MODULE-5

5(A):Symmetrical Fault Analysis

SYLLABUS

Symmetrical Fault Analysis: ZBUS Formulation by Step by step building algorithm without mutual coupling between the elements by addition of link and addition of branch, Illustrative Examples, ZBUS Algorithm for Short Circuit Studies excluding numerical.

5.1.1 ZBUS Formulation by Step by step building algorithm without mutual coupling between the elements by addition of link and addition of branch

It is a step-by-step programmable technique which proceeds branch by branch. It has the advantage that any modification of the network does not require complete rebuilding of ZBUS Consider that Z BUS has been formulated upto a certain stage and another branch is now added. Then

$$Z_{BUS}(old) \xrightarrow{Z_b = branch\ impedance} Z_{BUS}(new)$$

Upon adding a new branch, one of the following situations is presented.

1. Zb is added from a new bus to the reference bus (i.e. a new branch is added and the dimension of ZBUS goes up by one). **This is type-1 modification.**
2. Zb is added from a new bus to an old bus (i.e., a new branch is added and the dimension of ZBUS goes up by one). **This is type-2 modification.**
3. Zb connects an old bus to the reference branch (i.e., a new loop is formed but the dimension of ZBUS does not change). **This is type-3 modification.**
4. Zb connects two old buses (i.e., new loop is formed but the dimension of ZBUS does not change).**This is type-4 modification.**
5. Zb connects two new buses (ZBUS remains unaffected in this case). This situation can be avoided by suitable numbering of buses and from now onwards will be ignored.

Notation: i, j — old buses; r — reference bus; k — new bus.

Type-1 Modification: Figure **5.1** shows a passive (linear) n-bus network in which branch with impedance Zb is added to the new bus k and the reference bus r. Now

$$V_k = Z_b I_k$$

$$Z_{ki} = Z_{ik} = 0; i = 1, 2, \dots, n$$

$$\therefore\ Z_{kk} = Z_b$$

Hence

$$Z_{BUS}(new) = \left[\begin{array}{c|c} Z_{BUS}(old) & \begin{matrix} 0 \\ | \\ | \\ | \\ 0 \end{matrix} \\ \hline 0 \ - \ - \ - \ 0 & Z_b \end{array}\right] \text{---------------(1)}$$

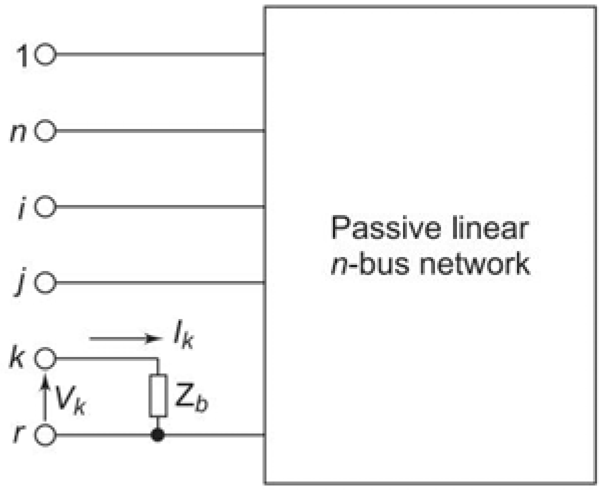


Fig.5.1 Type-1 Modification

Type-2 Modification: Zb is added from new bus k to the old bus j as in **Fig. 5.2.** It follows from this figure that

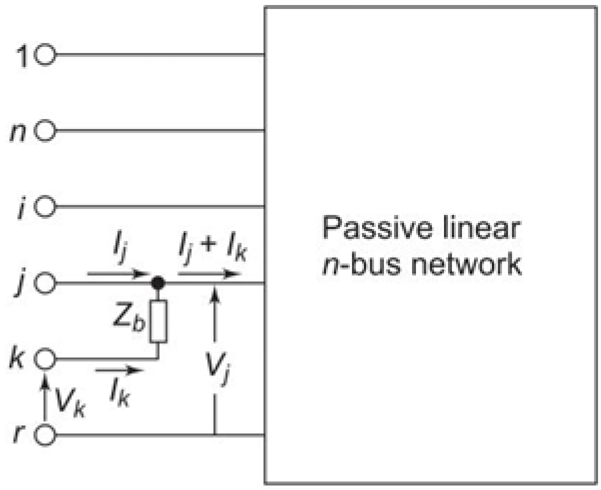


Fig.5.2 Type-2 Modification

$$V_k = Z_b I_k + V_j = Z_b I_k + Z_{j1} I_1 + Z_{j2} I_2 + \cdots + Z_{jj}(I_j + I_k) + \cdots + Z_{jn} I_n$$

Rearranging,

$$V_k = Z_{j1} I_1 + Z_{j2} I_2 + \cdots + Z_{jj} I_j + \cdots + Z_{jn} I_n + (Z_{jj} + Z_b) I_k$$

Consequently

$$Z_{\text{BUS}}(\text{new}) = \left[\begin{array}{c|c} & Z_{1j} \\ & Z_{2j} \\ Z_{\text{BUS}}(\text{old}) & \vdots \\ & Z_{nj} \\ \hline Z_{ji} Z_{j2} \ldots Z_{jn} & Z_{jj} + Z_b \end{array}\right] \text{-----------------------(2)}$$

Type-3 Modification: Zb connects an old bus (j) to the reference bus (r) as in **Fig. 5.3.** This case follows from **Fig. 5.2** by connecting bus k to the reference bus r, i.e. by setting $V_k = 0$.

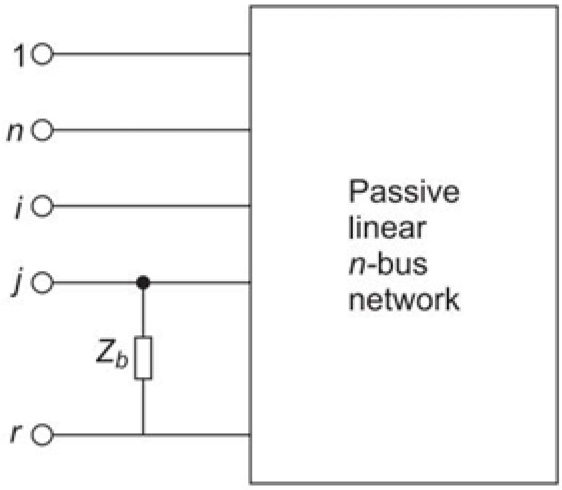


Fig.5.3 Type-3 Modification

Thus,

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ \hline 0 \end{bmatrix} = \left[\begin{array}{c|c} & Z_{1j} \\ & Z_{2j} \\ Z_{\text{BUS}}(\text{old}) & \vdots \\ & Z_{nj} \\ \hline Z_{j1} Z_{j2} \ldots Z_{jn} & Z_{jj} + Z_b \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_k \end{bmatrix} \text{--------- (3)}$$

Eliminate Ik in the set of equations contained in the matrix operation (3),

$$\boldsymbol{0 = Z_{j1}I_1 + Z_{j2}I_2 + \cdots + Z_{jn}I_n + (Z_{jj} + Z_b)I_k}$$

$$\boldsymbol{or \quad I_k = -\frac{1}{Z_{jj} + Z_b}(Z_{j1}I_1 + Z_{j2}I_2 + \cdots + Z_{jn}I_n) ------(4)}$$

Now,

$$\boldsymbol{V_i = Z_{i1}I_1 + Z_{i2}I_2 + \cdots + Z_{in}I_n + Z_{ij}I_k ----- (5)}$$

Substituting Eqn. (5) in Eqn. (4)

$$\boldsymbol{V_i = \left[Z_{i1} - \frac{1}{Z_{jj}+Z_b}(Z_{ij}Z_{j1})\right] I_1 + \left[Z_{i2} - \frac{1}{Z_{jj}+Z_b}(Z_{ij}Z_{j2})\right] I_2 + \cdots + \left[Z_{in} - \frac{1}{Z_{jj}+Z_b}(Z_{ij}Z_{jn})\right] I_n}$$
$$\boldsymbol{--------- (6)}$$

Equation (2) can be written in matrix form as

$$\boldsymbol{Z_{BUS}(new) = Z_{BUS}(old) - \frac{1}{Z_{jj} + Z_b}\begin{bmatrix} Z_{1j} \\ \vdots \\ Z_{nj} \end{bmatrix}[Z_{j1} \quad \cdots \quad Z_{jn}] ------(7)}$$

Type-4 Modification: Zb connects two old buses as in **Fig.5.4.** Equations can be written as follows for all the network buses.

