Measurements

ERROR / UNCERTAINTY

Random	vs Systematic
e.g. measure something ↓ 17.46, 17.42, 17.44 ↓ fix by taking mean	e.g. cloth tape measure stretched out ↓ check how much its stretched then adjust

Repeatability

Do experiment again and have same results → repeatable

Reproducibility

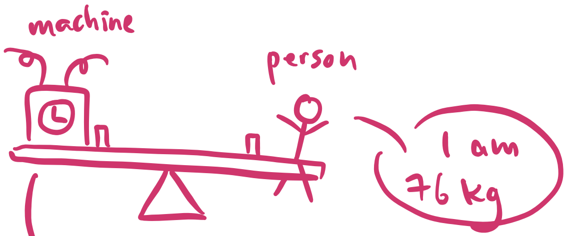
Experiment done through different techniques / done by different people and have same results → reproducible

PRECISION VS ACCURACY

PRECISION	ACCURACY
Values are close to each other	Values are close to the real value

Resolution

smallest change that can be measured with the device (e.g. ruler → mm)



trial 1: 72kg
2: 76 kg
3: 79 kg

is this...

a) Precise? No

b) Repeatable? No

c) Reproducable? No (we need the person)

d) Resolution? 1kg

e) Accurate? Yes

1: 4011 mm
2: 4015 mm
3: 4017mm

is this...

a) Precise? Yes

b) Repeatable? Yes

c) Reproducable? No

d) Resolution? 1mm

e) Accurate? Yes

start up!

1. 25.7s
2. 26.1s
3. 35.2s

SONY says its 30 s

is this...

a) Precise? No

b) Repeatable? Yes

c) Reproducable? Yes

d) Resolution? 0.1s

e) Accurate? Yes

Estimating uncertainty using Significant Figures

Assume uncertainty = ±1 in the last s.f.

e.g. charge of e^- = 1.60×10^{-19} C

uncertainty = $\pm 0.01 \times 10^{-19}$ C

distance = 3.5×10^3 m

uncertainty = $\pm 0.1 \times 10^3$

When measuring with an instrument, uncertainty = ±0.5 of the smallest division

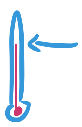
e.g. measuring with thermometer graduations are 1°C apart

uncertainty = ±0.5°C

READING vs MEASUREMENT

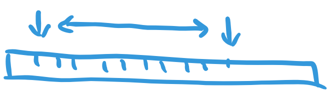
READING
- 1 judgement
↓
1 uncertainty

e.g. thermometer (temp.)



MEASUREMENT
- Start and end points
↓
2 judgements
↓
2 uncertainties

e.g. ruler (distance)



⚠ Temperature change is a MEASUREMENT!

measuring cylinder: 1
protractor: 2
stopwatch: 2
top pan balance: 1

Time measurements taken by a human

Uncertainty = 0.2s (for human reaction time)

∴ Total uncertainty = ±1.0 mm

Significant figures:

99.8 } 3 s.f.
99.9
100.0 } 4 s.f.
100.1

THAT'S A-OK!

