Intuitions and perceptual constraints on causal learning from dynamics

Tianwei Gong (tia.gong@ed.ac.uk)
Department of Psychology, University of Edinburgh
Edinburgh, Scotland

Neil R. Bramley (neil.bramley@ed.ac.uk)
Department of Psychology, University of Edinburgh
Edinburgh, Scotland

Abstract

Many of the real world phenomena that cognizers must grapple with are continuous, not only in the values they can take, but also in how these values change over time. The mind must somehow abstract from these inputs to extract useful discrete concepts such as objects, events and causal relationships. We investigate several factors that affect basic inferences about causal relationships between continuous variables based on observations in continuous time. In a novel experiment, we explore the ways in which causal judgments are sensitive to factors that relate to causal inductive biases (e.g. causal lags, the direction of variation) and causal perception (e.g. the range and rapidity of variation). We argue standard statistical time-series models have limited utility in accounting for human sensitivity to these factors. We suggest further work is needed to fully understand the cognitive processes that underlie causal induction from time-series information.

Keywords: causal learning, continuous time, continuous variables, time-series data, dynamics.

Introduction

From the waxing and waning of daylight in our eyes, to the ebb and flow of the neighborhood hubbub in our ears, analogue inputs, tied to real world causal dynamics, flow in through our senses continuously. It remains unclear how people extract information from continuous inputs to make causal inferences. We here investigate several factors that might influence this foundational aspect of causal model-based reasoning. In particular, we test the idea that reasoning about causal relationships in dynamic systems depends on two interacting dimensions: (1) An ability to perceive and extract causal signals from background noise, and (2) Intuitive causal theories about the nature of causal relata and their functional relationships.

Following Davis, Bramley, and Rehder (2020a), we adapt the Ornstein-Uhlenbeck (OU) process (Uhlenbeck & Ornstein, 1930) and use it as a generative causal model that can simulate a variety of naturalistic continuous dynamics. OU networks combine a causal graph with functions expressing the continuous influence of cause variables on effect variables by way of an augmented OU processes—producing a form of mean-regressive Brownian motion. Unlike standard causal graphical models (Pearl, 2000), OU networks model causal dynamics in continuous time, with effects continually noisily regressing toward a moving target defined as a function of earlier values of their cause(s). Critically, this behavior is governed by several parameters that, when manipulated, change the properties of the resultant causal dynamics. For example, we can manipulate the lag, i.e. how long influences from cause X take to arrive at Y, but also the strength or "rigidity" of the influence—how rapidly Y approaches whatever basin of attraction is created by the earlier values of X (see Figure 1). We use this as a test-bed to explore human causal judgments.

Studies into causal cognition have long focused on "prepackaged" contingency information, such as covariation of binary variables across independent trials (Cheng, 1997; Griffiths & Tenenbaum, 2009). However, recently several studies have investigated more naturalistic situations that involve continuous variables and time-series data (Soo & Rottman, 2018, 2020; Zhang & Rottman, 2021) produced by continuous dynamic causal systems (Davis et al., 2020a; Davis, Bramley, & Rehder, 2020b; Bramley, Gerstenberg, Mayrhofer, & Lagnado, 2019). Some of these studies have shown that people can leverage moment-by-moment transitions (i.e. changes in the values of variables between successive observations) to identify the presence and direction of causal relationships (Soo & Rottman, 2018). It has also been shown that people can often identify the causal structure of dynamic systems that involves three continuous variables (Davis et al., 2020a, 2020b) if they can freely intervene on and control each variable in real time, although they make systematic errors in identifying chain structures. Participants in these tasks appeared to follow an intervention strategy of creating occasional dramatic and rapid changes in variables and monitoring the behavior of other variables shortly afterward. They were able to learn better when the variables changed rapidly and affected one another rigidly rather than slowly or gradually (Figure 1; Davis et al., 2020b).

This paper aims to extend this previous work by considering factors that are particularly interesting for continuous-time causal systems with continuous variables. These novel empirical results can help inspire new computational models to describe the underlying cognitive process, as we will discuss towards the end of the paper. We focus on four factors as listed below.

