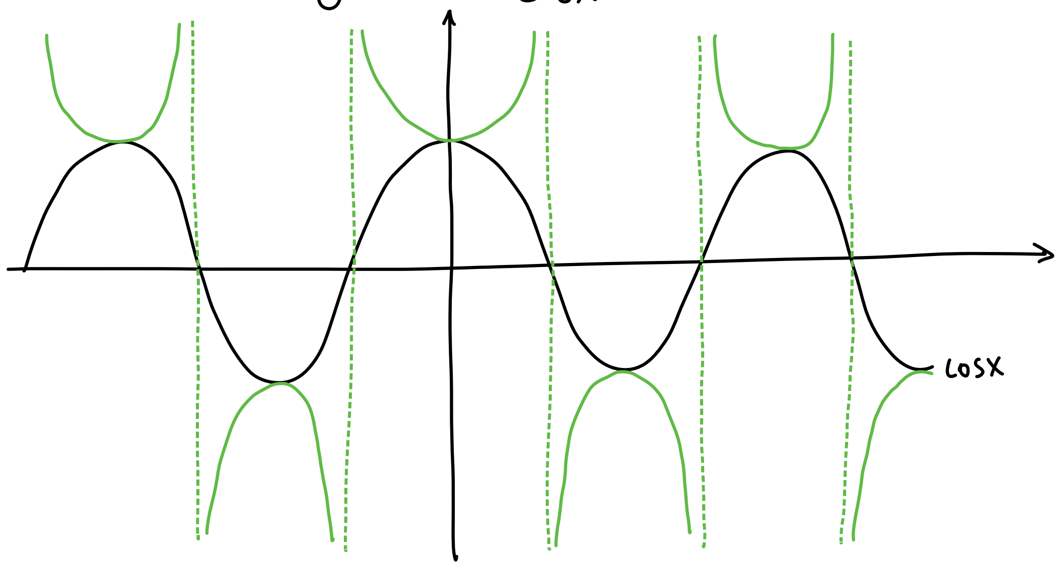
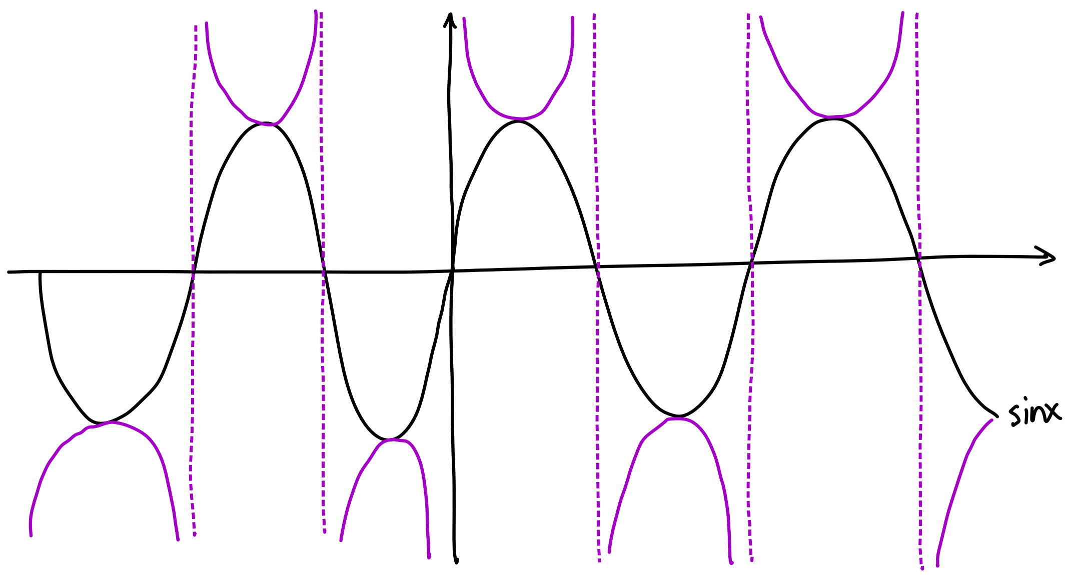
Secant, Cosecant and Cotangent