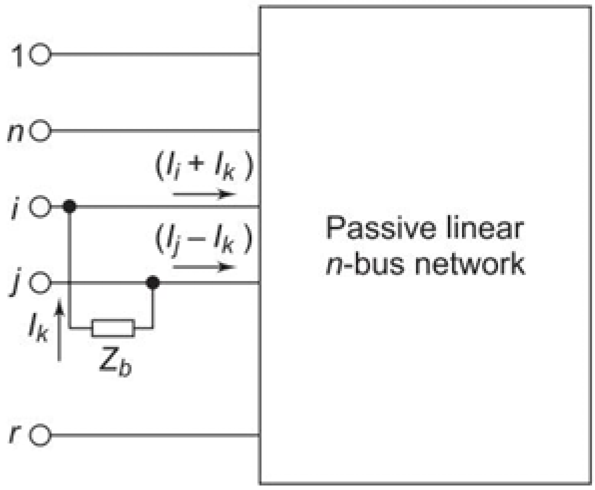


Fig.5.4 Type-4 Modification

$$\boldsymbol{V_i = Z_{i1}I_1 + Z_{i2}I_2 + \cdots + Z_{1i}(I_i + I_k) + Z_{ij}(I_j - I_k) + \cdots + Z_{in}I_n} ------(\mathbf{8})$$

Similar equations follow for other buses. The voltages of the buses i and j are, however, constrained by the equation **(Fig. 5.4).**

$$\boldsymbol{V_j = Z_bI_k + V_i}$$

$$\boldsymbol{or} \quad \boldsymbol{Z_{j1}I_1 + Z_{j2}I_2 + \cdots + Z_{ji}(I_i + I_k) + Z_{jj}(I_j - I_k) + \cdots + Z_{jn}I_n}$$
$$\boldsymbol{= Z_bI_k + Z_{i1}I_1 + Z_{i2}I_2 + \cdots + Z_{ii}(I_i + I_k) + Z_{ij}(I_j - I_k) + \cdots + Z_{in}I_n} ----(\mathbf{9})$$

Rearranging

$$\boldsymbol{0 = (Z_{i1} - Z_{j1})I_1 + \cdots + (Z_{ii} - Z_{ji})I_i + (Z_{ij} - Z_{jj})I_j + \cdots + (Z_{in} - Z_{jn})I_n}$$
$$\boldsymbol{+ (Z_b + Z_{ii} + Z_{jj} - Z_{ij} - Z_{ji})I_k} ------(\mathbf{10})$$

Collecting equations similar to Eqn (8) and Eqn. (10), we can write

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ - \\ 0 \end{bmatrix} = \left[\begin{array}{c|c} & (Z_{1i} - Z_{1j}) \\ & (Z_{2i} - Z_{2j}) \\ Z_{\text{BUS}} & | \\ & (Z_{ni} - Z_{nj}) \\ \hline (Z_{i1} - Z_{j1}) \ldots (Z_{in} - Z_{jn}) & Z_b + Z_{ii} + Z_{jj} - 2Z_{ij} \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ \hline I_j \end{bmatrix}$$

------(11)

Eliminating I_k in Eqn. (11) on lines similar to what was done in type-2 modification, it follows that

With the use of four relationships Eqns. (1), (2), (6) and (11) bus impedance matrix can be built by a step-by-step procedure (bringing in one branch at a time) as illustrated in Example-1. This procedure being a mechanical one can be easily computerized.

When the network undergoes changes, the modification procedures can be employed to revise the bus impedance matrix of the network. The opening of a line (Zij) is equivalent to adding a branch in parallel to it with impedance – Zij (see Example-1).

5.1.2 Z_{BUS} Algorithm for Short Circuit Studies excluding Numerical

- So far we have carried out short circuit calculations for simple systems whose passive networks can be easily reduced. In this section we extend our study to large systems.
- In order to apply the four steps of short circuit computation developed earlier to large systems, it is necessary to evolve a systematic general algorithm so that a digital computer can be used.
- Consider an n-bus system shown schematically in **Fig. 5.6** operating at steady load.
- The **first step** towards short circuit computation is to obtain prefault voltages at all buses and currents in all lines through a load flow study.
- Let us indicate the prefault bus voltage vector as

$$\boldsymbol{V_{BUS}^0} = \begin{bmatrix} \boldsymbol{V_1^0} \\ \boldsymbol{V_2^0} \\ \vdots \\ \boldsymbol{V_n^0} \end{bmatrix} ----- (\mathbf{12})$$

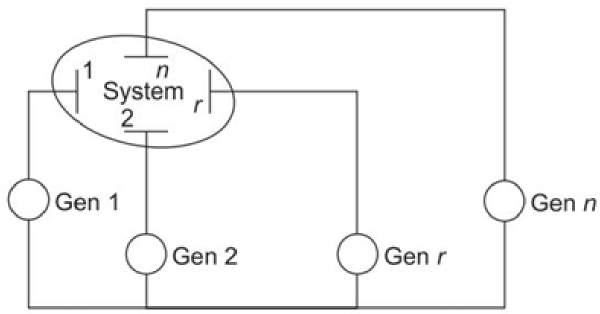


Fig.5.6 n-bus system under steady load

Let us assume that the rth bus is faulted through a fault impedance Z^f. The postfault bus voltage vector will be given by

$$\boldsymbol{V_{BUS}^f} = \boldsymbol{V_{BUS}^0} + \Delta \boldsymbol{V} ----(\mathbf{13})$$

Where ΔV is the vector of changes in bus voltages caused by the fault.

As **step 2**, we drawn the passive Thevenin network of the system with generators replaced by transient/subtransient reactances with their emfs shorted **(Fig. 5.7).**

As per **step 3**, we now excite the passive Thevenin network with $-V_r^0$ in series with Zf as in **Fig. 5.7.** The vector ΔV comprises the bus voltages of this network.

Now

$$\Delta \boldsymbol{V} = \boldsymbol{Z_{BUS}}\ \boldsymbol{J^f} ----(\mathbf{14})$$

Where

$$\boldsymbol{Z_{BUS}} = \begin{bmatrix} \boldsymbol{Z_{11}} \dots \boldsymbol{Z_{1n}} \\ \vdots \quad\quad \vdots \\ \boldsymbol{Z_{n1}} \dots \boldsymbol{Z_{nn}} \end{bmatrix}$$

$$= \boldsymbol{Bus\ Impedance\ Matrix\ of\ the\ Passive\ Thevenin\ Network} --- (\mathbf{15})$$

J^f = bus current injection vector

Since the network is injected with current – I^f only at the rth bus, we have

$$\boldsymbol{J^f} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \boldsymbol{I_r^f} = -\boldsymbol{I^f} \\ \vdots \\ \mathbf{0} \end{bmatrix} ----(\mathbf{16})$$

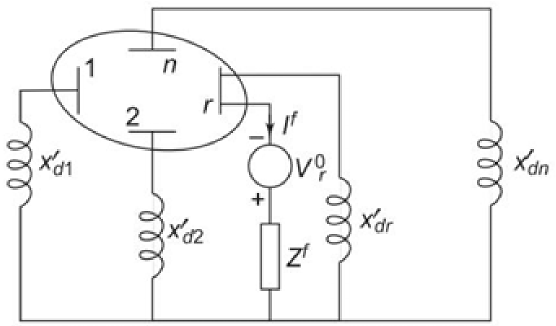


Fig.5.7 Network of the System of Fig. 5.6 for computing changes in bus voltages caused by the fault

Substituting Eqn. (16) in Eqn. (14), we have for the rth bus

$$\Delta \boldsymbol{V_r} = -\boldsymbol{Z_{rr}}\ \boldsymbol{I^f}$$

By step 4, the voltage at the rth bus under fault is

$$\boldsymbol{V_r^f} = \boldsymbol{V_r^0} + \Delta \boldsymbol{V_r^0} = \boldsymbol{V_r^0} - \boldsymbol{Z_{rr}}\ \boldsymbol{I^f} ----(\mathbf{17})$$

However, this voltage must equal

$$\boldsymbol{V_r^f} = \boldsymbol{Z^f I^f} ----(\mathbf{18})$$

We have from Eqs (17) and (18)

$$\boldsymbol{Z^f I^f} = \boldsymbol{V_r^0} - \boldsymbol{Z_{rr}}\ \boldsymbol{I^f}$$

$$\boldsymbol{or} \qquad \boldsymbol{I^f} = \frac{\boldsymbol{V_r^0}}{\boldsymbol{Z_{rr}} + \boldsymbol{Z^f}} ----- (\mathbf{19})$$

At the ith bus (from Eqns (14) and (16))

$$\Delta \boldsymbol{V}_i = -\boldsymbol{Z}_{ir}\ \boldsymbol{I}^f$$

$$\boldsymbol{V}_i^f = \boldsymbol{V}_i^0 - \boldsymbol{Z}_{ir}\ \boldsymbol{I}^f, \qquad \boldsymbol{i} = \mathbf{1}, \mathbf{2}, \dots, \boldsymbol{n} ----(\mathbf{20})$$

Substituting for I^f from Eqn. (19), we have

$$\boldsymbol{V}_i^f = \boldsymbol{V}_i^0 - \frac{\boldsymbol{Z}_{ir}}{\boldsymbol{Z}_{rr} + \boldsymbol{Z}^f}\boldsymbol{V}_r^0 \quad ----(\mathbf{21})$$

For i = r in Eqn. (21)

$$\boldsymbol{V}_r^f = \frac{\boldsymbol{Z}^f}{\boldsymbol{Z}_{rr} + \boldsymbol{Z}^f}\boldsymbol{V}_r^0 \quad ----(\mathbf{22})$$

In the above relationship$\boldsymbol{V}_i^0\boldsymbol{s}$, the prefault bus voltages are assumed to be known from a load flow study. Z_{BUS} matrix of the short-circuit study network of **Fig.5.7** can be obtained by the inversion of its Y_{BUS} or the Z_{BUS} matrix as in **Example-2** building algorithm presented in Sec.5.1.

It should be observed here that the SC study network of **Fig.5.7** is different from the corresponding load flow study network by the fact that the shunt branches corresponding to the generator reactances do not appear in the load flow study network.

Further, in formulating the SC study network, the load impedances are ignored, these being very much larger than the impedances of lines and generators. Of course synchronous motors must be included in Z_{BUS} Postfault currents in lines are given by formulation for the SC study.

