

MODULE 2

Code Division for Multiple Access (CDMA)

Code Division for Multiple Access (CDMA): Basic CDMA Mechanism, Fundamentals of CDMA codes, Spreading Codes based on PN sequences, Correlation Properties of Random CDMA Spreading Sequences, Advantages of CDMA.

Orthogonal Frequency Division Multiplexing (OFDM): Introduction, Motivation and Multicarrier basics, OFDM basics, OFDM Example, MIMO OFDM, OFDM Peak to Average Power ratio, SC-FDMA. [Text1: 5.1 to 5.5, 5.7, 7.1, 7.2, 7.3, 7.5, 7.7, 7.8]

1. Aditya K Jagannatham, “Principles of Modern Wireless Communication systems, Theory and Practice”, Mc Graw Hill Education (India) Private Limited, 2017, ISBN 978-81- 265-4231-4.

2.1 Introduction to CDMA

CDMA stands for **Code Division for Multiple Access**.

- ❖ 2nd generation IS-95 cellular standard used in North America under name CDMAone.
- ❖ In 3rd generation cellular standards its called as Wideband CDMA (WCDMA), High-Speed Downlink Packet Access(HSDPA), High Speed Uplink Packet Access (HSUPA), CDMA 2000, and 1x Evolution Data Optimized (1xEV-DO).
- ❖ In a wireless network, mobile phones and other wireless-communication devices are required to share the common radio channel over the air. This is shown in Figure 2.1. This is because the radio channel is common for all the users/ devices and the available wireless frequency bands are limited. Thus, it is necessary to devise a mechanism for multiple users to access this common radio channel, which is termed as a **Multiple Access (MA) technology**.

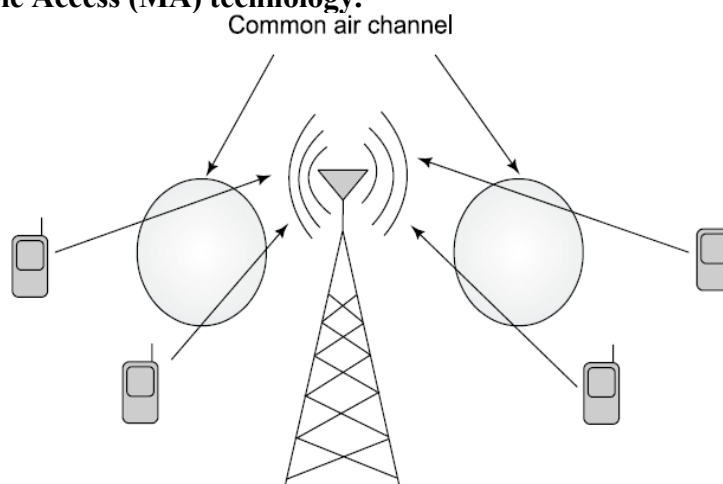


Figure 2.1 Multiple access for wireless cellular networks

- Each generation of cellular standards is characterized by a particular multiple-access technology. For instance, the first generation, i.e., 1G cellular standards were based on Frequency Division Multiple Access (FDMA).
 - ❖ In **FDMA**, different users are allotted different frequency bands. Thus, the users are multiplexed in the frequency domain and they access the radio channel in their respective frequency bands of bandwidth B . This is shown in Figure 2.2.

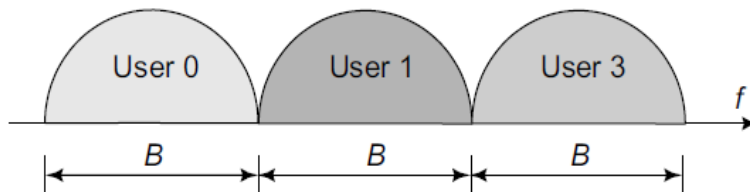


Figure 2.2 Frequency division for multiple access

- ❖ The second generation or 2G cellular standards are based on digital **Time Division for Multiple Access (TDMA)** in which different users are allocated different time slots of duration T for accessing the wireless channel. Thus, the different users are multiplexed in the time domain as shown in Figure 2.3.

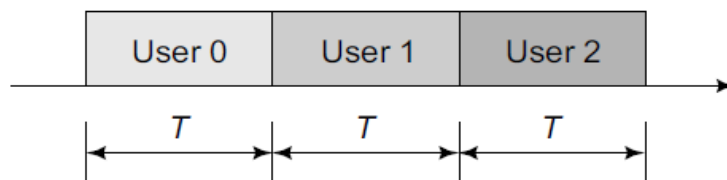


Figure 2.3 Time division for multiple access

2.2 Basic CDMA Mechanism

- ❖ CDMA, as the name suggests, is a multiple-access technology based on code division. In other words, different users are multiplexed using different codes.
- ❖ Consider a two-user scenario, i.e., two users accessing the radio channel simultaneously. Let a_0 denote the symbol of the user 0, while a_1 denotes the transmit symbol corresponding to the user 1.
- ❖ Let the code c_0 of the user 0 be given as $c_0 = [1, 1, 1, 1]$. The above code c_0 is of length $N = 4$ chips. Each element of the code is termed as a chip.
- ❖ The transmitted signal x_0 of the user 0 is then given by multiplying the code c_0 with the symbol a_0 as

$$\begin{aligned} x_0 &= a_0 \times [1, 1, 1, 1] \\ &= [a_0, a_0, a_0, a_0] \quad \text{----- 2.1} \end{aligned}$$

- ❖ The structure of the above transmit signal x_0 can be interpreted as follows. The symbol a_0 , of the user 0, is multiplied by the code c_0 to yield 4 chips $x_0(i)$, $0 \leq i \leq N - 1$. Similarly, let the code c_1 , given as $c_1 = [1, -1, -1, 1]$, correspond to the code of the user 1. Hence, the sequence of chips corresponding to the user 1 transmission is given as

$$\begin{aligned} x_1 &= a_1 \times [1, -1, -1, 1] \\ &= [a_1, -a_1, -a_1, a_1] \quad \text{----- 2.2} \end{aligned}$$

The signals x_0, x_1 corresponding to users 1, 2 respectively are now summed to yield the net signal x as

$$x = x_1 + x_2 = [(a_0 + a_1), (a_0 - a_1), (a_0 - a_1), (a_0 + a_1)] \text{ ---- 2.3}$$

This sum, or composite, signal is then transmitted on the downlink from which each of the Users 0, 1 detect their own signal. This is done as follows.

User 1 correlates the received signal x with his code c_0 , i.e., basically multiplies each chip of the received signal x with the corresponding chip of the code $c_0 = [1, 1, 1, 1]$ and sums across the chips as follows.

$$\begin{array}{rcccc}
 & a_0 + a_1 & a_0 - a_1 & a_0 - a_1 & a_0 + a_1 \\
 \times & 1 & 1 & 1 & 1 \\
 \hline
 & (a_0 + a_1) + (a_0 - a_1) + (a_0 - a_1) + (a_0 + a_1) = 4a_0 & & & \\
 \hline
 & & & & \text{----- 2.4}
 \end{array}$$

Thus, the result of the above correlation is $4a_0$, which is proportional to the transmitted Symbol a_0 . Similarly, at the user 2, the received signal x is correlated with the chip sequence $c_1 = [1, -1, -1, 1]$ of the user 1 as

$$\begin{array}{rcccc}
 & a_0 + a_1 & a_0 - a_1 & a_0 - a_1 & a_0 + a_1 \\
 \times & 1 & -1 & -1 & 1 \\
 \hline
 & (a_0 + a_1) - (a_0 - a_1) - (a_0 - a_1) + (a_0 + a_1) = 4a_1 & & & \\
 \hline
 & & & & \text{----- 2.5}
 \end{array}$$

to yield $4a_1$, which is proportional to the transmitted symbol a_1 of the user 1.

Thus, unlike in GSM or FDMA, in which the signals of different users are transmitted in different time slots or frequency bands, in CDMA, all the signals of the different users are contained in the single signal x over all time and frequency. However, in CDMA, the symbols of the different users are combined using different codes.

For instance, in the above example, the symbols a_0, a_1 of users 0, 1 are multiplied with codes c_0, c_1 prior to transmission. Thus, the users of the different signals are multiplexed over the common wireless channel employing different codes. Hence, this is termed Code Division for Multiple Access, i.e., multiple access based on different codes.

The key operations in CDMA can be summarized as follows.

1. Multiplying or modulation the symbols of the different users with the corresponding assigned unique code, similar to the procedure illustrated in equations (2.1), (2.2).
2. Combining or adding the code-modulated signals of all the users to form the composite signal as shown in Eq. (2.3), followed by subsequent transmission of the signal.
3. Finally, correlation of the composite received signal x at each user with the corresponding code of the user to recover the transmitted symbol. This is described in Eqs (2.4), (2.5).

2.3 Fundamentals of CDMA Codes

Computing the correlation r_{01} of the user codes c_0, c_1 yields

$$\begin{aligned}
 r_{01} &= \sum_{k=0}^3 c_0(k) c_1(k) \\
 &= 1 \times 1 + 1 \times (-1) + 1 \times (-1) + 1 \times 1 \\
 &= 1 + (-1) + (-1) + 1 = 0
 \end{aligned}$$

Thus, since the correlation between the codes c_0, c_1 is zero, the codes are, in fact, orthogonal. This is what helps us recover the symbols of the different users from the composite signal. This is a key property of the codes employed in CDMA wireless systems, and a fundamental principle on which the theory of CDMA is based.

Further, consider a fundamental property of the CDMA system arising because of the employment of these codes. Let the symbol rate for the symbols a_0 of the user 0 be 1 kbps. Hence, the time period T per symbol is

$$T = \frac{1}{1 \text{ kbps}} = 1 \text{ ms}$$

Hence, the corresponding bandwidth required for transmission is

$$B = \frac{1}{T} = 1 \text{ kHz}$$

However, now consider the transmission of the symbol a_0 multiplied with the corresponding code c_0 , i.e., $a_0 \times [1, 1, 1, 1] = [a_0, a_0, a_0, a_0]$. Thus, for each symbol a_0 , one has to transmit 4 chips. Thus, to keep the symbol rate constant at 1 kbps, the time of each chip T_c has to be set as $T_c = \frac{1}{4} T = 0.25 \text{ ms}$. Thus, the bandwidth required for this system is

$$B_{\text{CDMA}} = \frac{1}{T_c} = \frac{1}{0.25 \text{ ms}} = 4 \text{ kHz}$$

Thus, modulating with the code c_0 of length $N = 4$, results in an increase of the required bandwidth by a factor of N , i.e., from 1 kHz to 4 kHz. This is shown schematically in Figure 2.4. Thus, it basically results in a **spreading** of the original signal bandwidth and, hence, is termed a **spreading code**. Also, since the resulting signal occupies a large bandwidth, CDMA systems are also termed **spread spectrum or wideband systems**.

How many such orthogonals exist for a given spreading code length N ? The answer is there are N such orthogonal codes. For instance, consider the case $N = 4$.

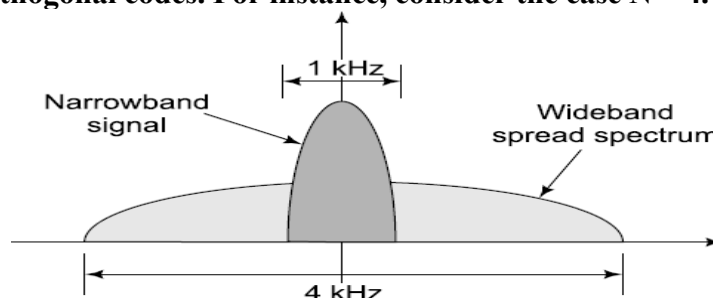


Figure 2.4 Spread spectrum communication

The different orthogonal spreading codes are

$$\begin{aligned}
 c_0 &= 1 \ 1 \ 1 \ 1 \\
 c_1 &= 1 \ -1 \ -1 \ 1 \\
 c_2 &= 1 \ -1 \ 1 \ -1 \\
 c_3 &= 1 \ 1 \ -1 \ -1
 \end{aligned}$$

The codes c_0, c_1, c_2, c_3 are orthogonal to each other. For example, consider c_1, c_2 . The correlation r_{12} between codes c_1, c_2 is

$$\begin{aligned}
 r_{12} &= \sum_{k=0}^3 c_0(k) c_1(k) \\
 &= 1 \times 1 + (-1) \times (-1) + (-1) \times 1 + 1 \times (-1) \\
 &= 1 + 1 + (-1) + (-1) = 0
 \end{aligned}$$

This implies that given a spreading sequence length N , there exist N orthogonal codes and hence, N users can be multiplexed together. This is important, since the bandwidth increases by a factor of N due to transmission employing the codes.

No inefficiency is introduced in the system because of the increase in bandwidth, because this increase in bandwidth by a factor of N is compensated by the parallel transmission of the signals corresponding to the N users over the same bandwidth. Thus, the spectral efficiency of the system is not compromised. This is schematically illustrated in Figure 2.5.

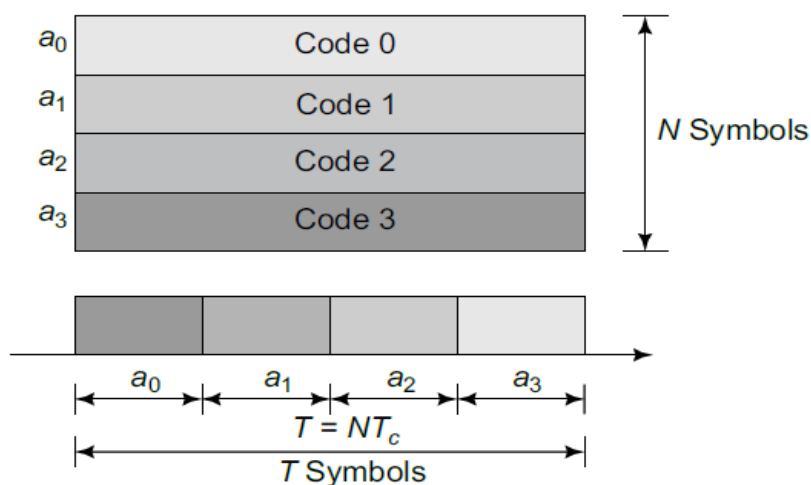


Figure 2.5 Parallel transmission of N symbols over N codes in CDMA over time interval $T = N T_c$ (above) and comparison with transmission of N symbols in time $T = N T_c$ in a conventional single carrier or time division system.

