Generational Economics - Assignment 2

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1 Exercise 1

Assume a Diamond model as described in the reader where the consumer has preferences $U_t = c_t^y * c_t^o + 1$, there is no technological progress and production is described by $y_t = k^\alpha$.

i. Compute the capital-labour ratio in the steady state. What is the effect of an increase in α on the steady-state capital-labour ratio?

$$k_{t+1} = \frac{s_t}{1+g}$$

$$f(k_t) = y_t = k_t^\alpha \text{ and } g = 0$$

$$k_{t+1} = \frac{1}{2+p}(k_t^\alpha - k_t \alpha k_t^{\alpha-1})$$

To complete the steady state we use the difference equation: $k_{t+1} = k_t = k$.

$$\begin{aligned} k_{t+1} = k_t &= \frac{(1-\alpha)k_t^\alpha}{2+p} \\ k^{1-\alpha} &= \frac{1-\alpha}{2+p} \\ k &= (\frac{1-\alpha}{2+p})^{\frac{1}{1-\alpha}} \quad \text{is the steady state.} \end{aligned}$$

An increase in α *will let* k *decrease and convert to 0. That means that in the steady state the capital-labour ration would move to an increase in labour relative to capital.*

ii. Suppose the government introduces a tax ? on interest income. All revenues of this tax are transferred to the elderly in the same period as a pension benefit. So the budget constraints become:

$$c_t^y = w - s_t \tag{1}$$

$$c_{t+1}^o = (1 + (1 - \varsigma)r)s_t + T \tag{2}$$

Note that the level of the pension benefit T is given for the individual and cannot be affected by saving more or less.
Derive the Euler equation.

First, we write down the utility function and the production function:
$U_t = C_t^y C_{t+1}^o$

We combine this with the consumption function at t for the young and t+1 for old, we get:
$U_t = (w - s_t)((1 + (1 - \varsigma)r)s_t + T)$

If we derive this equation:
$\frac{\partial U}{\partial s_t} = 0$
$\frac{\partial U}{\partial s_t} = w(1 + (1 - \varsigma)r) - 2s_t(1 + (1 - \varsigma)r) = 0$

$s_t = \frac{w(1+(1-\varsigma)r)-T}{2(1+(1-\varsigma)r)}$

By substituting s_t in both C_{t+1}^o and C_t^y, we can derive the Euler Equation:
$\frac{C_{t+1}^o}{C_t^y} = \frac{w(1+(1-\varsigma)r)+T}{\frac{w(1+(1-\varsigma)r)}{1+(1-\varsigma)r}+T} = 1 + (1 - \varsigma)r$

2 Excersise 2

a. Assume that production is described by the following neoclassical production function without labour-augmenting technological progress: $Y = F(K, L)$
Derive the dynamic equation for capital per person (k) assuming a constant savings rate.

Since Δ is equal to difference between the savings and the effective depreciation and we assume there is no labour-augmenting technology (g=0), we find:
$\Delta k(t) = \frac{\Delta K(t)}{L(t)} - \frac{K(t)}{L(t)}\frac{\Delta L(t)}{L(t)} = \frac{sY(t)-\delta K(t)}{L(t)} - K(t)n$

$\Delta k(t) = sf(k(t)) - (\delta + n)k(t)$

b. Assume a Solow-Swan model as described in the reader, with a Cobb-Douglas production function $Y = 1.5K^{0.5}L^{0.5}$ and $x = 0.5$, $n = 0.5$, $\delta = 0.25$, $\sigma = 0.5$.

i. Compute the capital-labour ration k^* in the steady state.

$(1 + g) = (1 + n)(1 + \alpha) = 1.5 * 1.5 = 2.25 \Leftrightarrow g = 1.25$
$f(k_t) = AK^{0.5}L^{-0.5} = Ak^{0.5} = 1.5k^{0.5}$

The steady state can then by calculated: $i = (\delta+g)k$ where $i = \sigma f(k) = 0.75k^{0.5}$
$0.75k^{0.5} = 1.5k \Leftrightarrow k^*_{ss} = 0.25$

ii. Is the steady state dynamically efficient or dynamically inefficient?

The Golden Rule (GR) can be derived using the following equation:
$f'(k) = \delta + g \Leftrightarrow 0.75k^{-0.5} = 1.5 \Leftrightarrow k_{GR} = 0.25$

$k_{SS} = k_{GR} \Rightarrow$ *Neither dynamically efficient nor dynamically inefficient.*