MODULE – 1

NETWORK TOPOLOGY

SYLLABUS

Network Topology: Introduction and Basic Definitions of Elementary Graph Theory, Tree, Cut-Set, Loop Analysis, Formation of Incidence Matrices, Primitive Network - Impedance Form and admittance Form, Formation of YBUS by Singular Transformation, YBUS by Inspection Method, Illustrative Examples.

1.1 Introduction

- The solution of a given linear network problem requires the formation of a set of equations describing the response of the network.
- The mathematical model so derived, must describe the characteristics of the individual network components, as well as the relationship which governs the interconnection of the individual components.
- In the bus frame of reference the variables are the node voltages and node currents. The independent variables in any reference frame can be either currents or voltages. Correspondingly, the coefficient matrix relating the dependent variables and the independent variables will be either an impedance or admittance matrix.
- The formulation of the appropriate relationships between the independent and dependent variables is an integral part of a digital computer program for the solution of power system problems.
- The formulation of the network equations in different frames of reference requires the knowledge of graph theory.

1.2 Basic Definitions of Elementary Graph Theory

- The geometrical interconnection of the various branches of a network is **called the network topology.**
- To describe the geometrical features of a network, it is replaced by single line segments **called elements,** whose terminals are **called nodes.** The resulting figure is **called the graph** of the given network.
- **A linear graph** depicts the geometrical interconnection of the elements of a network.
- **A connected graph** is one in which there is at least one path between every pair of nodes.
- If each element of a connected graph is assigned a direction, it is **called an oriented graph.**

- Power networks are so structured that out of the m total number of nodes, one node (normally described by 0) is always at ground potential and the remaining n = m –1 nodes are the buses at which the source power is injected.
- Figure 1.2 shows the oriented linear graph of the power network of **Fig.1.1**
- Here the overall line admittance between any two buses (nodes in the corre sponding graph) are represented by a single line element.
- Also, each source and the shunt admittance connected across it are represented by a single line element.
- In fact, this combination represents the most general network element, and is described under the subheading Primitive Network.
- A connected subgraph containing all the nodes of a graph but having no closed paths is **called a tree.**
- The elements of a tree are **called branches or tree branches.**
- The number of branches b that form a tree are given by **b = m – 1 = n** (number of buses)
- Those elements of the graph that are not included in the tree are **called links or link branches,** and they form a subgraph, not necessarily connected, **called co-tree.**
- The number of links *l* of a connected graph with e elements is ***l* = e – b = e – m + 1**
- There may be more than one possible trees (and therefore, co-trees) of a graph.
- A tree and the corresponding co-tree of the graph of **Fig. 1.2** are shown in **Fig. 1.3.**
- The reader should try and find some other tree and co-tree pairs.
- If a link is added to the tree, the corresponding graph contains one closed path **called a basic loop.** Thus, a graph has as many basic loops as the number of links.
- **A loop** is distinguished from a basic loop, as it can be any loop in the original graph. Therefore, the number of loops is greater than, or at the most equal to, the number of basic loops in a graph.

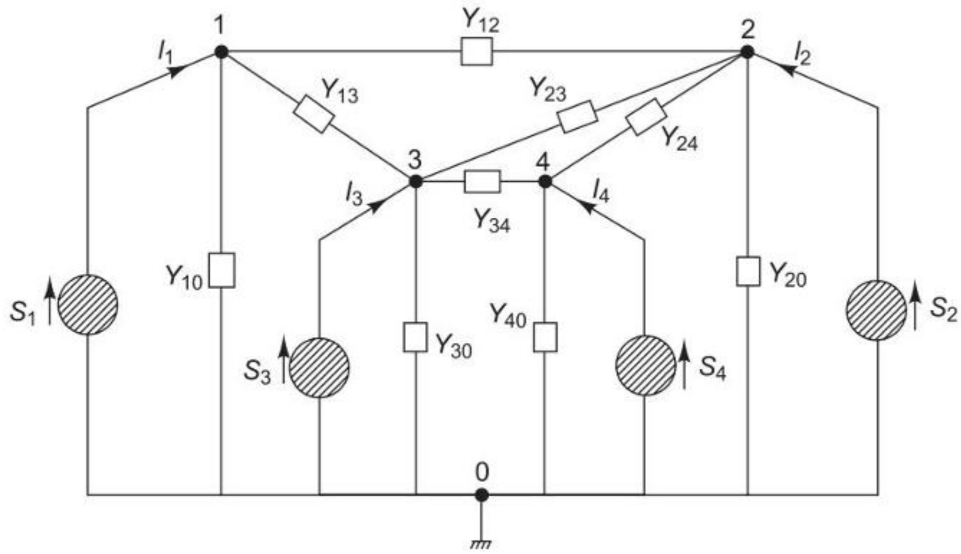


Fig.1.1 Reduced Circuit Diagram of 4-Bus Power System

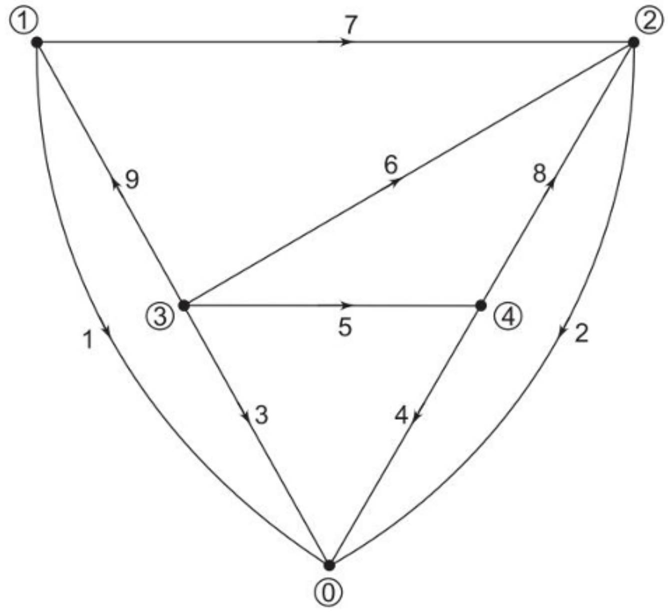


Fig.1.2 Oriented Linear Graph of the circuit in Fig.1.1

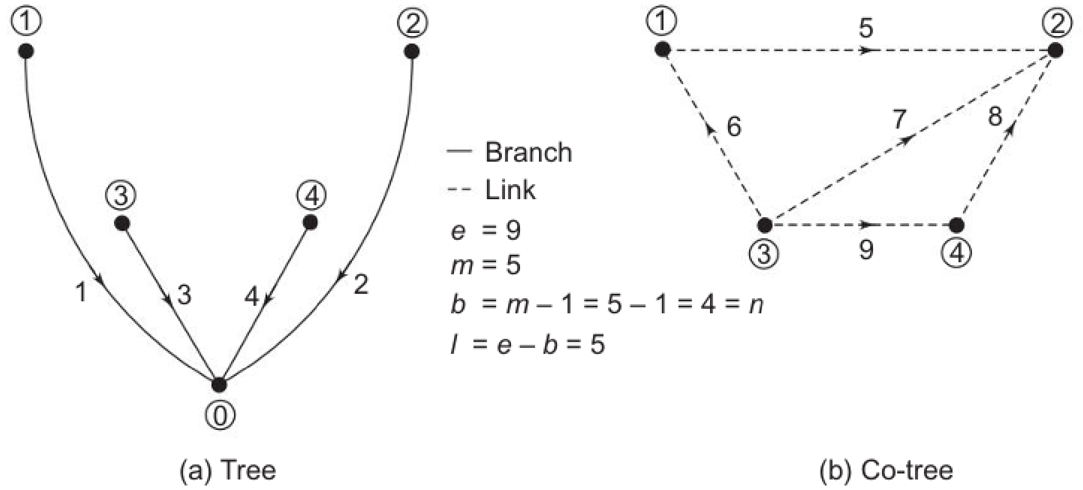


Fig.1.3 Tree and Co-Tree of the Oriented Connected Graph

Example:

A representative power system and its oriented graph areas shown in **Fig. 1.4**, with:
e = number of elements = 6
n = number of nodes = 4
b = number of branches = n-1 = 3
l = number of links = e-b = 3
Tree = T(1,2,3) and
Co-tree = T(4,5,6)

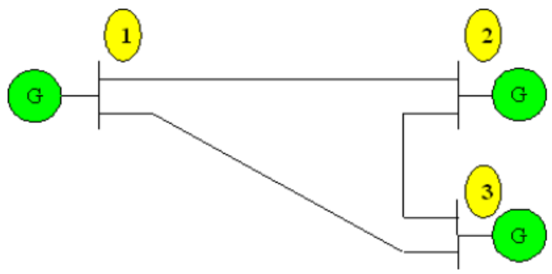


Fig 1.4: Single Line Diagram of a Power System

- **Sub-Graph:** Sg is a sub-graph of graph G if the following conditions are satisfied:
  - Sg is itself a graph
  - Every node of Sg is also a node of graph G
  - Every branch of Sg is a branch of graph G

- **Cutset:** It is a set of branches of a connected graph G which satisfies the following conditions:
  - The removal of all branches of the cutset causes the remaining graph to have two separate unconnected sub-graphs.
  - The removal of all but one of the branches of the set, leaves the remaining graph connected.

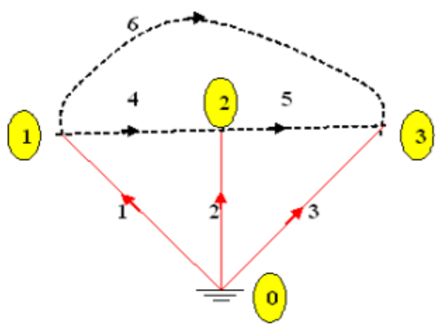


Fig.1.5 Oriented Graph of Fig.1.4

- Referring to **Fig 1.5,** the set {3,5,6} constitutes a cutset since removal of them isolates node 3 from rest of the network, thus dividing the graph into two unconnected subgraphs.
- However, the set (2,4,6) is not a valid cutset.
- The KCL (Kirchhoff's Current Law) for the cutset is stated as follows: In any lumped network, the algebraic sum of all the branch currents traversing through the given cutset branches is zero.
- **Tree:** It is a connected sub-graph containing all the nodes of the graph G, but without any closed paths (loops).
  - There is one and only one path between every pair of nodes in a tree. The elements of the tree are called twigs or branches. In a graph with n nodes,
  - The number of branches: b = n-1
  - For the graph of **Fig 1.5,** some of the possible trees could be T(1,2,3), T(1,4,6), T(2,4,5), T(2,5,6), etc.