2.4 Spreading Codes based on Pseudo-Noise (PN) Sequences

Consider the code $c_2 = [1, -1, 1, -1]$. Observe that the code looks like a random sequence of $+1$, -1 , or a pseudo-noise (PN) sequence. This is so termed since it only resembles a noise sequence, but is not actually a noise sequence.

One method to generate such long spreading codes based on PN sequences for a significantly large N is through the employment of a **Linear Feedback Shift Register (LFSR)**.

Consider the shift register architecture shown in Figure 2.6, where the element D represents delays. Thus, the digital circuit therein contains $D = 4$ delay elements or shift registers. The input on the left is denoted by X_i , and the outputs of the different delays are X_{i-1} , X_{i-2} , X_{i-3} , X_{i-4} . Let X_{i-4} also denote the final output of the system. Also observe that the xor $X_{i-4} \oplus X_{i-3}$ is fed back as X_i which is the input to the first shift register. Thus, the governing equation of the circuit is

$$X_i = X_{i-3} \oplus X_{i-4}$$

which is a linear equation. Thus, since it implements a linear relation, with feedback and uses delay elements or shift registers, such a circuit is also termed a **Linear Feedback Shift Register (LFSR) architecture**. Since the next input, i.e., X_i depends on X_{i-1} , X_{i-2} , X_{i-3} , X_{i-4} , this can also be thought of as the current state of the system.

Consider initializing the system in the state $X_{i-1} = 1$, $X_{i-2} = 1$, $X_{i-3} = 1$, $X_{i-4} = 1$. Thus, we have the corresponding X_i given as

$$X_i = X_{i-3} \oplus X_{i-4} = 1 \oplus 1 = 0$$

This X_i becomes X_{i-1} at the next instant and similarly, X_{i-2} , X_{i-3} are shifted to the right. As X_{i-3} , X_{i-4} respectively. Continuing in this fashion, the entire sequence of state of the above LFSR is summarized. It can be seen that the LFSR goes through the sequence of 15 States 1111, 0111, 0011, 0001, 1000, 0100, 0010, 1001, 1100, 0110, 1011, 0101, 1010, 1101, 1110, before reentering the state 1111. Subsequently, the entire sequence of states repeats again. Observe that this goes through $2^D - 1 = 2^4 - 1 = 15$ states. The maximum number of Possible states for $D = 4$ is $2^D = 16$. However, the LFSR can be seen to go through all the possible states except one, which is the 0000 or the all-zero state.

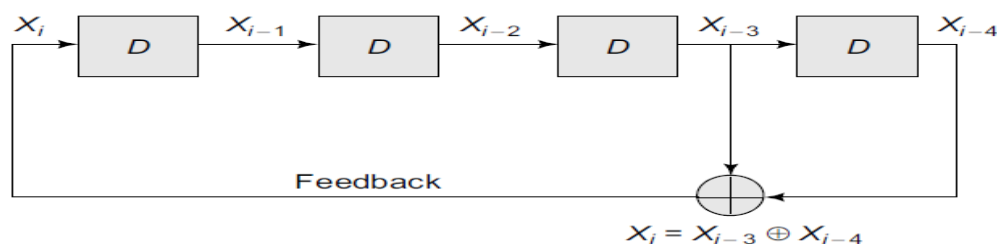


Figure 2.6 Linear feedback shift register

$$X_i = X_{i-3} \oplus X_{i-4} = 0 \oplus 0 = 0$$

If the LFSR is initialized in the 0000 state, it continues in the 0000 state, since the corresponding X_i is leading to the next state of 0000. Thus, the LFSR never gets out of the all zero states! Therefore, it is desired that the LFSR never enter the all-zero state.

Such an LFSR circuit which goes through the maximum possible $2^D - 1$ states, without entering the all-zero state is termed a **maximum-length shift register circuit or maximum length LFSR**. The generated PN sequence is termed a **maximum-length PN sequence**.

Thus, the maximum-length PN sequence is of length $2^D - 1$. For instance, for the above LFSR, the maximum-length PN sequence is the sequence of outputs X_{i-4} given as

$$\text{PN Sequence} = 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 1\ 0\ 1\ 0$$

We can map the bits 1, 0 to the BPSK symbols -1 , $+1$ to get the modulated PN sequence,

$$\text{PN sequence} = -1\ -1\ -1\ -1\ +1\ +1\ +1\ -1\ +1\ +1\ -1\ -1\ +1\ -1\ +1\ \text{-----} \quad 2.6$$

2.4.1 Properties of PN Sequences

Property 1-Balance Property:

Consider the BPSK-modulated PN sequence shown in Eq. (2.6). The PN sequence is of maximal length $2^D - 1 = 15$ corresponding to $D = 4$. Counting the number of -1 and $+1$ chips in the sequence, it can be seen that the number of -1 s is one more than the number of $+1$ s. This is termed the **balance property of the PN sequence**.

This fundamentally arises from the noise like properties of PN sequences.

If we are generating random noise of $+1$, -1 chips, with $P(X_i = +1) = P(X_i = -1) = 1/2$, we expect to find on an average that half the chips are $+1$ and the rest are -1 .

In the above case, however, as the total number of chips is an odd number, i.e., 15, it is not possible to have an exactly even number of $+1$, -1 s. Hence, the number of $+1$, -1 s is close to half the total number, i.e., eight -1 s and seven $+1$ s. Thus, the balance property basically supports the notion of a noise like PN chip sequence.

Property 2-Run-Length Property:

A run is defined as a string of continuous values. There are a total of 8 runs in this PN sequences. For instance, the first run $-1, -1, -1, -1$ is a run of length 4. Thus, there is one run of length 4. Similarly, there is one run $+1, +1, +1$ of length 3, and two runs of length 2, viz., $-1, -1, +1, +1$. Finally, it can also be seen that there are 4 runs of length 1, viz., two runs of $+1$ and two runs of -1 . Thus, there are a total of $2^{(D-1)} = 8$ runs.

Out of the 8 runs, it can be seen that 1, i.e., $1/8$ of the runs are of length 3, $1/4$ of the runs are of length 2 and $1/2$ of the runs are of length 1. This is termed the **run-length property** of PN sequences and can be generalized as follows.

Consider a maximal length PN sequence of length $2^D - 1$. Out of the total number of runs in the sequence, $1/2$ of the runs are of length 1, $1/4$ of the runs are of length 2, $1/8$ of the runs are of length 3, and so on.

This is again in tune with the noise like properties of PN sequences.

For instance, consider a random IID sequence of $+1, -1$. In such a sequence, one would expect the average number of $+1$ or -1 to be half the total chips. Further, the number of strings $+1, +1$ or $-1, -1$, i.e., runs of length two would be expected to comprises $1/4$ of the total runs. This arises since the probability of seeing two consecutive $+1, +1$ symbols is

$$P(X_i = +1, X_{i+1} = +1) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Property 3-Correlation Property:

The correlation property is one of the most important properties of PN sequences.

Consider again the BPSK chip sequence shown in Eq. (2.6) and denote it by $c_0(n)$. Let us now look at the correlation properties of this sequence. Consider the correlation $r_{00}(0)$, i.e., the correlation of the sequences c_0 with itself (the meaning of the (0) will become clear soon). This correlation is given as

$$\begin{aligned} r_{00}(0) &= \frac{1}{N} \sum_{i=0}^{N-1} c_0(n) c_0(n) \\ &= \frac{1}{N} \sum_{i=0}^{N-1} 1 \\ &= \frac{1}{N} \times N = 1 \end{aligned}$$

Now, consider a circularly shifted version of the PN sequence, shifted by $n_0 = 2$. Let it be denoted by $c_0(n - 2)$. This circularly shifted sequence by 2 chips can be readily seen to be given as

$$\begin{aligned} \text{PN Sequence} &= -1 + 1, -1 - 1 - 1 - 1 + 1 + 1 \\ &\quad + 1 - 1 + 1 + 1 - 1 - 1 + 1 \quad \text{----- 2.7} \end{aligned}$$

Let us denote the correlation between $c_0(n)$ and $c_0(n - 2)$ by $r_{00}(2)$, where the (2) can now be seen to represent a circular shift of 2. The correlation can be seen to be given as

$$\begin{aligned} r_{00}(2) &= \frac{1}{N} \sum_{i=0}^{N-1} c_0(n) c_0(n - 2) \\ &= \frac{1}{15} \{ (-1) \times (-1) + (-1) \times (1) + (-1) \times (-1) + (-1) \times (-1) + (1) \times (-1) + \\ &\quad (1) \times (-1) + (1) \times (1) + (-1) \times (1) + (1) \times (1) + (1) \times (-1) + (-1) \times (1) + \\ &\quad (-1) \times (1) + (1) \times (-1) + (-1) \times (-1) + (1) \times (1) \} \\ &= \frac{1}{15} (1 - 1 + 1 + 1 - 1 - 1 + 1 - 1 + 1 - 1 - 1 - 1 - 1 - 1 + 1 + 1) \\ &= \frac{1}{15} \times (-1) = -\frac{1}{15} \\ &= -\frac{1}{N} \end{aligned}$$

In fact, one can compute the correlation for other such nonzero delays, and can demonstrate that the correlation is always $-1/N$. This autocorrelation property of the PN sequence, i.e., of the sequence with a delayed version of itself, is shown pictorially in Figure 2.7.

Thus, it can be seen that while the correlation of the sequence with itself corresponding to a lag of 0 is 1, for any other nonzero shift, it assumes a very low value of $-1/N$, which tends to the limit 0 as the spreading length $N \rightarrow \infty$. This autocorrelation property of the PN sequences can be summarized as follows.

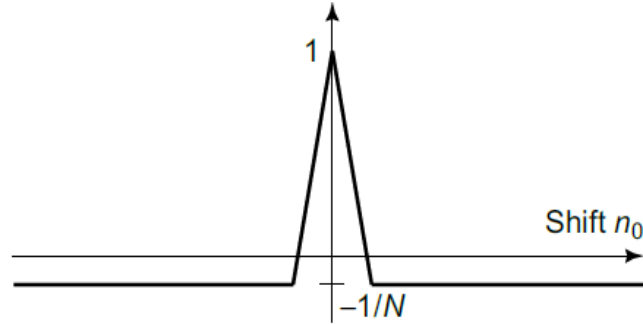


Figure 2.7 Autocorrelation of PN sequence

$$r_{00}(n_0) = \frac{1}{N} \sum_{i=0}^{N-1} = \begin{cases} 1 & \text{if } n_0 = 0 \\ -\frac{1}{N} & \text{otherwise} \end{cases}$$

2.5 Correlation Properties of Random CDMA Spreading Sequences

CDMA spreading sequences can be chosen as PN sequences, which have noise like properties.

One can choose a chip sequence $c_k(i)$, $0 \leq i \leq N-1$ for the user k such that

$P(c_k(i) = +1) = P(c_k(i) = -1) = 1/2$. Thus, we have,

$$E\{c_k(i)\} = \frac{1}{2} \times (+1) + \frac{1}{2} \times (-1) = 0.$$

choose such sequences as containing Independent Identically Distributed (IID) chips, i.e., satisfying the property

$$E\{c_k(i) c_k(j)\} = E\{c_k(i)\} E\{c_k(j)\} = 0 \times 0 = 0$$

The above property implies that each chip $c_k(i)$ is uncorrelated with chip $c_k(j)$. Further, one can choose independent sequences for different users, that is, to say

$$E\{c_k(i) c_l(j)\} = E\{c_k(i)\} E\{c_l(j)\}$$

Let us examine the correlation properties of such random spreading sequences. As before, Let $r_{00}(k)$ denote the autocorrelation of the chip sequence of the user $k=0$, corresponding to a lag $k \neq 0$. This can be expressed as

$$r_{00}(k) = \frac{1}{N} \sum_{i=0}^{N-1} c_0(i) c_0(i-k)$$

The average or expected value of $r_{00}(k)$ can be seen to be given as

$$\begin{aligned} E\{r_{00}(k)\} &= E\left\{\frac{1}{N} \sum_{i=0}^{N-1} c_0(i) c_0(i-k)\right\} \\ &= \frac{1}{N} \sum_{i=0}^{N-1} E\{c_0(i) c_0(i-k)\} \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{N} \sum_{i=0}^{N-1} E \{c_0(i)\} E \{c_0(i-k)\} \\
 &= \frac{1}{N} \sum_{i=0}^{N-1} 0 = 0
 \end{aligned}$$

Thus, the average value or the expected value of the correlation $E \{r_{00}(k)\}$ is zero for lags $k \neq 0$. This is expected from the random properties of the spreading sequence. To compute the variance of the autocorrelation $r_{00}(k)$, consider $r_{00}^2(k)$ given as

$$\begin{aligned}
 r_{00}^2(k) &= \frac{1}{N^2} \left(\sum_{i=0}^{N-1} c_0(i) c_0(i-k) \right) \left(\sum_{j=0}^{N-1} c_0(j) c_0(j-k) \right) \\
 &= \frac{1}{N^2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} c_0(i) c_0(i-k) c_0(j) c_0(j-k)
 \end{aligned}$$

Now, let us consider the quantity $c_0(i) c_0(i-k) c_0(j) c_0(j-k)$.

