IA-3-Copy1

October 30, 2020

1 Problem 1:

In this problem, we are going to learn the optimal strategy of selecting the right path. Consider the a naive setting of the shortest path problem with only 2 arcs from source 1 to sink 2, as shown in Figure below.

The optimal path can be obtained by solving $\min(c_1, c_2)$ where c_1 and c_2 are the cost on the arcs respectively. While it is a naive problem, we can still formulate it as an integer program (whose LP relaxation is tight):

$$\begin{aligned} \min \quad & c_1x_1 + c_2x_2 \\ \text{s.t.} \quad & x_1 + x_2 = 1 \\ & x_1, x_2 \in \{0, 1\}. \end{aligned}$$

After learning an optimal strategy, when we see a new instance with (c_1, c_2), instead of solving an optimization problem(even a simple one), we can use the strategy to select the path.

1.0.1 (1) Generate 50 traning samples and 50 test samples with c_1 uniformly distributed in $[2, 5]$ and c_2 uniformly distributed in $[3, 7]$. For each data point with (c_1, c_2), solve for the optimal path (x_1^*, x_2^*). [For optimization part, we used a solver.]

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
#set random seed
np.random.seed(23)
```

[2]:
```
## Generate training set for c1 and c2
c1_training= np.random.uniform(2,5,50)
c2_training= np.random.uniform(3,7,50)
## Generate test set for c1 and c2
c1_test= np.random.uniform(2,5,50)
c2_test= np.random.uniform(3,7,50)
```

[3]:
```
# function for optimal path
from gurobipy import *
def get_solution(c1,c2):
```

```python
    m = Model("P1 model")

    # Create variables
    x1 = m.addVar(vtype=GRB.BINARY, name="x12")
    x2 = m.addVar(vtype=GRB.BINARY, name="x13")

    # Set objective
    m.setObjective(c1*x1+c2*x2, GRB.MINIMIZE)

    # Add constraint:
    m.addConstr(x1 + x2  == 1, "c0")
    m.params.outputflag = 0; # suppress the output
    m.optimize()
    return [round(i.x) for i in m.getVars()]
```

[4]:
```python
# solve for the optimal path (training)
path_training=[]
for i in range (0,50):
    path=get_solution(c1_training[i],c2_training[i])
    path_training.append(path)
```

```
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```

[5]:
```python
# solve for the optimal path (testing)
path_test=[]
for i in range (0,50):
    path=get_solution(c1_test[i],c2_test[i])
    path_test.append(path)
```

[6]:
```python
# Number of path in each set
print("The number of [1,0] path in training set is", path_training.count([1,0]))
print("The number of [0,1] path in training set is", path_training.count([0,1]))


print("The number of [1,0] path in test set is", path_test.count([1,0]))
print("The number of [0,1] path in test set is", path_test.count([0,1]))
```

```
The number of [1,0] path in training set is 39
The number of [0,1] path in training set is 11
The number of [1,0] path in test set is 44
The number of [0,1] path in test set is 6
```

1.0.2 (2) Consider the strategy learning task as a binary classification problem with classes labeled by two feasible paths, i.e., $(x_1^*, x_2^*) = (1, 0)$ or $(x_1^*, x_2^*) = (0, 1)$. Develop classification models using Logistic Regression, SVM and Classification tree.

[7]:
```
#Training set
#Set up a dataframe for training set
import warnings
warnings.filterwarnings('ignore')
import pandas as pd
training_all={'c1_training': c1_training, 'c2_training': c2_training,␣
 ↪'path_training':path_training}
training_df = pd.DataFrame(training_all)

#Label the path with "1" and "0" for [1,0] and [0,1] respectively
training_df['y_train']=""
for i in range (0,50):
    training_df['y_train'][i] = training_df['path_training'][i][0]
#Convert the type from object to numeric
training_df['y_train'] = pd.to_numeric(training_df['y_train'])
```

[8]:
```
#store independent variables of training data
X_train = training_df[['c1_training','c2_training']]

#store dependent variables of training data
y_train = training_df['y_train']
```

[9]:
```
#test set
#Set up a dataframe for test set
test_all={'c1_test': c1_test, 'c2_test': c2_test, 'path_test':path_test}
test_df = pd.DataFrame(test_all)

#Label the path with "1" and "0" for [1,0] and [0,1] respectively
test_df['y_test']=""
for i in range (0,50):
    test_df['y_test'][i] = test_df['path_test'][i][0]
#Convert the type from object to numeric
test_df['y_test'] = pd.to_numeric(test_df['y_test'])
```

[10]:
```
#store independent variables of training data
X_test = test_df[['c1_test','c2_test']]

#store dependent variables of training data
y_test = test_df['y_test']
```

Develop classification model using Logistic Regression

[11]:
```python
#Develop classification model using Logistic Regression
from sklearn.linear_model import LogisticRegression
lr = LogisticRegression()
lr.fit(X_train, y_train)
y_pred_lr = lr.predict(X_test)

#Store the prediction result into the dataframe
test_df['y_pred_lr']=y_pred_lr
```

Develop classification model using SVM

[12]:
```python
#Develop classification model using SVM
from sklearn.svm import SVC

svm = SVC(kernel='linear')
svm.fit(X_train, y_train)
y_pred_svm = svm.predict(X_test)

#Store the prediction result into the dataframe
test_df['y_pred_svm']=y_pred_svm
```

Develop classification model using Classification tree

[13]:
```python
#Develop classification model using Classification tree
from sklearn.tree import DecisionTreeClassifier
dt = DecisionTreeClassifier()
dt.fit(X_train, y_train)
y_pred_dt = dt.predict(X_test)

#Store the prediction result into the dataframe
test_df['y_pred_dt']=y_pred_dt
```

1.0.3 (3) Evaluate the performance of the classification models above.