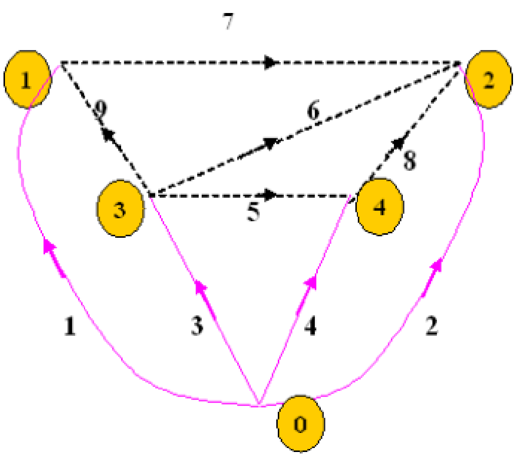
- **Co-Tree :** The set of branches of the original graph G, not included in the tree is called the co-tree.
  - The co-tree could be connected or non-connected, closed or open. The branches of the co-tree are **called links.**
  - By convention, the tree elements are shown as solid lines while the co-tree elements are shown by dotted lines as shown in Fig.1c for tree T(1,2,3).
  - With e as the total number of elements, the number of links: $l = e - b = e - n + 1$
  - For the graph of **Fig 1.5**, the co-tree graphs corresponding to the various tree graphs are as shown in the table below:

Tree	**T(1,2,3)**	**T(1,4,6)**	**T(2,4,5)**	**T(2,5,6)**
Co-Tree	**T(4,5,6)**	**T(2,3,5)**	**T(1,3,6)**	**T(1,3,4)**

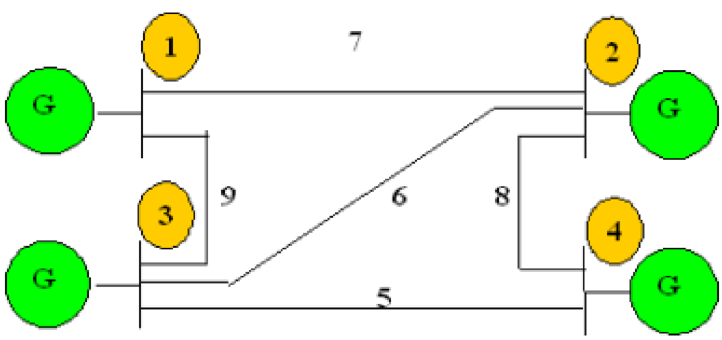
- **Basic loops:** When a link is added to a tree it forms a closed path or a loop.
  - Addition of each subsequent link forms the corresponding loop.
  - **A loop** containing only one link and remaining branches is **called a basic loop or a fundamental loop.**
  - These loops are defined for a particular tree. Since each link is associated with a basic loop, the number of basic loops is equal to the number of links.

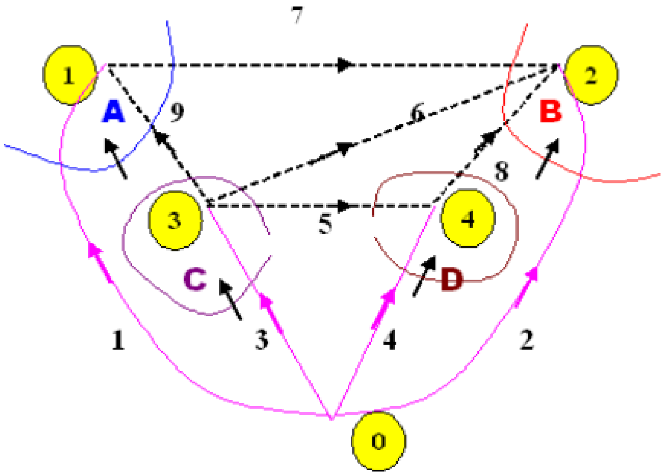
- **Basic cut-sets:** Cut-sets which contain only one branch and remaining links are **called basic cutsets or fundamental cut-sets.**
  - The basic cut-sets are defined for a particular tree.
  - Since each branch is associated with a basic cut-set, the number of basic cut-sets is equal to the number of branches.

Example: Obtain the oriented graph for the system shown in Fig. below. Select any four possible trees. For a selected tree show the basic loops and basic cut-sets.



Oriented Graph

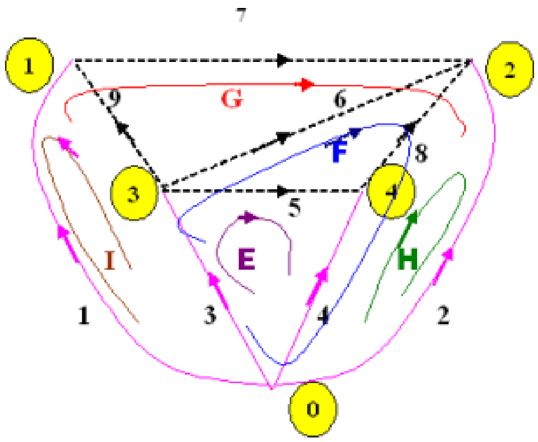




Basic Cutsets

For the system given, the oriented graph is as shown in figure E1b. some of the valid Tree graphs could be T(1,2,3,4), T(3,4,8,9), T(1,2,5,6), T(4,5,6,7), etc.

The basic cutsets (A,B,C,D) and basic loops (E,F,G,H,I) corresponding to the oriented graph of Fig.E1a and tree, T(1,2,3,4) are as shown in Figures respectively.



Basic Loops

1.3 Formation of Incidence Matrices

Element–Node incidence matrix: Aˆ

The incidence of branches to nodes in a connected graph is given by the element-node incidence matrix, Aˆ.

An element a_{ij} of Aˆ is defined as under:

a_{ij} = 1 if the ith element is incident to and oriented away from the jth node.

= -1 if the ith element is incident to and oriented towards the jth node.

= 0 if the ith element is not at all incident on the n jth node.

Thus the dimension of Aˆ is e×n,

Where e is the number of elements (e is no. of rows) and

n is the number of nodes (n is no. of columns) in the network.

Examples:

Consider again the sample system with its oriented graph as in **fig.1.6** the corresponding element-node incidence matrix, is obtained as under:

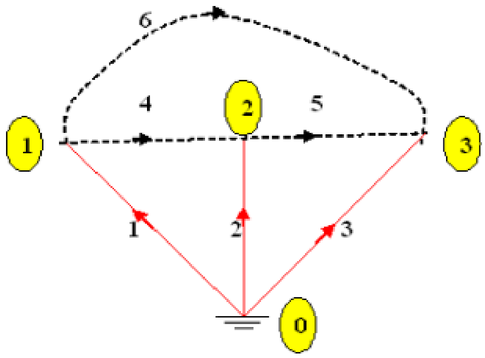


Fig.1.6 Oriented Graph

$\hat{A}$ =

Nodes / Elements	0	1	2	3
1	1	-1		
2	1		-1	
3	1			-1
4		1	-1	
5			1	-1
6		1		-1

Bus Incidence Matrix: A

A =

Buses / Elements	1	2	3
1	-1		
2		-1	
3			-1
4	1	-1	
5		1	-1
6	1		-1

=

A_b	Branches
A_l	Links

Bus Incidence Matrix:

For the specific system of **Fig.1.7** below, we obtain the following relations between the nine element voltages and the four bus (i.e. tree branch) voltages V_1, V_2, V_3 and V_4 .

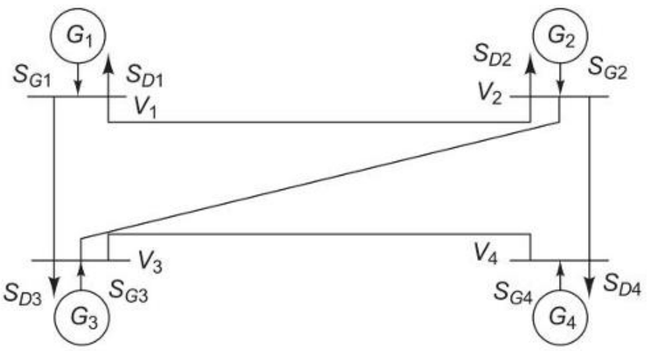


Fig.1.7 One Line Diagram of 4-Bus Power System

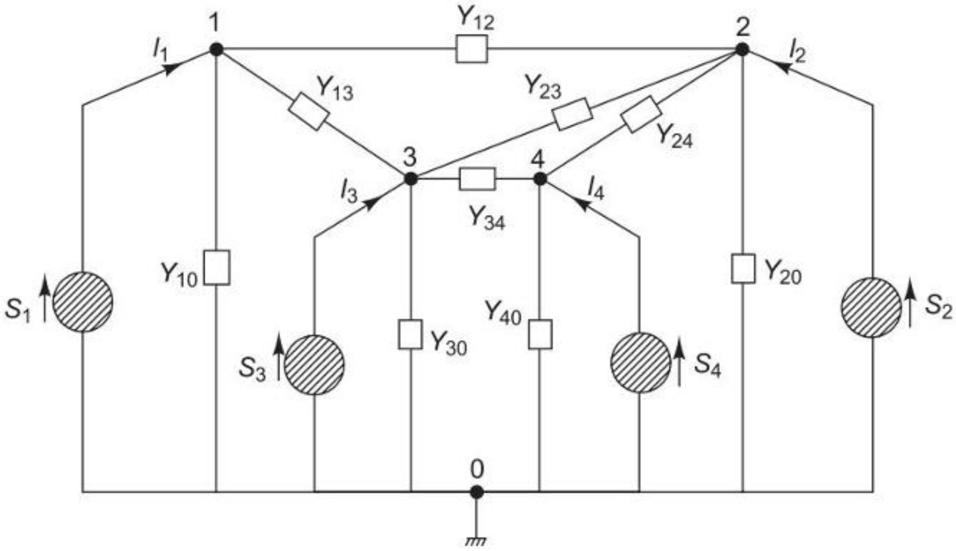


Fig.1.8 Reduced Circuit Diagram of 4-Bus Power System

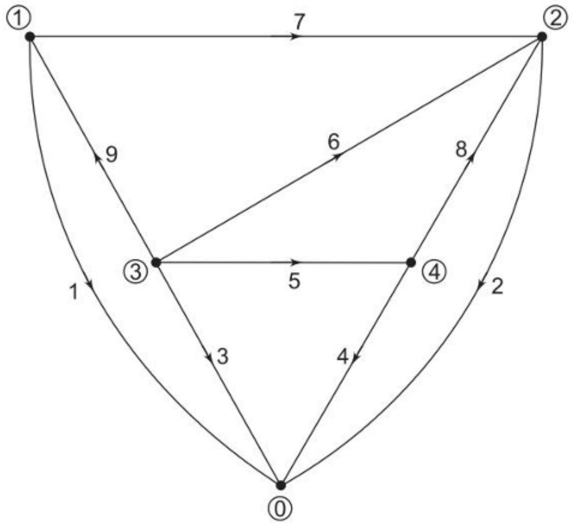


Fig.1.9 Oriented Graph

$V_{b1} = V_1$; $V_{b2} = V_2$; $V_{b3} = V_3$; $V_{b4} = V_4$

$V_{l5} = V_1 - V_2$; $V_{l6} = V_1 - V_3$; $V_{l7} = V_2 - V_3$; $V_{l8} = V_2 - V_4$; $V_{l9} = V_3 - V_4$;

