MODULE – 2

LOAD FLOW ANALYSIS

SYLLABUS

Load Flow Analysis: Introduction, Classification of Buses, Power Flow Equation, Operating Constraints, Data for Load Flow, Gauss Seidal Iterative Method, Illustrative Examples.

2.1 Introduction

- Load flow studies are important in planning and designing future expansion of power systems.
- The study gives steady state solutions of the voltages at all the buses, for a particular load condition.
- Generally, load flow studies are limited to the transmission system, which involves bulk power transmission. The load at the buses is assumed to be known.
- Load flow studies throw light on some of the important aspects of the system operation, such as: violation of voltage magnitudes at the buses, overloading of lines, overloading of generators, stability margin reduction, indicated by power angle differences between buses linked by a line, effect of contingencies like line voltages, emergency shutdown of generators, etc.
- Load flow studies are required for deciding the economic operation of the power system.
- The load flow problem consists of finding the power flows (real and reactive) and voltages of a network for given bus conditions.
- At each bus, there are four quantities of interest to be known for further analysis: the real and reactive power, the voltage magnitude and its phase angle.
- Because of the nonlinearity of the algebraic equations, describing the given power system, their solutions are obviously, based on the iterative methods only.

2.2 Classification of Buses

Sl. No.	Bus Types	Specified Variables	Unspecified variables	Remarks
1	Slack/ Swing Bus	$\lvert V\rvert$, δ	P_G, Q_G	$\lvert V\rvert$, δ: are assumed if not specified as 1.0 and 0^0
2	Generator/ Machine/ PV Bus	P_G, $\lvert V\rvert$	Q_G, δ	A generator is present at the machine bus
3	Load/ PQ Bus	P_G, Q_G	$\lvert V\rvert$, δ	About 80% buses are of PQ type
4	Voltage Controlled Bus	P_G,Q_G, $\lvert V\rvert$	δ, a	'a' is the % tap change in tap-changing transformer

Slack Bus/Swing Bus/Reference Bus

- This bus is distinguished from the remaining types by the fact that real and reactive powers at this bus are not specified.
- Instead, voltage magnitude (normally set equal to 1 pu) and voltage phase angle (normally set equal to zero) are specified.
- Usually, there is only one bus of this type in a given power system.
- The slack bus is numbered 1, for convenience.

PQ Bus/Load Bus

- At this type of bus, the net powers Pi and Qi are known (PDi and QDi are known from load forecasting and PGi and QGi are specified).
- The unknowns are |Vi| and δi.
- A pure load bus (no generating facility at the bus, i.e. PGi = QGi = 0) is a PQ bus.
- PQ buses are the most common, comprising almost 80% of all the buses in a given power system.

PV Bus/Generator Bus

- This bus has always a generator connected to it. Thus P_{Gi} and $|V_i|$ are specified.
- Hence the net power P_i is known (as P_{Di} is known from load forecasting).
- Hence the knowns are P_i and $|V_i|$ and unknowns are Q_i and δi.
- PV buses comprise about 10% of all the buses in a power system.

Voltage Controlled Bus

Frequently the PV bus and the voltage controlled bus are grouped together.
But they have physical differences and slightly different calculation strategies.
The voltage controlled bus has also voltage control capabilities, and uses a tap adjustable transformer and/or a static var compensator instead of a generator.
Hence $P_{Gi} = Q_{Gi} = 0$ at these buses.
Thus $P_i = -P_{Di}$, $Q_i = -Q_{Di}$ and $|V_i|$ are known at these buses and the unknown is δi.

No.	*Bus Type*	*Known*	*Unknown*
1.	Slack Bus	$\lvert V_1 \rvert, \delta_1$	P_1, Q_1
2.	PQ Bus	P_2, Q_2	$\lvert V_2 \rvert, \delta_2$
3.	PQ Bus	P_3, Q_3	$\lvert V_3 \rvert, \delta_3$
4.	PV Bus	$P_4, \lvert V_4 \rvert$	Q_4, δ_4

2.3 Power Flow Equation

The complex power injected by the source into the ith bus of a power system is

$$S_i = P_i + jQ_i = V_i I_i^*, \; i = 1, 2, ..., n$$

Since it is convenient to work with Ii instead of I_i^*, we take the complex conjugate of the above equation,

$$P_i - jQ_i = V_i^* I_i, \; i = 1, 2, ..., n$$

Substituting

$$I_i = \sum_{k=1}^{n} (Y_{ik} V_k) - - - - - (\mathbf{1})$$

from Eqn. (1) in the above equation, we have

$$P_i - jQ_i = V_i^* \left(\sum_{k=1}^{n} (Y_{ik} V_k) \right), \quad i = 1, 2, ..., n$$

Equating real and imaginary parts, we get

$$\sum_{k=1}^{n} (Y_{ik} V_k)$$

$$P_i \text{ (Real power)} = \text{Real} \left(V_i^* \left(\sum_{k=1}^{n} (Y_{ik} V_k) \right) \right) \quad \text{------------ (2.a)}$$

$$Q_i \text{ (Reactive power)} = - \text{Imaginary} \left(V_i^* \left(\sum_{k=1}^{n} (Y_{ik} V_k) \right) \right) \quad \text{---------- (2.b)}$$

Let $\quad V_i = |V_i| e^{j\delta_i}, \; V_k = |V_k| e^{j\delta_k}$

$$Y_{ik} = |Y_{ik}| \, e^{j\theta_{ik}}$$

then

$$P_i \text{ (Real power)} = |V_i| \sum_{k=1}^{n} |V_k| \, |Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i) \quad \text{------------ (3.a)}$$

$$Q_i \text{ (Reactive power)} = -|V_i| \sum_{k=1}^{n} |V_k| \, |Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i)$$

$$(i = 1, 2, ..., n) \quad \text{------- (3.b)}$$

Equations 3(a) and (b) are called power flow equations. There are n real and n reactive power flow equations giving a total of 2n power flow equations.

At each bus there are four variables, viz., $|V_i|$, δ_i, P_i and Q_i, giving a total of 4n variables (for n buses).

If at every bus two variables are specified (thus specifying a total of 2n variables), the remaining two variables at every bus (a total of 2n remaining variables) can be found by solving the 2n power flow Eqns. 3(a) and (b).

When a physical system is considered, specifying variables at every bus depends on what devices are connected to that bus.

2.4 Operating Constraints

For Static Load Flow Equations (SLFE) solution to have practical significance, all the state and control variables must be within specified practical limits. These limits are dictated by specifications of power system hard ware and operating constraints, and are described below:

(1) Voltage magnitude $|V_i|$ must satisfy the inequality

$$|V_i|_{min} \leq |V_i| \leq |V_i|_{max} \text{ -----------(4.a)}$$

This limit arises due to the fact that the power system equipment is designed to operate at fixed voltages with allowable variations of ± (5–10)% of rated values.

(2) Certain of the δ_is (state variables) must satisfy

$$|\delta_i - \delta_k| \leq |\delta_i - \delta_k|_{max} \text{ -----------(4.b)}$$

This constraint limits the maximum permissible power angle of transmission line connecting buses *i* and *k* and is imposed by considerations of stability.

(3) Owing to physical limitations of P and/or Q generation sources, P_{Gi} and Q_{Gi} are constrained as follows:

$$P_{Gi,min} \leq P_{Gi} \leq P_{Gi,max}$$

$$Q_{Gi,min} \leq Q_{Gi} \leq Q_{Gi,max} - - - - - (4.c)$$

Also, we have

$$\sum_i P_{Gi} = \sum_i (P_{Di}) + P_L - - - - - (5.a)$$

$$\sum_i Q_{Gi} = \sum_i (Q_{Di}) + Q_L - - - - - (5.b)$$

Where P_L and Q_L are system real and reactive power losses.