Postfault currents in lines are given by

$$\boldsymbol{I}_{ij}^f = \boldsymbol{Y}_{ij}\left(\boldsymbol{V}_i^f - \boldsymbol{V}_j^f\right) ------(\mathbf{23})$$

For calculation of postfault generator current, examine **Figs. 5.8(a) and (b).** From the load flow study **(Fig. 5.8(a))**

Prefault Generator Output = $P_{Gi} \setminus jQ_{Gi}$

$$\therefore \quad \boldsymbol{I}_{Gi}^0 = \frac{\boldsymbol{P}_{Gi} - \boldsymbol{j}\boldsymbol{Q}_{Gi}}{\boldsymbol{V}_i^0}\ (\boldsymbol{Prefault\ Generator\ Output} = \boldsymbol{P}_{Gi} + \boldsymbol{j}\boldsymbol{Q}_{Gi} ----(\mathbf{24})$$

$$\boldsymbol{E}'_{Gi} = \boldsymbol{V}_i + \boldsymbol{j}\boldsymbol{X}'_{Gi}\boldsymbol{I}_{Gi}^0 ----(\mathbf{25})$$

From the SC study, $\boldsymbol{V}_i^f$ is obtained. It then follows from **Fig. 5.8(b)** that

$$\boldsymbol{I}_{Gi}^f = \frac{\boldsymbol{E}'_{Gi} - \boldsymbol{V}_i^f}{\boldsymbol{j}\boldsymbol{X}'_{Gi}} ----(\mathbf{26})$$

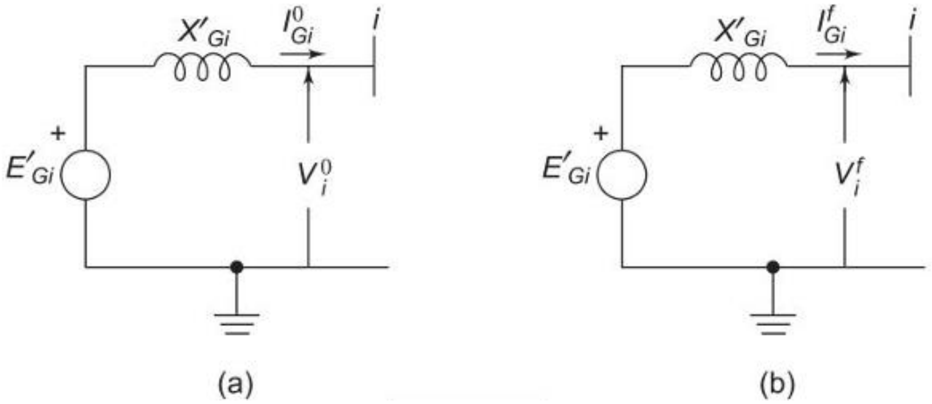


Fig. 5.8

5.1.3 Illustrative Examples

Example-1: For the three-bus network shown in **Fig.5.5** build ZBUS

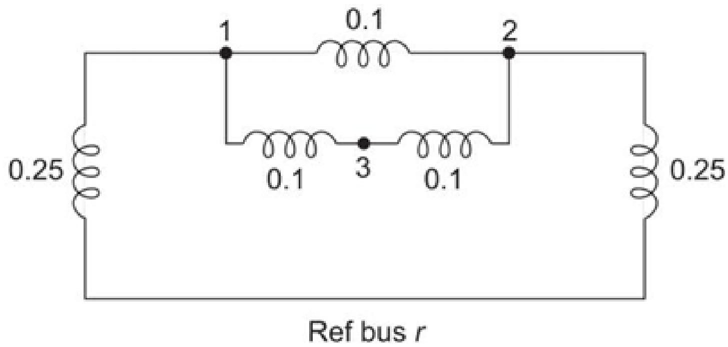


Fig.5.5

Solution:

Step 1: Add branch $z_{1r} = 0.25$ (from bus 1 (new) to bus r)

$$Z_{BUS} = [0.25] \quad \text{(i)}$$

Step 2: Add branch $z_{21} = 0.1$ (from bus 2 (new) to bus 1 (old)); type-2 modification

$$Z_{BUS} = \begin{array}{c} 1 \\ 2 \end{array} \begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0.35 \end{bmatrix} \quad \text{(ii)}$$

Step 3: Add branch $z_{13} = 0.1$ (from bus 3 (new) to bus 1 (old)); type-2 modification

$$Z_{BUS} = \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ 0.25 & 0.25 & 0.35 \end{bmatrix} \quad \text{(iii)}$$

Step 4: Add branch z_{2r} (from bus 2 (old) to bus r); type-3 modification

$$Z_{\text{BUS}} = \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.35 & 0.25 \\ 0.25 & 0.25 & 0.35 \end{bmatrix} - \frac{1}{0.35+0.25}\begin{bmatrix} 0.25 \\ 0.35 \\ 0.25 \end{bmatrix}[0.25\ 0.35\ 0.25]$$

$$= \begin{bmatrix} 0.1458 & 0.1042 & 0.1458 \\ 0.1042 & 0.1458 & 0.1042 \\ 0.1458 & 0.1042 & 0.2458 \end{bmatrix}$$

Step 5: Add branch z_{23} = 0.1 (from bus 2 (old) to bus 3 (old)); type-4 modification

$$Z_{\text{BUS}} = \begin{bmatrix} 0.1458 & 0.1042 & 0.1458 \\ 0.1042 & 0.1458 & 0.1042 \\ 0.1458 & 0.1042 & 0.2458 \end{bmatrix} - \frac{1}{0.1+0.1458+0.2458-2\times 0.1042}$$

$$= \begin{bmatrix} -0.1042 \\ 0.0417 \\ -0.0417 \end{bmatrix}\begin{bmatrix} -0.1042 & 0.0417 & -0.0417 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1397 & 0.1103 & 0.1250 \\ 0.1103 & 0.1397 & 0.1250 \\ 0.1250 & 0.1250 & 0.1750 \end{bmatrix}$$

Opening a line (line 3-2): This is equivalent to connecting an impedance – 0.1 between bus 3 (old) and bus 2 (old) i.e. type-4 modification.

$$Z_{\text{BUS}} = Z_{\text{BUS}}\,(\text{old}) - \frac{1}{(-0.1)+0.175+0.1397-2\times 0.125}$$

$$\begin{bmatrix} 0.0147 \\ -0.0147 \\ 0.0500 \end{bmatrix}[0.0147\ -\ 0.0147\ \ 0.0500]$$

$$= \begin{bmatrix} 0.1458 & 0.1042 & 0.1458 \\ 0.1042 & 0.1458 & 0.1042 \\ 0.1458 & 0.1042 & 0.2458 \end{bmatrix}; \text{(same as in step 4)}$$

Example-2: To illustrate the algorithm discussed above, we shall re-compute the short circuit Solution. Consider the four-bus system of **Fig. 5.9.** Buses 1 and 2 are generator buses and 3 and 4 are load buses. The generators are rated 11 KV, 100 MVA, with transient reactance of 10% each. Both the transformers are 11/110 KV, 100 MVA with a leakage reactance of 5%. The reactances of the lines to a base of 100 MVA, 110 KV are indicated on the figure. Obtain the short circuit solution for a three-phase solid fault on bus 4 (load bus). Assume prefault voltages to be 1 pu and prefault currents to be zero.

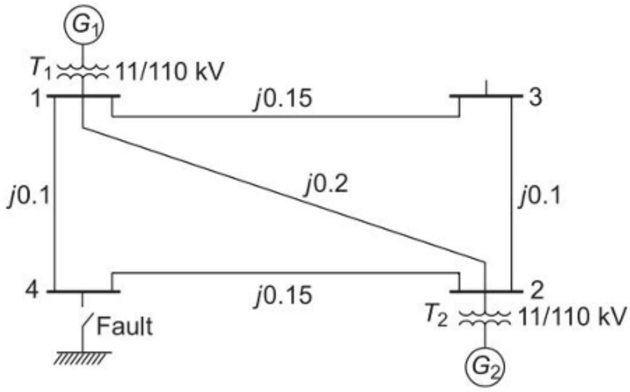


Fig. 5.9 Four-Bus System

Solution:

First of all the bus admittance matrix for the network of **Fig. 5.10** is formed as follows:

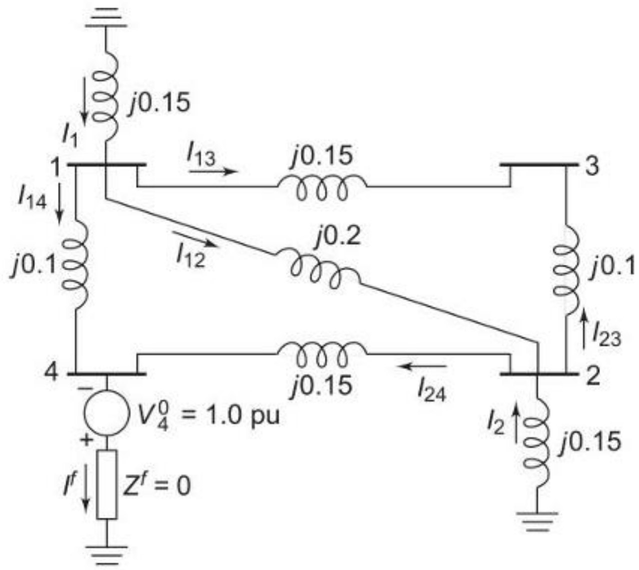


Fig. 5.10

$$Y_{11} = \frac{1}{j0.15} + \frac{1}{j0.15} + \frac{1}{j0.1} + \frac{1}{j0.2} = -j28.333$$

$$Y_{12} = Y_{21} = \frac{-1}{j0.2} = j5.000$$

$$Y_{13} = Y_{31} = \frac{-1}{j0.15} = j6.667$$

$$Y_{14} = Y_{41} = \frac{-1}{j0.1} = j10.000$$

$$Y_{22} = \frac{1}{j0.15} + \frac{1}{j0.15} + \frac{1}{j0.1} + \frac{1}{j0.2} = -j28.333$$

$$Y_{23} = Y_{32} = \frac{-1}{j0.1} = j10.000$$

$$Y_{24} = Y_{42} = \frac{-1}{j0.15} = j6.667$$

$$Y_{33} = \frac{1}{j0.15} + \frac{1}{j0.1} = -j16.667$$

$$Y_{34} = Y_{43} = 0.000$$

$$Y_{44} = \frac{1}{j0.1} + \frac{1}{j0.15} = -j16.667$$

$$\boldsymbol{Y}_{\text{BUS}} = \begin{bmatrix} -j28.333 & j5.000 & j6.667 & j10.000 \\ j5.000 & -j28.333 & j10.000 & j6.667 \\ j6.667 & j10.000 & -j16.667 & j0.000 \\ j10.000 & j6.667 & j0.000 & -j16.667 \end{bmatrix}$$