If $i \neq j$, the expected value of this quantity can be simplified as

$$\begin{aligned}
 E \{c_0(i) c_0(i-k) c_0(j) c_0(j-k)\} &= E \{c_0(i) c_0(i-k)\} E \{c_0(j) c_0(j-k)\} \\
 &= E \{c_0(i)\} E \{c_0(i-k)\} E \{c_0(j)\} E \{c_0(j-k)\} \\
 &= 0
 \end{aligned}$$

If $i = j$, the same quantity can be simplified as

$$\begin{aligned}
 E \{c_0(i) c_0(i-k) c_0(j) c_0(j-k)\} &= E \{c_0(i) c_0(i-k) c_0(i) c_0(i-k)\} \\
 &= E \{(c_0(i))^2\} E \{(c_0(i-k))^2\} \\
 &= 1 \times 1 = 1
 \end{aligned}$$

Thus, the variance of $r_{00}(k)$, i.e., $E \{r_{00}^2(k)\}$ can be simplified as

$$\begin{aligned}
 E \{r_{00}^2(k)\} &= \frac{1}{N^2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} E \{c_0(i) c_0(i-k) c_0(j) c_0(j-k)\} \\
 &= \frac{1}{N^2} \sum_{i=0}^{N-1} E \{c_0^2(i) c_0^2(i-k)\} \\
 &= \frac{1}{N^2} \sum_{i=1}^{N-1} 1 = \frac{1}{N^2} \times N \\
 &= \frac{1}{N}
 \end{aligned}$$

Thus, the variance or basically the power of $r_{00}(k)$, the autocorrelation of the random CDMA spreading sequence is $E \{r_{00}^2(k)\} = 1/N$. Also, once again, the autocorrelation corresponding to a lag of $k = 0$ can be readily seen to be given as

$$\begin{aligned}
 E \{r_{00}(0)\} &= E \left\{ \frac{1}{N} \sum_{i=0}^{N-1} c_0(i) c_0(i) \right\} \\
 &= \frac{1}{N} \sum_{i=0}^{N-1} E \{c_0^2(i)\} \\
 &= \frac{1}{N} \sum_{i=0}^{N-1} 1 \\
 &= \frac{1}{N} \times N = 1
 \end{aligned}$$

Summarizing the autocorrelation properties of the random spreading sequence as follows.

For $k = 0$, $r_{00}(k) = 1$. For $k \neq 0$, $r_{00}(k)$ is a random variable with $E \{r_{00}(k)\} = 0$ and variance $E \{r_{00}^2(k)\} = 1/N$.

Let us now examine the cross-correlation properties of the random CDMA spreading sequences, i.e., the correlation between the spreading sequences $c_0(i)$, $0 \leq i \leq N-1$ and $c_1(j)$, $0 \leq j \leq N-1$. We denote by $r_{01}(k)$ the cross-correlation between spreading sequences c_0, c_1 corresponding to a lag k as

$$r_{01}(k) = \frac{1}{N} \sum_{i=0}^{N-1} c_0(i) c_1(i-k)$$

The expected value for any lag k can be computed as

$$\begin{aligned}
 E \{r_{01}(k)\} &= \frac{1}{N} E \left\{ \sum_{i=0}^{N-1} c_0(i) c_1(i-k) \right\} \\
 &= \frac{1}{N} \sum_{i=0}^{N-1} E \{c_0(i) c_1(i-k)\} \\
 &= \frac{1}{N} \sum_{i=0}^{N-1} E \{c_0(i)\} E \{c_1(i-k)\} \\
 &= \frac{1}{N} \sum_{i=0}^{N-1} 0 \times 0 = 0
 \end{aligned}$$

The variance $E \{r_{01}^2(k)\}$ for any delay k is given as

$$\begin{aligned}
 E \{r_{01}^2(k)\} &= \frac{1}{N^2} E \left\{ \left(\sum_{i=0}^{N-1} c_0(i) c_1(i-k) \right) \left(\sum_{j=0}^{N-1} c_0(j) c_1(j-k) \right) \right\} \\
 &= \frac{1}{N^2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} E \{c_0(i) c_1(i-k) c_0(j) c_1(j-k)\}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{N^2} \sum_{i=0}^{N-1} E \{c_0^2(i)\} E \{c_1^2(i-k)\} \\
 &= \frac{1}{N^2} \sum_{i=0}^{N-1} 1 \\
 &= \frac{1}{N^2} \times N = \frac{1}{N}
 \end{aligned}$$

$E \{c_0(i) c_1(i-k) c_0(j) c_1(j-k)\}$ is nonzero only if $i = j$ in the above derivation.

The cross-correlation $r_{01}(k)$ between two random CDMA spreading sequences c_0 c_1 is a random variable with $E \{r_{01}(k)\} = 0$ and variance $E \{r_{01}^2(k)\} = 1/N$.

These random spreading codes do not satisfy the definition of exact orthogonality. However, they are **approximately orthogonal**, in that the average value of the Correlation is zero and the power in the correlation is proportional to $1/N$ which tends to 0 as $N \rightarrow \infty$.

2.6 Advantages of CDMA

2.6.1 Advantage 1: Jammer Margin

An important advantage of CDMA over conventional cellular systems is jammer suppression.

A jammer is basically a malicious user in a communication network who transmits with a very high power to cause interference, thus leading to disruption of communication links. This is shown schematically in Figure 2.8. Jammers are of significant concern, especially in the context of highly secure communication systems such as those used for military and defense purposes. The effect of jammer suppression in a CDMA system can be understood as follows.

Consider a communication system in which the signal $x(n)$ of the power P is received in the presence of additive white Gaussian noise $w(n)$ of power σ_w^2 . The baseband system model for this communication system can be expressed as

$$y(n) = x(n) + w(n)$$

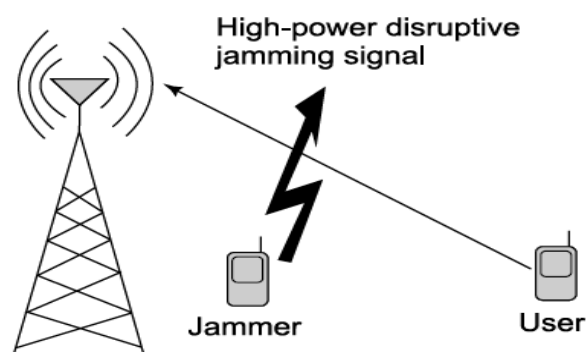


Figure 2.8 Disruption by jammer in wireless communication

Hence, the SNR at the receiver is $SNR = \frac{P}{\sigma_w^2}$. However, in the presence of a jamming signal $x_j(n)$ of power P_j , the received signal $y(n)$ is

$$y(n) = x(n) + x_j(n) + w(n)$$

Thus, the jammer interferes with the signal reception and the signal-to-interference-noise

power ratio (SINR) can be calculated as $SINR = \frac{P}{P_j + \sigma_w^2}$. Thus, the jammer has a significant disruptive impact on the communication signal. Consider now a CDMA system in which the transmitted signal $x(n)$ is a spread-spectrum signal.

The SINR for a CDMA scenario is given as

$$SINR = \frac{P}{\frac{P_j}{N} + \frac{\sigma_w^2}{N}} \quad \text{----- 2.11}$$

The jamming power P_j is suppressed by a factor of N . As the spreading factor N increases, the jammer suppression increases, minimizing the impact of the jammer on the communication system. This is termed **jammer suppression** in CDMA systems.

The gain of N is also termed the jammer margin. Thus, the jammer margin is equal to N , i.e., the spreading length of the CDMA codes.

2.6.2 Advantage 2: Graceful Degradation

Consider the expression for the SINR at the user 0 derived in Eq. (2.10). At this point, assume that another user, i.e., a user with index $K + 1$ joins the network. Let P_{k+1} denote the corresponding transmission power of this $(K + 1)^{th}$ user and a_{k+1} , $c_{k+1}(n)$ denote his transmitted symbol and spreading code respectively. The SINR of the user 0 now changes to

$$\begin{aligned} SINR &= \frac{P_0}{\frac{P_1}{N} + \frac{P_2}{N} + \dots + \frac{P_K}{N} + \frac{P_{K+1}}{N} + \frac{\sigma_n^2}{N}} \\ &= N \times \frac{P_0}{\sum_{k=0}^{K+1} P_k + \sigma_n^2} \end{aligned}$$

Thus, the addition of a new user $K + 1$ with power P_{k+1} only causes an incremental interference of P_{k+1}/N at the user 0. Further, in general, at any user $i \neq (K + 1)$, the additional interference due to the introduction of this new user is P_{k+1}/N . Therefore, the addition of the new user $K + 1$ does not adversely affect any single user. Rather, the additional interference caused by this new user is shared amongst all the existing users in the system leading to interference distribution. This sharing of the interference by all the existing users leads to a graceful degradation of the SINR at each user. This is termed the graceful degradation property of CDMA systems.

2.6.3 Advantage 3: Universal Frequency Reuse

Consider a cellular network organized into cells as shown in Figure 2.9. Consider two adjacent cells C_0 , C_1 shown in the figure. Assume now that the same frequency f is allotted for transmission to users in both C_0 , C_1 .

Let $x_0(n)$ with power P_0 denote the signal of the user on the frequency f in the cell 0, while $x_1(n)$ with power P_1 denotes the signal of the user in the cell 1. Since both the signals are being transmitted on the identical frequency f , they will interfere with each other.

The received signal $y_0(n)$ at the user 0 is given as

$$y_0(n) = \underbrace{x_0(n)}_{\text{Signal}} + \underbrace{x_1(n)}_{\text{Interferer from } C_1} + \underbrace{w(n)}_{\text{Noise}}$$

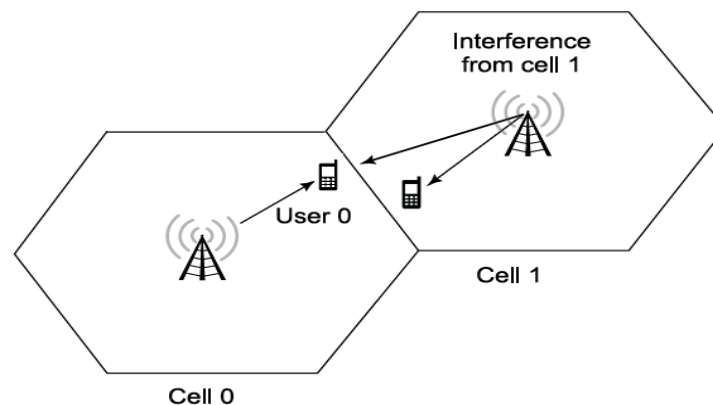


Figure 2.9 Intercell interference for the user 0 on the cell edge

Hence, the SINR at the user 0 is given as
$$\text{SINR} = \frac{P_0}{P_1 + \sigma_w^2}$$
. This is similar to the jamming interference case described in Eq. (2.11). Thus, if the same frequency f is allocated in adjacent cells, it will cause heavy interference and degradation of user SINR results from adjacent cell interference.

Thus, in a typical 1G or 2G cellular network such as GSM, only a fraction of the total available frequencies are allocated in each cell, carefully avoiding the allocation of the same frequency in adjacent cells.

For instance, as can be seen from the hexagonal-lattice based cellular structure in Figure 2.10, each hexagonal cell has 6 neighbours. Hence, to avoid adjacent cell interference, any of the frequencies allocated to C_0 cannot be allocated to its neighbours C_1 , C_2 , ..., C_6 . This holds true for all the cells in the network. Hence, only $1/7$ of the total available frequency bands can be allocated to each cell. This factor $1/7$ is termed the frequency-reuse factor of the cellular network. Thus, since only a fraction of the frequencies are used in the cell, the total spectral efficiency is proportional to the frequency-reuse factor, resulting in a rate which is $1/7$ compared to that of using all the available bandwidth, since the capacity is linearly related to bandwidth.

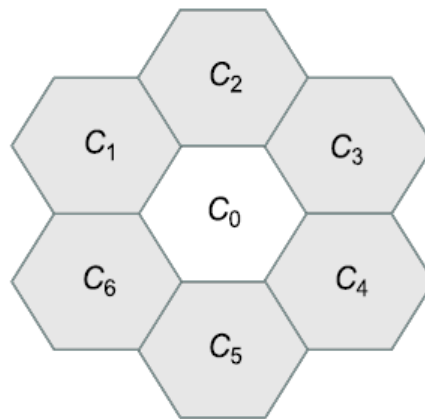


Figure 2.10 Grid or lattice of hexagonal cells

However, now consider the same scenario in the context of a CDMA network. Again, assume that the same frequency f is allotted for transmission to users in both C_0 , C_1 .

However, let $x_0(n)$ with power P_0 is now transmitted on code $c_0(n)$, while $x_1(n)$ with power P_1 is transmitted in the cell 1 on the random code $c_1(n)$. Hence, now similar to the jammer scenario in a CDMA system, the interference caused by the user on the identical frequency f in the adjacent cell is now reduced by a factor of N to P_1/N . Therefore, the SINR is now given as,

$$\text{SINR} = \frac{P_0}{\frac{P_1}{N} + \frac{\sigma_w^2}{N}}$$

Thus, the interference of each user is limited to a fraction $1/N$ of the interferer power. Hence, basically the jammer margin in defence applications, can be used for adjacent cell interference power suppression in modern cellular networks! This is a great advantage of CDMA, which implies that the same frequency bands can be used in all cells across the network.

Another way of stating this is that the fraction of bands used in each cell is 1, i.e., all the bands. Therefore, this is termed **universal frequency reuse** or equivalently, as a cellular network with frequency reuse factor 1. Thus, compared to GSM, which uses only $1/7$ of the frequency bands in each cell, CDMA can use all the available frequency bands in each cell. This right away leads to an increase of the spectral efficiency and resulting capacity by a factor of 7. Thus, CDMA-based cellular networks have a much higher capacity compared to conventional 1G and 2G cellular networks. This has led to a widespread adoption and embrace of CDMA-based technologies for mobile communication.

2.6.4 Multipath Diversity and Rake Receiver

Another important advantage of CDMA is its ability to achieve diversity gain via multipath scatter components. This is termed multipath diversity and is achieved through coherent combining of the multipath-signal components employing a rake receiver.

Consider a multipath frequency-selective channel with several delayed signal paths. A multipath frequency-selective channel can be modelled as a Finite Impulse Response (FIR) channel filter with channel taps $h(0), h(1), \dots, h(L-1)$. The received symbol $y(n)$ can be expressed as

$$\begin{aligned}
 y(n) &= h(0)x(n) + h(1)x(n-1) + \dots + h(L-1)x(n-L+1) + w(n) \\
 &= \sum_{l=0}^{L-1} h(l)x(n-l) + w(n)
 \end{aligned}$$

The above equation represents a frequency-selective or intersymbol interference-limited channel since the current output $y(n)$ depends not only on the current symbol $x(n)$, but also $L-1$ previous input symbols $x(n), x(n-1), \dots, x(n-L+1)$.

