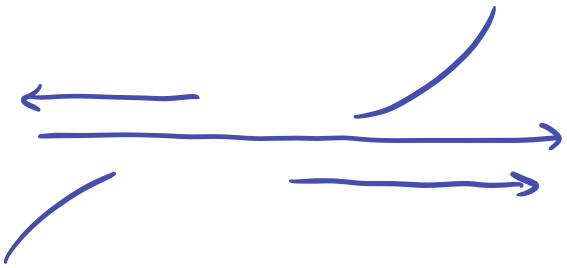
Cubic polynomials

$f(x) = ax^3 + bx^2 + cx + d$

assume $a > 0$

when $x \to \infty$, $f(x) \to \infty$



when $x \to -\infty$, $f(x) \to -\infty$

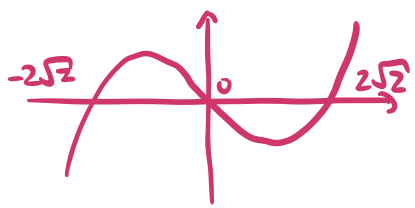
SKETCHING

Label all the intercepts and that's it!

$y = x^3 - 8x$

$= x(x^2 - 8) = 0 \Rightarrow$ roots are $0, \pm 2\sqrt{2}$

sketch:



No need to label vertexes of cubic or higher degree polynomials.

Quartic polynomials

$f(x) = ax^4 + bx^3 + cx^2 + dx + e$

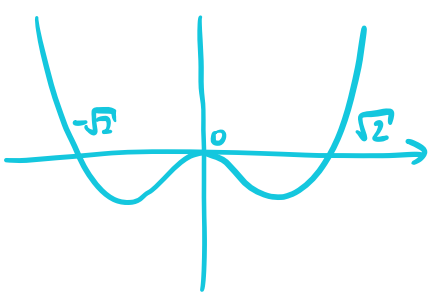
assume $a > 0$

when $x \to \infty$, $f(x) \to \infty$

when $x \to -\infty$, $f(x) \to \infty$

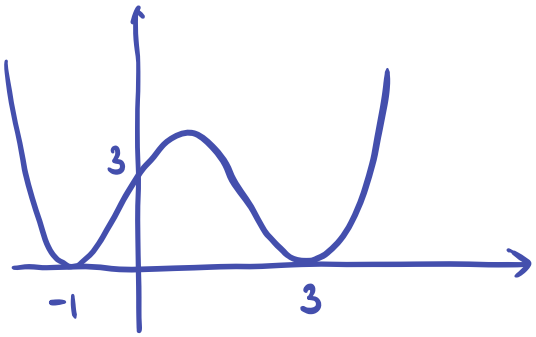
sketch $y = x^4 - 2x^2$

$= x^2(x^2 - 2)$



P. 66 challenge!

$f(x) = ax^4 + bx^3 + cx^2 + dx + e$



$e = 3$ $\quad f(x) = ax^4 + bx^3 + cx^2 + dx + 3$

$= A(x+1)^2(x-3)^2$

$= A(x^2+2x+1)(x^2-6x+9)$

$= A(x^4 - 6x^3 + 9x^2 - 12x^2 + 18x + x^2 - 6x + 9)$

$= A(x^4 - 6x^3 - 2x^2 + 12x + 9)$

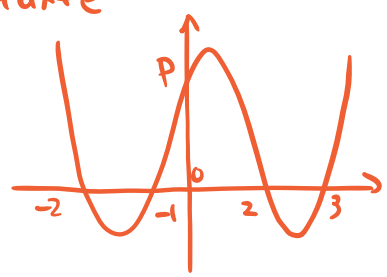
$= Ax^4 - 6Ax^3 - 2Ax^2 + 12Ax + 9A$

$9A = 3$ $\quad$ by comparing coefficients,

$A = \frac{1}{3}$ $\quad a = \frac{1}{3}$, $b = -2$, $c = 2/3$, $d = 4$, $e = 3$

Ex 4B (p. 66)

$y = x^4 + bx^3 + cx^2 + dx + e$



a $\quad y = (x+2)(x+1)(x-2)(x-3)$

$= (x^2-4)(x^2-2x-3)$

$= (x^4 - 2x^3 - 3x^2 - 4x^2 + 8x + 12)$

$= x^4 - 2x^3 - 7x^2 + 8x + 12$

$\therefore P(0, 12)$

b $\quad \begin{cases} b = -2 \\ c = -7 \\ d = 8 \\ e = 12 \end{cases}$

TRANSFORMATIONS

Translation

$f(x) \mapsto f(x-a)$ horizontal translation

"mapped to" ie translated to

$f(x) \mapsto f(x)+a$ vertical translation

Stretching $(a \neq -1)$

$f(x) \mapsto f(ax)$ horizontal "stretch" by factor $\frac{1}{a}$

⚠ If $a < 1$, it is a "compression"

