

Simultaneous equations

Consider $y = dx + e$ and

$$y = ax^2 + fx + g$$

$$dx + e = ax^2 + fx + g$$

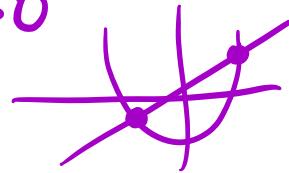
$$ax^2 + (f-d)x + (g-e) = 0$$

let $(f-d) = b$, $(g-e) = c$

$$\Delta = b^2 - 4ac$$

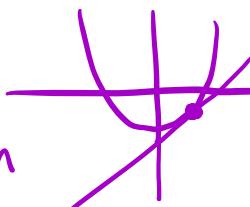
CASE 1: $\Delta > 0$

2 intersections



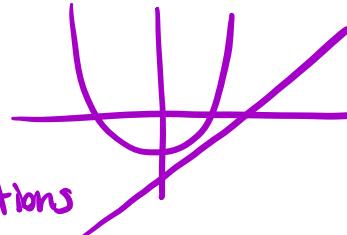
CASE 2: $\Delta = 0$

1 intersection



CASE 3: $\Delta < 0$

0 intersections



Consider $x - 2y = 8$ and $xy = 24$ or $y = \frac{24}{x}$ } reciprocal function

$$x = 2y + 8$$

$$y = \frac{24}{2y+8}$$

$$y^2 + 4y = 12$$

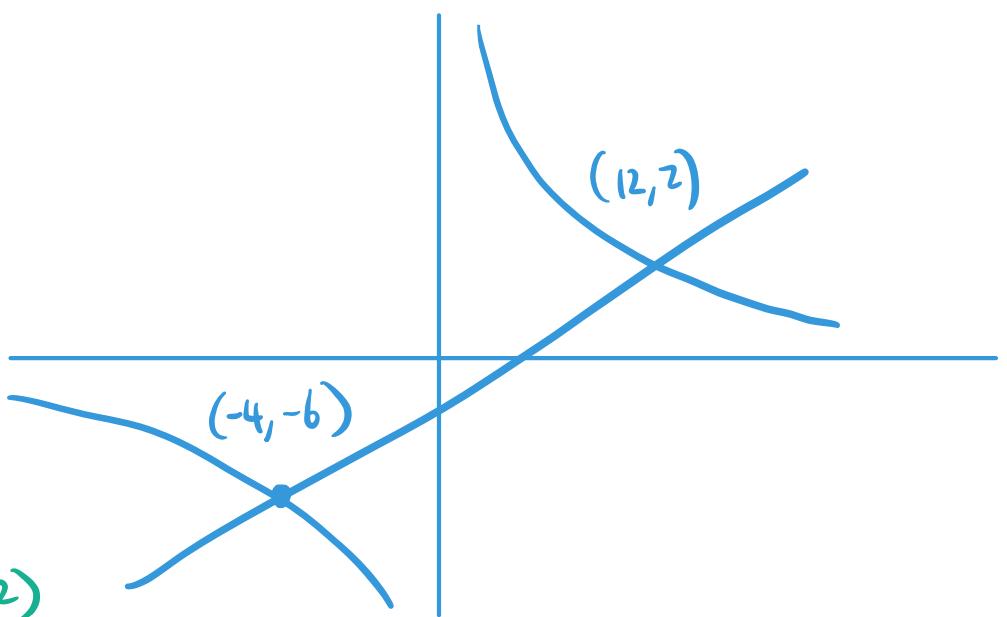
$$y^2 + 4y - 12 = 0$$

$$(y+6)(y-2) = 0$$

$$y = -6, 2$$

$$x = -4, 12$$

$$(-4, -6), (12, 2)$$



P.42 EX3B

1 a) $\begin{cases} x+y=11 \\ xy=30 \end{cases}$

$$y = -x + 11$$

$$-x^2 + 11x = 30$$

$$x^2 - 11x + 30 = 0$$

$$(x-6)(x-5)=0$$

$$x=5, 6$$

$$y=6, 5$$

b) $\begin{cases} 2x+y=1 \\ x^2+y^2=1 \end{cases}$

$$y = -2x + 1$$

$$x^2 + 4x^2 - 4x + 1 = 1$$

$$5x^2 - 4x = 0$$

$$x(5x-4) = 0$$

$$x=0, \frac{4}{5}$$

$$y=1, -\frac{3}{5}$$

c) $\begin{cases} y=3x \\ 2y^2 - xy = 15 \end{cases}$

$$18x^2 - 3x^2 = 15$$

$$x^2 = 1$$

$$x = \pm 1$$

