Acquisition and Analysis of Neural Data

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Analytical Problem Set 3 Deadline: Tuesday 4th May, 2021 at 23:55

Please scan and upload your solution on Moodle as a single PDF file by the due date. Make sure that all text is clearly readable. For example, you could use the 'Adobe Scan' application on your mobile phone. Each student must submit their own solution. Do not forget to write your name and to mention if you collaborated with someone. Please specify how much time you needed to finish the exercise.

Spike-train auto-correlation and cross-correlation

The spike train auto-correlation function

$$Q_{\rho\rho}(\tau) = \frac{1}{T}\int_0^T \mathrm{d}t\ \langle \rho(t)\rho(t+\tau)\rangle - \langle r\rangle^2 \tag{1}$$

is used to detect patterns in spike trains (e.g., oscillations) and to classify cells. Similarly, the spike train cross-correlation function

$$Q_{\rho\rho'}(\tau) = \frac{1}{T}\int_0^T \mathrm{d}t\ \langle \rho(t)\rho'(t+\tau)\rangle - \langle r\rangle \langle r'\rangle \tag{2}$$

is used to find relationships between a pair of spike trains, e.g., to assess whether two neurons fire synchronously or phase-locked, which could be evidence for synaptic connectivity.

1. Auto-correlation histogram of a Poisson process (3 points)
Sketch the expected (i.e., trial averaged) unnormalized auto-correlation histogram N_m (defined in class) of a Poisson process with refractory period $t_{\mathrm{ref}} = 3$ ms in the interval [-20, 20] ms. See also Dayan and Abbott book (Section 1.4, text after Equation 1.35). Although approximate, the sketch should include relevant values on the appropriate axes. Consider bin size $\Delta t = 0.1$ ms, trial length $T = 1$ s, and firing rate $\langle r\rangle = 100$ spikes/s. Repeat the sketch for $\langle r\rangle =$ 300 spikes/s (high rate) and $\langle r\rangle = 10$ spikes/s (low rate). What happens in these two limiting cases? Note: $\langle r\rangle$ is the effective rate of the process after introducing refractoriness (see also numerical exercise 3).

2. Cross-correlation of oscillatory signals (4 points)
Calculate the cross-correlation function $Q_{r_1 r_2}(\tau)$ between two oscillatory firing rates

$$r_1(t) = \left[1 + a_1 \sin(\omega\, t)\right]\, \bar{r}_1\,,$$
$$r_2(t) = \left[1 + a_2 \sin(\omega\, t + \varphi)\right]\, \bar{r}_2$$

where $\bar{r}_1$, $\bar{r}_2$ are constant firing rates, ω is an angular frequency, φ is a phase shift, and $0 < a_1 < 1$ and $0 < a_2 < 1$ are dimensionless constants. Approximate Q for the case in which the integration time T is much longer than the length of one oscillation cycle, i.e., $T \gg 2\pi/\omega$. Hint: Use the definition in Equation 2 (without trial averaging) and remember the following trigonometric identity

$$\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right].$$

3. Symmetry of the auto-correlation (3 points)

(a) Prove the symmetry property of the spike-train auto-correlation function, i.e., $Q_{\rho\rho}(\tau) = Q_{\rho\rho}(-\tau)$.
Hint: Use variable substitution.

(b) Briefly discuss the integration limits.

Numerical Problem Set 3 Deadline: Tuesday 4^{th} May, 2021 at 23:55

Please upload your solution on Moodle by the due date. Return both a Python code file (or notebook) and a PDF or HTML file including figures and code. Please use clear, descriptive variable names and comments. Each student must submit their own files. Do not forget to write your name and to mention if you cooperated with someone. Please also indicate how much time you needed to complete the exercise.

Spike-train statistics

The Poisson process plays a central role in the analysis of spiking neurons and networks. In this exercise, you get familiar with some basic properties of the Poisson process. Feel free to recycle you previous code (at least if it gave you the correct results).

1. Coefficient of variation and Fano factor (4 points)
The coefficient of variation CV and the Fano factor F are measures that characterize the variability (or regularity) of a spike train. Both measures quantify the dispersion of the data in relation to its mean.

The coefficient of variation is defined as:

$$CV = \frac{\sigma_\tau}{\langle \tau \rangle}$$

where σ_τ and $\langle \tau \rangle$ are the standard deviation and the mean of the inter-spike intervals τ (ISIs). The CV can be computed within a trial, or pooling ISIs across trials.