The load flow problem can now be fully defined as follows:

Assume a certain nominal bus load configuration. Specify $P_{Gi} + jQ_{Gi}$ at all PQ buses (this specifies $P_i + jQ_i$ at these buses, specify P_{Gi} (this specifies P_i) and $|V_i|$ at all PV buses; and specify $|V_1|$ and δ_1 (= 0) at the slack bus. Thus all the 2n variables of vector *x* are specified. Thus 2n SLFE, which are nonlinear algebraic equations can be solved (iteratively) to determine the values of the 2n variables of the vector y, comprising voltages and angles at the PQ buses; reactive powers and angles at the PV buses; and active and reactive powers at the slack bus.

The next logical step is to compute the line flows.

From the above definition we can state two versions of the load flow problem. In both the cases, bus 1 is assumed as the slack bus.

Case I: We assume all the remaining buses are PQ buses. We have:
Given: $V_1, S_2, S_3, \ldots, S_n$
Find: $S_1, V_2, V_3, \ldots, V_n$

Case II: We assume both PV and PQ buses. We number the buses so that buses 2, 3, ..., *m* are PQ buses and *m* + 1, ..., *n* are PV buses.

Thus,
Given: $V_1, S_2, \ldots, S_m, (P_{m+1}, |V_{m+1}|, \ldots, P_n, |V_n|)$
Find: $S_1, V_2, \ldots, V_m, (Q_{m+1}, \delta_{m+1}), \ldots, (Q_n, \delta_n)$

Since the load flow equations are essentially non-linear, they have to be solved through iterative numerical techniques.

At the cost of solution accuracy, it is possible to linearize load flow equations by making suitable assumptions and approximations so that fast and explicit solutions become possible.

Such techniques have value, particularly for planning studies where load flow solutions have to be carried out repeatedly but a high degree of accuracy is not needed.

Once a load flow problem is formulated it can be seen that there exists a range of every independent variable in the vector *x* for which there is no solution, or there are multiple solutions.

2.5 Data for Load Flow

The various data required are as under:

1. System Data:
- It includes:
- Number of buses-n,
- Number of PV buses,
- Number of loads,
- Number of transmission lines,
- Number of transformers,
- Number of shunt elements,
- The slack bus number,
- Voltage magnitude of slack bus (angle is generally taken as 0o),

- Tolerance limit,
- Base MVA and
- Maximum permissible number of iterations.

2. Generator Bus Data: For every PV bus i, the data required includes the
- Bus number,
- Active power generation P_{Gi},
- The specified voltage magnitude
- Minimum reactive power limit $Q_{i,min}$, and
- Maximum reactive power limit $Q_{i,max}$.

3. Load Data: For all loads the data required includes the
- Bus number,
- Active power demand P_{Di}, and
- The reactive power demand Q_{Di}.

4. Transmission Line Data: For every transmission line connected between buses i and k the data includes the
- Starting bus number i,
- Ending bus number k,
- Resistance of the line,
- Reactance of the line and
- Half line charging admittance.

5. Transformer Data: For every transformer connected between buses i and k the data to be given includes:
- Starting bus number i,
- Ending bus number k,
- Resistance of the transformer,
- Reactance of the transformer, and
- The off nominal turns-ratio a.

6. Shunt Element Data: The data needed for the shunt element includes:
- The bus number where element is connected, and
- The shunt admittance (G_{sh} + j B_{sh}).

2.6 Gauss Seidal Iterative Method

The Gauss–Siedel (GS) method is an iterative algorithm for solving a set of non-linear algebraic equations.

We consider a system of n equations in n unknowns $x_1, \dots, x_n$

We rewrite these n equations in the form: $x_i = f_i(x_1, \dots, x_n), \quad i = 1, \dots, n$

To find the solution, we assume initial values for $\boldsymbol{x_1}, \ldots, \boldsymbol{x_n}$ based on guidance from practical experience in a physical situation.

Let the initial values be $\boldsymbol{x_1^0}, \ldots, \boldsymbol{x_n^0}$

Then, we get the first approximate solution by substituting these initial values in the above n equations as follows:

$$\begin{aligned} x_1^1 &= f_1\left(x_1^0, x_2^0 \ldots, x_n^0\right) \\ &\vdots \\ x_i^1 &= f_i\left(x_1^1, x_2^1, \ldots, x_{i-1}^1, x_i^0, \ldots, x_n^0\right) \\ &\vdots \\ x_n^1 &= f_n\left(x_1^1, x_2^1, \ldots, x_{n-1}^1, x_n^0\right) \end{aligned}$$

This completes one iteration. In general, we get the kth approximate solution in the kth iteration as follows:

$$x_i^k = f_i\left(x_1^k, \ldots, x_{i-1}^k, x_i^{k-1}, x_{i+1}^{k-1}, \ldots, x_n^{k-1}\right), \quad i = 1, 2, \ldots, n$$

We note that the value of $\boldsymbol{x_i}$ for the *k*th iteration is obtained by substituting $\boldsymbol{x_1^k}, \boldsymbol{x_2^k}, \ldots, \boldsymbol{x_{i-1}^k}$ (obtained in kth iteration) and $\boldsymbol{x_i^{k-1}}, \ldots, \boldsymbol{x_n^{k-1}}$ (obtained in (*k*-1)th iteration) in the equation $\boldsymbol{x_i = f_i}$.

If $\left|\boldsymbol{x_i^k - x_i^{k-1}}\right| < \boldsymbol{\varepsilon}$ (a very small number), for all *i* values, then the solution is said to converge.

The iterative process is repeated till the solution converges within prescribed accuracy. The convergence is quite sensitive to the starting values assumed.

Fortunately, in a load flow study, a starting vector close to the final solution can be easily identified with previous experience.

To solve a load flow problem by GS method, we consider two cases depending on the type of buses present.

In the first case we assume all the buses other than the slack bus are PQ buses.

In the second case, we assume the presence of both PQ and PV buses, other than the slack bus. Later on we also include the presence of voltage controlled bus, whose voltage is controlled by a regulating transformer.

Case I: The slack bus is numbered one, and the remaining (n – 1) buses are PQ buses (*i* = 2, ..., *n*).

With slack bus voltage assumed, the remaining (n – 1) bus voltages are found through iterative process as follows:

The complex bus power injected into the ith bus is

$$S_i = P_i + jQ_i = V_i I_i^*$$

or

$$I_i = \frac{P_i - jQ_i}{V_i^*} \quad \text{----------- (6)}$$

From Eqn. (1)

$$V_i = \frac{1}{Y_{ii}}\left(I_i - \sum_{\substack{k=1 \\ k \neq i}}^{n} (Y_{ik} V_k) \right), \quad i = 2, \ldots, n \quad \text{-------- (7)}$$

Substituting I i from Eqn. (6) into Eqn. (7)

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} (Y_{ik} V_k) \right], \quad i = 2, \ldots, n \quad \text{----------- (8)}$$

We assume a starting value for V_i (i=1,2,…,n). These values are then updated using Eqn. (8). The voltages substituted at the right hand side of Eqn. (8) are the most recently (updated) values for the corresponding buses. Iterations are repeated till no bus voltage magnitude changes by more than a prescribed value.

If instead of updating voltages at every step of an iteration, updating is carried out at the end of a complete iteration, **the process is known as Gauss Iterative Method**. It converges much slower and may sometimes fail to do so.