Causal lag

In reasoning about causal relationships between *events*, people are clearly sensitive to temporal delays. They generally make stronger causal attributions when a putative cause is followed by a putative effect after a short delay than after a long delay, if no specific mechanistic information is conveyed

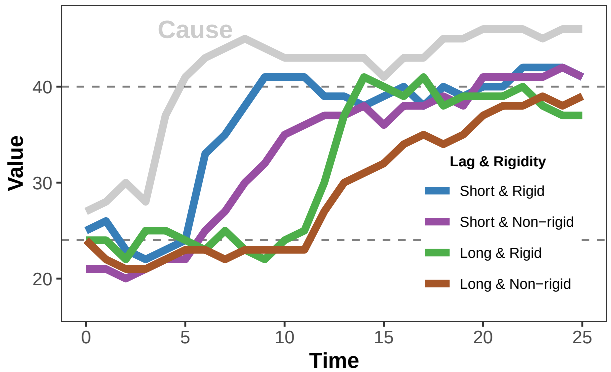

Figure 1: Examples of an effect variable (colored) following a change to its cause (gray) under different causal lags (t=2 vs. t=8) and degrees of rigidity. Dashed lines indicate the asymptotic values of the effect before and after the change in the value of the cause.

(Buehner & McGregor, 2006; Shanks, Pearson, & Dickinson, 1989; Lagnado & Speekenbrink, 2010). This could be explained from a cognitive perspective by the idea that, the longer delay between two events, the harder it is for learners to sustain memory of the first in working memory or potential association with the second (Buehner & May, 2003). From a normative perspective also, the longer the delay, the more likely it is that alternative candidate events have taken place during the interval that may then compete to explain the effect (Lagnado & Speekenbrink, 2010). It is unclear whether people also hold similar intuitions about "lags" when judging influences between continuous variables. For example, would people be more likely to make a causal attribution when the effect Y's value at t is influenced by cause X's value at $t-1$ seconds compared to when it follows X at $t-10$ seconds. Previous studies generally fix the causal lag (usually as $t-1$) and have not compared different intervals (Soo & Rottman, 2018; Davis et al., 2020b). Davis et al. (2020b) manipulated a related but distinct quantity: varying the refresh rate of their causal systems between 100ms to 300ms in different conditions but scaling the causal dynamics to be otherwise identical (with causal influences still lagging exactly one refresh).

Granger causality (Granger, 1969) is an established statistical technique designed to identify potential causality in time-series data, with a mechanism to accommodate causal lag. To assess if one variable "Granger causes" another, one searches across a range of fixed lags deemed to be mechanistically plausible, e.g. $X_{t-1} \dots X_{t-m}$, and tests whether inclusion of any of these terms statistically improve prediction of Y_t over and above its own lagged autocorrelation (modeled by including $Y_{t-1} \dots Y_{t-m}$ as a covariate). If a statistical relation is found for one or more of these lags, the causal influence is deemed to be supported. As such, Granger causality does not inherently privilege longer or shorter lags. This lag-indifference may be appropriate for minimizing bias when modeling domains that are poorly understood but may not reflect human expectations. We will investigate whether laypeople are similarly indifferent, or if their causal judgments decay with lag as they do with discrete cases.

Rigidity

For a given change in a cause, a *rigid* causal relationship would lead to the effect asymptoting to its new value sooner than a *non-rigid* relationship (see Figure 1). Davis et al. (2020b) found that people are more likely to infer a causal link between two variables if the effect responds rigidly to the cause's change, essentially overwhelming the variables' random motion more dramatically or saliently. However, they tested this in an active learning setting where participants' actions complicated interpretation of accuracy patterns. In particular, in the non-rigid condition, participants faced a tradeoff between either not waiting long enough between their actions to observe full influence of genuine causal effects, or else waiting a long time for outcomes to manifest, so performing fewer interventions. We thus test rigidity in a passive learning setting to make sure that observed rigid vs. non-rigid stimuli only differ in their detailed temporal dynamics and not the final magnitude of the change (Figure 2).

