

# Active fault-tolerant control for quadrotors subjected to a complete rotor failure

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**Abstract**—This paper deals with the active fault-tolerant control for quadrotors which are subjected to a total rotor failure. Previous studies assume that the fault has been detected and isolated and then design a fault-tolerant controller. The present paper proposes a complete active fault-tolerant control system which copes with not only fault detection and isolation but also fault-tolerant control. A novel and efficient fault detection and isolation approach is proposed for the total rotor failure case. An incremental nonlinear dynamic inversion approach is introduced to design the fault-tolerant controller for the quadrotor in the presence of the fault. The complete active fault-tolerant control system enables the quadrotor to achieve any position even after the complete loss of one rotor.

## I. INTRODUCTION

Autonomous Unmanned Aerial Vehicles (UAV)s such as quadrotors have attracted a considerable amount of attention because of their autonomy and being able to accomplish complicated missions. Quadrotors are able to take off, land vertically, and hover at a fixed point. They have been applied to a variety of areas, including scientific research, commercial services, civil development and military missions [1]. Since the environment these vehicles operate in contains a high degree of disturbances and uncertainties, the object of control and estimation of these vehicles is becoming increasingly challenging. Therefore, more advanced control and estimation approaches are required [1]. Furthermore, due to the increasing requirement for control and estimation systems to be more secure and reliable, Fault Detection and Diagnosis (FDD) and Fault Tolerant Control (FTC) are becoming more and more critical and significant [1].

Both sensor faults and actuator faults can affect the safety and reliability of the quadrotors. Sensor faults have been studied by some researchers using approaches such as observers [1], [2] and filters [3]. Different approaches have been also proposed to deal with actuator failures such as actuator fault detection [4] and actuator fault tolerant control [5], [6], [7], [8]. However, these papers only considers the partial failure of the rotors which means that the rotors experience a partial Loss Of Effectiveness (LOE) fault.

Few researchers have studied the complete failure of one rotor of the quadrotor [9]. Freddi, Lanzon and Longhi [10] first studied the fault-tolerant control for quadrotors subjected to a complete failure of one of the rotors. They propose to

sacrifice the control of the yaw to recover the flight. Mueller and D'Andrea [11] propose a periodic solution for this problem. Lippiello, Ruggiero and Serra [12] propose to turn off the motor which is opposite to the broken one. However, none of the above research considers the detection of the fault. They assume the Fault Detection and Isolation (FDI) has been performed [9], [12] or to be the future work [11].

The present paper deals with the total failure of one of the rotors. It proposes a complete Active Fault-Tolerant Control (AFTC) system for quadrotors subjected to a total rotor failure. The AFTC system contains not only the detection and isolation of the fault but also the fault-tolerant control of the quadrotors. This paper proposes a novel and efficient FDI approach which allows for timely detection of the total rotor failure. The AFTC system contains two controllers one of which is for the fault-free case while the other is the fault-tolerant controller designed to achieve fault tolerance. When the FDI system detects a total rotor failure, the controller switches from the fault-free controller to the fault-tolerant controller. The performance of the proposed AFTC system is demonstrated to be effective.

The remainder of this paper is as follows: Section II presents the modeling of the quadrotor which takes the hub forces and friction forces into account. Section III presents the FDI system which is designed for the case of one total rotor failure. The fault-tolerant control system is introduced in Section IV. The performance of the approach is shown in Section V. Finally, Section VI concludes the paper.

## II. MODELS OF THE QUADROTOR

This section first presents the translational and rotational dynamics of the quadrotor, followed by some simplifications used for design of the controller and fault detection system. Finally, the new definition of the LOE factors is presented.

### A. Dynamics of the quadrotor

The reference frames of the quadrotor are shown in Fig. 1. The earth frame is denoted as  $\{\Sigma_e\}(O_e, x_e, y_e, z_e)$  and the body frame of the quadrotor, which is fixed to the quadrotor, is denoted as  $\{\Sigma_b\}(O_b, x_b, y_b, z_b)$ . Both  $z_e$  and  $z_b$  point down. The rotation of  $\Sigma_b$  with respect to  $\Sigma_e$  is described by the rotation matrix  $R$  as follows:

$$R = \begin{bmatrix} C\theta C\psi & S\phi S\theta C\psi - C\phi S\psi & C\phi S\theta C\psi + S\phi S\psi \\ C\theta S\psi & S\phi S\theta S\psi + C\phi C\psi & C\phi S\theta S\psi - S\phi C\psi \\ -S\theta & S\phi C\theta & C\phi C\theta \end{bmatrix}$$

where  $S(\cdot)$ ,  $C(\cdot)$  and  $T(\cdot)$  denote  $\sin(\cdot)$ ,  $\cos(\cdot)$  and  $\tan(\cdot)$ , respectively.  $\phi$ ,  $\theta$  and  $\psi$  are the roll, pitch and yaw angles.