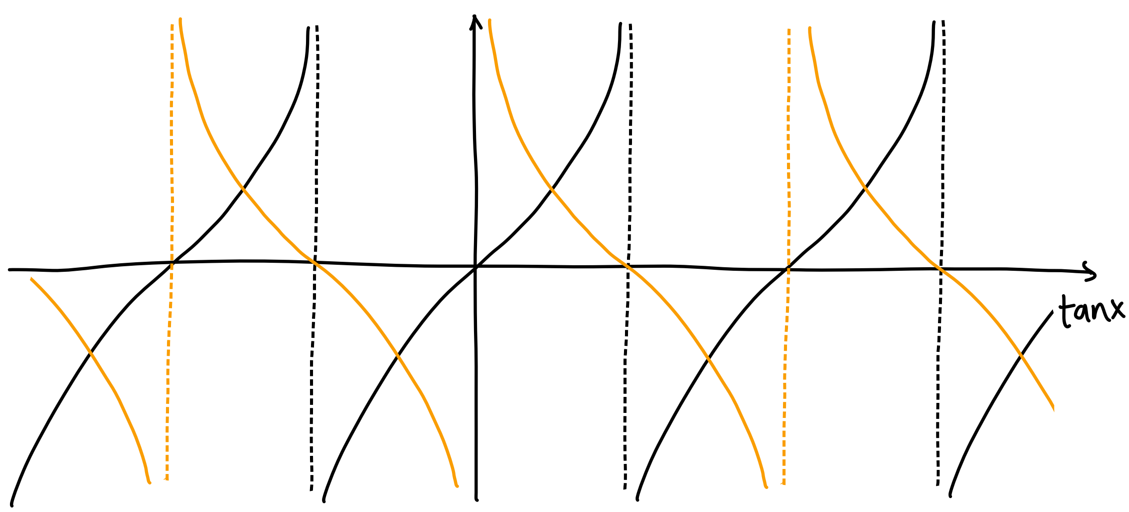
$$y = \sec x = \frac{1}{\cos x}$$



$$\operatorname{cosec} x = \frac{1}{\sin x}$$



$$\cot x = \frac{1}{\tan x}$$



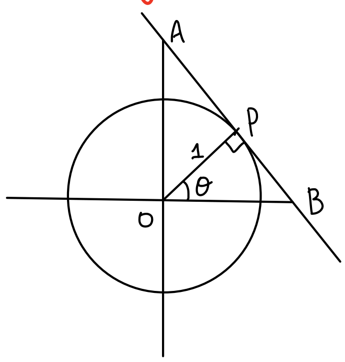
Ex 6A (p.144)

4. $\operatorname{cosec}(\pi - x) = \frac{1}{\sin(\pi - x)} = \frac{1}{\sin x} = \operatorname{cosec} x$

5. $\cot 30^\circ \sec 30^\circ = \left(\frac{1}{\tan 30^\circ}\right)\left(\frac{1}{\cos 30^\circ}\right) = \left(\frac{\cos 30^\circ}{\sin 30^\circ}\right)\left(\frac{1}{\cos 30^\circ}\right) = \frac{1}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2$

6. $\operatorname{cosec}\frac{2\pi}{3} + \sec\frac{2\pi}{3} = \frac{1}{\sin\frac{2\pi}{3}} + \frac{1}{\cos\frac{2\pi}{3}} = \frac{1}{\frac{\sqrt{3}}{2}} - \frac{1}{\frac{1}{2}} = \frac{2}{\sqrt{3}} - 2 = \frac{2\sqrt{3}}{3} - 2 = -2 + \frac{2}{3}\sqrt{3}$

Challenge



a) ΔOPB : $\cos\theta = \frac{OP}{OB} = \frac{1}{OB} \Rightarrow \sec\theta = OB$

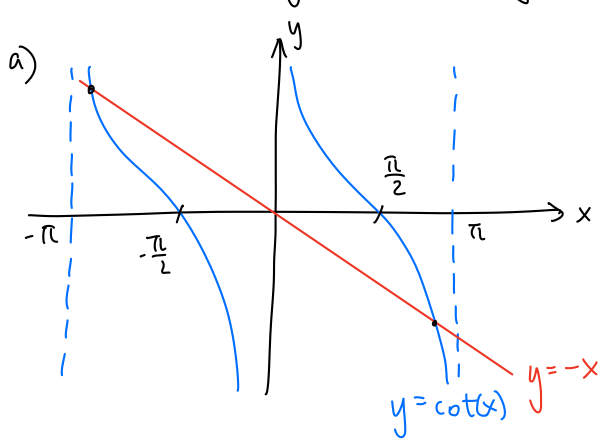
b) ΔOAP : $\sin\theta = \frac{OP}{OA} = \frac{1}{OA} \Rightarrow \operatorname{cosec}\theta = OA$

c) ΔOAP : $\tan\theta = \frac{OP}{AP} = \frac{1}{AP} \Rightarrow \cot\theta = AP$