By inversion we get $\boldsymbol{Z}_{\text{BUS}}$ as

$$\boldsymbol{Z}_{\text{BUS}} = \begin{bmatrix} j0.0903 & j0.0597 & j0.0719 & j0.0780 \\ j0.0597 & j0.0903 & j0.0780 & j0.0719 \\ j0.0719 & j0.0780 & j0.1356 & j0.0743 \\ j0.0780 & j0.0719 & j0.0743 & j0.1356 \end{bmatrix}$$

Now, the postfault bus voltages can be obtained using Eqn. (21) as

$$V_1^f = V_1^0 - \frac{Z_{14}}{Z_{44}} V_4^0$$

The prefault condition being no load, $V_1^0 = V_2^0 = V_3^0 = V_4^0 = 1$ pu

$$V_1^f = 1.0 - \frac{j0.0780}{j0.1356} \times 1.0 = 0.4248 \text{ pu}$$

$$V_2^f = V_2^0 - \frac{Z_{24}}{Z_{44}} V_4^0$$

$$= 1.0 - \frac{j0.0719}{j0.1356} \times 1.0 = 0.4698 \text{ pu}$$

$$V_3^f = V_3^0 - \frac{Z_{34}}{Z_{44}} V_4^0$$

$$= 1.0 - \frac{j0.0743}{j0.1356}\ 1.0 = 0.4521 \text{ pu}$$

$$V_4^f = 0.0$$

Using Eqn. (19) we can obtain the fault current as

$$I^f = \frac{1.000}{j0.1356} = -j7.37463 \text{ pu}$$

Let us also calculate the short circuit current in lines 1–3, 1–2, 1–4, 2–4 and 2–3.

$$I_{13}^f = \frac{V_1^f - V_3^f}{z_{13}} = \frac{0.4248 - 0.4521}{j0.15} = j0.182 \text{ pu}$$

$$I_{12}^f = \frac{V_1^f - V_2^f}{z_{12}} = \frac{0.4248 - 0.4698}{j0.2} = j0.225 \text{ pu}$$

$$I_{14}^f = \frac{V_1^f - V_4^f}{z_{14}} = \frac{0.4248 - 0}{j0.1} = -j4.248 \text{ pu}$$

$$I_{24}^f = \frac{V_2^f - V_4^f}{z_{24}} = \frac{0.4698 - 0}{j0.15} = -j3.132 \text{ pu}$$

$$I_{23}^f = \frac{V_2^f - V_3^f}{z_{23}} = \frac{0.4698 - 0.4521}{j.01} = -j0.177 \text{ pu}$$

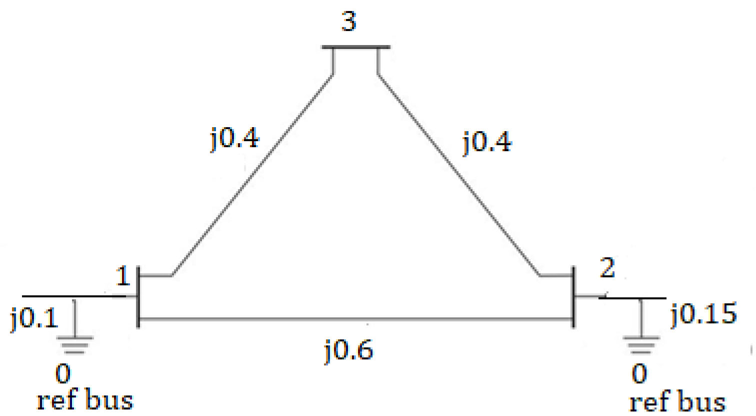
This, however, is a systematic method and can be easily adopted on the digital computer for practical networks of large size.

Further, another important feature of the method is that having computed Z_{BUS}, we can at once obtain all the required short circuit data for a fault on any bus.

For example, in this particular system, the fault current for a fault on bus 1 (or bus 2) will be

$$I^f = \frac{1.000}{Z_{11}(\text{or } Z_{22})} = \frac{1.00}{j0.0903} = -j11.074197 \text{ pu}$$

Example-3: Construct the bus impedance matrix Z_{BUS} by bus algorithm for the system shown in **Fig. 5.11**. All impedances are in pu. Take ground node 0 as the reference node. Add the elements in the order bus 1 to ref. bus, bus 2 to ref. bus, bus 3 to bus 1, bus 2 and bus 1, bus 3 and bus 2. **(VTU June/July 2024 QP)**



Solution:

Given Data:

- Reference Bus: 0 (grounded)
- Bus 1 connected to reference with j0.1
- Bus 2 connected to reference with j0.15
- Bus 3 connected to:
  - Bus 1 via j0.4
  - Bus 2 via j0.4
  - Bus 1 and Bus 2 connected via j0.6

Step 1: Add Bus 1 to reference bus

Z_{11} = j0.1

Zbus = [j0.1]

Step 2: Add Bus 2 to reference bus

This is a **new bus to reference connection**.

Z_{22} = j0.15

Since Bus 1 and Bus 2 are only connected to reference till now:

$$Z_{\text{bus}} = \begin{bmatrix} j0.1 & 0 \\ 0 & j0.15 \end{bmatrix}$$

Step 3: Add Bus 3 to Bus 1

This is a **new bus to existing bus** connection via j0.4.

- $Z_{33} = Z_{11} + j0.4 = j0.1 + j0.4 = j0.5$
- Mutual impedances:

$$Z_{31} = Z_{13} = Z_{11} = j0.12 = 0$$

So,

$$Z_{\text{bus}} = \begin{bmatrix} j0.1 & 0 & j0.1 \\ 0 & j0.15 & 0 \\ j0.1 & 0 & j0.5 \end{bmatrix}$$

Step 4: Modify for Bus 3 to Bus 2 via j0.4j0.4j0.4

This is **addition of a line** between Bus 3 and Bus 2.

Formula for addition of line between buses p and q:

$$Z' = Z + \frac{(Z_p - Z_q)(Z_p - Z_q)^T}{Z_{pp} + Z_{qq} + Z_{\text{pq}} + Z_{\text{qp}} + Z_{\text{added line}}}$$

But simpler method:

$$Z_{\text{bus new}} = Z_{\text{bus old}} - \frac{(Z_p - Z_q)(Z_p - Z_q)^T}{Z_{pp} + Z_{qq} + j0.4}$$

Let's calculate:

- p = 3, q = 2
- Z_{33} = j0.5, Z_{22} = j0.15, Z_{32} = 0

Denominator:

d = Z_{33} + Z_{22} + j0.4 = j0.5 + j0.15 + j0.4 = j1.05

Vector:

$$Z_p - Z_q = \begin{bmatrix} Z_{13} - Z_{12} \\ Z_{23} - Z_{22} \\ Z_{33} - Z_{32} \end{bmatrix} = \begin{bmatrix} j0.1 - 0 \\ 0 - j0.15 \\ j0.5 - 0 \end{bmatrix} = \begin{bmatrix} j0.1 \\ -j0.15 \\ j0.5 \end{bmatrix}$$

$$(Z_p - Z_q)(Z_p - Z_q)^T:$$

$$= \begin{bmatrix} j0.1 \\ -j0.15 \\ j0.5 \end{bmatrix} \times \begin{bmatrix} j0.1 & -j0.15 & j0.5 \end{bmatrix}$$

Numerator:

$j^2 = -1$, so all elements will be real negative.

$$\begin{bmatrix} -0.01 & 0.015 & -0.05 \\ 0.015 & -0.0225 & 0.075 \\ -0.05 & 0.075 & -0.25 \end{bmatrix}$$

$$Z_{\text{bus new}} = Z_{\text{bus old}} - \frac{\text{above matrix}}{j1.05}$$

Dividing by j1.05 means each element will be divided by j1.05, introducing −j and flipping signs:

$$= Z_{\text{bus old}} + j\frac{\text{matrix}}{1.05}$$

$$Z_{11} = j0.1 + j\frac{0.01}{1.05} = j0.1 + j0.00952 = j0.10952$$

Step 5: Modify for Bus 1 to Bus 2 via j0.6

Again, adding line between bus 1 and 2.

Denominator:

d = Z_{11} + Z_{22} + j0.6

Vector:

Zp – Zq = $Z_1 - Z_2$

Proceed similarly as in Step 4.

$$Z_{\text{bus}} = \begin{bmatrix} j0.1061 & j0.0303 & j0.0833 \\ j0.0303 & j0.1242 & j0.0424 \\ j0.0833 & j0.0424 & j0.4288 \end{bmatrix}$$

Example-4: For the power system shown in **Fig. 5.12** the pu reactance are shown therein. For a solid three-phase fault on bus 3, calculate the following:

(a) Fault current

(b) V_1^f and V_2^f

(c) I_{12}^f , I_{13}^f and I_{23}^f

(d) I_{G-1}^f and I_{G-2}^f

Assume prefault voltage to be 1 pu.