[14]:
```python
# compare by plain accuracy
test_df['acc_lr'] = test_df['y_test']==test_df['y_pred_lr']
test_df['acc_svm'] = test_df['y_test']==test_df['y_pred_svm']
test_df['acc_dt'] = test_df['y_test']==test_df['y_pred_dt']
```

[15]:
```python
#Accuracy of training set
print('Accuracy score with using Logistic Regression: '+ str(lr.score(X_train,␣
 ↪y_train)))
print('Accuracy score with using SVM: '+ str(svm.score(X_train, y_train)))
print('Accuracy score with using Classification tree: '+ str(dt.score(X_train,␣
 ↪y_train)))
```

```
Accuracy score with using Logistic Regression: 1.0
```

```
Accuracy score with using SVM: 1.0
Accuracy score with using Classification tree: 1.0
```

[16]:
```python
#Accuracy of test set
print('Using Logistic Regression, ',sum(test_df['acc_lr']), 'out of 50 test␣
↪data are predicted correctly')
print('Using SVM, ',sum(test_df['acc_svm']), 'out of 50 test data are predicted␣
↪correctly')
print('Using Classification Tree, ',sum(test_df['acc_dt']), 'out of 50 test␣
↪data are predicted correctly')
```

```
Using Logistic Regression,  48 out of 50 test data are predicted correctly
Using SVM,  48 out of 50 test data are predicted correctly
Using Classification Tree,  47 out of 50 test data are predicted correctly
```

[17]:
```python
#Accuracy of test set
print('Accuracy score with using Logistic Regression: '+ str(lr.score(X_test,␣
↪y_test)))
print('Accuracy score with using SVM: '+ str(svm.score(X_test, y_test)))
print('Accuracy score with using Classification tree: '+ str(dt.score(X_test,␣
↪y_test)))
```

```
Accuracy score with using Logistic Regression: 0.96
Accuracy score with using SVM: 0.96
Accuracy score with using Classification tree: 0.94
```

In general, we can see that all three models can attain a high prediction accuracy, since all of them are with higher than 90% correct prediction in test set.

Logistic Regression model and SVM model seem to be more accurate than Classification tree model in this trial, but not significant and not necessarily in a way that in other trial with setting other random seed , Classification tree model could be as accurate as the others.

[18]:
```python
#Print out the result
pd.set_option('display.max_rows', 10)
test_df
```

[18]:
```
     c1_test   c2_test path_test  y_test  y_pred_lr  y_pred_svm  y_pred_dt  \
0   3.930381  5.952160    [1, 0]       1          1           1          1
1   3.705049  5.265256    [1, 0]       1          1           1          1
2   2.720546  6.177517    [1, 0]       1          1           1          1
3   2.887120  6.709040    [1, 0]       1          1           1          1
4   2.015075  3.323449    [1, 0]       1          1           1          1
..       …         …         …     …       …          …          …
45  2.894699  4.922747    [1, 0]       1          1           1          1
46  2.555579  6.963164    [1, 0]       1          1           1          1
47  4.342558  4.972993    [1, 0]       1          1           1          1
48  2.465644  6.311469    [1, 0]       1          1           1          1
49  3.803357  4.105130    [1, 0]       1          1           1          0
```

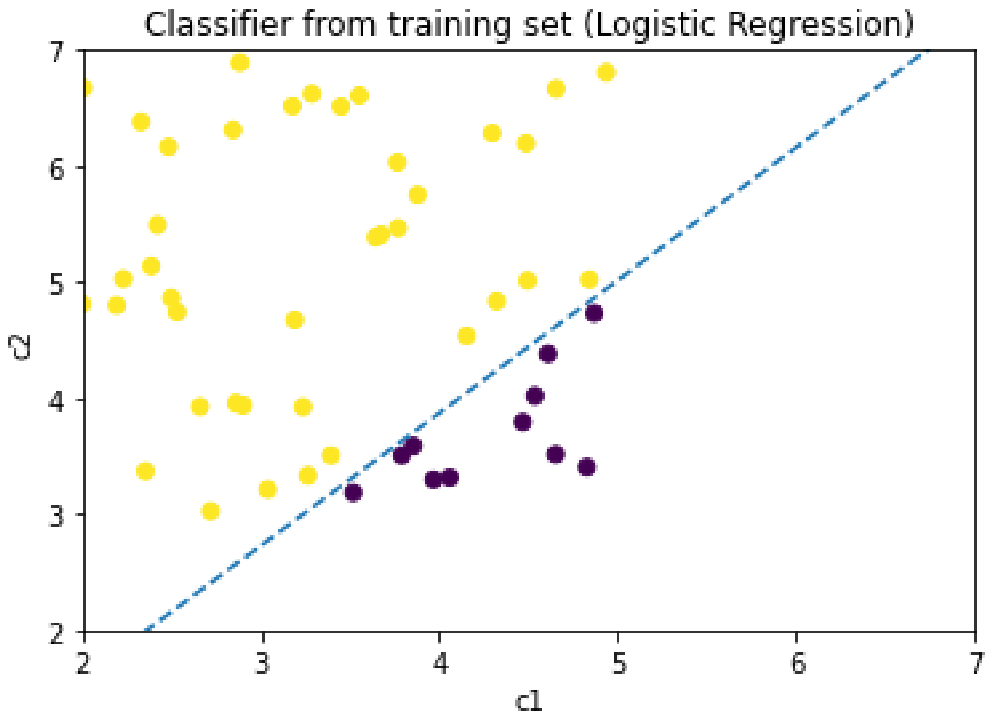
```
    acc_lr  acc_svm  acc_dt
0     True     True    True
1     True     True    True
2     True     True    True
3     True     True    True
4     True     True    True
..     …        …      …
45    True     True    True
46    True     True    True
47    True     True    True
48    True     True    True
49    True     True   False

[50 rows x 10 columns]
```

1.0.4 (4) Briefly explain the optimal strategy obtained from Logistic Regression, SVM, and Tree respectively.