Ex 6B

2. $-\pi \le x \le \pi$ $\quad y = \cot(x)$ and $y = -x$

a)



b) # of sols to $\cot(x) + x = 0$, $-\pi \le x \le \pi$

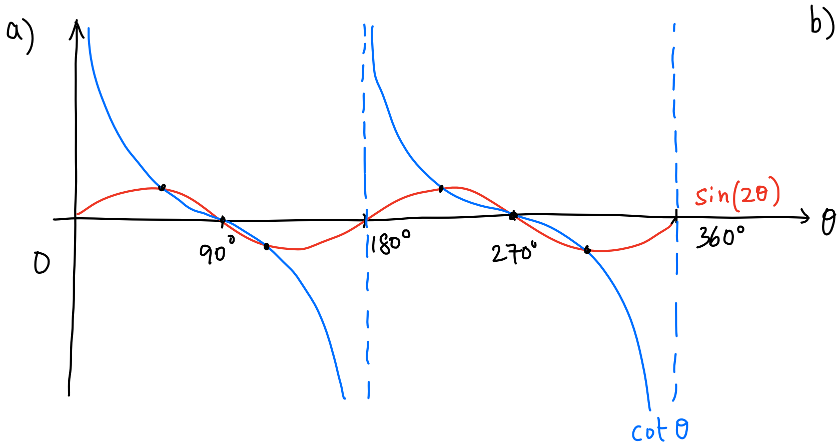
$\cot(x) = -x$

From graph, there are 2 intersections

therefore 2 solutions/roots.

4. $0 \le \theta \le 360^\circ$ $\quad \cot\theta$ and $\sin(2\theta)$

a)



b) # of sols to $\cot\theta = \sin(2\theta)$

From graph, there are 6 intersections

Therefore 6 solutions/roots

Ex 6C

5. e) $3\sec^2\theta - 4 = 0 \qquad 0 \le \theta \le 360°$

$\sec^2\theta = \frac{4}{3}$

$\cos^2\theta = \frac{3}{4}$

$\cos\theta = \frac{\sqrt{3}}{2} \quad \cos(360-\theta) = \frac{\sqrt{3}}{2} \qquad \cos\theta = -\frac{\sqrt{3}}{2} \quad \cos(360-\theta) = -\frac{\sqrt{3}}{2}$

$\theta = 30°, 330° \qquad \theta = 150°, 210°$

$\therefore \theta = 30°, 150°, 210°, 330°$

6. g) $-180° \le \theta \le 180° \qquad -360° \le 2\theta \le 360°$

$\operatorname{cosec}(2\theta) = 4$

$\sin(2\theta) = \frac{1}{4}$

$2\theta = \sin^{-1}\left(\frac{1}{4}\right) = 14.5°, 165.5°, -194.5°, -345.5°$

$\theta = 7.3°, 82.8°, -97.3°, -172.8°$

11. $0 \le x \le 360°$,

$\frac{1+\cot x}{1+\tan x} = 5$

$1 + \frac{1}{\tan x} = 5 + 5\tan x \qquad \begin{matrix} 5x & -1 \\ x & +1 \end{matrix}$

$\tan x + 1 = 5\tan x + 5\tan^2 x$

$5\tan^2 x + 4\tan x + 1 = 0$

$(5\tan x - 1)(\tan x + 1) = 0$

$\tan x = -1, \frac{1}{5}$

$\tan x = \frac{1}{5} \qquad \tan x = -1$

$x = 11.31, 191.31° \qquad x = 135°, 325°$

More Identities

$1 + \tan^2 x \equiv \sec^2 x$

$1 + \cot^2 x \equiv \operatorname{cosec}^2 x$

Ex 6D (p. 156)

1. c) $\tan^2\theta(\operatorname{cosec}^2\theta - 1) = \tan^2\theta(1 + \cot^2\theta - 1)$

$= \tan^2\theta \cot^2\theta = 1$

h) $(1-\sin^2\theta)(1+\tan^2\theta) = (1-\sin^2\theta)(\sec^2\theta)$

$= \sec^2\theta - \frac{\sin^2\theta}{\cos^2\theta} = \sec^2\theta - \tan^2\theta = 1$

