Hypothisis Testing

e.g: Assume: 10% of people are left handed ⟵ hypothesis

At a table of 20, 5 people are left handed ⟵ experiment

↳ use binomial distribution: $X \sim B(20, 0.1)$ ⟵ model

If $P(X \geq 5) < 0.05$ or 5%, ⟵ usually given, but if not, assume 5%

the hypothesis is probably wrong

level of significance (α)

(it is more likely the hypothesis is wrong than us getting ≥5 left handed people out of 20)

Hypothesis testing steps!

1. State hypothesis
2. Collect data
3. Model the data and find probability of this happening (Include more extreme cases!)
4. If $p < \alpha$, hypothesis is rejected!

Null hypothesis? Alternative hypothesis?

$H_0: p = 0.5$ null hypothesis: the initial assumption The coin is fair

$H_1: p > 0.5$ alternative hypothesis: the hypothesis if H_0 is incorrect $P(\text{heads}) > P(\text{tails})$

→ biased to one side: "One-tailed test"

5% students turn up late to school!

6 of 40 students are late! ($\alpha = 0.1$)

$$\underbrace{\begin{aligned} H_0&: p = 0.05 \\ H_1&: p > 0.05 \end{aligned}}_{\text{hypotheses}}$$

X = # of students late ⟵ test statistic

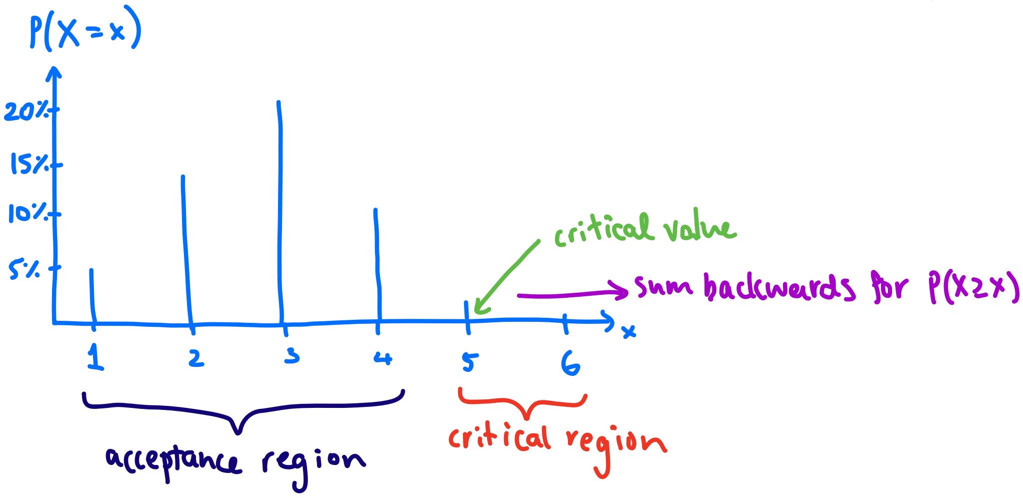
$X \sim B(40, 0.05)$

$$\left.\begin{aligned} P(X \geq 6) &= 1 - P(X < 6) \\ &= 1 - P(X \leq 5) \\ &= 1 - 0.986 \\ &= 0.014 < \alpha \end{aligned}\right\} \text{hypothesis test}$$

Critical Region: If P is within this region, reject null hypothesis ($< \alpha$)

Critical Value: First value of P within the critical region.

Actual Significance level: probability of incorrectly rejecting the null hypothesis



Summary

Scenario: Coin flip: 6/8 heads. Is it biased?

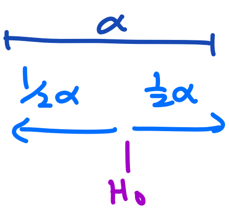
Steps:

1. Define test statistic (X) and parameter (P)
2. Write H_0 & H_1
3. Determine probability of observed test statistic assuming H_0 is true
4. 2 part conclusion:
   a) Do we reject H_0?
   b) Put in context of original problem

1. X = number of heads
   p = probability of heads
   $X \sim B(8, p)$
2. $H_0: p = 0.5$ $\quad H_1: p > 0.5$
3. Assume H_0 is true, $X \sim B(8, 0.5)$
   $P(X \geq 6) = 1 - P(X \leq 5) = 0.1445 = 14.45\%$
4. a: $14.45\% > 5\%$, ∴ insufficient evidence to reject H_0
   b: We cannot assume the coin is biased towards heads

two-tailed tests

Is this "different"?
keyword



H_1 of 2-tailed tests use $\neq$, not $<$ or $>$

$H_0: p = \frac{1}{3}$ $\quad H_1: p \neq \frac{1}{3}$

when doing 2-tailed tests, take ½ of significance (½ for each "tail")

Mixed Exercise 7 (p.109)

① X= number of times train is late
p= chance of train being late

$X \sim B(7, p)$ $H_0: p = 0.2$ $H_1: p > 0.2$

$P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.852 = 0.148 = 14.8\% > 5\%$

∴ not enough evidence to reject H_0

∴ new company is not more late than the old one

③ a) The tests are independent

b) X= # of cars failing $X \sim (5, 0.3)$ $P(X=0) = 0.16807 = 16.8\%$

c) p= chance of car in garage failing
$X \sim B(10, p)$ $H_0: p = 0.3$ $H_1: p < 0.3$

$P(X \leq 2) = 0.383 = 38.3\% > 5\%$
∴ cannot reject Ho
∴ cannot conclude that garage fails less than national average

⑤ X= # of women using Oriels powder
p= % chance of using oriels powder
$X \sim (20, p)$ $H_0: p = 0.5$ $H_1: p \neq 0.5$

$P(X \geq 12) = 1 - P(X \leq 11) = 1 - 0.868 = 13.2\% > 2.5\%$

∴ cannot reject H_0 ∴ there is evidence to support $p = 0.5$

⑦ a) i) A method of testing a hypothesis by comparing it to the null hypothesis
ii) the first value to fall inside the critical region
iii) the region where the null hypothesis is accepted.

b) X= # of times late $H_0: p = 0.2$ when $P(X) < 5\%$ or 0.05, critical
p= chance of late $H_1: p \neq 0.2$ critical region: $P(X=0)$
$X \sim (20, p)$