$$y = \pm 3$$

d) $\begin{cases} 3a+b=8 \\ 3a^2+b^2=28 \end{cases}$

$$b = -3a+8$$

$$3a^2 + 9a^2 - 48a + 64 = 28$$

$$12a^2 - 48a + 36 = 0$$

$$a^2 - 4a + 3 = 0$$

$$a=1, 3$$

$$b=5, -1$$

e) $\begin{cases} 2u+v=7 \\ uv=6 \end{cases}$

$$v = -2u + 7$$

$$-2u^2 + 7u = 6$$

$$2u^2 - 7u + 6 = 0$$

$$u=2, \frac{3}{2}$$

$$v=3, 4$$

f) $\begin{cases} 3x+2y=7 \\ x^2+xy=8 \end{cases}$

$$y = -\frac{3}{2}x + \frac{7}{2}$$

$$\frac{2x+3}{x-3}$$

$$x^2 - \frac{3}{2}x + \frac{7}{2} = 8$$

$$2x^2 - 3x - 9 = 0$$

$$x=3, -\frac{3}{2}$$

$$y=-1, \frac{23}{4}$$

2 a) $\begin{cases} 2x+2y=7 \\ x^2-4y^2=8 \end{cases}$

$$\begin{pmatrix} 3x & -19 \\ x & -3 \end{pmatrix}$$

$$y = -x + \frac{7}{2}$$

$$x^2 - 4(x - \frac{7}{2}x + \frac{49}{4}) = 8$$

$$x^2 - 4x^2 + 28x - 49 = 8$$

$$3x^2 - 28x + 57 = 0$$

$$x=3, \frac{19}{3}$$

$$y=\frac{1}{2}, -\frac{11}{6}$$

b) $\begin{cases} x+y=9 \\ x^2-3xy+2y^2=0 \end{cases}$

$$(x-2y)(x-y)=0$$

$$y = -x + 9$$

$$(x+2x-18)(x+x-9)=0$$

$$(3x-18)(2x-9)=0$$

$$x=6, \frac{9}{2}$$

$$y=3, \frac{9}{2}$$

$$c) \begin{cases} 5y - 4x = 1 \\ x^2 - y^2 + 5x = 41 \end{cases} \quad \left(\begin{array}{r} 3x - 18 \\ 3x + 57 \end{array} \right)$$