Consider now a CDMA signal in which $x(n) = a_0 c_0(n)$. Consider a single user and this can be easily extended to multi-user scenarios. Substituting this in the expression for $y(n)$ above, the received signal across a frequency selective channel in a CDMA system is given as

$$y(n) = \sum_{l=0}^{L-1} h(l) a_0 c_0(n-l) + w(n)$$

Let us correlate with $c_0(n)$ to recover the symbol corresponding to the user 0. This operation can be expressed as

$$\begin{aligned}
 d(0) &= \frac{1}{N} \sum_{n=0}^{N-1} y(n) c_0^*(n) \\
 &= \frac{1}{N} \sum_{n=0}^{N-1} \left(\sum_{l=0}^{L-1} h(l) a_0 c_0(n-l) + w(n) \right) c_0^*(n) \\
 &= \frac{1}{N} \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} h(l) a_0 c_0(n-l) c_0^*(n) + \underbrace{\frac{1}{N} \sum_{n=0}^{N-1} w(n) c_0^*(n)}_{\tilde{w}_0}
 \end{aligned}$$

the noise \tilde{w}_0 is Gaussian of power $E \{ \|\tilde{w}_0\|^2 \} = \frac{\sigma_w^2}{N}$.

The first term in the above expression can be split into two components: one corresponding to $l=0$ and the other corresponding to $l \neq 0$. Simplifying, we have

$$\begin{aligned}
 d(0) &= \frac{1}{N} \sum_{n=0}^{N-1} h(0) a_0 c_0(n) c_0^*(n) + \frac{1}{N} \sum_{l=1}^{L-1} \sum_{n=0}^{N-1} h(l) a_0 c_0(n-l) c_0^*(n) + \tilde{w}_0 \\
 &= h(0) a_0 \left(\frac{1}{N} \sum_{n=0}^{N-1} c_0(n) c_0^*(n) \right) + a_0 \sum_{l=1}^{L-1} h(l) \left(\frac{1}{N} \sum_{n=0}^{N-1} c_0(n-l) c_0^*(n) \right) + \tilde{w}_0 \\
 &= h(0) a_0 r_{00}(0) + a_0 \sum_{l=1}^{L-1} h(l) r_{00}(l) + \tilde{w}_0
 \end{aligned}$$

Consider the quantity $r_{00}(l)$. For $l \neq 0$, $r_{00}(l)$ is a random variable with mean 0 and power $1/N$. Observe that the power tends to 0 as $N \rightarrow \infty$. Hence, for large values of N , $r_{00}(l) \approx 0$. Employing this approximation in the above expression, and noting that $r_{00}(0) = 1$, we have

$$d(0) = h(0) a_0 + \tilde{w}_0$$

Even though there is intersymbol interference in the above channel, we are able to extract the signal corresponding to $h(0)$, i.e., delay 0 by correlation with the spreading code $c_0(n)$.

One can repeat this process by individually correlating with delayed versions of the spreading sequence $c_0(n-v)$, $1 \leq v \leq L-1$ to extract the multipath components corresponding to $h(1)$, $h(2)$, \dots , $h(L-1)$. Thus, correlating with $c_0(n-v)$, the resulting statistic $d(v)$ can be simplified as,

$$\begin{aligned}
 d(v) &= \frac{1}{N} \sum_{n=0}^{N-1} y(n) c_0^*(n-v) \\
 &= \frac{1}{N} \sum_{n=0}^{N-1} \left(\sum_{l=0}^{L-1} h(l) a_0 c_0(n-l) + w(n) \right) c_0^*(n-v) \\
 &= \frac{1}{N} \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} h(l) a_0 c_0(n-l) c_0^*(n-v) + \underbrace{\frac{1}{N} \sum_{n=0}^{N-1} w(n) c_0^*(n-v)}_{\tilde{w}_v}
 \end{aligned}$$

The noise \tilde{w}_v is Gaussian with variance $E\{|\tilde{w}_v|^2\} = \frac{\tilde{w}^2}{N}$. Further, splitting the first term into two components corresponding to $l=v$ and $l \neq v$, one can derive the expression for $d(v)$, $1 \leq v \leq L-1$ as

$$\begin{aligned}
 d(v) &= \frac{1}{N} \sum_{n=0}^{N-1} h(v) a_0 c_0(n-v) c_0^*(n-v) \\
 &\quad + \frac{1}{N} \sum_{l=0, l \neq v}^{L-1} \sum_{n=0}^{N-1} h(l) a_0 c_0(n-l) c_0^*(n-v) + \tilde{w}_v \\
 &= h(0) a_0 \left(\frac{1}{N} \sum_{n=0}^{N-1} c_0(n-v) c_0^*(n-v) \right) \\
 &\quad + a_0 \sum_{l=0, l \neq v}^{L-1} h(l) \left(\frac{1}{N} \sum_{n=0}^{N-1} c_0(n-l) c_0^*(n-v) \right) + \tilde{w}_v \\
 &= h(v) a_0 r_{00}(0) + a_0 \sum_{l=1}^{L-1} h(l) r_{00}(l-v) + \tilde{w}_v \\
 &\approx h(v) a_0 + \tilde{w}_v
 \end{aligned}$$

where we have again employed the approximation $r_{00}(l-v) \approx 0$, $l \neq v$ in the above

simplification. Now, one can process that extracted components $d(0), d(1), \dots, d(L-1)$ as follows. Employing vector notation, the components can be expressed as

$$\underbrace{\begin{bmatrix} d(0) \\ d(1) \\ \vdots \\ d(L-1) \end{bmatrix}}_{\mathbf{d}} = \underbrace{\begin{bmatrix} h(0) \\ h(1) \\ \vdots \\ h(L-1) \end{bmatrix}}_{\mathbf{h}} a_0 + \underbrace{\begin{bmatrix} \tilde{w}_0 \\ \tilde{w}_1 \\ \vdots \\ \tilde{w}_{L-1} \end{bmatrix}}_{\tilde{\mathbf{w}}}$$

The above system is now similar to the multiple receive antenna system, i.e., receive diversity system with channel coefficients $h(0), h(1), \dots, h(L-1)$. The model can be, therefore, be succinctly expressed in vector notation as

$$\mathbf{d} = \mathbf{h}a_0 + \tilde{\mathbf{w}}$$

The optimal combiner is the Maximum Ratio Combiner (MRC) given by $\frac{\mathbf{h}}{\|\mathbf{h}\|}$.

The power of each noise component \tilde{w}_v is $E\{|\tilde{w}_v|^2\} = 1$.

Denoting the symbol power $E\{|a_0|^2\}$ by P and combining the observation vector \mathbf{d} with the MRC yields the SNR Orthogonal Frequency-Division Multiplexing

$$\begin{aligned} \text{SNR} &= \frac{\|\mathbf{h}\|^2 P}{\frac{\sigma_w^2}{N}} \\ &= N \times \underbrace{\left(|h(0)|^2 + |h(1)|^2 + \dots + |h(L-1)|^2 \right)}_{\|\mathbf{h}\|^2} \frac{P}{\sigma_w^2} \end{aligned}$$

The above expression is similar to the SNR for the multiple receive antenna system, in that there is a factor $\|\mathbf{h}\|^2$ in the numerator, where \mathbf{h} is the vector of frequency-selective channel coefficients $\mathbf{h} = [h(0), h(1), \dots, h(L-1)]^T$. This is in addition to the spreading gain factor N . Thus, this is equivalent to the performance of a system with diversity order L . This is the essence of multipath diversity and CDMA is able to exploit this multipath diversity by correlating with the spreading code $c_0(n-v)$ corresponding to different lags $0 \leq v \leq L-1$, and combining the individual components employing MRC.

This receiver structure in CDMA is termed the rake receiver and the diversity gain thus achieved is termed multipath diversity since it is extracted from the multipath components.

This multipath diversity arising out of rake combining is a unique feature of CDMA and significantly improves its performance over wireless channels because of the higher diversity order of decoding.

2.7 Introduction

Orthogonal Frequency-Division Multiplexing (OFDM) forms the basis for 4G, i.e., Fourth Generation wireless communication systems.

OFDM is used in 4G wireless cellular standards such as Long-Term Evolution (LTE) and WiMAX (Worldwide Interoperability for Microwave Access).

OFDM is a key broadband wireless technology which supports data rates in excess of 100 Mbps. Similarly, the wireless local area (LAN) standards such as 802.11 a/g/n are based on OFDM.

2.8 Motivation and Multicarrier Basics

Consider a bandwidth $B = 2W$ available for communication, where W is the one-sided bandwidth, or, in other words, the maximum frequency. For a single carrier communication system, the symbol time T is given as

$$T = \frac{1}{B}$$

basically implying that symbols can be transmitted at intervals of $1/B$ seconds each. Therefore, the symbol rate is given as

$$\text{Rate} = \frac{1}{1/B} = B \quad \text{----- 2.12}$$

Such a system is termed a single-carrier communication system. In such a system, a single carrier is employed for the entire baseband bandwidth of B . The, symbols are transmitted as symbol $X(0)$ from $0 \leq t < T$, symbol $X(1)$ from $T \leq t < 2T$, and so on, i.e., roughly one symbol transmitted every $T = 1/B$ seconds.

Consider now dividing the total bandwidth B into N sub-bands of bandwidth B/N each as shown in Figure 2.11. Each subcarrier can now be represented by a subcarrier. Therefore, the subcarriers are placed at $\dots, -B/N, 0, B/N, \dots$, as shown in the figure. For instance, consider the bandwidth $B = 256$ kHz with $N = 64$ subcarriers. The bandwidth per sub-band is equal to $256/64 = 4$ kHz, which is also the frequency spacing between the subcarriers.

We now implement a multi-carrier transmission system as follows.

Consider the i^{th} subcarrier at the frequency $f_i = i(B/N)$, with $-(N/2 - 1) \leq i \leq N/2$.

Let X_i denote the data transmitted on the i^{th} subcarrier. Then, the signal $s_i(t)$ corresponding to the i^{th} subcarrier is given as

$$s_i(t) = X_i e^{j2\pi f_i t} = X_i e^{j2\pi i \frac{B}{N} t}$$

where f_i is the i^{th} subcarrier centre frequency, as described above, and $e^{j2\pi f_i t}$ is the i^{th} subcarrier. The above equation shows the data modulation process over the i^{th} subcarrier. The N different data symbols X_i are modulated over the N different subcarriers with centre frequencies f_i . Hence, there are a total of N data streams.

Next we illustrate the scheme for multicarrier transmission.

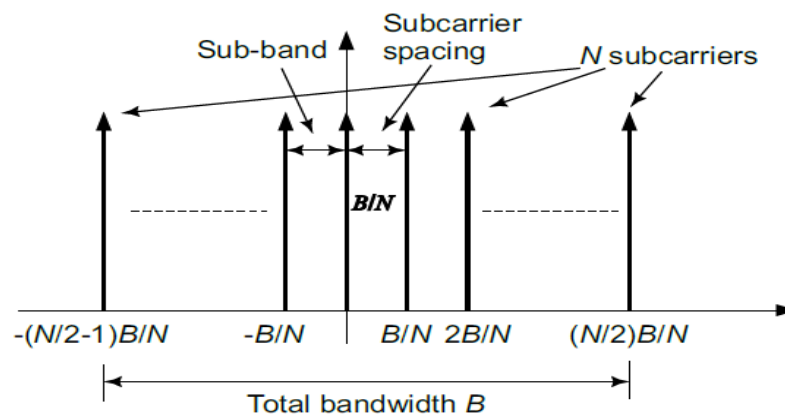


Figure 2.11 Multi-carrier concept

2.8.2 Multicarrier Transmission

Consider now the different modulated signals $s_i(t)$ corresponding to the N different subcarriers. These signals are then superposed and transmitted to form the **composite** signal $s(t)$ given as

$$\begin{aligned} s(t) &= \sum_i s_i(t) \\ &= \sum_i X_i e^{j2\pi f_i t} \\ &= \sum_i X_i e^{j2\pi i \frac{B}{N} t} \end{aligned} \quad \text{----- 2.12}$$

This composite signal $s(t)$ is then transmitted over the wireless channels. Thus, N different data streams are transmitted over N subcarriers in **parallel** in this multicarrier system. At the receivers, the individual data streams have then to be isolated from the composite signal $s(t)$ above. This is accomplished as follows. Consider the signal $y(t)$ received as

$$y(t) = s(t) = \sum_i X_i e^{j2\pi f_i t}$$

To illustrate the demodulation procedure at the receiver, we have assumed noise to be absent above. We will consider the general case of a noisy received signal later.

From the expression for the composite signal $s(t)$ in Eq. (2.12), it can be readily seen that the expression on the right-hand side is indeed the Fourier series representation $s(t)$, corresponding to the fundamental frequency $f_0 = (B/N)$ and the various X_i representing the Fourier coefficients. Indeed, all the frequencies $i(B/N)$ are multiples of the fundamental frequency $f_0 = 1/T_0 = B/N$. Therefore, to extract X_1 , which is the Fourier coefficient corresponding to the frequency $f_1 = 1f_0$, one needs to follow the procedure similar to compute the Fourier series as

$$f_0 \int_0^{T_0} y(t) \left(e^{j2\pi f_1 t} \right)^* dt = \frac{B}{N} \int_0^{\frac{N}{B}} \left(\sum_i X_i e^{j2\pi i \frac{B}{N} t} \right) e^{-j2\pi \frac{B}{N} t} dt$$

$$\begin{aligned}
 &= \frac{B}{N} \sum_i \int_0^{\frac{N}{B}} X_i e^{j2\pi(i-l)f_0 t} dt \\
 &= \underbrace{\frac{B}{N} \int_0^{\frac{N}{B}} X_l dt}_{i=l} + \frac{B}{N} \sum_{i \neq l} \int_0^{\frac{N}{B}} X_i e^{j2\pi(i-l)f_0 t} dt \\
 &= X_l + \frac{B}{N} \sum_{i \neq l} X_i \underbrace{\int_0^{\frac{N}{B}} e^{j2\pi(i-l)f_0 t} dt}_{=0} \\
 &= X_l
 \end{aligned}$$

where we have used the fact that $\int_0^{T_0} e^{j2\pi(i-l)f_0 t} dt = 0$ for $i \neq l$, since this is basically integrating a sinusoid of frequency $(i-l)f_0$, which is a multiple of the fundamental frequency f_0 over the period T_0 . Therefore, since there are an integer number of cycles of the sinusoid of frequency $(i-l)f_0$, this integral is 0.

