ max $= 1$

min $= -1$

period $= 360$

$\cos\theta$ is an even function $(f(-x) = f(x))$

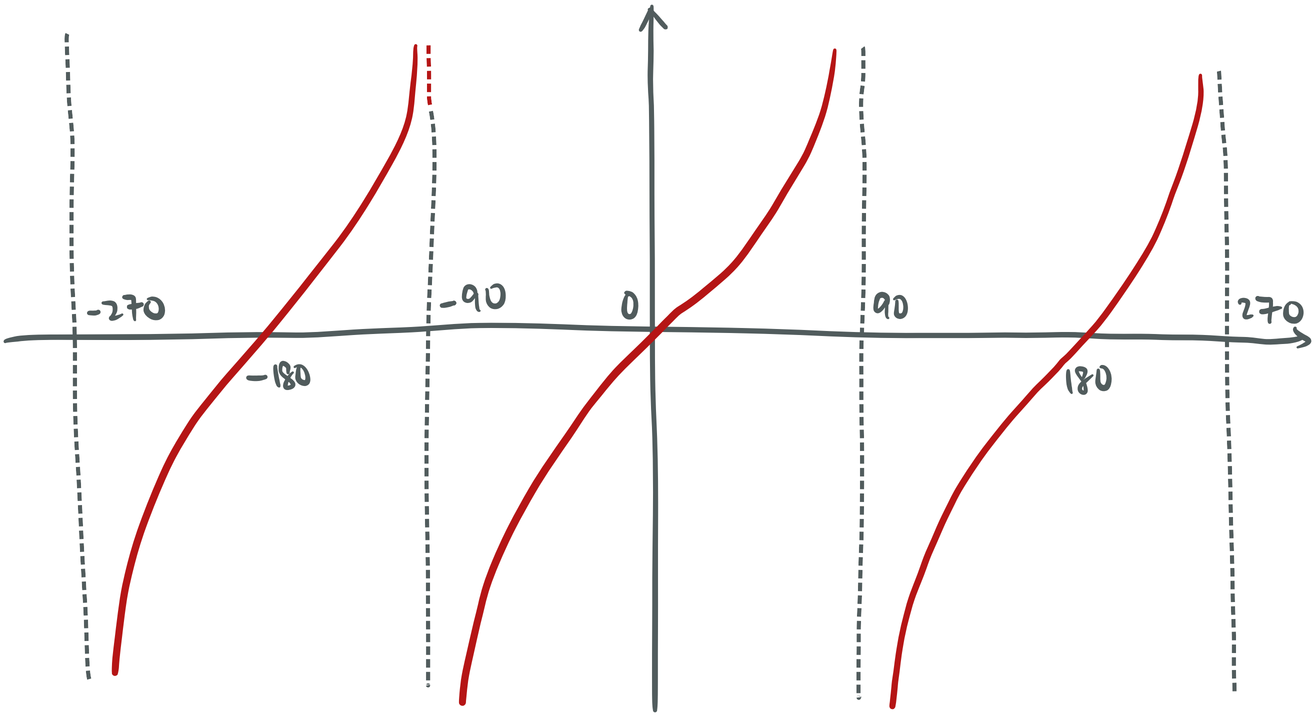
$\cos(360-\theta) = \cos\theta$ $\quad$ $\cos(90-\theta) = \sin\theta$

$\cos(180-\theta) = -\cos\theta$ $\quad$ $\cos(90+\theta) = -\sin\theta$

use this to find all solutions to $\cos^{-1}$

- **TANGENT** $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\because \cos\theta = 0$ for $\theta = 180n + 90$, $n \in \mathbb{Z}$ $\therefore \tan\theta$ is undefined when $\theta = 180n+90$




$\tan\theta$ no max
no min
period $= 180°$

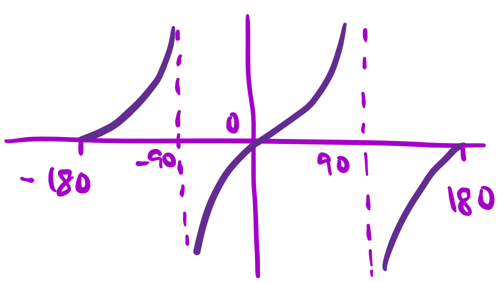
$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta$ $\therefore$ tan is an odd function