$$y = \frac{4}{5}x + \frac{1}{5}$$

$$x^2 - \left(\frac{16}{25}x^2 + \frac{8}{25}x + \frac{1}{25}\right) + 5x = 41$$

$$25x^2 - 16x^2 - 8x - 1 + 125x = 1025$$

$$9x^2 + 117x - 1026 = 0$$

$$(3x - 18)(3x + 57) = 0$$

$$x = 6, -19$$

$$3 \text{ a) } \begin{cases} x - y = 6 \\ xy = 4 \end{cases}$$

$$y = x - 6$$

$$x^2 - 6x - 4 = 0$$

$$(x-3)^2 - 13 = 0$$

$$x = 3 \pm \sqrt{13}$$

$$y = -3 \pm \sqrt{13}$$

$$\text{b) } \begin{cases} 2x + 3y = 13 \\ x^2 + y^2 = 78 \end{cases}$$

$$y = -\frac{2}{3}x + \frac{13}{3}$$

$$x^2 + \left(\frac{4}{9}x^2 - \frac{52}{9}x + \frac{169}{9}\right) = 78$$

$$9x^2 + 4x^2 - 52x + 169 = 702$$

$$13x^2 - 52x - 533 = 0$$

$$x^2 - 4x - 41 = 0$$

$$(x-2)^2 - 45 = 0$$

$$\begin{aligned} x &= 2 \pm \sqrt{45} \\ y &= -\frac{4}{3} + \frac{16}{3} \mp 2\sqrt{45} \pm 13\sqrt{5} \\ &= 4 \pm 11\sqrt{5} \end{aligned}$$

$$4 \begin{cases} x + y = 3 \\ x^2 - 3y = 1 \end{cases} \quad \left(\begin{array}{r} x + 5 \\ x - 2 \end{array} \right)$$

$$y = -x + 3$$

$$x^2 + 3x - 9 = 1$$

$$x^2 + 3x - 10 = 0$$

$$x = 2, -5$$

$$y = 1, -2$$

5 a) $\begin{cases} y = 2 - 4x \quad \text{--- ①} \\ 3x^2 + xy + 11 = 0 \quad \text{--- ②} \end{cases}$

Sub ① in ②

$$3x^2 - 4x^2 + 2x + 11 = 0$$

$$x^2 - 2x - 11 = 0$$

$$\text{b) } x = \frac{2 \pm \sqrt{4 + 44}}{2}$$

$$= 1 \pm 2\sqrt{3}$$

when $x = 1 + 2\sqrt{3}$,

$$y = -2 - 8\sqrt{3}$$

when $x = 1 - 2\sqrt{3}$,

$$y = -2 + 8\sqrt{3}$$

6 $\begin{cases} y = kx - 5 \quad \text{--- ①} \\ 4x^2 - xy = 6 \quad \text{--- ②} \end{cases}$ (1, p)

a) ①: $p = k - 5$

②: $4 - p = 6$

$$p = -2$$

$$k = 3$$

b) $\begin{cases} y = 3x - 5 \quad \text{--- ③} \\ 4x^2 - xy = 6 \quad \text{--- ④} \end{cases}$

③ into ④: $4x^2 - 3x^2 + 5x = 6$

$$\begin{array}{r} x + 6 \\ x - 1 \end{array}$$

$$x^2 + 5x - 6 = 0$$

$$x = 1, -6$$

$$y = -2, -23$$

$$\therefore (1, -2), (-6, -23)$$

CHALLENGE (p. 42 Ex3B)

$$\begin{cases} y-x=k & \text{--- ①} \\ x^2+y^2=4 & \text{--- ②} \end{cases}$$

$$\text{①: } x = y - k \quad \text{--- ①'}$$

$$\text{①' into ②: } y^2 - 2yk + k^2 + y^2 = 4$$

$$2y^2 - 2yk + (k^2 - 4) = 0$$

$$\Delta = 4k^2 - 8k^2 + 32 = 0$$

$$k^2 = 8$$

$$k = \pm 2\sqrt{2}$$

Ex3C (p. 45)

$$8 \begin{cases} 2x-y=1 \\ x^2+4ky+5k=0 \end{cases}$$

$$\text{a) } y = 2x - 1$$

$$\begin{aligned} & x^2 + 8kx - 4k + 5k = 0 \\ & x^2 + 8kx + k = 0 \end{aligned}$$

$$\text{b) } \Delta = 64k^2 - 4k = 0$$

$$\begin{aligned} & 4k(16k - 1) = 0 \\ & k = 0, \frac{1}{16} \\ & = \frac{1}{16} \end{aligned}$$