Logistic Regression:

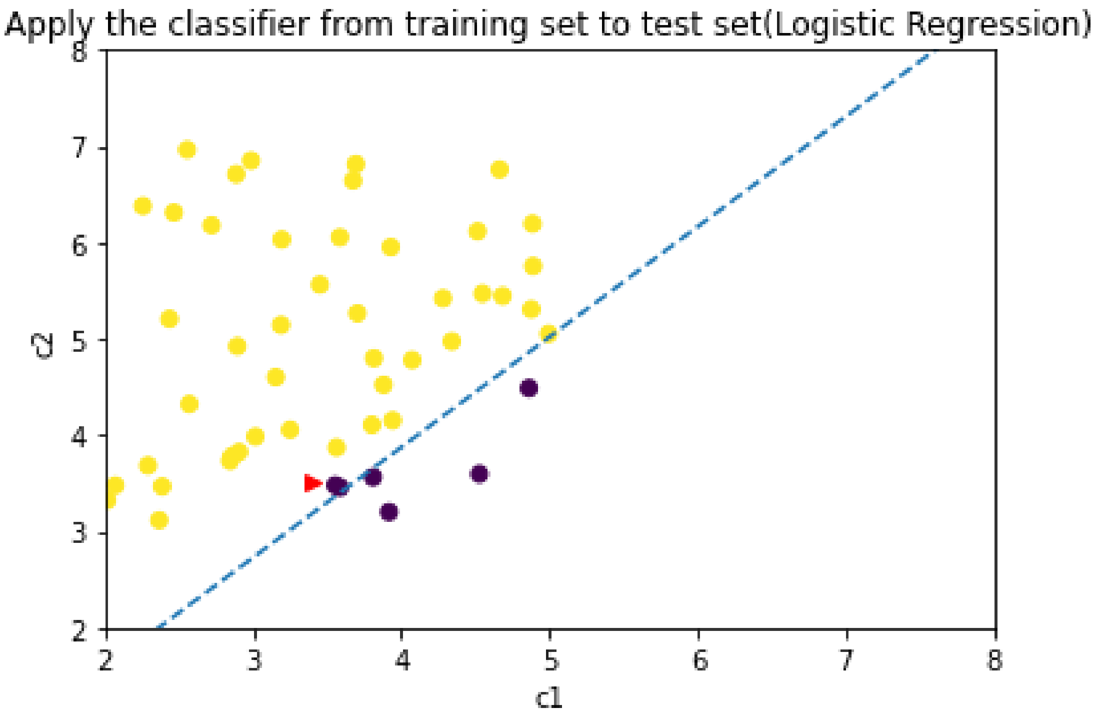
[19]:
```python
print(lr.coef_)
print(lr.intercept_)
# Graph for logistic regression (training set)
a = lr.coef_[0][0]
b = lr.coef_[0][1]
c = lr.intercept_[0]
x_plt = [1, 2, 3, 4, 5, 6, 7]
y_plt = [(-a/b) * i + (-c/b) for i in x_plt]

plt.scatter(training_df['c1_training'], training_df['c2_training'],␣
 ↪c=training_df['y_train'])
plt.title('Classifier from training set (Logistic Regression)')
plt.plot(x_plt, y_plt, '--')
plt.ylabel('c2')
plt.xlabel('c1')
plt.xlim(2,7)
plt.ylim(2,7)
plt.show()
```

```
[[-1.91504398  1.67297515]]
[1.18247585]
```



The boundary classifier (blue dotted line) was trained from the above training datapoints.

[20]:
```
# Graph for logistic regression (test set)
a = lr.coef_[0][0]
b = lr.coef_[0][1]
c = lr.intercept_[0]
x_plt = [1, 2, 3, 4, 5, 6, 7, 8]
y_plt = [(-a/b) * i + (-c/b) for i in x_plt]

plt.scatter(test_df['c1_test'], test_df['c2_test'], c=test_df['y_test'])
plt.plot(x_plt, y_plt, '--')
plt.ylabel('c2')
plt.xlabel('c1')
plt.title('Apply the classifier from training set to test set(Logistic␣
 ↪Regression)')
plt.plot(3.4,3.5, marker='>',c='red')
plt.xlim(2,8)
plt.ylim(2,8)
plt.show()
```



The graph above is divided by the boundary classifier trained from the training set (/previous graph).

The dots above are the datapoints from test set.

The model would predict the y_label "1" (which means the path [1, 0]) if the datapoint is at the left side of the line.

Meanwhile, the model would predict the y_label "0" (which means the path [0, 1]) if the datapoint is at the right side of the line.

The color of the datapoints represents the true optimal path/ true label of the testing set, yellow dot for the true optimal path [1, 0] and purple dot for the true optimal path [0, 1].