2.6.1 Algorithm for Load Flow Solution (Case I)

1. The slack bus voltage magnitude and angle are assumed, usually $V_1 = 1 \angle 0^o$ pu. With the load profile known at each bus (i.e. P_{Di} and Q_{Di} known), we allocate P_{Gi} and Q_{Gi} to all generating stations. With this step, bus injections ($P_i + jQ_i$) are known at all buses other than the slack bus.

2. Assembly of bus admittance matrix (Y_{BUS}): With the line and shunt admittance data stored in the computer, Y_{BUS} is assembled by using the algorithm developed earlier. Alternatively, Y_{BUS} is assembled using $Y_{BUS} = A^T YA$, where the input data is in the form of primitive admittance matrix Y and singular connection bus incidence matrix A.

3. Iterative computation of bus voltages (V_i ; i = 2, ..., n): To start the iteration, a set of initial voltage values is assumed. Since, in a power system the voltage spread is not too wide, it is normal practice to use a flat voltage start, i.e., initially all voltages are set equal to (1 + j0), except the slack bus voltage, which is fixed. This reduces the n equations to (n – 1) equations in complex numbers (Eqn. (8)) which are to be solved iteratively for finding complex voltages

V_2 , V_3 , ... , V_n . If complex number operations are not available in a computer, Eqn. (8) can be converted into 2(n – 1) equations in real unknowns (e_i, f_i **or** $|V_i|, \delta_i$ by writing

$$V_i = e_i + jf_i = |V_i| \varepsilon^{j\delta_i} \quad \text{-------- (9)}$$

We also define

$$A_i = \frac{P_i - jQ_i}{Y_{ii}}, \quad i = 2, \ldots, n \quad \text{--------- (10)}$$

$$B_{ik} = Y_{ik}/Y_{ii} \quad i = 2, \ldots, n; \quad k = 2, \ldots, n, k \neq i \quad \text{-------- (11)}$$

Now, for the (r + 1)th iteration, the voltage Eqn. (8) becomes

$$V_i^{(r+1)} = \left[\frac{A_i}{(V_i^r)^*} - \sum_{k=1}^{i-1} (B_{ik} V_k^{(r+1)}) - \sum_{k=i+1}^{n} (B_{ik} V_k^{(r)}) \right], \ i = 2, \ldots, n \quad \text{------ (12)}$$

The iterative process is continued till the change in magnitude of bus voltage, |ΔVi (r + 1)|, between two consecutive iterations is less than a certain tolerance for all bus voltages, i.e.

$$|\Delta V_i^{(r+1)}| = |V_i^{(r+1)} - V_i^{(r)}| \leq \varepsilon, \quad i = 2, \ldots, n \quad \text{-------- (13)}$$

Also, we see if $|V_i|_{\min} \leq |V_i| \leq |V_i|_{\max}, \quad i = 2, \ldots, n.$

If not, we fix $|V_i|$ at one of the extreme values, i.e.

$$|V_i|_{\min} \text{ if } |V_i| \leq |V_i|_{\min} \quad \text{or} \quad |V_i|_{\max} \text{ if } |V_i| \geq |V_i|_{\max}.$$

Depending on the nature of the problem, we can also check

$$\text{if } |\delta_i - \delta_k| \leq |\delta_i - \delta_k|_{\max} \ (i = 1, \ldots, n; k = 1, \ldots, n; i \neq k)$$

4. Computation of slack bus power: Substitution of all bus voltages computed in step 3 with V1 in Eqns. 3(a) and (b) with i = 1 yields real and reactive power at the slack bus i.e., $S_1 = P_1 + jQ_1$.

5. Computation of line flows: This is the last step in the load flow analysis wherein the power flows on the various lines of the network are computed. This also enables us to check whether any line is overloaded. Consider the line connecting buses i and k. The line and transformers at each end can be represented by a circuit with series admittance y_{ik} and two shunt admittances y_{ik0} and y_{ki0} as shown in **Fig. 2.1.**

The current fed by bus i into the line can be expressed as

$$I_{ik} = I_{ik1} + I_{ik0} = (V_i - V_k) y_{ik} + V_i y_{ik0} \quad \text{-------- (14)}$$

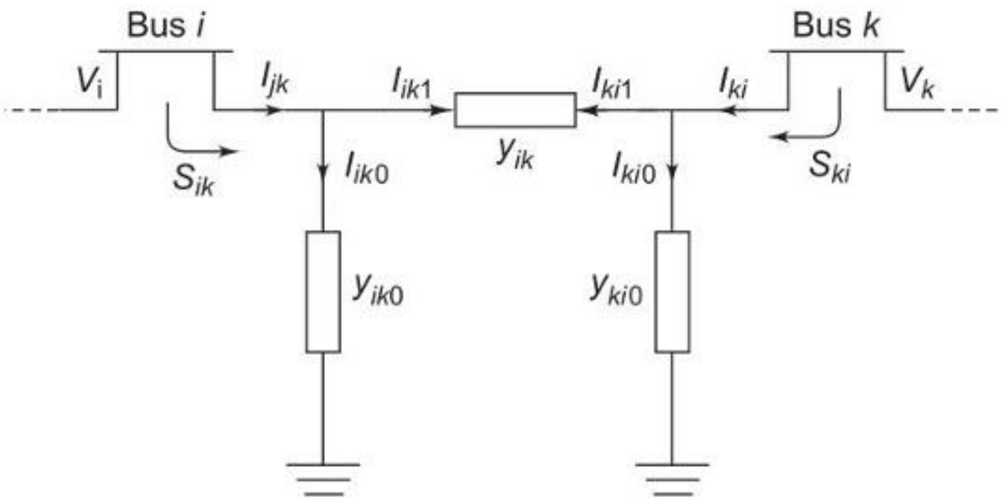


Fig. 2.1 π - Representation of a line and transformers connected between two buses

The power fed into the line from bus i is

$$S_{ik} = P_{ik} + jQ_{ik} = V_i I^*_{ik} = V_i(V_i^* - V_k^*)\, y_{ik}^* + V_i V_i^* y_{ik0}^* \quad \text{------- (15)}$$

Similarly, the power fed into the line from bus k is

$$S_{ki} = V_k\,(V^*_k - V_i^*)\, y_{ik}^* + V_k V_{iko}{}^* y^*_{ki0} \quad \text{------- (16)}$$

The power loss in the (i – k)th line is the sum of the power flows determined from Eqns. (15) and (16). Total transmission loss can be computed by summing all the line flows (i.e. Sik + Ski for all i, k).

If there are any mutually coupled transmission lines, then we use the Eqns. to find the currents between lines and then compute for the line flows as above.

It may be noted that the slack bus power can also be found by summing up the flows on the lines terminating at the slack bus.

2.6.2 Acceleration of Convergence

Convergence in the GS method can sometimes be speeded up by the use of the acceleration factor.

For the ith bus, the accelerated value of voltage at the (r + 1)st iteration is given by

$$V_i^{(r+1)}\,(\text{accelerated}) = V_i^{(r)} + \alpha\left(V_i^{(r+1)} - V_i^{(r)} \right)$$

Where α is a real number called the acceleration factor.

A suitable value of a for any system can be obtained by trial load flow studies. A generally recommended value is α = 1.6. A wrong choice of a may indeed slow down convergence or even cause the method to diverge.

This concludes the load flow analysis for PQ buses only. Next, we consider the presence of PV buses (see Example-3).

Case II: Here we consider (m – 1) PQ buses, (n – m) PV buses and the slack bus.

Given: V_1, (P_2, Q_2), ..., (P_m, Q_m), (P_{m+1}, $|V_{m+1}|$), ..., (P_n, $|V_n|$)

To find: S_1, V2, ..., V_m, (Q_{m+1}, δ_{m+1}), ..., (Q_n, δ_n).