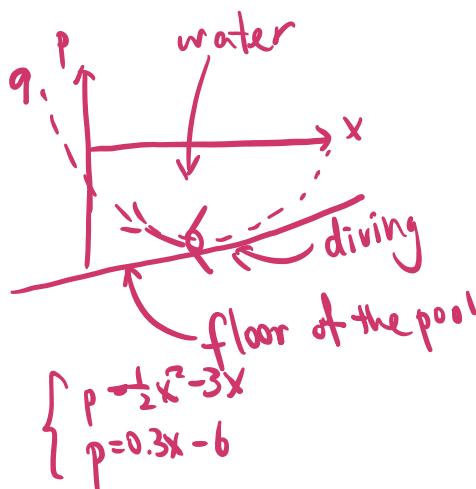
$$\text{c) } x^2 + \frac{1}{2}x + \frac{1}{16} = 0$$

$$16x^2 + 8x + 1 = 0$$

$$(4x+1)^2 = 0$$

$$x = -\frac{1}{4}$$

$$y = -\frac{3}{2}$$



$$\begin{cases} p = \frac{1}{2}x^2 - 3x \\ p = 0.3x - b \end{cases}$$

$$\frac{1}{2}x^2 - 3x = 0.3x - b$$

$$5x^2 - 30x = 3x - b$$

$$5x^2 - 33x + b = 0$$

$$\Delta = 1089 - 1200 = -111 < 0$$

∴ will not touch bottom of pool

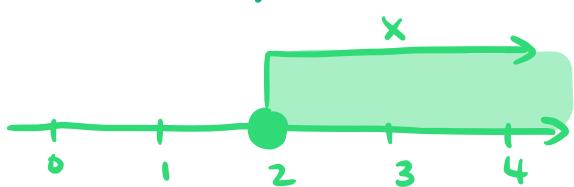
INEQUALITIES

The # line !

if $x > 2$,



if $x \geq 2$,

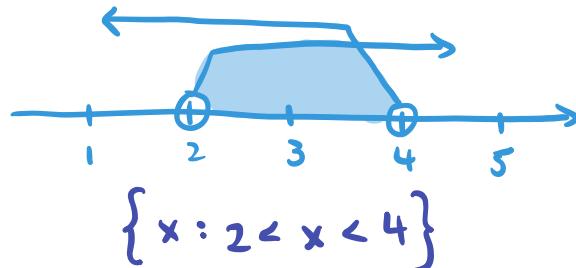


Set Notation:

$$\{x : x > 2\}$$

↑
"such that"
A SET

if $\{x : x > 2\} \cap \{x : x < 4\}$,



EX 3D P.48

3 a) $3(x-2) > x-4$ and $4x+12 > 2x+17$

$$3x-6 > x-4$$

$$x > 1$$

$$2x > 5$$

$$x > \frac{5}{2}$$

$$\therefore \left\{ x : x > \frac{5}{2} \right\}$$

b) $15-x < 2(11-x)$ and $5(3x-1) > 12x+19$

$$15-x < 22-2x$$

$$x < 7$$

$$15x-5 > 12x+19$$

$$x > 8$$

∴ NO SOLUTIONS

f) $5x+3 < 9$ or $5(2x+1) > 27$

$$5x < 6$$

$$x < \frac{6}{5}$$

$$10x+5 > 27$$

$$x > \frac{5}{2}$$

$$\left\{ x : x < \frac{6}{5} \right\} \cup \left\{ x : x > \frac{5}{2} \right\}$$

CHALLENGE (p. 48 EX3D)