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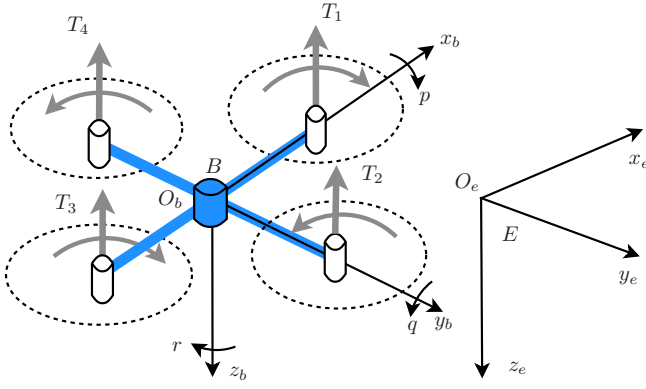


Fig. 1: The body fixed reference frame and the inertial reference frame used for deriving the model of the quadrotor

The position of the vehicle expressed in the earth frame is denoted as  $\mathbf{d} = (x, y, z)^T$ . The inertial velocity of the vehicle expressed in the earth frame is denoted as  $\mathbf{V}^E = (u, v, w)^T$ . Therefore, the dynamics of the position of the quadrotor can be described as follows:

$$\dot{\mathbf{d}} = \mathbf{V}^E \quad (1)$$

The dynamics of the velocity  $\mathbf{V}^E$  are:

$$\dot{u} = -(\mathbf{C}\phi\mathbf{S}\theta\mathbf{C}\psi + \mathbf{S}\phi\mathbf{S}\psi)\frac{T}{m} + \frac{f_x}{m} - \sum_{i=1}^4 H_{xi} \quad (2)$$

$$\dot{v} = -(\mathbf{C}\phi\mathbf{S}\theta\mathbf{S}\psi - \mathbf{S}\phi\mathbf{C}\psi)\frac{T}{m} + \frac{f_y}{m} - \sum_{i=1}^4 H_{yi} \quad (3)$$

$$\dot{w} = g - (\mathbf{C}\phi\mathbf{C}\theta)\frac{T}{m} + \frac{f_z}{m} \quad (4)$$

where  $m$  is the total mass of the vehicle,  $g = 9.81 \text{ m/s}^2$  denotes the gravity constant.  $T$  denotes the total thrust generated by the four rotors.  $H_{xi}$  and  $H_{yi}$  are the hub forces which are the resultants of the horizontal forces acting on the blades [13]. For the detailed calculation refer to [13].  $f_x$ ,  $f_y$  and  $f_z$  are the friction forces which are denoted as follows:

$$f_x = -\frac{1}{2}C_x A_c \rho u |u| \quad (5)$$

$$f_y = -\frac{1}{2}C_y A_c \rho v |v| \quad (6)$$

$$f_z = -\frac{1}{2}C_z A_c \rho w |w| \quad (7)$$

where  $C_x$ ,  $C_y$  and  $C_z$  are the friction coefficients,  $\rho$  is the air density and  $A_c$  is the helicopter center hub area.

The dynamics of the attitude angles are described by:

$$\dot{\phi} = p + q\mathbf{S}\phi\mathbf{T}\theta + r\mathbf{C}\phi\mathbf{T}\theta \quad (8)$$

$$\dot{\theta} = q\mathbf{C}\phi - r\mathbf{S}\phi \quad (9)$$

$$\dot{\psi} = q\frac{\mathbf{S}\phi}{\mathbf{C}\theta} + r\frac{\mathbf{C}\phi}{\mathbf{C}\theta} \quad (10)$$

where  $p$ ,  $q$  and  $r$  are the angular rates denoted by  $\boldsymbol{\omega} = [p, q, r]^T$ . The Euler angles  $\phi$  and  $\theta$  are assumed to satisfy the following conditions in this paper:

$$\phi \in [-\pi/2, \pi/2], \theta \in (-\pi/2, \pi/2) \quad (11)$$

Under these conditions, Eq. (10) is non-singular. It is assumed that the propellers are symmetrical and rigid. Therefore, the vehicle body inertia matrix can be denoted by a diagonal matrix as

$$\mathbf{I} = \text{diag}(I_x, I_y, I_z) \quad (12)$$

The rotational dynamics of the quadrotor including the propeller gyro effect are described as follows:

$$I_x \dot{p} = (I_y - I_z)qr + \tau_x + I_r q \omega_r - h \sum_{i=1}^4 H_{yi} \quad (13)$$

$$I_y \dot{q} = (I_z - I_x)rp + \tau_y - I_r p \omega_r + h \sum_{i=1}^4 H_{xi} \quad (14)$$

$$I_z \dot{r} = (I_x - I_y)pq + \tau_z + I_r \dot{\omega}_r + L(H_{x2} - H_{x4}) + L(-H_{y1} + H_{y3}) - k_r r \quad (15)$$

where  $h$  is the vertical distance between the center of gravity of the quadrotor and the propeller plane,  $L$  is the arm length,  $I_r$  is the inertia of the propeller,  $k_r$  is the aerodynamic drag coefficient.  $\omega_r$  is defined as

$$\omega_r = -\omega_1 + \omega_2 - \omega_3 + \omega_4 \quad (16)$$

where  $\omega_i$ ,  $i = 1, 2, 3, 4$  denote the rotational speeds of the four rotors. The thrust generated by each rotor is given by the following equations [14]:

$$T_i = b_i \omega_i^2, \quad i = 1, 2, 3, 4 \quad (17)$$

$$b_i = \rho \sigma a A R^2 \left[ \left( \frac{1}{6} + \frac{\mu_i^2}{4} \right) \theta_0 - (1 + \mu_i^2) \frac{\theta_{tw}}{8} - \frac{\lambda_i}{4} \right] \quad (18)$$

where  $b_i$  the thrust coefficient,  $\sigma$  and  $a$  are the solidity ratio and lift slope.  $A$  the disk area,  $R$  the radius of the propeller.  $\theta_0$  and  $\theta_{tw}$  are the pitch of incidence and twist pitch.  $\lambda_i$  and  $\mu_i$  are the inflow ratio and the advance ratio of the rotor denoted by [14]:

$$\lambda_i = \frac{\nu_1 - w}{\omega_i R}, \quad \mu_i = \frac{V}{\omega_i R} \quad (19)$$

In the equation, the horizontal velocity  $V = \sqrt{u^2 + v^2}$ ,  $\nu_1$  is the inflow velocity calculated by

$$\nu_1 = \sqrt{-V^2/2 + \sqrt{(V^2/2)^2 + (mg/(8\rho A))^2}}$$

For simplicity, it is assumed that the thrust and torque generated by the propellers are linear with respect to the square of their angular velocities such that the total thrust and torques can be denoted by:

$$\begin{bmatrix} T \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \\ 0 & -Lb_2 & 0 & Lb_4 \\ Lb_1 & 0 & -Lb_3 & 0 \\ -d_1 & d_2 & -d_3 & d_4 \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \quad (20)$$

where  $d_i$  is the torque coefficient which varies with the flight conditions.  $\omega_i$  the rotational speeds of the rotor given by [8]:

$$\omega_i = \frac{K}{\tau s + 1} \omega_{ci}, \quad i = 1, 2, 3, 4 \quad (21)$$

where  $\omega_{ci}$  are the command of rotational speeds given to the actuator.  $\tau$  is the time constant of the actuator and  $K$  is the gain of the actuator.

### B. Simplifications for the design of a fault detection system and controller

The above model is used to simulate the dynamics of the quadrotor. For the design of a controller and a fault detector, however, the following simplifications are made:

$$f_x = f_y = f_z = 0, \quad H_{xi} = H_{yi} = 0, \quad i = 1, 2, 3, 4. \quad (22)$$

The reason is that they are difficult to identify in practice and their magnitudes are small compared to the forces and moments generated by the four rotors.

For controller design,  $b_i$  and  $d_i$  are assumed to be constant such that

$$\begin{bmatrix} T \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & -Lb & 0 & Lb \\ Lb & 0 & -Lb & 0 \\ -d & d & -d & d \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} := \mathbf{A}_h \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \quad (23)$$

where  $b$  and  $d$  are the thrust coefficient and torque coefficients during the hovering of the quadrotor, respectively.