Algorithm (Case II)

We first repeat the iteration for PQ buses as in Case I, then continue the iteration for PV buses. At the PV buses, P and | V | are specified and Q and δ are unknowns to be determined. Therefore, the values of Q and δ are to be updated in every GS iteration through appropriate bus equations. This is accomplished in the following steps:

1. From Eqn. (2.b),

$$Q_i = -\operatorname{Im}\left\{V_i^* \sum_{k=1}^{n} Y_{ik}V_k\right\}, \quad i = m+1, \ldots, n$$

The revised value of Q_i is obtained from the above equation by substituting most updated values of voltages on the right hand side. For the (r + 1)th iteration we can write

$$Q_i^{(r+1)} = -\operatorname{Im}\left\{\left(V_i^{(r)}\right)^* \sum_{k=1}^{i-1} Y_{ik}V_k^{(r+1)} + \left(V_i^{(r)}\right)^* \sum_{k=i}^{n} Y_{ik}V_k^{(r)}\right\}$$

$$i = m+1, \ldots, n \qquad \text{-------(17)}$$

2. The revised value of d step 1.

$$\delta_i^{(r+1)} = \angle V_i^{(r+1)}$$

$$= \text{Angle}\left[\frac{A_i^{(r+1)}}{\left(V_i^{(r)}\right)^*} - \sum_{k=1}^{i-1} B_{ik}V_k^{(r+1)} - \sum_{k=i+1}^{n} B_{ik}V_k^{(r)}\right] \qquad \text{------- (18)}$$

where

$$A_i^{(r+1)} = \frac{P_i - jQ_i^{(r+1)}}{Y_{ii}}, \quad i = m+1, \ldots, n \qquad \text{----- (19)}$$

Physical limitations of Q generation require that Q demand at any bus must be in the range Q_{min} to Q_{max} . If at any stage during iteration, Q at any bus goes outside these limits, it is fixed at Q_{min} or Q_{max} as the case may be, and the bus voltage specification is dropped, i.e. the bus is now treated like a PQ bus. Thus Step 1 above branches out to Step 3 as follows:

3. If $Q_i^{r+1} \leq Q_{i,min}$, we set $Q_i^{r+1} = Q_{i,min}$ or if $Q_i^{r+1} \geq Q_{i,max}$, we set $Q_i^{r+1} = Q_{i,max}$ and treat the bus i as a PQ bus. We compute A_i^{r+1} and V_i^{r+1} from Eqns. (19) and (12) respectively.

Now, all the computational steps are summarized in the **detailed flow chart of Fig. 2.2.** It is assumed that out of n buses, the first is slack bus, then 2, 3, ..., m are PQ buses, and the remaining, m + 1, ..., n are PV buses.

Case III: We consider here the presence of voltage controlled buses in addition to PQ and PV buses other than the slack bus. This case will be dealt in Example-3.

Fig.2.2 (a to e) Flow chart for load flow solution by the Gauss–Siedel iterative method using Y_{BUS}

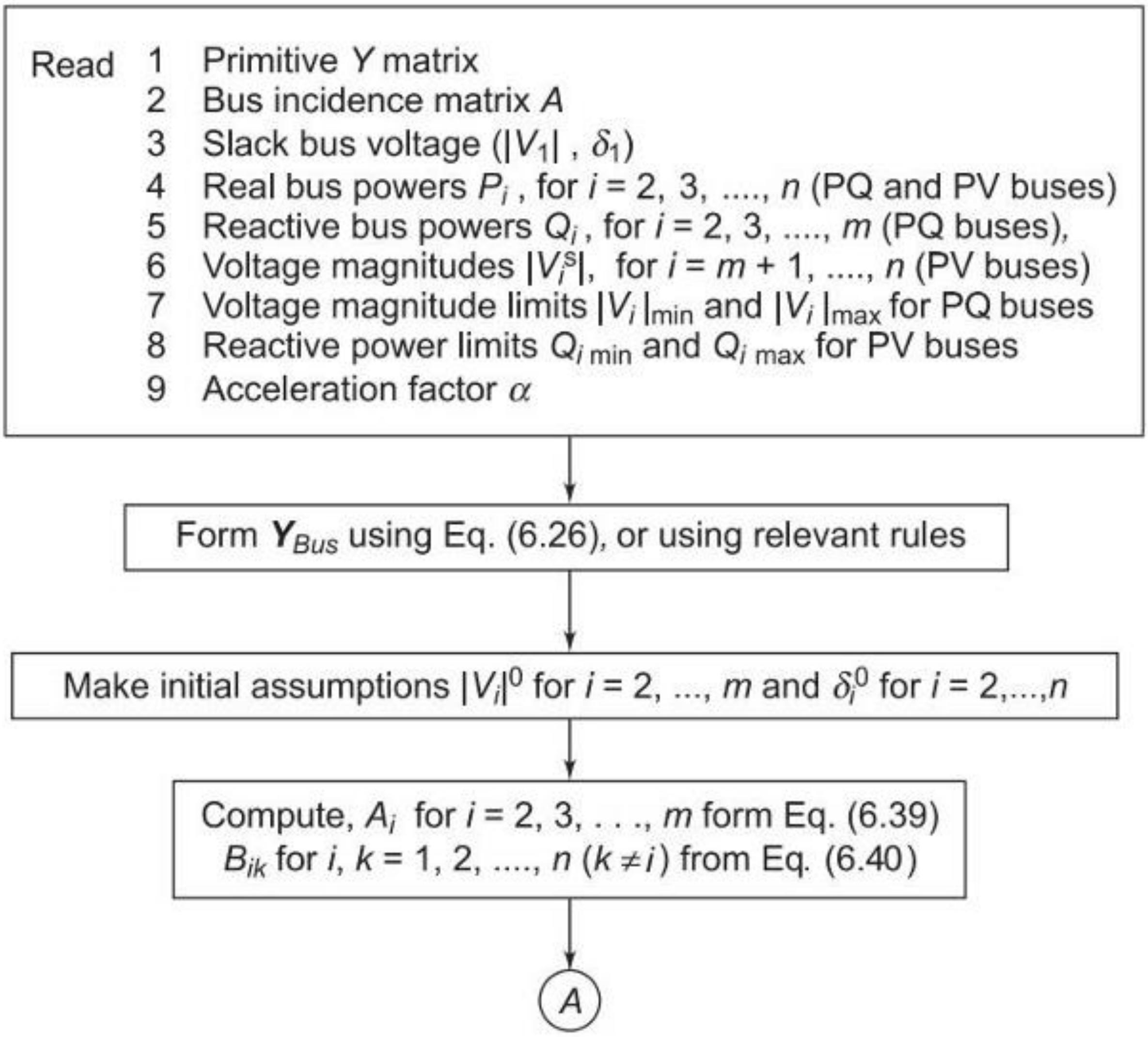


Fig.2.2 (a)

