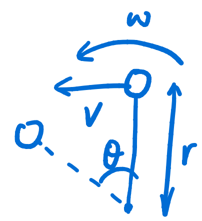
ENGINEERING:

ROTATIONAL DYNAMICS

Recap:



$\omega = \frac{\Delta\theta}{\Delta t} = \frac{v}{r}$ $\quad$ $1\text{ rpm} = 2\pi \text{ rad s}^{-1}$

Angular acceleration, α

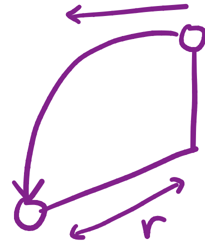
$$\boxed{\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\Delta v}{r\Delta t} \quad [\text{rad s}^{-2}]}$$

$\frac{\Delta v}{\Delta t} = a$ (tangential acceleration)

$$\boxed{a = r\alpha}$$

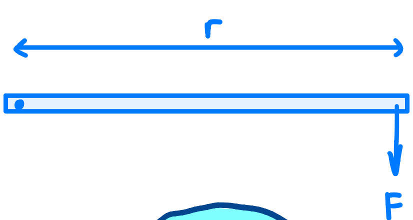
! NOT centrepetal acceleration

tangential velocity changing $\rightarrow \frac{v-u}{t} = a$



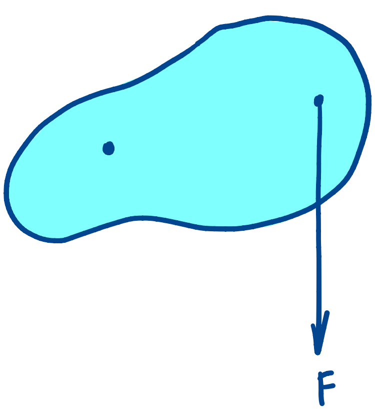
$\frac{\omega_2 - \omega_1}{t} = \alpha$ $\quad$ $a = r\alpha$

MOMENT OF INERTIA, I



Torque (or Moment) = Fr (we know this)

$F = \frac{T}{r}$



Some tiny molecule part of the object recieves force F

$F = ma = mr\alpha$

$\frac{T}{r} = mr\alpha$

$$\boxed{T = mr^2\alpha}$$

Angular version of F=ma

mr^2 is the moment of intertia of this particle

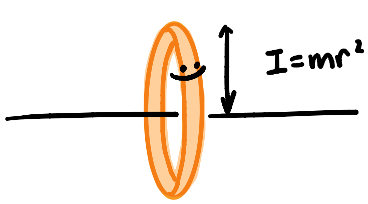
$$\boxed{I = mr^2} \quad [\text{kgm}^2]$$

Moment of inertia of the whole object

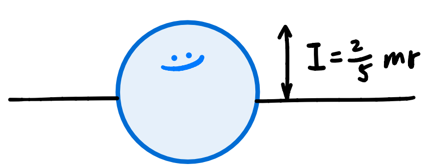
$$\boxed{I = \sum mr^2}$$

$T = I\alpha$

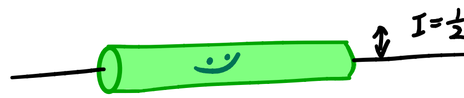
Force $\rightarrow T$, mass $\rightarrow I$, acceleration $\rightarrow \alpha$



