

## **MODULE – 4**

### **4(A) LOAD FLOW ANALYSIS**

#### **SYLLABUS**

**Economic Operation of Power System:** Introduction and Performance curves Economic generation scheduling neglecting losses and generator limits Economic generation scheduling including generator limits and neglecting losses Economic dispatch including transmission losses Derivation of transmission loss formula. Illustrative Examples.

#### **4.1 INTRODUCTION**

One of the earliest applications of on-line centralized control was to provide a central facility, to operate economically, several generating plants supplying the loads of the system. Modern integrated systems have different types of generating plants, such as coal fired thermal plants, hydel plants, nuclear plants, oil and natural gas units etc. The capital investment, operation and maintenance costs are different for different types of plants.

**The operation economics can again be subdivided into two parts:**

- i) Problem of economic dispatch, which deals with determining the power output of each plant to meet the specified load, such that the overall fuel cost is minimized.
- ii) Problem of optimal power flow, which deals with minimum – loss delivery, where in the power flow, is optimized to minimize losses in the system. In this chapter we consider the problem of economic dispatch.

**During operation of the plant, a generator may be in one of the following states:**

- i) Base supply without regulation: the output is a constant.
- ii) Base supply with regulation: output power is regulated based on system load.
- iii) Automatic non-economic regulation: output level changes around a base setting as area control error changes.
- iv) Automatic economic regulation: output level is adjusted, with the area load and area control error, while tracking an economic setting.

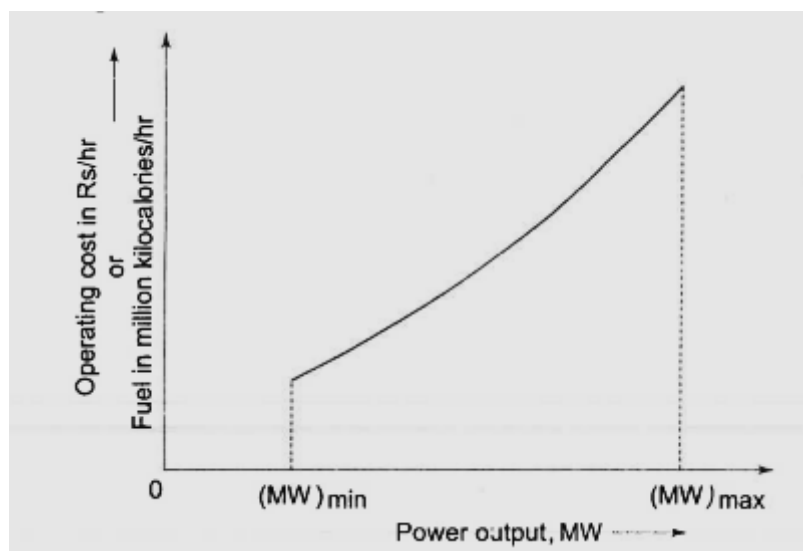
Regardless of the units operating state, it has a contribution to the economic operation, even though its output is changed for different reasons. The factors influencing the cost of generation are the generator efficiency, fuel cost and transmission losses. The most efficient generator may not give minimum cost, since it may be located in a place where fuel cost is high. Further, if the plant is located far from the load centers, transmission losses may be high and running the plant may become uneconomical. The economic dispatch problem basically determines the generation of different plants to minimize total operating cost.

Modern generating plants like nuclear plants, geo-thermal plants etc, may require capital investment of millions of rupees. The economic dispatch is however determined in terms of fuel cost per unit power generated and does not include capital investment, maintenance, depreciation, start-up and shut down costs etc.

## 4.2 PERFORMANCE CURVES

### Input-Output Curve

This is the fundamental curve for a thermal plant and is a plot of the input in British thermal units (Btu) per hour versus the power output of the plant in MW as shown in Fig.4.1.



**Fig .4.1 Input -Output Curve**

The input – output curve of a generating unit specifies the input energy rate  $F_i(P_{Gi})$  in MKcal/h or operating cost or cost of fuel used per hour  $C_i(P_{Gi})$  in Rs./h as a function of the generator power output ( $P_{Gi}$ ) in MW. The input – output curve can be determined experimentally. The input – output curve is concave upwards.

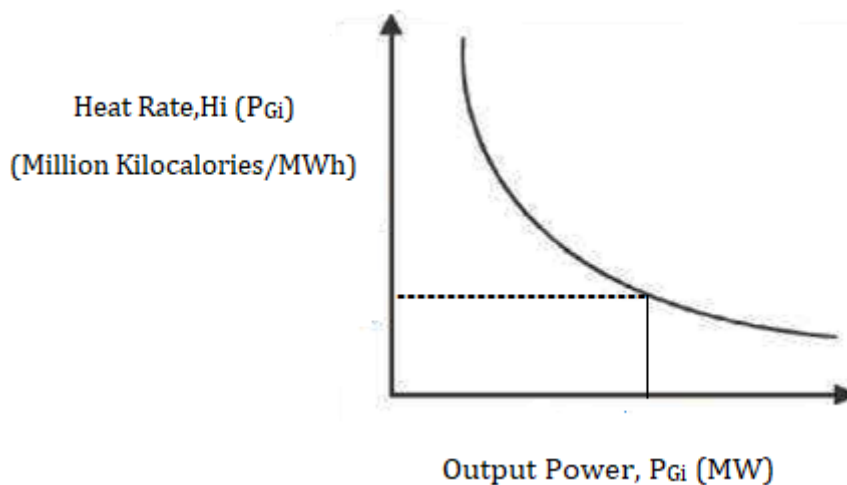
The input – output curve can be obtained from heat rate curve as

$$F_i(P_{Gi}) = P_{Gi} H_i(P_{Gi}) \text{ MKcal/h} \text{ -----(1)}$$

Where  $H_i(P_{Gi})$  is the heat rate in MKcal/MWh.

### Heat Rate Curve

The heat rate is the ratio of fuel input in Btu to energy output in KWh. It is the slope of the input – output curve at any point. The reciprocal of heat – rate is **called fuel –efficiency**. The heat rate curve is a plot of heat rate versus output in MW. A typical plot is shown in Fig.4.2.



**Fig .4.2 Heat Rate Curve**

The heat rate curve  $H_i(P_{Gi})$  which is the heat energy obtained by combustion of fuel in MKcal needed to generate one unit of electric energy (MWh). The approximate shape of heat rate curve can be determined experimentally.

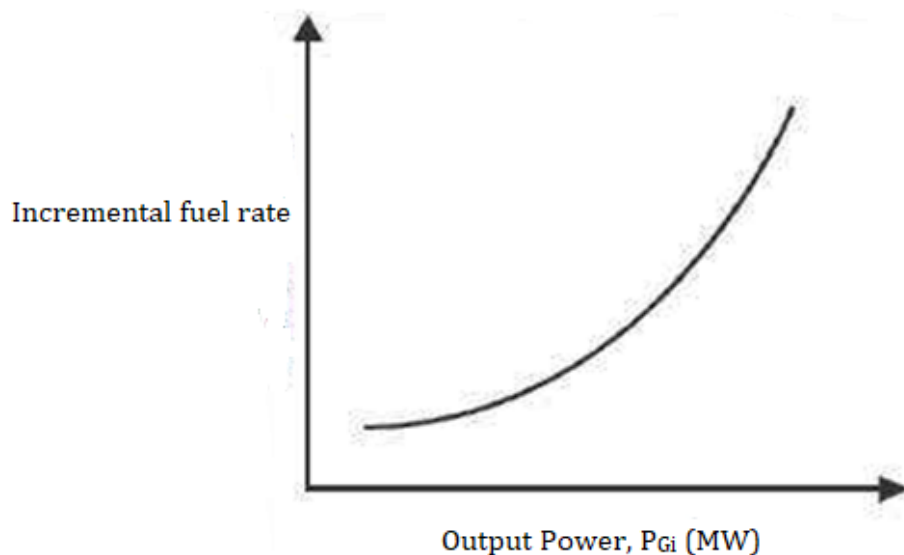
The generating unit efficiency can be defined as the ratio of electric energy output generated to fuel energy input.

#### Incremental Fuel Rate Curve

The incremental fuel rate is equal to a small change in input divided by the corresponding change in output.

$$\text{Incremental Fuel Rate} = \frac{\Delta \text{Input}}{\Delta \text{Output}}$$

The unit is again Btu / KWh. A plot of incremental fuel rate versus the output is shown in Fig. 4.3.