$$A = \{x : 3x+5 > 2\} = \{x : x > -1\}$$

$$B = \{x : \frac{x}{2} + 1 \leq 3\} = \{x : x \leq 4\}$$

$$C = \{x : 11 < 2x-1\} = \{x : x > 6\}$$

$$A \cap (B \cup C) = \{x : p < x \leq q\} \cup \{x : x > r\}$$

$$B \cup C = \{x : x \leq 4\} \cup \{x : x > 6\}$$

$$A \cap (B \cup C) = \{x : -1 < x \leq 4\} \cup \{x : x > 6\}$$

$$\therefore p = -1, q = 4, r = 6$$

QUADRATIC inequalities

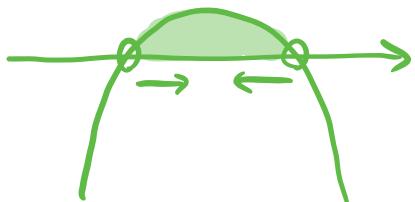
$$y = (x-\alpha)(x-\beta) > 0$$

$\{x : x < \alpha\} \cup \{x : x > \beta\}$

$$y = (x-\alpha)(x-\beta) < 0$$

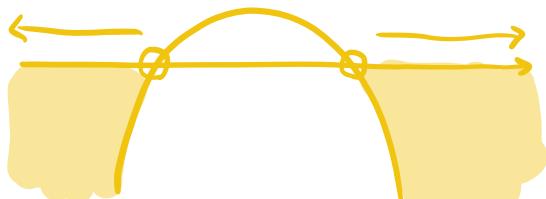
$\{x : \alpha < x < \beta\}$

$$y = -(x-\alpha)(x-\beta) > 0$$



$$\{x : \alpha < x < \beta\}$$

$$y = -(x-\alpha)(x-\beta) < 0$$

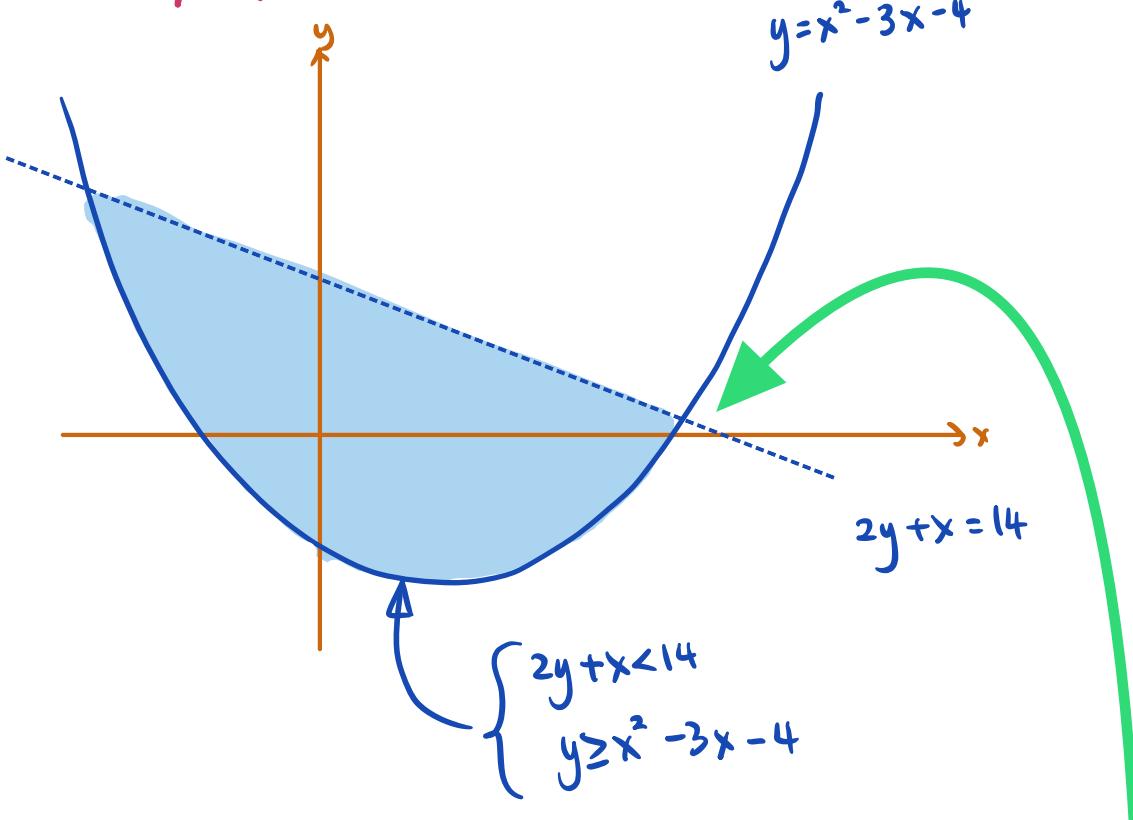


$$\{x : x < \alpha\} \cup \{x : x > \beta\}$$

EX 3E (p. 50)