In fact, this basically implies that the different sinusoids $e^{j2\pi f_0 t}$ and $e^{j2\pi l f_0 t}$ are orthogonal. It is this key property of orthogonality which helps extract the different streams X_i modulated over the different subcarriers. This property of orthogonality can be summarized as

$$\int_0^{N/B} e^{j2\pi(i-l)\frac{B}{N}t} dt = \begin{cases} 0 & i \neq l \\ \frac{N}{B} & i = l \end{cases}$$

Therefore, all the subcarriers other than the l^{th} subcarrier are orthogonal to the l^{th} subcarrier. Further, observe that multiplying with $(e^{j2\pi l f_0 t})^*$ and integrating is basically coherent demodulation, i.e., demodulation with the carrier matched to the subcarrier frequency $f_l = l(B/N)$. Thus, X_l , the data modulated on the different subcarriers, can be conveniently recovered by coherently demodulating with each of the subcarriers corresponding to $l = -(N/2 - 1), \dots, N/2$

The above scheme of transmission on multiple orthogonal subcarriers and the associated data recovery at the receiver is termed **MultiCarrier Modulation (MCM)**.

The window of time associated with detection of this multicarrier signal is $N/B = 1/f_0 = T_0$, which is basically the time period of integration. Hence, MCM basically transmits N symbols using N subcarriers in a time period of N/B . The symbol rate is, therefore, $\frac{N}{N/B} = B$.

Thus, the overall symbol rate in single carrier vs multicarrier systems is unchanged.

The transmitter and receiver block schematics for this MCM system are shown in figures 2.12 and 2.13 respectively.

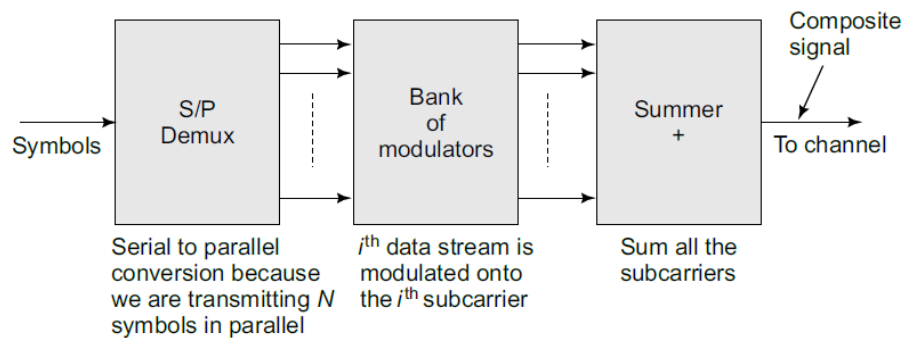


Figure 2.12 Multicarrier modulation transmitter

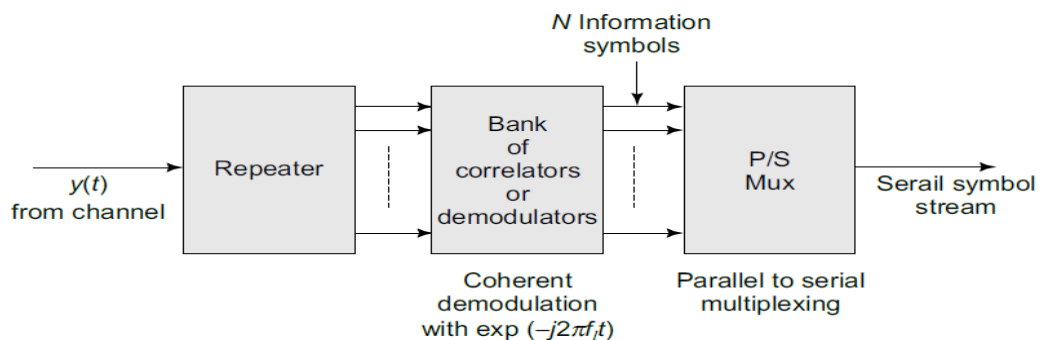


Figure 2.13 Multi-carrier modulation receiver

The symbol rate for both single carrier system and MCM systems is exactly identical, i.e., B . The single-carrier system transmits each symbol in time $1/B$, while the MCM system transmits N symbols in parallel in time N/B .

What then is the advantage of an MCM system over the single-carrier system?

To understand this, consider an example with a transmission bandwidth of $B = 1.024$ MHz, i.e., 1024 kHz. Bandwidth B is much greater than the coherence bandwidth B_c which is typically around 250 kHz, i.e., $B_c \approx 250$ kHz. Therefore, since the transmission bandwidth $B \gg B_c$, the single-carrier system experiences frequency-selective fading and inter-symbol interference.

Consider an OFDM system with employs $N = 256$ subcarriers in the same bandwidth. The bandwidth per subcarrier is $B_s = 1024/256 = 4$ kHz. The subcarrier bandwidth of 4 kHz is significantly lower than the coherence bandwidth of 250 kHz. Thus, since $B/N \ll B_c$, each subcarrier experiences **flat fading**. Hence, there is no inter-symbol interference in the data transmitted on any of the subcarriers.

The key benefit of this MCM system is that through parallel transmission using multiple narrowband subcarriers, it eliminates the Inter-Symbol interference (ISI), thus avoiding distortion of the received symbols.

Consider the MCM transmit signal $s(t)$. Observe that it is band-limited to the bandwidth B (total bandwidth). Therefore, the Nyquist sampling rate is B and the associated sampling time is $T_s = 1/B$. Consider now the composite MCM signal given in Eq. (2.9). The u^{th} sample at time instant $uT_s = u/B$ is given as

$$s(uT_s) = x(u) = \sum_i X_i e^{j2\pi i \frac{B}{N} \frac{u}{B}}$$

$$x(u) = \underbrace{\sum_i X_i e^{j2\pi i \frac{u}{N}}}_{\text{DFT}}$$

The sample $x(u)$ is basically the Inverse Discrete Fourier Transform (IDFT) coefficient of the information symbols $X(0), X(1), \dots, X(N-1)$ at the u^{th} time point. Thus, the Inverse Fast Fourier Transform (IFFT) can be conveniently employed to generate the sample MCM signal. Thus, it drastically reduces the complexity of implementing an OFDM system since it eliminates the need for the bank of modulators corresponding to the different subcarrier frequencies. This technique, where the MCM signal is generated by employing the IFFT operation is termed **Orthogonal Frequency Division Multiplexing**, or OFDM. At the receiver, to recover the information symbols, one can correspondingly employ an FFT operation.

Schematic figures of the OFDM transmitter and receiver with the IFFT and FFT blocks are given in figures 2.14 and 2.15 respectively.

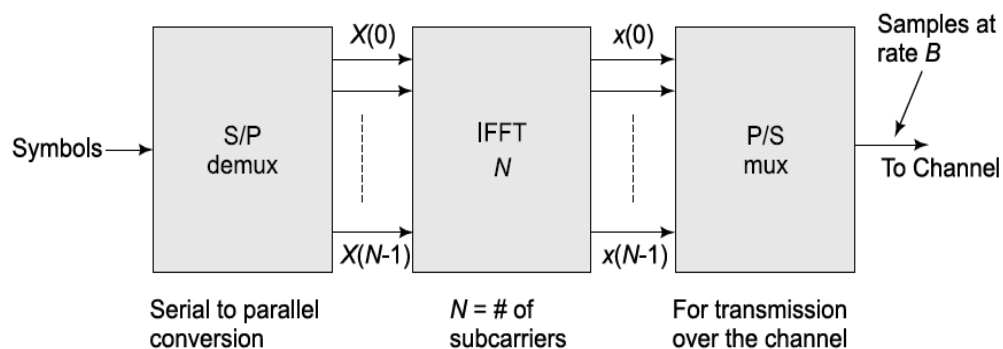


Figure 2.14 OFDM transmitter schematic with IFFT

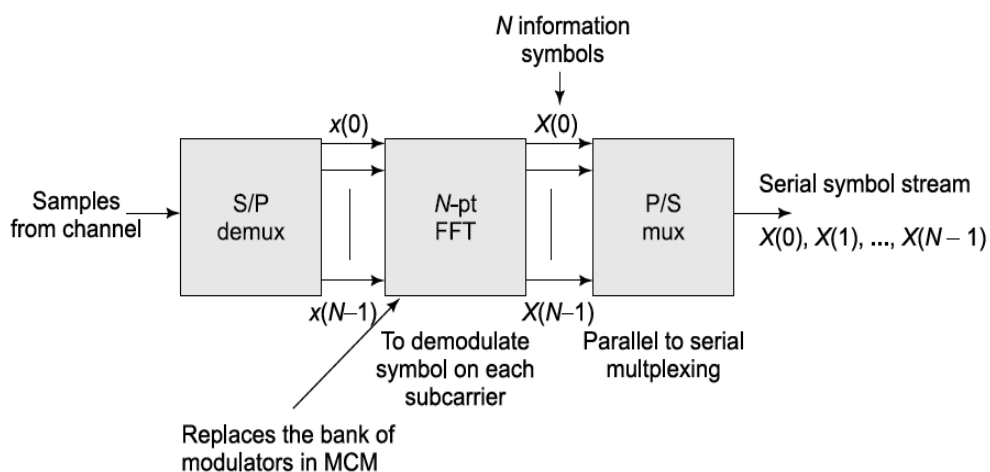


Figure 2.15 OFDM receiver schematic with FFT

2.8.2 Cyclic Prefix in OFDM

Consider a frequency-selective channel modelled with channel taps $h(0), h(1), \dots, h(L-1)$. Thus, the received symbol y at a given time instant n can be expressed as

$$y(n) = h(0)x(n) + \underbrace{h(1)x(n-1) + \dots + h(L-1)x(n-L+1)}_{\text{ISI component}},$$

from which it can be seen that the received symbol $y(n)$ at the time instant n experiences inter symbol interference from the previous $L-1$ transmitted symbols.

Consider now two OFDM symbols as follows. Let $x(0), x(1), \dots, x(N-1)$ denote the IFFT samples of the modulated symbols $X(0), X(1), \dots, X(N-1)$, while $\tilde{x}(0), \tilde{x}(1), \dots, \tilde{x}(N-1)$ denote the IFFT samples of the previous modulated symbol block $\tilde{X}(0), \tilde{X}(1), \dots, \tilde{X}(N-1)$. Thus, the samples corresponding to these two blocks of OFDM symbols are transmitted sequentially as

$$\underbrace{\tilde{x}(0), \tilde{x}(1), \dots, \tilde{x}(N-1)}_{\text{Previous block}}, \underbrace{x(0), x(1), \dots, x(N-1)}_{\text{Current block}}$$

Now, consider the received symbol $y(0)$ corresponding to the transmission of $x(0)$. This can be expressed as

$$y(0) = h(0)x(0) + \underbrace{h(1)\tilde{x}(N-1) + \dots + h(L-1)\tilde{x}(N-L+1)}_{\text{ISI from previous OFDM symbol}}.$$

From the above equation that the received symbol $y(0)$ experiences inter-symbol interference from $\tilde{x}(N-1), \tilde{x}(N-2), \dots, \tilde{x}(N-L+1)$. Thus, there is inter-OFDM symbol interference in this new OFDM system. The initial samples of the current OFDM symbol block are being subject to interference from the $N-1$ samples of the previous OFDM block. This is shown in Figure 2.16. Similarly, the received symbol $y(1)$ is given as

$$y(1) = h(0)x(1) + \underbrace{h(1)x(0) + h(2)\tilde{x}(N-1) + \dots + h(L-1)\tilde{x}(N-L+2)}_{\text{ISI from previous OFDM symbol}},$$

which can again be seen to experience inter-OFDM symbol interference from the previous OFDM block symbols $\tilde{x}(N-1), \tilde{x}(N-2), \dots, \tilde{x}(N-L+2)$.

Let us now consider a modified transmission scheme as follows. To each transmitted OFDM sample stream, we pad the last L_c symbols to make the transmitted stream as follows.

$$\underbrace{\tilde{x}(0), \tilde{x}(1), \dots, \tilde{x}(N-1)}_{\text{Previous block}}, \underbrace{x(N-L_c), x(N-L_c+1), \dots, x(N-1)}_{\text{Cyclic prefix}}, \underbrace{x(0), x(1), \dots, x(N-1)}_{\text{Current block}}.$$

Initial samples, of subject to inter OFDM symbol interference

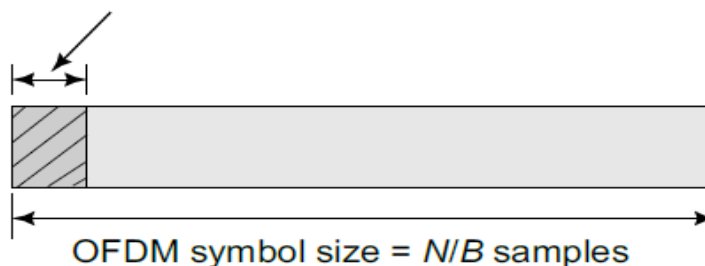


Figure 2.16 Inter-OFDM symbol interference

We are prefixing the transmitted sample block $x(0), x(1), \dots, x(N-1)$ of the current block with the L_c samples $x(N-L_c), x(N-L_c+1), \dots, x(N-1)$.

