Sum & Difference Formulas

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

10. $\tan A = \frac{1}{5}$ $\tan B = \frac{2}{3}$ Find $A+B$ in degrees

a) A & B are both acute

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{1/5 + 2/3}{1 - 2/15} = 1$$

If $\tan(A+B) = 1$, $0 < A, B < 90$

$A+B = 45°, 225°$ (Since both A & B $< 90°$, $A+B < 180°$)

$= 45°$

Double Angle Formulas

$\sin(A+A) = \sin(2A) = \sin A \cos A + \cos A + \sin A = 2\sin A \cos A$

$\cos(A+A) = \cos(2A) = \cos A \cos A - \sin A \sin A$

$= \cos^2 A - \sin^2 A$

$= 1 - 2\sin^2 A$

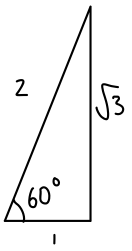
$= 2\cos^2 A - 1$

$$\tan(A+A) = \tan(2A) = \frac{\tan A + \tan A}{1 - \tan A \tan A} = \frac{2\tan A}{1 - \tan^2 A}$$

Ex 7B

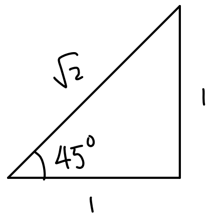
1. a) $\cos 15^\circ = \cos(60^\circ - 45^\circ) = \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$



$\cos 60^\circ = \frac{1}{2}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$



$\cos 45^\circ = \frac{\sqrt{2}}{2}$

$\sin 45^\circ = \frac{\sqrt{2}}{2}$

c) $\sin(120^\circ + 45^\circ) = \sin 120^\circ \cos 45^\circ + \cos 120^\circ \sin 45^\circ$

$\sin 120^\circ = \sin(180 - 120^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 120^\circ = \cos(2 \times 60^\circ) = 2\cos^2 60^\circ - 1 = 2\left(\frac{1}{4}\right) - 1 = -\frac{1}{2}$

$\therefore \sin(120^\circ + 45^\circ) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$

Simplifying $a\cos x \pm b\sin x$

given $f(x)$ and $g(x)$,

$af(x) + bg(x)$ is a linear combination of $f(x)$ and $g(x)$

let's consider linear combinations of $\sin x$ and $\cos x$!

$a\sin(x) \pm b\cos(x) = R\sin(x \pm \alpha)$ Evaluate this.

$a\cos(x) \pm b\cos(x) = R\cos(x \mp \alpha)$

$$R\sin(x+\alpha) = R(\sin x\cos\alpha + \cos x\sin\alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$$

$\downarrow$ Compare coefficients.

$$\begin{cases} a = R\cos\alpha & (1) \\ b = R\sin\alpha & (2) \end{cases}$$

$$R^2 = \frac{a^2}{\cos^2\alpha} = \frac{a^2}{1-\sin^2\alpha}$$

$$R^2 = \frac{a^2}{1 - \frac{b^2}{R^2}}$$

$$R^2 - b^2 = a^2$$

$$\underline{R = \sqrt{a^2+b^2}}$$

$(2)/(1)$: $\frac{R\sin\alpha}{R\cos\alpha} = \frac{a}{b}$

$$\tan\alpha = \frac{a}{b}$$

$$\underline{\alpha = \tan^{-1}\left(\frac{a}{b}\right)}$$

$$\underline{R = \frac{a}{\cos\alpha} = \frac{b}{\sin\alpha}}$$

$$\alpha = \tan^{-1}\left(\frac{a}{b}\right)$$

$$R = \frac{a}{\cos\alpha} = \frac{b}{\sin\alpha} = \sqrt{a^2+b^2}$$

Ex7E (p.184)(Ans p. 395)

1. $5\sin\theta + 12\cos\theta \equiv R\sin(\theta+\alpha)$

$R = \sqrt{5^2 + 12^2} = 13$ ✓ $\tan\alpha = \frac{12}{5}$ ✓

2. $\sqrt{3}\sin\theta + \sqrt{6}\cos\theta \equiv 3\cos(\theta - \alpha)$

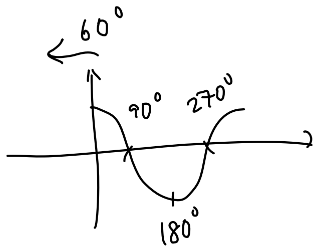
$\tan\alpha = \frac{\sqrt{3}}{\sqrt{6}}$

$\alpha = \tan^{-1}\left(\frac{\sqrt{3}}{\sqrt{6}}\right) = 35.3$ ✓

3. $2\sin\theta - \sqrt{5}\cos\theta \equiv -3\cos(\theta+\alpha)$

$\tan(-\alpha) = \frac{2}{\sqrt{5}}$

$\alpha = -\tan^{-1}\left(\frac{2}{-\sqrt{5}}\right) = 41.8^\circ$ ✓



4. a) $\cos\theta - \sqrt{3}\sin\theta \equiv R\cos(\theta+\alpha)$

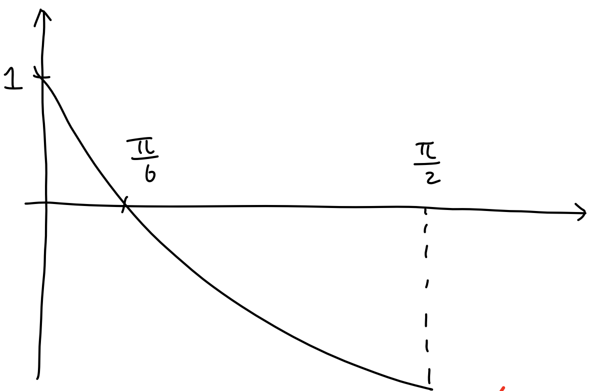
$\tan(\alpha) = \frac{\sqrt{3}}{1}$

$\alpha = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

$R = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$

$\therefore 2\cos(\theta + \frac{\pi}{3})$ ✓

b)



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