**Fig .4.3 Incremental Fuel Rate Curve**

### Incremental Cost Curve

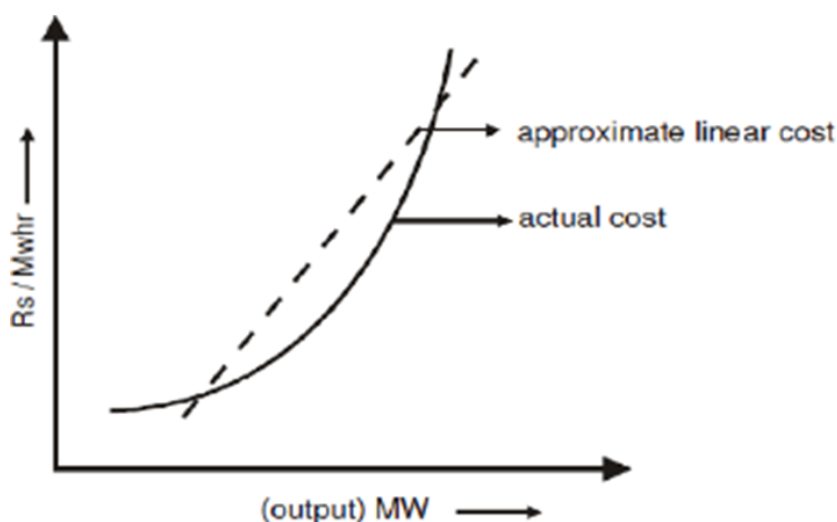
The incremental cost is the product of incremental fuel rate and fuel cost (Rs/Btu or \$/Btu). The curve is shown in Fig. 4.4. The unit of the incremental fuel cost is Rs/MWh or \$/MWh.

In general, the fuel cost  $F_i$  for a plant, is approximated as a quadratic function of the generated output  $P_{Gi}$ .

$$F_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \frac{Rs}{h} \text{ --- (2)}$$

The incremental fuel cost is given by

$$\frac{dF_i}{dP_i} = b_i + 2c_i P_{Gi} \frac{Rs}{MWh} \text{ --- (3)}$$



**Fig .4.4 Incremental Cost Curve**

The incremental fuel cost is a measure of how costly it will be produce an increment of power. The incremental production cost, is made up of incremental fuel cost plus the incremental cost of labour, water, maintenance etc. which can be taken to be some percentage of the incremental fuel cost, instead of resorting to a rigorous mathematical model. The cost curve can be approximated by a linear curve.

While there is negligible operating cost for a hydel plant, there is a limitation on the power output possible. In any plant, all units normally operate between  $P_{Gmin}$ , the minimum loading limit, below which it is technically infeasible to operate a unit and  $P_{Gmax}$ , which is the maximum output limit.

### 4.3 ECONOMIC GENERATION SCHEDULING NEGLECTING LOSSES AND GENERATOR LIMITS

The simplest case of economic dispatch is the case when transmission losses are neglected. The model does not consider the system configuration or line impedances. Since losses are neglected, the total generation is equal to the total demand  $P_D$ . Consider a system with  $n_g$  number of generating plants supplying the total demand  $P_D$ . If  $F_i$  is the cost of plant  $i$  in Rs/h, the mathematical formulation of the problem of economic scheduling can be stated as follows:

$$\text{Minimize } F_T = \sum_{i=1}^{n_g} F_i \text{ --- (4)}$$

$$\text{such that } \sum_{i=1}^{n_g} P_{Gi} - P_D = 0$$

where  $F_T$  = Total Cost

$P_{Gi}$  = Generation of Plant  $i$

$P_D$  = Total Demand

This is a constrained optimization problem, which can be solved by Lagrange's method.

### LAGRANGE METHOD FOR SOLUTION OF ECONOMIC SCHEDULE

The problem is restated below:

$$\text{Maximize } F_T = \sum_{i=1}^{n_g} F_i \text{ --- (5)}$$

$$\text{such that } P_D - \sum_{i=1}^{n_g} P_{Gi} = 0$$

The augmented cost function is given by

$$\mathcal{E} = F_T + \lambda \left( P_D - \sum_{i=1}^{n_g} P_{Gi} \right) \text{ --- (6)}$$

The minimum is obtained when

$$\frac{\partial \mathcal{E}}{\partial P_{Gi}} = 0 \text{ and } \frac{\partial \mathcal{E}}{\partial \lambda} = 0 \text{ --- (7)}$$

$$\frac{\partial \mathcal{E}}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda = 0 \text{ --- (8)}$$

$$\frac{\partial E}{\partial \lambda} - P_D - \sum_{i=1}^{n_g} P_{Gi} = 0 \text{ --- (9)}$$

The second equation is simply the original constraint of the problem. The cost of a plant  $F_i$  depends only on its own output  $P_{Gi}$ , hence

$$\frac{\partial F_T}{\partial P_{Gi}} = \frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}} \text{ --- (10)}$$

Using the above,

$$\frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}} = \lambda ; i = 1, \dots, n_g \text{ --- (11)}$$

We can write

$$b_i + 2c_i P_{Gi} = \lambda ; i = 1, \dots, n_g \text{ --- (12)}$$

The above equation is **called the co-ordination equation**. Simply stated, for economic generation scheduling to meet a particular load demand, when transmission losses are neglected and generation limits are not imposed, all plants must operate at equal incremental production costs, subject to the constraint that the total generation be equal to the demand. From we have

$$P_{Gi} = \frac{\lambda - b_i}{2c_i} \text{ --- (13)}$$

We know in a loss less system,

$$\sum_{i=1}^{n_g} P_{Gi} = P_D \text{ --- (14)}$$

Substituting eqn.(13) in eqn.(14), we get

$$\sum_{i=1}^{n_g} \frac{\lambda - b_i}{2c_i} = P_D \text{ --- (15)}$$

An analytical solution of  $\lambda$  is obtained from eqn.(15) as

$$\lambda = \frac{P_D + \sum_{i=1}^{n_g} \frac{b_i}{2c_i}}{\sum_{i=1}^{n_g} \frac{1}{2c_i}} \text{ --- (16)}$$

It can be seen that  $\lambda$  is dependent on the demand and the coefficients of the cost function.

#### 4.4 ECONOMIC GENERATION SCHEDULING INCLUDING GENERATOR LIMITS AND NEGLECTING LOSSES

The power output of any generator has a maximum value dependent on the rating of the generator. It also has a minimum limit set by stable boiler operation. The economic dispatch problem now is to schedule generation to minimize cost, subject to the equality constraint.

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

and the inequality constraint

$$P_{Gi(min)} \leq P_{Gi} \leq P_{Gi(max)} ; i = 1, \dots, n_g$$

The procedure followed is same as before i.e. the plants are operated with equal incremental fuel costs, till their limits are not violated. As soon as a plant reaches the limit (maximum or minimum) its output is fixed at that point and is maintained a constant. The other plants are operated at equal incremental costs.

#### 4.5 ECONOMIC DISPATCH INCLUDING TRANSMISSION LOSSES

When transmission distances are large, the transmission losses are a significant part of the generation and have to be considered in the generation schedule for economic operation.

The mathematical formulation is now stated as

$$\text{Minimize } F_T = \sum_{i=1}^{n_g} F_i \text{ --- (17)}$$

$$\text{such that } \sum_{i=1}^{n_g} P_{Gi} = P_D + P_L$$

Where  $P_L$  is the total loss.

The Lagrange function is now written as

$$\mathcal{E} = F_T - \lambda \left( \sum_{i=1}^{n_g} P_{Gi} - P_D - P_L \right) = 0 \text{ --- (18)}$$

The minimum point is obtained when

$$\frac{\partial \mathcal{E}}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda \left( 1 - \frac{\partial P_L}{\partial P_{Gi}} \right) = 0 ; i = 1, \dots, n_g \text{ --- (19)}$$

$$\frac{\partial E}{\partial \lambda} = \left( \sum_{i=1}^{n_g} P_{Gi} - P_D + P_L \right) = 0 \text{ (same as the constraint)}$$

since  $\frac{\partial F_T}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$  can be written as

$$\frac{dF_i}{dP_{Gi}} + \lambda \frac{\partial P_L}{\partial P_{Gi}} = \lambda \text{ --- (20)}$$

$$\therefore \lambda = \frac{dF_i}{dP_{Gi}} \left( \frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}} \right) \text{ --- (21)}$$

The term  $\left( \frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}} \right)$  is **called the penalty factor** of plant  $i$  ( $L_i$ ).

The coordination equations including losses is given by,

$$\lambda = \frac{dF_i}{dP_{Gi}} L_i; \quad i = 1, \dots, n_g \text{ --- (22)}$$

The minimum operation cost is obtained when the product of the incremental fuel cost and the penalty factor of all units is the same, when losses are considered.