Further, this prefix is cyclic in nature, since the same samples from the end of the block are being cycled towards the beginning. Therefore, this is known as the **cyclic prefix** and is an important aspect of OFDM systems. Consider now the received symbol corresponding to $x(0)$. This is given as

$$y(0) = h(0)x(0) + \underbrace{h(1)x(N-1) + \dots + h(L-1)x(N-L+1)}_{\text{ISI from same OFDM symbol}}$$

The inter-symbol interference can be seen to now be from $x(N-1), x(N-2), \dots, x(N-L+1)$, if $L_c \geq L-1$. Thus, with the cyclic prefix of appropriate length, i.e., $L_c \geq L-1$, inter-OFDM symbol interference can be avoided and inter-symbol interference is restricted to samples from the same OFDM symbol. Therefore, the samples $y(0), y(1), \dots, y(N-1)$ are given as

$$y(0) = h(0)x(0) + h(1)x(N-1) + \dots + h(L-1)x(N-L+1)$$

$$y(1) = h(0)x(1) + h(1)x(0) + \dots + h(L-1)x(N-L+2)$$

\vdots

$$y(N-1) = h(0)x(N-1) + h(1)x(N-2) + \dots + h(L-1)x(N-L)$$

The output $y(n)$ is a circular convolution between the channel filter $h(n)$ and the input $x(n)$. This can, therefore, be expressed as

$$[y(0), y(1), \dots, y(N-1)] = [h(0), h(1), \dots, h(L-1), 0, \dots, 0] *_{\text{N}} [x(0), x(1), \dots, x(N-1)]$$

where $*_{\text{N}}$ denotes circular convolution of modulo N . Therefore, the output y can be written as

$$y = h *_{\text{N}} x$$

Therefore, taking the DFT of $y(n)$ at the output, we have

$$Y(k) = H(k) X(k), 0 \leq k \leq N-1 \text{ ----- 2.13}$$

where $Y(k)$, $0 \leq k \leq N-1$, denotes the N -point DFT of $y(n)$. Similarly, $X(k)$ denotes the N -point DFT of $x(n)$. Further, observe that the samples $x(n)$ have been generated as the IDFT of $X(n)$. Therefore, the DFT of the samples $x(n)$ yields back the original transmitted symbols $X(n)$. The coefficients $H(k)$ denotes the DFT of the zero-padded channel filter,

$$h(0), h(1), \dots, h(L-1), \underbrace{0, \dots, 0}_{(N-L)} \text{ ----- 2.14}$$

Eq. (2.13) represents the **flat-fading channel** across the k^{th} subcarrier in the OFDM system. The quantity $Y(k)$ represents the output symbol, while $H(k)$ denotes the equivalent flat-fading channel coefficient. This holds true for each subcarrier k , i.e., for $0 \leq k \leq N-1$. Thus, the frequency-selective fading channel is converted into a group of narrowband flat-fading channels, one channel across each subcarrier. Observe that if a single carrier system was used, and the symbols $X(0), X(1), \dots, X(N-1)$ were transmitted directly then the received symbol $y(n)$ would be given as

$$y(n) = h(0)X(n) + h(1)X(n-1) + \dots + h(L-1)X(n-L+1)$$

Each symbol $X(n)$ would experience inter-symbol interference of $L-1$ past symbols. Therefore, using this novel scheme of OFDM, we have been able to totally eliminate the inter-symbol interference arising out of the frequency-selective nature of the channel.

The set of parallel flat-fading channels can be summarized by the expressions

$$Y(0) = H(0)X(0)$$

$$Y(1) = H(1)X(1)$$

$$\vdots$$

$$Y(N-1) = H(N-1)X(N-1)$$

This conversion of the frequency-selective wideband channel into N narrowband flat-fading channels is shown schematically in Figure 2.17. Also, the modified transmitter and receiver schematics with the blocks corresponding to the cyclic prefix are given in Figures 2.18 and 2.19 respectively. Now, considering the noise at the receiver, the received symbol $Y(k)$ can be expressed as

$$Y(k) = H(k)X(k) + N(k) \text{ ----- 2.14}$$

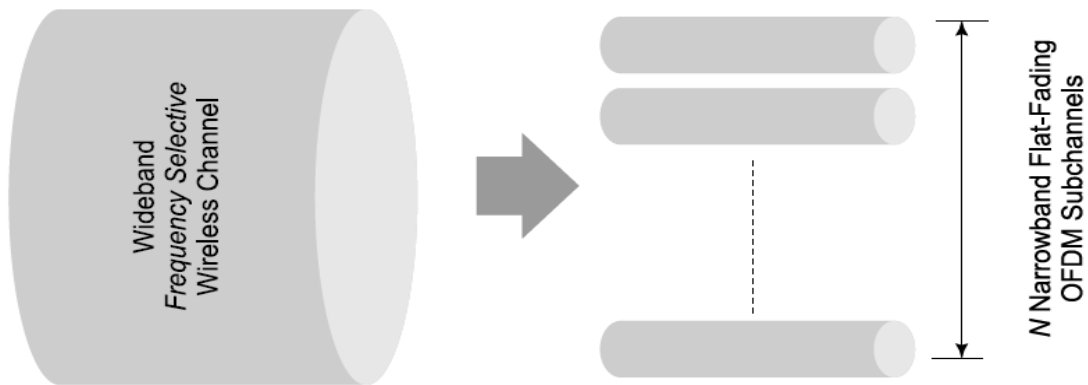


Figure 2.17 OFDM parallel subchannels

where $N(k)$ denotes the noise across the k^{th} subcarrier. A simple detection scheme for $X(k)$ is to use the zero-forcing detector for the subcarrier as

$$\hat{X}(k) = \frac{1}{H(k)} Y(k) = X(k) + \underbrace{\frac{N(k)}{H(k)}}_{\tilde{N}(k)}$$

For a simplistic BPSK or QPSK-modulated transmission, the coherent or matched filter detector can be simply obtained by multiplying with $H^*(k)$, i.e., the complex conjugate of $H(k)$ as

$$H^*(k) Y(k) = |H(k)|^2 X(k) + \underbrace{H^*(k) N(k)}_{N'(k)}$$

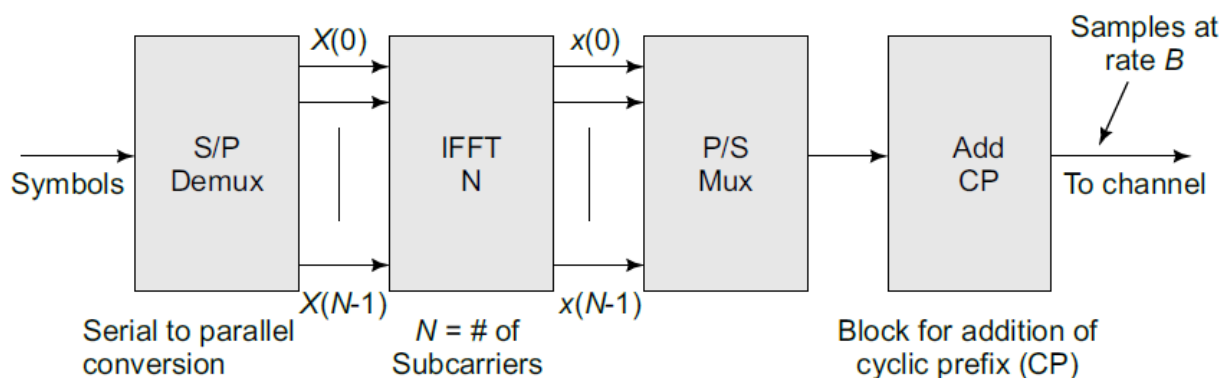


Figure 2.18 OFDM transmitter schematic with CP

Also, one can employ the MMSE detector as

$$\hat{X}_{\text{MMSE}}(k) = \frac{H^*(k)}{|H(k)|^2 + \sigma_n^2} Y(k)$$

The above equation gives the MMSE receiver across the k^{th} subcarrier in this OFDM system.

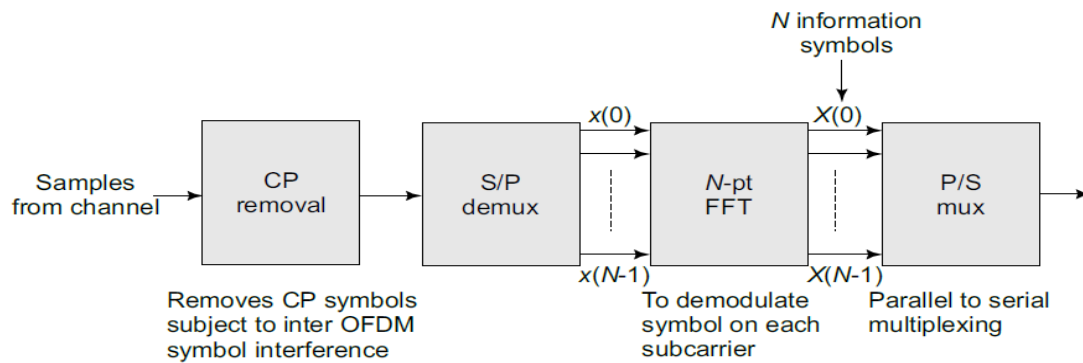


Figure 2.19 OFDM receiver schematic with CP

2.8.3 Impact of Cyclic Prefix on Data Rate

Consider the transmitted samples $x(n)$ with the cyclic prefix, as given below.

$$\underbrace{x(N - L_c), x(N - L_c + 1), \dots, x(N - 1)}_{\text{Cyclic prefix}} \underbrace{x(0), x(1), \dots, x(N - 1)}_{\text{Current block}}.$$

The minimum required length of the cyclic prefix is $L - 1$.

$L - 1$ is the delay spread of the wireless channel.

The length of the cyclic prefix should be greater than the delay spread of the channel.

However, since the samples in the tail, i.e., $x(N - L_c), x(N - L_c + 1), \dots, x(N - 1)$ are simply repeated in the beginning, they do not constitute any additional information. Hence, the effect of the addition of a long CP is lost in the throughput of the system.

The loss in efficiency can be calculated as

$$\begin{aligned} \text{Loss in efficiency} &= \frac{\text{Cyclic prefix}}{\text{Total OFDM symbol length}} \\ &= \frac{L - 1}{N + L - 1} \\ &= \frac{L - 1}{N + L - 1} \end{aligned}$$

However, as the block length N becomes very large, we have

$$\lim_{N \rightarrow \infty} \frac{L - 1}{N + L - 1} \rightarrow 0$$

Thus, the loss in throughput approaches 0 as the number of subcarriers N increases, for a fixed length of the delay spread L . As the number of subcarriers N increases, the symbol time N/B increases as shown in Figure 2.20.

Increasing N results in increasing OFDM symbol time, thus restricting the ISI to a small fraction of the OFDM symbol block, i.e., the fraction L/N is progressively smaller. However, as the block

length N increases, the decoding delay at the receiver also increases as one has to wait for arrival of the entire block of N samples before it can be demodulated. Hence, there is a trade-off for increasing N vs decoding delay.

Now, we present another intuitive framework to understand the effect of various parameters. The duration of the cyclic prefix has to be greater than the delay spread.

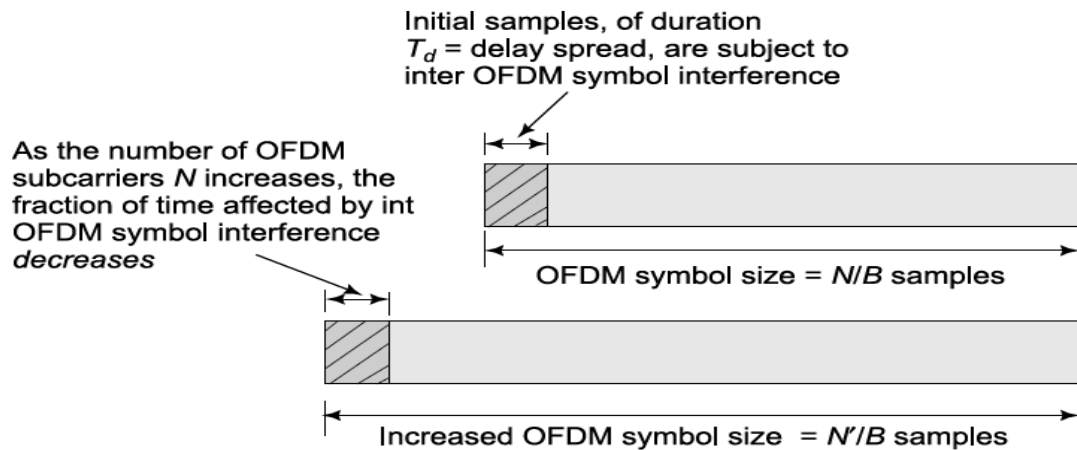


Figure 2.20 Inter OFDM symbol interference with increasing OFDM symbol time

$$L_c \times T_s \geq T_d$$

where T_s denotes the sample time and T_d denotes the delay spread. Also, the sample time $T_s = 1/B$, where B is the total bandwidth of the system and $T_d = 1/B_c$, where B_c is the coherence bandwidth of the system. The above condition implies

$$\begin{aligned}
 L_c &\geq \frac{T_d}{T_s}, \\
 &= \frac{B}{B_c}
 \end{aligned}$$

Combining this with the earlier condition that $N \gg L_c$ for efficiency in terms of the effective data rate, we have

$$N \gg L_c \geq \frac{B}{B_c}.$$

This can also be recast as $B_c \gg B/N$. Interestingly, this is the same condition for frequency flat fading across each subcarrier since this implies that the subcarrier bandwidth B/N is required to be much less than the coherence bandwidth B_c . Thus, an appropriately designed OFDM system converts a frequency-selective fading channel into a set of parallel narrowband flat-fading channels across the subcarriers.

