MODULE 1

Principles of Wireless Communications

Principles of Wireless Communications: The Wireless Communication Environment, Modelling of wireless systems, System model for narrowband Signals, Rayleigh fading Wireless Channel.
The Wireless Channel: Basics of Wireless Channel Modelling, Average Delay Spread in Outdoor Cellular Channels, Coherence bandwidth, Relation between ISI and Coherence Bandwidth, Doppler fading, Doppler Impact on a wireless Channel, Coherence Time.
[Text1: 3.1 to 3.4, 4.1 to 4.7]

1. **Aditya K Jagannatham, "Principles of Modern Wireless Communication systems, Theory and Practice ", Mc Graw Hill Education (India) Private Limited, 2017, ISBN 978-81- 265-4231-4.**

1.1 The Wireless Communication Environment

- In conventional **wireline communication** systems, there is a single signal-propagation path between the transmitter and the receiver, which is constrained by the propagation medium such as a coaxial cable or a twisted pair.
- In wireless systems, the signal can reach the receiver via direct, reflected, and scattered paths as shown in Figure 1.1.

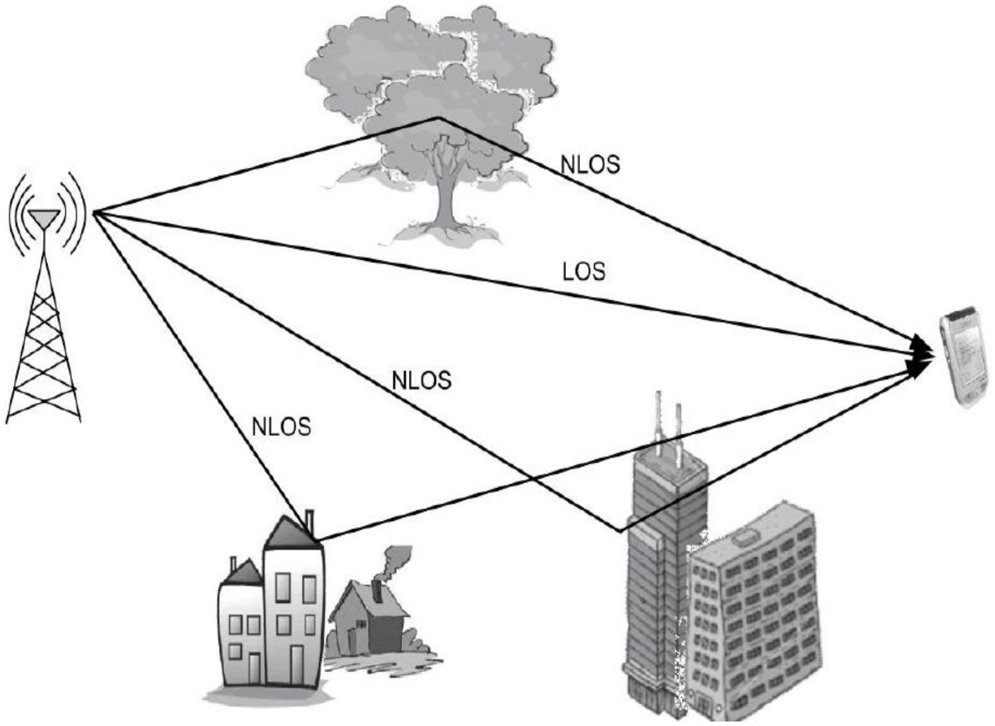


Fig. 1.1 Schematic of the wireless-propagation environment

- As a result, at the receiver, there is a superposition of multiple copies of the transmitted signal.

- These signal copies experience different attenuations, delays, and phase shifts arising from the varied propagation distances and properties of the scattering media.
- Hence, at the wireless receiver, there is interference of signals received from these multiple propagation paths, which is termed **multipath interference**.
- The **multipath interference**, in turn, results in an **amplification or attenuation** of the net received signal power observed at the receiver, and this variation in the received signal strength arising from the multipath propagation phenomenon is termed **multipath fading** or simply **fading**.
- Strong **destructive interference** at the receiver is referred to as a **deep fade**, and such a condition may result in temporary failure of communication due to a severe drop in the SNR at the receiver.

1.2 Modelling of Wireless Systems

- Let us start by considering the transmitted passband wireless signal s(t), which is transmitted across a wireless channel. Such a passband signal can be described analytically as,

$$s(t) = \mathrm{Re}\left\{ s_b\left(t\right) e^{j2\pi f_c t} \right\} \qquad \text{----------- 1.1}$$

The quantity $\mathbf{s_b(t)}$ is the complex baseband representation of the transmitted signal and **fc** is simply the carrier frequency employed for transmission.
- We will assume initially that the wireless channel is time invariant.
- Let us consider a channel with L multipath components.
- Observe that each path of the wireless channel basically has two characteristic properties.
    - Firstly, it delays the signal because of the propagation distance.
    - Secondly, there is an attenuation of the signal arising because of the scattering effect.
- Let the signal attenuation and delay of the i^{th} channel be denoted by the quantities a_i, τ_i respectively.
- The impulse response of an LTI system which attenuates a signal by a_i and delays it by τ_i is given as

$$h_i\left(\tau\right) = a_i \delta\left(\tau - \tau_i\right)$$

The above equation gives the impulse response of a single path of a wireless communication system.
- The wireless channel represents a linear input–output system, since the signal observed at the receiver is the sum of the different multipath signal copies impinging on the receive antenna.
- A typical Channel Impulse Response (CIR) of a multipath-scattering based wireless channel is given by the sum of the above impulse responses corresponding to the individual model,

$$h\left(\tau\right) = \sum_{i=0}^{L-1} a_i \delta\left(\tau - \tau_i\right) \qquad \text{---------- 1.2}$$

- The above impulse response is also termed the **tapped delay-line model** because of the nature of the arrival of several progressively delayed components of the signal.
- The above wireless channel model consists of L propagation paths arising from the several reflection and scattering multipath **Non-Line-Of-Sight (NLOS)** components.
- One of the multipath components can also be a **direct Line-Of-Sight (LOS)** component. Each such i^{th} path is characterized by two parameters, which are,
  1. The attenuation factor a_i
  2. The path delay τ_i
- Since the above wireless is a linear time-invariant (LTI) system, the received signal y(t) can be expressed as the convolution of the transmitted signal s (t) with the CIR h (t). Therefore, the received wireless signal y (t) is given as

$$y(t) = s(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)\, s(t-\tau)\, d\tau$$

By inserting the expression for the tapped delay-line channel in Eq. (1.2) in the above convolution, the expression for the received wireless signal y (t) across this tapped delay-line channel can be derived as

$$y(t) = \sum_{i=0}^{L-1} a_i \int_{-\infty}^{\infty} \delta(\tau - \tau_i)\, s(t-\tau)\, d\tau = \sum_{i=0}^{L-1} a_i s(t-\tau_i)\, d\tau$$

Further, this expression for the received signal can be written in terms of the transmitted baseband signal $s_b(t)$ by substituting the relation between s(t) and $s_b(t)$ in Eq. (1.1) in the above expression and simplifying it as

$$y(t) = \mathrm{Re}\left\{ \sum_{i=0}^{L-1} a_i s_b(t-\tau_i)\, e^{j2\pi f_c (t-\tau_i)} \right\}$$

$$= \mathrm{Re}\left\{ \underbrace{\left(\sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} s_b(t-\tau_i) \right)}_{y_b(t)} e^{j2\pi f_c t} \right\}$$

From the above expression, it can be seen that $y_b(t)$, the complex baseband signal equivalent of the received signal y (t), is simply given as

$$y_b(t) = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} s_b(t-\tau_i) \quad \text{--------- 1.3}$$

In addition to the attenuation and delay parameters in the passband channel model described earlier, the baseband system model consists of the addition phase $e^{-j2\pi f_c \tau_i}$ parameter.

This basically arises because of the path delay of the carrier signal $e^{j2\pi f_c t}$ corresponding to the i^{th} path.

- The received baseband signal consists of multiple delayed copies $s_b(t - \tau_i)$ of the transmitted signal $s_b(t)$. Each such i^{th} signal copy arising from the i^{th} multipath component is associated with the following three parameters.
  1. The attenuation factor a_i .
  2. The path delay τ_i .
  3. The phase factor $e^{-j2\pi f_c \tau_i}$

- The different signal copies for a typical baseband BPSK information signal sb (t) is shown in Figure 1.2. The quantity T denotes the symbol time, while Tm, which is the delay between the first and last arriving copies of the signal, is termed the **delay spread**.
- We will assume a narrowband channel, i.e., one in which Tm << T .

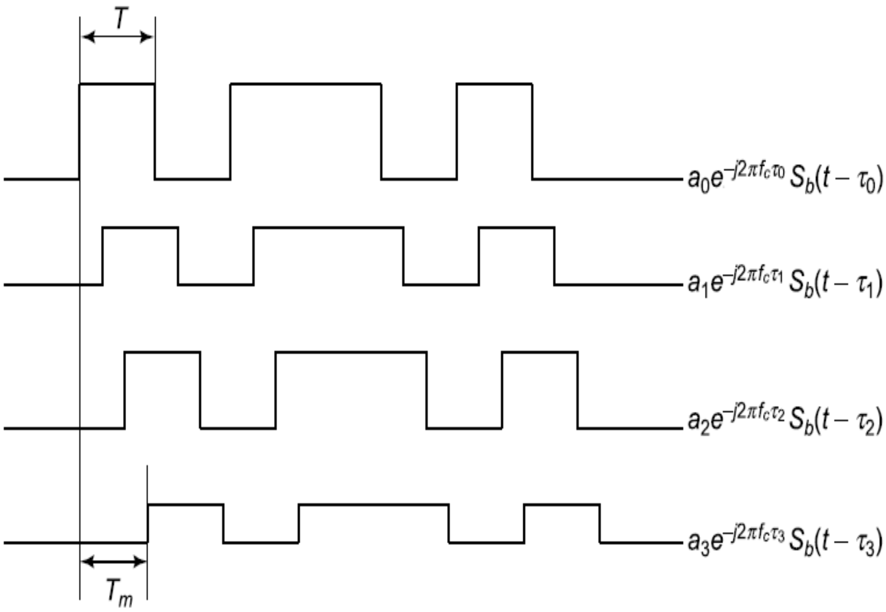


Figure 1.2 Multipath signal components at the receiver

Example 1: Consider a wireless signal with a carrier frequency of fc = 850 MHz, which is transmitted over a wireless channel that results in L = 4 multipath components at delays of 201, 513, 819, 1223 ns and corresponding to received signal amplitudes of 1, 0.6, 0.3, 0.2 respectively. Derive the expression for the received baseband signal $y_b(t)$ if the transmitted baseband signal is $s_b(t)$.

