

# A MPM with recurrent removal (Hastings *et al.* 2006) - preparatory document

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2025-12-19

## Introduction

$$N_T = L^T N_0 - \sum_{i=1}^T L^{T+1-i} H_i$$

This is a preparatory document that develops the above formula step by step, using a simple example. This not only serves as an illustration, but it also acts as a reference to check whether the developed function, the scripting of which is way more abstract, is correct.

First, I avoid using *popdemo* functions, to stay closer to the core formulation. Then, I provide the *popdemo* alternative, which is heavily used in further development. Note how the *popdemo::project()* function has the huge advantage that, for a given time span, all intermediate values are provided. Also, by using the option *return.vec = FALSE/TRUE*, one can switch between output per stage or summed over stages.

As a time span, we opt for five steps (years), i.e.  $T = 5$ .

## $L$ , $N_0$ and $H$

$L$  is

```
##          seedlings isolates meadows
## seedlings      0       0.1     0.1
## isolates      1       1.0     1.0
## meadows       0       0.3     1.2
```

$N_0$  is

```
##          [,1]
## seedlings  0.1
## isolates   0.1
## meadows    0.1
```

The area removed is  $H$ . Removal is performed at the **start of the year**. This is important to understand, because for one thing, it means that you will nowhere see the value of  $H$  in the ‘ghost matrix’ (see further); the first value you will encounter directly is  $LH$ , i.e. the end-of-year ‘ghost’ of what was newly removed at the start of the year.

$H$  is

```

##      [,1]
## [1,]  0.0
## [2,]  0.1
## [3,]  0.0

```

## Without removal

If there would be **no removal**, an MPM's base formula gives the population extent.

$$N_T = L^T N_0$$

Without *popdemo*:

```

N1 <- sum(L%^%1 %*% NO) # expm package
N2 <- sum(L%^%2 %*% NO) # matrix power ( $L^2 = \text{element-wise}$ ,  $L%^%2 = L \times L$ )
N3 <- sum(L%^%3 %*% NO)
N4 <- sum(L%^%4 %*% NO)
N5 <- sum(L%^%5 %*% NO)

print(c(sum(NO), N1, N2, N3, N4, N5))

```

```

## [1] 0.30000 0.47000 0.78500 1.32400 2.24250 3.80365

```

With *popdemo*. Note how  $N_0$  is automatically included in the output.

```

Nt <- project(L, vector = NO, time = 5, return.vec = T)

print(Nt)          # across stages

## 1 deterministic population projection over 5 time intervals.
##
## [1] 0.30000 0.47000 0.78500 1.32400 2.24250 3.80365

print(t(Nt@vec)) # per stage

##      [,1] [,2] [,3] [,4] [,5] [,6]
## seedlings 0.1 0.02 0.045 0.074 0.1250 0.21175
## isolates  0.1 0.30 0.470 0.785 1.3240 2.24250
## meadows   0.1 0.15 0.270 0.465 0.7935 1.34940

```

## With removal

Let's now include **removal**. The equation on top states that from this total projection, we now have to subtract a contingent of "what could have been", i.e. that which would have resulted from the removed portion ( $-L^T H$ ), for each of the time steps that have passed ( $\text{sum}_{i=1}^T$ ).

I refer to the removed cohorts and how they would have propagated as 'ghosts', their yearly numbers collected in the *ghost matrix*.

Without *popdemo*:

```

total = sum(L%^%5 *% NO)           # total (actual + removed) - as above

ghost1 = sum(L%^%(5+1-1) *% H)     # removed in year 1 (five years ago)
ghost2 = sum(L%^%(5+1-2) *% H)     # removed in year 2 (four years ago)
ghost3 = sum(L%^%(5+1-3) *% H)     # removed in year 3 (three years ago)
ghost4 = sum(L%^%(5+1-4) *% H)     # removed in year 4 (two years ago)
ghost5 = sum(L%^%(5+1-5) *% H)     # removed in year 5 (previous year)

total-sum(ghost1, ghost2, ghost3, ghost4, ghost5)

```

```
## [1] 1.453128
```

With *popdemo*:

```

total. = project(L, vector = NO, time = 5, return.vec = F)
ghost.1 = project(L, vector = H, time = 5+1-1, return.vec = F) # removed in year 1 (five years ago)
ghost.2 = project(L, vector = H, time = 5+1-2, return.vec = F) # removed in year 2 (four years ago)
ghost.3 = project(L, vector = H, time = 5+1-3, return.vec = F) # removed in year 3 (three years ago)
ghost.4 = project(L, vector = H, time = 5+1-4, return.vec = F) # removed in year 4 (two years ago)
ghost.5 = project(L, vector = H, time = 5+1-5, return.vec = F) # removed in year 5 (previous year)

tail(total.,1)-(tail(ghost.1,1)+tail(ghost.2,1)+tail(ghost.3,1)+tail(ghost.4,1)+tail(ghost.5,1))

```

```
## [1] 1.453128
```

This now opens the way to the generic formulation, in the next script (2\_eq3\_...).