### C. Definition of the LOE factors

In [8], a definition of the LOE factors is given. However, in that paper the thrust and torque coefficients are assumed to be constant. It is only effective when the quadrotor is in hover. In the present paper, a new definition is given which takes the varying  $b_i$  and  $d_i$  into consideration.

When the quadrotor is not in hover,  $b_i \neq b$ . Define

$$\eta_i := \frac{b_i}{b} = 1 + \frac{\theta_0 \mu_i^2 / 4 - \theta_{tw} \mu_i^2 / 8 - \lambda_i / 4 + \lambda_h / 4}{\theta_0 / 6 - \theta_{tw} / 8 - \lambda_h / 4} \quad (24)$$

where  $\lambda_h = \sqrt{mg/8\rho A}/(\omega_h R)$  with  $\omega_h$  the rotational speed in hover. The ratio  $\eta_i$  can be fitted to

$$\eta_i = a_0 + a_1 \lambda_i + a_2 \mu_i^2 \quad (25)$$

$a_0$ ,  $a_1$  and  $a_2$  need to be identified when using a different quadrotor. In this paper,  $a_0 = 1.40$ ,  $a_1 = -5.94$  and  $a_2 = 1.42$ .

In case of actuator failures, the actuator effectiveness decreases. Denote  $l_i$ ,  $i = 1, 2, 3, 4$  as the LOE factors of the four actuators. Assuming the drag torque generated by the rotor is proportional to the thrust it generates, the LOE factors can be given using Eq. (20) as follows:

$$\begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \mathbf{A}^{-1} \begin{bmatrix} T \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \quad (26)$$

where  $\mathbf{A}$  is given as follows:

$$\mathbf{A} = \mathbf{A}_h \cdot \text{diag}(\eta_1 \omega_1, \eta_2 \omega_2^2, \eta_3 \omega_3^2, \eta_4 \omega_4^2) \quad (27)$$

It can be seen that  $l_i = 0$ ,  $i = 1, 2, 3, 4$  when there is no actuator fault. When the  $i$ th rotor fails,  $l_i \neq 0$ .  $l_i = 1$  when the  $i$ th rotor fails completely.

### III. FAULT DETECTION AND ISOLATION OF THE TOTAL ROTOR FAILURE

In order to achieve AFTC, FDI must be performed first. Lanzon, Freddi and Longhi [9] first deal with the complete failure of one rotor. They assumed that the fault has already been detected. Mueller and D'Andrea [11] also deal with the complete failure of one rotor, where they consider the FDI as the future work. This paper designs a complete AFTC system which contains both the FDI and the FTC.

In this section, the estimation of the LOE factors is presented followed by the FDI of the complete rotor failures.

#### A. Estimation of the loss of effectiveness factors

First, we try to estimate the torques generated by the rotors. Rewrite the rotational dynamics of the quadrotor (Eqs.(13), (14) and (15)) as follows:

$$\tau_x = I_x \dot{p} - [(I_y - I_z)qr + I_r q \omega_r - h \sum_{i=1}^4 H_{yi}] \quad (28)$$

$$\tau_y = I_y \dot{q} - [(I_z - I_x)rp - I_r p \omega_r + h \sum_{i=1}^4 H_{xi}] \quad (29)$$

$$\tau_z = I_z \dot{r} - [(I_x - I_y)pq + I_r \dot{\omega}_r + L(H_{x2} - H_{x4}) + L(-H_{y1} + H_{y3}) - k_r r] \quad (30)$$

Considering (22), we can estimate the torques generated by the rotor using the following equations:

$$\hat{\tau}_x = I_x \hat{p} - [(I_y - I_z)\hat{q}\hat{r} + I_r \hat{q}\hat{\omega}_r] \quad (31)$$

$$\hat{\tau}_y = I_y \hat{q} - [(I_z - I_x)\hat{r}\hat{p} - I_r \hat{p}\hat{\omega}_r] \quad (32)$$

$$\hat{\tau}_z = I_z \hat{r} - [(I_x - I_y)\hat{p}\hat{q} + I_r \hat{\omega}_r - k_r \hat{r}] \quad (33)$$