Solution: As given in the expression for the received baseband signal in

$$y_b(t) = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} s_b(t - \tau_i)$$

, we need to compute the factors $a_i e^{-j2\pi f_c \tau_i}$ for i = 0, 1, 2, 3 to derive the expression for the received baseband signal yb (t). Accordingly, the factor $a_0 e^{-j2\pi f_c \tau_0}$ is given as

$$a_0 e^{-j2\pi f_c \tau_0} = 1 \times e^{-j2\pi 850 \times 10^6 \times 201 \times 10^{-9}}$$

$$= 0.59 + j0.81$$

Similarly, the other factors for i = 1, 2, 3 can be computed as 0.57 − j0.19, 0.18 − j0.24, −0.19 + j0.06 respectively. Hence, the receive baseband signal is, therefore, given as

$$y_b(t) = (0.59 + j0.81)s_b(t) + (0.57 - j0.19)s_b\left(t - 201 \times 10^{-9}\right)$$
$$+ (0.18 - j0.24)s_b\left(t - 513 \times 10^{-9}\right) + (-0.19 + j0.06)s_b\left(t - 1223 \times 10^{-9}\right)$$

Thus, the receiver sees L = 4 signal copies $s_b(t - \tau_i)$, each delayed by τ_i, attenuated and phase shifted by $a_i e^{-j2\pi f_c \tau_i}$.

1.3 System Model for Narrowband Signals

- For a sufficiently narrowband signal $s_b(t)$, the different delayed components $s_b(t-\tau_i)$ are approximately equal to each other, i.e., $s_b(t-\tau_i) \approx s_b(t)$.
- Hence, for a narrowband transmit signal $s_b(t)$, the expression for the received baseband signal $y_b(t)$

$$y_b(t) = \underbrace{\left(\sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}\right)}_{ae^{j\phi}} s_b(t) = h s_b(t) \qquad \text{------1.4}$$

Where $h = ae^{j\phi} = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}$ is termed the complex fading channel coefficient.

Hence, the output baseband signal $y_b(t)$ is related to the input baseband signal $s_b(t)$ by a complex attenuation factor $ae^{j\phi}$.

The fading nature of the wireless channel can now be readily observed from the above expression. The signal power at the receiver critically depends on the magnitude of the overall attenuation factor $ae^{j\phi}$.

For instance, consider a two-component multipath channel with identical magnitude and exactly out-of-phase components, i.e., $a_0 = a_1$ and $e^{-j2\pi f_c \tau_0} = -e^{-j2\pi f_c \tau_1}$.

- The received signal $y_b(t) = 0$, resulting in 0(i.e., −∞ dB) received signal power and the channel is in a **deep fade**.
- Thus, the fortunes of the signal processor at the receiver are hinged on this erratic factor $ae^{j\phi}$, which is also termed the **complex baseband fading coefficient** or simply, the **fading coefficient**.
- The narrowband assumption essentially implies that the carrier phase is sensitive to the delay spread while the baseband signal is not.
- One of the most common assumptions in communication systems, which states that “the bandwidth of the transmitted signal is usually orders of magnitude smaller than the carrier frequency fc”.
- Next, we initiate a statistical characterization of the fading coefficient.

Example 2: For the wireless channel given in Example 1, derive the corresponding received signal with the narrowband assumption.

Solution: As given in

$$y_b(t) = \underbrace{\left(\sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}\right)}_{ae^{j\phi}} s_b(t) = h s_b(t)$$

the received signal with the narrowband assumption is given as

$$y(t) = \left(\sum_{i=0}^{3} a_i e^{-j2\pi f_c \tau_i}\right) s_b(t)$$
$$= (1.14 + j0.44)\, s_b(t)$$

Thus, the net received baseband signal in this case can be expressed as $y_b(t) = hs_b(t)$, where h the channel coefficient is h = 1.14 + j0.44.

1.4 Rayleigh Fading Wireless Channel

- The complex fading coefficient h can be expressed in terms of its real and imaginary components as,

$$h = ae^{j\phi} = \sum_{i=0}^{L-1} (x_i + jy_i) = X + jY$$

Thus, X, Y , which are the real and imaginary components of the fading coefficient $ae^{j\phi}$, are derived from the summation of a large number of random multipath components x_i, y_i, especially in a rich urban setting which allows for a large number of scatterers.
Hence, it is reasonable to assume that X, Y are random in nature.

- A simplistic model for the statistical characterization of X, Y would be to assume that they are Gaussian and un-correlated.
The assumption of **Gaussianity** is lent support by the central limit theorem, which in simple terms states that a normalized random variable derived from the sum of a large number of independent identically distributed random components, converges to a Gaussian random variable.

- The above assumption is valid as L → ∞, i.e., the number of multipath components is fairly large. Hence, X, Y are distributed as N (0, ½) (assuming zero-mean and variance ½).

- Further, since X, Y are Gaussian in nature and un-correlated, it directly follows that they are independent. The joint distribution of X, Y is given by the standard multivariate Gaussian as

$$f_{X,Y}(x,y) = \frac{1}{\pi} e^{-(x^2+y^2)}$$

- One can now derive the statistics of the fading coefficient $ae^{j\phi}$ in terms of its amplitude and phase factors a, ϕ. It can be seen through elementary trigonometric properties that

$$x = a\cos\phi,\ y = a\sin\phi$$

- The joint distribution $f_{A,\phi}(a, \phi)$ can be derived from $f_{X,Y}(x, y)$ using the relation for multivariate PDF transformation as

$$f_{A,\Phi}(a,\phi) = \frac{1}{\pi}e^{-a^2}J_{X,Y}$$

where we have used the property that $x^2 + y^2 = a^2$ in the above expression.

- The quantity $J_{X,Y}$ is termed the Jacobian of X, Y and is given by the expression

$$J_{X,Y} = \left|\begin{bmatrix} \cos\phi & \sin\phi \\ -a\sin\phi & a\cos\phi \end{bmatrix}\right| = a$$

where |A| denotes the determinant of the matrix A.

Substituting the Jacobian in the expression for multivariate PDF transformation above, the joint PDF with respect to the random variables A, Φ can be derived as

$$f_{A,\Phi}(a,\phi) = \frac{a}{\pi}e^{-a^2}$$

The marginal distributions f_A, f_ϕ with respect to the amplitude and phase factor random variables A, Φ can be readily derived from the above joint distribution as

$$f_A(a) = \int_{-\pi}^{\pi} f_{A,\Phi}(a,\phi)\, d\phi = 2ae^{-a^2}, 0 \le a \le \infty$$

$$f_\Phi(\phi) = \int_0^\infty f_{A,\Phi}(a,\phi)\, da = \frac{1}{2\pi}\, e^{-a^2}\Big|_0^\infty = \frac{1}{2\pi}, -\pi < \phi \le \pi$$

We have now derived one of the most popular and frequently employed models for the wireless channel, termed a **Rayleigh fading wireless channel**.

This nomenclature arises from the distribution f_A of the amplitude factor a, which is the well known **Rayleigh density**, shown in Figure 1.3.

- Observe that the average power in the amplitude a of the Ralyeigh fading channel coefficient h is given as

$$\mathrm{E}\left\{|h|^2\right\} = \mathrm{E}\left\{a^2\right\} = \mathrm{E}\left\{X^2 + Y^2\right\} = 1$$

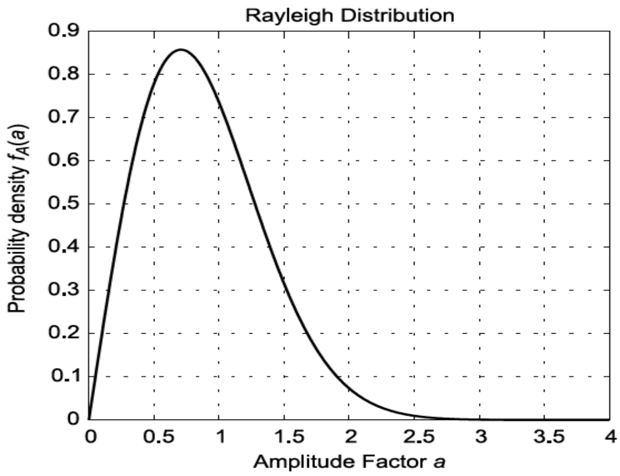


Figure 1.3 Rayleigh density for amplitude factor a

- The term Rayleigh refers to the distribution of the amplitude factor, the Rayleigh fading wireless channel characterizes both the amplitude factor as a Rayleigh fading random variable and the phase factor as uniformly distributed in $(-\pi, \pi)$.
- Finally, it can be readily seen that the joint distribution $f_{A,\phi}(a, \phi)$ is related to the marginals $f_A(a)$, $f_\phi(\phi)$ as,

$$f_{A,\Phi}(a,\phi) = f_A(a)\, f_\Phi(\phi)$$

essentially implying that the random variables A, Φ are independent.
This is a fairly important result since it suggests that the random varying nature of the phase factor of the arriving signal is independent of that of the amplitude, i.e., for a given amplitude a, all phase factors in $(-\pi, \pi)$ are equiprobable.

- Figure 1.4 shows a scatter plot of the real and imaginary components of 10000 randomly generated samples of the Rayleigh fading coefficient.
- From the circular symmetry of the plot, it can be readily seen that the phase of the Rayleigh coefficient is distributed uniformly in $(-\pi, \pi)$.

