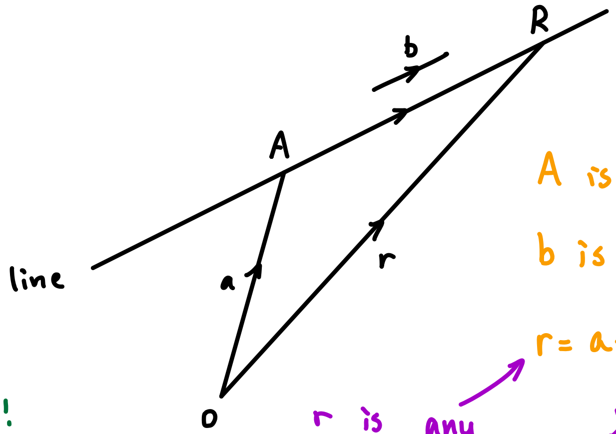
Vectors

Equation of a line



When λ varies from $-\infty$ to ∞, r takes every value on the line!

A is a point on the line

b is the direction of the line

$r = a + \lambda b$

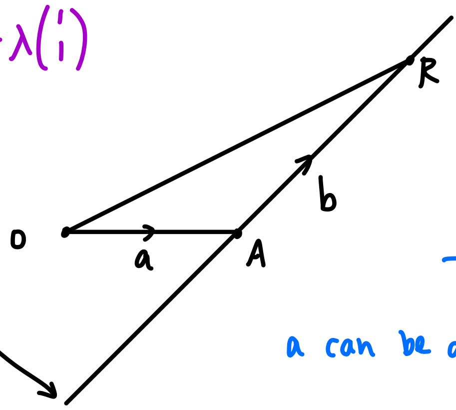
r is any point on the line

λ is a scalar

This equation works for both <u>2D AND 3D</u>

(with 3D vectors in 3D of course)

e.g. $r = \begin{pmatrix}2\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\1\end{pmatrix}$



a line in direction $\begin{pmatrix}1\\1\end{pmatrix}$ and passes through point $\begin{pmatrix}2\\0\end{pmatrix}$

the equation is <u>NOT UNIQUE!</u>

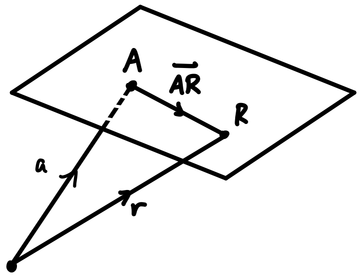
a can be any point the line passes through e.g. $\begin{pmatrix}3\\1\end{pmatrix}$

b can be any vector in that or -ve direction e.g. $\begin{pmatrix}-2\\-2\end{pmatrix}$

Cartesian form of the line:

if $r = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} + \lambda\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$, cartesian equation is $\underbrace{\frac{x-a_1}{b_1}}_{\lambda} = \underbrace{\frac{y-a_2}{b_2}}_{\lambda} = \underbrace{\frac{z-a_3}{b_3}}_{\lambda}$

Equation of a plane (in 3 dimensions obviously)



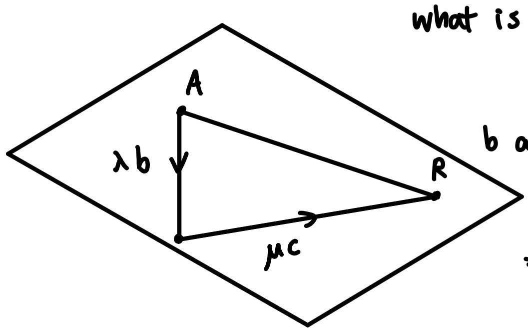
We need an expression $r = \boxed{\ ?\ ?\ ?\ }$ to express any point on the plane

$r = a + \overrightarrow{AR}$

↑

$\overrightarrow{AR}$ lies on the plane.

what is $\overrightarrow{AR}$?



(and non-zero)

b and c are non-parallel AND in the plane

$\Rightarrow$ THEREFORE, $r = a + \lambda b + \mu c$

point on plane   variable scalar

Example 7

$A(2,2,-1)$ $B(3,2,-1)$ $C(4,3,5)$ all lie on the plane

$\overrightarrow{AB}$ is a vector in the plane $= \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\overrightarrow{AC}$ is a vector in the plane $= \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}$

$\therefore$ equation of plane is $r = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}$

point on plane   vectors on plane

Example 8

Plane: $r = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ $P = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$

Verify that P lies on the plane

if a point lies on the plane, it satisfies the plane equation (all 3 equations are consistent)

$r = \begin{pmatrix} 3 + 2\lambda + \mu \\ 4 + \lambda - \mu \\ -2 + \lambda + 2\mu \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ if P lies on the plane

$$\begin{cases} 2\lambda + \mu = -1 & \text{--- (1)} \\ \lambda - \mu = -2 & \text{--- (2)} \\ \lambda + 2\mu = 1 & \text{--- (3)} \end{cases}$$

(1)+(2): $3\lambda = -3$

$\lambda = -1$

$\mu = -1 - 2\lambda = -1 + 2 = 1$

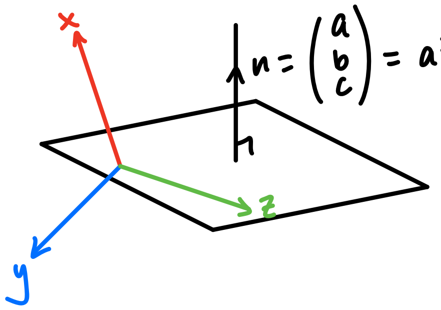
(1) and (2) are consistent

(3) LHS $= \lambda + 2\mu = -1 + 2 = 1 =$ RHS

(3) is consistent with solutions

$\therefore$ P lies on the plane

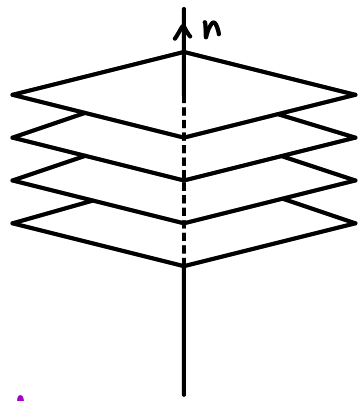
Normal Vector of a Plane



$n = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = ai + bj + ck$

a normal vector n describes infinite number of planes

$\downarrow$



choose a point the plane passes through to pinpoint the correct plane

n describes the direction of the plane

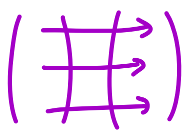
cartesian form of a plane : $ax+by+cz=d$

$\hookrightarrow n = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

Scalar Product

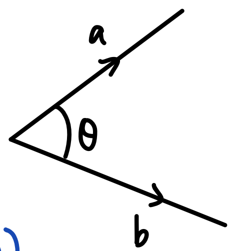
$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix} = ad + be + cf$

vector · vector = scalar



multiply across and add

$a \cdot b = |a||b| \cos\theta$



scalar product is used to find the angle between two vectors.

If $a \perp b$, $a \cdot b = 0$ $(\cos 90° = 0)$ $\rightarrow$ non-zero vectors a and b are $\perp$ <u>if and only if</u> $a \cdot b = 0$

If $a // b$, $a \cdot b = |a||b|$ $(\cos 0° = 1)$

$\Downarrow$

$a \cdot a = |a|^2$