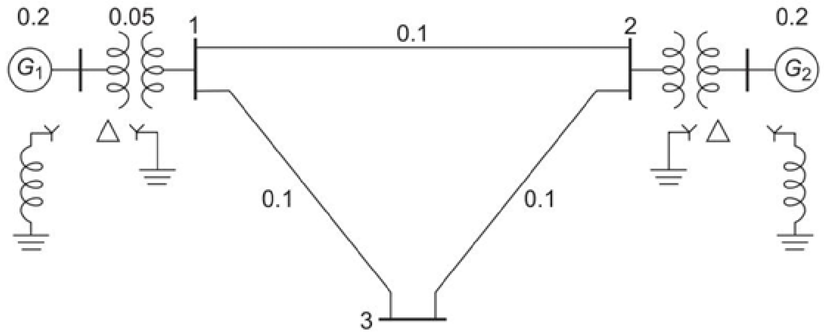


Fig.5.12

Solution:

The Thevenin passive network for this system is drawn in **Fig. 5.12** with its given in Eqn. (iv) of Example-1

(a) As per Eqn. (19)

$$I^f = \frac{V_r^{\,0}}{Z_{rr} + Z^f} \qquad or \qquad I^f = \frac{V_3^{\,0}}{Z_{33}} = \frac{1}{j0.175} = -j5.71$$

(b) As per Eqn. (20)

$$I_i^{\,f} = V_i^{\,0} - \frac{Z_{ir}}{Z_{rr} + Z^f} V_r^{\,0}$$

Now,

$$V_1^{\,f} = \left(1 - \frac{Z_{13}}{Z_{33}}\right) = \left(1 - \frac{0.125}{0.175}\right) = 0.286$$

$$and \qquad V_2^{\,f} = \left(1 - \frac{Z_{23}}{Z_{33}}\right) = \left(1 - \frac{0.125}{0.175}\right) = 0.286$$

The two voltages are equal because of the symmetry of the given power network

(c) As per Eqn. (23)

$$I_{ij}^f = Y_{ij}\left(V_i^f - V_j^f\right)$$

$$I_{12}^{f} = \frac{1}{j0.1}(0.286 - 0.286) = 0$$

$$and \qquad I_{13}^{f} = I_{31}^{f} = \frac{1}{j0.1}(0.286 - 0) = -j2.86$$

(d) As per Eqn. (26)

$$I_{Gi}^{f} = \frac{E'_{Gi} - V_i^f}{jX'_{Gi} + jX_T}$$

$$\boldsymbol{But\ \ E'_{Gi} = 1\ pu\ (prefault\ no\ load)}$$

$$\therefore \quad I_{G1}^{f} = \frac{1 - 0.286}{j0.2 + j0.05} = -j2.86$$

$$Simmilarly \qquad I_{G2}^{f} = j2.86$$

5(B): POWER SYSTEM STABILITY

SYLLABUS

Power System Stability: Numerical Solution of Swing Equation by Point by Point method and Runge Kutta Method, Illustrative Examples.

5.2.1 Numerical Solution of Swing Equation by Point by Point Method

- In most practical systems, after machine lumping has been done, there are still more than two machines to be considered from the point of view of system stability.
- Therefore, there is no choice but to solve the swing equation of each machine by a numerical technique on the digital computer.
- Even in the case of a single machine tied to infinite bus bar, the critical clearing time cannot be obtained from equal area criterion and we have to make this calculation numerically through swing equation.
- There are several sophisticated methods now available for the solution of the swing equation including the powerful Runge-Kutta method.
- Here we shall treat the **point-by-point method of solution** which is a conventional, approximate method like all numerical methods but a well tried and proven one.
- We shall illustrate the point-by-point method for one machine tied to infinite bus bar. The procedure is, however, general and can be applied to every machine of a multimachine system.

Consider the swing equation

$$\boldsymbol{\frac{d^2\delta}{dt^2} = \frac{1}{M}(P_m - P_{max}\sin\delta) = \frac{P_a}{M}}$$

$$\boldsymbol{\left(M = \frac{GH}{\pi}\ \ or\ \ in\ pu\ system\ M = \frac{H}{\pi f}\right)}$$

The solution d(t) is obtained at discrete intervals of time with interval spread of Dt uniform throughout. Accelerating power and change in speed which are continuous functions of time are discretized as below:

1. The accelerating power P_a computed at the beginning of an interval is assumed to remain constant from the middle of the preceding interval to the middle of the interval being considered as shown in **Fig. 5.2.1.**

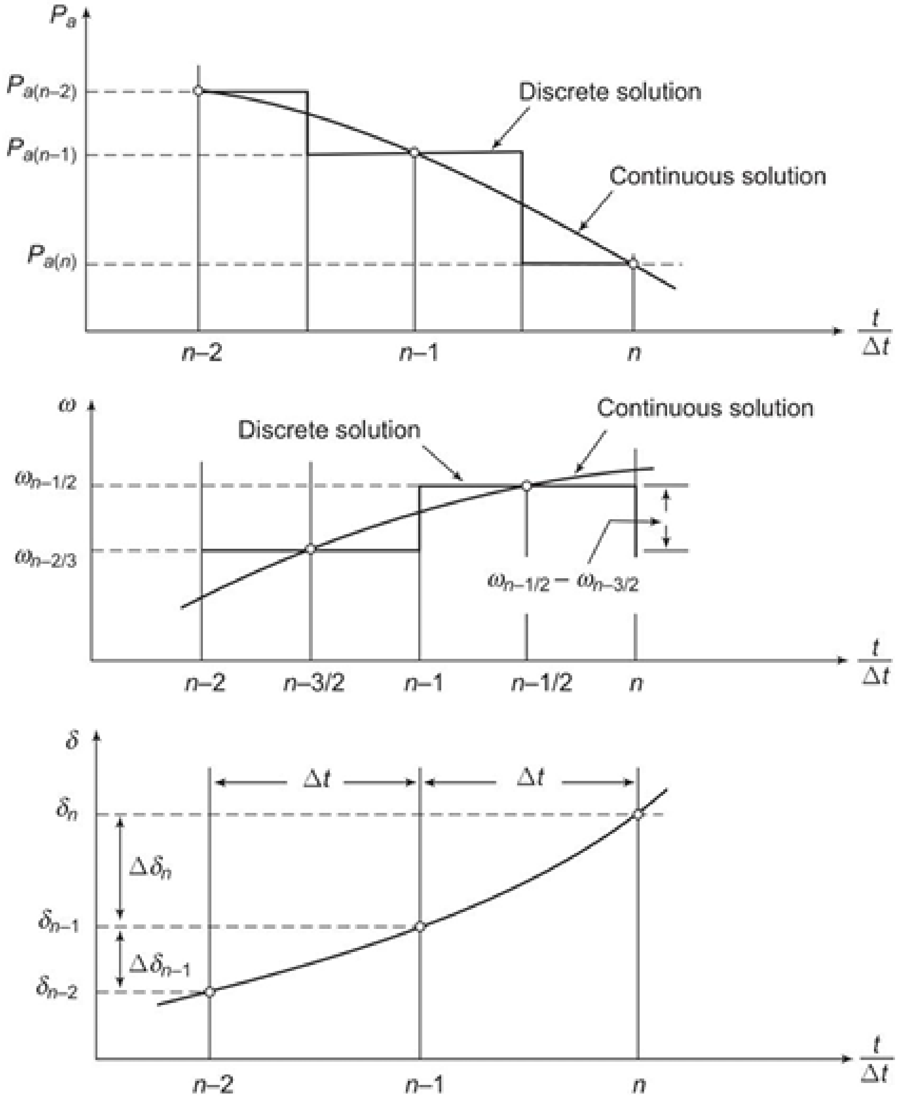


Fig. 5.2.1 Point-By-Point Solution of Swing Equation

2. The angular rotor velocity $\omega = d\delta/dt$ (over and above synchronous velocity ω_s) is assumed constant throughout any interval, at the value computed for the middle of the interval as shown in Fig. 5.2.1.

In Fig. 5.2,1, the numbering on $t/\Delta t$ axis pertains to the end of intervals. At the end of the (n – 1)th interval, the acceleration power is

$$P_{a(n-1)} = P_m - P_{max} \sin \delta_{n-1} ----(1)$$

Where δ_{n-1} has been previously calculated.

The change in velocity ($\omega = d\delta/dt$), caused by the $P_{a(n-1)}$, assumed constant over Δt from $(n - 3/2)$ to $(n - 1/2)$ is

$$\omega_{n-1/2} - \omega_{n-3/2} = \left(\frac{\Delta t}{M}\right) P_{a(n-1)} ----(2)$$

The change in δ during the $(n - 1)$th interval is

$$\Delta\delta_{n-1} = \delta_{n-1} - \delta_{n-2} = \Delta t\,\omega_{n-3/2} ----(3.a)$$

and during the nth interval

$$\Delta\delta_n = \delta_n - \delta_{n-1} = \Delta t\,\omega_{n-1/2} ----(3.b)$$

Subtracting Eqn. (3.a) from Eqn. (3.b) and using Eqn. (2), we get

$$\Delta\delta_n = \delta_{n-1} + \frac{(\Delta t)^2}{M} P_{a(n-1)} ----(4)$$

Using this, we can write

$$\delta_n = \delta_{n-1} + \Delta\delta_n ----(5)$$

The process of computation is now repeated to obtain $P_{a(n)}$, $\Delta\delta_{n+1}$ and δ_{n+1}.

