Engineering Drawing Software Package

MATHEMATICAL MODELING

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1. 3D to 2D projection

Assumptions:- we are representing a 3-D object with all the vertices and edges (wireframe model). We are considering a cartesian coordinate system with xyz axes
We are considering object in 1^{st} Quadrant so the front view projection is on $(x-y)$ plane, top view is on $(x-z)$ plane and side view is on $(y-z)$ plane

- 3 views of a object are neccesary and sufficient to make the 3D model of the object

Computing projections from the 3D object

we assume the input as :-
-> given all the coordinates of all the vertices
-> all the vertices pairs which have the edge between them while

drawing the projection on a plane first we will get the projections of all the vertices on that plane and then joining those vertices which have edge between them

Projection of a point (x, y, z) on a plane

a) projection on (x, y) plane -

multiply the point matrix (x,y,z) with $(1,1,0)$ to get projection on (x,y) plane

$$P_x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

b) projection on (x,z) plane -

multiply the point matrix (x,y,z) with $(1,0,1)$ to get projection on (x,z) plane

$$P_y = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} x \\ 0 \\ z \end{bmatrix}$$

c) projection on (y,z) plane -

multiply the point matrix (x,y,z) with $(0,1,1)$ to get projection on (y,z) plane

$$p = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ y \\ z \end{bmatrix}$$

d) projection on an auxilary plane -

Let consider the plane(p) of equation

$$(p)ax + by + cz + d = 0$$

and a point M(u,v,w) We look for the point $N = (x_0, y_0, z_0)$, the projection of M on the plane.Normal of the plane is the vector (a,b,c) so line by M(u,v,w) of equations

$$\frac{x-u}{a} = \frac{y-v}{b} = \frac{z-w}{c} = t$$

is the line perpendicular on the plane. so we have for a point on this line parametric equations

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is the line perpendicular on the plane. so we have for a point on this line parametric equations

$$x = u + at$$

$$y = v + bt$$

$$z = w + ct$$

Now we want the value of t_0 for with a point of this line is on the plane.

$$a(u + at_0) + b(v + bt_0) + c(w + ct_0) + d = 0$$

$$t_0 = -\frac{au + bv + cw + d}{a^2 + b^2 + c^2}$$

so we have

$$x_0 = u + at_0$$

$$y_0 v + bt_0$$

$$z_0 = w + ct_0$$

so finally the projection points are

$$x_0 = u - a\frac{au + bv + cw + d}{a^2 + b^2 + c^2}$$

$$y_0 = v - b\frac{au + bv + cw + d}{a^2 + b^2 + c^2}$$

$$z_0 = w - c\frac{au + bv + cw + d}{a^2 + b^2 + c^2}$$

After getting the porjections of all the vertices on the auxillary plane we can get the front view, top view and side view by joining the vertices pairs which have edge between them in the 3D model

2. 2D to 3D

we assume the input as -
given all the cartesian coordinates of all the end points in the diagram and the pairs of points which are connected on the same line
we have these inputs in all three planes$(xy)(yz)(xz)$

consider projection in any one of the plane let us say xy plane and it consists of points
$(x_1,y_1),(x_2,y_2),(x_3,y_3),(x_4,y_4),(x_5,y_5)$..............
consider the point (x_1,y_1) and now search for all the points in xz plane which have the x_1 as the x-coordinate.
let the points obtained be $(x_1,z_1),(x_1,z_2),(x_1,z_3)$... now check for (y_1,z_1) in the yz plane, if it exists then we can say that (x_1,y_1,z_1) is a vertex of 3D object and check for (y_1,z_2) in yz plane, if it exists then we can say that (x_1,y_1,z_2) is a vertex of 3D objects similarly check for $(y_1,z_3),(y_1,z_4),..$ if it exists then (x_1,y_1,z_k) is a vertex of 3D object

similarly check for all the points of xy plane e.g $(x_2,y_2),(x_3,y_3),(x_4,y_4)$.....

By this we can get all the points of 3D object

Edges between the vertices in 3D model -
consider any two vertices like $(x_1,y_1,z_1) and (x_2,y_2,z_2)$ if

$(x_1,y_1) and (x_2,y_2)$ are connected in xy plane and
$(y_1,z_1) and (y_2,z_2)$ are connected in yz plane and
$(x_1,z_1) and (x_2,z_2)$ are connected in xz plane

then there is an edge between $(x_1,y_1,z_1) and (x_2,y_2,z_2)$
similarly check for all the vertices of 3D model

By this finally we can get the 3D model of an object containing all vertices and edges