Radians

θ rad or $\theta^c \longrightarrow \theta$ radians

Degrees to Radians

π rad $\equiv 180^\circ$

$$\text{Degrees} \underset{\times \frac{180}{\pi}}{\overset{\times \frac{\pi}{180}}{\rightleftarrows}} \text{Radians}$$

Ex 5A p.116

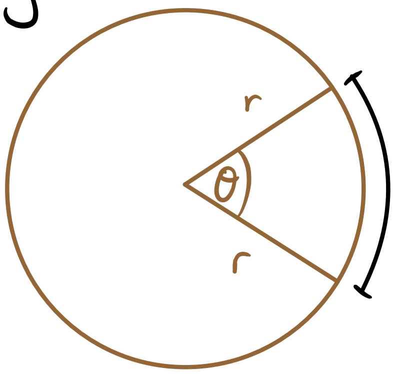
1 (a) $\frac{\pi}{20}$ rad $= \left(\frac{\pi}{20} \times \frac{180}{\pi}\right)^\circ = 9^\circ$

(f) 3π rad $= \left(3\pi \times \frac{180}{\pi}\right)^\circ = 540^\circ$

4 (f) $240^\circ = \left(240 \times \frac{\pi}{180}\right)$ rad $= \frac{4\pi}{3}$ rad

A lot of this chapter is the same stuff in book 1 trigonometry (but in radians).

Arc length



Arc length = $r\theta$

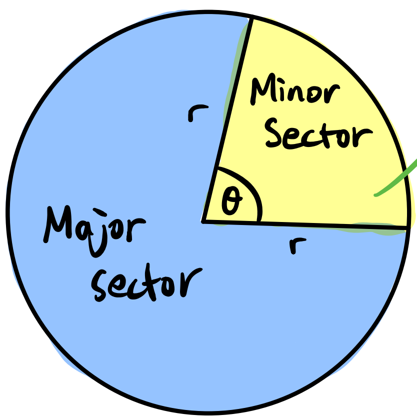
Only if θ is in radians?

Why?

$$\text{Arc length} = 2\pi r \times \frac{\theta}{360^\circ} = 2\pi r \times \frac{\theta \times \frac{\pi}{180}}{2\pi} = r \times \overbrace{\theta \times \frac{\pi}{180}}^{\theta \text{ in radians}} = r \times \theta^c$$

convert to radians

Sectors

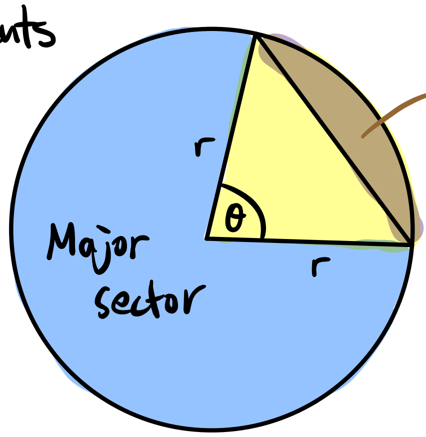


Area of sector

$= \frac{1}{2}r^2\theta$

θ still in radians

Segments



Area of Segment

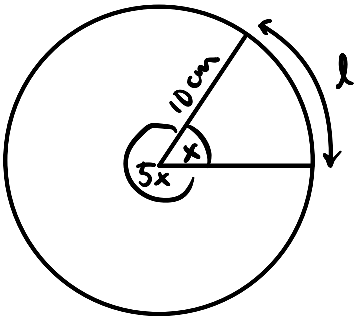
= area of sector - area of isosceles Δ

$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$

$= \frac{1}{2}r^2(\theta - \sin\theta)$ } Again, θ is in radians

Ex 5C p.120

2.

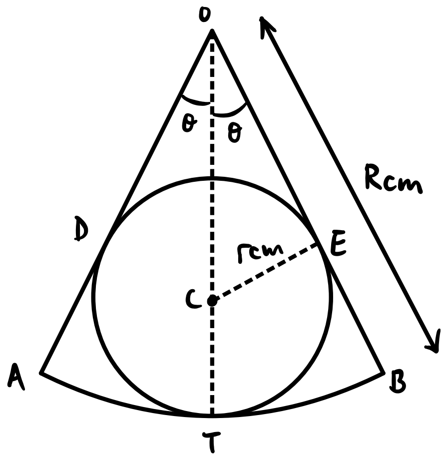


$5x + x = 2\pi$

$x = \frac{\pi}{3}$ rad

$l = r\theta = 10 \times \frac{\pi}{3} = \frac{10}{3}\pi$ cm

10



(a) $OC = OT - CT$

$= OB - CT$

$= R - r$

(b) $\sin\theta = \frac{r}{R-r}$

$R\sin\theta - r\sin\theta = r$

$R\sin\theta = r(1+\sin\theta)$

(c) if $\sin\theta = \frac{3}{4}$, $\theta = \sin^{-1}\left(\frac{3}{4}\right)$

Perimeter of OAB $= R\theta = R\sin^{-1}\left(\frac{3}{4}\right) + 2R = 21$cm

$R = \frac{21}{\sin^{-1}\left(\frac{3}{4}\right)+2} = 7.37$ cm

$r = \frac{R\sin\theta}{1+\sin\theta} = \frac{7.37\left(\frac{3}{4}\right)}{1+\frac{3}{4}}$