6. g) $\operatorname{cosec} A \sec^2 A = \operatorname{cosec} A(1 + \tan^2 A)$

$= \operatorname{cosec} A + \tan^2 A \operatorname{cosec} A$

$= \operatorname{cosec} A + \tan^2 A \left(\frac{1}{\sin A}\right)$

$= \operatorname{cosec} A + \tan A \left(\frac{1}{\cos A}\right)$

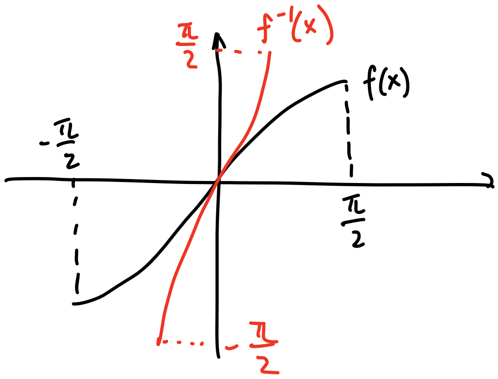
$= \operatorname{cosec} A + \tan A \sec A$

Inverse Functions

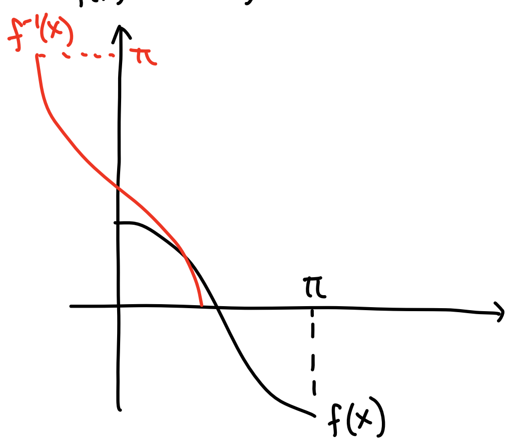
$f^{-1}(x)$ only exists if $f(x)$ is one-to-one

if $f(x) = \sin x$, $x \in \mathbb{R}$, $f^{-1}(x)$ does not exist

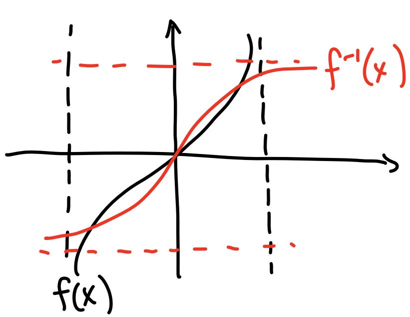
if $\underbrace{f(x) = \sin x, \ -\frac{\pi}{2} \le x \le \frac{\pi}{2}}_{\text{force } \sin x \text{ to be one-to-one}}$, $f^{-1}(x)$ does exist



$f(x) = \cos x$, $0 \le x \le \pi$



$f(x) = \tan x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$



Sum & Difference Formulas

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

10. $\tan A = \frac{1}{5}$ $\tan B = \frac{2}{3}$ Find $A+B$ in degrees

a) A & B are both acute

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{1/5 + 2/3}{1 - 2/15} = 1$$

If $\tan(A+B) = 1$, $0 < A, B < 90$

$A+B = 45^\circ, 225^\circ$ (Since both A & B $< 90^\circ$, $A+B < 180^\circ$)

$= 45^\circ$

Double Angle Formulas

$\sin(A+A) = \sin(2A) = \sin A \cos A + \cos A + \sin A = 2 \sin A \cos A$

$\cos(A+A) = \cos(2A) = \cos A \cos A - \sin A \sin A$

$= \cos^2 A - \sin^2 A$

$= 1 - 2\sin^2 A$

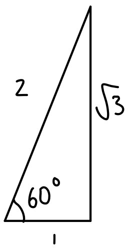
$= 2\cos^2 A - 1$

$$\tan(A+A) = \tan(2A) = \frac{\tan A + \tan A}{1 - \tan A \tan A} = \frac{2 \tan A}{1 - \tan^2 A}$$

Ex 7B

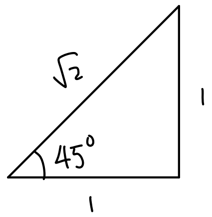
1. a) $\cos 15^\circ = \cos(60^\circ - 45^\circ) = \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$



$\cos 60^\circ = \frac{1}{2}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$



$\cos 45^\circ = \frac{\sqrt{2}}{2}$

$\sin 45^\circ = \frac{\sqrt{2}}{2}$

c) $\sin(120^\circ + 45^\circ) = \sin 120^\circ \cos 45^\circ + \cos 120^\circ \sin 45^\circ$

$\sin 120^\circ = \sin(180 - 120^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 120^\circ = \cos(2 \times 60^\circ) = 2\cos^2 60^\circ - 1 = 2\left(\frac{1}{4}\right) - 1 = -\frac{1}{2}$

$\therefore \sin(120^\circ + 45^\circ) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$