The time solution in discrete form is thus carried out over the desired length of time, normally 0.5 s.

Continuous form of solution is obtained by drawing a smooth curve through discrete values as shown in Fig. 5.2.1.

Greater accuracy of solution can be achieved by reducing the time duration of intervals.

The occurrence or removal of a fault or initiation of any switching event causes a discontinuity in accelerating power P_a.

If such a discontinuity occurs at the beginning of an interval, then the average of the values of P_a before and after the discontinuity must be used.

Thus, in computing the increment of angle occurring during the first interval after a fault is applied at t = 0, Eqn. (12.71) becomes

$$\Delta\delta_1 = \frac{(\Delta t)^2}{M} + \frac{P_{a0+}}{2} ----(4)$$

Where P_{a0+} is the accelerating power immediately after occurrence of fault.

Immediately before the fault the system is in steady state, so that $P_{a0-} = 0$ and δ_0 is a known value.

If the fault is cleared at the beginning of the nth interval, in calculation for this interval one should use for $P_{a(n-1)}$ the value $1/2\,[P_{a(n-1)-} + P_{a(n-1)+}]$, where $P_{a(n-1)-}$ is the accelerating power immediately before clearing and $P_{a(n-1)+}$ is that immediately after clearing the fault.

If the discontinuity occurs at the middle of an interval, no special procedure is needed.

The increment of angle during such an interval is calculated, as usual, from the value of P_a at the beginning of the interval.

Example-1: A 20 MVA, 50 Hz generator delivers 18 MW over a double circuit line to an infinite bus. The generator has kinetic energy of 2.52 MJ/MVA at rated speed. The generator transient reactance is X¢ d = 0.35 pu. Each transmission circuit has R = 0 and a reactance of 0.2 pu on a 20 MVA base. |E'| = 1.1 pu and infinite bus voltage V = 1.0 –0º. A three-phase short circuit occurs at the mid point of one of the transmission lines. Plot swing curves with fault cleared by simultaneous opening of breakers at both ends of the line at 2.5 cycles and 6.25 cycles after the occurrence of fault. Also plot the swing curve over the period of 0.5 s if the fault is sustained.

Solution:

Before we can apply the step-by-step method, we need to calculate the inertia constant M and the power angle equations under prefault and postfault conditions.
Base MVA = 20

$$Inertia\ Constant, M(pu) = \frac{H}{180f} = \frac{1.0 \times 2.52}{180 \times 50} = 2.8 \times 10^{-4} \quad s^2/\ elect\ degree$$

(1) Prefault

$$X_1 = 0.35 + \frac{0.2}{2} = 0.45$$

$$\therefore \quad P_{e1} = P_{max1}\sin\delta = \frac{1.1 \times 1}{0.45}\sin\delta = 2.44\sin\delta ----(1)$$

$$\text{Prefault Power Transfer} = \frac{18}{20} = 0.9\ pu$$

Initial power angle is given by

$$2.44\sin\delta_0 = 0.9$$
$$\delta_0 = 21.64^o$$

(2) During fault A positive sequence diagram is shown in Fig. 5.2.2(a). Converting star to delta, we obtain the network of Fig. 5.2.2(b), in which

$$X_2 = \frac{0.35 \times 0.1 + 0.2 \times 0.1 + 0.35 \times 0.2}{0.1} = 1.25\ pu$$

$$\therefore \quad P_{e2} = P_{max2}\sin\delta = \frac{1.1 \times 1}{1.25}\sin\delta = 0.88\sin\delta ----(2)$$

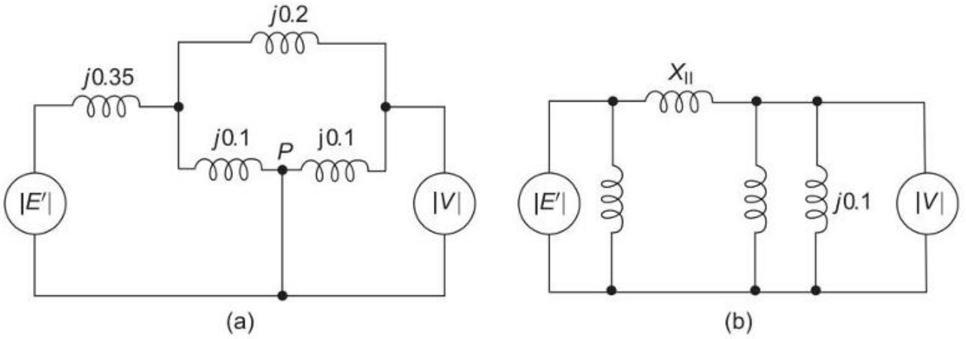


Fig.5.2.2

(3) Postfault

With the faulted line switched off,

$$X_3 = 0.35 + 0.2 = 0.55$$

$$\therefore \quad P_{e3} = P_{max3}\sin\delta = \frac{1.1\times 1}{0.55}\sin\delta = 2.0\sin\delta ----(3)$$

Let us choose Δt = 0.05 s. The recursive relationships for step-by-step swing curve calculation are reproduced below.

$$P_{a(n-1)} = P_m - P_{max}\,\sin\delta_{n-1} ------(4)$$

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{(\Delta t)^2}{M}P_{a(n-1)} ----- (5)$$

$$\delta_n = \delta_{n-1} + \Delta\delta_n ----(6)$$

Since there is a discontinuity in P_e and hence in P_a, the average value of P_a must be used for the first interval.

$$P_a(0_-) = 0\;pu\;and\;P_a(0_+) = 0.9 - 0.88\,sin\,21.64^o = 0.576\,pu$$

$$P_a\left(0_{average}\right) = \frac{0+0.576}{2} = 0.288\,pu$$

Sustained Fault

Calculations are carried out in Table 5.2.1 in accordance with the recursive relationship (4), (5) and (6) above.

The second column of the table shows P_{max} the maximum power that can be transferred at time t given in the first column. P_{max} in the case of a sustained fault undergoes a sudden change at t = 0_+ and remains constant thereafter.

The procedure of calculations is illustrated below by calculating the row corresponding to t = 0.15 s.

Table 5.2.1: Point-by-point computations of swing curve for sustained fault, Δt = 0.05 s

t s	P_{max} pu	$\sin\delta$	$P_e = P_{max}\sin\delta$ pu	$P_a = 0.9 - P_e$ pu	$\frac{(\Delta t)^2}{M} P_a = 8.929\, P_a$ deg	$\Delta\delta$ deg	δ deg
0_-	2.44	0.368	0.9	0.0	—	—	21.64
0_+	0.88	0.368	0.324	0.576	—	—	21.64
0_{avg}	—	0.368	—	0.288	2.57	2.57	21.64
0.05	0.88	0.41	0.361	0.539	4.81	7.38	24.21
0.10	0.88	0.524	0.461	0.439	3.92	11.30	31.59
0.15	0.88	0.680	0.598	0.301	2.68	13.98	42.89
0.20	0.88	0.837	0.736	0.163	1.45	15.43	56.87
0.25	0.88	0.953	0.838	0.06	0.55	15.98	72.30
0.30	0.88	0.999	0.879	0.021	0.18	16.16	88.28
0.35	0.88	0.968	0.852	0.048	0.426	16.58	104.44
0.40	0.88	0.856	0.754	0.145	1.30	17.88	121.02
0.45	0.88	0.657	0.578	0.321	2.87	20.75	138.90
0.50	0.88	—	—	—	—	—	159.65

$$(0.1\text{ s}) = 31.59^\circ$$
$$P_{max} = 0.88$$
$$\sin\delta(0.1\text{ s}) = 0.524$$
$$P_e(0.1\text{ s}) = P_{max}\sin\delta(0.1\text{ s}) = 0.88 \times 0.524 = 0.461$$
$$P_a(0.1\text{ s}) = 0.9 - 0.461 = 0.439$$
$$\frac{(\Delta t)^2}{M} P_a(0.1\text{ s}) = 8.929 \times 0.439 = 3.92^\circ$$
$$\delta(0.15\text{ s}) = \Delta\delta(0.1\text{ s}) + \frac{(\Delta t)^2}{M} P_a(0.1\text{ s})$$
$$= 7.38^\circ + 3.92^\circ = 11.33^\circ$$
$$\delta(0.15\text{ s}) = \delta(0.1\text{ s}) + \Delta\delta(0.15\text{ s})$$
$$= 31.59^\circ + 11.30^\circ = 42.89^\circ$$

δ(t) for sustained fault as calculated in Table 5.2.1 is plotted in **Fig. 5.2.3** from which it is obvious that the system is unstable.

Fault Cleared in 2.5 Cycles

Time to clear fault = 2.5 / 50 = 0.05 s

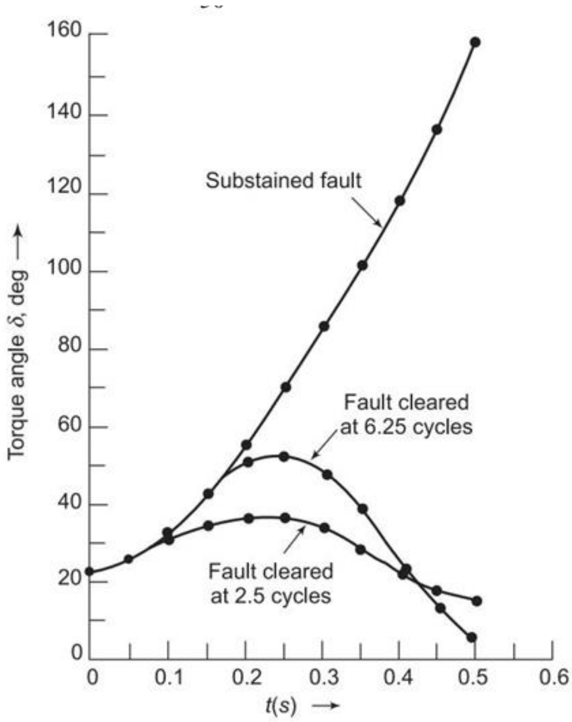


Fig. 5.2.3 Swing Curves for Example-1 for a sustained fault and for clearing in 2.5 and 6.25 cycles

P_{max} suddenly changes from 0.88 at t = 0.05_{-} to 2.0 at t = 0.05_{+}.