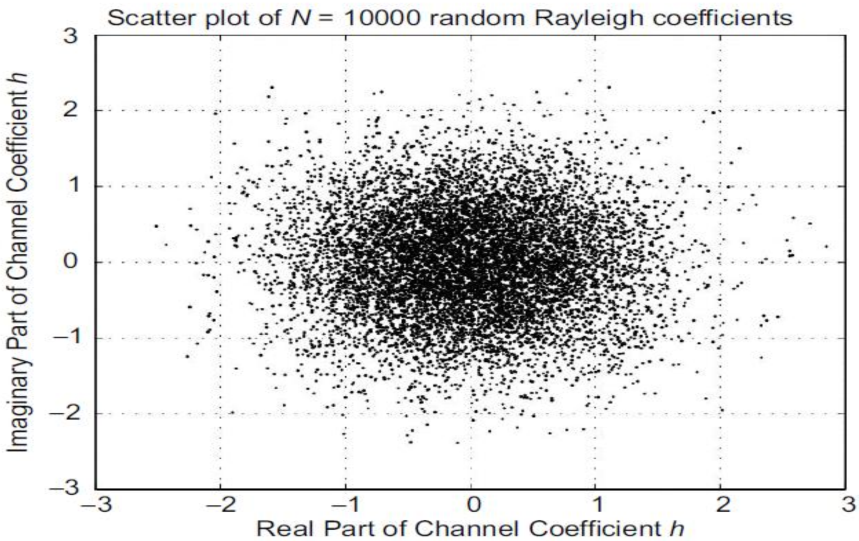


Figure 1.4 Scatter plot of the Rayleigh fading channel coefficient h

Example 3: Derive the probability density function of the channel power gain g = a^2, where a, as defined above, is the magnitude of the Rayleigh fading channel with E { a^2 } = 1.

Solution: The pdf of the magnitude of the channel coefficient a, where E { a^2 } = 1, is given by the Rayleigh distribution as

$$f_A(a) = 2ae^{-a^2},\ a \geq 0$$

Define the function w as g = w(a) = a^2. Then, from the standard result of the probability density of a function of a random variable, the distribution of g is given by the pdf transformation

$$f_G(g) = \frac{f_A\left(w^{-1}(g)\right)}{\left.\frac{dg}{da}\right|_{w^{-1}(g)}}.$$

Observe that since g = w (a) = a^2, we have w $^{-1}$(g) = a = √g. Hence, the above expression can simplified as

$$f_G(g) = \frac{f_A\left(\sqrt{g}\right)}{2\sqrt{g}} = \frac{2\sqrt{g}e^{-g}}{2\sqrt{g}} = e^{-g}$$

Thus, the expression for the power gain of the wireless channel has a rather simple expression given as f_G (g) = e^{-g} . However, it should be kept in mind that this is valid only for the case E { a^2 } = E {g} = 1. Further, one can confirm that E {g} = 1 as

$$\mathrm{E}\{g\} = \int_0^\infty e^{-g} = -e^{-g}\Big|_0^\infty = 1$$

Example 4: In the wireless Rayleigh fading channel described above, consider a transmit power Pt (dB) = 20 dB. What is the probability that the power at the receiver is greater than Pr (dB) = 10 dB ?

Solution: First, let us begin by computing the appropriate linear power values for the above given dB values. P_t(dB) = 10 $\log_{10}$ (P_t).

Hence, the linear transmit power P_t is given as P_t = 10P_t(dB)/10 = 10^2 = 100.

Similarly, the linear receiver power corresponding to P_r (dB) = 10 dB is given as P_r = 10^1 = 10.

Also, observe that given a power gain g, the received power is simply P_r = gP_t.
Hence, for a received power P_r > 10, it naturally implies that

$$\begin{aligned} gP_t &> 10 \\ \Rightarrow g &> \frac{10}{P_t} = \frac{10}{20} \\ &= \frac{1}{2} \end{aligned}$$

Thus, the probability that the received power is greater than 10 essentially corresponds to the probability that the random power gain g of the Rayleigh fading wireless channel is greater than 1/2. This probability can be readily computed as

$$\begin{aligned}\mathbf{Pr}\left(g > \frac{1}{2}\right) &= \int_{\frac{1}{2}}^{\infty} f_G(g)\, dg \\ &= \int_{\frac{1}{2}}^{\infty} e^{-g}\, dg \\ &= -e^{-g}\Big|_{\frac{1}{2}}^{\infty} \\ &= e^{-\frac{1}{2}} \\ &= 0.61\end{aligned}$$

1.4.1 Baseband Model of a Wireless System

- The baseband digital wireless communication system model for the above Rayleigh fading channel can be now readily derived as follows.
- Let x(k), y(k) be the k^{th} transmitted and received symbols respectively and h denote the Rayleigh fading channel coefficient. The baseband system model for symbol detection in this channel is given as

  $$y(k) = h_x(k) + n(k) \text{------ } 1.5$$

  where n(k) is the additive white Gaussian noise (AWGN).
- Without loss of generality, one can assume that the AWGN is of unit power, i.e., $E\{|n(k)|^2\} = 1$ (for if this does not hold then the whole system can be scaled by a constant scaling factor without affecting the performance, since the SNR is invariant under scaling by a constant factor).
- In particular, the information symbols x(k) are derived from a digital modulation constellation such as BPSK, QPSK, etc.
- For instance, if the symbol constellation is BPSK of average symbol power P , the transmitted symbol levels are given as $+\sqrt{P}$, $-\sqrt{P}$ for the information symbols 1, 0 respectively.
- Finally, one can derive the standard nonfading model for the conventional wireline systems as

  $$y(k) = x(k) + n(k) \text{-------- } 1.6$$

  where the Rayleigh fading factor a has simply been replaced by the constant 1 in the previous system model, which is essentially due to the fact the there is no multipath fading phenomenon in a wireline system.

The Wireless Channel

1.5 Basics of Wireless Channel Modelling

- The fading wireless channel comprises several multipath components arising from the presence of multiple Non-Line-Of-Sight (NLOS) Signal-propagation paths.
- These NLOS components arise from the scattering effects of objects in the wireless environment such as buildings, trees, vehicles, water bodies, etc.

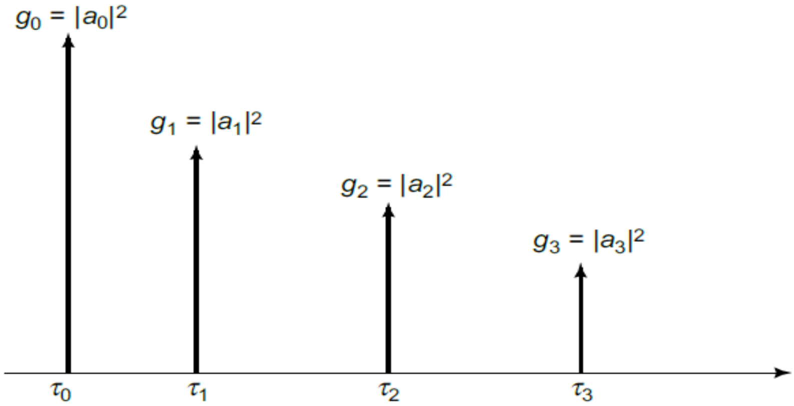


Figure 1.5 Schematic of an L = 4 tap wireless channel profile

- The impulse response of the standard multipath wireless channel can be modelled as,

$$h(t) = \sum_{i=0}^{L-1} a_i \delta(t - \tau_i)$$

where each $\delta(t - \tau_i)$ corresponds to delaying the signal by τ_i, and ai is the attenuation associated with the i^{th} path. The quantity L denotes the number of paths or multipath components. From the above impulse response, one can define the multipath power profile of the multipath channel as

$$\phi(t) = \sum_{i=0}^{L-1} |a_i|^2 \delta(t - \tau_i)$$
$$= \sum_{i=0}^{L-1} g_i \delta(t - \tau_i) \quad \text{--------- 1.7}$$

where $g_i = |a_i|^2$ is the power gain of the i^{th} path.
For instance, consider an L = 4 path multipath channel. The gains and the corresponding delays of the paths of this multipath channel can be listed as given in Table 1.1, and this is schematically shown in Figure 1.5.

The total energy corresponding to the transmitted wireless signal is received in increments at the receiver, with a part of it arriving in each multipath component. For instance, power with gain g_0 is received after a delay of τ_0, while a gain of g_1 is received after a delay τ_1, and so on, till the last path arriving at a delay of τ_{L-1} delivers

power with a gain of g_{L-1}.
Thus, the total power received in a multipath wireless channel occurs over a spread of time referred to as the delay spread.

This spread of the arriving power at the wireless receiver is schematically shown in Figure 1.6.
The delay spread of a wireless channel is a key parameter that characterizes the nature of the wireless environment and is denoted by the parameter σ_τ .
We describe the procedure for computation of the delay-spread parameter σ_τ of a wireless channel next.

