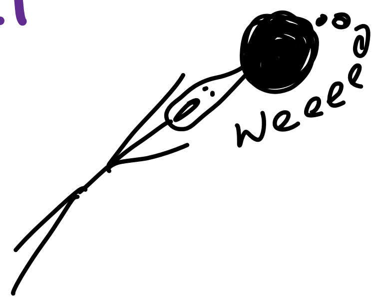
Gravitational Fields



Gravitational Forces :

Act on masses

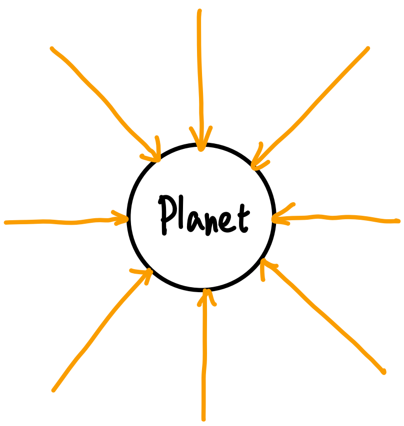
Infinite Range

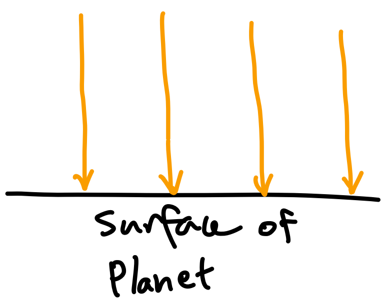
Weak (large mass to see effect compared to strong nuclear force)

Attractive towards centre of mass

 Can act over distance (no contact needed)

FIELD LINES





Shows : shape of field

strength of field

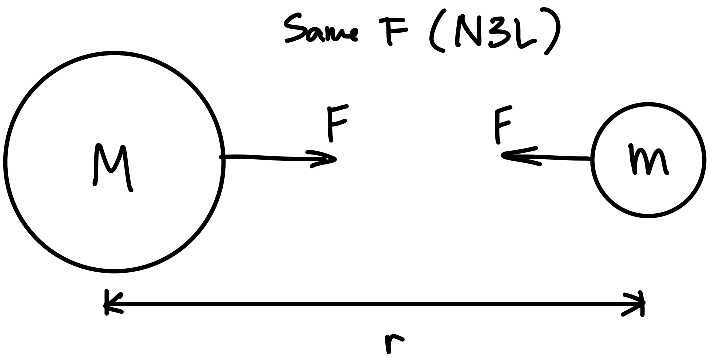
direction of force acting on a mass

$W = mg$

$g = \frac{W}{m}$ $[Nkg^{-1}]$

gravitational force per unit mass acting on that point in the gravitational field.

is a vector (since fields can cancel)



F depends on:

M, m and r $\rightarrow$ $F = \frac{GMm}{r^2}$

F [N]

G [Nm^2kg^{-2}] = 6.67×10^{-11}

M [kg]

m [kg]

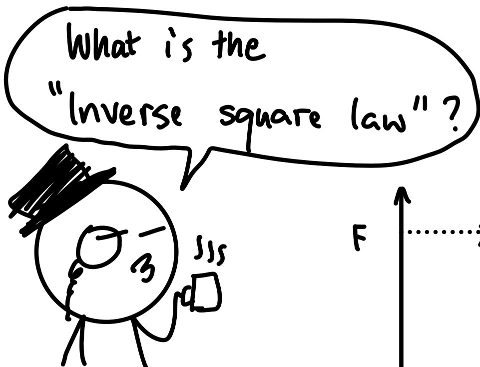
r [m]

Newton's Law of Gravitation

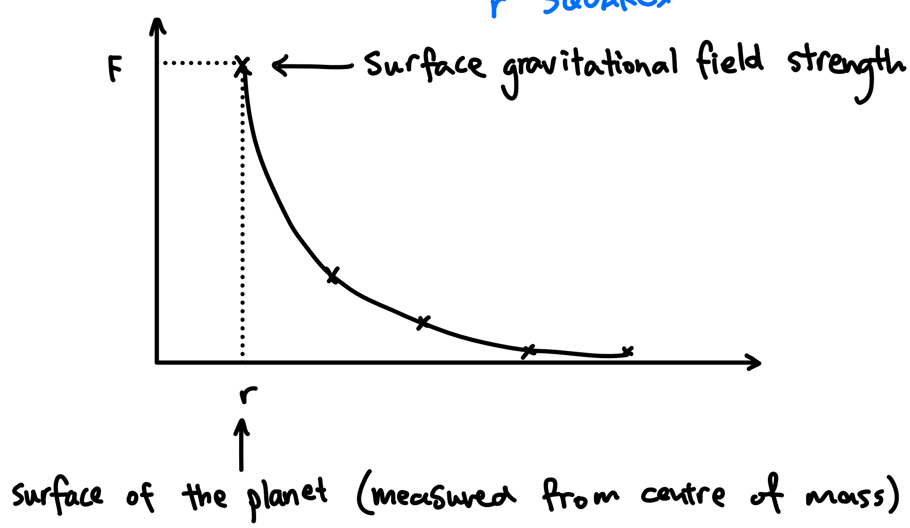
$\boxed{\text{Definition}}$

$\Downarrow$

Gravitational force between 2 objects is directly proportional to the product of the 2 masses and inversely proportional to the squared distance between them.