A rigorous general expression for the loss  $P_L$  is given by

$$P_L = \sum_m \sum_n P_{Gm} B_{mn} P_{Gn} + \sum_n P_{Gn} B_{no} + B_{oo} \text{ --- (23)}$$

Where  $B_{mn}$ ,  $B_{no}$ ,  $B_{oo}$  **called loss - coefficients**, depend on the load composition. The assumption here is that the load varies linearly between maximum and minimum values.

A simpler expression is

$$P_L = \sum_m \sum_n P_{Gm} B_{mn} P_{Gn} \text{ --- (24)}$$

The expression assumes that all load currents vary together as a constant complex fraction of the total load current. Experiences with large systems has shown that the loss of accuracy is not significant if this approximation is used. An average set of loss coefficients may be used over the complete daily cycle in the coordination of incremental production costs and incremental transmission losses.

In general,  $B_{mn} = B_{nm}$  and can be expanded for a two plant system as

$$P_L = B_{11} P_{G1} + 2 B_{12} P_{G1} P_{G2} + B_{22} P_{G2}^2 \text{ --- (23)}$$

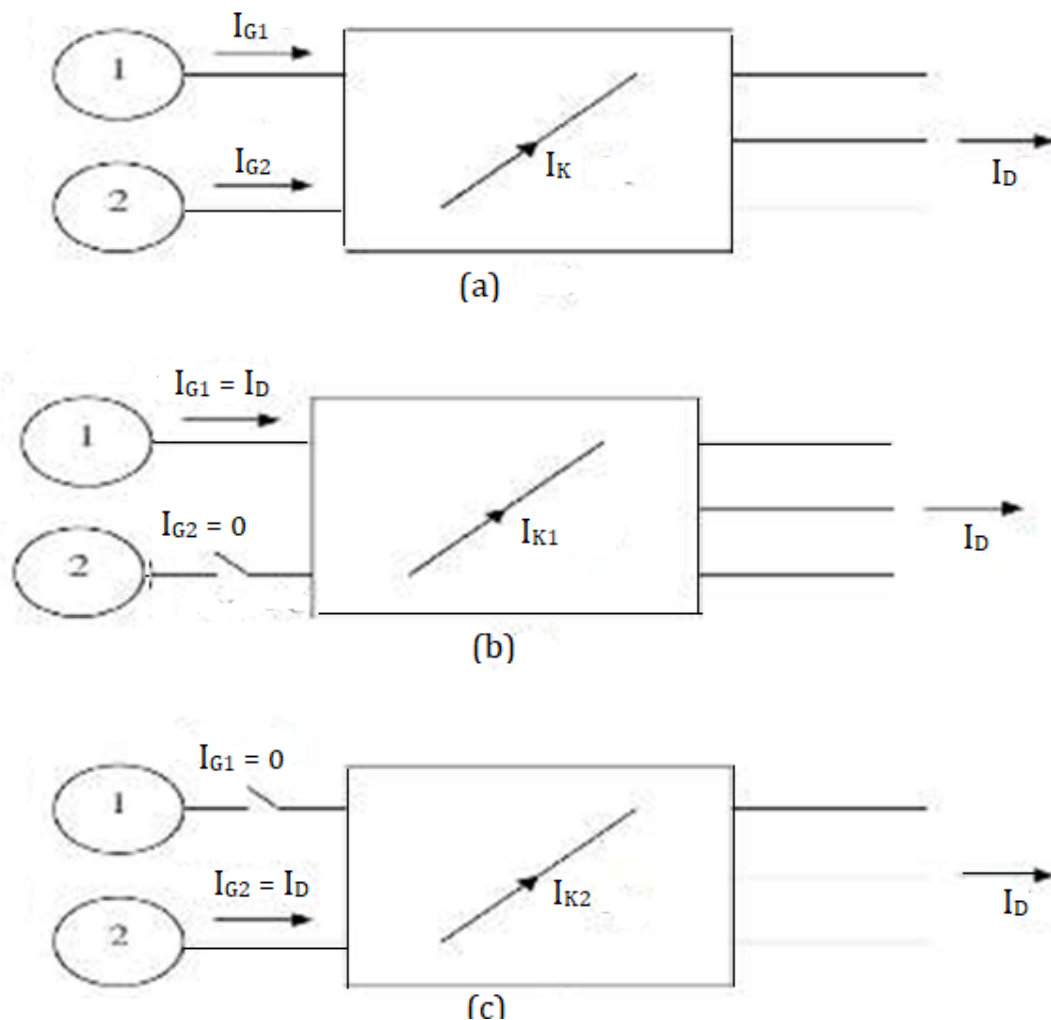
#### 4.6 DERIVATION OF TRANSMISSION LOSS FORMULA

An accurate method of obtaining general formula for transmission loss (loss coefficients) has been given by Kron. This, however, is quite complicated.

The method is elaborate and a simpler approach is possible by making the following assumptions:

- (i) All load currents have same phase angle with respect to a common reference
- (ii) The ratio  $X / R$  is the same for all the network branches.

Consider the simple case of two generating plants connected to an arbitrary number of loads through a transmission network as shown in Fig 4.5(a)



**Fig .4.5 Two plants connected to a no. of loads through a transmission network**

Let assume that the total load is supplied by only generator 1 as shown in fig. 4.5(b).

Let the current through a branch K in the network be  $I_{K1}$ .

We define

$$N_{K1} = \frac{I_{K1}}{I_D} \text{ --- (24)}$$

It is to be noted that  $I_{G1} = I_D$  in this case. Similarly with only plant 2 supplying the load current  $I_D$ , as shown in fig.4.5(c), we define

$$N_{K2} = \frac{I_{K2}}{I_D} \text{ --- (25)}$$

$N_{K1}$  and  $N_{K2}$  are **called current distribution factors** and their values depend on the impedances of the lines and the network connection. They are independent of  $I_D$ . When both generators are supplying the load, then by principle of superposition

$$I_K = N_{K1}I_{G1} + N_{K2}I_{G2} \text{ --- (26)}$$

Where  $I_{G1}$  and  $I_{G2}$  are the currents supplied by plants 1 and 2 respectively to meet the demand  $I_D$ .

Because of the assumptions made,  $I_{K1}$  and  $I_D$  have same phase angle, as do  $I_{K2}$  and  $I_D$ . Therefore, the current distribution factors are real rather than complex.

Let  $I_{G1} = |I_{G1}| \angle \sigma_1$  and  $I_{G2} = |I_{G2}| \angle \sigma_2$

Where  $\sigma_1$  and  $\sigma_2$  are phase angles of  $I_{G1}$  and  $I_{G2}$  with respect to a common reference.

We can write

$$\begin{aligned}
 |I_K|^2 &= (N_{K1}|I_{G1}|\cos\sigma_1 + N_{K2}|I_{G2}|\cos\sigma_2)^2 + (N_{K1}|I_{G1}|\sin\sigma_1 + N_{K2}|I_{G2}|\sin\sigma_2)^2 \\
 &= N_{K1}^2|I_{G1}|^2[\cos^2\sigma_1 + \sin^2\sigma_1] + N_{K2}^2|I_{G2}|^2[\cos^2\sigma_2 + \sin^2\sigma_2] + \\
 &\quad 2[N_{K1}|I_{G1}|\cos\sigma_1 N_{K2}|I_{G2}|\cos\sigma_2 + N_{K1}|I_{G1}|\sin\sigma_1 N_{K2}|I_{G2}|\sin\sigma_2] \\
 &= N_{K1}^2|I_{G1}|^2 + N_{K2}^2|I_{G2}|^2 + 2N_{K1}|I_{G1}|N_{K2}|I_{G2}|\cos(\sigma_1 - \sigma_2) \text{ --- (27)}
 \end{aligned}$$

$$\text{Now } |I_{G1}| = \frac{P_{G1}}{\sqrt{3}V_1\cos\phi_1} \text{ and } |I_{G2}| = \frac{P_{G2}}{\sqrt{3}V_2\cos\phi_2}$$

Where  $P_{G1}$  and  $P_{G2}$  are the real power outputs of plant 1 and 2;  $V_1$  and  $V_2$  are the line to line bus voltages of plant 1 and 2 and  $\phi_1$  and  $\phi_2$  are the power factor angles of plant 1 and 2

The total transmission loss in the system is given by

$$P_L = \sum_K 3 |I_K|^2 R_K \text{ --- (28)}$$

Where the summation is taken over all branches of the network and  $R_K$  is the branch resistance.