2.9 OFDM Example

Consider a practical WiMAX example to illustrate the impact of the various parameters in the design of a complete OFDM system. WiMAX, which stands for Worldwide Interoperability for

Microwave Access, is a prominent 4G wireless standard. The total number of subcarriers $N = 256$, with a bandwidth of 15.625kHz per subcarrier. Therefore,

$$\begin{aligned} B/N &= 15.625 \text{ kHz} \\ \Rightarrow B &= N \times 15.625 = 256 \times 15.625 \\ &= 4 \text{ MHz} \end{aligned}$$

The subcarrier bandwidth is less than the coherence bandwidth, i.e., $B_s = 15.625 \text{ kHz} \ll B_c = 250 \text{ kHz}$. Therefore each subcarrier experiences frequency flat fading. The OFDM symbol time without CP is

$$\frac{N}{B} = \frac{256}{4 \times 10^6} = 64 \mu s$$

The raw OFDM symbol time, corresponding to the $N = 256$ IFFT samples, is $64 \mu s$. WiMAX employs a cyclic prefix which is 12.5% of the symbol time. Therefore, the duration of the cyclic prefix is

$$\begin{aligned} \text{Duration of cyclic prefix} &= 12.5\% \text{ of symbol time} \\ &= \frac{12.5}{100} \times 64 \mu s, \\ &= 8 \mu s \end{aligned}$$

Thus, the total transmitted OFDM symbol duration with cyclic prefix is $64 \mu s + 8 \mu s = 72 \mu s$. Also, the number of samples in the CP is

$$\begin{aligned} \# \text{ Samples in CP} &= \frac{\text{CP duration}}{\text{Sample time}} \\ &= \frac{8 \mu s}{1/B} \\ &= 8 \mu s \times 4 \times 10^6 = 32 \end{aligned}$$

Thus, the length of the cyclic prefix $L_c = 32$ samples and the total number of samples is $256 + 32 = 288$. This break-up of the OFDM symbol in terms of the regular samples and the cyclic prefix is shown in Figure 2.21. Finally, the loss in spectral efficiency is

$$\begin{aligned} \text{Loss in spectral efficiency} &= \frac{32}{288} \\ &= \frac{8 \mu s}{72 \mu s} \\ &= 11.1\% \end{aligned}$$

This is the loss in spectral efficiency arising because of the addition of the cyclic prefix.

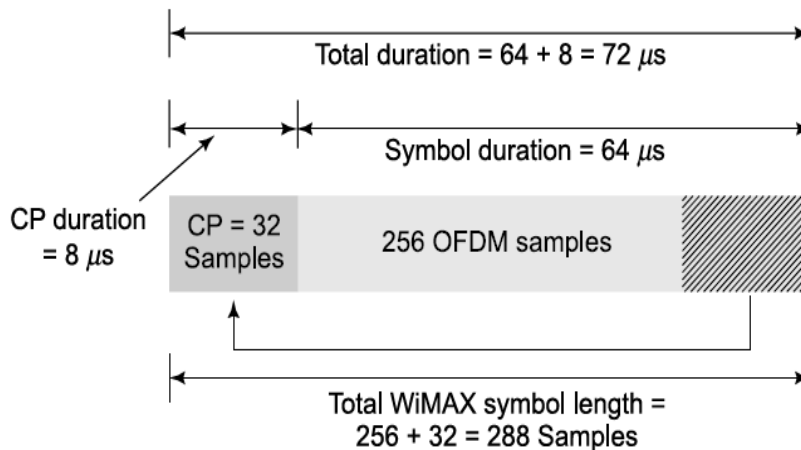


Figure 2.21 WiMAX OFDM symbol with cyclic prefix

2.10 MIMO-OFDM

- MIMO-OFDM is a combination of the Multiple-Input Multiple-Output (MIMO) wireless technology with that of OFDM, to further increase the rate in broadband multi-antenna wireless systems.
- Similar to OFDM, MIMO-OFDM converts a frequency-selective MIMO channel into multiple parallel flat fading MIMO channels. Hence, MIMO-OFDM significantly simplifies baseband receive processing by eliminating the need for a complex MIMO equalizer.
- The frequency-selective SISO channel is modelled as an FIR channel filter, with the output $y(n)$ at time instant n given as

$$\begin{aligned}
 y(n) &= \sum_{l=0}^{L-1} h(l) x(n-l) + w(n), \\
 &= h(0)x(n) + \underbrace{h(1)x(n-1) + \dots + h(L-1)x(n-L+1)}_{\text{ISI from previous symbols}} + w(n)
 \end{aligned}$$

where $w(n)$ denotes the noise. Hence, a MIMO frequency-selective channel can be modelled as a MIMO FIR filter, which can be described as

$$\begin{aligned}
 \mathbf{y}(n) &= \sum_{l=0}^{L-1} \mathbf{H}(l) x(n-l) + \mathbf{w}(n) \\
 &= \mathbf{H}(0)x(n) + \underbrace{\mathbf{H}(1)x(n-1) + \dots + \mathbf{H}(L-1)x(n-L+1)}_{\text{ISI from previous symbol vectors}} + \mathbf{w}(n)
 \end{aligned}$$

Therefore, the symbol vector $\mathbf{y}(n)$ at the time instant n is affected by inter-symbol vector interference from $x(n-1)$, $x(n-2)$, ..., $x(n-L+1)$. This is an L -tap frequency-selective MIMO channel.

In a MIMO frequency-selective channel, the interference occurs between current and previous transmit symbol vectors. In a MIMO-OFDM system, one needs to perform the IFFT operation at each transmit antenna.

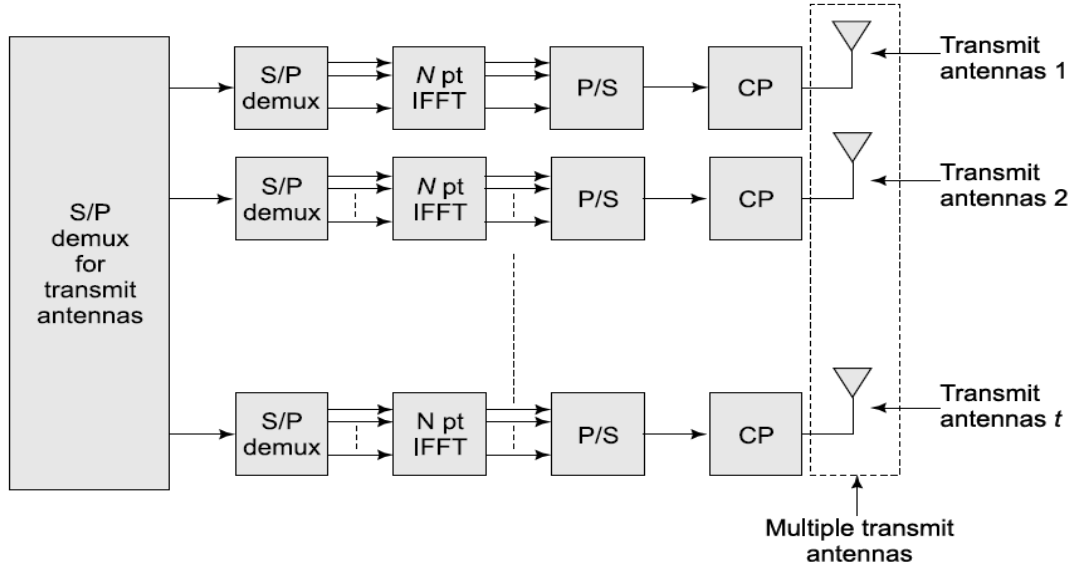


Figure 2.22 MIMO OFDM transmitter schematic

Hence, employing MIMO-OFDM, the MIMO frequency-selective channel can be converted into a set of parallel flat-fading MIMO channels.

$$\begin{aligned}\tilde{\mathbf{y}}(0) &= \tilde{\mathbf{H}}(0) \tilde{\mathbf{x}}(0) \\ \tilde{\mathbf{y}}(1) &= \tilde{\mathbf{H}}(1) \tilde{\mathbf{x}}(1) \\ &\vdots \\ \tilde{\mathbf{y}}(N-1) &= \tilde{\mathbf{H}}(N-1) \tilde{\mathbf{x}}(N-1)\end{aligned}$$

The model across the k^{th} subcarrier is $\tilde{\mathbf{y}}(k) = \tilde{\mathbf{H}}(k) \tilde{\mathbf{x}}(k)$, where $\tilde{\mathbf{y}}(k)$ and $\tilde{\mathbf{x}}(k)$ are the received and transmitted symbol vectors corresponding to the k^{th} subcarrier, and $\tilde{\mathbf{H}}(k)$ is the flat-fading channel matrix corresponding to the subcarrier k .

Each of the received vectors $\tilde{\mathbf{y}}(0), \tilde{\mathbf{y}}(1), \dots, \tilde{\mathbf{y}}(N-1)$ can be processed by a simple MIMO zero-forcing receiver or a MIMO-MMSE receiver for detection of the vectors $\tilde{\mathbf{x}}(0), \tilde{\mathbf{x}}(1), \dots, \tilde{\mathbf{x}}(N-1)$.

The zeroforcing MIMO receiver is given as

$$\begin{aligned}\hat{\tilde{\mathbf{x}}}_{\text{ZF}}(k) &= \left(\tilde{\mathbf{H}}(k) \right)^{\dagger} \tilde{\mathbf{y}}(k) \\ &= \left(\tilde{\mathbf{H}}^H(k) \tilde{\mathbf{H}}(k) \right)^{-1} \tilde{\mathbf{H}}^H(k) \tilde{\mathbf{y}}(k)\end{aligned}$$

Also, the MMSE receiver for the subcarrier k of the MIMO-OFDM system is given as

$$\begin{aligned}\hat{\tilde{\mathbf{x}}}_{\text{MMSE}}(k) &= \left(\tilde{\mathbf{H}}(k) \right)^{\dagger} \tilde{\mathbf{y}}(k) \\ &= P_d \left(P_d \tilde{\mathbf{H}}^H(k) \tilde{\mathbf{H}}(k) + \sigma_w^2 \mathbf{I} \right)^{-1} \tilde{\mathbf{H}}^H(k) \tilde{\mathbf{y}}(k)\end{aligned}$$

where P_d denotes the data power. The channel matrices $\tilde{\mathbf{H}}(0), \tilde{\mathbf{H}}(1), \dots, \tilde{\mathbf{H}}(N-1)$ corresponding to the OFDM subcarriers are given as follows. Let $h_{u,v}(k), \tilde{h}_{u,v}(k)$ denote the $(u, v)^{\text{th}}$ entries of the matrices $\mathbf{H}(k), \tilde{\mathbf{H}}(k)$ respectively. Then, $\tilde{h}_{u,v}(k)$ is given as the N -point DFT of the zero-padded coefficients

$$h_{u,v}(0), h_{u,v}(1), \dots, h_{u,v}(L-1), \underbrace{0, \dots, 0}_{(N-L)}$$

In effect, the channel matrix $\tilde{H}(k)$ is the k th frequency point corresponding to the FFT of the zero padded channel matrices $[H(0), H(1), \dots, H(L-1), 0_{r \times t}, \dots, 0_{r \times t}]$.

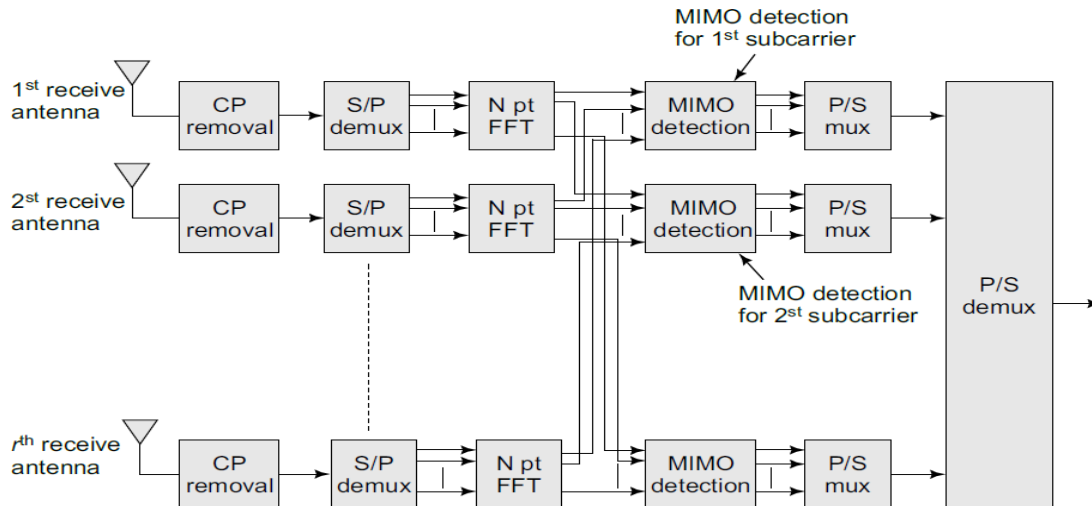


Figure 2.23 MIMO-OFDM receiver schematic

2.11 OFDM–Peak-to-Average Power Ratio (PAPR)

The Peak-to-Average Power Ratio (PAPR) is a critical problem in OFDM systems, which needs to be handled effectively in order to limit the distortion at the receiver.

Consider a non-OFDM or single-carrier system with BPSK modulated symbols. For example, let the symbol stream $x(0), x(1), x(2), \dots$ be given as $+a, -a, +a, \dots$ and so on. The power in each symbol equals a^2 . Further, also observe that this is the peak power at any given instant of time.

$$\text{Peak power} = \text{Average power} = \mathbb{E} \{ |x(k)|^2 \} = a^2$$

Thus, since the peak and average power are equal, the peak-to-average power ratio, or PAPR, is given as

$$\begin{aligned} \text{PAPR} &= \frac{\text{Peak power}}{\text{Average power}} \\ &= \frac{a^2}{a^2} \\ &= 1 = 0 \text{ dB} \end{aligned}$$

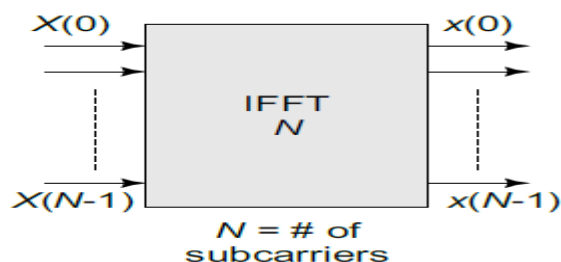


Figure 2.24 OFDM subcarrier loading

The above relation clearly shows that there is no significant deviation of the instantaneous power level from the mean power level.