or, in matrix form or, in matrix form **$V = A\, V_{BUS}$ -----------(1)**

Where V is the vector of element voltages of order $\mathbf{e} \times \mathbf{1}$ (e = number of elements), V_{BUS} is the vector of bus voltages of order $\mathbf{b} \times \mathbf{1}$ (b = number of branches = number of buses = n) and A is the bus incidence matrix of order $\mathbf{e} \times \mathbf{b}$ given by

$$\begin{array}{c|cccc}
\overset{Bus \rightarrow}{e \downarrow} & 1 & 2 & 3 & 4 \\
\hline
1 & 1 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 \\
3 & 0 & 0 & 1 & 0 \\
4 & 0 & 0 & 0 & 1 \\
\hline
5 & 1 & -1 & 0 & 0 \\
6 & 1 & 0 & -1 & 0 \\
7 & 0 & 1 & -1 & 0 \\
8 & 0 & 1 & 0 & -1 \\
9 & 0 & 0 & 1 & -1
\end{array}
\quad \begin{array}{c} \text{Buses} \\ \text{Elements} \end{array}
\quad \frac{\text{Branches}}{\text{links}} = \left[\frac{A_b}{A_l}\right] = \left[\frac{I}{A_l}\right]$$

-------- **(2)**

This matrix is rectangular, and therefore, singular. Its elements a_{ik} are found as per the following rules:

a_{ik} = 1 if *i* th element is incident to and oriented away from the *k* th node (bus)

= – 1 if *i* th element is incident to but oriented towards the *k* th node

= 0 if the *i* th element is not incident to the *k* th node

Substituting Eqn. (1) into $\mathbf{V_{BUS} = Z_{BUS}\, I_{BUS}}$, we get

$$\mathbf{I + J = Y\, A\, V_{BUS}} \quad \text{-------------- } \mathbf{(3)}$$

Premultiplying by A^T, $\quad \mathbf{A^T I + A^T J = A^T\, Y\, A\, V_{BUS}} \quad$ -------------- **(4)**

each component of the n-dimensional vector $\mathbf{A^T I}$ is the algebraic sum of the element currents leaving nodes 1, 2, ..., n(n = number of buses). Therefore the application of KCL must result in

$$\mathbf{A^T I = 0} \quad \text{-------------- } \mathbf{(5)}$$

Similarly, each component of the vector $\mathbf{A^T J}$ can be recognized as the algebraic sum of all source currents injected into nodes 1, 2, ..., n. These components are therefore the bus currents. Hence we can write

$$\mathbf{A^T J = I_{BUS}} \quad \text{-------------- } \mathbf{(6)}$$

Equation (4) then is simplified to

$$\mathbf{I_{BUS} = (A^T\, Y\, A)\, V_{BUS}} \quad \text{-------------- } \mathbf{(7)}$$

Thus, following an alternative systematic approach, we obtain the same nodal current equation as Eqn. $\mathbf{I_{BUS} = Y_{BUS}\, V_{BUS}}$. The bus admittance matrix can then be obtained from the singular transformation of the primitive **Y** matrix, i.e.,

$$\mathbf{Y_{BUS} = A^T\, Y\, A} \quad \text{--------------- } \mathbf{(8)}$$

A computer programme can be developed to write the bus incidence matrix A from the interconnected data of the directed elements of the power system. Standard matrix transpose and multiplication subroutines can then be used to compute **YBUS** from Eqn. (8).

Example: For the sample network-oriented graph shown in **Fig.1.10**, by selecting a tree, T(1,2,3,4), obtain the incidence matrices A and Aˆ .
Also show the partitioned form of the matrix-A.

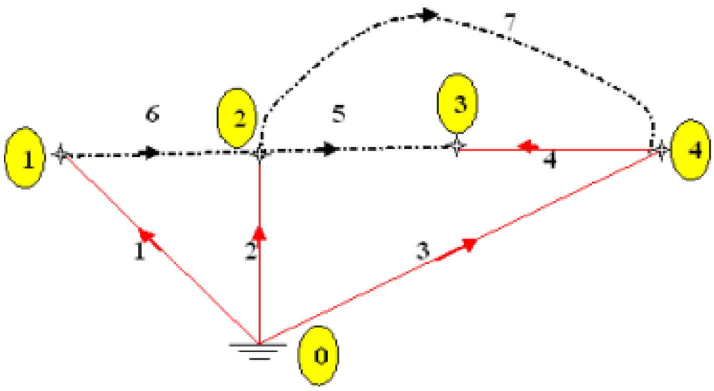


Fig.1.10 Sample Network-Oriented Graph

$$\hat{A} = \text{Elements} \overset{\text{nodes}}{\begin{bmatrix} e\backslash n & 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & -1 & 0 & 0 & 0 \\ 2 & 1 & 0 & -1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & 0 & -1 & 1 \\ 5 & 0 & 0 & 1 & -1 & 0 \\ 6 & 0 & 1 & -1 & 0 & 0 \\ 7 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}} \qquad \mathbf{A} = \text{Elements} \overset{\text{buses}}{\begin{bmatrix} e\backslash b & 1 & 2 & 3 & 4 \\ 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & -1 & 1 \\ 5 & 0 & 1 & -1 & 0 \\ 6 & 1 & -1 & 0 & 0 \\ 7 & 0 & 1 & 0 & -1 \end{bmatrix}}$$

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two submatrices as under:

$$A_b = \text{branches} \overset{\text{buses}}{\begin{bmatrix} b\backslash b & 1 & 2 & 3 & 4 \\ 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & -1 & 1 \end{bmatrix}} \qquad A_l = \text{links} \overset{\text{buses}}{\begin{bmatrix} l\backslash b & 1 & 2 & 3 & 4 \\ 5 & 0 & 1 & -1 & 0 \\ 6 & 1 & -1 & 0 & 0 \\ 7 & 0 & 1 & 0 & -1 \end{bmatrix}}$$

Example: For the sample-system shown in **Fig. 1.11**, obtain an oriented graph. By selecting a tree, T(1,2,3,4), obtain the incidence matrices A and $\hat{A}$. Also show the partitioned form of the matrix-A.

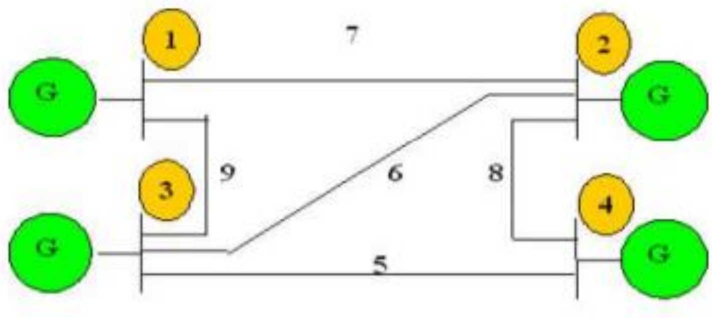


Fig.1.11

Consider the oriented graph of the given system as shown in **fig.1.12**, below.

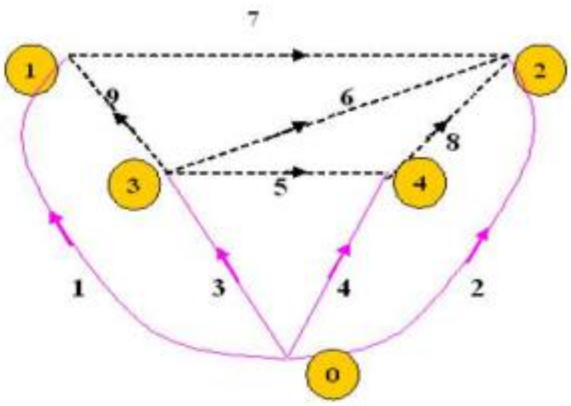


Fig.1.4b Oriented Graph of System of Fig 1.12

Corresponding to the oriented graph above and a Tree, T(1,2,3,4), the incidence matrices A and A can be obtained as follows:

$\hat{A}$ =

e\n	0	1	2	3	4
1	1	-1			
2	1		-1		
3	1			-1	
4	1				-1
5				1	-1
6			-1	1	
7		1	-1		
8			-1		1
9		-1		1	

A =

e\b	1	2	3	4
1	-1			
2		-1		
3			-1	
4				-1
5			1	-1
6		-1	1	
7	1	-1		
8		-1		1
9	-1		1	

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two sub-matrices as under:

$A_b =$

e\b	1	2	3	4
1	-1			
2		-1		
3			-1	
4				-1

$A_l =$

e\b	1	2	3	4
5			1	-1
6		-1	1	
7	1	-1		
8		-1		1
9	-1		1	

1.4 Primitive Network

In general, a network element may contain active or passive components. Network elements represented both in impedance and admittance form are as shown in fig.a & fig.b respectively.

Primitive Network is a set of unconnected elements which gives information regarding the characteristics of individual elements.

Fig.1.13 shows a general network element, connected between nodes r and s, with its alternative impedance and admittance form.

