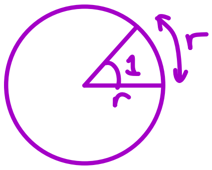
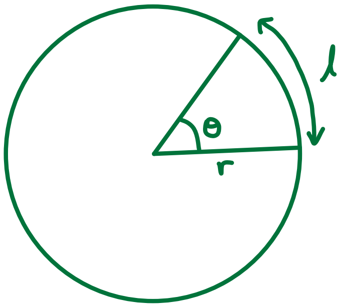
Circular Motion

Radians:

(You know this)

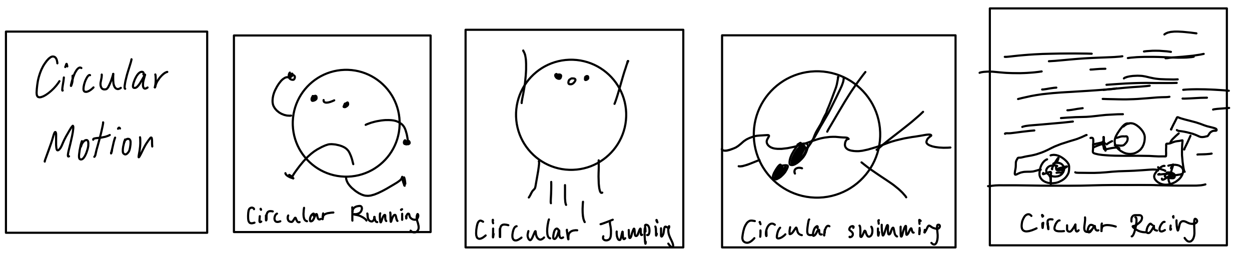


$360^\circ = 2\pi$ rad

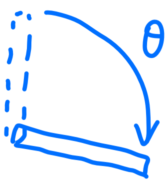


$l = r\theta$

(θ in radians)



Angular Displacement and Velocity

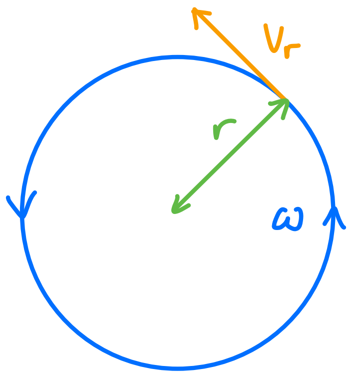


θ radians clockwise

$\omega = \frac{\Delta\theta}{\Delta t} =$  Angular velocity [rad s^{-1}]

Omega Ω's little brother

f vs T → frequency = $\frac{1}{\text{period}}$ $f = \frac{1}{T}$



Circumference = $2\pi r$ = distance

$\omega = \frac{\theta}{t} = \frac{2\pi}{t}$

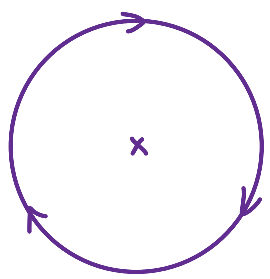
$t = \frac{2\pi}{\omega}$

$v = \frac{\text{distance}}{\text{time}}$

$= \frac{2\pi r}{\frac{2\pi}{\omega}} = 2\pi r\left(\frac{\omega}{2\pi}\right) = \underline{\underline{r\omega}}$

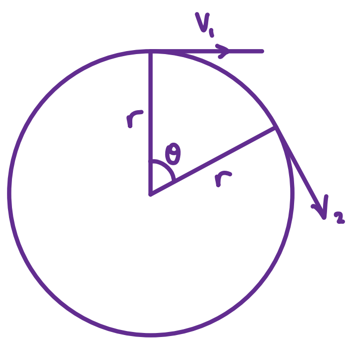
$V_r = r\omega$ velocity of circular motion = radius × angular velocity