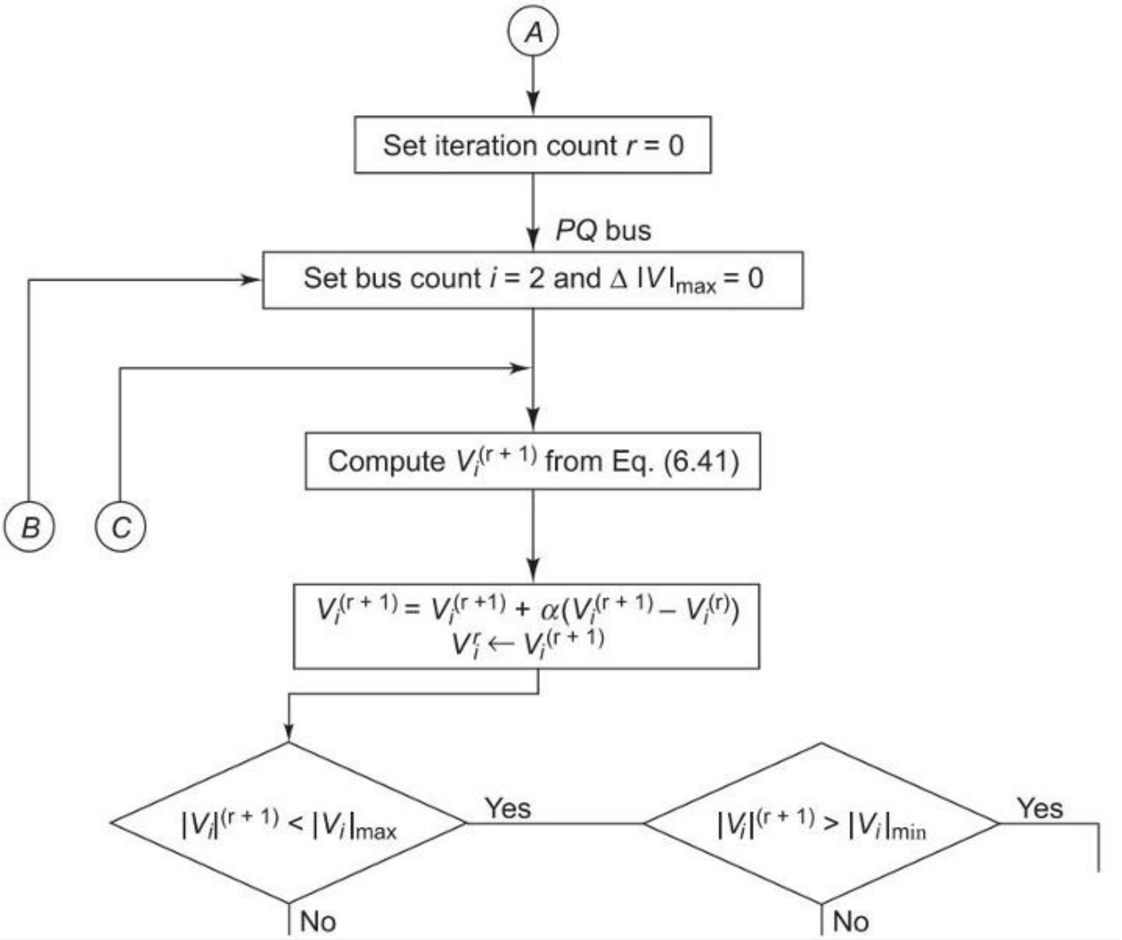


Fig.2.2 (b)

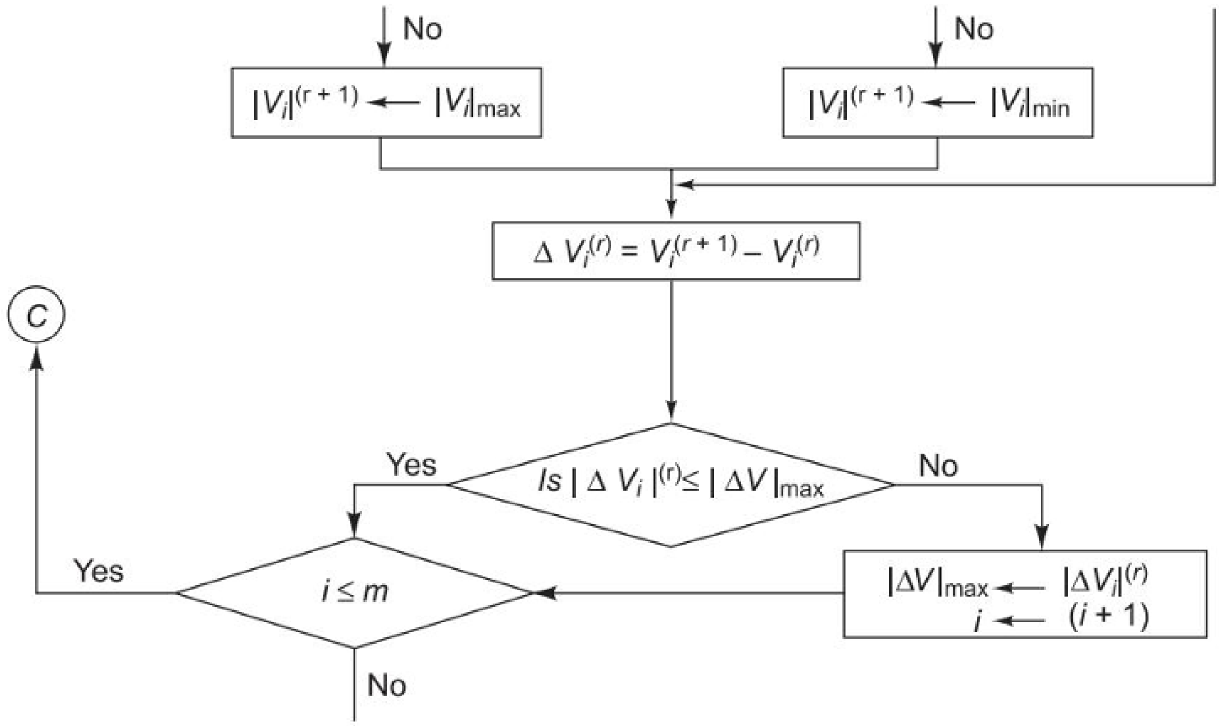


Fig.2.2 (c)

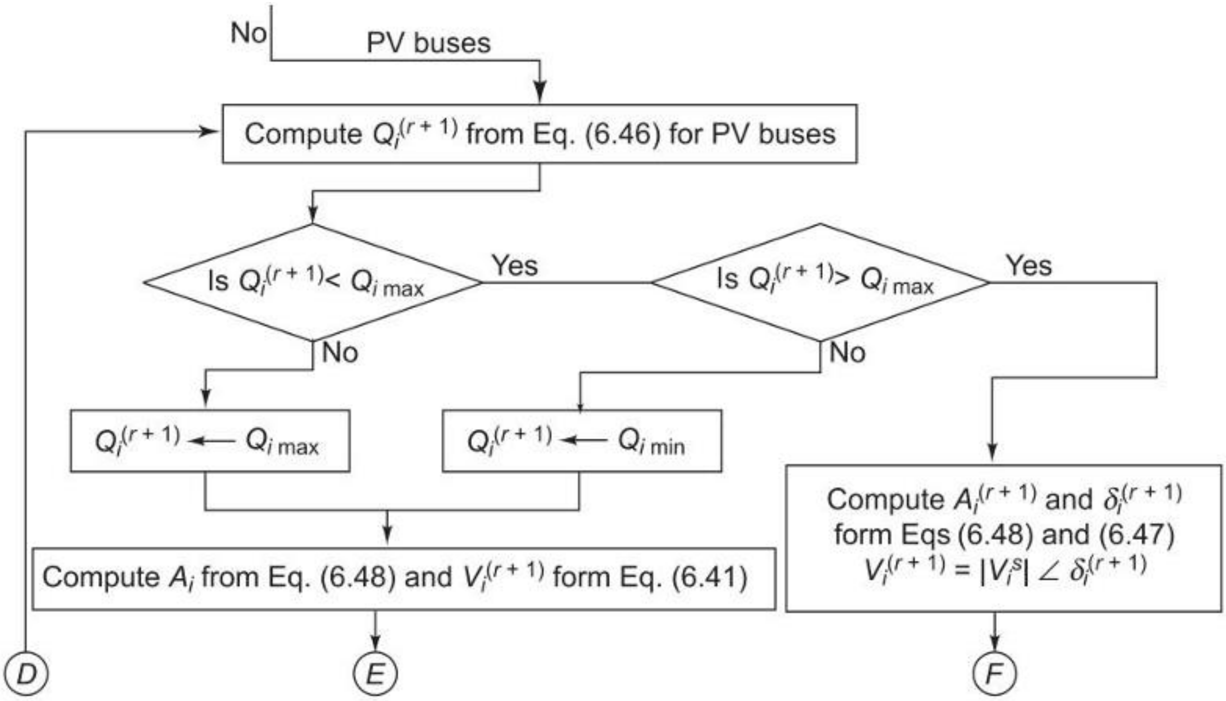


Fig.2.2 (d)