We can note the two wrongly predicted datapoints [pointed by red arrow and noted from answer for question (3)].

SVM:

[21]:
```
print(svm.coef_)
print(svm.intercept_)

# Graph for SVM (training set)
d = svm.coef_[0][0]
e = svm.coef_[0][1]
f = svm.intercept_[0]
x_plt_svm = [1, 2, 3, 4, 5, 6, 7, 8]
```

```
y_plt_svm = [(-d/e) * i + (-f/e) for i in x_plt_svm]


plt.scatter(svm.support_vectors_[:,0],svm.support_vectors_[:,1], c='pink',␣
 ↪s=150)
plt.scatter(training_df['c1_training'], training_df['c2_training'],␣
 ↪c=training_df['y_train'])
plt.plot(x_plt_svm, y_plt_svm, '--')
plt.ylabel('c2')
plt.xlabel('c1')
plt.title('Classifier from training set (SVM)')
plt.xlim(2,8)
plt.ylim(2,8)
plt.show()
```

```
[[-1.9880577  1.7547084]]
[1.10148766]
```

The training datapoints with pink circles are those support vectors which decide the margin and hence the decision boundary (linear classifier).

[22]:
```
# Graph for SVM (test set)
d = svm.coef_[0][0]
e = svm.coef_[0][1]
```

```python
f = svm.intercept_[0]
x_plt_svm = [1, 2, 3, 4, 5, 6, 7, 8]
y_plt_svm = [(-d/e) * i + (-f/e) for i in x_plt_svm]

plt.scatter(test_df['c1_test'], test_df['c2_test'], c=test_df['y_test'])
plt.plot(x_plt_svm, y_plt_svm, '--')
plt.ylabel('c2')
plt.xlabel('c1')
plt.title('Apply the classifier from training set to test set(SVM)')
plt.plot(3.4,3.5, marker='>',c='red')
plt.xlim(2,8)
plt.ylim(2,8)
plt.show()
```

The graph above is divided by the decision boundary from the training set (previous graph).

The dots above are the datapoint from test set.

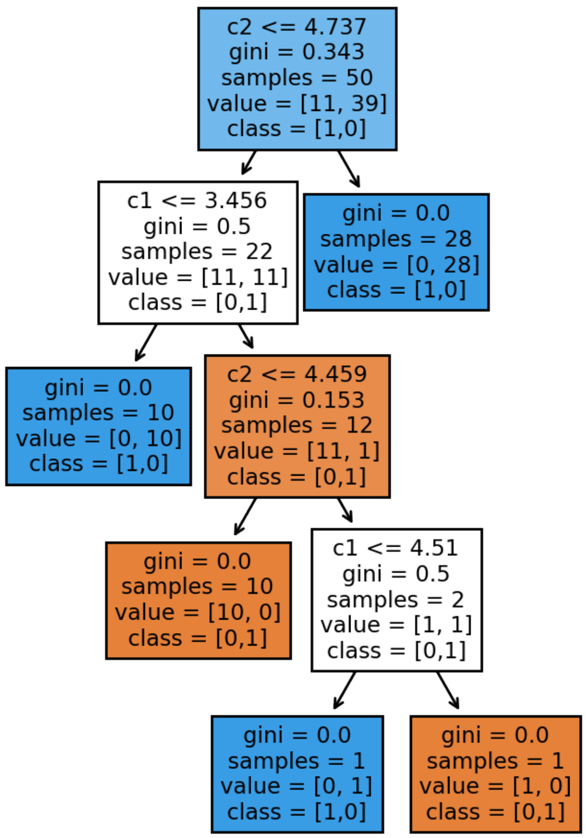
The model would predict the y_label "1" (which means the path [1, 0]) if the datapoint is at the left side of the line.

Meanwhile, the model would predict the y_label "0" (which means the path [0, 1]) if the datapoint is at the right side of the line.

The color of the datapoints represents the true optimal path/ true label of the testing set, yellow dot for the true optimal path [1, 0] and purple dot for the true optimal path [0, 1].

[23]:
```
# optimal strategy obtained from Tree
import sklearn
from sklearn import tree
fig, axes = plt.subplots(figsize = (4,6), dpi=200)
tree.plot_tree(dt, filled=True, feature_names=['c1', 'c2'],␣
↪class_names=['[0,1]','[1,0]'])
fig.text(0, 0,'')
```

[23]:
```
Text(0, 0, '')
```



The above graph shows the decision rules in the tree model.

For example, the model will first examine if c2 is smaller or equal to 4.737. If so (go to the left), the model would then examine if c1 is smaller than or equal to 3.456 . If so, it would predict the optimal path as [1,0]. If c1 is greater than 3.456 (go to the right), then it would further go down the tree plot to predict the optimal path.

1.0.5 (5) Insight from the result

This is a naive setting of the shortest path problem with only 2 arcs from source 1 to sink 2, which means that there are only 2 features (c1 and c2) and only two labels (/pahts) to classify in this classification problem. All three models can get a high accurary score (all higher than 90%) as mentioned above.

The "true" decision boundry is linear in this setting (i.e. c1 = c2), therefore it might explain why logistic regression and linear SVM models could be more accurate than classification tree model in this naive setting.