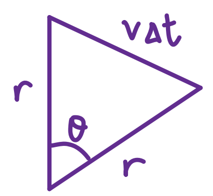
Centripetal Force



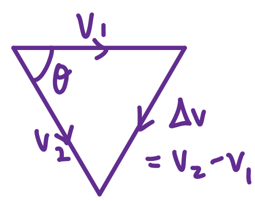
Constant speed

accelerating due to change in velocity (direction)





2 triangles are similar! (when θ is small)



$\frac{\Delta v}{v\Delta t} = \frac{v}{r}$

$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$

$a_c = \frac{v^2}{r}$

Centri → centre
petal → seeking
centre-seeking force

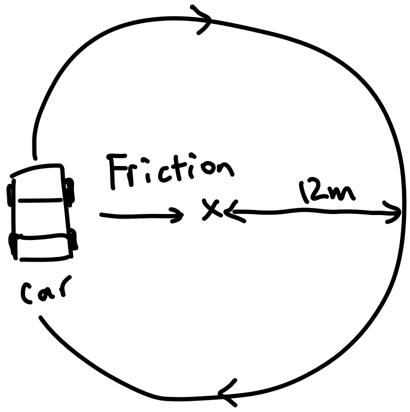
Centripetal Acceleration
towards centre

$v = \omega r \qquad a_c = \frac{\omega^2 r^2}{r} = \omega^2 r$

Conclusion: Magnitude of $a_c = \frac{v^2}{r} = \omega^2 r$

Direction of a_c always toward centre $\qquad F_c = ma_c = \frac{mv^2}{r}$

↳ resultant force (centripetal force) towards centre

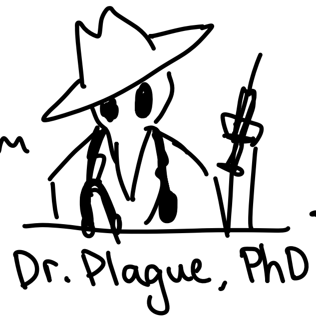


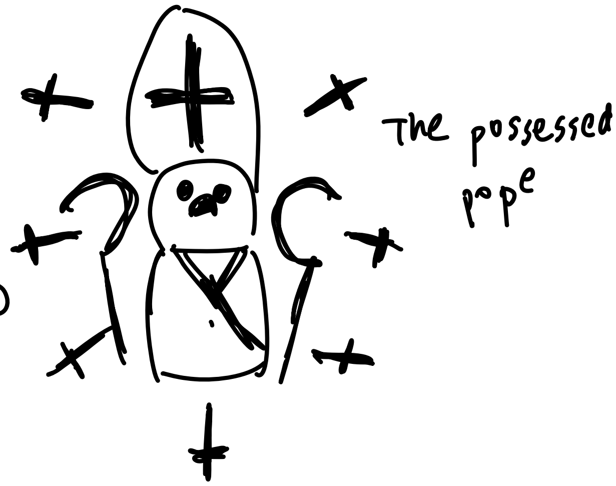
2100kg car

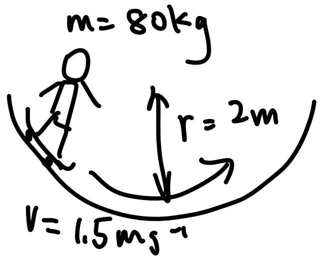
8 seconds to move 190°

$$F = m\omega^2 r = 2100 \times \left(\left(\frac{190^\circ \times 2\pi}{360^\circ}\right) \div 8\right)^2 \times 12$$