$F \propto \frac{1}{r^2}$ ← INVERSELY proportional to r SQUARED

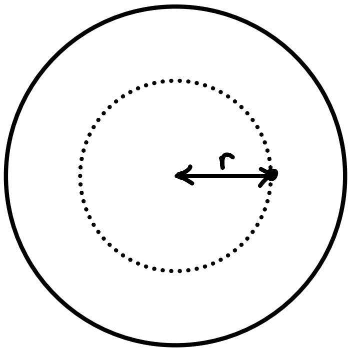


Gravitational strength in a radial field

$F = mg$ $\quad$ $F = \frac{GMm}{r^2}$ $\quad$ $mg = \frac{GMm}{r^2}$ $\longrightarrow$ $g = \frac{GM}{r^2}$

What if we went INSIDE the earth? (assuming uniform density)

"O" oh~

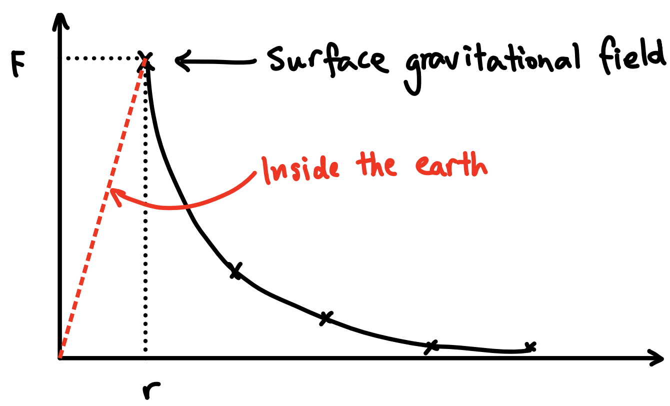


Mass of sphere of radius $r - \frac{4}{3}\pi r^3 \rho$

$$g = \frac{G(\frac{4}{3}\pi r^3 \rho)}{r^2} = \frac{4}{3}G\pi\rho r$$

LINEAR

$\therefore g \propto r$ if earth has uniform density



No field at centre because mass pulling in all directions

surface of the planet (measured from centre of mass)

Gravitational Potential

↳ V

GPE = mgh (only when g is constant)

Electricity: Electrical Potential (V) → energy per unit charge

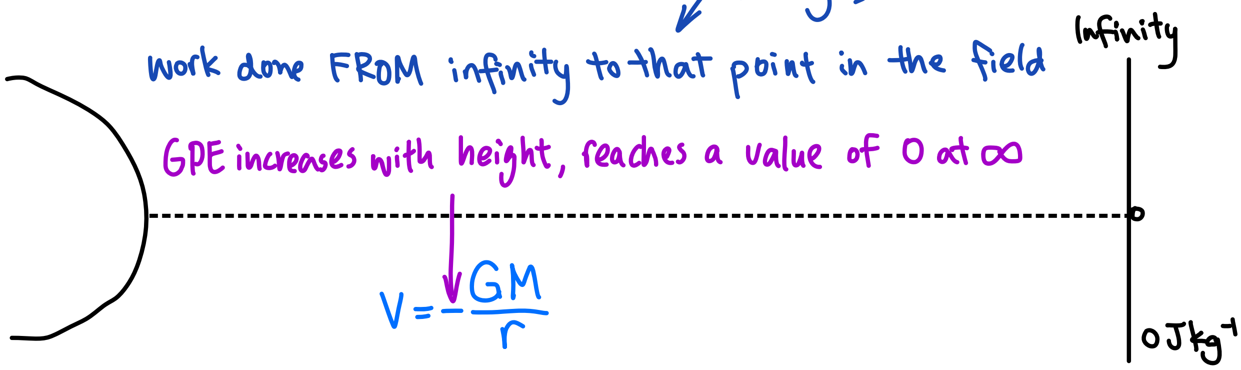
Gravitational Potential (V) → energy per unit mass

↳ NOT SAME AS GPE!

[Jkg^{-1}]

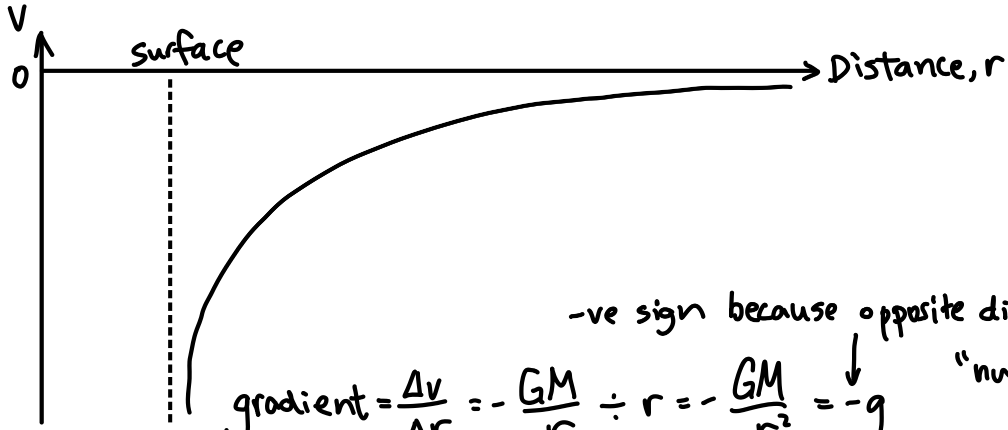
work done FROM infinity to that point in the field