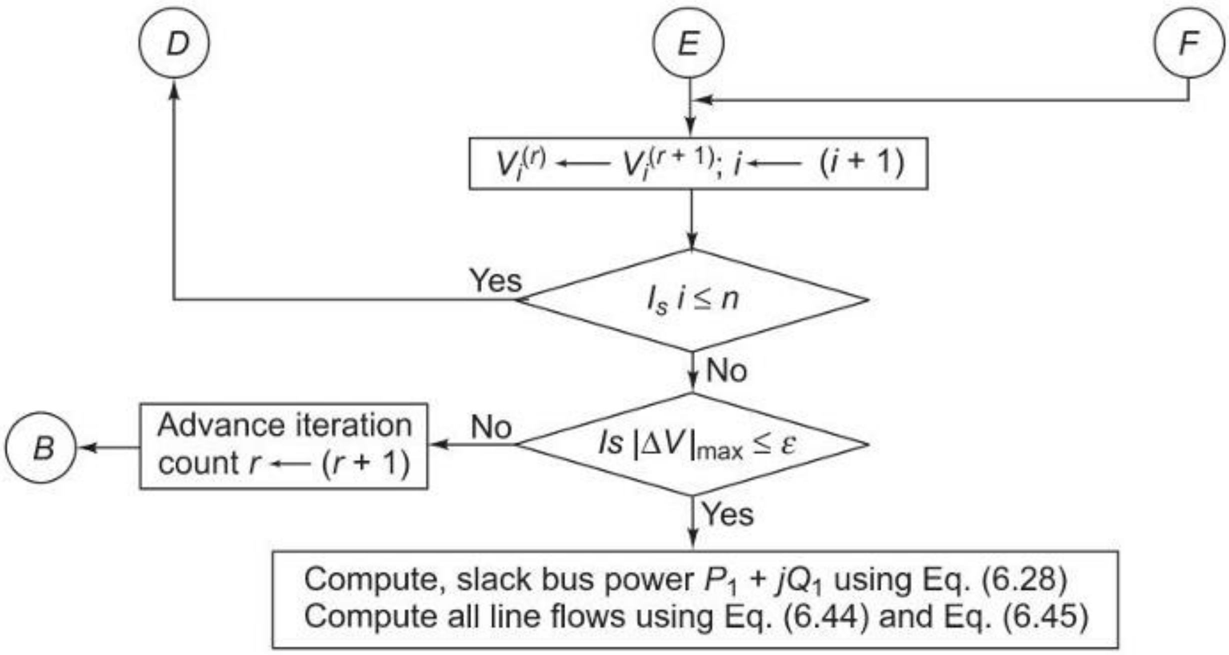


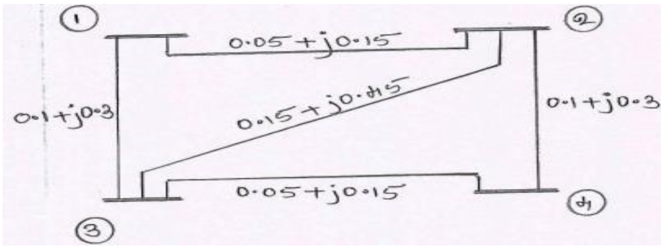
Fig.2.2 (e)

2.7 Illustrative Examples

Example-1:

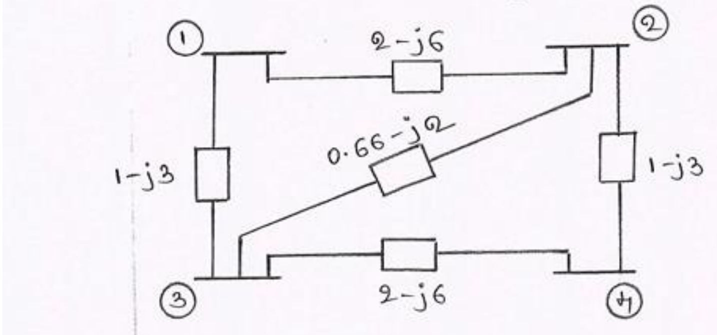
For a bus power system network shown below in fig, the generators are connected at all four buses, while loads are at buses 2 and 3. The real and reactive powers are listed below in table. Assuming a flat voltage start compute the unknown variables in all the buses other than the slack at the end of first GS iteration. Take acceleration factor as 1.4. (VTU QP)

Bus No.	P_i (pu)	Q_i (pu)	V_i (pu)
1	-	-	$1.04\angle 0^o$
2	0.5	-0.2	-
3	-1.0	0.5	-
4	0.3	-0.1	-



Solution:

Solⁿ:- PU admittance diagram.



$$Y_{bus} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.66-j11 & -0.66+j2 & -1+j3 \\ -1+j3 & -0.66+j2 & 3.66-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

Assume initial voltages as

$V_1^0 = 1.04\angle 0$, $V_2^0 = 1\angle 0$ $V_3^0 = 1\angle 0$ $V_4^0 = 1\angle 0$

we have

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ \neq i}}^{n} Y_{ik}V_k\right]$$

$P_2 = 0.5$, $P_3 = -1$ $P_4 = 0.3$

$Q_2 = -0.2$ $Q_3 = 0.5$ $Q_4 = -0.1$

$$\therefore V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - \left[Y_{21}V_1 + Y_{23}V_3 + Y_{24}V_4\right]\right]$$

$$= \frac{1}{3.66 - j11}\left[\frac{0.5 + j0.2}{1} - \left[(-2+j6)\times 1.04 + (-0.66+j2) + (-1+j3)\right]\right]$$

$$V_2^1 = 1.019 + j0.046 \text{ pu}$$

$$V_{2\,acc}^1 = V_2^0 + \alpha\left[V_2^1 - V_2^0\right]$$

$$= 1 + 1.4\left[1.019 + j0.046 - 1\right]$$

$$V_{2\,acc}^1 = 1.026 + j0.065 \text{ pu}$$

$$= 1.028 \angle 3.618^\circ \text{ pu}$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^*} - \left[Y_{31}V_1 + Y_{32}V_2 + Y_{34}V_4\right]\right]$$

$$= \frac{1}{3.66 - j11}\left[\frac{-1 - j0.5}{1} - \left[(-1+j3)\times 1.04 + \left[(-0.66+j2)\times(1.026+j0.065)\right] + (-2+j6)\right]\right]$$

$$V_3^1 = 1.029 - j0.083 \text{ pu.}$$

$$V_{3\,acc}^1 = 1 + 1.4\left(1.029 - j0.083 - 1\right)$$

$$= 1.04 - j0.1162 \text{ pu}$$

$$V_{3\,acc}^1 = 1.047 \angle -6.37^\circ \text{ pu}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^*} - \left[Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3\right]\right]$$

$$= \frac{1}{3 - j9}\left[\frac{0.3 + j0.1}{1} - \left[0 + (-1+j3)(1.026+j0.065) + (-2+j6)(1.04 - j0.1162)\right]\right]$$

$$V_4^1 = 1.035 - j0.022 \text{ pu}$$

$$V_{4\,acc}^1 = 1 + 1.4\left[1.035 - j0.022 - 1\right]$$

$$= 1.049 - j0.031 \text{ pu}$$

$$V_{4\,acc}^1 = 1.049 \angle -1.71^\circ \text{ pu}$$

At the end of first iteration

$V_1^1 = 1.04 \angle 0^\circ$ pu

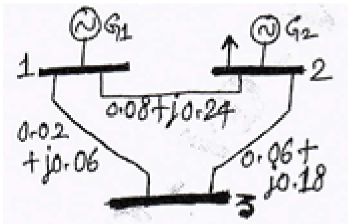
$V_{2acc}^1 = 1.028 \angle 3.618^\circ$ pu