1. a) $(x-8)(x-3) < 0$ b) $(x+4)(x-3) < 0$ c) $(x-5)(x+2) > 0$
 $\{x : 3 < x < 8\}$ $\{x : -4 < x < 3\}$ $\{x : x < -2\} \cup \{x : x > 5\}$
- d) $(x+4)(x+3) > 0$ e) $(2x+7)(x-1) < 0$ f) $(2x-5)(x+3) > 0$
 $\{x : x \leq -4\} \cup \{x : x \geq -3\}$ $\left\{x : -\frac{7}{2} < x < 1\right\}$ $\{x : x < -3\} \cup \left\{x : x > \frac{5}{2}\right\}$
- g) $\begin{pmatrix} 2x & -3 \\ 2x & -1 \end{pmatrix}$ h) $\begin{pmatrix} 3x & -1 \\ x & -2 \end{pmatrix}$ i) $(x+3)(x-3) < 0$
 $(2x-3)(2x-1) \leq 0$ $(3x-1)(x-2) < 0$ $\{x : -3 < x < 3\}$
 $\left\{x : \frac{1}{2} \leq x \leq \frac{3}{2}\right\}$ $\{x : \frac{1}{3} < x < 2\}$
- j) $\begin{pmatrix} 3x & -2 \\ 2x & +5 \end{pmatrix}$ k) $x(x-5) > 0$ l) $x(2x+3) \leq 0$
 $(3x-2)(2x+5) > 0$ $\{x : x < 0\} \cup \{x : x > 5\}$ $\left\{x : -\frac{3}{2} \leq x \leq 0\right\}$
 $\{x : x < -5/2\} \cup \{x : x > 3/2\}$

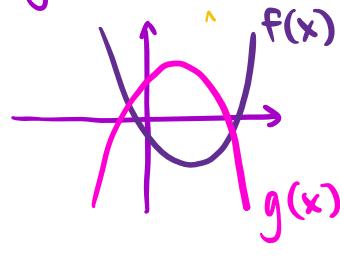
INEQUALITY REGIONS



Ex 3f (p.52) Challenge

$$f(x) = x^2 - 4x - 12$$

$$g(x) = 6 + 5x - x^2$$



a) $2x^2 - 9x - 18 = 0$

$$(2x+3)(x-6) = 0$$

$$x = -\frac{3}{2}, 6$$

$$y = -\frac{9}{2}, 0$$

$$\therefore \left(-\frac{3}{2}, -4.5 \right), (6, 0)$$

b) $\{x : -\frac{3}{2} < x < 6\}$

DON'T FORGET

use a
DOTTED line to
indicate " $<$ " or " $>$ "