GPE increases with height, reaches a value of 0 at ∞



$$V = -\frac{GM}{r}$$

The graph of Gravitational Potential

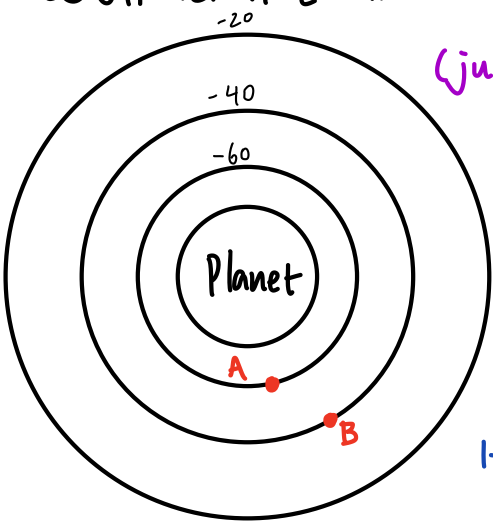


-ve sign because opposite direction

"numerically equivalent"

$$\text{gradient} = \frac{\Delta v}{\Delta r} = -\frac{GM}{r} \div r = -\frac{GM}{r^2} = -g$$

EQUIPOTENTIAL LINES



(just like contour lines)

move from A to B

$-60\,\text{J kg}^{-1}$ (A) $\quad -40\,\text{J kg}^{-1}$ (B)

$\Delta v = -40 - (-60) = 20\ \text{J kg}^{-1}$

If object is 3 kg, w.d. = $20 \times 3 = 60\text{J}$

$\therefore$ Work Done $= \Delta v \times m$ or Work Done $= m(V_B - V_A)$

$\Delta V = V_{new} - V_{old}$ → signs work out!

(If falling towards planet, -ve work → work by gravity)

Example

Rocket from surface to 3500 km work?

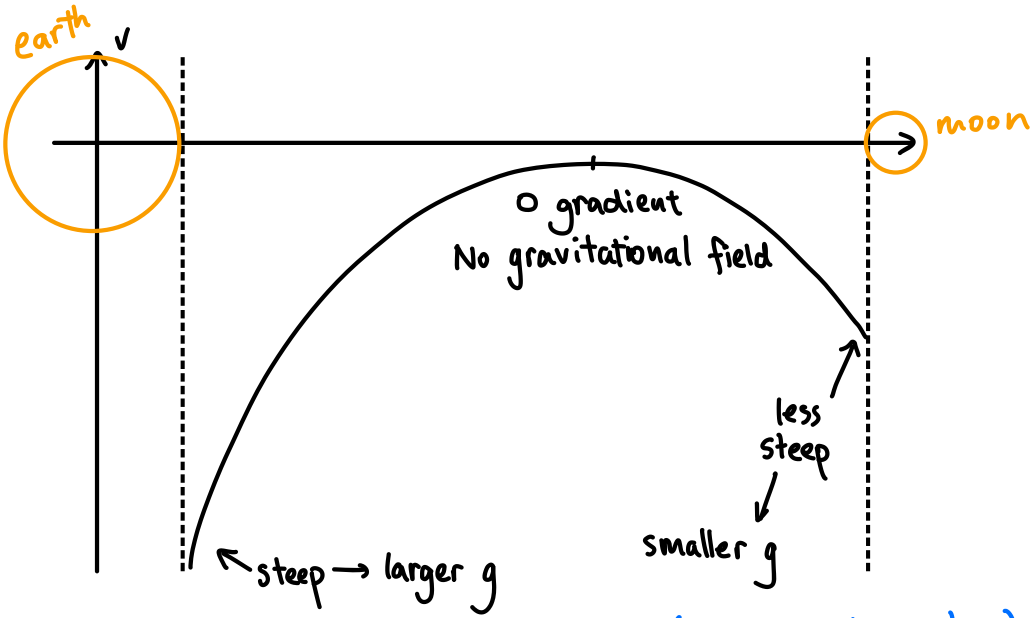
50000 kg

$M_{earth} = 6.0\times10^{24}$ kg $\quad r_{earth} = 6400\text{km} = 6.4\times10^6$ m

$$V = -\frac{GM}{r} = -\frac{6.67\times10^{-11}\times6.0\times10^{24}}{6.4\times10^6} = -62531250\ \text{J kg}^{-1} \text{ at surface}$$

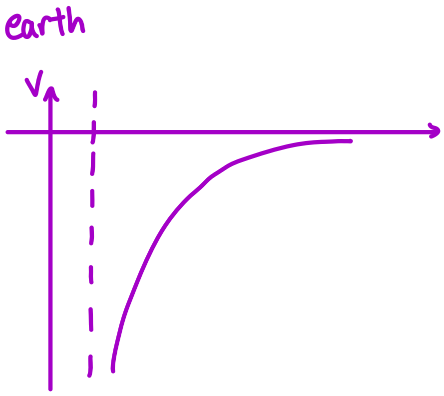
$$V = -\frac{GM}{r} = \frac{-6.67\times10^{-11}\times6.0\times10^{24}}{6.4\times10^6 + 3.5\times10^6} = -40424242.42\ \text{J kg}^{-1} \text{ at 3500 km}$$