where  $\hat{(\cdot)}$  is the estimated value of a variable. The estimates of  $\hat{p}$ ,  $\hat{q}$  and  $\hat{r}$  can be obtained using an Inertial Measurement Unit (IMU) sensor.  $\hat{\omega}_r$  is denoted as:

$$\hat{\omega}_r = -\hat{\omega}_1 + \hat{\omega}_2 - \hat{\omega}_3 + \hat{\omega}_4 \quad (34)$$

where  $\hat{\omega}_i$  can be obtained using the commanded  $\omega_i$  and the actuator dynamics which is described in Eq. (21). We still need to estimate  $\dot{p}$ ,  $\dot{q}$  and  $\dot{r}$  to obtain  $\hat{\tau}_x$ ,  $\hat{\tau}_y$  and  $\hat{\tau}_z$ . In [8], a sliding mode differentiator is used. Here, we compute  $\hat{\dot{\omega}}$  ( $\hat{\dot{p}}$ ,  $\hat{\dot{q}}$  and  $\hat{\dot{r}}$ ) by taking the difference of the angular rates between two successive steps which is denoted as follows:

$$\hat{\dot{\omega}} = (\hat{\omega}_{k+1} - \hat{\omega}_k) / \Delta t \quad (35)$$

where  $\Delta t$  is the time interval between time step  $k$  and  $k+1$ . However, this method is sensitive to noise. Therefore, in this paper, the following second-order filter is used to obtain  $\hat{\dot{\omega}}$ :

$$(\omega_n^2 s) / (s^2 + 2\xi \omega_n s + \omega_n^2) \quad (36)$$

where  $\xi$  and  $\omega_n$  can be chosen to obtain the derivative. There is a trade-off between noise reduction and signal delay. In this paper,  $\xi = 0.8$ ,  $\omega_n = 20$  rad/s. By now, the estimation of torques are obtained.

Next, we estimate the force generated by the rotors. This can be readily achieved by rewriting Eq. (4) as follows:

$$T = [m(g - \dot{w}) + f_z] / (C\phi C\theta) \quad (37)$$

Therefore, the estimate of  $T$ , considering Eq. (22), can be obtained as follows:

$$\hat{T} = m(g - \hat{w}) / (C\hat{\phi}C\hat{\theta}) \quad (38)$$

where  $\hat{\phi}$  and  $\hat{\theta}$  can be obtained using an optic track system.  $\hat{w}$  can be obtained using the similar method as obtaining  $\hat{\omega}$ .

Having obtained the estimate of the total force and torques generated by the rotors, an estimate of the LOE factors can be obtained as follows:

$$\begin{bmatrix} \hat{l}_1 \\ \hat{l}_2 \\ \hat{l}_3 \\ \hat{l}_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \mathbf{A}^{-1} \begin{bmatrix} \hat{T} \\ \hat{\tau}_x \\ \hat{\tau}_y \\ \hat{\tau}_z \end{bmatrix} \quad (39)$$

#### B. Fault detection and isolation logic

The estimated LOE factors can be used to detect and isolate the total rotor failure. If one of the estimated LOE factors exceeds a certain threshold, it is considered as a total rotor failure. Define four indicators  $I_i$  which monitor the four estimated LOE factors:

$$I_i = \begin{cases} 0, l_i < l_T \\ 1, l_i \geq l_T \end{cases} \quad (40)$$

where  $l_T$  is a predefined threshold which is selected based on fault-free simulations.

Let  $i_0$  denote the rotor which is detected to be subjected to a total failure. After the detection of a total rotor failure,  $\mathbf{A}$  is singular due to  $\omega_{i_0} = 0$ . Since the goal of this paper is to deal with one rotor failure, after the detection of one total rotor failure, the LOE factors are set to the following:

$$l_i = \begin{cases} 0, i \neq i_0 \\ 1, i = i_0 \end{cases} \quad (41)$$

### IV. FAULT-TOLERANT CONTROL IN THE PRESENCE OF THE COMPLETE ROTOR FAILURE

In the previous section, the FDI of one total rotor failure is presented, which is the precondition of the FTC. In this section, first the reduced kinematics are introduced. Then, the Incremental Nonlinear Dynamic Inversion (INDI) controller used to control the quadrotor is introduced.

#### A. Reduced attitude kinematics and the equilibria

Considering (22), the translational dynamics can be rewritten into the following:

$$\ddot{\mathbf{d}} = g\mathbf{e}_3 - \mathbf{R}\mathbf{e}_3 \frac{T}{m} \quad (42)$$

where  $\mathbf{e}_3 = [0, 0, 1]^T$ .

