Problem 2

$\boxed{x_i \sim e^{\lambda}}$ for λ rate

Part a. $p(x_i|\lambda) = \prod_{i=1}^{n} \lambda e^{-\lambda x_i}$, $x > 0$

$$= (\lambda)^n \cdot e^{\sum_1^n -\lambda x_i}$$

$$= (\lambda)^n \cdot e^{-\lambda \cdot \sum_1^n x_i}$$ likelihood

Prior

Part. b. $\lambda \sim N\left(\frac{1}{10}, 1\right) = \frac{1}{\sqrt{2\pi \cdot 1^2}} \cdot e^{\left\{-\frac{(\lambda - 0.1)^2}{2 \cdot 1^2}\right\}} = \dfrac{e^{\left\{-\frac{(\lambda-0.1)^2}{2}\right\}}}{\sqrt{2\pi}}$

Posterior $p(\lambda | 0.1, 1) \propto p(\lambda) p(x_i | \lambda)$

$$\propto \frac{1}{\sqrt{2\pi}} \cdot e^{\left\{-\frac{(\lambda-0.1)^2}{2}\right\}} \cdot (\lambda)^n \cdot e^{\left\{-\lambda \cdot \sum_i^n x_i\right\}}$$

$$\propto (\lambda)^n \cdot e^{\left\{-\frac{(\lambda-0.1)^2}{2} - \lambda \cdot \sum_i^n x_i\right\}}$$

Gamma

$$p(x|\alpha, \beta) = \frac{\beta^{\alpha}}{(\alpha-1)!} \cdot x^{\alpha-1} \cdot e^{-\beta x}$$

Exponential

$$p(x|\lambda) = \lambda \cdot e^{-\lambda x}$$