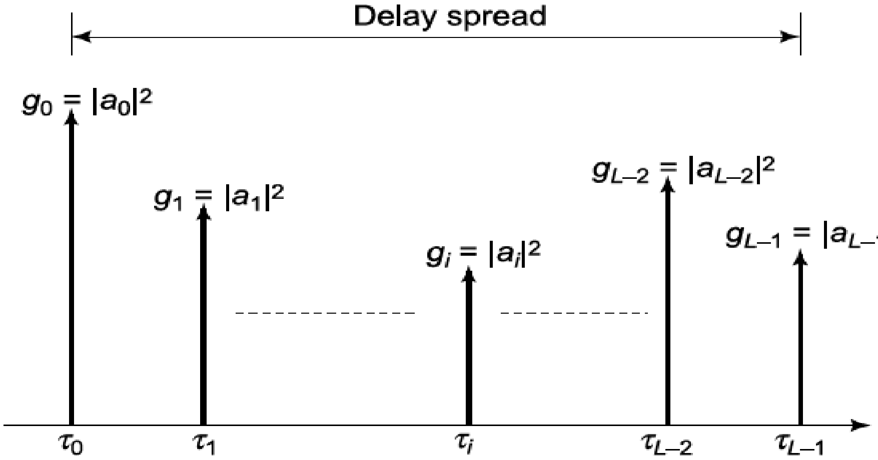


Figure 1.6 Schematic of a typical wireless channel power profile and delay spread

Table 1.1 Path gains and associated delays for an L = 4 multipath channel

Gain	Delay
$\|a_0\|^2$	τ_0
$\|a_1\|^2$	τ_1
$\|a_2\|^2$	τ_2
$\|a_3\|^2$	τ_3

1.5.1 Maximum Delay Spread σ_τ^{max}

- A framework to quantify the delay spread of a wireless channel is through the maximum delay spread of the channel denoted by $\boldsymbol{\sigma_\tau^{max}}$ **.**
- Consider a wireless channel with L multipath components, with the first path arriving at a delay of τ_0 and the last signal copy arriving at τ_{L-1} . The maximum delay spread is simply defined as

$$\sigma_{\tau}^{\max} = \tau_{L-1} - \tau_0 \quad \text{------ 1.8}$$

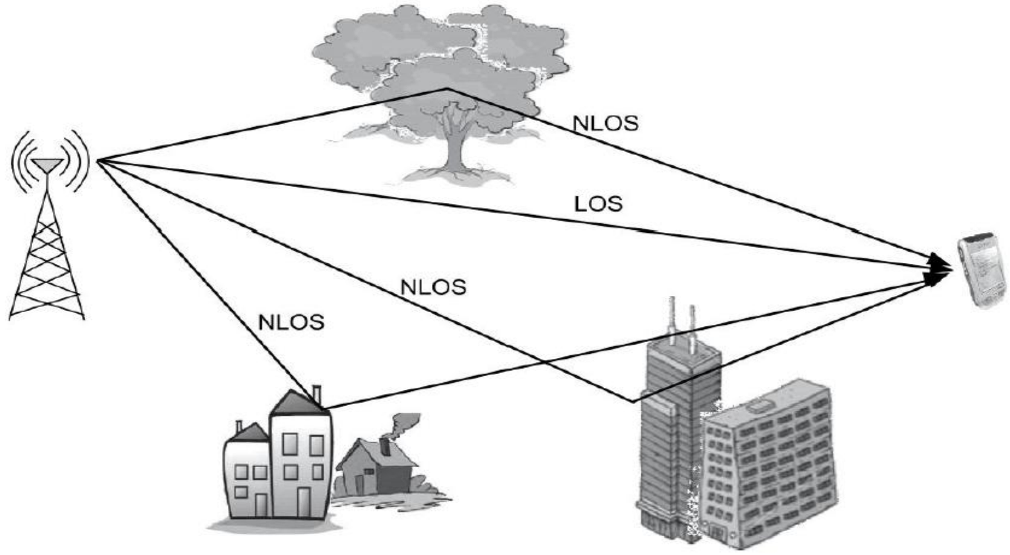


Figure 1.7 Schematic of the wireless-propagation environment

- The time interval between the arrival of the first and last signal copies at the receiver. This is a simple measure of the spread of the energy in the wireless channel, while effectively capturing the multipath signal arrival.
- A larger value of **σ_{τ}^{max}** naturally implies a richer scatter environment and larger differential propagation delays between the paths.
- The delay spread does NOT depend on the absolute delays τ_0, τ_{L-1}, but the difference $\tau_{L-1} - \tau_0$. Thus, the distance of the mobile receiver node from the base station has no impact on the delay spread, which leads to a larger propagation delay.
- For instance, consider a scenario where there is a single propagation path, corresponding to a large delay τ_0 for a mobile at a large distance from the base station. Since there is only a single path in this case, the first and last components correspond to the single component arriving at a delay of τ_0.
- Hence, the corresponding delay spread is $\tau_0 - \tau_0 = 0$.
  Thus, the delay spread indeed depends critically on the presence of multipath components and the richness of the scatter environment, which basically affects the total number of multipath scatter-signal components arriving at the receiver.

Example 4: Consider an L = 4 multipath channel with the delays τ_0, τ_{L-1} corresponding to the first and last arriving paths, given as $\tau_0 = 0$ µs and $\tau_{L-1} = 5$ µs. Such a wireless-channel power profile is shown schematically in Figure 1.8. What is the maximum delay spread σ_{τ}^{max} corresponding to this wireless channel ?

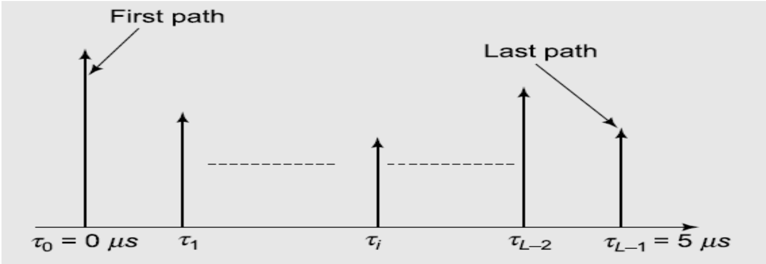


Figure 1.8 Power profile for Example 4

Solution:

$\sigma_\tau^{\max} = \tau_{L-1} - \tau_0$ for L multipath components, it can be seen that the maximum delay spread is

$$\sigma_\tau^{\max} = 5\ \mu s - 0\ \mu s = 5\ \mu s$$

1.5.2 RMS Delay Spread σ_τ^{RMS}

- In typical wireless channels, the paths which arrive later are significantly lower in power due to the larger propagation distances and weaker reflections as shown in Figure 1.9.
- This results in a large value of the maximum delay spread σ_τ^{max} even though several of the later paths comprise weak scatter components with negligible power.
- Thus, the maximum delay spread metric is not a reliable indicator of the true power spread of the arriving multipath signal components in such scenarios, since it does not weight the delays in proportion to the signal power in the multipath components.
- For this purpose, the RMS delay spread is a more realistic indicator of the spread of the signal power in the arriving components. Further, since it weights the delays of the signal components with respect to the power in the arriving paths, it is not susceptible to distortion in scenarios with a large number of trailing weak components, unlike the maximum delay spread.

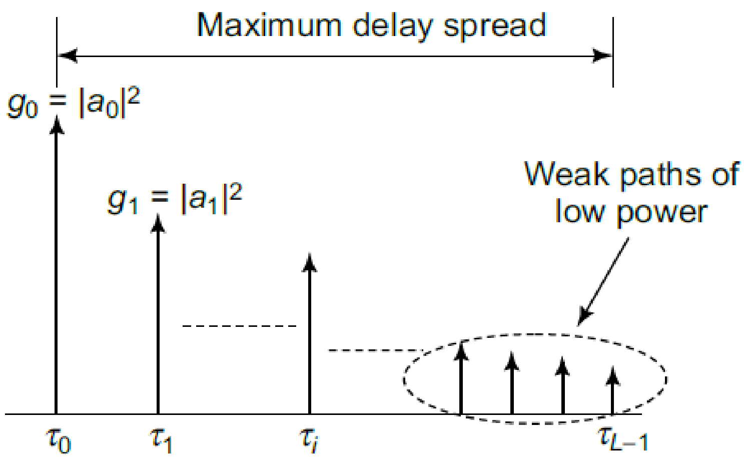


Figure 1.9 Power profile with weak trailing paths of very low power

Consider the power profile comprising of L multipath components defined in Eq. (1.7), with $g_i = |a_i|^2$, $0 \le i \le L - 1$ denoting the power gain of each multipath component. We define a new quantity b_i as

$$b_i = \frac{g_i}{g_0 + g_1 + \ldots + g_{L-1}},$$

where gi denotes the total power corresponding to the i^{th} path, while $g_0 + g_1 + \ldots + g_{L-1}$ denotes the total power in the multipath power profile.
Thus, the ratio b_i denotes the fraction of power in the i^{th} multipath component.
One can now conveniently employ this quantity b_i proportionally with the multipath delay components.
The various b_i define a power distribution for the above multipath power profile since each $b_i > 0$ and $b_0 + b_1 + \ldots b_{L-1} = 1$.

Therefore, the average delay $\bar{\tau}$ can be computed to the mean of the above power distribution as

$$\bar{\tau} = b_0\tau_0 + b_1\tau_1 + \ldots + b_{L-1}\tau_{L-1} = \sum_{i=0}^{L-1} b_i\tau_i$$

$$= \sum_{i=0}^{L-1} \frac{g_i}{\sum_{j=0}^{L-1} g_j}\tau_i$$

$$= \frac{\sum_{i=0}^{L-1} g_i\tau_i}{\sum_{j=0}^{L-1} g_j}$$

It can be seen that the average delay $\bar{\tau}$ is obtained by weighing each delay τ in proportion to the fraction of the power bi. Finally, the RMS delay spread $\boldsymbol{\sigma_\tau^{RMS}}$ can be computed as the standard deviation of the power distribution, which is defined as

$$\left(\sigma_\tau^{\mathrm{RMS}}\right)^2 = b_0\left(\tau_0 - \bar{\tau}\right)^2 + b_1\left(\tau_1 - \bar{\tau}\right)^2 + \ldots + b_{L-1}\left(\tau_{L-1} - \bar{\tau}\right)^2$$

$$= \sum_{i=0}^{L-1} b_i\left(\tau_i - \bar{\tau}\right)^2$$

$$\sigma_\tau^{\mathrm{RMS}} = \sqrt{\frac{\sum_{i=0}^{L-1} g_i\left(\tau_i - \bar{\tau}\right)^2}{\sum_{i=0}^{L-1} g_i}} \qquad \text{------ 1.9}$$

$$= \sqrt{\frac{\sum_{i=0}^{L-1} |a_i|^2\left(\tau_i - \bar{\tau}\right)^2}{\sum_{i=0}^{L-1} |a_i|^2}} \qquad \text{---------- 1.10}$$

Thus, the RMS metric to characterize the delay spread defined above is not sensitive to spurious multipath components of weak signal power since it weights each delay in proportion to its power, thereby automatically suppressing the contribution of weaker paths.

Example 5: Consider the multipath power profile of a wireless channel shown in Figure 1.10 comprising L = 4 multipath components. Compute the RMS delay spread σ_τ^{RMS} for this wireless channel.