$$= 4330\ \text{N}\ (3sf)$$



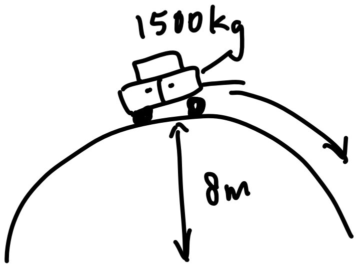




$$F_c = \frac{mv^2}{r} = \frac{80 \times 1.5^2}{2} = 90\text{N} = R - W$$

$$R = W + 90$$

$$= 80 \times 9.8 + 90 = 874\text{N}$$



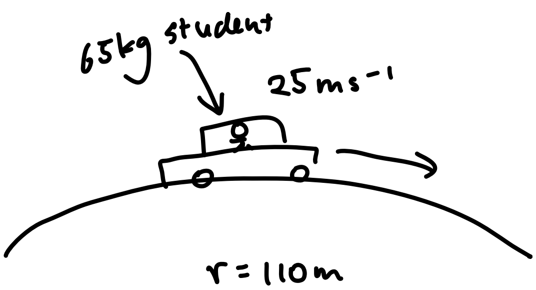
Lose contact when $F_c > W$

$$W = mg = 1500 \times 9.8 = 14700\text{N}$$

$$F_c = \frac{mv^2}{r} = \frac{1500v^2}{8} = 187.5v^2 > 14700$$

$$v^2 > 78.4$$

$$v > 8.85\ \text{ms}^{-1}$$

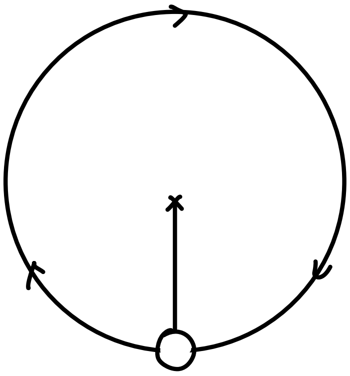


$$f_c = \frac{mv^2}{r} = \frac{65(25)^2}{110} = 369.3\dot{1}\dot{8}\ \text{N}$$

$$W = mg = 65 \times 9.81 = 637.65$$

$$W - R = f_c$$

$$R = W - f_c = 637.65 - 369.3\dot{1}\dot{8} = 268\ \text{N}$$



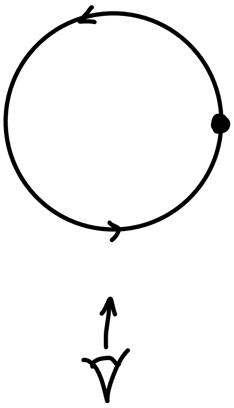
$f_c = \frac{mv^2}{r}$ = constant

Top: $T + mg = f_c$

Bottom: $T - mg = f_c$

T larger at bottom.

Simple Harmonic Motion ☹



particle oscillates

x = displacement from centre = $A\cos\omega t$

A = max displacement / amplitude

velocity $v = \frac{dx}{dt} = -A\omega\sin\omega t$ $\qquad V_{max} = \omega A \qquad a_{max} = \omega^2 A$

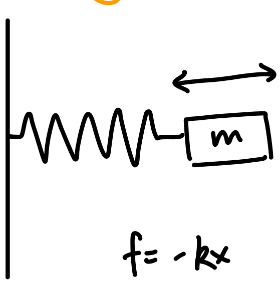
acceleration $a = \frac{dv}{dt} = -A\omega^2\cos\omega t = -\omega^2 x$

condition for SHM

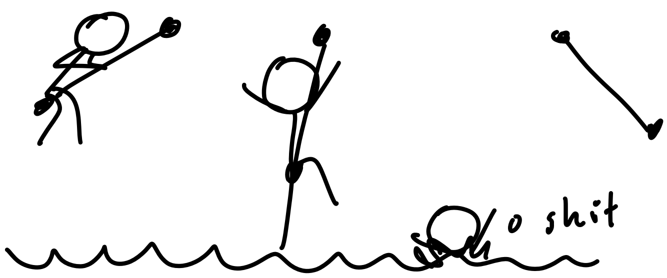
$x_{max} = A \qquad V_{max} = -A\omega \qquad a_{max} = -A\omega^2$

$a \propto x$ a in opposite direction of x

Spring Systems



Horizontal Spring



$f = -kx$

$ma = -kx$

$a = -\frac{kx}{m}$

we also know that $a = -\omega^2 x$

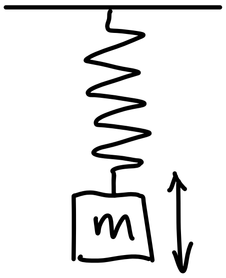
not radius, displacement

$k = m\omega^2$

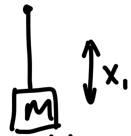
(k = spring constant)