$I = mr^2$



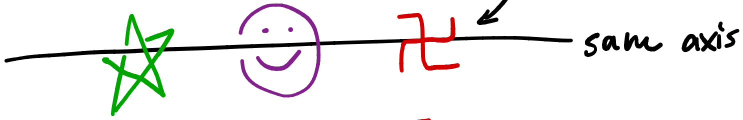
$I = \frac{2}{5}mr^2$



$I = \frac{1}{2}mr^2$

Moment of Inertia of a system

NOT nazis wdym



$I_1 + I_2 + I_3 = I_T$ inertia of the combination

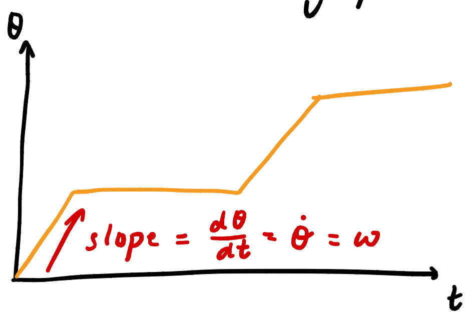
Rotational SUVAT

Original	Ripoff
$v = u + at$	$\omega_2 = \omega_1 + \alpha t$
$v^2 = u^2 + 2as$	$\omega_2^2 = \omega_1^2 + 2\alpha\theta$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$
$s = \frac{(u+v)t}{2}$	$\theta = \frac{(\omega_1+\omega_2)t}{2}$

key

	linear		rotational	
[m]	s	→	θ	[rad]
[ms⁻¹]	u	→	ω_1	[rads⁻¹]
[ms⁻¹]	v	→	ω_2	[rads⁻¹]
[ms⁻²]	a	→	α	[rads⁻²]
[s]	t	→	t	[s]

θ-t graph



ω-t graph

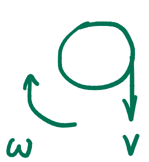
area $= \int \omega \, dt = \theta$

same as s-t and v-t graphs

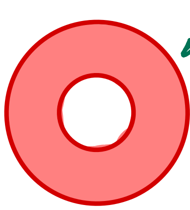
Rotational Kinetic Energy

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}m(r\omega^2) = \frac{1}{2}mr^2\omega^2 = \frac{1}{2}I\omega^2 = \text{Rotational } E_K$$

more ripoffs lmao



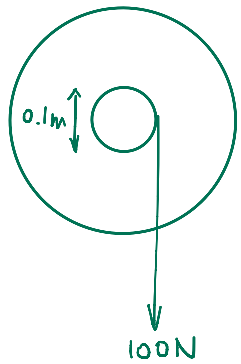
FLYWHEEL:



hoop stores Rotational KE by spinning

Questions (from Rotational Dynamics)

10 a) Flywheel



$T = 100 \times 0.05 = 5\,Nm$

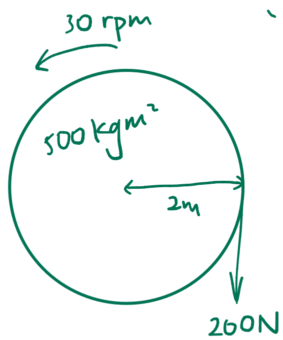
$I = 8.0\,kgm^2$

$T = I\alpha$

$\alpha = \frac{T}{I} = \frac{5}{8} = 0.625\ rads^{-2}$

b) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2 = 0 + \frac{0.625}{2}(30)^2 = 281.25\ rad = \frac{281.25}{2\pi} = 44.76\ rev$

11



a) Initial $\omega = 30 \div 60 \times 2\pi = \pi\ rads^{-1}$

b) $T = Fr = 200 \times 2 = 400\ Nm$

c) $T = I\alpha$

$\alpha = \frac{T}{I} = \frac{400}{500} = 0.8\ rads^{-2}$ deceleration

d) $\omega_2 = \omega_1 + \alpha t$ $\qquad t = \frac{\omega_2 - \omega_1}{\alpha} = \frac{-\pi}{-0.8} = \frac{5}{4}\pi$

$\omega_2^2 = \omega_1^2 + 2\alpha\theta$

$\theta = \frac{\omega_2^2 - \omega_1^2}{2\alpha} = \frac{-(\pi)^2}{-2(0.8)} = 6.1685\ rad \simeq 6.17\ rad = 0.982\ rev$