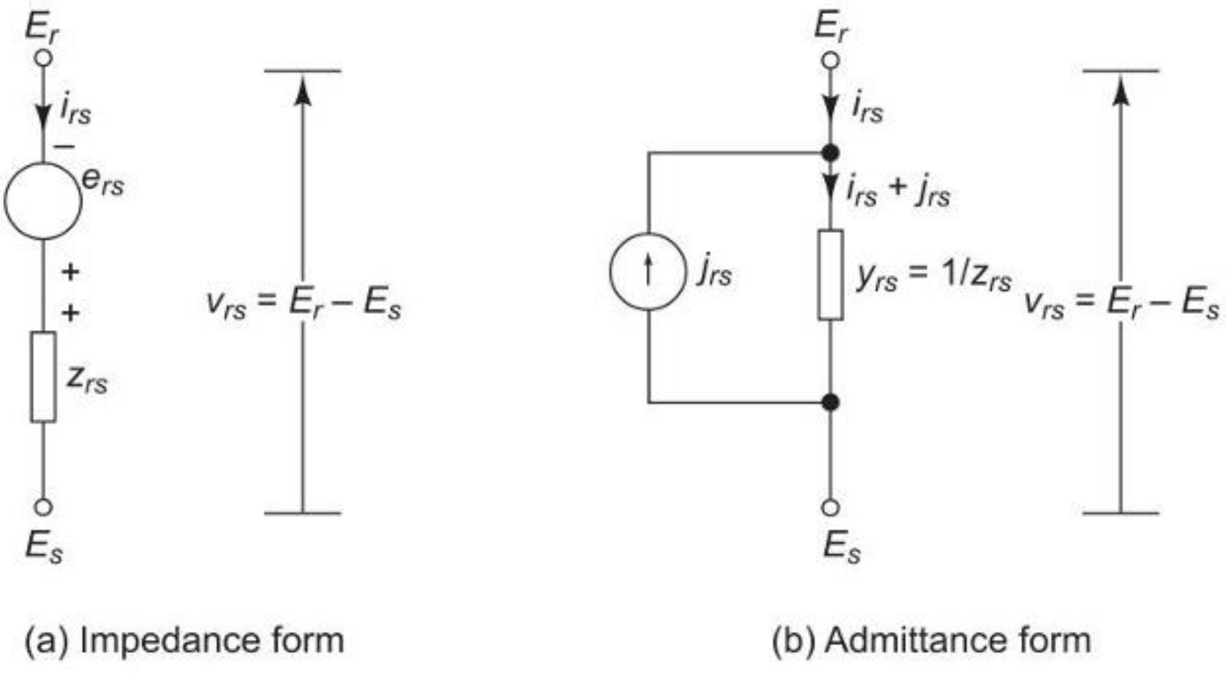


Fig.1.13 Representation of a network element

The impedance form is a voltage source e_{rs} in series with an impedance z_{rs} , while the admittance form is a current source j_{rs} in parallel with an admittance y_{rs} .

The element current is i_{rs} , and the element voltage is $v_{rs} = E_r - E_s$, where E_r and E_s are the voltages of the element nodes r and s respectively.

It may be remembered here that for steady state AC performance, all element variables (v_{rs} , E_r , E_s , i_{rs} , j_{rs}) are phasors and the element parameters (z_{rs} , y_{rs}) are complex numbers.

The voltage relation for **Fig. 1.13** can be written as

$$v_{rs} + e_{rs} = z_{rs}\, i_{rs} \text{ --------- } (9)$$

Similarly, the current relation for Fig. 1.5 can be written as

$$i_{rs} + j_{rs} = y_{rs}\, v_{rs} \text{ --------- } (10)$$

The forms of Fig. 1.13(a) and (b) are equivalent, wherein the parallel source current in admittance form is related to the series voltage in impedance form by

$$j_{rs} = y_{rs}\, e_{rs}$$

Also $y_{rs} = 1/z_{rs}$

A set of unconnected elements is defined as a primitive network.

The performance equations of a primitive network are given by:

In impedance form, $\mathbf{V + E = ZI}$ ----------------- **(11)**

In admittance form, $\mathbf{I + J = YV}$ ----------------- **(12)**

Here **V** and **I** are the element voltage and current vectors respectively and **J** and **E** are the source vectors. **Z** and **Y** are referred to as the primitive impedance and admittance matrices, respectively. These are related as $\mathbf{Z = Y^{-1}}$.

If there is no mutual coupling between elements **Z** and **Y** are diagonal matrices, where the diagonal entries are the impedances/admittances of the network elements and are reciprocal.

Example: Given that the self impedances of the elements of a network referred by the bus incidence matrix given below are equal to: $Z_1=Z_2=0.2$, $Z_3=0.25$, $Z_4=Z_5=0.1$ and $Z_6=0.4$ units, draw the corresponding oriented graph, and find the primitive network matrices. Neglect mutual values between the elements.

A =

-1	0	0
0	-1	0
0	0	-1
1	-1	0
0	1	-1
1	0	-1

Example: Consider three passive elements whose data is given in Table below. Form the primitive network impedance matrix.

Element Number	Self Impedance (Zpq-pq)		Mutual Impedance (Zpq-rs)	
	Bus Code (pq)	Impedance in pu	Bus Code (rs)	Impedance in pu
1	1-2	j0.452		
2	2-3	j0.387	1-2	j0.165
3	1-3	j0.619	1-2	j0.234

1.6 Formation of Y_{BUS} and Z_{BUS}

- The bus admittance matrix, Y_{BUS} plays a very important role in computer aided power system analysis.
- It can be formed in practice by either of the methods as under:
  - Rule of Inspection or Inspection Method
  - Singular Transformation
  - Non-Singular Transformation
  - Z_{BUS} Building Algorithms

1.6.1 Formation of Y_{BUS} by Inspection Method or Rule of Inspection

Self Admittance:

- Each diagonal term Y_{pp} is called the self admittance of node p.
- It is equal to the algebraic sum of all the admittances connected to that node (including offline admittances, if any).

Mutual Admittance:

- Each off-diagonal element Y_{pq} is called the mutual admittance between nodes p and q.
- Its value is equal to the negative sum of all admittances connected between these nodes.
- Further, we have the condition $Y_{pq} = Y_{qp}$.

Consider the 3-node admittance network as shown in **fig.1.14** below. Using the basic branch relation: I = (YV), for all the elemental currents and applying Kirchhoff's Current Law principle at the Nodal points, we get the relations as under:

At node 1: $I_1 = Y_1V_1 + Y_3(V_1 - V_3) + Y_6(V_1 - V_2)$

At node 2: $I_2 = Y_2V_2 + Y_5(V_2 - V_3) + Y_6(V_2 - V_1)$

At node 3: $0 = Y_3(V_3 - V_1) + Y_4V_3 + Y_5(V_3 - V_2)$

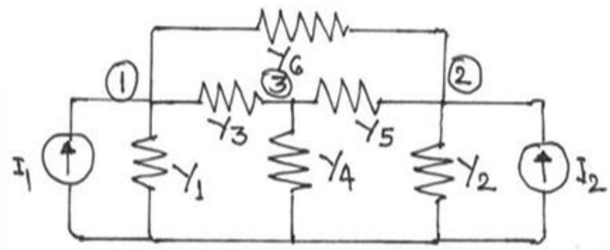


Fig.1.14

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} = \begin{vmatrix} (Y_1+Y_3+Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix}$$

$$\mathbf{I_{BUS} = Y_{BUS}\,E_{BUS}}$$

Diagonal Elements: A diagonal element (Yii) of the bus admittance matrix, YBUS, is equal to the sum total of the admittance values of all the elements incident at the bus/node i.

Off Diagonal Elements: An off-diagonal element (Yij) of the bus admittance matrix, YBUS, is equal to the negative of the admittance value of the connecting element present between the buses i and j, if any.

This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

Yii = S yij (j = 1,2,…….n)

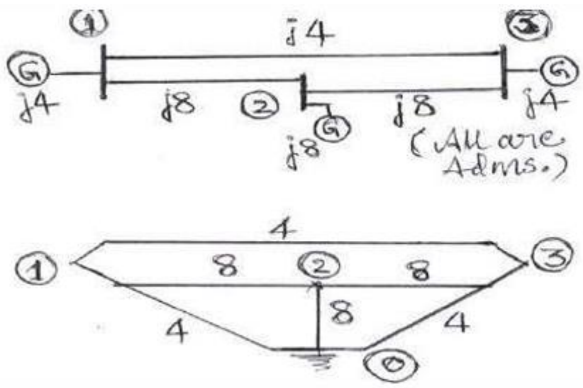
Yij = –yij (j = 1,2,…….n)

For i = 1,2,….n Where n = no. of buses of the given system

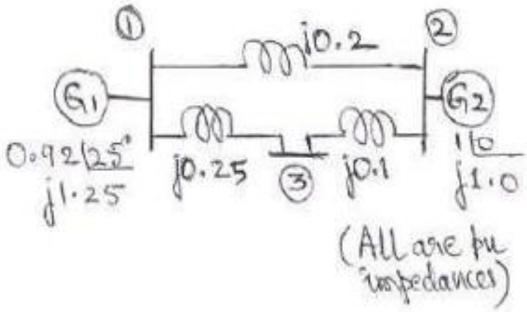
yij is the admittance of element connected between buses i and j and yii is the admittance of element connected between bus i and ground (reference bus).

Example: Obtain the bus admittance matrix for the admittance network shown aside by the rule of inspection.

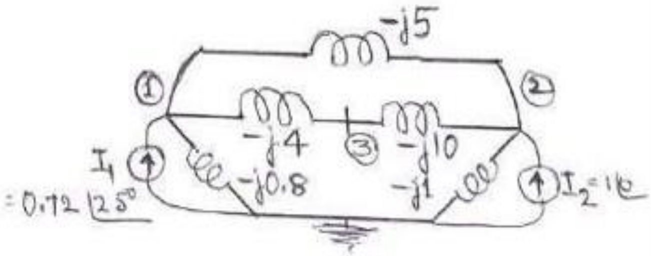
$$Y_{BUS} = j\begin{vmatrix} 16 & -8 & -4 \\ -8 & 24 & -8 \\ -4 & -8 & 16 \end{vmatrix}$$



Example: Obtain YBUS for the impedance network shown aside by the rule of inspection. Also, determine YBUS for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.



$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$



$$Z_{BUS} = Y_{BUS}^{-1}$$

1.6.2 Formation of Y_{BUS} by Singular Transformation

The matrix pair Y_{BUS} and Z_{BUS} form the network models for load flow studies. The Y_{BUS} can be alternatively assembled by the use of singular transformation given by a graph theoretical approach. This approach is of great theoretical and practical significance.