Substituting we get

$$\begin{aligned}
 P_L &= \frac{P_{G1}^2}{|V_1|^2 (\cos\phi_1)^2} \sum_K N_{K1}^2 R_K + \frac{2 P_{G1} P_{G2} \cos(\sigma_1 - \sigma_2)}{|V_1| |V_2| \cos\phi_1 \cos\phi_2} \sum_K N_{K1} N_{K2} R_K \\
 &\quad + \frac{P_{G2}^2}{|V_2|^2 (\cos\phi_2)^2} \sum_K N_{K2}^2 R_K \text{ --- (29)}
 \end{aligned}$$

$$P_L = P_{G1}^2 B_{11} + 2 P_{G1} P_{G2} B_{12} + P_{G2}^2 B_{22} - - - - - (30)$$

$$\text{Where } B_{11} = \frac{1}{|V_1|^2 (\cos \Phi_1)^2} \sum_K N_{K1}^2 R_K$$

$$B_{12} = \frac{\cos(\sigma_1 - \sigma_2)}{|V_1| |V_2| \cos \Phi_1 \cos \Phi_2} \sum_K N_{K1} N_{K2} R_K$$

$$B_{22} = \frac{1}{|V_2|^2 (\cos \Phi_2)^2} \sum_K N_{K2}^2 R_K$$

The loss – coefficients are **called the B – Coefficients** and have unit MW<sup>-1</sup>.

For a general system with n plants the transmission loss is expressed as

$$P_L = \frac{P_{G1}^2}{|V_1|^2 (\cos \Phi_1)^2} \sum_K N_{K1}^2 R_K + \dots + \frac{P_{Gn}^2}{|V_n|^2 (\cos \Phi_n)^2} \sum_K N_{Kn}^2 R_K \\ + 2 \sum_{\substack{p,q=1 \\ m \neq n}}^n \frac{P_{G1} P_{G2} \cos(\sigma_p - \sigma_q)}{|V_p| |V_q| \cos \Phi_p \cos \Phi_q} \sum_K N_{Kp} N_{Kq} R_K - - - - - (31)$$

It can be recognized as

$$P_L = P_{G1}^2 B_{11} + \dots + P_{Gn}^2 B_{nn} + 2 \sum_{\substack{p,q=1 \\ m \neq n}}^n P_{GP} B_{pq} P_{Gq} - - - - - (32)$$

In compact form

$$P_L = \sum_{p=1}^n \sum_{q=1}^n P_{GP} B_{pq} P_{Gq} - - - - - (32)$$

$$B_{pq} = \frac{\cos(\sigma_p - \sigma_q)}{|V_p| |V_q| \cos \Phi_p \cos \Phi_q} \sum_K N_{Kp} N_{Kq} R_K - - - - - (33)$$

B-Coefficients can be treated as constants over the load cycle by computing them at average operating conditions without significant loss of accuracy.

#### 4.7 ILLUSTRATIVE EXAMPLES

**Example-1:** The fuel cost function in Rs/hr for three thermal plants are given by,

$$F_1 = 350 + 7.2 P_{G1} + 0.004 P_{G1}^2 \text{ Rs/hr}$$

$$F_2 = 500 + 7.3 P_{G2} + 0.0025 P_{G2}^2 \text{ Rs/hr}$$

$$F_3 = 600 + 6.74 P_{G3} + 0.003 P_{G3}^2 \text{ Rs/hr}$$

Determine the optimal generation scheduling neglecting the losses for a load of 450MW. Also, calculate the cost of production of 450MW for the obtained schedule. (VTU June/July 2024 QP)

**Solution:**

Solve this step by step using the **Lambda-Iteration Method** (Equal incremental cost principle), since transmission losses are neglected.

**Step 1: Incremental cost (dF/dP):**

$$\frac{dF_1}{dP_{G1}} = 7.2 + 0.008 P_{G1}$$

$$\frac{dF_2}{dP_{G2}} = 7.3 + 0.005 P_{G2}$$

$$\frac{dF_3}{dP_{G3}} = 6.74 + 0.006 P_{G3}$$

At optimum,

$$\frac{dF_1}{dP_{G1}} = \frac{dF_2}{dP_{G2}} = \frac{dF_3}{dP_{G3}} = \lambda$$

and

$$P_{G1} + P_{G2} + P_{G3} = 450$$

**Step 2: Express  $P_{G1}, P_{G2}, P_{G3}$  in terms of  $\lambda$ :**

From each incremental cost:

$$P_{G1} = \frac{\lambda - 7.2}{0.008}$$

$$P_{G2} = \frac{\lambda - 7.3}{0.005}$$

$$P_{G3} = \frac{\lambda - 6.74}{0.006}$$

**Step 3: Total generation = 450 MW:**

$$\frac{\lambda - 7.2}{0.008} + \frac{\lambda - 7.3}{0.005} + \frac{\lambda - 6.74}{0.006} = 450$$

Simplify each term:

$$125(\lambda - 7.2) + 200(\lambda - 7.3) + 166.67(\lambda - 6.74) = 450$$

Expanding:

$$125\lambda - 900 + 200\lambda - 1460 + 166.67\lambda - 1124.66 = 450$$

$$(125 + 200 + 166.67)\lambda - (900 + 1460 + 1124.66) = 450$$

$$491.67\lambda - 3484.66 = 450$$

$$491.67\lambda = 450 + 3484.66 = 3934.66$$

$$\lambda = \frac{3934.66}{491.67} \approx 8.006 \text{ Rs/MWhr}$$

**Step 4: Calculate Individual Generations:**

$$P_{G1} = \frac{8.006 - 7.2}{0.008} = 100.75 \text{ MW}$$

$$P_{G2} = \frac{8.006 - 7.3}{0.005} = 141.2 \text{ MW}$$

$$P_{G3} = \frac{8.006 - 6.74}{0.006} = 208.05 \text{ MW}$$

**Total Generation = 100.75 + 141.2 + 208.05 = 450 MW**

**Step 5: Total Cost Calculation:**

$$F_1 = 350 + 7.2(100.75) + 0.004(100.75)^2 = 350 + 725.4 + 40.6 \approx 1116 \text{ Rs/hr}$$

$$F_2 = 500 + 7.3(141.2) + 0.0025(141.2)^2 = 500 + 1031 + 49.9 \approx 1581 \text{ Rs/hr}$$

$$F_3 = 600 + 6.74(208.05) + 0.003(208.05)^2 = 600 + 1402 + 129.9 \approx 2132 \text{ Rs/hr}$$

$$\text{Total Cost} = 1116 + 1581 + 2132 = 4829 \text{ Rs/hr}$$

**Final Answer:**

**Optimal generation schedule:**

$$P_{G1} = 100.75 \text{ MW}$$

$$P_{G2} = 141.2 \text{ MW}$$

$$P_{G3} = 208.05 \text{ MW}$$

**Total Production Cost = 4829 Rs / hr**

**Example-2:** In a system with two plants, the incremental fuel costs are given by,

$$\frac{dF_1}{dP_{G1}} = 0.001P_{G1} + 20 \text{ Rs/MWhr}$$

$$\frac{dF_2}{dP_{G2}} = 0.015P_{G2} + 22.5 \text{ Rs/MWhr}$$

The system is running under optimal schedule with  $P_{G1} = P_{G2} = 100\text{MW}$ . If  $\frac{\partial P_L}{\partial P_{G2}} = 0.2$ . Find  $\frac{\partial P_L}{\partial P_{G1}}$  and plant penalty factors. (VTU June/July 2024 QP)

**Solution:**

**Step 1: Penalty factor formula:**

$$\lambda = \frac{\frac{dF_1}{dP_{G1}}}{1 - \frac{\partial P_L}{\partial P_{G1}}} = \frac{\frac{dF_2}{dP_{G2}}}{1 - \frac{\partial P_L}{\partial P_{G2}}}$$

**Step 2: Calculate incremental costs at given outputs:**

$$\frac{dF_1}{dP_{G1}} = 0.01 \times 100 + 20 = 21 \text{ Rs/MWh}$$

$$\frac{dF_2}{dP_{G2}} = 0.015 \times 100 + 22.5 = 24 \text{ Rs/MWh}$$

$$\frac{21}{1 - \frac{\partial P_L}{\partial P_{G1}}} = \frac{24}{1 - 0.2}$$

$$\frac{21}{1 - \frac{\partial P_L}{\partial P_{G1}}} = \frac{24}{0.8} = 30$$

$$1 - \frac{\partial P_L}{\partial P_{G1}} = \frac{21}{30} = 0.7$$

$$\frac{\partial P_L}{\partial P_{G1}} = 1 - 0.7 = 0.3$$

**Step 3: Penalty Factors:**

$$\text{Penalty Factor, } \mu = \frac{1}{1 - \frac{\partial P_L}{\partial P_G}}$$

$$\text{Penalty Factor of Plant 1, } \mu_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G1}}} = \frac{1}{1 - 0.3} = 1.4286$$

$$\text{Penalty Factor of Plant 2, } \mu_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{G2}}} = \frac{1}{1 - 0.2} = 1.25$$

## **4(B) UNIT COMMITMENT**

### **SYLLABUS**

**Unit Commitment:** Introduction, Constraints and unit commitment solution by prior list method and dynamic forward DP approach (Flow chart and Algorithm only).