2 Problem 2:

Now, we consider a (not-so-naive) shortest path problem as shown in Figure below:

Formulate the shortest path problem as:

$$
\begin{aligned}
\min \quad & c_{13}x_{13} + c_{12}x_{12} + c_{25}x_{25} + c_{34}x_{34} + c_{35}x_{35} + c_{46}x_{46} + c_{56}x_{56} \\
\text{s.t.} \quad & x_{12} + x_{13} = 1 \\
& x_{12} = x_{25} \\
& x_{13} = x_{34} + x_{35} \\
& x_{34} = x_{46} \\
& x_{35} + x_{25} = x_{56} \\
& x_{12}, x_{13}, x_{25}, x_{34}, x_{35}, x_{46}, x_{56} \in \{0, 1\}.
\end{aligned}
$$

By learning an optimal strategy, when we see a new instance with parameters $(c_{25}, c_{35}, c_{34}, c_{46}, c_{56})$, instead of solving the optimization problem, we can apply the optimal strategy to select the path.

[24]:
```
import numpy as np
from gurobipy import *
import pandas as pd
from sklearn.linear_model import LogisticRegression
from sklearn.svm import LinearSVC
from sklearn.tree import DecisionTreeClassifier
import sklearn
from sklearn import tree
import warnings
warnings.filterwarnings('ignore')

#set random seed
np.random.seed(23)
```

2.0.1 (1) Generate 50 traning samples and 50 test samples with c_1 uniformly distributed in $[2,5]$ and c_2 uniformly distributed in $[3,7]$. For each data point with (c_1, c_2), solve for the optimal path (x_1^*, x_2^*). [For optimization part, we used a solver.]

[25]:
```
## Generate training samples for  ( 25, 35, 34, 46, 56)
c25_training= np.random.uniform(4,8,50)
c35_training= np.random.uniform(8,10,50)
c34_training= np.random.uniform(14,20,50)
c46_training= np.random.uniform(2,4,50)
c56_training= np.random.uniform(8,10,50)

## Generate test samples for ( 25, 35, 34, 46, 56)
c25_test= np.random.uniform(4,8,50)
c35_test= np.random.uniform(8,10,50)
c34_test= np.random.uniform(14,20,50)
c46_test= np.random.uniform(2,4,50)
c56_test= np.random.uniform(8,10,50)
```

[26]:
```
# The optimization model is provided to generate data
from gurobipy import *
def get_solution(c25,c35,c34,c46,c56):
    m = Model("P2 model")

    # Create variables
    x12 = m.addVar(vtype=GRB.BINARY, name="x12")
    x13 = m.addVar(vtype=GRB.BINARY, name="x13")
    x25 = m.addVar(vtype=GRB.BINARY, name="x25")
    x34 = m.addVar(vtype=GRB.BINARY, name="x34")
    x35 = m.addVar(vtype=GRB.BINARY, name="x35")
    x46 = m.addVar(vtype=GRB.BINARY, name="x46")
    x56 = m.addVar(vtype=GRB.BINARY, name="x56")

    c13 = 8
    c12 = 12
    # Set objective
    m.setObjective(c13*x13+c12*x12+c25*x25+c34*x34+c35*x35+c46*x46+c56*x56, GRB.
 ↪MINIMIZE)

    # Add constraint:
    m.addConstr(x12 + x13  == 1, "c0")
    m.addConstr(x12 == x25, "c1")
    m.addConstr(x13 == x34 + x35, "c1")
    m.addConstr(x34 == x46, "c1")
    m.addConstr(x35 + x25 == x56, "c1")
    m.params.outputflag = 0; # suppress the output
    m.optimize()
```

```python
    return [round(i.x) for i in m.getVars()]
```

[27]:
```python
# solve for the optimal path (training)
path_training=[]
for i in range (0,50):
    ␣
 ↪path=get_solution(c25_training[i],c35_training[i],c34_training[i],c46_training[i],c56_train
    path_training.append(path)
```

[28]:
```python
# solve for the optimal path (test)
path_test=[]
for i in range (0,50):
    ␣
 ↪path=get_solution(c25_test[i],c35_test[i],c34_test[i],c46_test[i],c56_test[i])
    path_test.append(path)
```

[29]:
```python
# Number of path in training set
print("The number of [0, 1, 0, 1, 0, 1, 0] path in training set is",␣
 ↪path_training.count([0, 1, 0, 1, 0, 1, 0]))
print("The number of [0, 1, 0, 0, 1, 0, 1] path in training set is",␣
 ↪path_training.count([0, 1, 0, 0, 1, 0, 1]))
print("The number of [1, 0, 1, 0, 0, 0, 1] path in training set is",␣
 ↪path_training.count([1, 0, 1, 0, 0, 0, 1]))
```

```
The number of [0, 1, 0, 1, 0, 1, 0] path in training set is 8
The number of [0, 1, 0, 0, 1, 0, 1] path in training set is 31
The number of [1, 0, 1, 0, 0, 0, 1] path in training set is 11
```

[30]:
```python
# Number of path in test set
print("The number of [0, 1, 0, 1, 0, 1, 0] path in test set is", path_test.
 ↪count([0, 1, 0, 1, 0, 1, 0]))
print("The number of [0, 1, 0, 0, 1, 0, 1] path in test set is", path_test.
 ↪count([0, 1, 0, 0, 1, 0, 1]))
print("The number of [1, 0, 1, 0, 0, 0, 1] path in test set is", path_test.
 ↪count([1, 0, 1, 0, 0, 0, 1]))
```

```
The number of [0, 1, 0, 1, 0, 1, 0] path in test set is 8
The number of [0, 1, 0, 0, 1, 0, 1] path in test set is 30
The number of [1, 0, 1, 0, 0, 0, 1] path in test set is 12
```

2.0.2 (2) Consider the strategy learning task as a classification problem with classes labeled by three feasible paths. Develop classification models using Logistic Regression, SVM and Classification tree.