Algorithm for the Formation of Y_{BUS} Matrix

(a) Assuming no Mutual Coupling between Transmission Lines

Initially all the elements of Y_{BUS} are set to zero. Addition of an element of admittance y between buses i and j affects four entries in Y_{BUS}, viz., Yii, Yij, Yji, Yjj, as follows:

$$Y_{ii\ new} = Y_{ii\ old} + y\ ; \quad Y_{ij\ new} = Y_{ij\ old} - y\ ; \quad Y_{ji\ new} = Y_{ji\ old} - y\ ; \quad Y_{jj\ new} = Y_{jj\ old} + y$$

Addition of an element of admittance y from bus i to ground will only affect Y_{ii}, i.e.

$$Y_{ii\ new} = Y_{ii\ old} + y$$

Example: Consider the sample four-bus system in **Fig. 1.1**. Initially, set all the elements of Y_{BUS} to zero.

1. Addition of y_{10} affects only Y_{11}

$$Y_{11\ new} = Y_{11\ old} + y_{10} = 0 + y_{10} = y_{10}$$

2. Addition of y_{12} affects Y_{11}, Y_{12}, Y_{21}, Y_{22}

$$Y_{11\ new} = Y_{11\ old} + y_{12} = y_{10} + y_{12}$$

$$Y_{12\ new} = Y_{12\ old} - y_{12} = 0 - y_{12} = -y_{12} \text{ ------------ (i)}$$

$$Y_{21\ new} = Y_{21\ old} - y_{12} = 0 - y_{12} = -y_{12} \text{ ------------ (ii)}$$

$$Y_{22\ new} = Y_{22\ old} - y_{12} = 0 + y_{12}$$

3. Addition of y_{13} affects Y_{11}, Y_{13}, Y_{31}, Y_{33}

$$Y_{11\,new} = Y_{11\,old} + y_{13} = y_{10} + y_{12} + y_{13} \text{ ------------- (iii)}$$

$$Y_{13\,new} = Y_{13\,old} - y_{13} = 0 - y_{13} = -y_{13} \text{ ------------------ (iv)}$$

$$Y_{31\,new} = Y_{31\,old} - y_{13} = 0 - y_{13} = -y_{13} \text{ ---------- (v)}$$

$$Y_{33\,new} = Y_{33\,old} + y_{13} = 0 + y_{13} = y_{13}$$

4. Addition of y_{20} affects only Y_{22}

$$Y_{22\,new} = Y_{22\,old} + y_{20} = y_{12} + y_{20}$$

5. Addition of y_{23} affects Y_{22}, Y_{23}, Y_{32}, Y_{33}

$$Y_{22\,new} = Y_{22\,old} + y_{23} = (y_{20} + y_{12}) + y_{23} = y_{20} + y_{12} + y_{23}$$

$$Y_{23\,new} = Y_{23\,old} - y_{23} = 0 - y_{23} = -y_{23} \text{ ------------- (vi)}$$

$$Y_{32\,new} = Y_{32\,old} - y_{23} = 0 - y_{23} = -y_{23} \text{ ----------- (vii)}$$

$$Y_{33\,new} = Y_{33\,old} + y_{23} = y_{13} + y_{23}$$

6. Addition of y_{24} affects Y_{22}, Y_{24}, Y_{42}, Y_{44}

$$Y_{22\,new} = Y_{22\,old} + y_{24} = (y_{20} + y_{12} + y_{23}) + y_{24} \text{ ---------- (viii)}$$

$$= y_{20} + y_{12} + y_{23} + y_{24}$$

$$Y_{24\,new} = Y_{24\,old} - y_{24} = 0 - y_{24} = -y_{24} \text{ ---------- (ix)}$$

$$Y_{42\,new} = Y_{42\,old} - y_{24} = 0 - y_{24} = -y_{24} \text{ ---------- (x)}$$

$$Y_{44\,new} = Y_{44\,old} + y_{24} = 0 + y_{24} = y_{24}$$

7. Addition of y_{30} affects only Y_{33}

$$Y_{33\,new} = Y_{33\,old} + y_{30} = (y_{13} + y_{23}) + y_{30} + y_{13} + y_{23}$$

8. Addition of y_{34} affects Y_{33}, Y_{34}, Y_{43}, Y_{44}

$$Y_{33\,new} = Y_{33\,old} + y_{34} = (y_{30} + y_{13} + y_{23}) + y_{34} \text{ -------- (xi)}$$

$$= y_{30} + y_{13} + y_{23} + y_{34}$$

$$Y_{34\,new} = Y_{34\,old} - y_{34} = 0 - y_{34} = -y_{34} \text{ ------- (xii)}$$

$$Y_{43\,new} = Y_{43\,old} - y_{34} = 0 - y_{34} = -y_{34} \text{ ------- (xiii)}$$

$$Y_{44\,new} = Y_{44\,old} + y_{34} = y_{24} + y_{34}$$

9. Addition of y_{40} affects only Y_{44}

$$Y_{44\,new} = Y_{44\,old} + y_{40} = (y_{24} + y_{34}) + y_{40} = y_{40} + y_{24} + y_{34} \text{ ------------- (xiv)}$$

The final values of the elements of the bus admittance matrix are given by appropriate equation from Eqns. (i) through (xiv).

Further $Y_{14} = Y_{41} = 0$

(b) Assuming Mutual Coupling between Transmission Lines

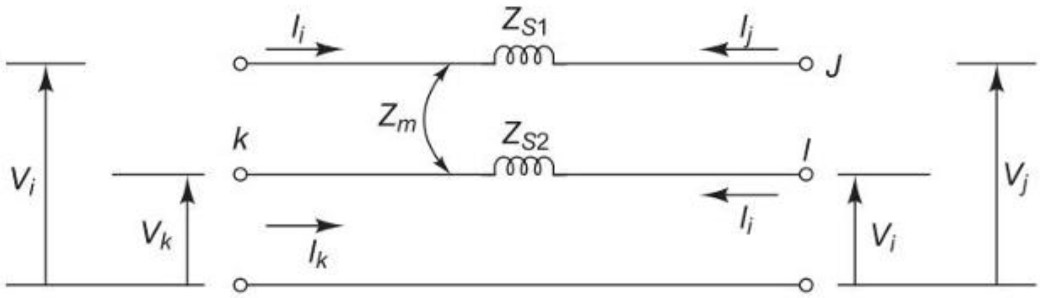


Fig. 1.15 Mutually Coupled Transmission Lines

The equivalent circuit of mutually coupled transmission lines is shown in **Fig. 1.15.** Shunt elements are omitted for simplicity; this effect can be included in a straight forward manner as seen in above Example.

The mutual impedance between the transmission lines is zm, and the series impedances are z_{S1} and z_{S2}. From **Fig. 1.15**, we have

$$V_i = z_{s1} I_i + z_m I_k + V_j$$
$$V_k = z_{s2} I_k + z_m I_i + V_l$$

or

$$\begin{bmatrix} V_i \\ V_k \end{bmatrix} - \begin{bmatrix} V_j \\ V_l \end{bmatrix} = \begin{bmatrix} z_{s1} & z_m \\ z_m & z_{s2} \end{bmatrix} \begin{bmatrix} I_i \\ I_k \end{bmatrix}$$

or

$$\begin{bmatrix} I_i \\ I_k \end{bmatrix} = \begin{bmatrix} y_{s1} & y_m \\ y_m & y_{s2} \end{bmatrix} \begin{bmatrix} V_i - V_j \\ V_k - V_l \end{bmatrix}$$ **------(1.a)**

Similarly, we have

$$\begin{bmatrix} I_j \\ I_l \end{bmatrix} = \begin{bmatrix} y_{s1} & y_m \\ y_m & y_{s2} \end{bmatrix} \begin{bmatrix} V_j - V_i \\ V_l - V_k \end{bmatrix}$$

where

$$\begin{bmatrix} y_{s1} & y_m \\ y_m & y_{s2} \end{bmatrix} = \begin{bmatrix} z_{s1} & z_m \\ z_m & z_{s2} \end{bmatrix}^{-1}$$ **------- (1.b)**

From Eqns. 1(a) and (b) the elements of Y_{BUS} become

$$\left\{\begin{array}{l} Y_{ii\,\text{new}} = Y_{ii\,\text{old}} + y_{s1} \\ Y_{jj\,\text{new}} = Y_{jj\,\text{old}} + y_{s1} \\ Y_{kk\,\text{new}} = Y_{kk\,\text{old}} + y_{s2} \\ Y_{ll\,\text{new}} = Y_{ll\,\text{old}} + y_{s2} \end{array}\right\}$$

$$\left\{\begin{array}{l} Y_{ij\,\text{new}} = Y_{ji\,\text{new}} = Y_{ij\,\text{old}} - y_{s1} \\ Y_{kl\,\text{new}} = Y_{lk\,\text{new}} = Y_{kl\,\text{old}} - y_{s2} \end{array}\right\} \text{------------------------------ (2)}$$

$$\left\{\begin{array}{l} Y_{ik\,\text{new}} = Y_{ki\,\text{new}} = Y_{ik\,\text{old}} + y_{m} \\ Y_{jl\,\text{new}} = Y_{lj\,\text{new}} = Y_{jl\,\text{old}} + y_{m} \end{array}\right\}$$

$$\left\{\begin{array}{l} Y_{il\,\text{new}} = Y_{li\,\text{new}} = Y_{il\,\text{old}} - y_{m} \\ Y_{jk\,\text{new}} = Y_{kj\,\text{new}} = Y_{jk\,\text{old}} - y_{m} \end{array}\right\} \text{------------------------------- (3)}$$

Example: Figure 1.16 shows the one-line diagram of a simple four-bus system. Table 1.1 gives the line impedances identified by the buses on which these terminate. The shunt admittance at all the buses is assumed to be negligible.

(a) Find Y_{BUS} , assuming that the line shown dotted is not connected.

(b) What modifications need to be carried out in Y_{BUS} if the line shown dotted is connected?