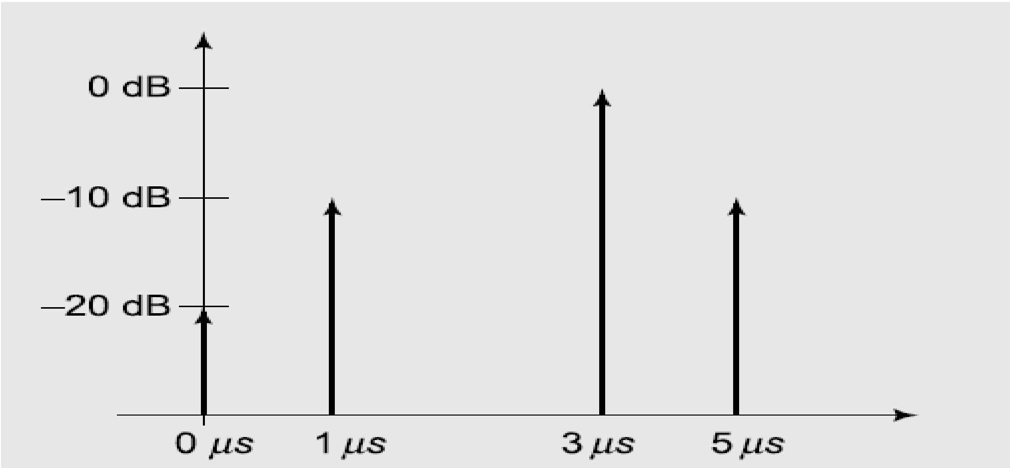


Figure 1.10 Power profile for Example 5

Solution: Consider the first path corresponding to τ_0 = 0 μs. The power associated with this path is g (dB) = −20 dB. Hence, the linear power can be obtained as

$$10 \log_{10} g_0 = -20 \text{ dB}$$
$$\Rightarrow \log_{10} g_0 = -2$$
$$\Rightarrow g_0 = 10^{-2} = 0.01$$

Also, the amplitude a_0 associated with this path can be derived as

$$a_0 = \sqrt{g_0} = 0.1$$

Thus, one can compute the corresponding power gi and amplitude ai for each of the L = 4 multipath components corresponding to $0 \leq i \leq 3$.

Table 1.2 Table of gains for Example 5

τ	dB Gain	g	$a = \sqrt{g}$
$0\,\mu s$	-20 dB	0.01	0.1
$1\,\mu s$	-10 dB	0.1	0.3162
$3\,\mu s$	0 dB	1	1
$5\,\mu s$	-10 dB	0.1	0.3162

The mean delay $\overline{\tau}$ for this channel as

$$\overline{\tau} = \frac{\sum_{i=1}^{L-1} g_i \tau_i}{\sum_{i=0}^{L-1} g_i}$$

$$= \frac{0.01 \times 0 + 0.1 \times 1 + 1 \times 3 + 0.1 \times 5}{0.01 + 0.1 + 1 + 0.1}\ \mu s$$

$$= 2.9752\,\mu s$$

The RMS delay spread can be computed as

$$\sigma_{\tau}^{\mathrm{RMS}} = \sqrt{\frac{\sum_{i=0}^{L-1} g_i \left(\tau_i - \overline{\tau}\right)^2}{\sum_{i=0}^{L-1} g_i}}$$

$$\sigma_{\tau}^{\mathrm{RMS}} = \frac{0.01 \times (0-2.9752)^2 + 0.1 \times (1-2.9752)^2 + 1 \times (3-2.9752)^2 + 0.1 \times (5-2.9752)^2}{0.01+0.1+1+0.1}$$

$$= 0.8573\,\mu s$$

The RMS delay spread is 0.8573 µs, which is much more realistic compared the maximum delay spread $\sigma_{\tau}^{\mathrm{RMS}}$ = 5 µs. This is because the initial path at 0 µs is of a significantly smaller power of −20 dB compared to the rest of the components. Since the RMS delay spread weighs each delay by the appropriate power, it is not susceptible to this distortion.

1.5.3 RMS Delay Based on Average Power Profile

- Consider the instantaneous power $|h(\tau)|^2$ corresponding to the delay τ . The average power associated with this delay can be defined as

$$\phi(\tau) = \mathrm{E}\left\{|h(\tau)|^2\right\}$$

The above quantity $\phi(\tau)$ can be thought of as the average power associated with the delay τ at various instants of time. It can also be thought of as the power at delay τ for the wireless channels of different users in an area. The former is averaging over time, while the latter represents an averaging over the ensemble of channels.

The fractional power associated with the delay τ as

$$f(\tau) = \frac{\phi(\tau)}{\int_0^\infty \phi(\tau)\, d\tau}$$

where $f(\tau)$ denotes the power distribution density corresponding to the delay τ, i.e., $f(\tau)\,\Delta\tau$ is the fraction of power in a delay interval of $\Delta\tau$ around τ.

The average $\overline{\tau}$ can, therefore, be defined as

$$\overline{\tau} = \int_0^\infty \tau f(\tau)\, d\tau = \frac{\int_0^\infty \tau\phi(\tau)\, d\tau}{\int_0^\infty \phi(\tau)\, d\tau}$$

The RMS delay spread for the above power profile $\phi(\tau)$ is defined as

$$\sigma_\tau^{\text{RMS}} = \sqrt{\int_0^\infty (\tau - \overline{\tau})^2 f(\tau)\, d\tau}$$

$$= \sqrt{\frac{\int_0^\infty (\tau - \overline{\tau})^2 \phi(\tau)\, d\tau}{\int_0^\infty \phi(\tau)\, d\tau}}$$

Example 6: Consider the average power profile $\phi(\tau) = \alpha e^{-\tau/\beta}$, where α = 3 dB, β = 1 µs. Compute the RMS delay spread σ_τ^{RMS} for this profile which is schematically shown in Figure 1.11.

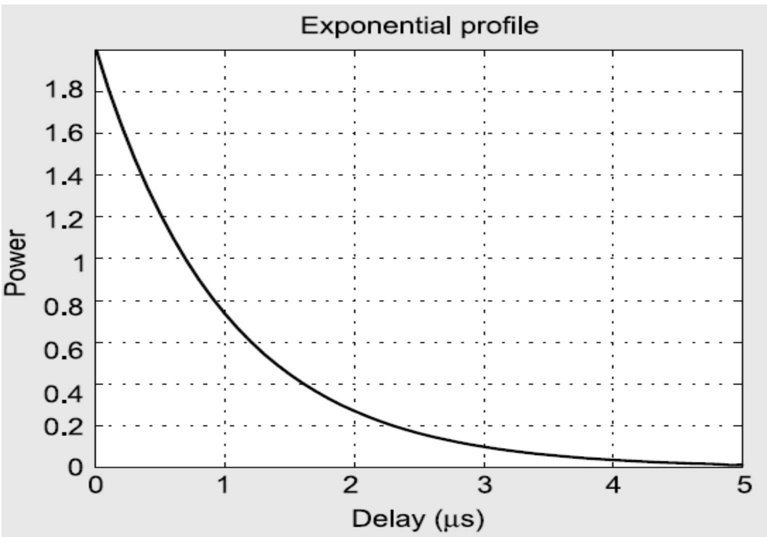


Figure 1.11: Power profile for Example 6

Solution: Firstly, given that α (dB) = 3 dB. Hence, we have α = 2. Therefore, $\phi(\tau) = 2e^{-\tau/\beta}$. To compute the normalized delay profile f (τ), the normalization factor can be computed as

$$\int_0^\infty \phi(\tau)\, d\tau = \int_0^\infty 2e^{-\tau/\beta}\, d\tau$$

$$= 2\beta e^{-\tau/\beta}\Big|_0^\infty$$

= 2β

The fractional power profile f (τ) can be obtained as

$$f(\tau) = \frac{2e^{-\tau/\beta}}{2\beta} = \frac{1}{\beta}e^{-\tau/\beta}$$

The average delay $\bar{\tau}$ is given as

$$\bar{\tau} = \int_0^\infty \tau f(\tau)\, d\tau$$

$$= \int_0^\infty \frac{\tau}{\beta}e^{-\tau/\beta}\, d\tau$$

$$= \tau e^{-\tau/\beta}\Big|_0^\infty + \int_0^\infty e^{-\tau/\beta}\, d\tau$$

$$= \beta e^{-\tau/\beta}\Big|_0^\infty$$

$$= \beta = 1\ \mu s$$

The mean delay spread $\bar{\tau}$ = β = 1 μs. To compute the RMS delay spread $\boldsymbol{\sigma_\tau^{RMS}}$, we begin with the computation of E {τ^2 } defined as

$$\mathrm{E}\{\tau^2\} = \int_0^\infty \tau^2 f(\tau)\, d\tau$$

$$= \int_0^\infty \frac{\tau^2}{\beta}e^{-\tau/\beta}\, d\tau$$

$$= \tau^2 e^{-\tau/\beta}\Big|_0^\infty + \int_0^\infty 2\tau e^{-\tau/\beta}\, d\tau$$

$$= 2\beta\tau e^{-\tau/\beta}\Big|_0^\infty + \int_0^\infty 2\beta e^{-\tau/\beta}\, d\tau$$

$$= 2\beta^2 e^{-\tau/\beta}\Big|_0^\infty = 2\beta^2$$

$$\sigma_\tau^{\mathrm{RMS}} = \sqrt{\mathrm{E}\left\{\tau^2\right\} - \bar{\tau}^2}$$
$$= \sqrt{2\beta^2 - \beta^2}$$
$$= \beta = 1\ \mu s.$$

The RMS delay spread $\boldsymbol{\sigma_\tau^{RMS}}$ for the above average power profile $\boldsymbol{\alpha e^{-\tau/\beta}}$ can be computed as $\boldsymbol{\sigma_\tau^{RMS}}$ = 1 µs.

1.6 Average Delay Spread in Outdoor Cellular Channels

- Consider an outdoor cellular wireless communication scenario. The cell radii of typical cells are in the range of 1–5 km, i.e., outdoor wireless signal-propagation distances are of the order of a few kilometres.

- Consider two paths illustrated in Figure 1.12, where the direct and scatter distances are given as d_0 = 2 km, d_0 = 3 km respectively. Hence, the propagation delays τ_0, τ_1 are given as

$$\tau_0 = \frac{2\,\mathrm{km}}{c},\ \ \tau_1 = \frac{3\,\mathrm{km}}{c},$$

where c = 3 × 10^8 m/s.