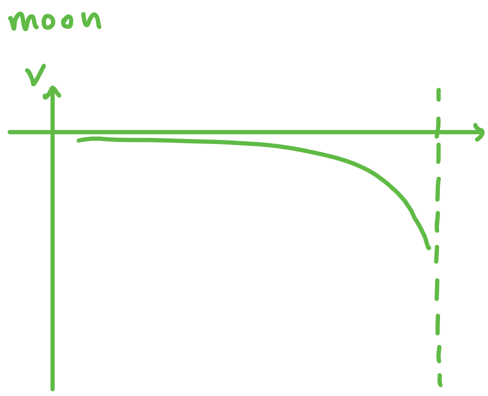
$$\text{work} = \Delta v\, m = (-40424242.42 + 62531250)\,50000 = 1.105\times10^{12}$$



g = gradient

add together (V is a scalar value)





Escape Velocity of earth

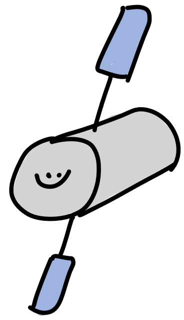
$$V \text{ at surface} = \frac{GM}{r} = \frac{6.67\times10^{-11} \times 6.0\times10^{24}}{6.4\times10^{6}} = 62531250 \text{ Jkg}^{-1}$$

target : 0 Jkg^{-1} at infinity   work = 62531250 m

$$62531250\,m = \frac{1}{2}mv^2$$

$$v = \sqrt{2\times62531250} = 11183.13... \approx 11200 \text{ ms}^{-1}$$

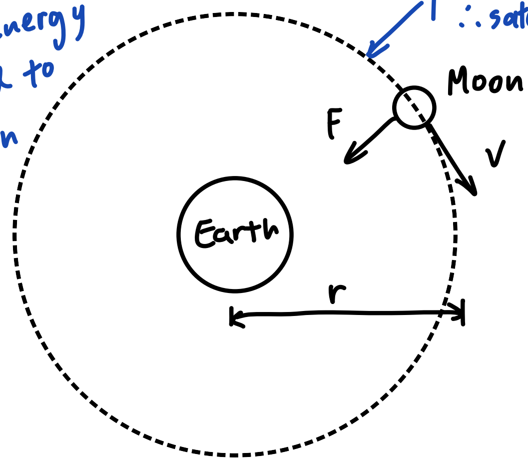
What is a SATELLITE?



Objects orbiting other masses

path is equipotential
∴ satellite has constant energy

No energy needed to maintain orbit



Circular Motion

$F = \frac{mv^2}{r}$ (From CM topic)

(F provided by gravitational force)

$\therefore \frac{mv^2}{r} = \frac{GMm}{r^2}$

$\boxed{v^2 = \frac{GM}{r}}$

v is the velocity needed to maintain an orbit of radius r around mass M

Danger! v = velocity
V = gravitational potential } Don't mess up!

Gravitational force is also

$F = mg = \frac{mv^2}{r}$

$\boxed{g = \frac{v^2}{r}}$

v is also $v = \frac{2\pi r}{T}$ $\left(\frac{\text{circumfrence}}{\text{period}}\right)$

+

$v^2 = \frac{GM}{r}$

$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$

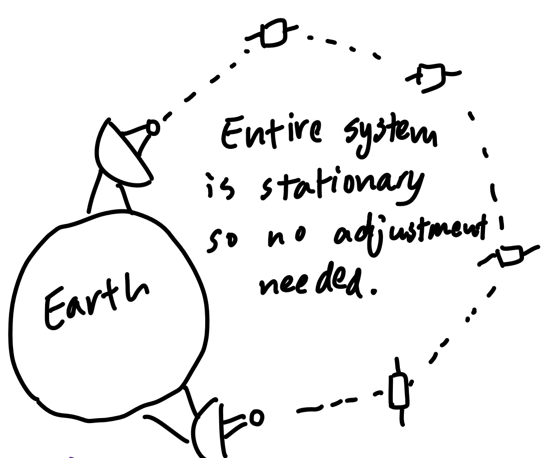
$T^2 \propto r^3$ → $\boxed{T^2 = \frac{4\pi^2 r^3}{GM}}$

Kepler's 3rd Law

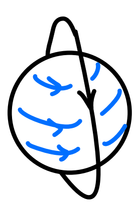
Satellite Orbits

- Geostationary / Geosynchronous
  ↓
  remains in same position relative to the earth's surface
  (Time period = 1 day)

Communications satellites are geostationary



- Polar Orbits
  ↓
  orbits over the poles of the earth



perpendicular to rotation of earth
sees EVERYTHING given time.
(covers all areas of the earth's surface)
weather monitoring, espionage (spying), mapping