P.56 Mixed EX 3

$$1 \begin{cases} 2kx - y = 4 \\ 4kx + 3y = -2 \end{cases}$$

$$a) 2kx = y + 4$$

$$2kx = -\frac{5}{2}y - 1$$

$$y + 4 = -\frac{5}{2}y - 1$$

$$\frac{5}{2}y = -5$$

$$y = -2$$

$$2 \begin{cases} x + 2y = 3 \\ x^2 - 4y^2 = -33 \end{cases}$$

$$x = 2y + 3$$

$$9 - 12y + 4y^2 - 4y^2 = -33$$

$$12y = 42$$

$$y = 7/2 \text{ or } 3.5$$

$$x = -4$$

$$b) 2kx + 2 = 4$$

$$x = \frac{1}{k}$$

$$3 \begin{cases} x - 2y = 1 \\ 3xy - y^2 = 8 \end{cases}$$

$$a) x = 2y + 1$$

$$6y^2 + 3y - y^2 = 8$$

$$5y^2 + 3y - 8 = 0$$

$$b) \begin{pmatrix} 5y & +8 \\ y & -1 \end{pmatrix}$$

$$(5y+8)(y-1) = 0$$

$$y = -8/5, 1$$

$$x = -13/5, 3$$

$$4 \quad a \begin{cases} x + y = 2 \\ x^2 + xy - y^2 = -1 \end{cases}$$

$$y = 2 - x$$

$$x^2 + 2x - x^2 - 4 + 4x - x^2 = -1$$

$$x^2 - 6x + 3 = 0$$

$$b) (x - 3)^2 = 6$$

$$x = 3 \pm \sqrt{6}$$

$$5 \quad a \begin{cases} 3^x = 9^{y-1} \\ 3^x = 3^{2y-2} \end{cases}$$

$$x = 2y - 2$$

$$b \begin{cases} x = 2y - 2 \\ x^2 = y^2 + 7 \end{cases}$$

$$4y^2 - 8y + 4 = y^2 + 7$$

$$3y^2 - 8y - 3 = 0$$

$$\begin{pmatrix} 3x + 1 \\ x - 3 \end{pmatrix}$$

$$x = -1/3, 3$$

$$y = -8/3, 4$$

$$6 \begin{cases} x+2y=3 \\ x^2-2y+4y^2=18 \end{cases}$$

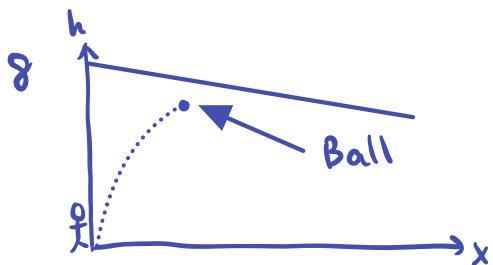
$$x = 3 - 2y$$

$$9 - 12y + 4y^2 - 2y + 4y^2 = 18$$

$$8y^2 - 14y - 9 = 0 \quad \begin{pmatrix} 4y & -9 \\ 2y & +1 \end{pmatrix}$$

$$y = \frac{9}{4}, -\frac{1}{2}$$

$$x = -\frac{3}{2}, 4$$



ceiling: -20cm every 1m, starting from 7.5m

$$\text{ceiling } h = -0.2x + 7.5$$

$$\text{ball } h = -0.3x^2 + 2.5x + 1.5$$

$$-0.2x + 7.5 = -0.3x^2 + 2.5x + 1.5$$

$$0.3x^2 - 2.7x + 6 = 0$$

$$\Delta = 2.7^2 - (0.3)(24) = 7.29 - 7.2 = 0.09 > 0$$

\therefore the curves intersect

\therefore the ball will hit the ceiling

$$10 \quad (x-1)(x-4) < 2(x-4)$$

$$x^2 - 5x + 4 < 2x - 8$$