$\omega = 2\pi f = \frac{2\pi}{T}$

Vertical Spring



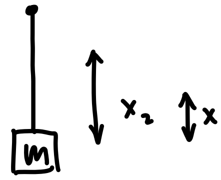
rest length



equilibrium

$F = -kx_1 = -mg$

$a = 0$



extension

$F = -kx_2$

$\Sigma F = F - mg = -kx_2 - mg = ma$

$ma = -kx_2 + kx_1 = -k(x_2 - x_1)$

$ma = -kx$

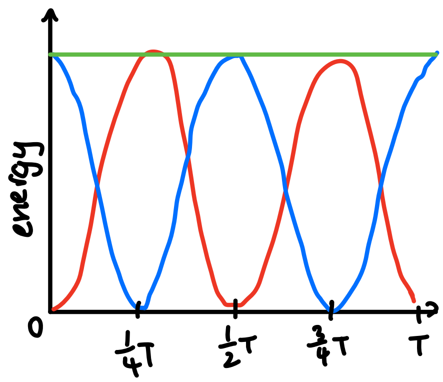
$a = -\frac{k}{m}x$ or $-\omega^2 x$

omegaman

vertical springs also follow SHM

Describing the energy changes of 1 oscillation

high	GPE
mid	KE
low	EPE



total energy

elastic energy

kinetic energy

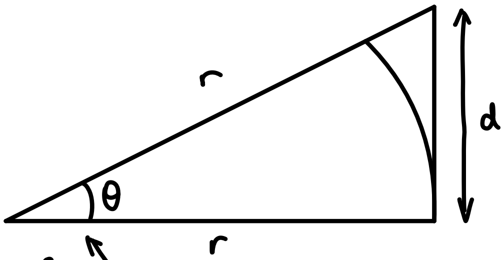
$\downarrow$

$\sin^2$ graph

(v is sin, so KE is $\sin^2$)

$\frac{1}{2}mv^2$

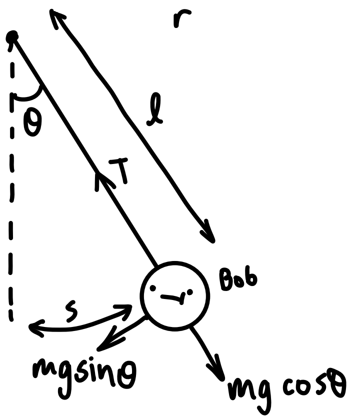
Small angle approximations



arc length $= r\theta \approx d$

$\sin\theta = \frac{d}{r} \approx \frac{r\theta}{r} = \theta$

$\therefore \sin\theta \approx \theta$ (up to $\theta = 0.3$)



$s \approx l\sin\theta \approx l\theta$

$\theta = \frac{s}{l}$

force $= -mg\sin\theta \approx -mg\theta \approx -mg\frac{s}{l}$

acceleration $\approx -g\frac{s}{l}$ or $-\frac{g}{l}s \propto -s$

$\therefore$ SHM :)

$a = -\omega^2 x$ comparing coefficients: $\omega^2 = \frac{g}{l}$

$\omega = \frac{2\pi}{T}$

$\left(\frac{2\pi}{T}\right)^2 = \frac{(2\pi)^2}{T^2} = \frac{g}{l}$

$T^2 = \frac{(2\pi)^2 l}{g}$

$T = 2\pi\sqrt{\frac{l}{g}}$ formula for simple bendulum

(mass is irrelevant!)