The energy of a satellite

KINETIC ENERGY

$\frac{1}{2}mv^2$

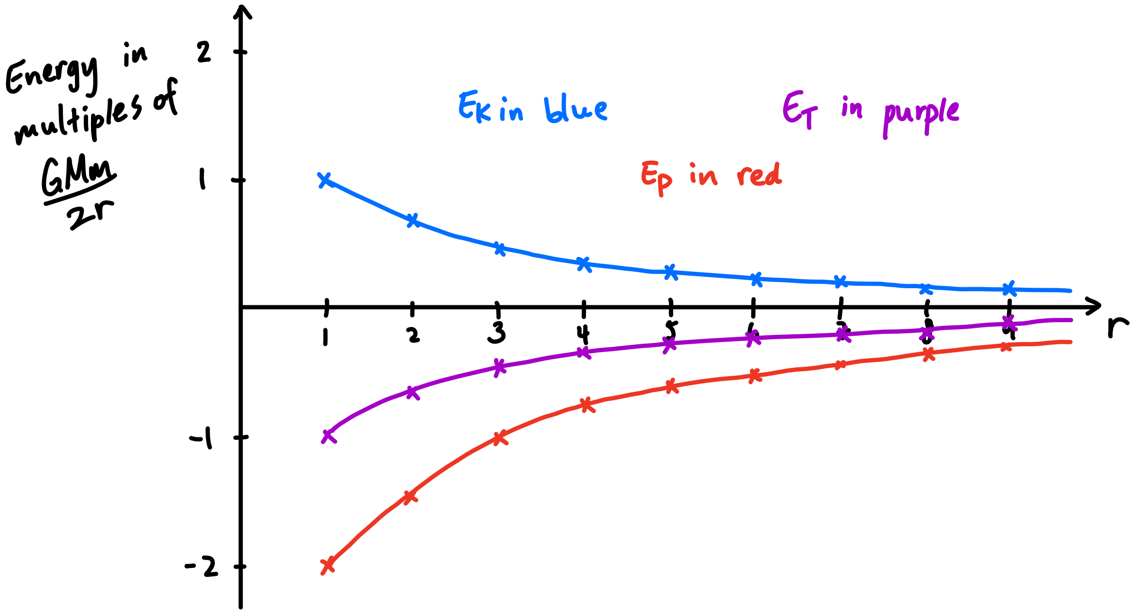
$= \frac{GMm}{2r}$ (since $v^2 = \frac{GM}{r}$)

GRAVITATIONAL POTENTAL ENERGY

$-\frac{GMm}{r}$ (from $V = \frac{GM}{r}$)

TOTAL ENERGY = $E_K + E_P$

$= \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$



ELECTRIC FIELDS

- exist around charges

2 types of charges: positive (+) and negative (−)

objects can be +ve, -ve or neutral

-ve charge transferred
(electrons move, not nucleus)

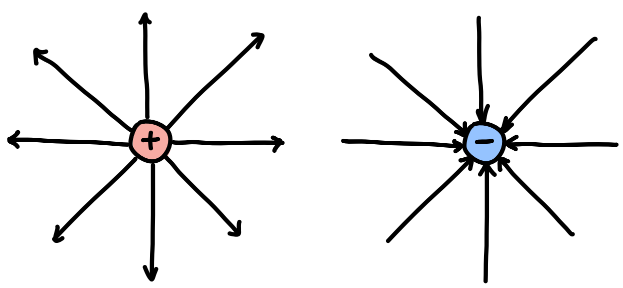
SIMILAR CHARGES REPEL

OPPOSITE CHARGES ATTRACT

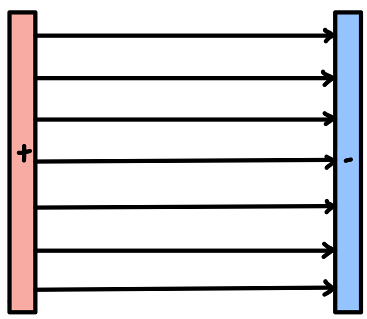
infinite range

stronger than gravitational

Field lines



arrows show direction of force acting on a +ve charge at that point

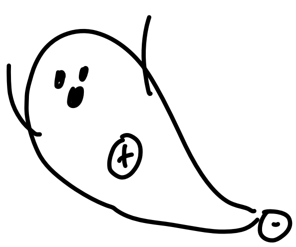


uniform electric field

created between 2 parallel charged plates

parallel lines → same direction

evenly-spaced lines → uniform strength



Electric field strength, E

- Force acting per unit positive charge at that point in the field

$E = \frac{F}{q}$ $[NC^{-1}]$ vector!

$E = \frac{kQ}{r^2}$ (Force per charge)

Force between 2 charges

depends on

- separation distance (inverse square)
- size/magnitude of the charges } charge of the charges
- type of the charge } charge of the charges
- the medium transferring the charge

↓

PERMITTIVITY

$\boxed{F = \frac{kQq}{r^2}}$ takes into account of permittivity of medium

$\boxed{k = \frac{1}{4\pi\varepsilon_0}}$

permittivity of free space $[Fm^{-1}]$ (Farad per metre)

$\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$

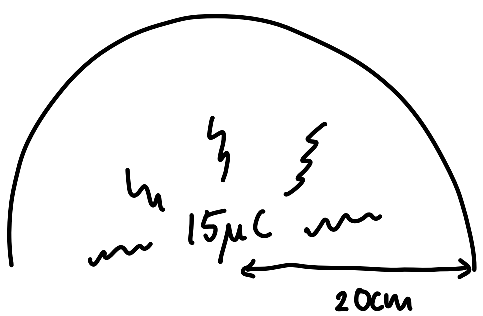
$k = 8.991804694 \times 10^{9} \approx 9.00 \times 10^{9}\ Nm^2C^{-2}$ or mF^{-1}

Example

force between charges 12nC and 15nC 10cm apart

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = \frac{1}{4\pi \times 8.85 \times 10^{-12}} \times \frac{12 \times 10^{-9} \times 15 \times 10^{-9}}{0.1^2} = 1.62 \times 10^{-4}$$

Van Der Graaff Generator



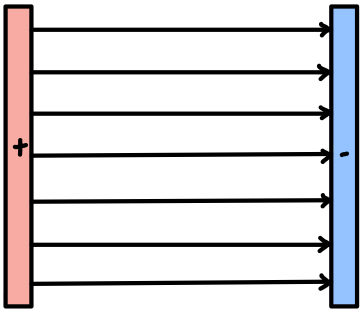
$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = \frac{15 \times 10^{-6} \times 1}{4\pi \times 8.85 \times 10^{-12} \times 0.2^2} = 3370000\ N$$