12 a) $I = mr^2 = 7 \times 1.3^2 = 11.83\ kgm^2$

b) $a = \frac{30}{5} = 6\,ms^{-2}$ $\qquad a = r\alpha$ $\qquad \alpha = \frac{a}{r} = \frac{6}{1.3} = 4.62\ rads^{-2}$

c) $T = I\alpha = 11.83 \times 4.62 = 54.6\ Nm$

d) $I_{arm} = \frac{1}{3}mL^2 = \frac{1}{3} \times 9 \times 0.9^2 = 2.43$

15. a) $\frac{14000 \times 2\pi}{60} = \frac{1400}{3}\pi = 1466\ rads^{-1}$

b) $I = \frac{1}{2}mr^2 = \frac{1}{2} \times 272 \times 0.38^2 = 19.6384\ kgm^2$

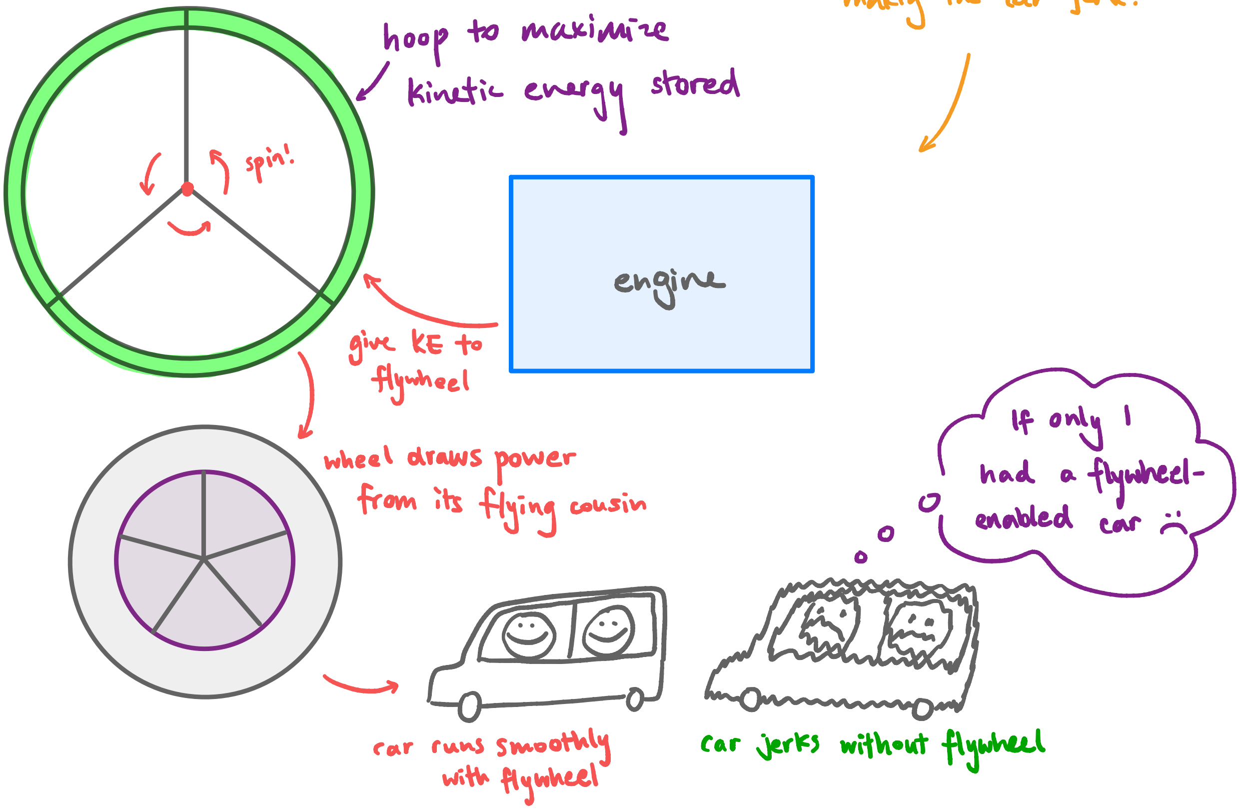
$KE = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 19.6384 \times \left(\frac{1400}{3}\pi\right)^2 = 21105197.14 \simeq 2.11 \times 10^7\ J$

16. a) $I = \frac{2}{5}mr^2 = \frac{2}{5} \times 6 \times 10^{24} \times (6.4 \times 10^6)^2 = 9.8304 \times 10^{37}$

b) K.E. stored in a rotating object.

