

**Topic:** Finding center and radius of a circle**Question:** Find the center and radius of the circle.

$$4x^2 + 4y^2 + 4x - 12y + 1 = 0$$

**Answer choices:**

A      Center at  $\left(\frac{1}{2}, \frac{3}{2}\right)$       Radius of  $\frac{3}{2}$

B      Center at  $\left(-\frac{1}{2}, \frac{3}{2}\right)$       Radius of  $\frac{3}{2}$

C      Center at  $\left(\frac{1}{2}, \frac{3}{2}\right)$       Radius of  $\frac{1}{2}$

D      Center at  $\left(-\frac{1}{2}, \frac{3}{2}\right)$       Radius of  $\frac{1}{2}$



**Solution: B**

In order to find the center and radius, we need to change the equation of the circle into standard form,  $(x - h)^2 + (y - k)^2 = r^2$ , where  $h$  and  $k$  are the coordinates of the center and  $r$  is the radius.

In order to get the equation into standard form, we have to complete the square with respect to both variables.

Our first step is to remove the coefficients of the second degree terms of both  $x$  and  $y$ , which we'll do by multiplying both sides of our equation by  $1/4$ .

$$\frac{1}{4} (4x^2 + 4y^2 + 4x - 12y + 1 = 0)$$

$$x^2 + y^2 + x - 3y + \frac{1}{4} = 0$$

Grouping  $x$ s and  $y$ s together and moving the constant to the right side, we get

$$(x^2 + x) + (y^2 - 3y) = -\frac{1}{4}$$

Completing the square requires us to take the coefficient on the first degree terms, divide them by 2, then square the result before adding the result back to both sides.

Coefficient on  $x$  is 1

$$1 \div 2 = \frac{1}{2}$$

Coefficient on  $y$  is  $-3$

$$-3 \div 2 = -\frac{3}{2}$$



$$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\left(-\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$\left(x^2 + x + \frac{1}{4}\right) + \left(y^2 - 3y + \frac{9}{4}\right) = -\frac{1}{4} + \frac{1}{4} + \frac{9}{4}$$

$$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2$$

Therefore, the center of the circle is at

$$(h, k) = \left(-\frac{1}{2}, \frac{3}{2}\right)$$

and its radius is

$$r = \frac{3}{2}$$



**Topic:** Finding center and radius of a circle**Question:** Find the center and radius of the given circle.

$$x^2 + y^2 - 6y = 5$$

**Answer choices:**

- A Center is (0,3). Radius is  $\sqrt{14}$ .
- B Center is (0, - 3). Radius is  $\sqrt{14}$ .
- C Center is (0,3). Radius is 14.
- D Center is (0, - 3). Radius is 14.



**Solution: A**

$x^2$  is already a perfect square, so we'll complete the square in  $y$  by adding 9 to both sides of the equation.

$$x^2 + y^2 - 6y + 9 = 5 + 9$$

$$x^2 + y^2 - 6y + 9 = 14$$

Factor to get the equation in the form  $(x - h)^2 + (y - k)^2 = r^2$ .

$$(x - 0)^2 + (y - 3)^2 = (\sqrt{14})^2$$

The center of the circle  $(h, k)$  is  $(0, 3)$  and the radius is  $\sqrt{14}$ . Rule out  $-\sqrt{14}$  because a radius can't be negative.