$f(x) \mapsto af(x)$ vertical "stretch" by factor $\frac{1}{a}$

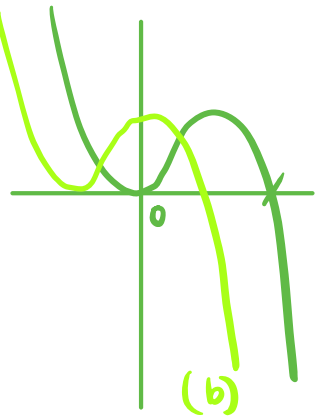
Reflection $(a = -1)$

$f(x) \mapsto f(ax) = f(-x)$ reflect in y-axis

$f(x) \mapsto af(x) = -f(x)$ reflect in x-axis

Ex 4E

③ a) $f(x) = x^2(1-x)$



c) $f(x+1) = (x^2+2x+1)(1-x-1)$

$= -x^3 - 2x - x = 0$

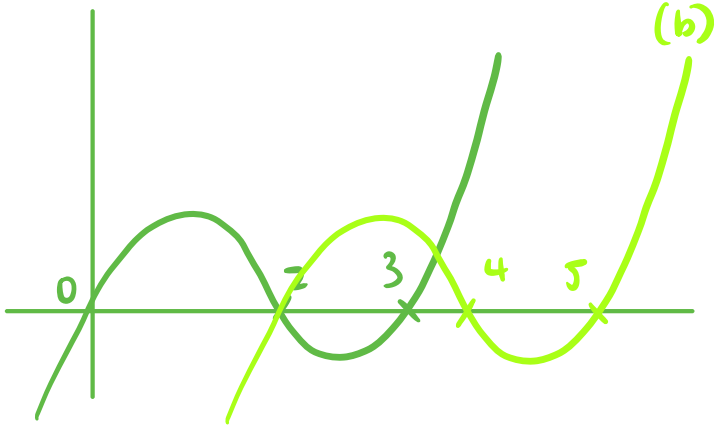
$x(x^2+2x+1) = 0$

$x = 0, -1$

⑨ a) $f(x) = x^3 - 5x^2 + 6x$

$= x(x^2 - 5x + 6)$

$= x(x-2)(x-3)$



① $f(x) = x^3$ point of inflection = (0,0)

$\downarrow$

$f(x-3)+2 = (x-3)^3 + 2$ point of inflection = (3,2)

② Q(-5,-7) on $y = f(x)$

a) $Q'(-5-2, -7-5) = Q'(-7, -12)$

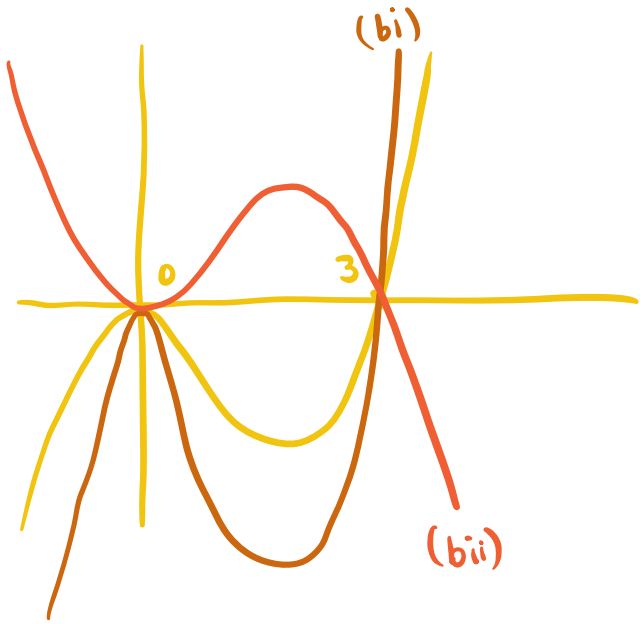
b) $Q''(-3, -6)$

translation $= (-3-(-5), -6-(-7)) = (2, 1)$

$\therefore \quad y = f(x-2) + 1$

Ex 4F (p.78)

④ a) $y = x^2(x-3)$



Ex 4G (p.80)

② a) $f(x)+2$ •

c) $2f(x)$ •

f) $f(\frac{1}{2}x)$ •

g) $\frac{1}{2}f(x)$ •