When one of the rotors of the quadrotor fails completely, full attitude control of the quadrotor is not possible [9]. The goal of this paper is to control the position of the quadrotor. Therefore, the controllability of the yaw angle is sacrificed. Consequently, Eq. (42) can not be directly used to design the controller for  $x$  and  $y$  after the total rotor failure.

According to Mueller and D'Andrea [11], a constant primary axis exists about which the quadrotor rotates with a

constant angular velocity. Let  $\mathbf{n} = [n_x, n_y, n_z]^T$  denote the unit vector in the body frame. The dynamics of  $\mathbf{n}$  are:

$$\dot{\mathbf{n}} = -\boldsymbol{\omega} \times \mathbf{n} \quad (43)$$

In the case of one complete loss of a rotor, there exists multiple equilibria or primary axes satisfying Eq. (43). One of the equilibria for  $\boldsymbol{\omega}$ , given by Lanzon, Freddi and Longhi [9], is  $[0, 0, r_e]^T$ , which means that  $p$  and  $q$  are both equal to zero while  $r$  equals to a constant non-zero number  $r_e$ . For this equilibrium, the primary axis is  $\mathbf{n} = [0, 0, \pm 1]^T$  where the sign of  $n_z$  depends on the sign of  $r_e$ .

#### B. Control loops using the INDI

In this subsection, an INDI controller is designed to achieve FTC. This approach is first proposed by Smith [15] and later modified by Bacon, Ostroff and Joshi [16]. The robustness of this approach in the absence of actuator dynamics can be found in Sieberling, Chu and Mulder [17] and the robustness analysis including the actuator dynamics is referred to Lu, van Kampen and Chu [18].

When there are no faults, the controller can be designed based on the translational dynamics (Eq. (42)) and the rotational dynamics (Eqs. (8), (9), (10), (13), (14) and (15)). For the sake of page limits, the design procedure of the nominal controller is omitted in this paper.

The remainder of this section is to introduce the design of the fault-tolerant controller under the condition that the fault has been detected and isolated (introduced in section III). The control structure using the INDI is based on two loops:

- 1) Outer control loop which controls the primary axis.
- 2) Inner control loop which controls the rotational dynamics of the quadrotor.

These two control loops are presented in the following.

1) *Outer control loop*: The objective of the outer control loop is to follow the position command by controlling the direction of the primary axis.

The desired direction of the primary axis  $\mathbf{n}_d = [n_{xd}, n_{yd}, n_{zd}]^T$  can be designed by [11]

$$\mathbf{n}_d = m\mathbf{R}^{-1}(\ddot{\mathbf{d}}_d - g\mathbf{e}_3)/(n_z T_c) \quad (44)$$

where  $\ddot{\mathbf{d}}_d = [\ddot{x}_d, \ddot{y}_d, \ddot{z}_d]^T$  is the desired acceleration. It can be obtained using a linear controller which depends on the error between the position and its desired value.  $T_c$  is the command for the total thrust which can be designed using Eq. (4) by

$$T_c = -m(\ddot{z}_d - g)/(\cos \phi \cos \theta) \quad (45)$$

Next, the control of the primary axis is presented. Rewrite Eq. (43) into the following:

$$\begin{bmatrix} \dot{n}_x \\ \dot{n}_y \\ \dot{n}_z \end{bmatrix} = \begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} := \mathbf{B} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (46)$$

It can be seen that  $\mathbf{B}$  is not invertible. In order to design the Nonlinear Dynamic Inversion (NDI) controller, the above

equation is rewritten into:

$$\begin{bmatrix} \dot{n}_x \\ \dot{n}_y \end{bmatrix} = \begin{bmatrix} 0 & -n_z \\ n_z & 0 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} + \begin{bmatrix} n_y \\ -n_x \end{bmatrix} r \quad (47)$$

$$:= \mathbf{B}_2 \begin{bmatrix} p \\ q \end{bmatrix} + \begin{bmatrix} n_y \\ -n_x \end{bmatrix} r \quad (48)$$

Since  $\mathbf{B}_2$  is invertible, the desired angular rates  $p_d$  and  $q_d$  can be readily designed using the standard NDI procedure.

2) *Inner control loop*: The control objective of the inner loop is to calculate the desired torques based on the desired angular rates.