$\tan(360-\theta) = -\tan\theta$

$\tan(180-\theta) = -\tan\theta$

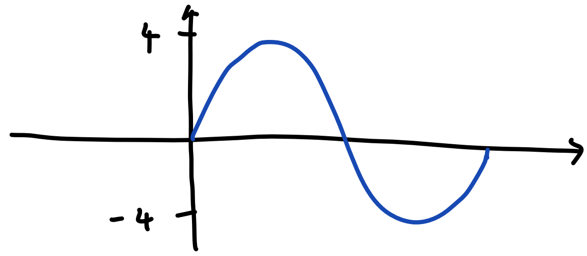
Ex 9F (p.194)

2. $y = \tan\theta$ $-180 \leq \theta \leq 180$



# Exercise 9G (p. 197)

① b) $4\sin x$

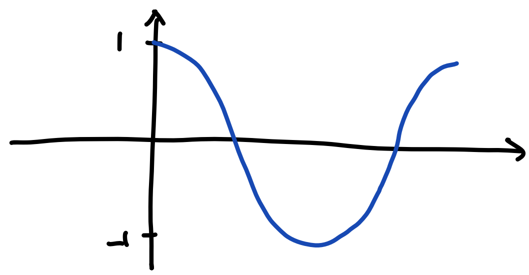



$\sin x \longmapsto 4\sin x$

scale factor of 4

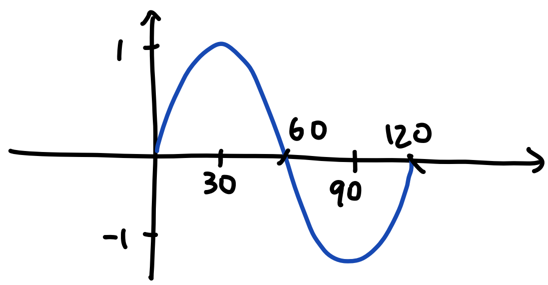
max=4 at $x=90°$,
min=-4 at $x=270°$

c) $\cos(-x)$
since cosine is an even func,
$\cos(-x) = \cos x$




max=1 at $x=0, 360°$
min=-1 at $x=180°$

f) $\sin(3x)$

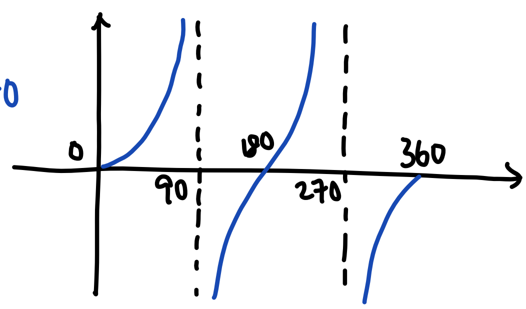


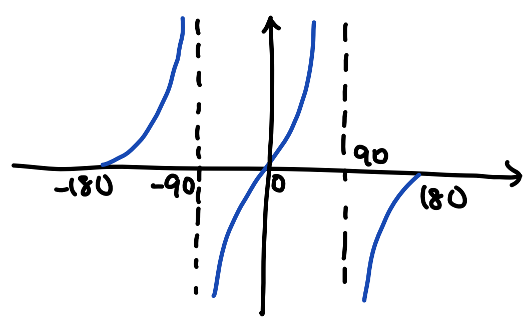

$\sin x \longmapsto \sin(3x)$
horizontal compression of 1/3

max = 1 at $x=30°$
min = -1 at $x=90°$

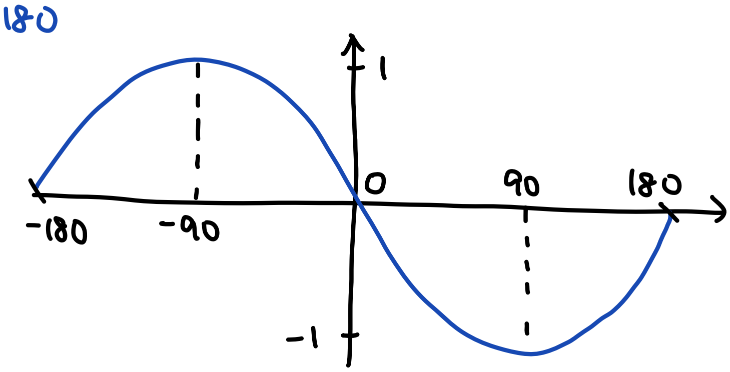
④ b) sketch $\tan\theta$ $0 \le \theta \le 360$ $\tan(\theta+180)$ $-180 \le \theta \le 180$

$y=\tan(\theta+180)$ $-180 \le \theta \le 180$
horizontal translation
180 to the left

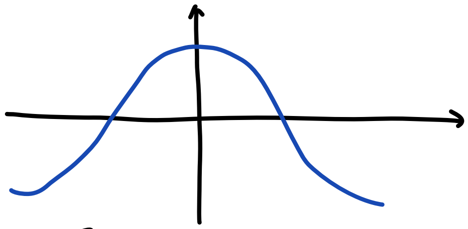



d) $y=\sin(-\theta)$ $-180 \le \theta \le 180$
reflection
in y axis