The Fano factor is defined as:

$$F = \frac{\sigma_n^2}{\langle n \rangle}$$

where σ_n^2 and $\langle n \rangle$ are the variance and the mean of the spike counts n in time windows of length T_W. If the counting window spans the entire trial ($T_\mathrm{W} = T$), spike counts n are pooled across trials and F is a single number. By contrast, if $T_\mathrm{W} < T$, each trial is subdivided in $N_\mathrm{W} = \lfloor T/T_\mathrm{W} \rfloor$ non-overlapping windows of length T_W. In this case, the Fano factor can be either *trial resolved* or *time resolved*. In the first case, spike counts are pooled within trials and across windows (trial resolved, one F value per trial). In the latter case, spike counts are pooled within windows and across trials (time resolved, one F value per time window).

Download SpikeTimes.dat from the Moodle webpage and load it with the numpy function loadtext. The file contains a matrix of spike times in milliseconds. The dimensions are [m x n], where m=100 is the maximum spike-time index and n=100 is the number of trials. Note: each trial has a different number of spikes, the remaining entries are filled with nan. The trial length is $T = 5.5$ s, the resolution is 0.1 ms. It is convenient to convert the data in units of seconds.

(a) Make a raster plot of the data. Use the pyplot function eventplot. You may need to set the axes limits manually for correct visualization.

(b) Plot a histogram of the ISIs pooled across trials. Hint: use the numpy function diff. How are the ISIs distributed?

(c) Compute the coefficient of variation CV from the pooled ISIs.

(d) Compute and plot both the trial-resolved and the time-resolved Fano factor F in non-overlapping windows of lengths $T_\mathrm{W} \in [0.3, 0.5, 1]$ s. Interpret the results. How does the window size affect the Fano factor in this dataset?

2. Homogeneous and inhomogeneous Poisson process (4 points)
Download and load the file PoissonSpikeTrains.mat using the function loadmat in scipy.io. The arrays SpikeTimes_hom and SpikeTimes_inh contain spike times in milliseconds for an homogeneous and an inhomogeneous Poisson process, respectively. The Python program generatePoissonTrains.py explains how these spike times were generated. For both the homogeneous and the inhomogeneous process:

(a) Construct and plot the ISIs histograms. How are they different?

(b) Compute the CV and the Fano factor. For the fano Factor use non-overlapping windows of length $T_W = 100$ ms (only one trial).

(c) Compute and plot the spike-train auto-correlation histogram by discretizing the auto-correlation function

$$Q_{\rho\rho}(\tau) = \frac{1}{T} \int_0^T \mathrm{d}t \, \langle \rho(t)\rho(t+\tau) \rangle - \langle r \rangle^2 .$$

Use a bin size $\Delta t = 0.1$ ms and multiply the result by Δt such that the auto-correlation in zero equals the average firing rate $\langle r \rangle$. Use the Numpy function correlate. Plot the auto-correlation both with and without the central peak. Also try different x-axis limits to see if you can observe any interesting structure. If you want, you can additionally implement step-by-step the algorithm outlined in Dayan and Abbott (Section 1.4, text after Equation 1.35). With the correct scaling, the two procedures should lead to the same result.

(d) Why do the CV and the Fano factor deviate from the theoretical value of 1?

3. Poisson process with refractory period (2 points)
Load the variables SpikeTimes_ref and rates_ref from the data file PoissonSpikeTrains.mat. The array SpikeTimes_ref contains the spike times in milliseconds generated from six homogeneous Poisson processes with refractoriness. Each process has a different *driving* rate, i.e., the rate before refractoriness is introduced (more details in generatePoissonTrains.py). The driving rates are given in the variable rates_ref. The refractory period is 5 ms.

(a) Estimate the effective firing rate r_{eff} for each driving rate r_{drive}. Plot r_{eff} against r_{drive}. Which level does r_{eff} approach for large r_{drive}?

(b) Construct and plot the ISI histograms for each driving rate.

(c) Compute the CV and the Fano factor for each driving rate and plot them against r_{drive}. Why do they decrease with increasing rates?

(d) Calculate and plot the spike-train autocorrelation function for each riving rate. Use a bin size $\Delta t = 1$ ms. Why are there multiple peaks for large driving rates?

4. Extra: Gamma Process (3 extra points)
A Gamma process of order k (with a constant rate) can be generated from a homogeneous Poisson process by retaining only each k^{th} spike.

(a) Generate Gamma processes of order 2, 3, 5, and 10, each with a rate of $r = 100$ spikes/s (the rate of the underlying homogeneous Poisson process needs to be adjusted accordingly). For each Gamma process, generate 1000 spikes.

(b) Compute ISI histograms and CVs. Compare your results to the theoretical distribution

$$p_{\text{ISI}}(\tau) = \frac{r(r\tau)^{k-1} e^{-r\tau}}{(k-1)!}$$

where τ is the inter-spike interval and k is the order of the Gamma process.

(c) Compile your results into a figure where the left column shows example spike trains of 1 s length of each Gamma process, and the right column shows the corresponding ISI distributions, with the CV values next to them.

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