Direction of change

Continuous variables can increase or decrease over time. Previous research provides people with time-series data where variables both increase and decrease within the same episode (Soo & Rottman, 2018; Davis et al., 2020a). In this study, we are simply curious whether people are equally sensitive to increases and decreases and to matched or inverted changes of cause relative to effect. People are frequently found to have a preference for positive over equally informative negative evidence (Newman, Wolff, & Hearst, 1980) and to have a prior expectation of positive linear relationships (Sanborn, Griffiths, & Shiffrin, 2010). People tend to test positive examples of a hypothesis (Coenen, Rehder, & Gureckis, 2015), and focus more on the positive aspects of their observations (e.g. the "A-cell bias", Kao & Wasserman, 1993). As an intuitive example, direct associations are far more often described as if ascending: "Y increases as X increases" than as if descending "Y decreases as X decreases". We will test whether people are influenced by the direction of change by including all four combinations of X's increase or decrease over time with Y's increase or decrease over time.

Perceived magnitude of change

In situations where people experience real-time continuous dynamics, we have to also consider the role of low-level information processing, i.e. *perception*. That is, in order to discover relationships between variables, learners must first perceive that the variables have changed meaningfully—e.g. surprisingly, relative to their baseline behavior of nonsystematic fluctuations or drift.

Sensitivity to continuous change is a domain in which it seems likely that people will deviate dramatically from naive idealized observer accounts, which are typically presumed to

Table 1: Parameter Settings of OU Processes.

Condition	Level
Lag	short: $k=2$ vs. long: $k=8$
Rigidity	rigid: $\omega=0.8$ vs. non-rigid: $\omega=0.2$
Slope	positive: $\beta=1$ vs. negative: $\beta=-1$
Range$_Y$	boundary vs. middle, controlled by α

have perfect numerical precision and computing resources. For example, it would likely be easier for us to notice a change in the quantity of green algae in a fish tank when it accumulates from 0% (a perfectly clean tank) to 10% than accumulating from a higher baseline (e.g. 50%-60%). It would also be easier to detect the change from 90% and 100% (as the observer can focus on the change in the remaining clean space). This is known as a boundary effect in numerical estimation and relates to proximity of both lower and upper bounds (Kim & Opfer, 2017; Thompson & Opfer, 2008). Therefore, we plan to compare four ranges of Y's change that differ in how close they are to the limits of the range of the variables in question (Figure 2c). We hypothesize that people are more likely to detect causal relationships correctly when the effect in question starts or ends nearer to its boundary (Figure 2c).

OU process

As sketched at the start, we adapt OU processes to generate the stimuli in our experiment (Davis et al., 2020a; Uhlenbeck & Ornstein, 1930). A standard OU process models mean-regressive Brownian motion of the sort one might recognize in a coke can bouncing around the seat well of a car in motion. By combining a causal graph (here just $X \to Y$) with OU-processes, and replacing the fixed mean of these with a function of cause X, we can model how an effect Y changes and fluctuates in continuous time while being causally influenced by its parent X. OU processes provide a mathematically straightforward formula for producing rich continuous dynamics. Compared to using simple regressions (Zhang & Rottman, 2021), the OU process can easily capture the "stickiness" feature where Y is "dragged" to the expected value gradually with noise.

$$P(\Delta v_y^t | v^t, \omega, k, \beta, \alpha, \sigma) = \omega[(\beta \cdot v_x^{t-k} + \alpha) - v_y^t] + N(0, \sigma) \quad (1)$$

Eq.1 formalizes the extended OU process we use such that Δv_y^t—the change in Y from t to $t+1$—noisily depends on the difference between the target Y value and current Y value v_y^t. We further assume the target Y value is determined by a linear function of its cause X at time $t-k$, where k captures the time lag, and the slope β and the intercept α potentially rescale and offset Y relative to X. ω then controls how "rigidly" the change in the effect occurs — for example, $\omega=0.8$ would mean that Y moves 80%$\pm\sigma$ of the way toward its target value every time interval while $\omega=0.2$ means moves only 20%$\pm\sigma$ of the way and so adjusts to a change in its target value more gradually. Critically, the variables additionally fluctuate according to random noise here drawn from Gaussian distribution with a mean of zero and a standard deviation of σ.