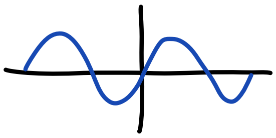
⑥ a) i) $\underbrace{\cos\theta = \cos(-\theta)}_{\text{cos is an even function}}$



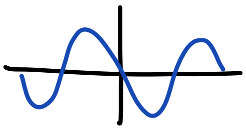
SYMMETRICAL

ii) $\sin\theta = -\sin(-\theta)$

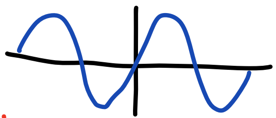
$\sin\theta \longmapsto \sin(-\theta) \longmapsto -\sin(-\theta)$



reflect in y-axis

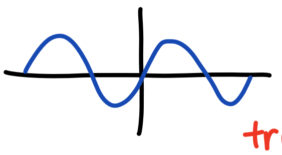


reflect in x-axis

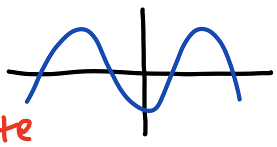


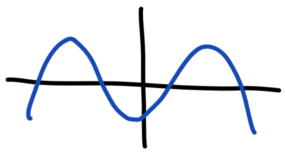
iii) $\sin(\theta - 90) = -\cos\theta$

$\sin\theta \longmapsto \sin(\theta - 90)$ $\quad -\cos\theta$



translate 90 to right



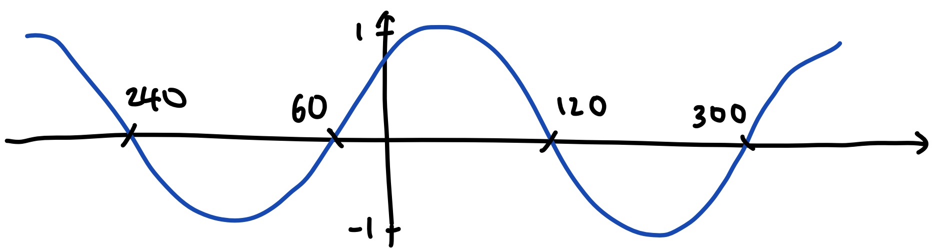


b) $\sin(90-\theta) = \sin(-(\theta-90)) = -\sin(\theta-90) = -(-\cos\theta) = \cos\theta$

c) Given $\cos(\theta - 90) = \sin\theta$,

$\cos(90-\theta) = \cos(-(\theta-90)) = \cos(\theta-90) = \sin\theta$

⑧



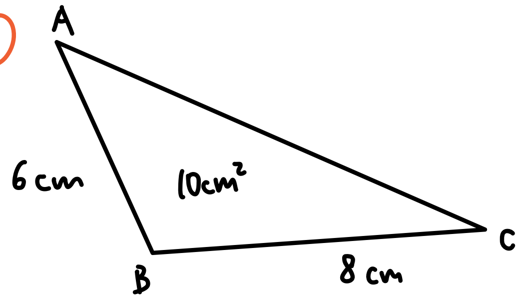

$y = \sin(x+k)$, where k is constant

a) 1 possible value: $k = 60$

b) there are infinite possibilities because the graph repeats (has period of 360°)