Since the discontinuity occurs at the beginning of an interval, the average value of P_a will be assumed to remain constant from 0.025 s to 0.075 s.

The rest of the procedure is the same and complete calculations are shown in Table 5.2.2.

The swing curve is plotted in Fig. 5.2.3 from which we find that the generator undergoes a maximum swing of 37.5^{o} but is stable as d finally begins to decrease.

Fault Cleared in 6.25 Cycles

Time to clear fault = 6.25 / 50 = 0.125 s

Since the discontinuity now lies in the middle of an interval, no special procedure is necessary, as in deriving Eqns. (4)–(6) discontinuity is assumed to occur in the middle of the time interval.

The swing curve as calculated in Table 5.2.3 is also plotted in Fig. 5.2.3.

It is observed that the system is stable with a maximum swing of 52.5^{o} which is much larger than that in the case of 2.5 cycle clearing time.

Table 5.2.2: Computations of swing curve for fault cleared at 2.5 cycles (0.05 s), Δt = 0.05 s

t s	P_{max} pu	$\sin\delta$	$P_e = P_{max}\sin\delta$ pu	$P_a = 0.9 - P_e$ pu	$\frac{(\Delta t)^2}{M} P_a$ $= 8.929\,P_a$ deg	$\Delta\delta$ deg	δ deg
0_-	2.44	0.368	0.9	0.0	—	—	21.64
0_+	0.88	0.368	0.324	0.576	—	—	21.64
0_{avg}	—	0.368	—	0.288	2.57	2.57	21.64
0.05_-	0.88	0.41	0.36	0.54	—	—	24.21
0.05_+	2.00	0.41	0.82	0.08	—	—	24.21
0.05_{avg}				0.31	2.767	5.33	24.21
0.10	2.00	0.493	0.986	– 0.086	– 0.767	4.56	29.54
0.15	2.00	0.56	1.12	– 0.22	– 1.96	2.60	34.10
0.20	2.00	0.597	1.19	– 0.29	– 2.58	0.02	36.70
0.25	2.00	0.597	1.19	– 0.29	– 2.58	– 2.56	37.72
0.30	2.00	0.561	1.12	– 0.22	– 1.96	– 4.52	34.16
0.35	2.00	0.494	0.989	– 0.089	– 0.79	– 5.31	29.64
0.40	2.00	0.41	0.82	0.08	0.71	– 4.60	24.33
0.45	2.00	0.337	0.675	0.225	2.0	– 2.6	19.73
0.50							17.13

To find the critical clearing time, swing curves can be obtained, similarly, for progressively greater clearing time till the torque angle d increases without bound.

In this example, however, we can first find the critical clearing angle using Eqn. (12.67) and then read the critical clearing time from the swing curve corresponding to the sustained fault case.

The values obtained are: Critical clearing angle = 118.62°

Critical clearing time = 0.38 s

$$\cos\delta_{cr} = \frac{P_m(\delta_{max} - \delta_0) - P_{max\,II}\cos\delta_0 + P_{max\,III}\cos\delta_{max}}{P_{max\,III} - P_{max\,II}} \qquad (12.67)$$

Table 5.2.3: Computations of swing curve for fault cleared at 6.25 cycles (0.125 s), Δt = 0.05 s

t s	P_{max} pu	$\sin\delta$	$P_e = P_{max}\sin\delta$ pu	$P_a = 0.9 - P_e$ pu	$\frac{(\Delta t)^2}{M} P_a = 8.929\, P_a$ deg	$\Delta\delta$ deg	δ deg
0_-	2.44	0.368	0.9	0.0	–	–	21.64
0_+	0.88	0.368	0.324	0.576	–	–	21.64
0_{avg}	–	0.368	–	0.288	2.57	2.57	21.64
0.05	0.88	0.41	0.361	0.539	4.81	7.38	24.21
0.10	0.88	0.524	0.461	0.439	3.92	11.30	31.59
0.15	2.00	0.680	1.36	– 4.46	– 4.10	7.20	42.89
0.20	2.00	0.767	1.53	– 0.63	– 5.66	1.54	50.09
0.25	2.00	0.78	1.56	– 0.66	– 5.89	– 4.35	51.63
0.30	2.00	0.734	1.46	– 0.56	– 5.08	– 9.43	47.28
0.35	2.00	0.613	1.22	– 0.327	– 2.92	– 12.35	37.85
0.40	2.00	0.430	0.86	0.04	0.35	– 12.00	25.50
0.45	2.00	0.233	0.466	0.434	3.87	– 8.13	13.50
0.50	2.00						5.37

(2) A 20 MVA, 50 HZ generator delivers 18 MW over a double circuit line to an infinite bus. The generator has kinetic energy of 2.52 MJ/MVA at rated speed. The generator has a transient reactance of 0.35 pu. Each transmission line has a reactance of 0.2 pu on a 20 MVA base. The generator excitation voltage $|E'| = 1.1$ pu and infinite bus v/g $V = 1\angle 0^\circ$ pu. A three phase short circuit occurs at the midpoint of one of the lines. Plot the swing curve with the fault cleared by simultaneous opening of breakers at both ends of the line at 2.5 cycles after the occurrence of fault. Take a step size of time as 0.05 sec. Also, calculate the critical clearing angle. Use point by point method.

Solution :-

Let base MVA = 20

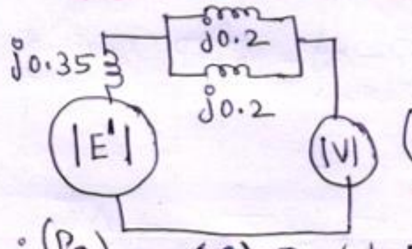
Inertia constant, $M\,(pu) = \dfrac{H}{\pi f} = \dfrac{2.52}{180 \times 50}$

$= 2.8 \times 10^{-4}\ s^2/\text{elect.deg.}$

Fault clearing time, $t_c = 2.5$ cycles $= \dfrac{2.5}{50}$

$= 0.05$ sec.

Prefault Condition :- (Before fault)



$X_{bf} = 0.35 + (0.2 \| 0.2) = 0.45$ pu

$(P_e)_{bf} = \dfrac{|E'|\,|V|}{X_{bf}} \sin\delta = \dfrac{1.1 \times 1}{0.45} \sin\delta$

$= 2.44 \sin\delta$

$\therefore (P_e)_{max(bf)} = 2.44$ pu

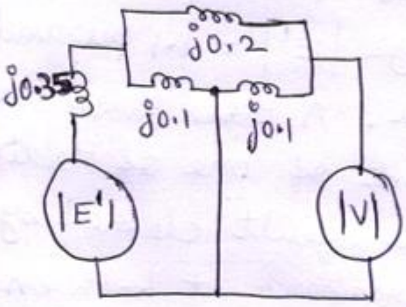
Prefault power transfer, $(P_e)_0 = P_m = \frac{18}{20} = 0.9$ pu

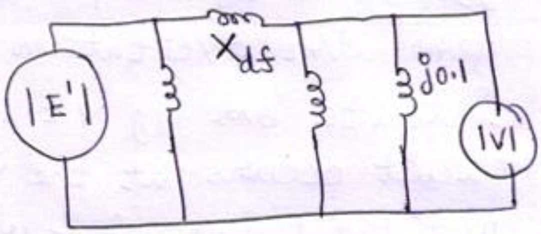
Initial power angle is given by

$$2.44 \sin\delta_0 = 0.9$$

$$\therefore \delta_0 = 21.64^\circ$$

During fault :- converting star to delta





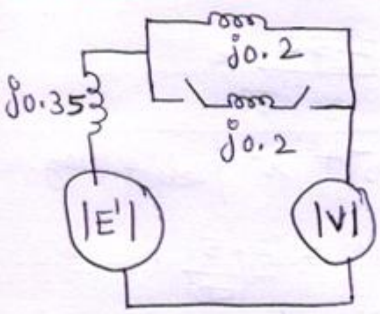
$$X_{df} = \frac{(0.35\times0.1)+(0.2\times0.1)+(0.35\times0.2)}{0.1} = 1.25 \text{ pu}$$

$$(P_e)_{df} = \frac{|E'||V|}{X_{df}}\sin\delta = \frac{1.1\times1}{1.25}\sin\delta = 0.88\sin\delta$$

$$\therefore (P_e)_{max(df)} = 0.88 \text{ pu}$$

Post fault :- (after fault)

$$X_{af} = 0.35 + 0.2 = 0.55 \text{ pu}$$



$$(P_e)_{af} = \frac{|E'||V|}{X_{af}}\sin\delta = \frac{1.1\times1}{0.55}\sin\delta$$

$$= 2.0\sin\delta$$

$$\therefore (P_e)_{max(af)} = 2.0 \text{ pu}$$

Let $\Delta t = 0.05$ sec.

The recursive relationship for step-by-step swing curve calculation are reproduced below.

$$P_{a(n-1)} = P_m - P_{max}\sin\delta_{n-1}$$

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{(\Delta t)^2}{M}P_{a(n-1)} \quad ; \quad \delta_n = \delta_{n-1} + \Delta\delta_n$$

Since there is a discontinuity in P_e and hence in P_a, the average value of P_a must be used for the first interval.