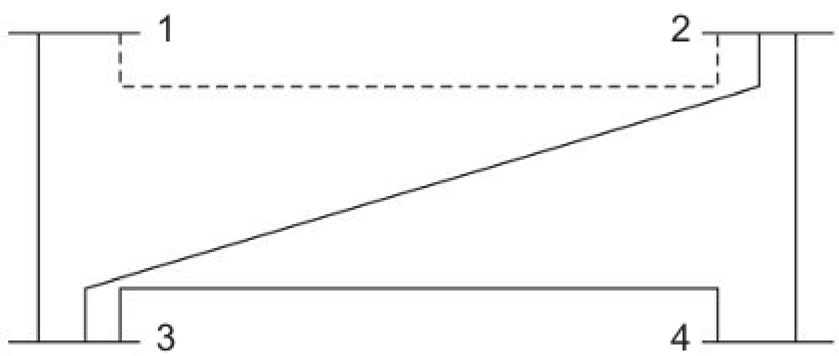


Fig. 1.16 One-line diagram of a simple four-bus system

Solution:

(a) From Table 1.1, Table 1.2 is obtained from which Y can be written as

Table 1.1

Line, bus to bus	*R, pu*	*X, pu*
1–2	0.05	0.15
1–3	0.10	0.30
2–3	0.15	0.45
2–4	0.10	0.30
3–4	0.05	0.15

Table 1.2

Line	*G, pu*	*B, pu*
1–2	2.000	– 6.0
1–3	1.000	– 3.0
2–3	0.666	– 2.0
2–4	1.000	– 3.0
3–4	2.000	– 6.0

$$\boldsymbol{Y}_{\text{BUS}} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \tag{i}$$

$$\boldsymbol{Y}_{\text{BUS}} = \begin{bmatrix} y_{13} & 0 & -y_{13} & 0 \\ 0 & (y_{23}+y_{24}) & -y_{23} & -y_{24} \\ -y_{13} & -y_{23} & (y_{31}+y_{32}+y_{34}) & -y_{34} \\ 0 & -y_{24} & -y_{34} & (y_{43}+y_{42}) \end{bmatrix} \tag{ii}$$

$$\boldsymbol{Y}_{\text{BUS}} = \begin{bmatrix} 1-j3 & 0 & -1+j3 & 0 \\ 0 & 1.666-j5 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix} \tag{iii}$$

(b) The following elements of Y_{BUS} of part (a) are modified when a line is added between buses 1 and 2.

$$\begin{aligned} Y_{11\text{ new}} &= Y_{11\text{ old}} + (2 - j6) = 3 - j9 \\ Y_{12\text{ new}} &= Y_{12\text{ old}} - (2 - j6) = -2 + j6 = Y_{21\text{ new}} \\ Y_{22\text{ new}} &= Y_{22\text{ old}} + (2 - j6) = 3.666 - j11 \end{aligned} \qquad \text{(iv)}$$

Modified $\boldsymbol{Y}_{\text{BUS}}$ is written as

$$\boldsymbol{Y}_{\text{BUS}} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.666-j11 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix} \qquad \text{(v)}$$

Example of Singular Transformation

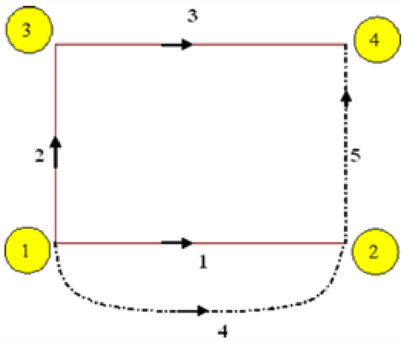
Bus Admittance Matrix, Y_{BUS} and Bus Impedance Matrix, Z_{BUS}

- The primitive network matrices are the most basic matrices and depend purely on the impedance or admittance of the individual elements.
- However, they do not contain any information about the behaviour of the interconnected network variables.
- Hence, it is necessary to transform the primitive matrices into more meaningful matrices which can relate variables of the interconnected network.

In the bus frame of reference, the performance of the interconnected network is described by n independent nodal equations, where n is the total number of buses (n+1nodes are present, out of which one of them is designated as the reference node).

Example: For the network of Fig, form the primitive matrices [z] & [y] and obtain the bus admittance matrix by singular transformation.

Choose a Tree T(1,2,3). The data is given in Table.



Elements	Self Impedance	Mutual Impedance
1	j0.6	-
2	j0.5	j0.1 (with element 1)
3	j0.5	-
4	j0.4	j0.5 (with element 1)
5	j0.2	-

Example of formation of Y_{BUS} by Singular Transformation

Find the Y_{BUS} using singular transformation for the system of **Fig. 1.1** whose graph is shown in **Fig. 1.2.**

Solution: As there is no mutual coupling between any two lines, the primitive Y matrix is diagonal and given by the following.

$$\mathbf{Y} = \begin{array}{c|ccccccccc} \text{Elements} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline 1 & y_{10} & & & & & & & & 0 \\ 2 & & y_{20} & & & & & & & \\ 3 & & & y_{30} & & & & & & \\ 4 & & & & y_{40} & & & & & \\ 5 & & & & & y_{12} & & & & \\ 6 & & & & & & y_{13} & & & \\ 7 & & & & & & & y_{23} & & \\ 8 & & & & & & & & y_{24} & \\ 9 & 0 & & & & & & & & y_{34} \end{array}$$

Using A from Eq. (2), we get

$$\boldsymbol{YA} = \begin{bmatrix} y_{10} & 0 & 0 & 0 \\ 0 & y_{20} & 0 & 0 \\ 0 & 0 & y_{30} & 0 \\ 0 & 0 & 0 & y_{40} \\ y_{12} & -y_{12} & 0 & 0 \\ y_{13} & 0 & -y_{13} & 0 \\ 0 & -y_{23} & y_{23} & 0 \\ 0 & y_{24} & 0 & -y_{24} \\ 0 & 0 & y_{34} & -y_{34} \end{bmatrix}$$

Finally,

$$\boldsymbol{Y}_{\text{BUS}} = \boldsymbol{A}^T\,\boldsymbol{YA}$$

$$= \begin{bmatrix} (y_{10}+y_{12}+y_{13}) & -y_{12} & -y_{13} & 0 \\ -y_{12} & (y_{20}+y_{12}+y_{23}+y_{24}) & -y_{23} & -y_{24} \\ -y_{13} & -y_{23} & (y_{30}+y_{13}+y_{23}+y_{34}) & -y_{34} \\ 0 & -y_{24} & -y_{34} & (y_{40}+y_{24}+y_{34}) \end{bmatrix}$$

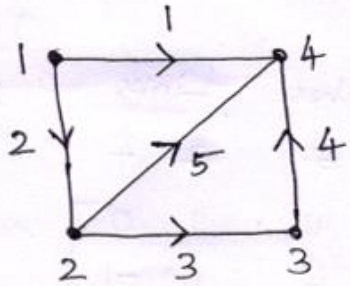
The elements of this matrix, of course, agree with those previously calculated in previous Example. If there is mutual coupling between elements i and j of the network, the primitive Y matrix will not be diagonal.

Thus, the element corresponding to the i_{th} row and j^{th} column (also, j^{th} row and i^{th} column) will be equal to the mutual admittance between the elements i and j. Thus Y_{BUS} is also modified as given by Eqn. (8).

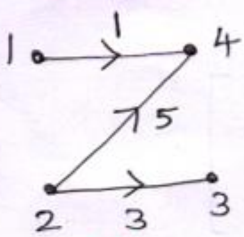
Module - 1

Illustrative Examples

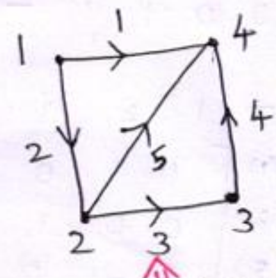
① Consider an oriented graph of the power system network shown below fig. Choose branches 1, 3 and 5 as twigs. Build a bus incidence matrix A and basic cut-set matrix B for the oriented graph. Select node 2 as reference.



Solution:



Tree



oriented Graph.

Bus Incidence Matrix, $A = \begin{array}{c} \text{Nodes (or) Buses} \rightarrow \begin{array}{ccc} 1 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \end{array}$

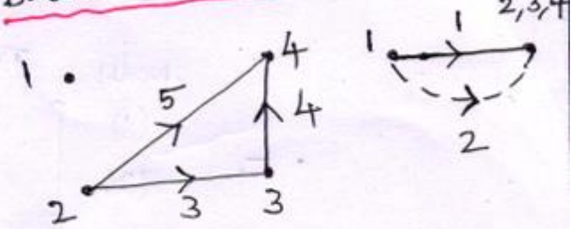
Basic cut - sets

cut - set 1 : $\{ \underline{1}, 2 \}$

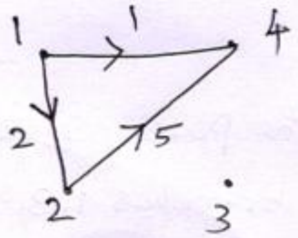
cut-set 2 : $\{ \underline{3}, 4 \}$

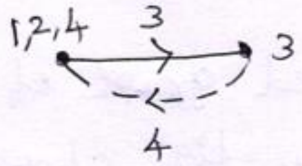
cut - set 3 : $\{ \underline{5}, 4, 2 \}$

Basic cut set 1

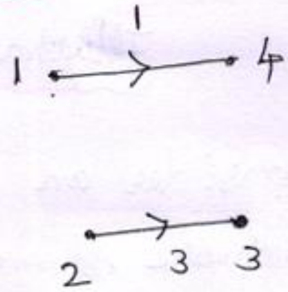


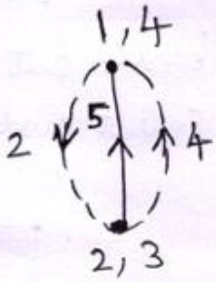
Basic cut-set 2





Basic cut-set 3





Basic (or) fundamental cut-set Matrix, $B = \left[U \,\vdots\, B_l \right]$

$$= \begin{array}{c} \\ \\ 1 \\ 3 \\ 5 \end{array} \begin{array}{c} \text{Tree Branches} \qquad \text{Links} \\ \begin{array}{ccc:cc} 1 & 3 & 5 & 2 & 4 \end{array} \\ \left[\begin{array}{ccc:cc} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 \end{array}\right] \end{array}$$

(or)

$$B = \begin{array}{cc} & \text{Branches} \rightarrow \\ 1 & (1) \\ 2 & (3) \\ 3 & (5) \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \left[\begin{array}{ccccc} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 1 & 1 \end{array}\right] \end{array}$$

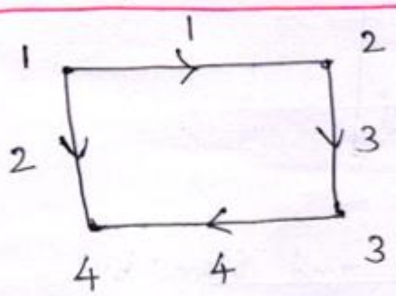
Bus Incidence Matrix, $A = \left[A_b \,\vdots\, A_l \right]$

$$= \begin{array}{c} \text{Nodes} \\ \\ (1) \\ (3) \\ (4) \end{array} \begin{array}{c} \text{Tree Branches} \qquad \text{Links} \\ \begin{array}{ccc:cc} 1 & 3 & 5 & 2 & 4 \end{array} \\ \left[\begin{array}{ccc:cc} 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ -1 & 0 & -1 & 0 & -1 \end{array}\right] \end{array}$$

(2) A power system consists of four buses. The generators are connected between buses 1-2, 1-4, 2-3 and 3-4 which have reactances of $j0.25$, $j0.5$, $j0.4$ and $j0.1$ respectively. Develop a bus admittance matrix by direct inspection method. Choose bus 1 as reference.