We fixed $\sigma=1$ and varied all other parameters systematically to create our set of experimental conditions. As shown in Table 1, we consider different levels of *Lag* (k), *Rigidity* (ω), *Slope* (β) and *Range*$_Y$ (i.e. the intercept, α). Additionally, we vary the behavior of the cause X (*Direction*$_X$) such that it either increases or decreases abruptly shortly after the beginning of each trial (Figure 2b). X's behavior is also controlled by an OU process but since it has no parent we introduce it as being manually controlled (i.e. through external intervention) and have it approach a basin point μ that either increases or decreases abruptly a few seconds into the trial with $\omega=0.6$ and $\sigma=1$ (see Figure 2b). *Direction*$_Y$ (whether Y ultimately increases or decreases) is thus jointly determined by *Direction*$_X$ and the sign of *Slope* β. Critically, under our current parametric set, differences in *Lag* and *Rigidity* do not affect the final value that Y arrives at by the end of a trial (Figure 2c).

Experiment

Methods

Participants 100 participants (47 female, 52 male, 1 non-binary, aged 42$\pm$12) were recruited via Prolific Academic and were paid £1.20. The task took around 10 minutes. The anonymized pre-registration (https://osf.io/dqyez), data and analysis code (https://osf.io/ybp5m/), and the experiment demo (https://bit.ly/3tDr4uz) are available online.

Design & Procedure Each stimulus contained 32 frames visualizing X and Y's values from $t=1$ to 32. In each case, cause X starts initially following an OU process that reverts to basin point μ_1, and switches to a new basin point μ_2 at $t=7$ (Figure 2). The effect Y responds according to Eq.1. Each frame lasts 750ms.

For the cover story, participants were asked to imagine playing the role of a "forestry manager" who needs to identify the causal relationship between different pairs of Plant A and Plant B following observations in which Plant A's quantity is manipulated artificially (i.e. by the forestry commission). The quantity of each kind of plant varies across the trial. Each pair of Plant A and B had unique colors to ensure participants understand that plants from different trials are not related.

Plant A is displayed in a rectangular "forest" in which the leaves accumulate from the bottom to the top (Figure 2a). We did this to make this quantity perceptually similar to the vertical sliders used in past research (Soo & Rottman, 2018; Davis et al., 2020a; Zhang & Rottman, 2021) where the quantity and range can be perceived by simply considering the height and boundaries. Plant B (the potential effect) is displayed in a circular birds-eye view of a "forest" in which the leaves appear in random positions (Figure 2a). The quantity needs to be determined by estimating the number of leaves (or the area they cover). We used slightly different presentations for Plant A and B to to aid separability (Soo & Rottman, 2018). Leaves