ABSOLUTE VS PERCENTAGE UNCERTAINTY

e.g. Light bulb: 5.0 ± 0.4 Ω

Absolute uncertainty = 0.4 Ω

% uncertainty = 8%

e.g. Wire stretching: from 4.3 ± 0.1 cm to 5.5 ± 0.1 cm

sol: add the abs. uncertainties

extension = 5.5 − 4.3 = 1.2 cm

uncertainty = 0.1 + 0.1 = ±0.2 cm

} extension = 1.2 ± 0.2 cm

Multiplying and Dividing Data

sol: Add the % uncertainties

force: 15 ± 3% N

stationary obj mass: 6.0 ± 0.3 kg = 6.0 ± 5% kg

acceleration = $\frac{F}{m} = \frac{15}{6.0} = 2.5\ ms^{-2}$

% uncertainty = 3 + 5 = ± 8%

abs uncertainty = 2.5 × 0.08 = ± 0.2 ms^{-2}

% uncertainty should be quoted to **2 S.F.**

UNCERTAINTY OF REPEATED MEASUREMENTS

1.51 1.67 1.45 1.62 1.59

uncertainty = $\frac{\text{range}}{2} = \frac{1.67 - 1.45}{2} = \pm 0.11$

ONLY APPLICABLE IF UNCERTAINTY FROM MEASUREMENT DEVICE IS SMALLER THAN THE CALCULATED VALUE

ABSOLUTE & % UNCERTAINTY WS

1. Four students measure the same length of string and their results are as follows:

$$l_1 = 38.6\text{cm},\ l_2 = 38.7\text{cm},\ l_3 = 38.6\text{cm},\ l_4 = 38.5\text{cm}$$

What is the average or *mean* measurement?

38.6 cm

What is the *range* of measured values?

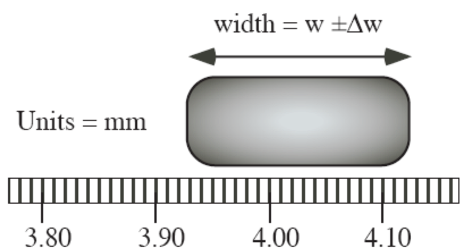
0.2cm

Repeated measurement of the same quantity can improve the overall acceptable value of that measurement. What is the *mean* and its *absolute uncertainty* for the length of string?

mean = 38.6 u = ± 0.1 cm

38.6 ± 0.1 cm

2. What is the *width* and its *absolute uncertainty* of the object being measured in the sketch below?



4.12 - 3.93 = 0.19 mm

u = 2 × 0.005 = 0.01 mm

3. The digital stopwatch was started at a time $t_0 = 0$ and then was used to measure ten swings of a simple pendulum to a time of $t = 17.26s$.

1	7	2	6

If the time for ten swings of the pendulum is 17.26*s* what is the *minimum absolute uncertainty* in this measurement?

± 0.01 s

What is the *time* (period T) *of one complete pendulum swing* and its *absolute uncertainty*?

17.26 / 10 = 1.726 ± 0.001 s

4. Given two masses, $m_1 = (100.0 \pm 0.4)$ g and $m_2 = (49.3 \pm 0.3)$ g, what is their sum, $m_1 + m_2$, and what is their difference, $m_1 - m_2$, both expressed with uncertainties.

sum = 149.3 ± 0.7g

Δ = 50.7 ± 0.7g

5. What is the uncertainty in the calculated area of a circle whose radius is determined to be $r = (14.6 \pm 0.5)$ cm?

$\% U_r = \frac{0.5}{14.6} = 3.42\%$

$\% U_{r^2} = (3.42 \times 2)\% = 6.84\%$

$U_{r^2} = (14.6)^2 \times 0.0684 = 213.16 \pm 14.58\,cm^2$

6. An electrical resistor has a 2% tolerance and is marked $R = 1800\ \Omega$. What is the range of acceptable values that the resistor might have? An electrical current of $I = (2.1 \pm 0.1)$ mA flows through the resistor. What is the *uncertainty* in the calculated *voltage* across the resistor where the voltage is given as $V = IR$?

Range:

$1764\,\Omega \leq x \leq 1836\,\Omega$

$\% U_I = \frac{0.1}{2.1} = 5\%$

$V = IR = (0.0021 \times 1800) = 3.78\,V$

$U = 3.78\,(2+5)\% = \pm 0.26\,V$

7. An accelerating object has an initial speed of $u = (12.4 \pm 0.1)$ ms^{-1} and a final speed of $v = (28.8 \pm 0.2)$ ms^{-1}. The time interval for this change in speed is $\Delta t = (4.2 \pm 0.1)$ s. Acceleration is defined as $a = \frac{v - u}{\Delta t}$. Calculate the *acceleration* and its *uncertainty*

$a = \frac{28.8 - 12.4}{4.2} = 3.9\,ms^{-2}$

uncertainty of $\Delta v = \pm 0.3\,ms^{-1}$

% uncertainty of $\Delta v = 1.83\%$

% uncertainty of $\Delta t = 2.38\%$

uncertainty of $a = 0.16\,ms^{-2}$

8. What are the *volume* and its *uncertainty* for a sphere with a radius of $r = (21 \pm 1)$ mm?

9. Einstein's famous equation relates energy and mass with the square of the speed of light, where $E=mc^2$. What is the *percentage of uncertainty* and the *absolute uncertainty* of the *energy* for a mass m = 1.00 kg where the speed of light is $c = 3.00 \times 10^8$ ms^{-1}?