Example

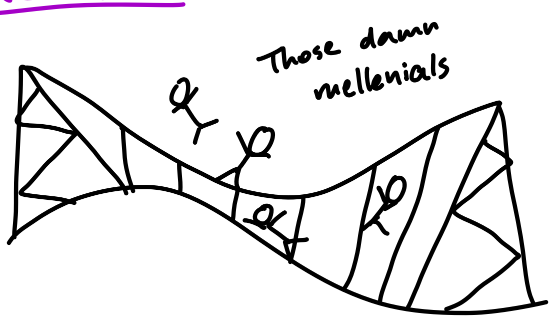
$T = 0.500$

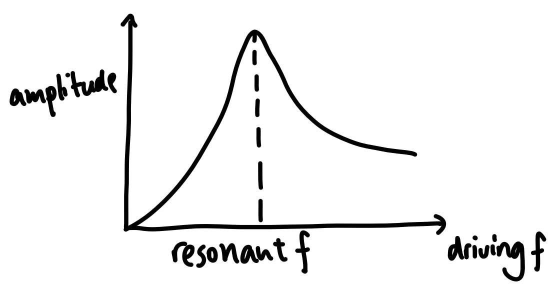
$4\pi^2\left(\frac{l}{g}\right) = T^2$

$l = \frac{T^2 g}{4\pi^2} = \frac{0.5^2 \times 9.81}{4\pi^2} = 0.0621\,m$

$l = 1m$ $T = 2\pi\sqrt{\frac{1}{9.8}} = 2.01s$

Resonance





f_d = driving frequency

f_r = resonant frequency

$f_d \ll f_r \rightarrow$ in phase

$f_d = f_r \rightarrow 90°$ out of phase

$f_d \gg f_r \rightarrow$ antiphase

FREE VIBRATION: system displaced and left to oscillate

FORCED VIBRATION: oscillation due to external periodic driving force.

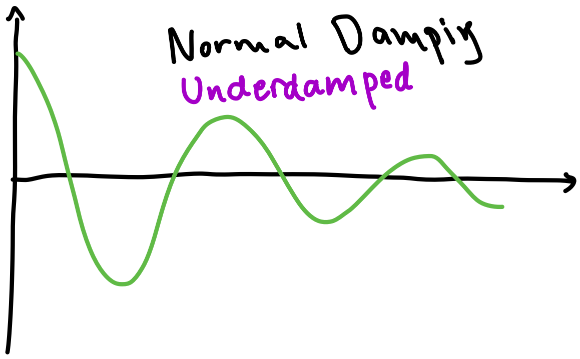
frequency determined by frequency of driving force

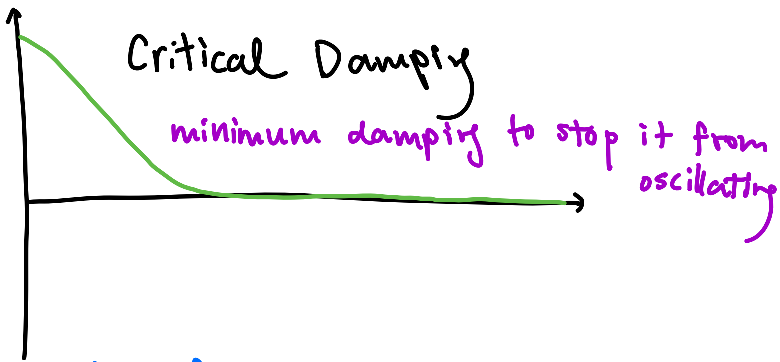
resonance when freq. of driving force = natural freq.

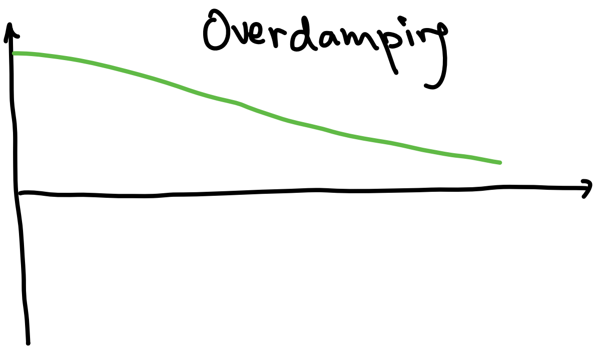
when vibrations of large amplitude produced

4 point question?

DAMPING







When force opposes motion

Reduces sharpness of resonance