The desired torques  $\tau_{xc}$  and  $\tau_{yc}$  can be designed by

$$\tau_{xc} = I_x(v_p - \dot{p}_0), \quad \tau_{yc} = I_y(v_q - \dot{q}_0) \quad (49)$$

where  $v_p$  and  $v_q$  are designed by a linear controller depending on the error between  $p$ ,  $q$  and their desired values  $p_d$  and  $q_d$  respectively.  $\dot{p}_0$  and  $\dot{q}_0$  can be obtained by passing  $p$  and  $q$  through a second-order filter as in (36) respectively.

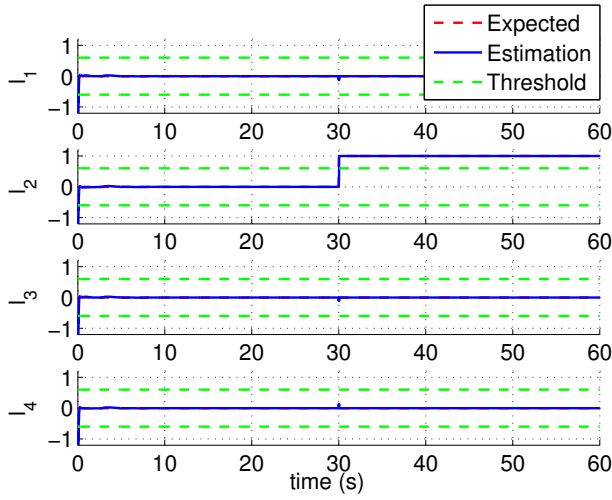


Fig. 2: Estimation of loss of effectiveness factors for case 1

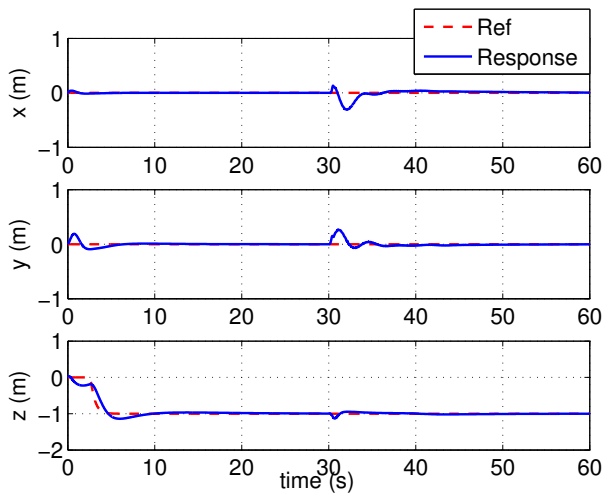


Fig. 3: The position and its command for case 1

### C. Control allocation

When the command for the total force and torques are generated, control allocation has to be performed to assign the force and torques to the four rotors. When there is no fault, the control allocation can be performed using Eq. (23). When there is a complete loss of one rotor, Eq. (23) can not be used due to singularity. Assuming rotor 2 fails without loss of generality, the control allocation can be performed using the following equation:

$$\begin{bmatrix} \omega_{c1}^2 \\ \omega_{c3}^2 \\ \omega_{c4}^2 \end{bmatrix} = \begin{bmatrix} b & b & b \\ 0 & 0 & Lb \\ Lb & -Lb & 0 \end{bmatrix}^{-1} \begin{bmatrix} T_c \\ \tau_{xc} \\ \tau_{yc} \end{bmatrix} \quad (50)$$

where  $\omega_{c1}$ ,  $\omega_{c3}$  and  $\omega_{c4}$  are the commands for rotor 1, 3 and 4 respectively.

## V. SIMULATION RESULTS

In this section, two cases are performed to demonstrate the performance of the proposed AFTC system. In the first case, the quadrotor climbs to an altitude of 1 m and then hovers there. In the second case, the quadrotor is required to follow a position command even after the detection of the fault. In both cases, the fault (complete loss of rotor 2) occurs at  $t = 30$  s.