$V_{3acc}^1 = 1.047 \angle -6.37^\circ$ pu

$V_{4acc}^1 = 1.049 \angle -1.71^\circ$ pu

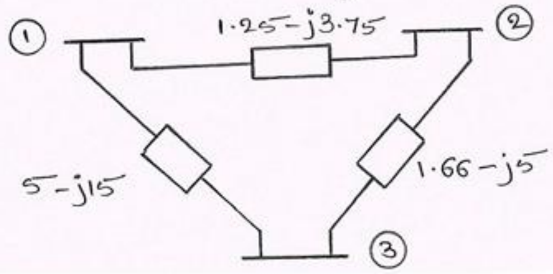
Example-2: For the power system network shown below in Fig., the line impedance are marked in pu. The bus data of the power system are shown below Table. Compute the voltage in all buses other than slack at the end of first iteration using Gauss-Seidel method. Take $0<Q_2<0.35$ pu. (VTU QP)

Bus No.	Voltage (pu)	Generation		Load	
		P_G	Q_G	P_D	Q_D
1	$0.05\angle 0^o$	-	-	-	-
2	1.03	0.2	-	0.5	0.2
3	-	0	0	0.6	0.25



Solution:

PU admittance diagram.



$$Y_{bus} = \begin{bmatrix} 6.25-j18.75 & -1.25+j3.75 & -5+j15 \\ -1.25+j3.75 & 2.91-j8.75 & -1.66+j5 \\ -5+j15 & -1.66+j5 & 6.66-j20 \end{bmatrix}$$

Assume initial voltages as

$V_1^0 = 1.05 \angle 0$, $V_2^0 = 1.03 \angle 0$ $V_3^0 = 1 \angle 0$

$P_2 = P_{G2} - P_{D2} = 0.2 - 0.5 = -0.3$ pu.

$Q_{D2} = 0.2$

$P_3 = -0.6$, $Q_3 = -0.25$

Calculate Q_2 at bus 2.

$$Q_{2cal} = -\text{Img}\left[V_2^* \sum_{k=1}^{n} Y_{2k} V_k\right]$$

$$= -\text{Img}\left[V_2^* \left[Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3\right]\right]$$

$$= -\text{Img}\left[1.03\left[(-1.25+j3.75)(1.05) + (2.91-j8.75)(1.03) + (-1.66+j5)\right]\right]$$

$$= 0.077 \text{ pu.}$$

$0 < 0.077 < 0.35$ ∴ Q_2 is within the limit

$$Q_2 = 0.077 - 0.2 = -0.123 \text{ pu.}$$

$$V_2' = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - \left[Y_{21}V_1 + Y_{23}V_3\right]\right]$$

$$= \frac{1}{2.91-j8.75}\left[\frac{-0.3+j0.123}{1.03} - \left[(-1.25+j3.75)(1.05) + (-1.66+j5)\right]\right]$$

$$= 0.99 - j0.025 \text{ pu}$$

$$V_2' = 0.99 \angle -1.48^\circ \text{ pu}$$

$$V_3' = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^*} - \left[Y_{31}V_1 + Y_{32}V_2\right]\right]$$

$$= \frac{1}{6.66-j20}\left[\frac{-0.6+j0.25}{1} - \left[(-5+j15)(1.05) + (-1.66+j5)(0.99-j0.025)\right]\right]$$

$$= 1.014 - j0.029 \text{ pu}$$

$$V_3' = 1.015 \angle -1.66^\circ \text{ pu.}$$

At the end of first itteration

$V_1' = 1.05 \angle 0^\circ$ pu

$V_2' = 0.99 \angle -1.48^\circ$ pu

$V_3' = 1.015 \angle -1.66^\circ$ pu.

Example-3: In Example-1, let bus 2 be a PV bus now with $|V_2|$ = 1.04 pu. Once again assuming a flat voltage start, find Q_2, δ_2, V_3, V_4 at the end of the first GS iteration.
Given: $0.2 \le Q_2 \le 1$ pu.

Solution:

From Eqn. (17), we get (Note: $\delta_2^0 = 0, ie., V_2^0 = 1.04 + j0$

$$\begin{aligned} Q_2^1 &= -\text{Im}\left((V_2^0)^* Y_{21} V_1 + (V_2^0)^* (Y_{22} V_2^0 + Y_{23} V_3^0 + Y_{24} V_4^0)\right) \\ &= -\text{Im}\,(1.04\,(-2 + j6)\,1.04 + 1.04\,((3.666 - j11)1.04 \\ &\quad + (-0.666 + j2) + (-1 + j3))) \\ &= -\text{Im}\,(-0.0693 - j0.2097) = 0.2079 \text{ pu} \end{aligned}$$

From Eqn. (18),

$$\delta_2^1 = \angle \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2^1}{\left(V_2^0\right)^*} - Y_{21} V_1 - Y_{23} V_3^0 - Y_{24} V_4^0 \right]$$

$$= \angle \frac{1}{Y_{22}} \left[\frac{0.5 - j0.2079}{1.04 - j0} - (-2 + j6)\,(1.04 + j0) - (-0.666 + j2) \right.$$

$$\left. (1 + j0) - (-1 + j3)\,(1 + j0) \right]$$

$$= \angle \left[\frac{4.2267 - j11.439}{3.666 - j11} \right] = \angle(1.0512 + j0.0339)$$

$$= 1.84658^\circ = 0.032 \text{ red}$$

$$\begin{aligned} V_2^1 &= 1.04 \left(\cos \delta_2^1 + j \sin \delta_2^1 \right) \\ &= 1.04\,(0.99948 + j0.0322) \\ &= 1.03946 + j0.03351 \end{aligned}$$

$$V_3^1 = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{\left(V_3^0\right)^*} - Y_{31} V_1 - Y_{32} V_2^1 - Y_{34} V_4^0 \right]$$

$$= \frac{1}{Y_{33}} \left[\frac{-1 - j0.5}{1 - j0} - (-1 + j3)1.04 - (-0.66 + j2) \right.$$

$$\left. (1.03946 + j0.03351) - (-2 + j6) \right]$$

$$= \frac{2.7992 - j11.6766}{3.666 - j11} = 1.0317 - j0.08937$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{\left(V_4^0\right)^*} - Y_{41}V_1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1 - j0} - (-1 + j3)\,(1.0394 + j0.0335) - (-2 + j6)\,(1.0317 - j0.08937)\right]$$

$$= \frac{2.9671 - j8.9962}{3 - j9} = 0.9985 - j0.0031$$

Now, suppose the permissible limits on Q_2 (reactive power injection) are revised as follows:

$$0.25 \leq Q2 \leq 1.0 \text{ pu}$$

It is clear that other data remaining the same, the calculated Q_2 (= 0.2079) is now less than the Q_{2min}. Hence Q_2 is set equal to Q_{2min} i.e.

$$Q_2 = 0.25 \text{ pu}$$

Bus 2, therefore, becomes a PQ bus from a PV bus. Therefore, $|V_2|$ can no longer remain fixed at 1.04 pu. The value of V_2 at the end of the first iteration is calculated as follows. (Note: V_2^0 = 1 + j0 by virtue of a flat start).