### **4.8 INTRODUCTION**

- Power systems have grown in size and complexity.
- In power system, the total generation on the system will generally be higher than total load on the system.
- The total load on the system will generally be higher during the day time and early evening and lower during the early morning and late evening.
- It is not economical to run all the units available all the time. So, the commitment of a generating unit is difficult.
- The cost of the system can be saved by turning off generators when they are not needed. Generating units can be broadly divided into two groups namely thermal and hydro plants. The operation of thermal units involves both full and maintenance cost but no fuel cost is required for hydro plants.
- Thermal units include steam plants, nuclear plant, diesel and gas turbine plants are used as peak loads.
- Because human activity follows cycles, most systems supplying services to a large population will experience cycles.
- This includes transportation systems, communication systems, as well as electric power systems.
- In the case of an electric power system, the total load on the system will generally be higher during the daytime and early evening when industrial loads are high, lights are on, and so forth, and lower during the late evening and early morning when most of the population is asleep.
- In addition, the use of electric power has a weekly cycle, the load being lower over weekend days than weekdays.
- But why is this a problem in the operation of an electric power system? Why not just simply commit enough units to cover the maximum system load and leave them running?
- Note that to “commit” a generating unit is to “turn it on;” that is, to bring the unit up to speed, synchronize it to the system, and connect it so it can deliver power to the network.
- The problem with “commit enough units and leave them on line” is one of economics.
- As will be shown in Example-1, it is quite expensive to run too many generating units.
- A great deal of money can be saved by turning units off (decommitting them) when they are not needed.

### **Need of Unit Commitment with Example:1**

**Enough units will be committed to supply the system load.**

1. To reduce the loss or fuel cost
2. By running the most economic unit, the load can be supplied by that unit operating closer to its best efficiency.
3. Difficulties to find unit commitment solution
4. Time consuming process
5. If the number of units is more, the number of combinations is more in a complex bus system.
6. If  $n$  be the number of units, then the number of combinations will be  $2^n - 1$ .

**Suppose one had the three units given here:**

#### **Unit 1:**

Min = 150 MW

Max = 600 MW

$H_1 = 510.0 + 7.2P_1 + 0.00142P_1^2 \text{ MBtu/h}$

#### **Unit 2:**

Min = 100 MW

Max = 400 MW

$H_2 = 310.0 + 7.85P_2 + 0.00194 P_2^2 \text{ MBtu/h}$

#### **Unit 3:**

Min = 50 MW

Max = 200 MW

$H_3 = 78.0 + 7.97P_3 + 0.00482 P_3^2 \text{ MBtu/h}$

**with fuel costs:**

Fuel cost, = 1.1 P/MBtu

Fuel cost, = 1.0 P/MBtu

Fuel cost, = 1.2 P/MBtu

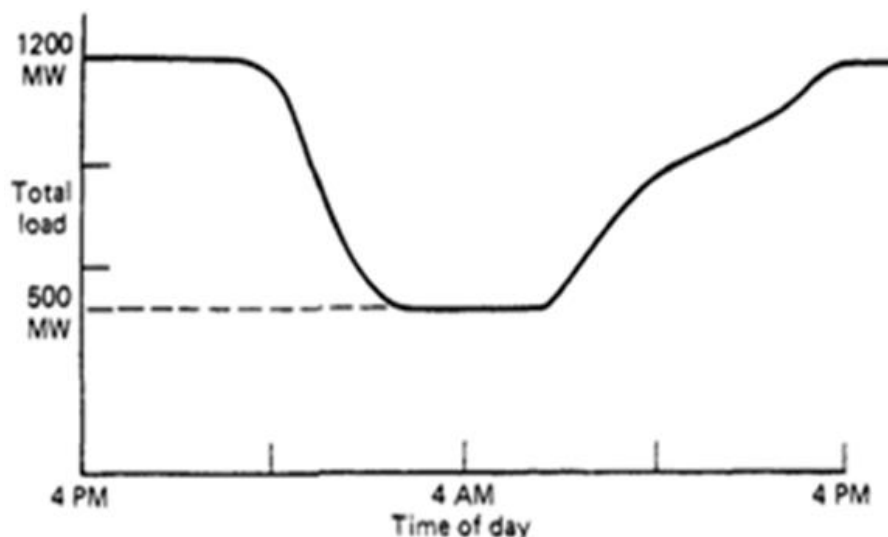
- If we are to supply a load of 550 MW, what unit or combination of units should be used to supply this load most economically?
- To solve this problem, simply try all combinations of the three units.
- Some combinations will be infeasible if the sum of all maximum MW for the units committed is less than the load or if the sum of all minimum MW for the units committed is greater than the load.
- For each feasible combination, the units will be dispatched using the techniques.
- The results are presented in Table.1.

**Table1: Unit Combinations and Dispatch for 550 MW Load of Example 1**

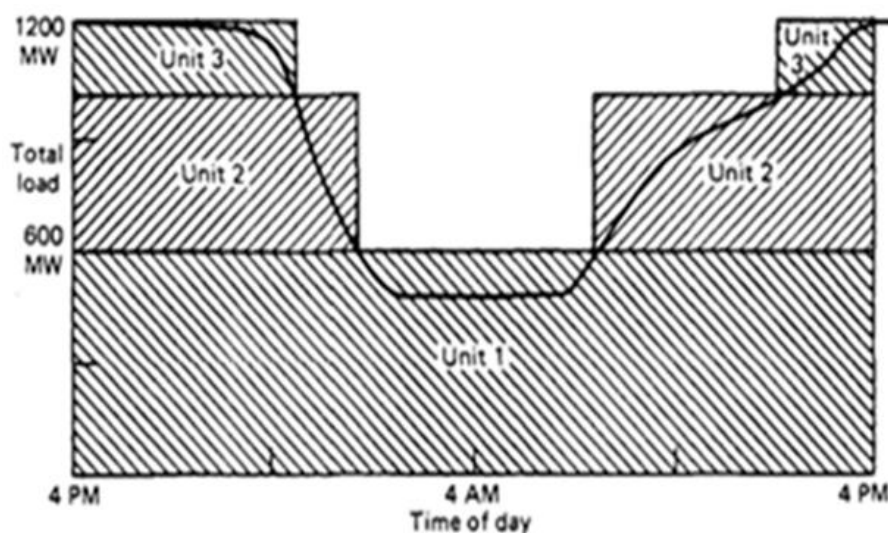
Unit1	Unit2	Unit3	Maximum Generation	Minimum Generation	Production Cost, $P_1$	Production Cost, $P_2$	Production Cost, $P_3$	Fuel Cost, $F_1$	Fuel Cost, $F_2$	Fuel Cost, $F_3$	Total Generation Cost, $F_1 + F_2 + F_3$
Off	Off	Off	0	0	Infeasible						
Off	Off	On	200	50	Infeasible						
Off	On	Off	400	100	Infeasible						
Off	On	On	600	150	0	400	150	0	3760	1658	5418
On	Off	Off	600	150	550	0	0	5389	0	0	5389
On	Off	On	800	200	500	0	50	4911	0	586	5497
On	On	Off	1000	250	295	255	0	3030	2440	0	5471
On	On	On	1200	300	267	233	50	2787	2244	586	5617

- Note that the least expensive way to supply the generation is not with all three units running, or even any combination involving two units.
- Rather, the optimum commitment is to only run unit 1, the most economic unit.
- By only running the most economic unit, the load can be supplied by that unit operating closer to its best efficiency.
- If another unit is committed, both unit 1 and the other unit will be loaded further from their best efficiency points such that the net cost is greater than unit 1 alone.
- Suppose the load follows a simple “peak-valley” pattern as shown in Figure 4.1(a).
- If the operation of the system is to be optimized, units must be shut down as the load goes down and then recommitted as it goes back up.
- We would like to know which units to drop and when.

- As we will show later, this problem is far from trivial when real generating units are considered.
- One approach to this solution is demonstrated in Example-2, where a simple priority list scheme is developed.