Applications of FLYWHEELS

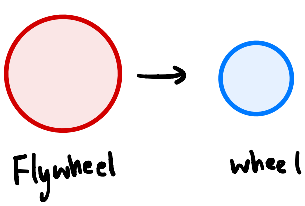
engines produce uneven torque, making the car "jerk".



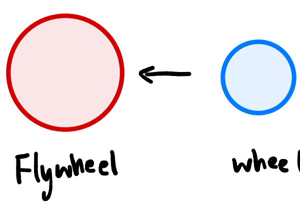
Regenerative braking

Kinetic energy recovery system (KERS) is a system that recovers kinetic energy.

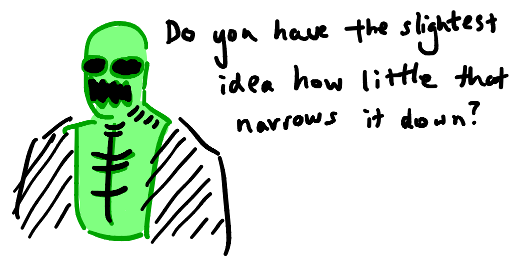
Acceleration



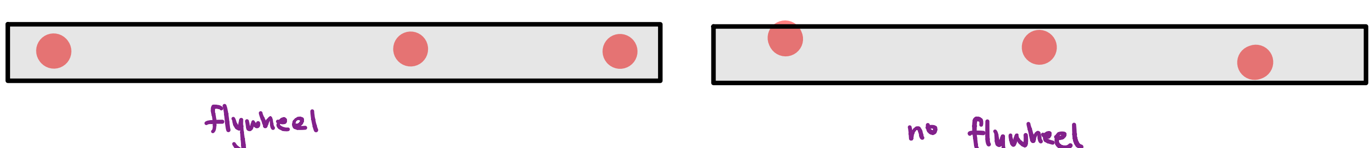
Deceleration



flywheel recovers the KE of decelerating wheel



Flywheels also help with consistent motion during manufacturing of products



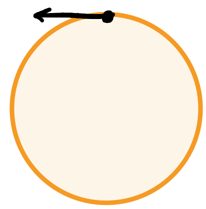
*results may be exaggerated

Angular Momentum, L

Normal momentum

$$\begin{array}{ccccc} p & = & m & & v \\ \updownarrow & & \updownarrow & & \updownarrow \\ L & = & I & & \omega \end{array}$$