Now, consider an OFDM system in which the different information symbols $X(0)$, $X(1)$, $X(2)$, ... given by $+a$, $-a$, $+a$, ... for instance, are loaded onto the subcarriers. This is shown schematically in Figure 2.24. The actual samples transmitted over the wireless channel are $x(0)$, $x(1)$, $x(2)$, ..., $x(N-1)$, which are the IFFT samples of the information symbols $X(0)$, $X(1)$, $X(2)$, ..., $X(N-1)$. Consider the k^{th} IFFT sample $x(k)$ given as

$$x(k) = \frac{1}{N} \sum_{i=0}^{N-1} X(i) e^{j2\pi \frac{ki}{N}}$$

where $X(i)$ denotes the information symbols. The average power in the symbols is given as

$$\begin{aligned} \text{Average power} &= E \{ |x(k)|^2 \} \\ &= \frac{1}{N} \sum_{i=0}^{N-1} E \{ |X(i)|^2 \} \underbrace{E \{ |e^{j2\pi \frac{ki}{N}}|^2 \}}_1 \\ &= \frac{1}{N^2} \sum_{i=0}^{N-1} E \{ |X(i)|^2 \} \\ &= \frac{1}{N^2} \sum_{i=0}^{N-1} a^2 \\ &= \frac{1}{N^2} a^2 N = \frac{a^2}{N} \end{aligned}$$

The average power of transmission is a^2/N .

The peak power can be found as follows. Observe that the peak of the OFDM sample arises for all symbols $X(i) = +a$ or $X(i) = -a$. This can be verified as follows.

$$\begin{aligned} |x(k)| &= \left| \frac{1}{N} \sum_{i=0}^{N-1} X(i) e^{j2\pi \frac{ki}{N}} \right| \\ &\leq \frac{1}{N} \sum_{i=0}^{N-1} |X(i) e^{j2\pi \frac{ki}{N}}| \\ &= \frac{1}{N} \sum_{i=0}^{N-1} \underbrace{|X(i)|}_a \underbrace{|e^{j2\pi \frac{ki}{N}}|}_1 \\ &= \frac{1}{N} \sum_{i=0}^{N-1} a \\ &= a \end{aligned}$$

Therefore, the peak power is given as a^2 . Hence, the peak-to-average power ratio in an OFDM system is given as

$$\text{OFDM PAPR} = \frac{a^2}{a^2/N} = N$$

From the above expression, it can be seen that the peak-to-average power ratio in an OFDM system is N , which is significantly higher compared to that of the single-carrier system, which is 1. PAPR rises with N , i.e., the number of subcarriers. Larger the number of subcarriers, larger is the PAPR. This high PAPR of the OFDM arises because of the IFFT operation. The data symbols across the subcarriers can add up to produce a high peak valued signal as seen above.

For instance, in an OFDM system with 512 subcarriers and BPSK modulation, the PAPR at the output can be as high as 10 dB. The PAPR of an OFDM system is characterized using the CCDF, i.e., the complementary cumulative distribution function.

The CCDF $\bar{F}_X(x)$ of a random variable X is given as the probability that $X > x$, expressed as

$$\bar{F}_X(x) = \Pr(X > x)$$

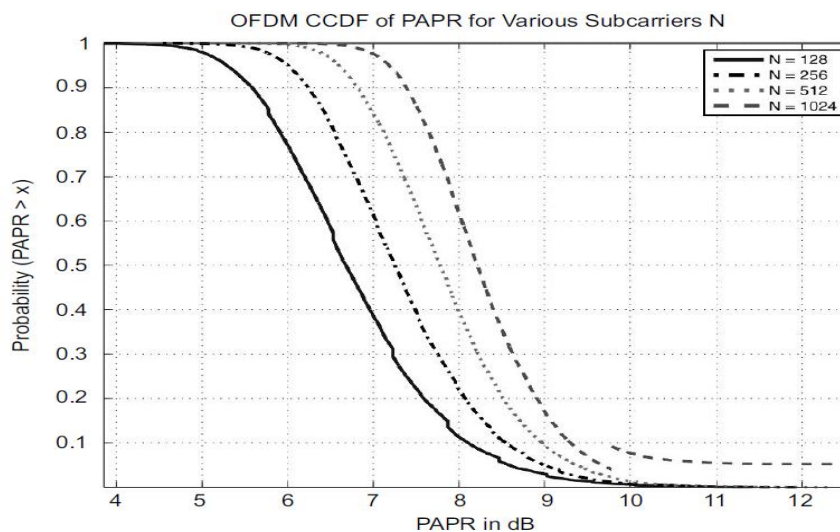


Figure 2.25 OFDM PAPR for various number of subcarriers N

The CCDF is related to the CDF, i.e., cumulative distribution function $F_X(x)$ of X as

$$\begin{aligned} F_X(x) &= \Pr(X \leq x) \\ &= 1 - \Pr(X > x) \\ &= 1 - \bar{F}_X(x) \end{aligned}$$

The CCDF of the PAPR then shows the probability that the PAPR, which is a random quantity, exceeds a particular threshold. A plot of the CCDF of the PAPR for various values of N , the total number of subcarriers, is shown in Figure 2.25.

The impact of PAPR on the OFDM system hardware can be understood as follows.

Every communication system has a receiver amplifier, which serves to amplify the amplitude of the receive signal, in order to boost its strength. However, the characteristic of the amplifier is linear only for a limited amplitude range of the signal. Typically, the amplifier operates around a bias point, as shown in Figure 2.26, which is roughly around the average power of the signal.

As long as the signal amplitude is restricted to the dynamic range of the amplifier around this bias point, for which the amplifier characteristic is linear, there is no nonlinear distortion at the output. However, in the case of OFDM, since the peak power deviates significantly from the average

power, there is a high chance that the signal crosses into the voltage region outside the dynamic range of the amplifier, thus resulting in a nonlinear distortion of the received signal. This nonlinear effect, arising out of amplifier saturation, leads to loss of orthogonality of the subcarriers and inter-carrier interference. The net result is a poor decoding performance and a rise in the bit-error rate.

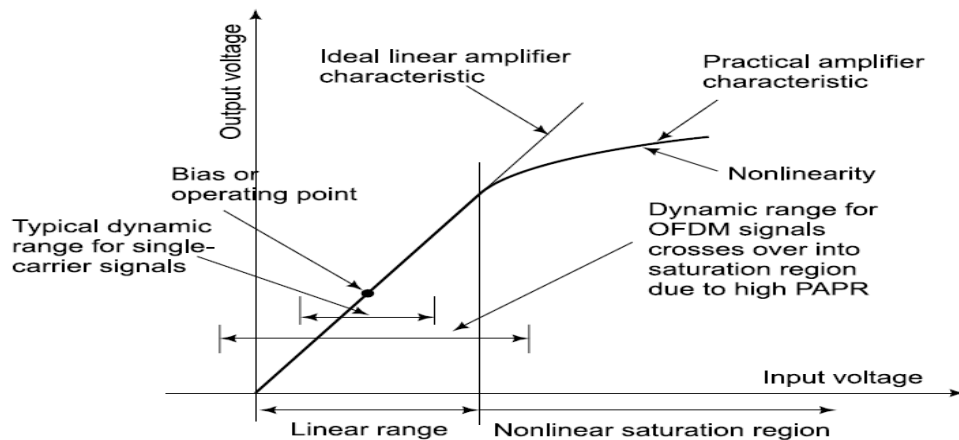


Figure 2.26 Nonlinear amplifier characteristic

2.12 SC-FDMA

SC-FDMA, which stands for Single-Carrier Frequency Division for Multiple Access, can be employed to reduce the peak-to-average power ratio in an OFDM system.

Consider the following hypothetical modification of the OFDM transmitter, shown in Figure 2.27, by the insertion of an N -point FFT block before the N -point IFFT block.

The FFT and the IFFT cancel the effect of each other and the net output is the exact input symbol stream, i.e., corresponding to a single-carrier system. This drastically reduces the PAPR, since, as seen previously, the PAPR of a single-carrier system is 0 dB.

However, instead of using an N -point FFT, one can use an M -point FFT, where $M < N$, to reduce the PAPR, while still retaining the properties of the OFDM system. This proposed SC-FDMA schematic is shown in Figure 2.28. Hence, introduction of the M -point FFT in SC-FDMA significantly reduces the PAPR of the system. This is the central principle of SC-FDMA.

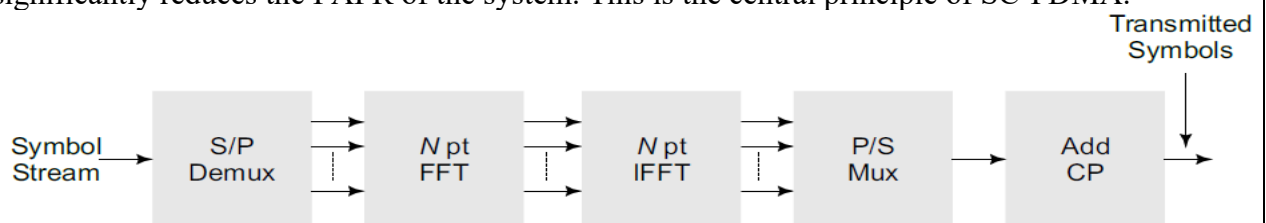


Figure 2.27 Hypothetical modification of OFDM transmitter

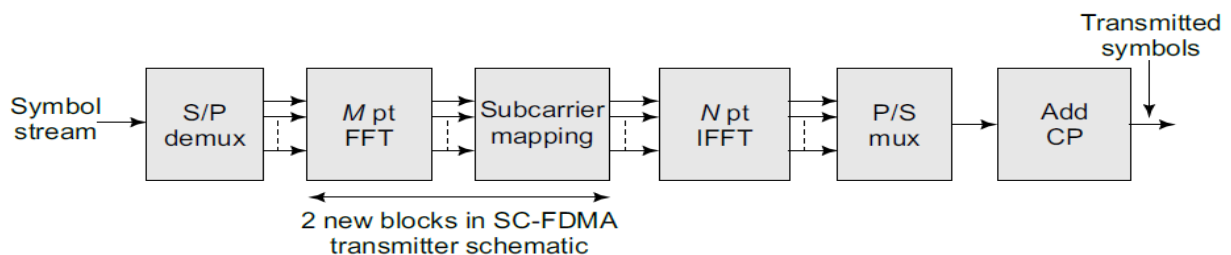


Figure 2.28 SC-FDMA transmitter schematic

2.12.1 SC-FDMA Receiver

The SC-FDMA receiver schematic is shown in Figure 2.29. The SC-FDMA receiver incorporates two new blocks compared to the OFDM receiver. The purpose of these additional blocks can be described as follows.

After the N -point FFT operation at the receiver, the signals are equalized across all the subcarriers, to remove the effect of the fading-channel coefficient across the subcarriers. Following the above operation, they are demapped from the subcarriers, which are N in number, to the original FFT block size of M . Finally, the M -point FFT is performed on these samples to generate the symbol stream.

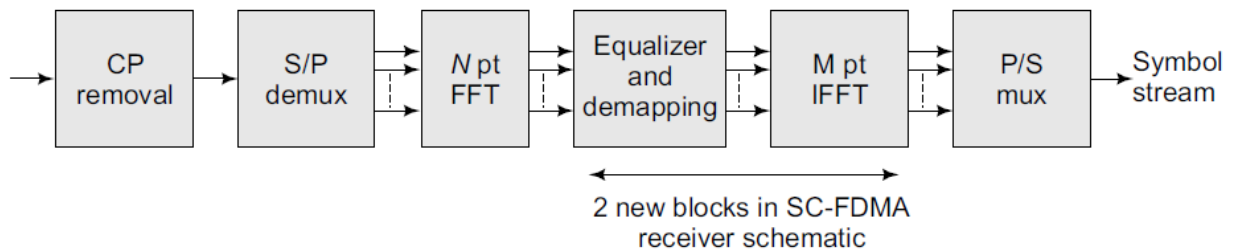


Figure 2.29 SC-FDMA receiver schematic

2.12.2 Subcarrier Mapping in SC-FDMA

Subcarrier mapping, in which the M samples at the output of the M -point FFT are mapped to the N subcarriers, is a key operation in SC-FDMA, a block representation of which can be seen in the SC-FDMA transmitter schematic in Figure 2.28.

The various possible SC-FDMA subcarrier mappings are illustrated through the following example. Consider $M = 4$ SC-FDMA symbols and $N = 12$ subcarriers.

Let $x(0)$, $x(1)$, $x(2)$, $x(3)$ denote the symbols and $X(0)$, $X(1)$, $X(2)$, $X(3)$ denote the corresponding $M = 4$ -point FFT samples which are to be loaded onto the subcarriers.

Let the number of subcarriers be $N = 12$.

In Interleaved FDMA (IFDMA) shown in Figure 2.30, the samples $X(i)$ are interleaved with zeros.

In LFDMA, which is also employed in the uplink of the 4G mobile standard LTE, the samples are loaded as a block onto the subcarriers, with appropriate zero padding. This is shown in Figure 2.30.

Post this subcarrier mapping, the rest of the procedure prior to transmission proceeds as shown in the SC-FDMA transmitter block.

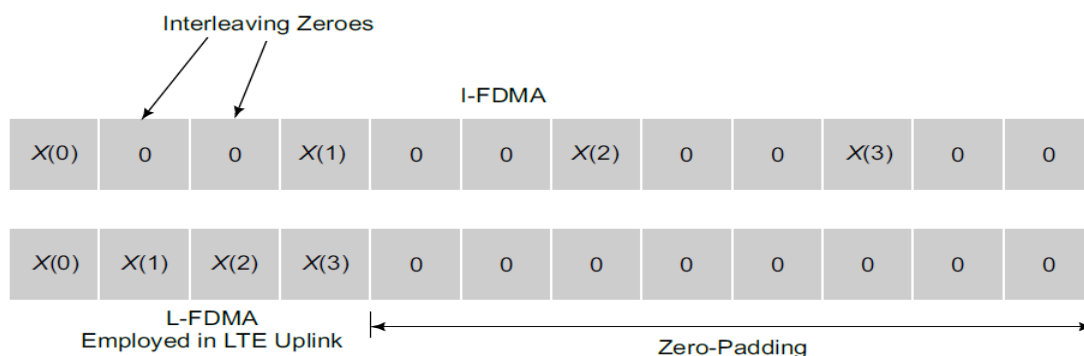


Figure 2.30 SC-FDMA various subcarrier mapping schemes