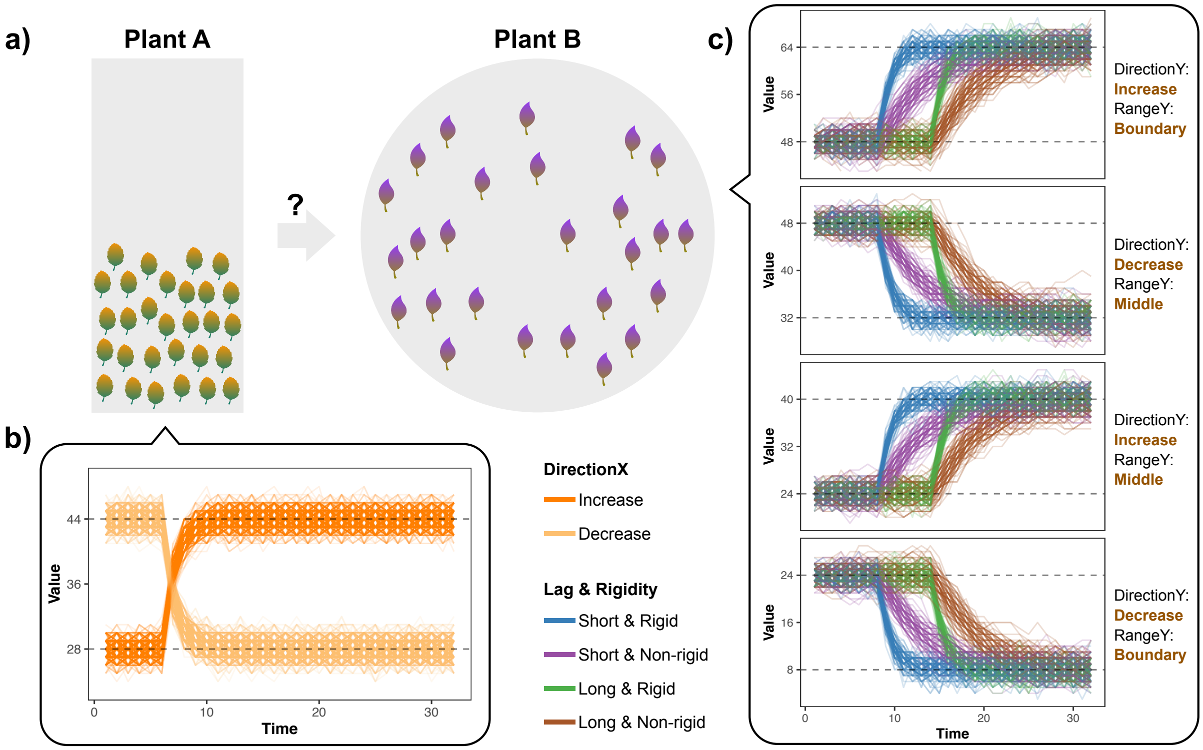

Figure 2: The cover story and experimental stimuli. a) Visualization of task interface. b) The actual runs of how X behaved in the experiment. c) How Y behaved in different conditions. Stimuli (Individual lines) were generated individually for each trial.

are never stacked or overlapping in the presentation meaning that the circular boundary also implied an upper bound on the range of variation for Plant B.

Participants answered "What is the relationship between Plant A and B" by choosing one of the three radio buttons labeled: "Positive (regular)", "Negative (inverse)", and "No relationship". The order of presentation of the radio buttons was randomized between participants. Each stimulus could be viewed only once and lasted a total of 24 seconds.

Each participant went through 16 trials, facing 16 stimuli representing all combinations of *Lag*, *Rigidity*, $Range_Y$, and $Direction_Y$. The $Direction_X$ and hence *Slope* was randomly selected for each trial. Each trial involved a unique sequence generated by the OU process (Figure 2b and c). The order of trials was randomized independently for each participant.

Before starting the task, participants were instructed that a positive (regular) relationship is one in which an increase in the quantity of Plant A causes an increase in the quantity of Plant B, or equally that a decrease in the quantity of Plant A causes a decrease in the quantity of Plant B. A negative (inverse) relationship was described as one such that an increase in the quantity of Plant A causes a decrease in the quantity of Plant B, or equally that a decrease in the quantity of Plant A causes an increase in the quantity of Plant B. We added "(regular)" and "(inverse)" after the word positive or negative in order to emphasize that the question is about the relationship itself not about the input or the the outcome of the causal influence in the scenario (cf. Davis et al., 2020a). It was also emphasized in the instructions that figuring out the relationship would require the learner to observe both Plant A and B and that it would be impossible to work out the relationship by focusing on just one kind of plant. Participants had to pass comprehension check questions before starting the task.

