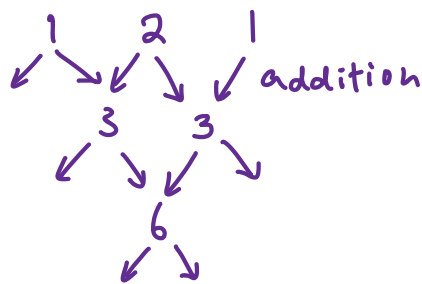
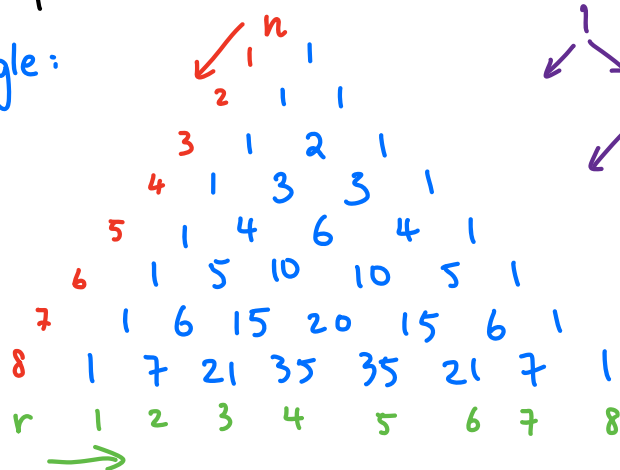


Binomial Expansion

Pascal's triangle:



$${}^nC_r \text{ or } \binom{n}{r} = \frac{n!}{(n-r)!r!} \quad \text{use the triangle to find it (or calculator)}$$

Binomial expansion:

$$(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^r b^{n-r} = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$$

$$= a^n + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \binom{n}{3} a^{n-3} b^3 + \dots + \binom{n}{n-2} a^2 b^{n-2} + \binom{n}{n-1} a b^{n-1} + b^n$$

$$\text{Example: } (a+b)^2 = a^2 + \binom{2}{1} ab + b^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + \binom{3}{1} a^2 b + \binom{3}{2} ab^2 + b^3 = a^3 + 3a^2 b + 3ab^2 + b^3$$

More on Combinations & Permutations

Combinations

$$\underbrace{{}^nC_r}_{\text{"n choose r"}} = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \# \text{ of ways } r \text{ items can be chosen from } n \text{ items}$$

★ Order is NOT important

e.g. ^A Apple ^B Banana ^O Orange (list of $n=3$)

What is the # of combinations of 2 items? \rightarrow AB, BO, AO $\xrightarrow{3 \text{ combinations}}$ ${}^3C_2 = 3$

Permutations

$${}^nP_r = \underbrace{n(n-1)\dots(n-r+1)}_{r \text{ factors}} = \frac{n!}{(n-r)!} = \# \text{ of ways } r \text{ items can be permuted from } n \text{ items}$$

★ Order IS important

e.g. A, B, O (list of $n=3$)

of permutations of 2 items \rightarrow AB, BA, BO, OB, AO, OA $\xrightarrow{6 \text{ combinations}}$ ${}^3P_2 = 6$

Challenge (p.164)

$$\begin{aligned} \text{a) } (a+b)^4 - (a-b)^4 &= [(a+b)^2 + (a-b)^2][(a+b)^2 - (a-b)^2] \\ &= [a^2 + 2ab + b^2 + a^2 - 2ab + b^2][a^2 + 2ab + b^2 - a^2 + 2ab - b^2] \\ &= [2a^2 + 2b^2][4ab] = 8ab(a^2 + b^2) \end{aligned}$$

$$\begin{aligned} \text{b) } a+b=17 \Rightarrow a=11 \quad 82896 &= (11+b)^4 - (11-b)^4 = 8(11)(b)(11^2 + b^2) \\ a-b=5 \quad b=6 &= (11)^3(6)(8) + (11)(6)^3(8) \end{aligned}$$

Estimating using binomial expansion

$$\Rightarrow \left(1 + \frac{x}{4}\right)^8 = 1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3 + \dots$$

what is 1.025^8 ?

$$\begin{aligned} &\Downarrow \\ &1 + \frac{x}{4} = 1.025 \\ &x = 0.1 \end{aligned} \quad \begin{aligned} &\downarrow \\ &\rightarrow 1 + 0.2 + \frac{7}{40} + \frac{7}{80} + \dots \\ &\approx 1.2184 \end{aligned}$$

$$\Rightarrow \left(1 + \frac{x}{2}\right)^{10} = 1 + 5x + \frac{45}{4}x^2 + 15x^3 + \dots$$

what is 1.005^{10} ?

$$\begin{aligned} &\Downarrow \\ &1 + \frac{x}{2} = 1.005 \\ &x = 0.01 \end{aligned} \quad \begin{aligned} &\downarrow \\ &\rightarrow 1 + 0.05 + \frac{45}{4}(0.01)^2 + 15(0.01)^3 + \dots \\ &\approx 1.05114 \end{aligned}$$