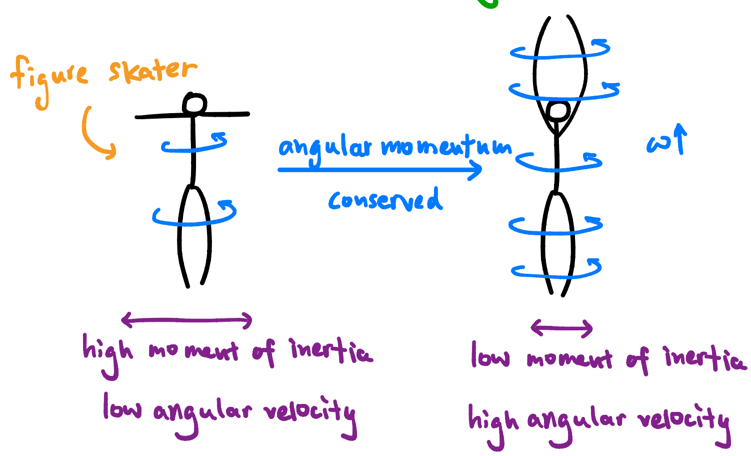
Angular momentum



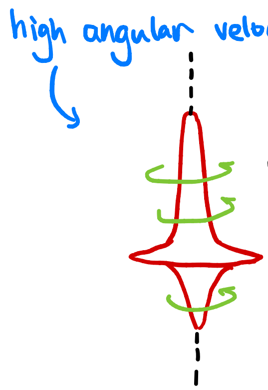
$L = mv \cdot r = m\omega r \cdot r$

$= mr^2 \omega = I\omega$

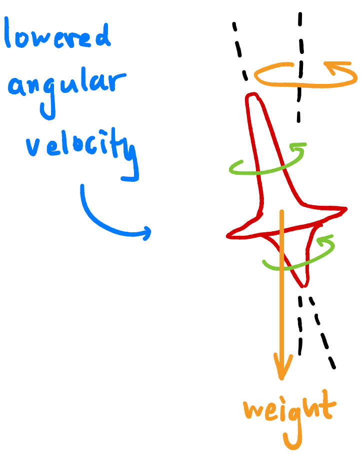
Principle of conservation of angular momentum



Gyroscopes



Axis of rotation is maintained as long as it keeps spinning fast (Conservation of angular momentum)



Axis of rotation rotates around the vertical
↳ "Precession"

Precession direction is same as angular velocity

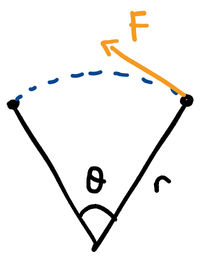
Angular Impulse

$\Delta L = L_2 - L_1 = I(\omega_2 - \omega_1) = I\Delta\omega$

$T = I\alpha = I\dfrac{\Delta\omega}{\Delta t} = \dfrac{\Delta L}{\Delta t} \Rightarrow \Delta L = T\Delta t$

STILL same eqns as linear motion $\Delta p = F\Delta t$ and $\Delta p = m(v-u)$

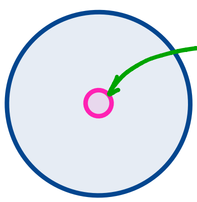
Work and Power in rotating systems



displacement = arclength = $r\theta$

Work = $F \times d = Fr\theta = T\theta$ (just like $W = Fs$. suprised?)

Power = rate of work = $\dfrac{\Delta W}{\Delta t} = \dfrac{T\Delta\theta}{\Delta t} = T\omega$ (just like $P = Fv$. suprised?)



Friction in the axle is called frictional torque (like frictional force)

Let's recap:

HOW BIG OF A RIPOFF IS ROTATIONAL DYNAMICS?

related with SUVAT or TOOAT

Linear motion	Linear quantity	Rotational motion	Rotational quantity	Rotational units
Mass	m	Moment of inertia	I	$kg\,m^{-2}$
Velocity	$v = \dfrac{\Delta s}{\Delta t}$	Angular velocity	$\omega = \dfrac{\Delta\theta}{\Delta t}$	$rad\,s^{-1}$
Acceleration	$a = \dfrac{\Delta v}{\Delta t}$	Angular acceleration	$\alpha = \dfrac{\Delta\omega}{\Delta t}$	$rad\,s^{-2}$
Displacement	s	Angular displacement	θ	rad
Translational kinetic energy	$E_T = \dfrac{1}{2}mv^2$	Rotational kinetic energy	$E_k = \dfrac{1}{2}I\omega^2$	J
Momentum	$p = mv$	Angular momentum	$L = I\omega$	N m s
Force	$F = ma$ $F = \dfrac{\Delta(mv)}{\Delta t}$	Turning moment (torque)	$T = Fr$ $T = I\alpha$ $T = \dfrac{\Delta(I\omega)}{\Delta t}$	N m
Work done	$W = Fs$	Work done	$W = T\theta$	J
Power	$P = Fv$	Power	$P = T\omega$	W
Impulse	$I = Ft$	Angular impulse	$\Delta L = I\omega_2 - I\omega_1$ $\Delta L = T\Delta t$	N m s

Literally everything is