$E = 3370000\ NC^{-1}$ (3sf)

(act on +5nC charge)

$3370000 \times 5 \times 10^{-9} = 0.0169\ N$

Field strength in a uniform field



Field strength increased by:

↓ separation distance

↑ potential difference

⇓

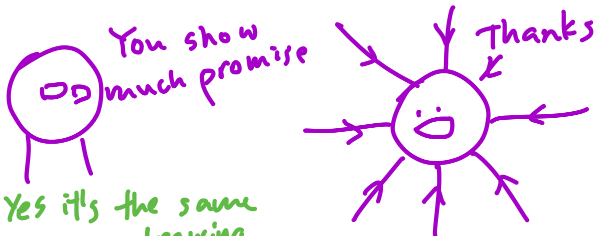
$$E = \frac{V}{d}$$

use it ONLY FOR UNIFORM FIELDS

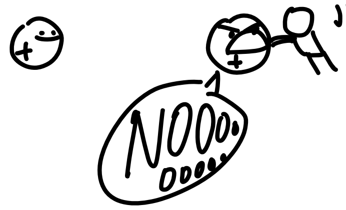
V = p.d. [V] d = separation distance [m]

E = field strength $\underbrace{[NC^{-1}] \text{ or } [Vm^{-1}]}_{\text{the same thing}}$

Electric Potential, V



The work done per unit +ve charge when moving it from infinity to that point in the field



$V = \frac{kQ}{r}$ (just like $V = -\frac{Gm}{r}$)

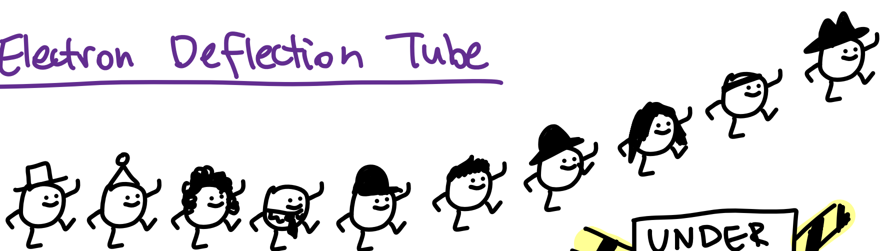
(no -ve sign needed)

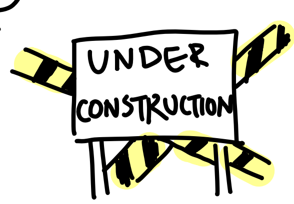
⊕ ⟶ ⊖

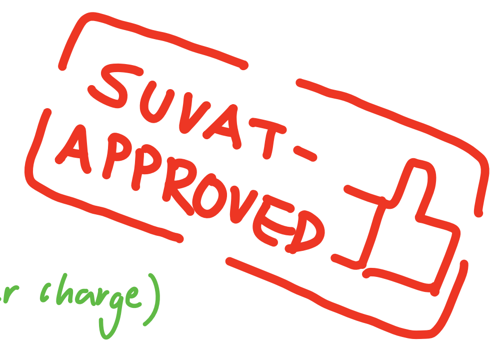
attracts

(potential becomes -ve because Q is -ve)

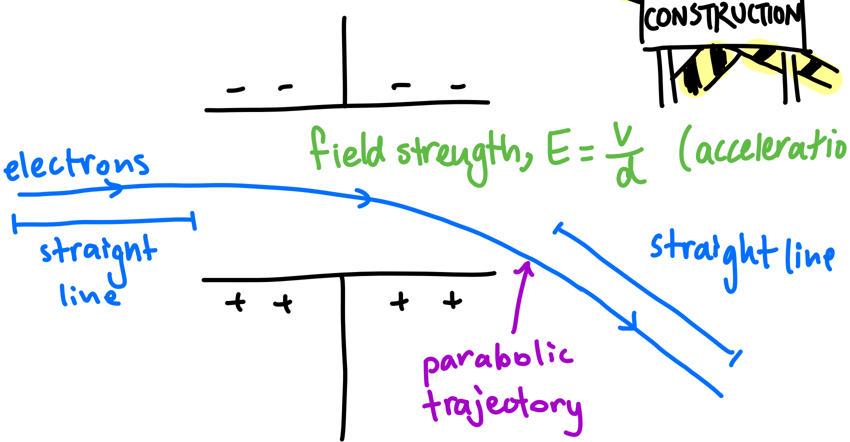
Electron Deflection Tube



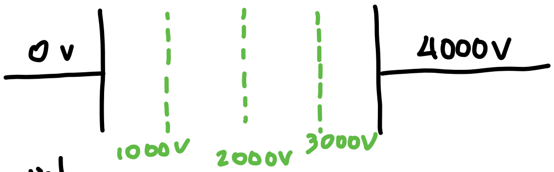


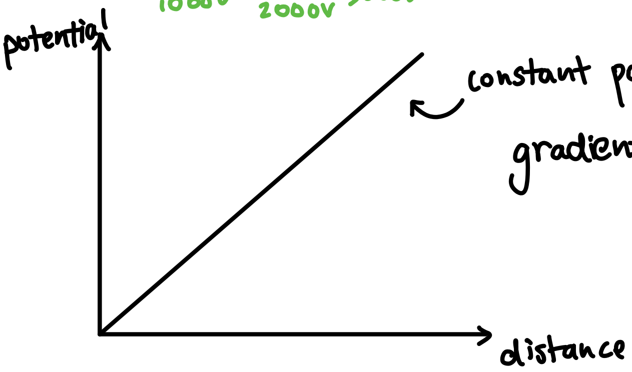


field strength, $E = \frac{V}{d}$ (acceleration per charge)