# MIXED EXERCISE 9 (p.198)

1




a) Area $= \frac{1}{2} ac \sin B$

$10 = \frac{1}{2}(6)(8)\sin B$

$B = \sin^{-1}(10/(3\times8))$

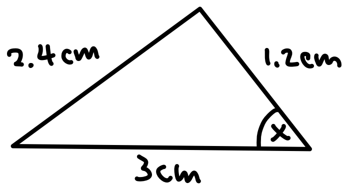
$= 24.6°$ or $\underline{155°}$ $(\angle ABC > 90)$

b) $|AC|^2 = 6^2 + 8^2 - 2(6)(8)\cos(155)$

$|AC| = \sqrt{36 + 64 - 96\cos 155}$

$= 13.7\text{cm}$

2 a)




a) $2.4^2 = 1.2^2 + 3^2 - 2(1.2)(3)\cos x$

$x = \cos^{-1}\left(\frac{1.2^2 + 3^2 - 2.4^2}{2\times1.2\times3}\right)$

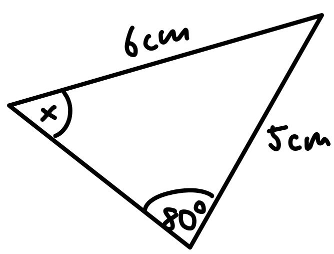
$= 49.5°$

b) Area $= \frac{1}{2} ab\sin C$

$= \frac{1}{2}(3)(1.2)\sin 49.5$

$= 1.37 \text{ cm}^2$

b)




a) $\frac{\sin x}{5} = \frac{\sin 80}{6}$

$x = \sin^{-1}\left(\frac{5\sin 80}{6}\right)$

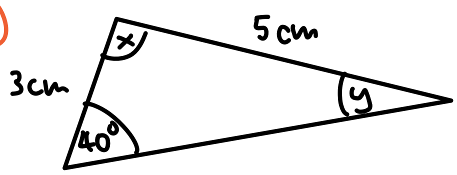
$= 55.2°$

b) Area $= \frac{1}{2} ab\sin C$

$= \frac{1}{2}(6)(5)\sin(180 - 55.2 - 80)$

$= 10.6 \text{ cm}^2$

c)




a) $\frac{\sin y}{3} = \frac{\sin 40}{5}$

$y = \sin^{-1}\left(\frac{3\sin 40}{5}\right)$

$= 22.7°$

$x = 180 - 40 - 22.7$

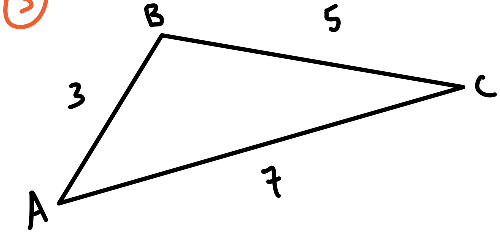
$= 117°$

b) Area $= \frac{1}{2} ab\sin C$

$= \frac{1}{2}(5)(3)\sin 117$

$= 6.66 \text{ cm}^2$

3




$49 = 9 + 25 - 2(15)\cos B$

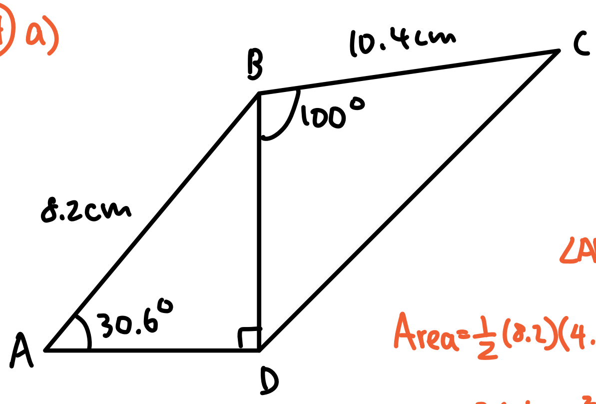
$B = \cos^{-1}\left(\frac{9 + 25 - 49}{30}\right)$