- Hence, the delay spread in this case is given as

$$\sigma_\tau^{\max} = \Delta\tau$$
$$= \tau_1 - \tau_0$$
$$= \frac{\Delta d}{c}$$
$$= \frac{3000\,\mathrm{m} - 2000\,\mathrm{m}}{3 \times 10^8}$$
$$= 3.33\ \mu s$$

- In typical outdoor cellular scenarios, where the distances and signal-propagation paths are of the orders of kilometres, the delay spreads are of the order of 1–3 µs.
- This value is of great importance in the design and analysis of practical wireless-communication systems. Also, similarly, corresponding to indoor distances of around 10 m, typical indoor delay spreads are of the order of 10–50 ns.

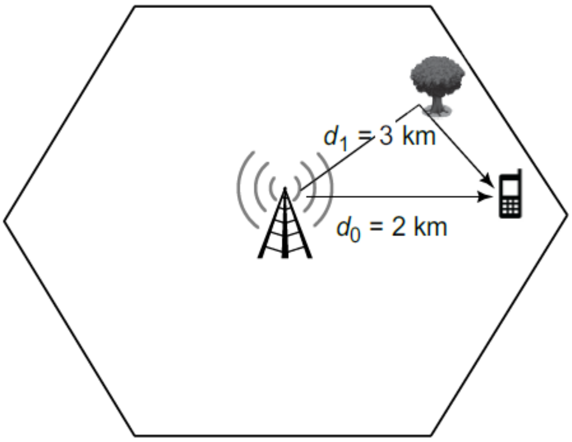


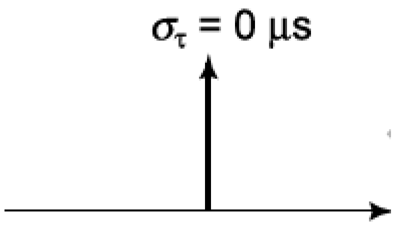
Figure 1.12 Typical delay spread in outdoor cellular channels

1.7 Coherence Bandwidth in Wireless Communications

- Let us define the frequency response H (f) of the wireless channel as

$$H(f) = \int_0^{\infty} h(\tau)\, e^{-j2\pi f\tau}\, d\tau$$

To understand the relation between these two fundamental quantities, i.e., the delay spread σ_τ and coherence bandwidth Bc of the wireless channel, let us begin by considering a simple case corresponding to $\sigma_\tau = 0$, shown in Figure below.



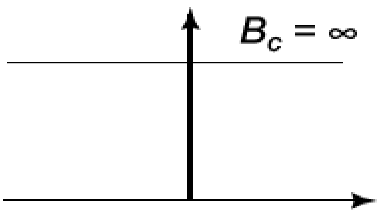
In this scenario, since the delay spread is zero, the wireless channel comprises a single propagation path. Hence, the delay profile h (τ) is given as

$$h(\tau) = \delta(\tau).$$

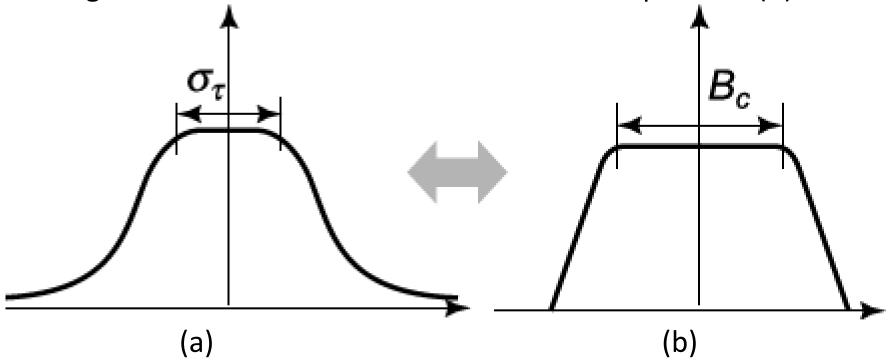
The corresponding frequency response H (f) is given as

$$H(f) = \int_0^{\infty} a_0 \delta(\tau)\, e^{-j2\pi f\tau}\, d\tau = 1$$

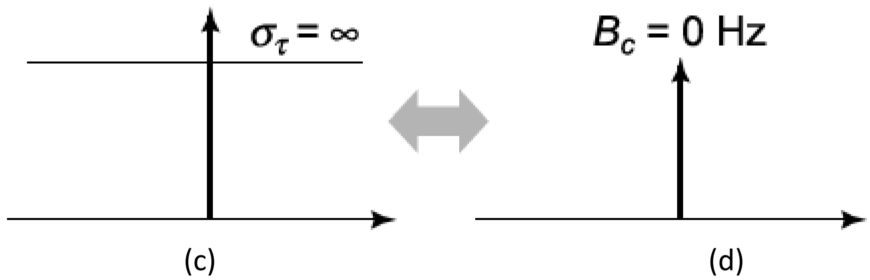
Thus, the frequency response is the constant 1 and |H (f)| = 1. This is basically a flat frequency response over the entire frequency band as shown in Figure below, i.e., of infinite bandwidth.



As the delay spread σ_τ increases in Figure (a), the time spread of this response increases, leading to a decrease in the bandwidth of the response H (f) as shown in Figure (b).



As the time spread of the response becomes ∞ as shown in Figure (c), the channel filter becomes an impulse δ (f) as shown in Figure (d) and the bandwidth of the channel filter reduces to 0.



- The **coherence bandwidth Bc** is then defined as the bandwidth of the response H (f), i.e., the frequency band over which the response H (f) is flat as shown in Figure 1.13.
- Consider any signal x (t) transmitted over the wireless channel, with corresponding Fourier transform X (f). The output response Y (f) of the output signal y (t) is given as

$$Y(f) = H(f)\,X(f) \qquad \text{------ 1.11}$$

which is shown in Figure 1.13.
The impact of the coherence bandwidth Bc on the signal x (t) can be understood as follows

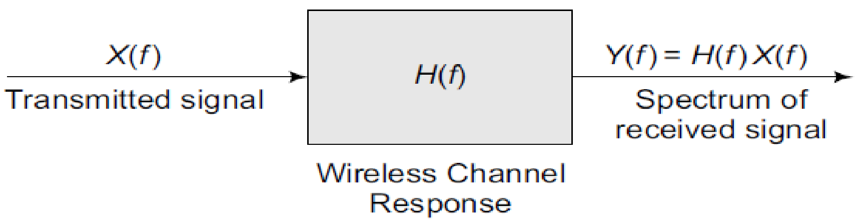


Figure 1.13 Linear input-output system model for the wireless channel

- As shown in Figure 1.14, if the bandwidth B_s of the signal x (t) is less than B_c, then X (f) spans the flat part of the channel response H (f).

- Hence, the output Y (f) = H (f) X (f) is simply a scaled version of X (f) corresponding to the magnitude of the flat part. Thus, the input signal spectrum X (f) is undistorted at the output. Such a wireless channel is termed a **flat-fading channel**.

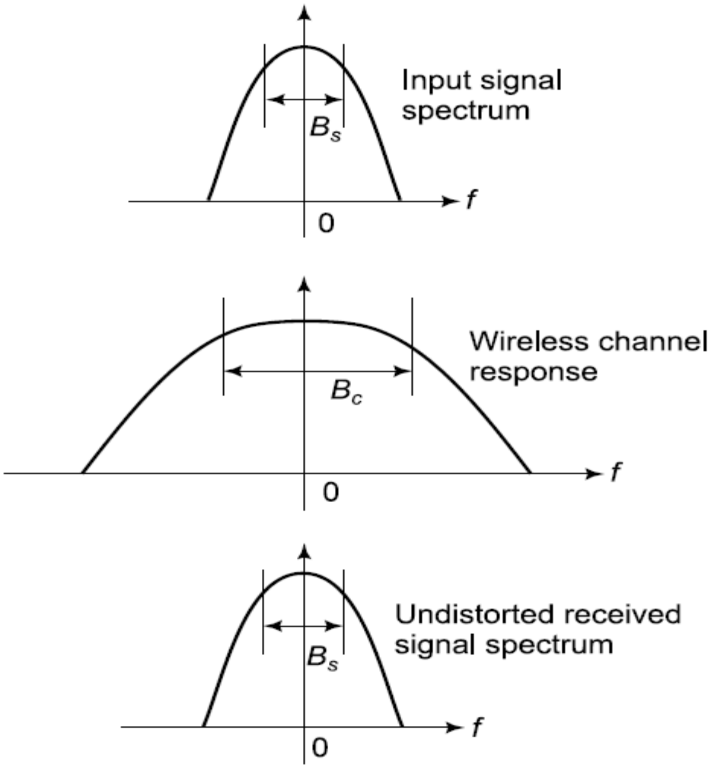


Figure 1.14 Signal bandwidth Bs less than coherence bandwidth Bc implying no distortion

- However, consider the case where the signal bandwidth Bs is greater than the coherence bandwidth Bc.
- In this scenario, different parts of the signal spectrum X (f) experience different attenuations, i.e., the attenuation is frequency-selective.
- Thus, the output spectrum Y (f) is a distorted version of the input spectrum X (f). Such a wireless channel is termed a **frequency-selective channel** due to the frequency-dependent nature of the attenuation of the signal. This is schematically shown in Figure 1.15.