Now we have 3 paths in total, let us define the path label:

Path “0” is equivalent to [0, 1, 0, 1, 0, 1, 0] / “Node1–>Node3–>Node4–>Node6”

Path “1” is equivalent to [0, 1, 0, 0, 1, 0, 1] / “Node1–>Node3–>Node5–>Node6”

Path “2” is equivalent to [1, 0, 1, 0, 0, 0, 1] / “Node1–>Node2–>Node5–>Node6”

[31]:
```
#Set up function to label path
def label (df):
    if df['path_raw'] == [0, 1, 0, 1, 0, 1, 0]:
        return 0
    if df['path_raw'] == [0, 1, 0, 0, 1, 0, 1]:
        return 1
    if df['path_raw'] == [1, 0, 1, 0, 0, 0, 1]:
        return 2
```

[32]:
```
#Training set
#Set up a dataframe for training set
training_all={'c25_training': c25_training, 'c35_training': c35_training,␣
 ↪'c34_training': c34_training, 'c46_training': c46_training, 'c56_training':␣
 ↪c56_training, 'path_raw': path_training}
training_df = pd.DataFrame(training_all)

#Label the path
training_df['path_label'] = training_df.apply(label, axis=1)
```

[33]:
```
#store independent variables of training data
X_train =␣
 ↪training_df[['c25_training','c35_training','c34_training','c46_training','c56_training']]

#store dependent variables of training data
y_train = training_df['path_label']
```

[34]:
```
#Training set
#Set up a dataframe for training set
test_all={'c25_test': c25_test, 'c35_test': c35_test, 'c34_test': c34_test,␣
 ↪'c46_test': c46_test, 'c56_test': c56_test, 'path_raw': path_test}
test_df = pd.DataFrame(test_all)

#Label the path
test_df['path_label'] = test_df.apply(label, axis=1)
```

[35]:
```
#store independent variables of test data
X_test = test_df[['c25_test','c35_test','c34_test','c46_test','c56_test']]

#store dependent variables of test data
y_test = test_df['path_label']
```

Develop classification model using Logistic Regression

[36]:
```python
#Develop classification model using Logistic Regression
lr2 = LogisticRegression()
lr2.fit(X_train, y_train)
y_pred_lr2 = lr2.predict(X_test)

#Store the prediction result into the dataframe
test_df['y_pred_lr2']= y_pred_lr2
```

Develop classification model using SVM

[37]:
```python
#Develop classification model using SVM (here we use LinearSVC)
svm2 = SVC(kernel='linear')
svm2.fit(X_train, y_train)
y_pred_svm2 = svm2.predict(X_test)

#Store the prediction result into the dataframe
test_df['y_pred_svm2'] = y_pred_svm2
```

Develop classification model using Classification tree

[38]:
```python
#Develop classification model using Classification tree
dt2 = DecisionTreeClassifier()
dt2.fit(X_train, y_train)
y_pred_dt2 = dt2.predict(X_test)

#Store the prediction result into the dataframe
test_df['y_pred_dt2'] = y_pred_dt2
```

2.0.3 (3) Evaluate the performance of the classification models above.

[39]:
```python
# compare by plain accuracy
test_df['acc_lr'] = test_df['path_label']==test_df['y_pred_lr2']
test_df['acc_svm'] = test_df['path_label']==test_df['y_pred_svm2']
test_df['acc_dt'] = test_df['path_label']==test_df['y_pred_dt2']
```

[40]:
```python
#Accuracy of training set
print('Accuracy score with using Logistic Regression: '+ str(lr2.score(X_train,␣
 ↪y_train)))
print('Accuracy score with using SVM: '+ str(svm2.score(X_train, y_train)))
print('Accuracy score with using Classification tree: '+ str(dt2.score(X_train,␣
 ↪y_train)))
```

```
Accuracy score with using Logistic Regression: 0.98
Accuracy score with using SVM: 0.98
Accuracy score with using Classification tree: 1.0
```

[41]:
```
#Accuracy of test set
print('Using Logistic Regression, ',sum(test_df['acc_lr']), 'out of 50 test␣
↪data are predicted correctly')
print('Using SVM (LinearSVC), ',sum(test_df['acc_svm']), 'out of 50 test data␣
↪are predicted correctly')
print('Using Classification Tree, ',sum(test_df['acc_dt']), 'out of 50 test␣
↪data are predicted correctly')
```

```
Using Logistic Regression,  43 out of 50 test data are predicted correctly
Using SVM (LinearSVC),  42 out of 50 test data are predicted correctly
Using Classification Tree,  36 out of 50 test data are predicted correctly
```

[42]:
```
#Accuracy of test set
print('Accuracy score with using Logistic Regression: '+ str(lr2.score(X_test,␣
↪y_test)))
print('Accuracy score with using SVM: '+ str(svm2.score(X_test, y_test)))
print('Accuracy score with using Classification tree: '+ str(dt2.score(X_test,␣
↪y_test)))
```

```
Accuracy score with using Logistic Regression: 0.86
Accuracy score with using SVM: 0.84
Accuracy score with using Classification tree: 0.72
```

In general, we can see that all three models can attain a reasonably accurate prediction, as all of them attain at least 70% correct prediction in test set.

Here in this setting, Logistic Regression model and SVM (LinearSVC is used here) model seem to be more accurate than Classification tree model in this trial significantly when we compare the plain accurary.