Results

Since all stimuli in fact display a causal relationship, we code answers aligned with the underlying OU processes ($\beta = 1$ for positive and $\beta = -1$ for negative) as correct (1), and others including judgments of no connection, as incorrect (0). This accuracy serves as the primary index in this paper.

To examine the manipulated factors, we fit mixed-effects logistic regression models using `lmerTest` in R, with Subject as a random effect.[1] Under simple regressions with one predictor each, lower *Lag* ($z = 4.45$, $p < .001$), higher *Rigidity* ($z = 3.36$, $p < .001$), increasing $Direction_X$ ($z = 3.37$, $p < .001$), increasing $Direction_Y$ ($z = 3.87$, $p < .001$) and higher (closer to the boundaries) $Range_Y$ ($z = 5.59$, $p < .001$) all relate to higher accuracy (Figure 3), while there was no evidence that *Slope* (positive or negative) made any difference to accuracy ($z = 1.01$, $p = .31$). This suggests there was no general difference between identifying positive and negative relationships (Figure 3). Since *Slope* could be seen as an interaction effect of $Direction_X$ and $Direction_Y$, we later only focus on the five factors (*Lag*, *Rigidity*, $Direction_X$,

[1]Concretely, we included random intercepts and slopes for each subject (Brauer & Curtin, 2018; Barr, 2013).

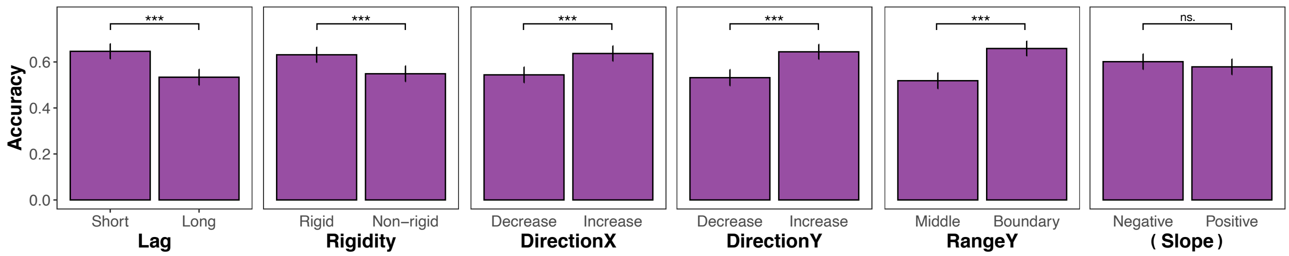

Figure 3: Accuracy separated by levels of different factors.

$Direction_Y$, $Range_Y$) to test whether there are interactions between conditions.[2]

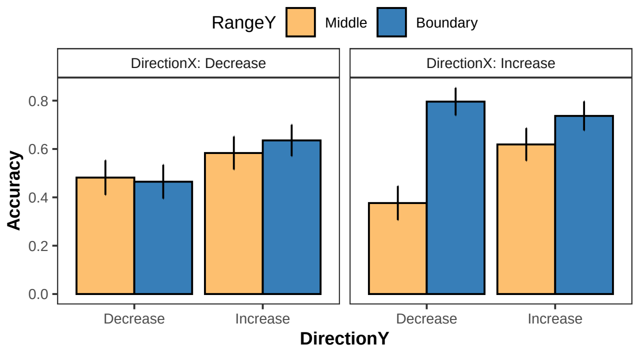

Figure 4: The interaction effect between $Direction_X$, $Direction_Y$, and $Range_Y$

Quantity changes We first focus on the two- and three-way interactions of three of the factors—$Direction_X$ * $Direction_Y$ * $Range_Y$—that determined the start and end points of X and Y (Figure 4). There was an interaction between $Direction_Y$ and $Range_Y$ ($z = 2.19$, $p = .03$), such that the boundary effects were stronger between Y's decreasing from around 24 to 8 vs. from 48 to 32, than between increasing from 48 to 64 vs. from 24 to 40 (Figure 2). Interestingly, there was an interaction between $Direction_X$ and $Range_Y$ such that the boundary effect was stronger when the cause increased than decreased ($z = 4.85$, $p < .001$). This might indicate that participants' processing of X and Y happened sequentially rather than in parallel, where the deficit of reasoning through X would influence the reasoning of Y. There was also a three-way interaction ($z = 3.52$, $p < .001$) such that the boundary effect was largest when X increased and when Y decreased from 24 to 8 vs. from 48 to 32 (Figure 4).