$$x^2 - 7x + 12 < 0 \quad \begin{pmatrix} x-4 \\ x-3 \end{pmatrix}$$

$$(x-4)(x-3) < 0$$

$$\{x : 3 < x < 4\}$$

$$7 \begin{cases} kx^2 - xy + (k+1)x = 1 \\ -\frac{k}{2}x + y = 1 \end{cases}$$

$$a \quad y = \frac{k}{2}x + 1$$

$$\therefore kx^2 - \frac{k}{2}x^2 - x + (k+1)x = 1$$

$$\frac{k}{2}x^2 + kx - 1 = 0$$

$$\Delta = k^2 + 2k = 0$$

$$k = 0, -2 = -2$$

$$b \quad -x^2 - 2x - 1 = 0$$

$$x = -1$$

$$y = 2 \quad \therefore (-1, 2)$$

$$9a \quad 3x-8 > x+13$$

$$2x > 21$$

$$\therefore \{x : x > 10.5\}$$

$$b \quad x^2 - 5x - 14 > 0 \quad \begin{pmatrix} x-7 \\ x+2 \end{pmatrix}$$

$$(x+2)(x-7) > 0 \quad \begin{pmatrix} x+2 \\ x-7 \end{pmatrix}$$

$$\therefore \{x : x < -2\} \cup \{x : x > 7\}$$

$$11a \quad (x-1)(x+2) = 18$$

$$x^2 + x - 2 = 18 \quad \begin{pmatrix} x+5 \\ x-4 \end{pmatrix}$$

$$x^2 + x - 20 = 0$$

$$x = -5, 4$$

$$b \quad (x-1)(x+2) > 18$$

$$\{x : x < -5\} \cup \{x : x > 4\}$$

$$12 \text{ a } 6x - 7 < 2x + 3$$

$$\therefore \{x : x < 5/2\}$$

$$\text{b } 2x^2 - 11x + 5 < 0 \\ (2x-1)(x-5) < 0 \\ \begin{pmatrix} 2x-1 & \\ & x-5 \end{pmatrix} \\ \therefore \{x : 1/2 < x < 5\}$$

$$\text{c } 5 < \frac{20}{x}$$

$$x < 4, x \neq 1$$

$$\therefore \{x : x < 4\} \cup \{x : x \neq 1\}$$

d $\{x : 1/2 < x < 5/2\}$

$$13 \quad \frac{8}{x^2} + 1 \leq \frac{9}{x}$$

$$8 + x^2 \leq 9x \\ x^2 - 9x + 8 \leq 0 \\ (x-8)(x-1) \leq 0$$

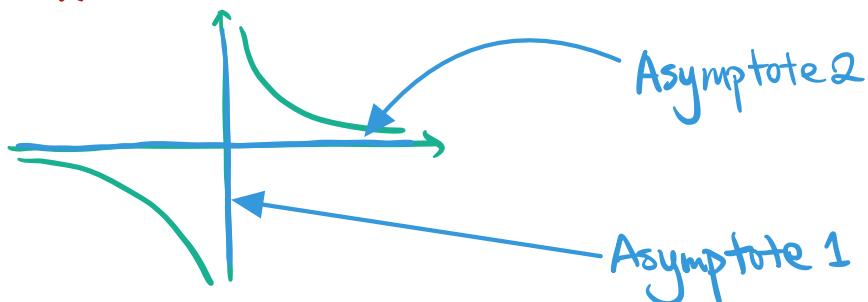
$$\therefore \{x : 1 \leq x \leq 8\}$$

$$14 \quad kx^2 + 8x + 5 = 0$$

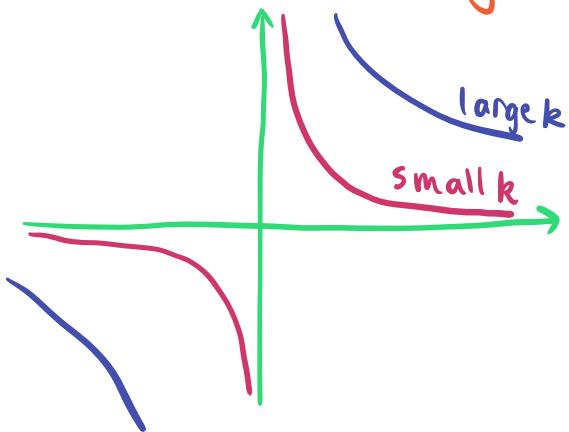
$$\Delta = 64 - 20k \geq 0 \\ 20k \leq 64 \\ k \leq 3.2$$

Reciprocal functions () { }

$f(x) = \frac{k}{x}$ is undefined at $x=0$



when k increases,
distance from origin increases



DON'T FORGET

when solving equations,
 x CANNOT be 0!
if you get it, reject it!