1. 38.6 cm
   0.2 cm
   38.6±0.1 cm
2. 0.19±0.01 mm
3. 17.26±0.01 s
   1.726±0.001s
4. 149.3±0.7 g
   50.7±0.7 g
5. 670±46 cm^2
6. 1836 – 1764 ohms
   3.8±0.3 V
7. 3.9±0.2 ms^{-2}
8. 39,000±5600 mm^3

Plotting graphs

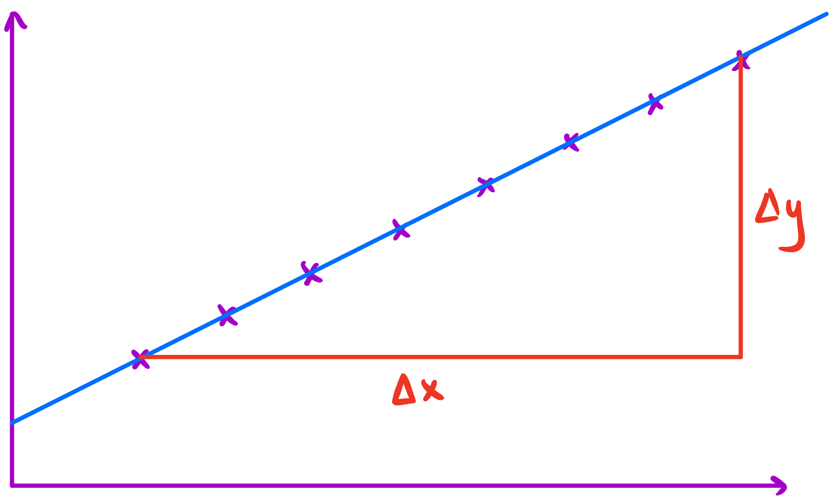
4 KEY POINTS : Scales increment — Increment by 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50 etc

Scales range — Data more than 50% of paper/graph

Point plotting — How accurate the points are

Origin — Does the graph need an origin?
⇓
NOT NEEDED!
JUST ZOOM IN ON DATA

when finding gradient:



DON'T FORGET

you MUST

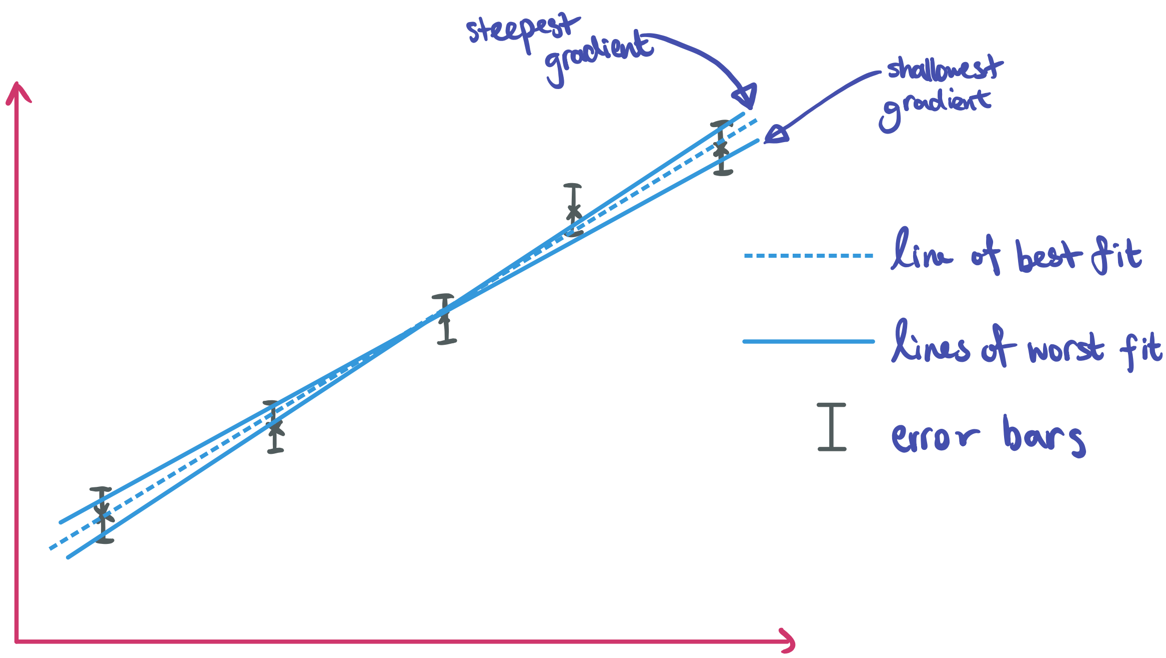
draw this triangle

AND

it must be at least

HALF of the graph

ERROR BARS



Estimation of physical measurements

estimating ≠ guessing!

use prior experience or logic

TIP 1

Large lengths from small lengths

(eg. 1 table = 20 pencils)

TIP 2

Geometric mean

e.g. Mass of a moose?

heavier than a person (10^2 kg)

lighter than a car (10^3 kg)

Geometric mean = $\sqrt{10^2 \times 10^3} = \underline{3 \times 10^2 \text{ kg}}$

pretty close!

out of syllabus! useful, but not needed

TIP 3

1 s.f. is ok for estimations.

Sanity check: does the estimate make sense?

Estimation problems

1. volume of classroom: $10m \times 15m \times 3m = 450\ m^3$
2. area on roof for solar panels: $10m \times 10m = 100\ m^2$
3. thickness of rubber tread worn off per mile: $\frac{5\text{ mm}}{50000\text{ miles}} = 1 \times 10^{-4}$ mm
4. # of people in a field of 100m×50m: $\frac{100m \times 50m}{0.5m \times 0.5m} = 20000$ people
5. total mass of oceans on earth: $510{,}100{,}000\ km^2 \times 70\% \times 3.688\ km \times 100000 \times 100000 \times 100000$
   $= 1 \times 10^{24}$ g $= 1 \times 10^{21}$ kg $= 1 \times 10^{18}$ tonnes $= 1 \times 10^{15}$ Gg $= 1 \times 10^{12}$ Tg
6. mass of atmosphere $= 10^{19}$ kg
   density of atmosphere $= 1\ kg/m^3$
   volume $= 10^{19}\ m^3$
   $= 10^{10}\ km^3$
   $5.101 \times 10^8 \times x = 10^{10}$
   $x = 20$ km
7. £ per person if distribute 1 mole £ among everyone: $\frac{£6.022 \times 10^{23}}{7{,}800{,}000{,}000} = 80{,}00\ 0{,}000{,}00\ 0{,}000$
   $=$ £80 trillion

Uncertainties Rules (Summary)

Is it multiple measurements (average)?

- YES → uncertainty $= \frac{1}{2}(\text{range}) = \frac{1}{2}(\text{max} - \text{min})$ only if this result is bigger
- NO → Is it a time measurement made by a human?
  - YES → uncertainty = human reaction time ≈ 0.2 s
  - NO → uncertainty = 1 in the last s.f.

Adding / Subtracting: Add the abs. uncertainties

Multiplying / Dividing: Add the % uncertainties

Risk Assessment

Hazard	Risk	Steps taken to reduce risk
COVID-19	Transmission between students and/or teachers	Face masks & hand sanitization
HCl which causes burns	Spilling HCl on students or teachers	Keep lid on, wear gloves
etc...	etc...	etc...