Uniform fields





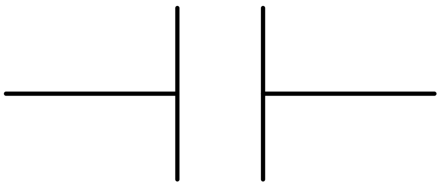
constant potential gradient

gradient = field strength

GRAVITATIONAL VS ELECTRIC FIELDS

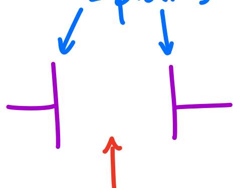
	GRAVITATIONAL	ELECTRIC
Field strength	Force per unit mass $g = \frac{F}{m}$ [Nkg^{-1}] [ms^{-2}]	Force per unit +ve charge $E = \frac{F}{Q}$ [NC^{-1}] [Vm^{-1}]
Field lines	Direction of force on a mass points towards centre of large mass	Direction of force on + charge point from + to − or point from high to low potential
Force between 2 objects	Newton's Law of Gravitation $F = \frac{GMm}{r^2}$ $G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$	~~Newton~~ Coulomb's Law $F = \frac{kQq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ $k = \frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9\ Nm^{-2}C^{-2}$ $\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$
Potential	work per unit mass to move from infinity to that point $V = -\frac{GM}{r}$	work per unit + charge to move from infinity to that point $V = \frac{kQ}{r}$
Potential Energy	$E_p = -\frac{GMm}{r}$	$E_p = \frac{kQq}{r}$
Radial field strength	$g = \frac{GM}{r^2}$	$E = \frac{kQ}{r^2}$
Uniform field strength		$E = \frac{V}{d}$
	weaker attractive only 1 type of mass	stronger attractive/repulsive 2 types of charge

Capacitance



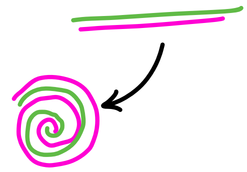
The capacitor
(NOT A CELL!)

2 plates



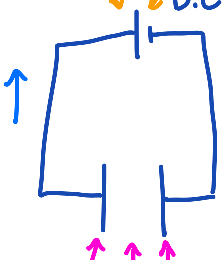
insulator/dielectric
(sometimes air, sometimes something else)
some capacitors have
+ve and -ve terminals
because of this material

usually rolled up into
a cylindrical shape



Charging

+ve -ve
D.C. power supply



e^- move this way

current drops when p.d. across plate = emf

e^- get repelled into +ve terminal of cell.
(plate becomes +ve)

e^- cannot pass through insulator

e^- stored on plate (plate becomes -ve)

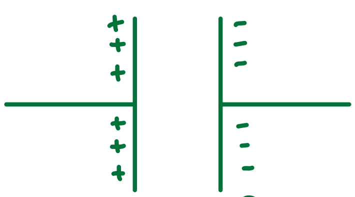
Now the capacitor has a p.d.

+ve -ve

Exam answers: - e^- from -ve terminal of cell move onto one of the plates.

"explain how capacitors work" (≈ 4 marks)

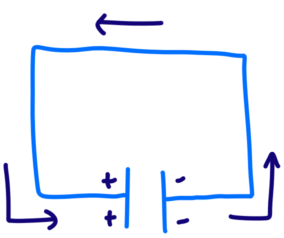
- build up of -ve charge on one plate repels the -ve charges on the other plate.
- as the charge increases, it becomes more difficult for more e^- to move onto the plate
- current drops to 0 when p.d. across capacitor equals emf



+Q charge
-Q charge

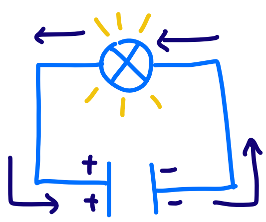
This has stored Q charge

Discharging



connect to itself

electrons flow the other way



connect to appliance

appliance turns on.

current drops when p.d. across plates becomes 0.

Capacitance, C

symbol, not unit (Coulomb is C)

Since the charge the capacitor stores depends on the voltage, capacitance is the amount of charge per volt that a capacitor can store

$$\boxed{C = \frac{Q}{V}}$$

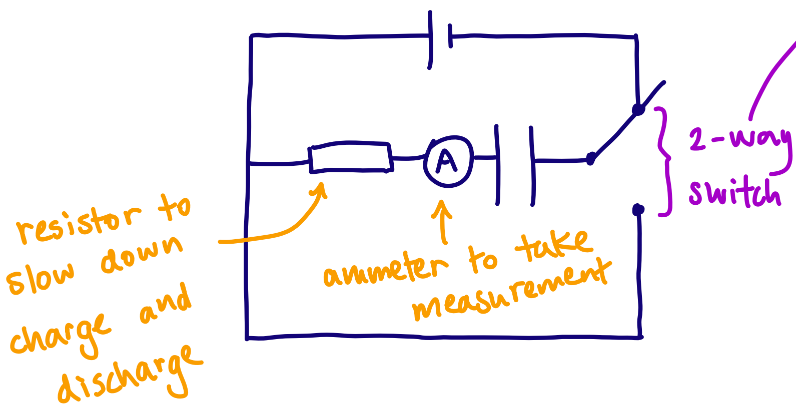
C = capacitance Q = charge V = p.d. across capacitor