[43]:
```
#Print out the result
test_df
```

[43]:
```
    c25_test  c35_test   c34_test  c46_test  c56_test               path_raw  \
0   6.408535  9.694227  19.366879  2.506404  8.386006  [0, 1, 0, 0, 1, 0, 1]
1   4.339591  8.684791  19.874508  2.321841  8.845113  [1, 0, 1, 0, 0, 0, 1]
2   4.755159  8.289720  14.501771  3.741455  9.610198  [0, 1, 0, 0, 1, 0, 1]
3   7.092343  8.367297  16.687302  3.375479  9.715388  [0, 1, 0, 0, 1, 0, 1]
4   6.361216  8.327014  15.364057  2.760406  9.866211  [0, 1, 0, 1, 0, 1, 0]
..       …         …          …         …         …                    …
45  7.244055  8.790472  14.496991  2.602558  8.017081  [0, 1, 0, 0, 1, 0, 1]
46  6.864591  9.688125  19.094762  2.455682  9.868167  [0, 1, 0, 0, 1, 0, 1]
47  7.323541  8.596973  18.323895  3.457574  8.312303  [0, 1, 0, 0, 1, 0, 1]
48  4.587840  8.388095  19.942889  2.539947  8.424251  [0, 1, 0, 0, 1, 0, 1]
49  6.361003  9.742304  16.675067  3.499646  8.333306  [0, 1, 0, 0, 1, 0, 1]

    path_label  y_pred_lr2  y_pred_svm2  y_pred_dt2  acc_lr  acc_svm  acc_dt
0            1           1            1           1    True     True    True
```

```
1            2            2            2            2     True     True     True
2            1            1            1            0     True     True    False
3            1            1            1            1     True     True     True
4            0            1            1            1    False    False    False
..         …        …          …        …        …      …      …
45           1            1            1            1     True     True     True
46           1            1            1            1     True     True     True
47           1            1            1            1     True     True     True
48           1            1            2            2     True    False    False
49           1            1            1            1     True     True     True

[50 rows x 13 columns]
```

2.0.4 (4) Briefly explain the optimal strategy obtained from Logistic Regression, SVM, and Tree respectively.

Since we have 5 features in this problem (namely, 25, 35, 34, 46, 56), therefore it is hard to draw the boundary for logistic regression model and the SVM model. Alternatively, we will print out the boundary in equation format for respective model.

Logistic regression:

[44]:
```python
lr2_coef = lr2.coef_
lr2_intercept = lr2.intercept_
```

[45]:
```python
for j in range (0,3):
    print('Equation ' + str(j+1))
    print('0 = '+ str(lr2_intercept[j]) + ' + ('+ str(lr2_coef[j][0])+')c25' + ' + ('+ str(lr2_coef[j][1])+')c35' + ' + ('+ str(lr2_coef[j][2])+')c34' + ' + ('+ str(lr2_coef[j][3])+')c46' + ' + ('+ str(lr2_coef[j][4])+')c56')
    print('')
```

```
Equation 1
0 = 3.9776100633675733 + (0.3045763325013224)c25 + (0.9507897154602677)c35 +
(-1.0692425173642752)c34 + (-0.5846034680570129)c46 + (0.4797521800344944)c56

Equation 2
0 = 9.12416693964876 + (1.1001618791580012)c25 + (-1.7320148253479917)c35 +
(0.4209324019359374)c34 + (0.17372558898803972)c46 + (-0.6677300690943335)c56

Equation 3
0 = -13.101777003016299 + (-1.4047382116593992)c25 + (0.7812251098874555)c35 +
(0.648310115428761)c34 + (0.4108778790689498)c46 + (0.18797788905997642)c56
```

The above hyperplanes define the boundary in the logistic regression model, which helps predict the test datapoints being classified into label '0', label '1' and label '2'.

SVM model:

[46]:
```
#Support Vectors
print(svm2.support_vectors_)
```

```
[[ 7.91370766  9.90317552 14.51297458  3.68923497  8.78958372]
 [ 6.50473215  9.37589275 16.50216864  2.52710877  9.66974945]
 [ 5.71440725  9.81095045 16.03314966  2.49196842  9.1147183 ]
 [ 4.31013637  9.01460839 14.14027958  2.23835261  9.78165024]
 [ 5.93222074  9.75653171 14.93128848  3.65573443  9.23799347]
 [ 6.06919154  9.80320439 17.86076126  3.47608005  8.13563861]
 [ 7.06183904  9.64140297 15.44109141  3.58875861  8.24321727]
 [ 6.47220939  8.29500612 14.87217173  3.10280481  9.38908271]
 [ 5.20163876  8.4695951  17.10988667  2.23930504  8.62740206]
 [ 5.15173776  8.478742   15.69209829  2.36609864  8.93781656]
 [ 7.31500594  9.59720041 16.90335183  3.28112702  9.95366337]
 [ 4.47690678  8.18545201 15.9727241   3.92544294  8.78631819]
 [ 7.32482102  9.00706955 15.14023191  2.66053931  8.06025152]
 [ 6.19190117  9.19210251 17.12834268  2.43498831  9.96142369]
 [ 6.23082704  9.20497569 19.75786886  3.15309739  8.60497423]
 [ 4.96233412  8.0128566  15.78939742  2.96137345  9.77292716]
 [ 5.12958338  9.65543185 15.77424082  3.85452003  9.61913271]
 [ 4.66855681  8.93152585 17.17493012  2.22760247  8.62526976]
 [ 5.56976987  9.75795162 17.88496993  2.5758659   9.20059421]
 [ 4.71298907  8.87119834 17.38871269  3.90648217  8.64824709]]
```

These support vectors decide the margin and hence the decision boundary.