Solution :-

SLD of the power system n/w | oriented graph:



$$Z_{12} = j0.25 \; ; \; Z_{14} = j0.5$$

$$Z_{23} = j0.4 \; ; \; Z_{34} = j0.1$$

Bus Admittance Matrix,

$$Y_{BUS} = \begin{array}{c|c} & \text{Buses} \rightarrow \; 2 \quad 3 \quad 4 \\ \downarrow & \\ \begin{array}{c} 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} Y_{22} & Y_{23} & Y_{24} \\ Y_{32} & Y_{33} & Y_{34} \\ Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \end{array}$$

Diagonal elements are

$$Y_{22} = \frac{1}{Z_{23}} + \frac{1}{Z_{12}} = \frac{1}{j0.4} + \frac{1}{j0.25} = -6.5j$$

$$Y_{33} = \frac{1}{Z_{34}} + \frac{1}{Z_{23}} = \frac{1}{j0.1} + \frac{1}{j0.4} = -12.5j$$

$$Y_{44} = \frac{1}{Z_{14}} + \frac{1}{Z_{34}} = \frac{1}{j0.5} + \frac{1}{j0.1} = -12j$$

off - diagonal elements are,

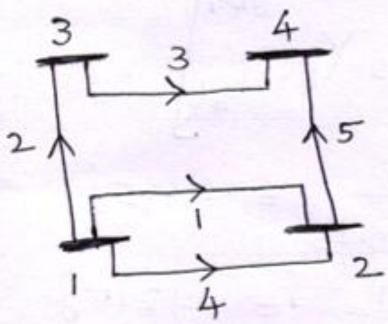
$$Y_{23} = Y_{32} = -y_{23} = -\frac{1}{Z_{23}} = \frac{-1}{j0.4} = 2.5j$$

$$Y_{24} = Y_{42} = -y_{24} = -\frac{1}{Z_{24}} = 0$$

$$Y_{34} = Y_{43} = -y_{34} = -\frac{1}{Z_{34}} = \frac{-1}{j0.1} = 10j$$

$$\therefore Y_{Bus} = \begin{bmatrix} -6.5j & 2.5j & 0 \\ 2.5j & -12.5j & 10j \\ 0 & 10j & -12j \end{bmatrix}$$

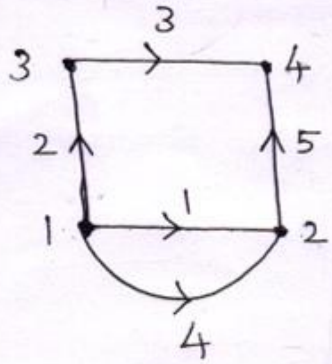
(3) Build bus incidence matrix A and then bus admittance matrix Y_{Bus} using Singular transformation method for the power system network shown below in fig. Choose bus1 as reference. The line data of the power system are given in table below.



Line NO	Bus code (p-q)	Z(pu)	Mutual temperature Z_m (pu)
1	1 - 2	0.6	0.2 (line2)
2	1 - 3	0.5	—
3	3 - 4	0.5	—
4	1 - 2	0.4	0.1 (line1)
5	2 - 4	0.2	—

Solution :-

Oriented Graph

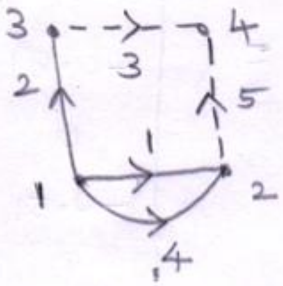


Bus Incidence Matrix, A

$$= \begin{array}{c} \text{Branches} \downarrow \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \text{Nodes} \rightarrow \\ \begin{matrix} 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \end{array}$$

$b = m - 1 = 4 - 1 = 3$
$= n$

$$A = \begin{array}{c} \text{Nodes} \downarrow \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \text{Tree Branches} \rightarrow \quad \text{Links} \rightarrow \\ \begin{matrix} 1 & 2 & 4 & 3 & 5 \end{matrix} \\ \left[\begin{array}{ccc|cc} 1 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 \end{array}\right] \end{array}$$



$l = e - b = 5 - 3$
$= 2$

Primitive Impedance Matrix, Z

$$= \begin{bmatrix} 0.6 & 0.2 & 0 & 0.1 & 0 \\ 0.2 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0.1 & 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}$$

Primitive Admittance Matrix, $Y = \frac{1}{Z}$

$$= \begin{bmatrix} \frac{1}{0.6} & \frac{1}{0.2} & 0 & \frac{1}{0.1} & 0 \\ \frac{1}{0.2} & \frac{1}{0.5} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{0.5} & 0 & 0 \\ \frac{1}{0.1} & 0 & 0 & \frac{1}{0.4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{0.2} \end{bmatrix}$$

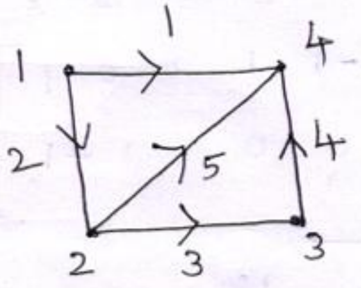
$$= \begin{bmatrix} 2.02 & -0.81 & 0 & -0.51 & 0 \\ -0.81 & 2.32 & 0 & 0.20 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ -0.51 & 0.20 & 0 & 2.63 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Bus Admittance Matrix, $Y_{BUS} = A^T Y A$

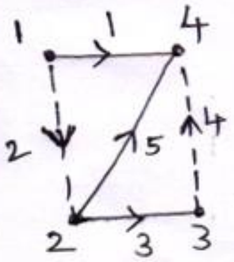
$$= \underset{3\times5}{\begin{bmatrix} -1 & 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix}} \underset{5\times5}{\begin{bmatrix} 2.02 & -0.81 & 0 & -0.51 & 0 \\ -0.81 & 2.32 & 0 & 0.20 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ -0.51 & 0.20 & 0 & 2.63 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}} \underset{5\times3}{\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}}$$

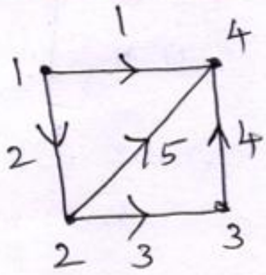
$$= \underset{3\times3}{\begin{bmatrix} 8.6364 & -0.6061 & -5 \\ -0.6061 & 4.3232 & -2 \\ -5 & -2 & 7 \end{bmatrix}}$$

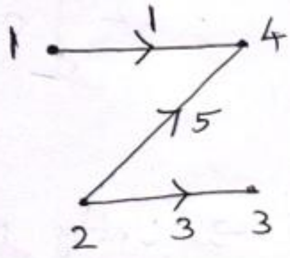
(4) Consider an oriented graph of the power system shown below in fig. choose branches 1, 3 and 5 as twigs to form a tree. Build a basic loop incidence matrix C for the given oriented graph. Select node 2 as reference.



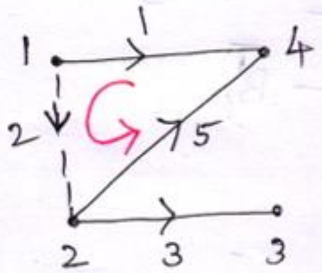
Solution:- Oriented Graph Tree

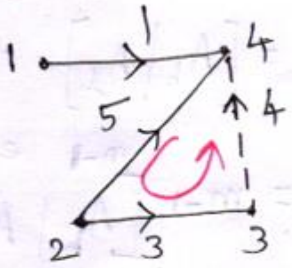






Basic Loop-1 : $\{2, 1, 5\}$ Basic Loop-2: $\{4, 3, 5\}$





Basic (or) fundamental Loop matrix, $C = [C_b | U]$

$\therefore C_b = -B_l^T$

Basic (or) fundamental cut-set matrix, $B = [U \mid B_l]$

$\therefore B = A_b^{-1} A = A_b^{-1}[A_b | A_l] = [U | A_b^{-1} A_l]$

$$\therefore B = \begin{array}{c} \\ 1 \\ 3 \\ 5 \end{array} \begin{array}{c} \text{Tree Branches} \qquad \text{Links} \\ \begin{array}{ccc|cc} 1 & 3 & 5 & 2 & 4 \end{array} \\ \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 \end{array}\right] \end{array} = \left[U \mid B_l\right]$$

$$C = \begin{array}{c} \\ BL\text{-}1 \\ BL\text{-}2 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \left[\begin{array}{ccccc} -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 \end{array}\right] \end{array}$$

$$\therefore C = \begin{array}{c} \\ 2 \\ 4 \end{array} \begin{array}{c} \text{Tree Branches} \qquad \text{Links} \\ \begin{array}{ccc|cc} 1 & 3 & 5 & 2 & 4 \end{array} \\ \left[\begin{array}{ccc|cc} -1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{array}\right] \end{array}$$

Note:

Where

$U \rightarrow$ Unity matrix.

$B_l = A_b^{-1} A_l$ $\qquad C = \left[C_b \mid U\right]$

$A = \left[A_b \mid A_l\right]$ $\qquad C_b = -B_l^T$

$$A = m-1 \overset{\quad m-1 \qquad e-m+1}{\left[A_b \mid A_l\right]}$$

$$B = m-1 \overset{\quad m-1 \qquad e-m+1}{\left[U \mid B_l\right]}$$

(5) For the transmission network shown in fig(a), assume that the shunt admittance at each bus are lumped into a single admittance. The oriented system graph is shown in fig(b) with (0) representing the ground bus. Pick a tree and write the basic (or) fundamental loop matrix, C & Basic cut-set matrix, B.