$= 120°$ All other angles are $< 60°$ ($\because \angle$ sum of $\triangle$)

$\therefore$ largest $\angle$ is $120°$

Area $= \frac{1}{2} ab\sin C = \frac{1}{2}(3)(5)\sin 120 = \frac{15\sqrt{3}}{4}$ or $6.50 \text{ cm}^2$

4 a)




$$\frac{|BD|}{\sin 30.6} = \frac{8.2}{\sin 90}$$

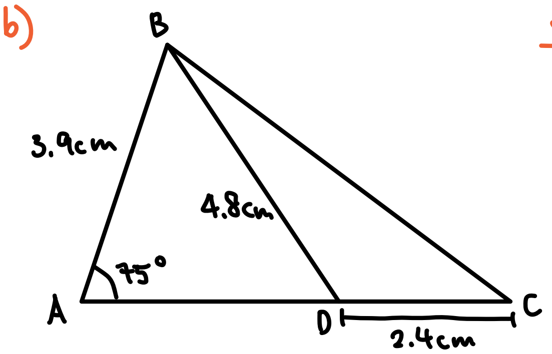
$$|BD| = \frac{8.2\sin 30.6}{\sin 90} = 4.17$$

$$\angle ABD = 180-90-30.6 = 59.4°$$

$$\text{Area} = \tfrac{1}{2}(8.2)(4.17)\sin 59.4 + \tfrac{1}{2}(4.17)(10.4)\sin 100$$

$$= 36.1 \text{ cm}^2$$

b)




$$\frac{\sin\angle ADB}{3.9} = \frac{\sin 75}{4.8}$$

$$\angle ADB = \sin^{-1}\left(\frac{3.9\sin 75}{4.8}\right)$$

$$= 51.7°$$

$$\angle BDC = 180 - 51.7 = 128.3°$$

$$\Delta BCD \text{ area} = \tfrac{1}{2}(4.8)(2.4)\sin 128.3$$

$$= 4.52 \text{ cm}^2$$

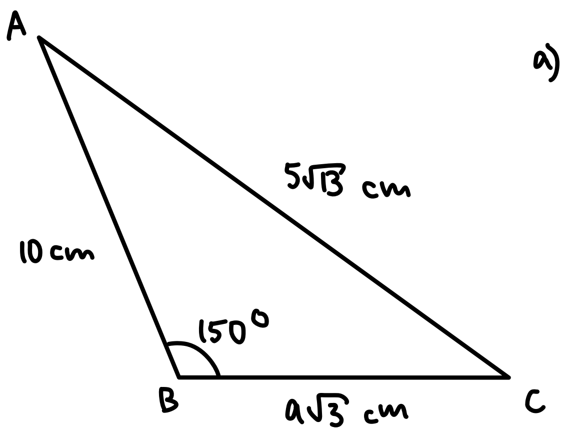
$$\angle ABD = 53.3°$$

$$\Delta ABD \text{ area} = \tfrac{1}{2}(3.9)(4.8)\sin 53.3$$

$$= 7.50 \text{ cm}^2$$

Total area $= 12.0\text{cm}^2$

5




a) $(5\sqrt{13})^2 = 10^2 + (a\sqrt{3})^2 - 2(10)(a\sqrt{3})\cos 150$

$20a\sqrt{3}\cos 150 = 100 + 3a^2 - 325$

$225 = 20a\sqrt{3}\cos 150 + 3a^2$

$3a^2 + (20\sqrt{3}\cos 150)a - 225 = 0$

$a = 15, -5$ reject