$= 3.16$ cm

Small Angle Approximations

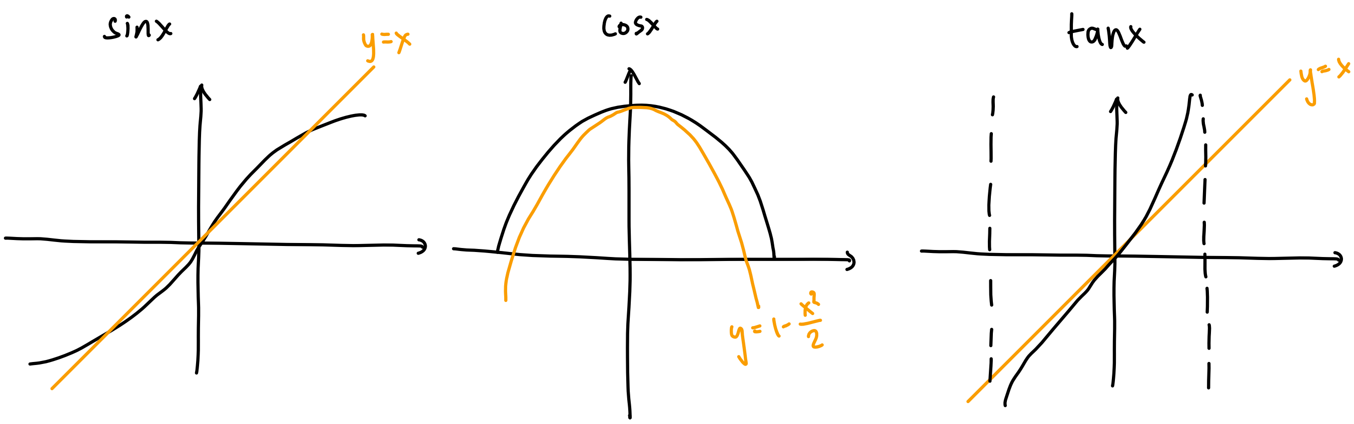
When θ is in radians and θ is small,

These are APPROXIMATIONS! DO NOT USE "=" ! USE "≈"!

$$\left.\begin{aligned} \sin\theta &\approx \theta \\ \cos\theta &\approx 1-\frac{\theta^2}{2} \\ \tan\theta &\approx \theta \end{aligned}\right\}$$

If you plot these on a graph, you can see them get very close!

What is "small"? "Small" = is very close to zero



Taylor/ Maclaurin Expansion (Out of syllabus for now)

$$\left.\begin{aligned} \sin x &= x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\dots \\ \cos x &= 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\dots \\ \tan x &= x+\frac{x^3}{3}+\frac{2x^5}{15}+\dots \end{aligned}\right\}$$

When x is small, The terms are negligible

Ex 5F

Only approximate — Actually equal

1. a) $\frac{\sin 4\theta-\tan 2\theta}{3\theta} \approx \frac{4\theta-2\theta}{3\theta} = \frac{2}{3}$ for small values of θ

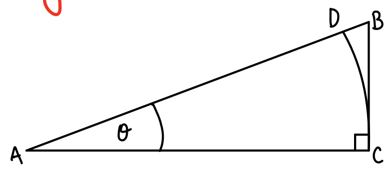
c) $\frac{3\tan\theta-\theta}{\sin 2\theta} \approx \frac{3\theta-\theta}{2\theta} = 1$ for small values of θ

2. a) $\frac{\sin 3\theta}{\theta\sin 4\theta} \approx \frac{3\theta}{\theta 4\theta} = \frac{3}{4\theta}$

c) $\frac{\tan 4\theta+\theta^2}{3\theta-\sin 2\theta} \approx \frac{4\theta+\theta^2}{3\theta-2\theta} = 4+\theta$

Challenge (p. 135)

1.



a) $CD = AC\theta$

b) $\sin\theta = \frac{BC}{AB} \approx \frac{CD}{AD} = \frac{CD}{AC} = \theta$

$\tan\theta = \frac{BC}{AC} \approx \frac{CD}{AC} = \theta$

2. a) $\sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}}$

$= 1 - \frac{1}{2}x^2 + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{1}{2!}\right)x^4 - \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{1}{3!}\right)x^6 + \ldots$

$\approx 1 - \frac{x^2}{2}$

b) $\sin^2\theta + \cos^2\theta = 1$

$\downarrow$

$\cos\theta = \sqrt{1-\sin^2\theta} \approx \sqrt{1-\theta^2}$

$= 1 + \frac{\theta^2}{2}$