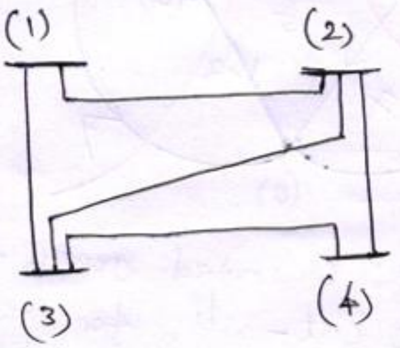


Fig (a): Transmission Network

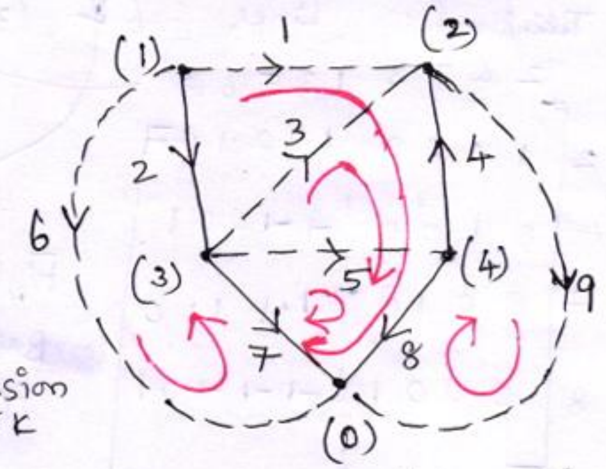


Fig (b): oriented System Graph, Tree and co-tree

Solution :-

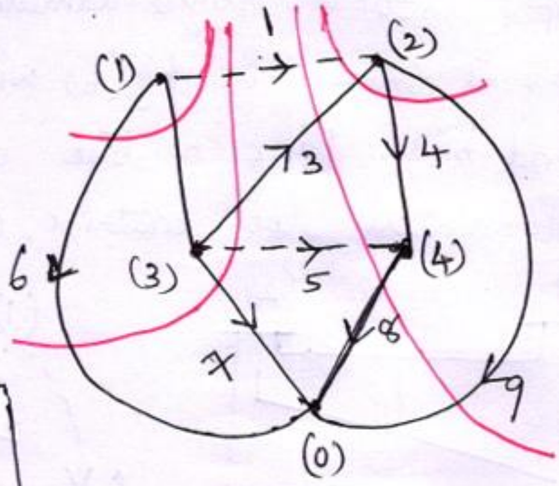
The following tree is chosen with the tree branches (2, 4, 7, 8), shown by solid lines. The links are (1, 3, 5, 6, 9) shown by dotted lines. The orientations for the fundamental loops are shown with dotted lines.

The Basic (or) fundamental loop matrix, C is ~~give~~ written as

$$C = \begin{array}{c} \text{Links} \downarrow \\ \\ 1 \\ 3 \\ 5 \\ 6 \\ 9 \end{array} \begin{array}{c} \text{Tree Branches} \quad\quad\quad \text{Links} \\ \begin{array}{cccc|ccccc} 2 & 4 & 7 & 8 & 1 & 3 & 5 & 6 & 9 \end{array} \\ \left[\begin{array}{cccc|ccccc} -1 & -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \end{array}$$

The oriented graph is redrawn in fig (c) with the curved lines defining the basic (or) fundamental cut-sets.

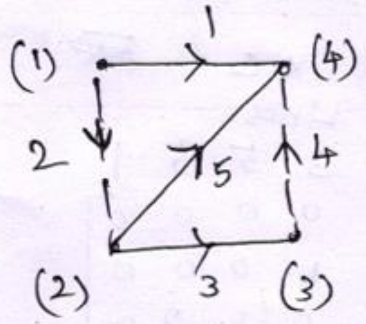
The basic cut-set matrix, B is written by inspection as



Fig(C): oriented graph & Basic cut-sets for the Transmission network.

$$B = \begin{array}{c} \\ 2 \\ 4 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc|ccccc} \text{Tree Branches} & & & & \text{Links} & & & & \\ 2 & 4 & 7 & 8 & 1 & 3 & 5 & 6 & 9 \end{array} \\ \left[\begin{array}{cccc|ccccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & -1 & 0 & 1 \end{array}\right] \end{array}$$

(6) Consider the graph shown in fig. choose the tree whose branches are (1, 3, 5). Find the Basic (or) fundamental cut-set and loop matrices B and C using the incidence matrix, A.



Solution :- Choosing (2) as the reference node, we write the reduced incidence matrix A as

$$A = \begin{array}{c} \text{Nodes} \downarrow \\ (1) \\ (3) \\ (4) \end{array} \begin{array}{c} \text{Tree Branches} \quad \text{Links} \\ \begin{array}{ccccc} 1 & 3 & 5 & 2 & 4 \end{array} \\ \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ -1 & 0 & -1 & 0 & -1 \end{array}\right] \end{array} = \left[A_b \,\vdots\, A_l\right]$$

$$A_b^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$$\therefore A_b^{-1} A_l = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ -1 & 1 \end{bmatrix}$$

Hence Basic cut-set matrix, $B = \left[U \,\vdots\, B_l\right]$

$$B = \begin{array}{c} \\ \\ 1 \\ 3 \\ 5 \end{array} \begin{array}{c} \text{Tree Branches} \quad \text{Links} \\ \begin{array}{ccccc} 1 & 3 & 5 & 2 & 4 \end{array} \\ \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 \end{array}\right] \end{array}$$

Since $C_b = -B_l^T$ = Basic Loop Matrix, C , we have

$$C = \begin{array}{c} \\ \\ 2 \\ 4 \end{array} \begin{array}{c} \text{Tree Branches} \quad \text{Links} \\ \begin{array}{ccccc} 1 & 3 & 5 & 2 & 4 \end{array} \\ \left[\begin{array}{ccc|cc} -1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 \end{array}\right] \end{array}$$

The nature of matrices of B and C can be independently verified from the graph.

(7) consider the network in fig where two branches have mutual coupling as shown. Find the primitive impedance matrices Z, Y and the Y_{BUS} matrix. choose (0) as reference bus.

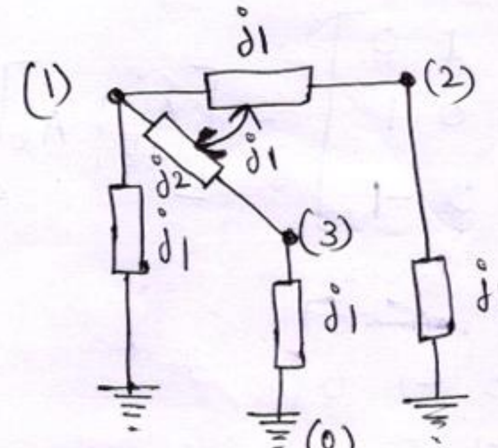


Fig (a) : Network

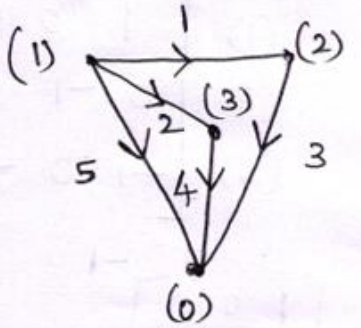


Fig (b) : Graph

solution :-

The primitive impedance matrix, Z is

$$Z = j \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

The Primitive admittance matrix, y is calculated as Z^{-1}.

$$y = j \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} -2 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

The incidence matrix obtained by taking (0) as reference node is

$$A = \begin{array}{c} \\ (1) \\ (2) \\ (3) \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

A^T is given by

$$A^T = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccc} (1) & (2) & (3) \end{array} \\ \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

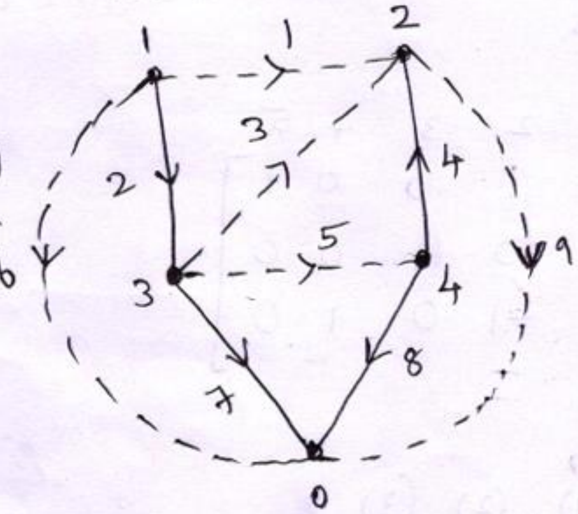
Hence Bus Admittance Matrix, $Y_{BUS} = A\,y\,A^T$

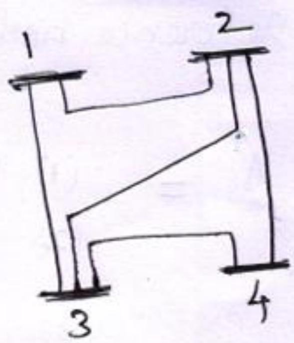
$$\therefore Y_{BUS} = \underset{3\times5}{\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \end{bmatrix}} \; j \underset{5\times5}{\begin{bmatrix} -2 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}} \underset{5\times3}{\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}}$$

$$= j \underset{3\times3}{\begin{bmatrix} -2 & 1 & 0 \\ 1 & -3 & 1 \\ 0 & 1 & -2 \end{bmatrix}}$$

(8) Build reduced incidence matrix, A. for the Tn n/w

oriented Graph.





Transmission n/w.

Solution :- By inspection, the matrix A is written as

Edges (Branches & Links)

$$
\begin{array}{c c}
 & \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\
\text{nodes} & \\
A = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 & 0 \end{bmatrix}
\end{array}
$$

$$
\begin{array}{c c}
 & \begin{array}{cccc} \text{Branches} & & & \end{array} \quad \begin{array}{ccccc} \text{Links} & & & & \end{array} \\
\text{Nodes} & \begin{array}{cccc} 2 & 4 & 7 & 8 \end{array} \quad \begin{array}{ccccc} 1 & 3 & 5 & 6 & 9 \end{array} \\
A = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}
\end{array}
$$