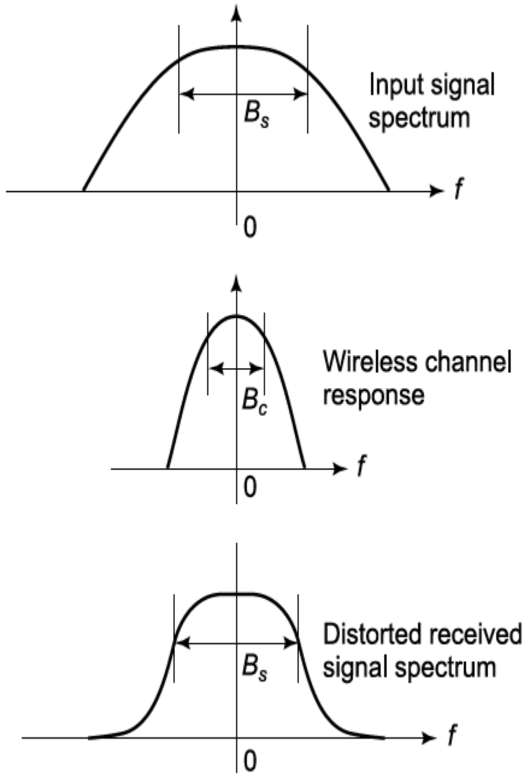


Figure 1.15 Signal bandwidth Bs greater than coherence bandwidth Bc leading to distortion in spectrum of received signal

- Thus, the impact of the frequency spectrum H (f) of the wireless channel on the input signal x (t) can be summarized as
  Bs ≤ Bc ⇒ No distortion in received signal, i.e., flat fading
  Bs ≥ Bc ⇒ Distortion in received signal, i.e., frequency-selective fading ------ 1.12

  To derive an empirical relationship between the delay spread and coherence bandwidth of a typical wireless channel, consider a wireless delay profile

$$h(\tau) = \sum_{l=0}^{L-1} a_l \delta(\tau - \tau_l)$$

The response H (f) of this channel is given as

$$\begin{aligned} H(f) &= \int_0^{\infty} h(\tau) e^{-j2\pi f\tau} d\tau \\ &= \int_0^{\infty} \left(\sum_{l=0}^{L-1} a_l \delta(\tau - \tau_l) \right) e^{-j2\pi f\tau} d\tau \\ &= \sum_{l=0}^{L-1} \left(\int_0^{\infty} a_l \delta(\tau - \tau_l) e^{-j2\pi f\tau} d\tau \right) \\ &= \sum_{l=0}^{L-1} a_l e^{-j2\pi f\tau_l} \end{aligned}$$

Thus, the frequency response of the channel is given as the sum of L harmonics, with the l^{th} component changing at the rate τ_l. Consider now the highest frequency harmonic corresponding to $a_{L-1} e^{-j2\pi f\tau_{L-1}}$, i.e., with phase varying at the rate τ_{L-1}.

Its values at frequencies 0 and $\frac{1}{4\tau_{L-1}}$ are given as

$$f = 0 \Rightarrow a_{L-1}e^{-j2\pi f \tau_{L-1}} = a_{L-1}e^{0} = 1$$

$$f = \frac{1}{4\tau_{L-1}} \Rightarrow a_{L-1}e^{-j2\pi \frac{1}{4\tau_{L-1}}\tau_{L-1}} = a_{L-1}e^{-j\pi\frac{1}{2}} = -ja_{L-1}$$

- Thus, it can be seen that as f changes from 1 to $\frac{1}{4\tau_{L-1}}$, the phase changes significantly. This leads to a significant change in the response H (f) from f = 0 to f = $\frac{1}{4\tau_{L-1}}$. Thus, $\frac{1}{4\tau_{L-1}}$ is a point of significant change in the frequency response, where it changes significantly compared to the response at f = 0, as shown in Figure 1.16.

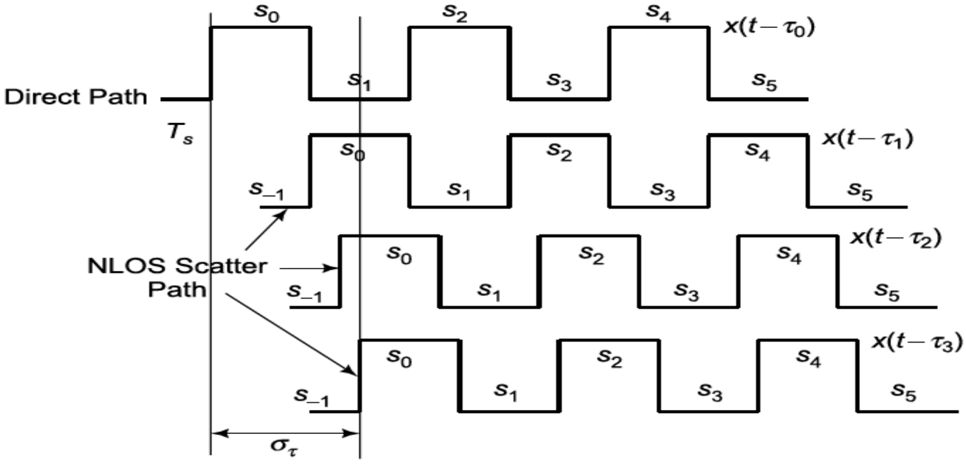


Figure 1.16 Severe ISI caused by multiple scatter components

Thus, the bandwidth of the response H (f) is approximately given as

$$f_c = \frac{1}{4\tau_{L-1}}. \quad \text{-------- 1.13}$$

Hence, the coherence bandwidth of the filter H (f) is approximately given as

$$B_c \approx 2 \times \frac{1}{f_c} = \frac{1}{2\tau_{L-1}} \quad \text{--------- 1.14}$$

τ_{L-1} is the maximum delay spread σ_τ^{max} of the channel. Thus, the coherence bandwidth Bc can be related to the delay spread σ_τ as

$$B_c \approx \frac{1}{\sigma_\tau} \quad \text{--------- 1.15}$$

Thus, it can be seen that the coherence bandwidth Bc decreases as the delay spread σ_τ increases.

Finally, the approximate delay spread corresponding to outdoor channels with a typical delay spread of 2 μs can be derived as

$$B_c = \frac{1}{2 \times 1 \times 10^{-6}} = 250\,\mathrm{kHz} \quad \text{-------- 1.16}$$

Thus, the typical delay spread of outdoor cellular wireless channels is Bc = 250 kHz.

1.8 Relation Between ISI and Coherence Bandwidth

- We will explore the relation between the Inter-Symbol Interference (ISI) distortion at the receiver and the coherence bandwidth Bc of the wireless channel.
- Consider a Pulse Amplitude Modulated (PAM) signal x (t) of symbol time T_s transmitted by the base station. Let us also consider the presence of a scatter component at a delay of $\tau_1 = T_d$ in addition to the direct line-of-sight component with a delay $\tau_0 = 0$. This is shown in Figure 1.17.

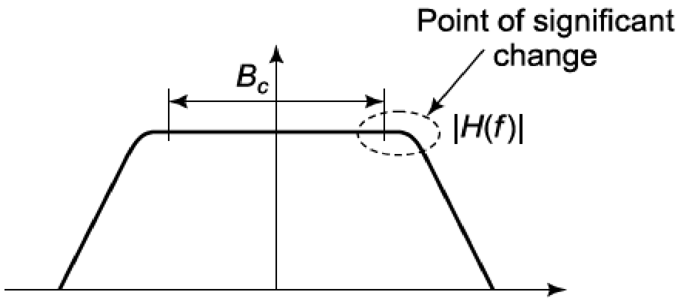


Figure 1.17 Coherence bandwidth showing point of change of response

- The net signal sensed by the receiver is the sum of the direct and scatter components, i.e., x (t) and x $(t - \tau_0)$.
- From Figure 1.17 if the delay spread $\sigma_\tau = \tau_1 - \tau_0$ is comparable to the symbol time Ts, when these two signals are superposed at the receiver, the symbol s_0 from x (t) adds to a different symbol from x $(t - \tau_0)$.
- For instance, in the figure, s_0 adds to s_{-1}, i.e., the previous symbol. The delay spread increases, and the number of interfering paths correspondingly increases, the severity of ISI increases, with several symbols superposing at the receiver. This can be clearly seen in Figure 1.18.

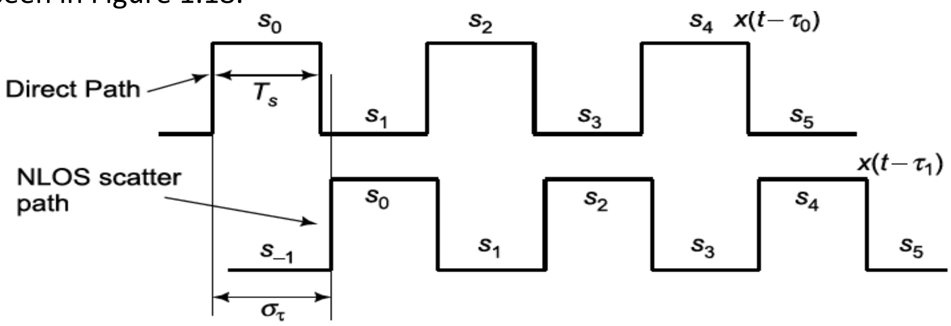


Figure 1.18 Relation between ISI and delay spread

The ISI at the receiver is related to the interplay between the symbol time Ts and delay spread Td.
For instance, when the symbol time Ts is much larger than the delay spread Td as shown in Figure 1.19, there is no ISI.
However, as the delay spread Td becomes comparable to Ts, it leads to ISI. Thus one can empirically state the criterion for ISI as

$$T_d \geq \frac{1}{2} T_s$$

Also, the symbol time Ts is related to the bandwidth Bs of the signal as Ts = 1/Bs.
The delay spread T_d is related to the coherence bandwidth Bc as $B_c = \frac{1}{2}\frac{1}{T_d}$.
The criterion for inter-symbol interference above can be recast in terms of the Bandwidths Bs, Bc as the same as the condition for frequency-selective signal distortion as illustrated in Eq. (1.12).

$$\frac{1}{2}\frac{1}{B_c} \geq \frac{1}{2}\frac{1}{B_s}$$
$$\Rightarrow B_s \geq B_c$$

The frequency-selective distortion and inter-symbol interference are essentially both sides of the same coin.
In the **time domain**, if the delay spread is much larger compared to the symbol time, it results in inter-symbol interference.
Correspondingly, in the **frequency domain**, this implies that the bandwidth of the signal is much larger than the coherence bandwidth of the channel.
Thus, in effect, one is trying to push a signal of much higher bandwidth through a channel filter, with a much smaller bandwidth. This results in frequency-selective distortion.
Thus, to correct for the inter-symbol interference at the receiver, one needs to intuitively multiply by the inverse of the channel response filter, i.e., 1/H(f) , to convert the frequency selective channel into a system with a net flat-fading response. This process, termed equalization is the different frequency components are being equalized to a common flat-level.