**Fig.4.1(a) Simple "Peak-Valley" Load Pattern**



**Fig.4.1.(b) Unit Commitment Schedule using Shut-Down Rule**

- Figure 4.1(b) shows the unit commitment schedule derived from this shut-down rule as applied to the load curve of Figure 4.1(a).
- So far, we have only obeyed one simple constraint: Enough units will be committed to supply the load.
- If this were all that was involved in the unit commitment problem-that is, just meeting the load-we could stop here and state that the problem was "solved."

- Unfortunately, other constraints and other phenomena must be taken into account in order to claim an optimum solution.
- These constraints will be discussed in the next section, followed by a description of some of the presently used methods of solution.

### **Shut – Down Rule - Example: 2**

- Suppose we wish to know which units to drop as a function of system load.
- Let the units and fuel costs are the same as in Example 1, with the load varying from a peak of 1200 MW to a valley of 500 MW.
- To obtain a “shut-down rule,” simply use a brute-force technique wherein all combinations of units will be tried (as in Example-1) for each load value taken in steps of 50 MW from 1200 to 500.
- The results of applying this brute-force technique are given in Table.2. Our shut-down rule is quite simple.
- When load is above 1000 MW, run all three units; between 1000 MW and 600 MW, run units 1 and 2; below 600 MW, run only unit 1.

**Table 2: Shut – Down Rule Derivations of Example 2**

Load	Optimum Combinations		
	Unit 1	Unit 2	Unit 3
1200	On	On	On
1150	On	On	On
1100	On	On	On
1050	On	On	On
1000	On	On	Off
950	On	On	Off
900	On	On	Off
850	On	On	Off
800	On	On	Off
750	On	On	Off
700	On	On	Off
650	On	On	Off
600	On	On	Off
550	On	On	Off
500	On	On	Off
450	On	On	Off
400	On	On	Off
350	On	On	Off
300	On	On	Off
250	On	On	Off
200	On	On	Off
150	On	Off	Off
100	On	Off	Off
50	On	Off	Off

#### 4.9 Simple Enumeration Constraints - Constraints in Unit Commitment

- Each individual power system may impose different rules on the scheduling of units depends on generation make up and load curve characteristics etc.
- **The constraints to be considered for unit commitment are**
  - Spinning Reserve
  - Thermal Constraints
    - ✓ Minimum up time
    - ✓ Minimum down time
    - ✓ Crew Constraint
  - Other Constraint:
    - ✓ Hydro Constraint
    - ✓ Must run constraint
    - ✓ Fuel Constraint
- Many constraints can be placed on the unit commitment problem. The list presented here is by no means exhaustive.
- Each individual power system, power pool, reliability council, and so forth, may impose different rules on the scheduling of units, depending on the generation makeup, load-curve characteristics, and such.
- **Spinning Reserve**
  - **Spinning reserve** is the term used to describe the total amount of generation available from all units synchronized (i.e., spinning) on the system, minus the present load and losses being supplied.
  - Spinning reserve must be carried so that the loss of one or more units does not cause too far a drop in system frequency.
  - Quite simply, if one unit is lost, there must be ample reserve on the other units to make up for the loss in a specified time period.

**Spinning Reserve** is the reserve generating capacity running at no load.

#### **Reserve has**

- To avoid transmission system limitations or bottling of reserves
- To allow some parts of the system to run as islands (some area electrically disconnected).

**Reserve capacity:** Capacity in excess of that required to carry peak load

**Cold reserves:** It is that reserve generating capacity which is available for service but is not in operation.

**Hot reserves:** It is that reserve generating capacity which is in operation but is not in service.

### **Reserve generating capacity:**

- The amount of power that can be produced at a given point in time by generating units that are kept available in case of special need.
- The capacity may be used when unusually high power demand occurs or when other generating units are off-line for maintenance, repairs or refueling.
- **Reserve generating capacity** include quick-start diesel or gas turbine unit or hydro units and pumped storage hydro units that can be brought on-line, synchronized and brought up to full capacity quickly.
- Automatic generation control system is used to make up for a generation unit failure and to restore frequency and interchange power through tie-line quickly in the event of generating unit outage.

### **Reserve margin:**

- The percentage of installed capacity exceeding the expected peak demand during a specified period.
- Typical rules for spinning reserve set by regional reliability council .
- Reserve must be given percentage of forecasted peak demand.
- Reserve must be capable of making up the loss of the most heavily loaded unit in a given period of time.
- Calculate reserve requirements as a function of the probability of not having sufficient generation to meet the load.
- Spinning reserve must be allocated to obey certain rules, usually set by regional reliability councils (in the United States) that specify how the reserve is to be allocated to various units.
- Typical rules specify that reserve must be a given percentage of forecasted peak demand, or that reserve must be capable of making up the loss of the most heavily loaded unit in a given period of time.
- Others calculate reserve requirements as a function of the probability of not having sufficient generation to meet the load.
- Not only must the reserve be sufficient to make up for a generation-unit failure, but the reserves must be allocated among fast-responding units and slow-responding units.
- This allows the automatic generation control system to restore frequency and interchange quickly in the event of a generating-unit outage.
- Beyond spinning reserve, the unit commitment problem may involve various classes of “scheduled reserves” or “off-line” reserves.
- These include quick-start diesel or gas-turbine units as well as most hydro-units and pumped-storage hydro-units that can be brought on-line, synchronized, and brought up to full capacity quickly.

- As such, these units can be “counted” in the overall reserve assessment, as long as their time to come up to full capacity is taken into account.
- Reserves, finally, must be spread around the power system to avoid transmission system limitations (often called “bottling” of reserves) and to allow various parts of the system to run as “islands,” should they become electrically disconnected.

### Two Region Systems- Example: 3

- Suppose a power system consisted of two isolated regions: a western region and an eastern region.
- Five units, as shown in Figure.2, have been committed to supply 3090 MW.
- The two regions are separated by transmission tie lines that can together transfer a maximum of 550 MW in either direction.
- This is also shown in Figure.4.2. What can we say about the allocation of spinning reserve in this system?
- The data for the system in Figure.2 are given in Table 3. With the exception of unit 4, the loss of any unit on this system can be covered by the spinning reserve on the remaining units. Unit 4 presents a problem, however.
- If unit 4 was to be lost and unit 5 was to be run to its maximum of 600 MW, the eastern region would still need 590 MW to cover the load in that region.
- The 590 MW would have to be transmitted over the tie lines from the western region, which can easily supply 590 MW from its reserves.
- However, the tie capacity of only 550 MW limits the transfer.
- Therefore, the loss of unit 4 cannot be covered even though the entire system has ample reserves.
- The only solution to this problem is to commit more units to operate in the eastern region.

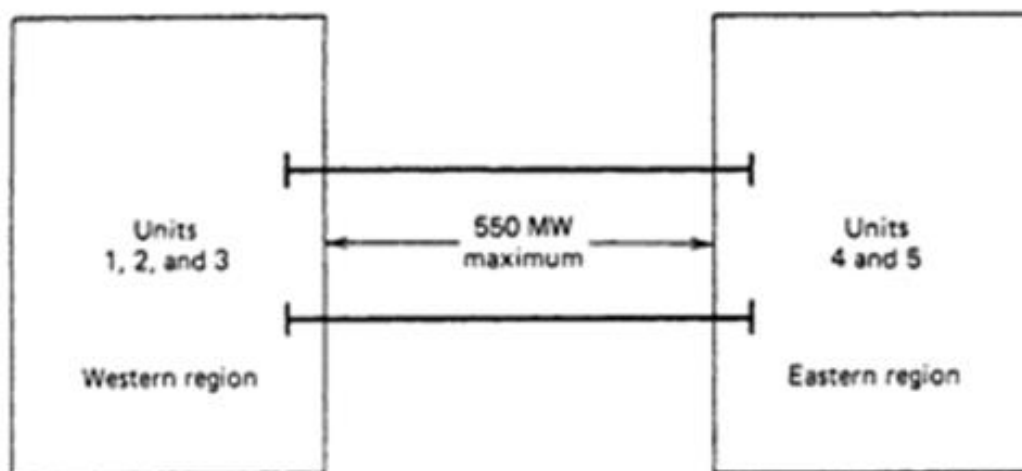


Fig.4.2 Two Region System

**Table3: Data for the System in Figure 4.2**

Region	Unit	Unit Capacity (MW)	Unit Output (MW)	Regional Generation (MW)	Spinning Reserve	Regional Load (MW)	Interchange (MW)
Western	1	1000	900	1740	100	1900	160 in
	2	800	420		380		
	3	800	420		380		
Eastern	4	1200	1040	1350	160	1190	160 out
	5	600	310		290		
<b>Total</b>	1 - 5	4400	3090	3090	1310	3090	

#### ➤ Thermal Unit Constraints

- ✓ Thermal units usually require a crew to operate them, especially when turned on and turned off.
- ✓ A thermal unit can undergo only gradual temperature changes, and this translates into a time period of some hours required to bring the unit on-line.
- ✓ As a result of such restrictions in the operation of a thermal plant, various constraints arise, such as:
  - **Minimum Up Time,**
  - **Minimum Down Time and**
  - **Crew Constraints**
- ❖ **Minimum Up Time:** once the unit is running, it should not be turned off immediately.
- ❖ **Minimum Down Time:** once the unit is decommitted, there is a minimum time before it can be recommitted.
- ❖ **Crew Constraints:** if a plant consists of two or more units, they cannot both be turned on at the same time since there are not enough crew members to attend both units while starting up.
- ✓ In addition, because the temperature and pressure of the thermal unit must be moved slowly, a certain amount of energy must be expended to bring the unit on-line.
- ✓ This energy does not result in any MW generation from the unit and is brought into the unit commitment problem as a start-up cost.
- ✓ The start-up cost can vary from a maximum “**cold-start**” value to a much smaller value if the unit was only turned off recently and is still relatively close to operating temperature.
- ✓ There are **two approaches** to treating a thermal unit during its down period.
- ✓ The first allows the unit’s boiler to cool down and then heat back up to operating temperature in time for a scheduled turn on.