[47]:
```
svm2_coef = svm2.coef_
svm2_intercept = svm2.intercept_
```

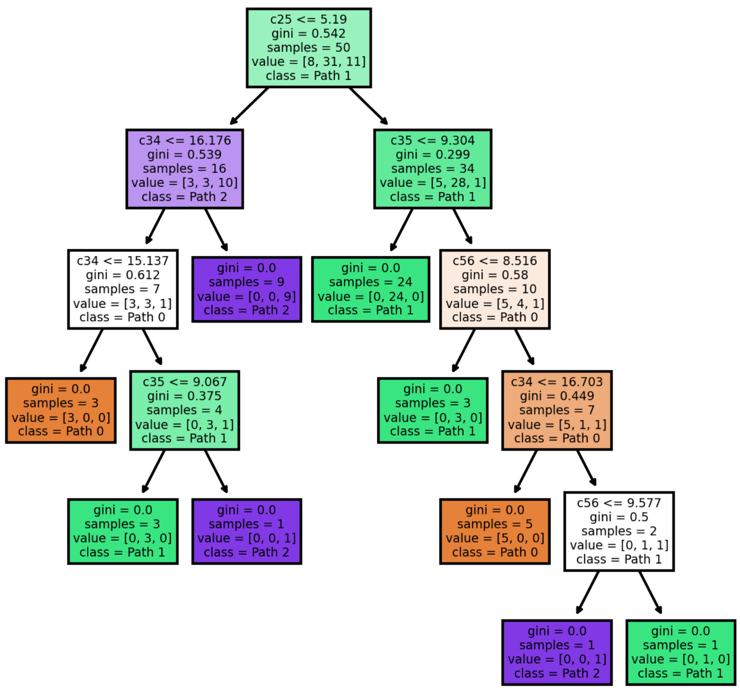
[48]:
```
for j in range (0,3):
    print('Equation ' + str(j+1))
    print('0 = '+ str(svm2_intercept[j]) + ' + ('+ str(svm2_coef[j][0])+')c25'␣
 ↪+ ' + ('+ str(svm2_coef[j][1])+')c35' + ' + ('+ str(svm2_coef[j][2])+')c34'␣
 ↪+ ' + ('+ str(svm2_coef[j][3])+')c46' + ' + ('+ str(svm2_coef[j][4])+')c56')
    print('')
```

```
Equation 1
0 = -1.8753134020847058 + (-0.3079191233273164)c25 + (1.5317340958812231)c35 +
(-0.9360084384441976)c34 + (-0.47007008700403574)c46 + (0.5901559744736424)c56

Equation 2
0 = 11.977770582709203 + (1.024940223531777)c25 + (0.24971689600321412)c35 +
(-0.8921600489035697)c34 + (-0.9222018766894156)c46 + (-0.30332611684387345)c56

Equation 3
0 = 13.261722503131503 + (1.7445918451370268)c25 + (-1.4337705919450556)c35 +
(-0.2765615364768905)c34 + (-0.0221345827919901)c46 + (-0.5113728116170435)c56
```

The above hyperplanes define the boundary in the SVM model, which helps predict the test datapoints being classified into label '0', label '1' and label '2'.

[49]:
```
# optimal strategy obtained from Tree
fig, axes = plt.subplots(nrows = 1,ncols = 1,figsize = (5,5), dpi=300)
tree.plot_tree(dt2, filled=True, feature_names=['c25', 'c35', 'c34', 'c46',␣
↪'c56'], class_names=['Path 0','Path 1','Path 2'])
fig.text(0, 0,'')
```

[49]:
```
Text(0, 0, '')
```



The above graph shows the decision rules in the tree model.

For example, the model will first examine if c25 is smaller or equal to 5.19.

If so (go to the left), the model would then examine if c34 is smaller than or equal to 16.176 . If so, it would further go down the tree plot to predict the optimal pat. Otherwise, it would predict the optimal path as Path 2.

[Path "0" is equivalent to [0, 1, 0, 1, 0, 1, 0] / "Node1–>Node3–>Node4–>Node6"]

[Path "1" is equivalent to [0, 1, 0, 0, 1, 0, 1] / "Node1–>Node3–>Node5–>Node6"]

[Path "2" is equivalent to [1, 0, 1, 0, 0, 0, 1] / "Node1–>Node2–>Node5–>Node6"]

2.0.5 (5) Insight from the results

Insight 1: (Accuracy score difference between problem 1 and problem2)

The general accuracy of all three models in this probelm are smaller than those in problem 1 (naive-setting). One possible reason could be that we have more features (higher dimension) in probelm 2, i.e. we have five features (25, 35, 34, 46, 56) in problem 2 versus two only (c1 and c2) in problem 1. Also we have more arcs in problem 2, rendering problem 2 being more complicated than problem 1.

Insight 2: (Accuracy score difference between decision tree and the other two models within problem2)

Referring to tree graph above, it is possible that the decision tree model could give its prediction result without considering all features (25, 35, 34, 46, 56). Meanwhile, the 'true' decision in problem 2 should be with consideration of all 5 features. This could be possible explanation for significant lower accuracy score in decision tree model (72% versus around 85% in Logistic Regression model and SVM model).

[ ]:
```

```