Detailed dynamics We now investigate factors that influenced the trajectory of how Y arrived at its final value: *Lag*×*Rigidity*. There was no interaction between *Lag* and *Rigidity* ($z = 1.19$, $p = .23$). As for whether *Lag* or *Rigidity* interact with $Direction_X$, $Direction_Y$, or $Range_Y$, we found no two- or three-way interactions.

[2]All main effects remained significant when we tested interactions so we do not report them again here.

Choice frequency We finally checked the frequency of three choices (positive, negative, no relationship) under different conditions. Since the relationship cannot be determined by any single factor apart from *Slope*, we normatively would not expect any other single factor to predict the frequency of positive vs. negative choice. As shown in Figure 5, the relative frequency of positive vs. negative did not vary much depending on *Lag* (multinomial regression: $z = 1.44$, $p = .15$) or *Rigidity* ($z = 0.35$, $p = .73$). The only difference is that people chose "no relationship" more often when the causal lag was long or when Y's change was non-rigid. However, the frequency of positive vs. negative did vary depending on $Direction_X$ and $Direction_Y$. Participants were more inclined to choose positive (regular) relationships when X increased, and choose negative (inverse) relationships when X decreased ($z = 5.45$, $p < .001$). Similarly, they chose more positive (regular) relationships when Y increased, and more negative (inverse) relationships when Y decreased ($z = 3.11$, $p = .002$). This shows that judgments were influenced by the change direction of each variable taken separately in spite of instruction to focus on the relationship between two variables. Finally, the difference between positive vs. negative was larger in the boundary than the central range condition ($z = 3.78$, $p < .001$). This is aligned with the finding that participants performed best with the stimuli in which Y decreased to the lower bound as X increased.

Discussion

Everyday experience is one of endless small changes that occur from moment to moment. Making useful causal discoveries under these continuous dynamics may involve complex and hierarchical cognitive processes that cognitive science is just beginning to explain. In this paper we described a study that systematically investigates how several elemental aspects of continuous dynamics combine in shaping real-time dynamic causal inferences.

As has been found with discrete variables (Buehner & McGregor, 2006; Shanks et al., 1989; Lagnado & Speekenbrink, 2010), people more reliably identified a relationship when its causal lag was short than long. This is already a departure from standard statistical models such as Granger causality that do not inherently favor any particular lag (Granger, 1969). Since the trajectory of Y's change was identical for short and long lags except for the onset time, it is unlikely that Y's changes under the long lag were harder to perceive.

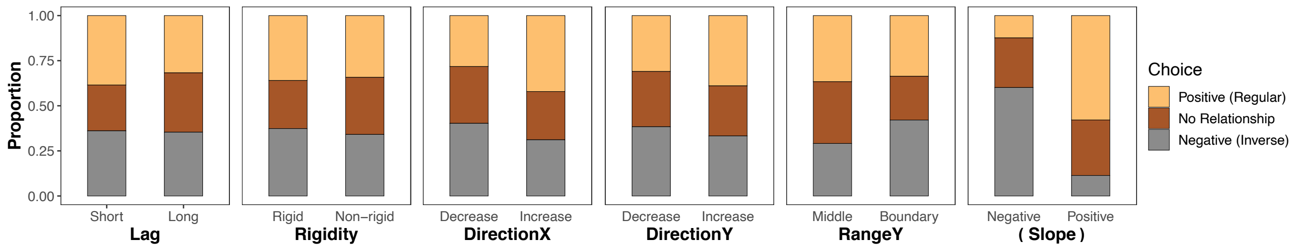

Figure 5: Choice frequency separated by levels of different factors

This could be more indicative of a memory constraint or prior expectation that causal lags will be short, making causal attributions less certain under longer lag times. It is also possible that people concluded there was no relationship toward the end of the observation and so paid little attention to what happened afterward.