- ✓ **The second (called banking)** requires that sufficient energy be input to the boiler to just maintain operating temperature.
- ✓ The costs for the two can be compared so that, if possible, the best approach (**cooling or banking**) can be chosen.

$$\text{Start-up cost when cooling} = C_c (1 - e^{-t/\alpha}) F + C_f$$

Where

$C_c$  = cold-start cost (MBtu)

$F$  = fuel cost

$C_f$  = fixed cost (includes crew expense, maintenance expenses) (in p)

$\alpha$  = thermal time constant for the unit

$t$  = time (h) the unit was cooled

$$\text{Start-up cost when banking} = C_t t F + C_c$$

Where

$C_t$  = cost (MBtu/h) of maintaining unit at operating temperature

- ✓ Up to a certain number of hours, the cost of banking will be less than the cost of cooling, as is illustrated in Figure 4.3.
- ✓ Finally, the capacity limits of thermal units may change frequently, due to maintenance or unscheduled outages of various equipment in the plant; this must also be taken into account in unit commitment.

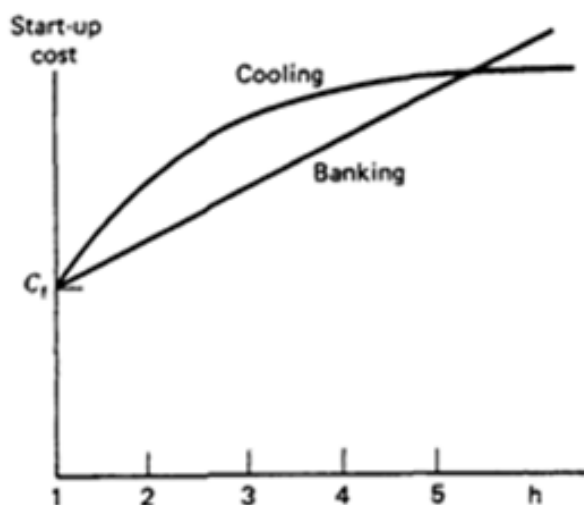


Fig.4.3 Time – Dependent Start-Up Costs

#### ❖ Other Constraints

##### ▪ Hydro-Constraints

- ✓ Unit commitment cannot be completely separated from the scheduling of hydro-units.
- ✓ We will assume that the hydrothermal scheduling (or “coordination”) problem can be separated from the unit commitment problem.
- ✓ We, of course, cannot assert flatly that our treatment in this fashion will always result in an optimal solution.

### ▪ **Must Run**

Some units are given a must-run status during certain times of the year for reason of voltage support on the transmission network or for such purposes as supply of steam for uses outside the steam plant itself.

### ▪ **Fuel Constraints**

A system in which some units have limited fuel, or else have constraints that require them to burn a specified amount of fuel in a given time, presents a most challenging unit commitment problem.

## 4.10 Unit Commitment Solution Methods

The commitment problem can be very difficult. As a theoretical exercise, let us postulate the following situation.

- We must establish a loading pattern for M periods.
- We have N units to commit and dispatch.
- The M load levels and operating limits on the N units are such that any one unit can supply the individual loads and that any combination of units can also supply the loads.

Next, assume we are going to establish the commitment by enumeration (brute force). The total number of combinations we need to try each hour is,

$$C(N, 1) + C(N, 2) + \dots + C(N, N - 1) + C(N, N) = 2^N - 1$$

Where  $C(N, j)$  is the combination of N items taken j at a time. That is,

$$C(N, j) = \frac{N!}{(N-j)!j!}$$

$$j! = 1 \times 2 \times 3 \times \dots \times j$$

For the total period of M intervals, the maximum number of possible combinations is  $(2^N - 1)^M$ , which can become a horrid number to think about.

**For example**, take a 24-h period (e.g., 24 one-hour intervals) and consider systems with 5, 10, 20, and 40 units. The value of  $(2^N - 1)^{24}$  becomes the following.

N	$(2^N - 1)^{24}$
5	$6.2 \times 10^{35}$
10	$1.73 \times 10^{72}$
20	$3.12 \times 10^{144}$
40	Too big

- ✓ These very large numbers are the upper bounds for the number of enumerations required. Fortunately, the constraints on the units and the load-capacity relationships of typical utility systems are such that we do not approach these large numbers.
- ✓ Nevertheless, the real practical barrier in the optimized unit commitment problem is the high dimensionality of the possible solution space.

**The most talked-about techniques for the solution of the unit commitment problem are:**

- Priority-List Methods
- Dynamic Programming (DP)
- Lagrange Relation (LR)

#### **4.10.1 Priority List Methods**

- The simplest unit commitment solution method consists of creating a priority list of units.
- As we saw in Example 4, a simple shut-down rule or priority-list scheme could be obtained after an exhaustive enumeration of all unit combinations at each load level.
- The priority list of Example 5B could be obtained in a much simpler manner by noting the full-load average production cost of each unit, where the full-load average production cost is simply the net heat rate at full load multiplied by the fuel cost.

#### **Priority List Methods with Example**

Construct a priority list for the units of Example-1. (Use the same fuel costs as in Example-1)

**The imposition of a priority list arranged in order of the full-load average cost rate would result in a theoretically correct dispatch and commitment only if:**

1. No load costs are zero.
2. Unit input-output characteristics are linear between zero output and full load.
3. There are no other restrictions.
4. Start-up costs are a fixed amount.

**Step-1: First, the full-load average production cost will be calculated:**

Unit	Full Load Average Production cost (₹/MWh)
1	9.79
2	9.48
3	11.188

**Step-2: A strict priority order for these units, based on the average production cost, would order them as follows:**

Unit	₹/MWh	Min MW	Max MW
2	9.79	100	400
1	9.48	150	600
3	11.188	50	200

**Step-3: The commitment scheme would (ignoring min up/down time, start-up costs, etc.) simply use only the following combinations.**

Combination	Min MW from combination	Max MW from combination
2+1+3	300	1200
2+1	250	1000
2	100	400

- ✓ Note that such a scheme would not completely parallel the shut-down sequence described in Example 2, where unit 2 was shut down at 600 MW leaving unit 1.
- ✓ With the priority-list scheme, both units would be held on until load reached 400 MW, then unit 1 would be dropped.

**Most priority-list schemes are built around a simple shut-down algorithm that might operate as follows.**

- At each hour when load is dropping, determine whether dropping the next unit on the priority list will leave sufficient generation to supply the load plus spinning-reserve requirements. If not, continue operating as is; if yes, go on to the next step.
- Determine the number of hours,  $H$ , before the unit will be needed again. That is, assuming that the load is dropping and will then go back up some hours later.
- If  $H$  is less than the minimum shut-down time for the unit, keep commitment as is and go to last step; if not, go to next step.
- Calculate two costs.
- The first is the sum of the hourly production costs for the next  $H$  hours with the unit up.
- Then recalculate the same sum for the unit down and add in the start-up cost for either cooling the unit or banking it, whichever is less expensive.
- If there is sufficient savings from shutting down the unit, it should be shut down, otherwise keep it on.
- Repeat this entire procedure for the next unit on the priority list. If it is also dropped, go to the next and so forth.
- ✓ Various enhancements to the priority-list method can be made by grouping of units to ensure that various constraints are met.
- ✓ We will note later that dynamic-programming methods usually create the same type of priority list for use in the DP search.