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{\left(V_2^0\right)^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{0.5 - j0.25}{1 - j0} - (-2 + j6)1.04 - (-0.666 + j2) - (-1 + j3)\right]$$

$$= \frac{4.246 - j11.49}{3.666 - j11} = 1.0559 + j0.0341$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{\left(V_3^0\right)^*} - Y_{31}V_1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{-1 - j0.5}{1 - j0} - (-1 + j3)\,1.04 - (-0.666 + j2)\,(1.0559 + j0.0341) - (-2 + j6)\right]$$

$$= \frac{2.8112 - j11.709}{3.666 - j11} = 1.0347 - j0.0893 \text{ pu}$$

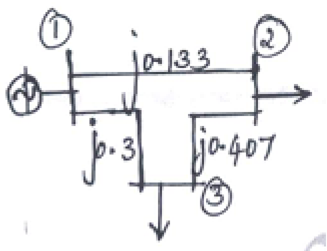
$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{\left(V_4^0\right)^*} - Y_{41}V_1 - Y_{42} - V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1 - j0} - (-1 + j3)\,(1.0509 + j0.0341) - (-2 + j6)\,(1.0347 - j0.0893)\right]$$

$$= \frac{4.0630 - j9.4204}{3 - j9} = 1.0775 + j0.0923 \text{ pu}$$

Example-4: For the power system network shown in Fig., the line reactances are in pu. The bus data are shown in Table. Determine the voltages at bus-2 and bus-3 at the end first iteration using Gauss-Seidel method. Take acceleration factor α = 1.6 (VTU July 2024 QP)

Bus No.	Type	Bus Voltage		Generation		Load	
		|V|	δ	P_G	Q_G	P_L	Q_L
1	Slack	1.05	-	-	-	-	-
2	PQ	-	-	0	0	0.7	0.8
3	PQ	-	-	0	0	0.5	0.2



Solution:

Step 1: Given Data

- Bus-1: Slack bus, $V_1=1.05\angle 0^o$.
- Bus-2: PQ bus, $P_L=0.7$, $Q_L=0.8$, $P_2=-0.7$, $Q_2=-0.8$, initial $V_2=1+j0$
- Bus-3: PQ bus, $P_L=0.5$, $Q_L=0.2$, $P3=-0.5$, $Q_3=-0.2$, initial $V_3=1+j0$

Step 2: Line Reactances:

$Z_{12}=j0.133$, $Z_{13}=j0.3$, $Z_{23}=j0.407$

Step 3: Form Y_{BUS} matrix

Admittances:

$$Y_{12} = -j\frac{1}{0.133} = -j7.519,$$
$$Y_{13} = -j\frac{1}{0.3} = -j3.333,$$
$$Y_{23} = -j\frac{1}{0.407} = -j2.457.$$

Diagonal Elements:

$$Y_{11} = j(7.519 + 3.333) = j10.852,$$
$$Y_{22} = j(7.519 + 2.457) = j9.976,$$
$$Y_{33} = j(3.333 + 2.457) = j5.790.$$

Thus,

$$Y_{\text{bus}} = \begin{bmatrix} j10.852 & -j7.519 & -j3.333 \\ -j7.519 & j9.976 & -j2.457 \\ -j3.333 & -j2.457 & j5.790 \end{bmatrix}$$

Step 4: Iteration Formula

For PQ buses:

$$V_i^{k+1} = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i}{(V_i^k)^*} - \sum_{j=1, j\neq i}^{n} Y_{ij} V_j^{k+1} \right)$$

Let's compute bus-2 and bus-3 voltages for the first iteration.

Iteration 1 - Bus 2:

Given:

$$P_2 = -0.7, Q_2 = -0.8, V_2^0 = 1 + j0, V_3^0 = 1 + j0, V_1 = 1.05\angle 0 = 1.05 + j0.$$

$$S_2 = P_2 + jQ_2 = -0.7 - j0.8$$

$$\frac{S_2}{(V_2^0)^*} = (-0.7 - j0.8) \cdot (1 - j0) = -0.7 - j0.8$$

$$\sum_{j\neq 2} Y_{2j} V_j = (-j7.519)(1.05 + j0) + (-j2.457)(1 + j0)$$

$$= -j7.895 - j2.457 = -j10.352$$

$$V_2^{k+1} = \frac{1}{j9.976} [-0.7 - j0.8 - (-j10.352)]$$

$$= \frac{1}{j9.976} [-0.7 + j9.552]$$

$$= \frac{-0.7 + j9.552}{j9.976}$$

Multiply numerator and denominator by -j:

$$= \frac{(-0.7)(-j) + j9.552(-j)}{(-j)(j9.976)} = \frac{0.7j + 9.552}{9.976}$$

$$= \frac{9.552 + j0.7}{9.976}$$

$$= 0.9577 + j0.0702$$

Acceleration:

$$V_2^{\text{new}} = V_2^0 + \alpha(V_2^1 - V_2^0) = 1 + j0 + 1.6 \times [(0.9577 + j0.0702) - (1 + j0)]$$
$$= 1 + j0 + 1.6 \times (-0.0423 + j0.0702)$$
$$= 1 - 0.0677 + j0.1123 = 0.9323 + j0.1123$$

Iteration 1 - Bus 3:

$$P_3 = -0.5, Q_3 = -0.2, S_3 = -0.5 - j0.2$$
$$\frac{S_3}{(V_3^0)^*} = -0.5 - j0.2$$
$$\sum_{j \neq 3} Y_{3j} V_j = (-j3.333)(1.05) + (-j2.457)(V_2^{\text{new}})$$
$$= -j3.5 + (-j2.457)(0.9323 + j0.1123)$$

First compute:

$$-j2.457 \times (0.9323 + j0.1123) = -j2.457 \times 0.9323 - 2.457 \times 0.1123$$
$$= -j2.291 + 0.2756 - j^2(2.457 \times 0.1123) = -j2.291 + 0.2756 + 0.276$$
$$= 0.5516 - j2.291$$

Total Sum:

$$-j3.5 + (0.5516 - j2.291) = 0.5516 - j5.791$$
$$V_3^1 = \frac{1}{j5.790}[-0.5 - j0.2 - (0.5516 - j5.791)]$$
$$= \frac{-1.0516 + j5.591}{j5.790}$$

Multiply numerator and denominator by -j:

$$= \frac{-1.0516(-j) + j5.591(-j)}{5.790} = \frac{1.0516j + 5.591}{5.790}$$
$$= \frac{5.591}{5.790} + j\frac{1.0516}{5.790}$$
$$= 0.9656 + j0.1816$$

Acceleration:

$$V_3^{\text{new}} = 1 + j0 + 1.6 \times [(0.9656 + j0.1816) - (1 + j0)]$$
$$= 1 - 0.0542 + j0.2906 = 0.9458 + j0.2906$$

Final Answer after 1st iteration:

V_2 = 0.9323 + j0.1123

V_3 = 0.9458 + j0.2906