or $Q = CV$

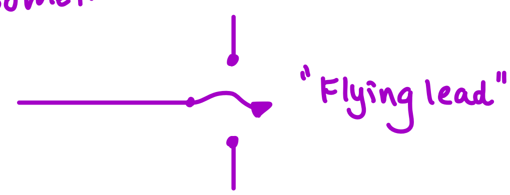
units: Farads, F $[F] = \frac{[C]}{[V]} = [CV^{-1}]$

most capacitors have capacitance of μF

Typical charge/discharge circuit

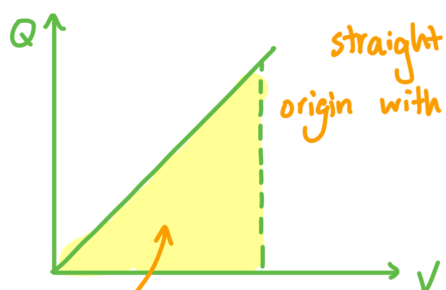


sometimes drawn like this



Energy stored by capacitors

$C = \frac{Q}{V}$



straight line through origin with +ve gradient (capacitance)

energy stored by capacitor $= \frac{1}{2}QV$

or

$\frac{1}{2}CV^2$

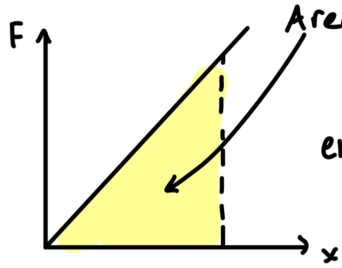
we know the capacitance!

Area = $VQ = E$

uncanny similarity

Sound familiar?

$F = kx$

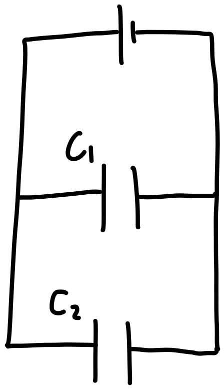


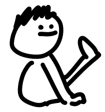
Area = Fx = Work / Energy

energy stored by spring $= \frac{1}{2}Fx$ or $\frac{1}{2}kx^2$

Combining Capacitors (making capacitors useful)

Parallel capacitors



& 

Same p.d. across C_1 & C_2 $(V_T = V_1 = V_2)$

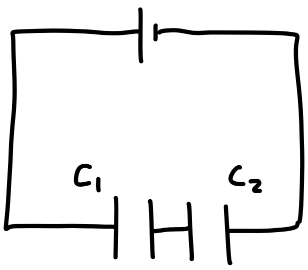
Total charge (Q_T) equals $Q_1 + Q_2$ $(Q_T = Q_1 + Q_2)$

$Q = CV$

$C_T V = C_1 V + C_2 V$

$$\boxed{C_T = C_1 + C_2}$$

Series Capacitors



$V_T = V_1 + V_2$

$Q_T = Q_1 = Q_2$

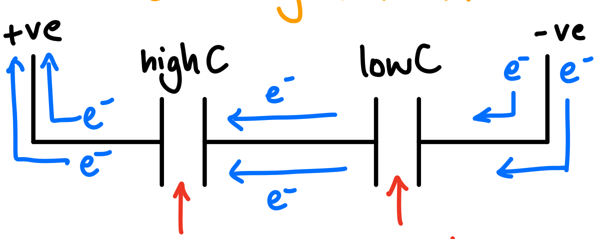
$V = \frac{Q}{C}$

$$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$\boxed{\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}}$$

opposite of resistance calculations!

But why is that?

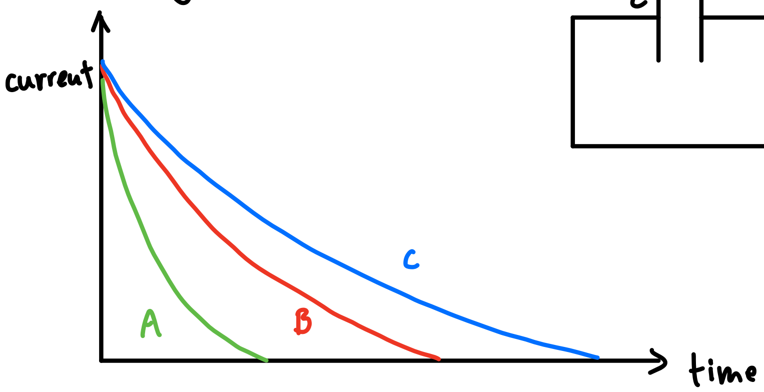


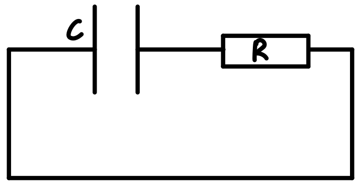
⚠ only $2e^-$ have moved through the cell
↳ total charge, Q_T, is $2e^-$, not $4e^-$

e^- are being repelled

capacitors have the same charge <u>always</u>! even when they have different capacitance

Discharge Curves





$C = 1000\mu F$ $R = 100\,k\Omega$

$C = 1000\mu F$ $R = 50\,k\Omega$

$C = 500\mu F$ $R = 50\,k\Omega$

Both C and R dictate rate of discharge