Accuracy was higher when the effect changed rigidly. This replicated the finding on Davis et al. (2020b) but in a cleaner passive learning setting where we controlled the total amount of change. One possibility is that people have an independent representation of rigidity and follow their causal intuitions by expecting effects to respond rigidly, especially when the cause changed rigidly, as in the current study. Another possibility is that rigidity works interdependently with other factors to influence how perceptually salient the change is. Future work could look closer at both explanations. Although people were asked to judge the relationship between two variables, their judgments were influenced by the direction of change of a single variable, with a general increase in accuracy when both were positive. Again, purely statistical models seldom differentiate between increases and decreases of a single variable in time. Participants were more accurate when they observed that the cause increased or the effect increased, in alignment with heightened sensitivity to positive evidence in general (Kao & Wasserman, 1993; Newman et al., 1980). We also see hints of use of a heuristic of using the increase or decrease of one variable as a stand-in for the direction of the relationship. It could be that people may have prototypes for how positive or negative relationships typically look in time-series data. This finding is of practical relevance to issues of creating effective data visualizations to communicate causal relationships (Soo & Rottman, 2020). One may want to transform variables so as to visualize them in ways that make use of inductive biases rather than require overriding them.

Participants' judgments were also influenced by the perceivability of change. We found they performed better when Y's change was easier to detect, i.e. closer to the lower or upper bound of the range. This is consistent with the idea that numerical quantities are often represented logarithmically in cognition (Kim & Opfer, 2017; Thompson & Opfer, 2008). This reminds us that it is important to consider potential for non-linearity in how values with different formats are perceived and represented in the human mind when studying causal learning in continuous settings.

Our results highlight several ways that general-purpose statistical models like linear regression may fail to account for basic phenomena in human cognition (Yarkoni, 2022). This is connected to research outside the narrow field of causality that studies how people interact with realistically complex systems (Adolph & Hoch, 2019; Smith & Thelen, 2003), and the dynamic, nonlinear generative models they may form when reasoning in these environments (see Clark, 2013, for review). In future work, we plan to develop quantitative accounts of effects obtained from the current experiment and also experiments with varied formats and cover stories. This includes incorporating priors, exploring a wider range of time dynamics and contrasting models that represent causality either continuously or discretely. It is possible that people still use continuous representations (i.e. as something like an OU process in the head) but make different functional or parameterization assumptions that lead them to deviate from directly reverse engineering the generative model behind these stimuli. It is also possible that people abstract away to higher-level and more discrete representations, such as marking substantial changes as events linked by parametric causal delays (Davis et al., 2020b; Bramley, Gerstenberg, Mayrhofer, & Lagnado, 2018; Gong & Bramley, 2020). For example, someone might simplify complex dynamics by abstracting events such as (X increased dramatically at $t = 1$) triggering another (Y increased/decreased around $t = 5$). This would unlock reasoning at the level of events providing a different set of computational opportunities and constraints (Davis et al., 2020b; Gong & Bramley, 2020). In doing this, we hope these empirical findings can speak to the large topic of how bounded human learners (Simon, 1982), succeed in identifying useful causal representations of a continuous dynamic world.

In sum, we systematically tested factors that seemed likely to influence judgments about the nature of a single causal relationship. We found that, indeed, judgment patterns reflected plausible inductive biases—preferences for shorter lags, and positive changes—but also more perceptual factors, such as sensitivity to range of change relative to each variable's bounds, and the rigidity with which an effect adjusts to changes in its cause. The insensitivity of standard time-series models to these factors suggests they may have limited utility in accounting for human causal induction. We argue that careful work to reverse engineer how human inductive biases and perceptual constraints combine in shaping causal representation is necessary for understanding the cognitive process of causal induction in a continuous world.

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