Example 7: Consider the 2G Global System for Mobile Communications (GSM) standard with a signal bandwidth of BGSM = 200 kHz. Does the GSM signal experience frequency selective or flat fading? Is there inter-symbol interference at the GSM receiver? Answer the same questions as above in the context of the 3G Wideband Code Division for Multiple Access (WCDMA) standard with a signal bandwidth BWCDMA = 5 mHz.

Solution: The coherence bandwidth Bc corresponding to outdoor cellular wireless channels is Bc ≈ 250 kHz. Hence, since the 2G GSM signal bandwidth BGSM = 200 kHz < Bc = 250 kHz, typically the GSM signal experiences only frequency-flat and not frequency-selective fading.
There is no inter-symbol interference at the GSM receiver.
Since the WCDMA signal bandwidth B_{WCDA} = 5 MHz > Bc = 250 kHz, the WCDMA signal experiences frequency-selective fading, leading to inter-symbol interference at the receiver. Inter-symbol interference is a boon and not a curse for CDMA systems. This is due to the fact that the CDMA receiver can easily remove the effects of inter-symbol interference through the RAKE receiver.

1.9 Doppler Fading in Wireless Systems

- The Doppler shift is a fundamental principle related to the electromagnetic radio-wave propagation.
- In this context, the Doppler shift associated with an electromagnetic wave is defined as the perceived change in the frequency of the wave due to relative motion between the transmitter and receiver. This is schematically shown in Figure 1.19.

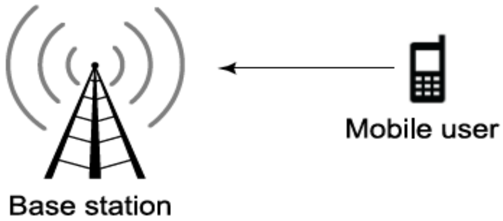


Figure 1.19 Doppler fading due to user mobility

The perceived frequency is higher than the true frequency if the transmitter is moving towards the receiver and lower otherwise.

- Doppler fading is inherent in wireless communications due to the untethered nature of mobile transceivers, which enables mobility in wireless systems, leading to relative motion between the transmitter and the receiver. This is different compared to the conventional wired communications, where the tethered nature of the fixed radio-access medium does not allow for mobility.

1.9.1 Doppler Shift Computation

Consider the scenario shown pictorially in Figure 1.20, where the mobile station is moving with a velocity v at an angle θ with the line joining the mobile and base station.

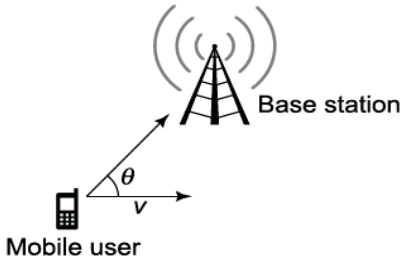


Figure 1.20 Doppler scenario

Let the carrier frequency be fc. The Doppler shift for this scenario is given as

$$f_d = \left(\frac{v}{c}\cos\theta\right) f_c \qquad \text{----------------1.14}$$

where $c = 3 \times 10^8$m/s is the velocity of light, i.e., velocity of an electromagnetic wave in free space. The Doppler shift increases with the velocity v.

Moreover, it depends critically on the angle θ between the direction of motion and the line joining the transmitter and receiver.

For instance, the Doppler shift is maximum when θ = 0, π, i.e., when the relative motion is along the line joining the transmitter and receiver.

However, when $\theta = \pi/2$, i.e., the motion is perpendicular to the receive direction, the Doppler shift is zero.
Also, the Doppler shift is positive in the sense that the perceived frequency is higher if $0 \le \theta \le \pi/2$, in which case $\cos\theta > 0$.
On the other hand, it is negative, leading to a lower perceived frequency that the transmit frequency is $\pi/2 \le \theta \le \pi$.

Example 8: Consider a vehicle moving at 60 miles per hour at an angle of $\theta = 30^\circ$ with the line joining the base station. Compute the Doppler shift of the received signal at a carrier frequency of fc = 1850 MHz.

Solution: We convert the velocity v from units of miles per hour to the standard metres per second. Noting that a mile is equal to 1.61 km, the required velocity in meters per second can be derived as

60 mph = 60 × 1.61 kmph
= 60 × 1.61 × 5/18 m/s = 26.8 m/s

The doppler shift f_d can be computed as

$$f_d = \left(\frac{v}{c}\cos\theta\right) f_c$$

$$f_d = \frac{26.8}{3\times 10^8} \times \cos(30^\circ) \times 1850 \times 10^6$$

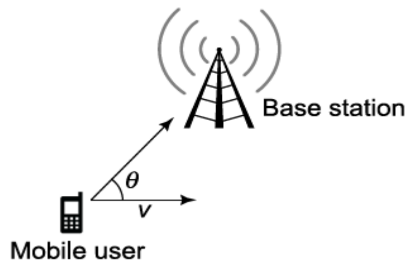
= 143 Hz

Since the mobile user is moving towards the base station, the Doppler shift is positive, i.e., the perceived frequency fr is higher compared to the carrier frequency fc and is given as

fr = fc + fd = 1850 MHz + 143 kHz.

1.10 Doppler Impact on a Wireless Channel

Consider the impulse response of the i^{th} component of the multipath channel given as $a_i\delta(t - \tau_i)$.
Let the vehicle be moving with velocity v at an angle θ with respect to the line joining the mobile and base station.



Observe that the distance between the base station and the mobile is changing constantly due to the motion of the user. Therefore, as a result, the delay of the i^{th} signal component is also changing. Let the initial distance for the i^{th} signal component be di. The initial propagation delay is therefore,

$$\tau_i = \frac{d_i}{c}$$

After a small interval of time t, this distance decreases by vt cos θ, since v cos θ is the component of the velocity in the direction of the base station. Hence, the delay of the i^{th} component after time t is correspondingly given as

$$\tau_i(t) = \frac{d_i - vt\cos\theta}{c} = \frac{d_i}{c} - \frac{vt}{c}\cos\theta = \tau_i - \frac{vt}{c}\cos\theta$$

The flat-fading wireless-channel coefficient has been defined as

$$h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}$$

The equivalent model for the flat-fading channel coefficient h taking into account the velocity v of the user can now be derived by simply replacing the delay τ_i of the i^{th} component by $\tau_i(t)$. Naturally, the resulting channel coefficient is a function of the time t. This model for the time-varying channel coefficient h is given as

$$h(t) = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c\left(\tau_i - \frac{v\cos\theta}{c}t\right)} = \sum_{i=0}^{L-1} e^{-j2\pi f_c \tau_i} e^{j2\pi f_c \frac{v\cos\theta}{c}t} = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i} e^{j2\pi f_d t} \quad \text{------ 1.15}$$

where the last equality follows by substituting $f_d = f_c \frac{v\cos\theta}{c}$.

Observe now that the quantity $e^{j2\pi f_d t}$ represents the time-varying phase of the wireless channel. The rate of variation of the phase is given by the Doppler frequency f_d. Thus, to summarize, the mobility of the user in a wireless communication system leads to a Doppler shift, which in turn results in a time-varying wireless channel coefficient.

This time-varying nature of the wireless channel is also termed time selectivity and the time-varying wireless channel is termed a time-selective channel.

As frequency selectivity refers to different signal attenuations in different bands, time selectivity refers to different attenuations at different instants of time. Further, a channel can be both time- and frequency-selective. Such channels are termed **doubly selective wireless channels.**

1.11 Coherence Time of the Wireless Channel

Consider the i th multipath component of the time-varying channel coefficient in Eq. (1.15), which is given as

$$a_i(t) = a_i e^{-j2\pi f_c \tau_i} e^{j2\pi f_d t}$$

The value of this ith component corresponding to t = 0, π/2 can be obtained as

$$t = 0 \Rightarrow a_i(0) = a_i e^{-j2\pi f_c \tau_i}$$

$$t = \frac{1}{4f_d} \Rightarrow a_i e^{-j2\pi f_c \tau_i} e^{j2\pi f_d \frac{1}{4f_d}} = j a_i e^{-j2\pi f_c \tau_i}$$

The channel changes significantly from time t = 0 to t = 1/4f_d since the phase changes by pi/2 . This time duration in which the channel changes significantly due to the mobility of the user is termed the coherence time, Tc. Further, although f_d depends on the angle of motion θ, a conservative estimate, i.e., minimum coherence time can be obtained by setting θ = π/2 , in other words, corresponding to the fastest rate of change f_d for a given velocity v. This value of the coherence time T_c can be defined as

$$T_c = \frac{1}{4f_d^{\max}}, \; f_d^{\max} = \frac{v}{c} f_c$$

The impact of coherence time can be understood as follows. Consider a wireless channel which is changing with time. The coherence time Tc is the approximate duration of time for which the wireless channel can be assumed to be constant. This can also be expressed as

$$T_c = \frac{1}{2B_d} \quad \text{------ 1.16}$$

where B_d = 2f_d is the Doppler spread of the wireless channel.

Example 9: Consider the scenario described above in Example 8, i.e., a mobile user in a vehicle moving at 60 miles per hour. Compute the coherence time Tc at the carrier frequency fc = 1.85 GHz.

Solution: To compute the coherence time Tc, we start by computing the maximum Doppler shift f_d^{max} corresponding to θ = 0◦.

$$f_d^{\max} = \frac{26.8}{3 \times 10^8} \times 1850 \times 10^6$$
$$= 165 \text{ Hz}$$

Hence, the corresponding Doppler spread is given as Bd = 2 × f_d^{max} = 330 Hz. Hence, the coherence time Tc is given from the relation

$$T_c = \frac{1}{2B_d}$$

$$T_c = \frac{1}{2 \times 330}$$

= 1.5ms

The value of Tc in practical wireless systems, at vehicular velocities around 60 mph and carrier frequencies in the 2 GHz range is of the order of milliseconds (ms).
Thus, the Doppler spread of a wireless system gives the wireless-system designer an idea of the rate of change of the wireless-channel coefficient.
A larger Doppler spread B_d corresponds to a smaller coherence time Tc leading to a faster rate of channel variation.