Part 6.

$$\ln\left(L(\beta|X,\vec{y})\right) = \ln\left[\prod_{i=1}^{n}\left(\frac{e^{X_i\beta}}{1+e^{X_i\beta}}\right)^{y_i}\cdot\prod_{i=1}^{n}\left(\frac{1}{1+e^{X_i\beta}}\right)^{1-y_i}\right]$$

Prop
$\ln(x\cdot y) = \ln(x) + \ln(y)$

$$= \ln\left(\prod_{i=1}^{n}\left(\frac{e^{X_i\beta}}{1+e^{X_i\beta}}\right)^{y_i}\right) + \ln\left(\prod_{i=1}^{n}\left(\frac{1}{1+e^{X_i\beta}}\right)^{1-y_i}\right)$$

$$= \sum_{i=1}^{n}\ln\left(\left(\frac{e^{X_i\beta}}{1+e^{X_i\beta}}\right)^{y_i}\right) + \sum_{i=1}^{n}\ln\left(\left(\frac{1}{1+e^{X_i\beta}}\right)^{1-y_i}\right)$$

$$= \sum_{i=1}^{n} y_i\cdot\ln\left(\frac{e^{X_i\beta}}{1+e^{X_i\beta}}\right) + \sum_{i=1}^{n}(1-y_i)\cdot\ln\left(\frac{1}{1+e^{X_i\beta}}\right)$$

Prop
$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$

$$= \sum_{i=1}^{n} y_i\cdot\underbrace{\ln\left(e^{X_i\beta}\right)}_{\text{evaluates}} - y_i\left(1+e^{X_i\beta}\right) + \sum_{i=1}^{n}(1-y_i)\cdot\underbrace{\ln(1)}_{0} - (1-y_i)\cdot\ln\left(1+e^{X_i\beta}\right)$$

$$= \sum_{i=1}^{n} y_i\cdot X_i\beta + \sum_{i=1}^{n} -(1-y_i)\cdot\ln\left(1+e^{X_i\beta}\right) + \sum_{i=1}^{n} -y_i\left(1+e^{X_i\beta}\right)$$

$$= \sum_{i=1}^{n} y_i\cdot X_i\beta + \sum_{i=1}^{n}(y_i-1)\ln\left(1+e^{X_i\beta}\right) + \sum_{i=1}^{n} -y_i\left(1+e^{X_i\beta}\right)$$

$$= \sum_{i=1}^{n} y_i\cdot X_i\beta + \sum_{i=1}^{n}(-1)\cdot\ln\left(1+e^{X_i\beta}\right) + \sum_{i=1}^{n}(y_i)\cdot\ln\left(1+e^{X_i\beta}\right) - \sum_{i=1}^{n}(y_i)\left(1+e^{X_i\beta}\right)$$

$$\ell(\beta|X,\vec{y}) = \sum_{i=1}^{n} y_i\cdot X_i\beta - \sum_{i=1}^{n}\ln\left(1+e^{X_i\beta}\right)$$