### 4.10.2 Dynamic Programming Solution Method

#### Introduction

- Dynamic programming has many advantages over the enumeration scheme, the chief advantage being a reduction in the dimensionality of the problem.
- Suppose we have found units in a system and any combination of them could serve the (single) load.
- **There would be a maximum of  $2^4 - 1 = 15$  combinations to test. However, if a strict priority order is imposed, there are only four combinations to try:**

Priority 1 unit

Priority 1 unit + Priority 2 unit

Priority 1 unit + Priority 2 unit + Priority 3 unit

Priority 1 unit + Priority 2 unit + Priority 3 unit + Priority 4 unit

#### In the dynamic-programming approach that follows, we assume that:

1. A state consists of an array of units with specified units operating and the rest off-line.
2. The start-up cost of a unit is independent of the time it has been off-line (i.e., it is a fixed amount)
3. There are no costs for shutting down a unit.
4. There is a strict priority order, and in each interval a specified minimum amount of capacity must be operating.

**A feasible state** is one in which the committed units can supply the required load and that meets the minimum amount of capacity each period.

#### ❖ Forward DP Approach

- One could set up a dynamic-programming algorithm to run backward in time starting from the final hour to be studied, back to the initial hour.
- Conversely, one could set up the algorithm to run forward in time from the initial hour to the final hour.
- The forward approach has distinct advantages in solving generator unit commitment. For example, if the start-up cost of a unit is a function of the time it has been off-line (i.e., its temperature), then a forward dynamic-program approach is more suitable since the previous history of the unit can be computed at each stage.
- There are other practical reasons for going forward.
- The initial conditions are easily specified and the computations can go forward in time as long as required.
- A forward dynamic-programming algorithm is shown by the flowchart in Figure 4.4.

The recursive algorithm to compute the minimum cost in hour K with combination I is,

$$F_{\text{cost}}(K, I) = \min_{\{L\}} [P_{\text{cost}}(K, I) + S_{\text{cost}}(K-1, L: K, I) + F_{\text{cost}}(K-1, L)]$$

Where

$F_{\text{cost}}(K, I)$  = least total cost to arrive at state (K, I)

$P_{\text{cost}}(K, I)$  = production cost for state (K, I)

$S_{\text{cost}}(K-1, L: K, I)$  = transition cost from state (K-1, L) to state (K, I)

State (K, I) is the  $I^{\text{th}}$  combination in hour K

For the forward dynamic programming approach, we define a **strategy** as the transition, or path, from one state at a given hour to a state at the next hour.

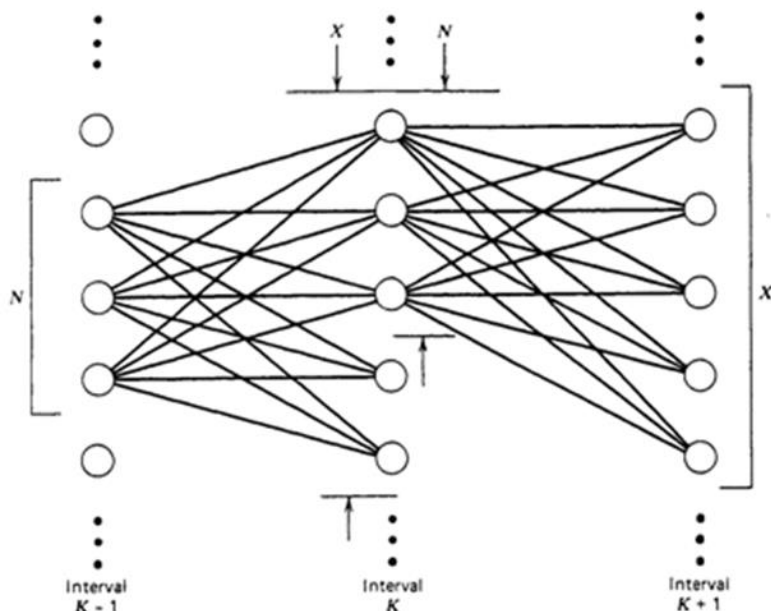
Note that two new variables, X and N, have been introduced in flow chart (Fig.4.4).

X = number of states to search each period

N = number of strategies, or paths, to save at each step

These variables allow control of the computational effort (see Figure.4.5).

For complete enumeration, the maximum number of the value of X or N is  $2^n - 1$ .



**Fig.4.5 Restricted Search Paths in Dynamic Programming with N=3 and X=5**

- For example, with a simple priority-list ordering, the upper bound on X is n, the number of units.
- Reducing the number N means that we are discarding the highest cost schedules at each time interval and saving only the lowest N paths or strategies.

- There is no assurance that the theoretical optimal schedule will be found using a reduced number of strategies and search range (the X value).
- Only experimentation with a particular program will indicate the potential error associated with limiting the values of X and N below their upper bounds.

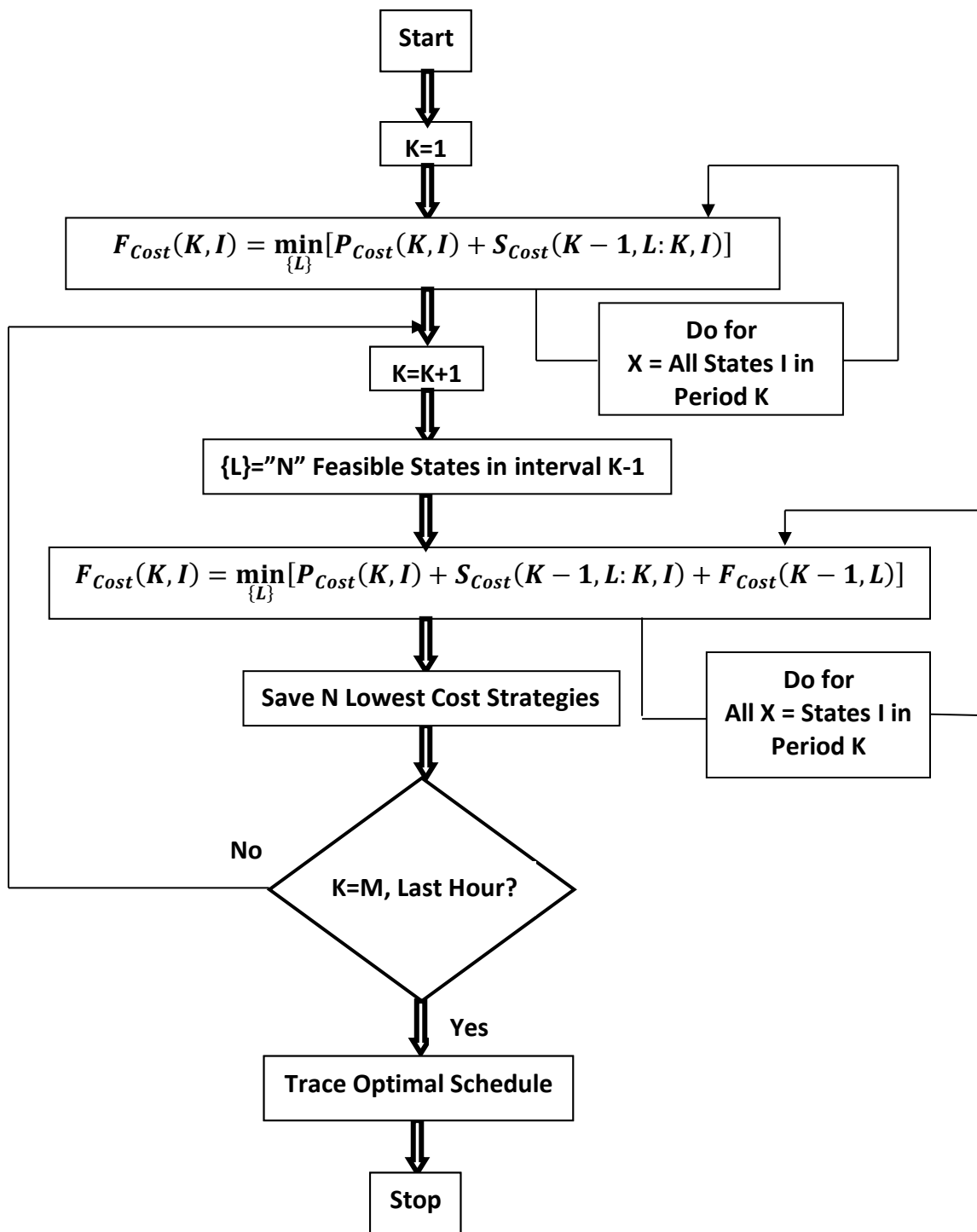


Fig.4.4 Flow Chart of Unit Commitment via Forward Dynamic Programming