$(P_a)_0^- = 0$ pu and $(P_a)_0^+ = 0.9 - 0.88 \sin 21.64$
$= 0.576$ pu

$\therefore (P_a)_0^{avg.} = \dfrac{0 + 0.576}{2} = 0.288$ pu $= (P_a)_0$

At $t=0$, $(P_a)_0^{avg.} = \dfrac{(P_a)_0^- + (P_a)_0^+}{2}$

$\therefore \Delta\delta_1 = \Delta\delta_0 + \dfrac{(\Delta t)^2}{M}(P_a)_0 = 0 + \dfrac{0.05^2}{2.8\times10^{-4}} \times 0.288$
$= 2.57^\circ$

$\therefore \delta_1 = \delta_0 + \Delta\delta_1 = 21.64 + 2.57 = 24.21^\circ$

$P_a(0.05_-) = (P_a)_{0.05}^- = 0.88 \sin 24.21 = 0.54$ pu

$P_a(0.05_+) = (P_a)_{0.05}^+ = 2.5 \sin 24.21 = 0.08$ pu

$\therefore$ at $t = 0.05$ sec, $P_a(0.05_{avg}) = (P_a)_{0.05}$

$= \dfrac{(P_a)_{0.05}^- + (P_a)_{0.05}^+}{2} = \dfrac{0.54 + 0.08}{2}$

$= 0.31$ pu

$\therefore \Delta\delta_2 = \Delta\delta_1 + \dfrac{(\Delta t)^2}{M}(P_a)_1 = 2.57 + \dfrac{0.05^2}{2.8\times10^{-4}} \times 0.31$
$= 5.33^\circ$

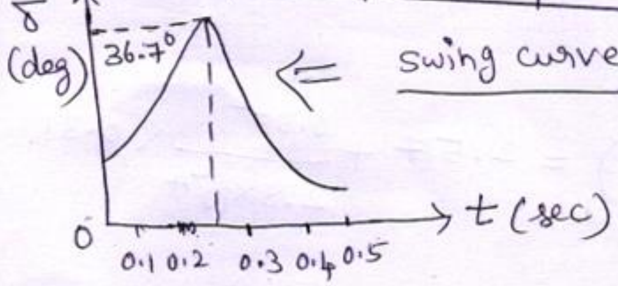
$\therefore \delta_2 = \delta_1 + \Delta\delta_2 = 24.21^\circ + 5.33^\circ = 29.54^\circ$

Computations of swing curve for fault cleared at 2.5 cycles (0.05 sec), $\Delta t = 0.05$ sec

t sec	P_{max} pu	δ deg.	$P_e = P_{max}\sin\delta$ pu	$P_a = 0.9 - P_e$ pu	$\Delta\delta$ deg.
0^-	2.44	21.64	0.9	0	—
0^+	0.88	21.64	0.324	0.576	—
0_{avg}	—	21.64	—	0.288	2.57
0.05^-	0.88	24.21	0.36	0.54	—
0.05^+	2.0	24.21	0.82	0.08	—
0.05_{avg}	2.0	24.21	—	0.31	5.33
0.10	2.0	29.54	0.986	−0.086	4.56
0.15	2.0	34.10	1.12	−0.22	2.60
0.20	2.0	36.70	1.19	−0.29	0.02
0.25	2.0	37.72	1.19	−0.29	−2.56
0.30	2.0	34.16	1.12	−0.22	−4.52
0.35	2.0	29.64	0.989	−0.089	−5.31
0.40	2.0	24.33	0.82	0.08	−4.60
0.45	2.0	19.73	0.675	0.225	−2.6
0.50	2.0	17.13	—	—	—

$\therefore$ Critical clearing angle, $\delta_{cr} = 118.62^\circ$

$\therefore$ Critical clearing time = 0.38 s



⇐ swing curve.

$$\cos\delta_{cr} = \frac{P_m(\delta_{max} - \delta_0) - P_{max\,df}\cos\delta_0 + P_{max\,af}\cos\delta_{max}}{P_{max\,af} - P_{max\,df}}$$

5.2.2 Numerical Solution of Swing Equation by Runge Kutta Method

* The Runge-kutta method is self-starting but generally more accurate than the Euler's method.
* Consider the DE $\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0$
* From the Taylor Series expansion of $y(x)$ around x_0,

$$y(x_0+h) = y(x_0) + hy'(x_0) + \frac{h^2}{2!}y''(x_0) + \cdots \quad \text{—} (1)$$

* The method consists in approximating the higher-order derivatives on the right hand sides. We develop the method as follows:
* First define $y' = \frac{dy}{dx} = f(x,y)$

$$y'' = \frac{d}{dx}\left[f(x,y)\right] = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{dy}{dx} = f_x + ff_y \quad \text{—} (2)$$

$$y''' = f_{xx} + f_{xy}y' + f(f_{yx} + f_{yy}y') + f_y(f_x + f_y f)$$

$$= f_{xx} + 2ff_{xy} + f^2 f_{yy} + f_y(f_x + f_y f) \quad \text{—} (3)$$

assuming $f_{xy} = f_{yx}$

similarly, $y'''' = f_{xxx} + f_{xxy}f + 2(f_x + ff_y)f_{xy} + 2f(f_{xyx} + f_{xxx}f)$

$$+ \cdots \quad \text{—} (4)$$

* Expansion of $y(x_0+h)$ in a Taylor series around $x = x_0$ gives $y(x_0+h) = y(x_0) + hy'(x_0) + \frac{h^2}{2!}y''(x_0) + \frac{h^3}{3!}y'''(x_0) + \cdots$

$$y(x_0+h) = y(x_0) + hf_0 + \frac{h^2}{2!}(f_x + ff_y)\Big|_0 +$$

$$\frac{h^3}{3!}\left[f_{xx} + 2ff_{xy} + f^2 f_{yy} + f_y(f_x + ff_y)\right]\Big|_0 + \cdots \quad \text{—} (5)$$

Second-Order Runge-Kutta Method

Define $K_1 = hf(x_0, y_0)$ —⑥

$K_2 = hf(x_0 + ph, y_0 + qK_1)$ —⑦

Let $y(x_0+h) = y(x_0) + aK_1 + bK_2$ —⑧

where p, q, a and b are constants. These constants are selected in such a manner that right hand sides of eqns ⑤ and ⑧ agree up to the terms in h^2. From eqn ⑦,

$$K_2 = h\left[f(x_0,y_0) + \left(ph\frac{\partial f}{\partial x} + qK_1\frac{\partial f}{\partial y}\right)\right]\Bigg|_0 + \cdots$$

$$= hf_0 + ph^2 f_x\big|_0 + h^2 q f_0 f_y\big|_0 + \text{terms of order greater than } h^2 \quad \text{—⑨}$$

Substituting eqns ⑥ and ⑨ in eqn ⑧ and combining terms

$$y(x_0+h) = y(x_0) + (ah+bh)f_0 + h^2\left(bpf_x\big|_0 + bqf_0f_y\big|_0\right) + \cdots \quad \text{—⑩}$$

Comparing the right hand sides of eqn. ⑤ and ⑩, we get

$$\left.\begin{aligned} a+b&=1\\ bp&=\tfrac{1}{2}\\ bq&=\tfrac{1}{2}\end{aligned}\right\} \Rightarrow \begin{aligned} p&=q\\ a+b&=1\\ bp&=\tfrac{1}{2}\end{aligned} \quad \text{—⑪}$$

Eqn. ⑪ consists of three eqns in four unknowns. One of the unknowns is fixed arbitrarily which then determines the rest uniquely. Selecting an arbitrary value for $a = \frac{1}{2}$, then $b = \frac{1}{2}$, $p=1$ and $q=1$. We get the algorithm as

$$y_1 = y_0 + \frac{1}{2}(K_1 + K_2) \quad \text{—⑫}$$

where $K_1 = hf(x_0, y_0)$; $K_2 = hf(x_0 + h, y_0 + K_1)$ —⑬

Generalizing we have

$$y_{i+1} = y_i + \frac{h}{2}\left[f(x_i, y_i) + f(x_{i+1}, \bar{y}_{i+1})\right] \quad \text{—⑭}$$

where $\bar{y}_{i+1} = y_1 + hf(x_i, y_i)$ —⑮

The application of the Runge-Kutta (R-K) method requires computation of K_1 and K_2 and the error involved is of the order of h^3, since the terms after h^2 were ignored.

Fourth - Order Runge - Kutta (R-K) Method

The general fourth - order Runge - kutta method consists in assuming

$$y = y_0 + ak_1 + bk_2 + ck_3 + dk_4 \quad \text{--- (16)}$$

Where

$$k_1 = hf(x_0, y_0)$$
$$k_2 = hf(x_0 + ph,\ y_0 + qk_1)$$
$$k_3 = hf(x_0 + rh,\ y_0 + sk_2)$$
$$k_4 = hf(x_0 + th,\ y_0 + uk_2)$$

Following the same procedure as for the second - order Runge - kutta method and neglecting terms of order greater than h^4 in the approximation, several fourth - order algorithms are possible depending on the choice of the arbitrary constant.

The widely used fourth - order Runge - kutta approximation is $y_{i+1} = y_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ --- (17)

Where

$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf\left(x_i + \frac{h}{2},\ y_i + \frac{k_1}{2}\right)$$
$$k_3 = hf\left(x_i + \frac{h}{2},\ y_i + \frac{k_2}{2}\right)$$
$$k_4 = hf(x_i + h,\ y_i + k_3)$$

The error involved in this approximation is of the order of h^5.