The parameters used in this paper are presented as follows. The inertia of the quadrotor in three axes is  $I_x = I_y = 6.2 \times 10^{-3}$  kg m<sup>2</sup>,  $I_z = 1.2 \times 10^{-2}$  kg m<sup>2</sup>.  $I_r = 6 \times 10^{-5}$  kg m<sup>2</sup>,  $k_r = 6 \times 10^{-3}$ . The friction coefficients  $C_x = C_y = C_z = 1.3$ .  $\rho = 1.225$  kg/m<sup>3</sup>,  $A_c = 0.005$  m<sup>2</sup>,  $h = 0.058$  m, arm length  $L = 0.232$  m. The parameters related to the thrust generated by the rotor are  $R = 0.15$  m,  $\sigma = 0.17$  rad,  $a = 5.7$ ,  $\theta_0 = 0.262$  rad,  $\theta_{tw} = 0.045$  rad,  $A = 0.071$  m<sup>2</sup>. The thrust and torque coefficients  $b = 3.13 \times 10^{-5}$  and  $d = 7.5 \times 10^{-7}$ . The parameters related to the actuator are  $K = 1$  and  $\tau = 0.02$  s.

### A. Results of the first case

In this case, the reference command given to the quadrotor is shown by the dotted lines in Fig. 3. Before the fault, the quadrotor is in hover.

The result of the estimation of LOE factors are given in Fig. 2.  $l_2$  exceeds the threshold at  $t = 30.03$  s, which indicates a complete failure of rotor 2. This shows the efficiency of the FDI system.

In [9], the initial yaw rate is close to the equilibrium (the initial value is 2.7 rad/s while the equilibrium value is around 3 rad/s). In [11], the fault-tolerant controller is enabled when the yaw rate exceeds 10 rad/s. In this paper, the fault-tolerant controller is enabled even when the yaw rate still significantly deviates from its equilibrium value.

The position response is shown in Fig. 3. As can be seen, after the fault,  $x$ ,  $y$  and  $z$  deviate from the command. Once the fault is detected, the fault-tolerant controller is used. Therefore, the hover is recovered after 5 s. The rotational speeds of the four rotors are shown in Fig. 4. It is seen that after the faults, the rotational speeds of the rotors oscillate.

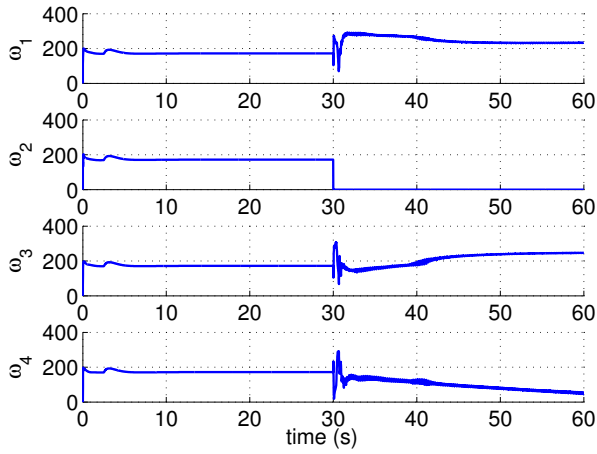


Fig. 4: Rotational speeds of the rotors for case 1

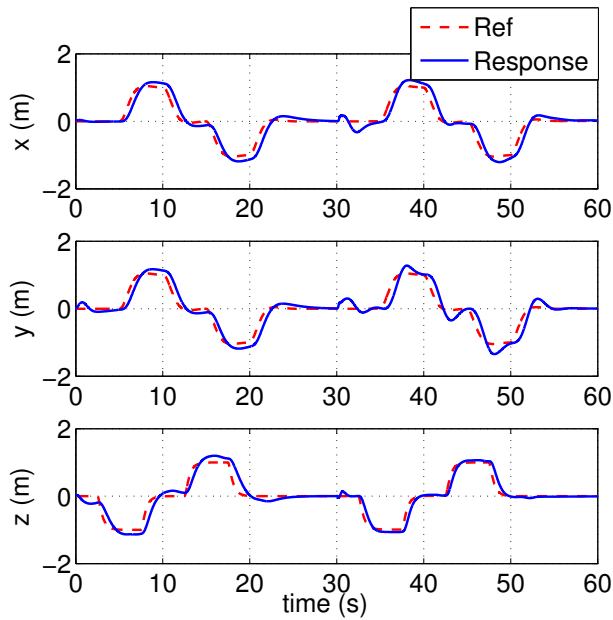


Fig. 5: The position and its command for case 2

### B. Results of the second case

In this case, the quadrotor follows the command to change its position and altitude from the initial point. After the fault, the command is the same to demonstrate the performance after the fault.

The LOE factor estimation is similar to that shown in Fig. 2. The position and its command is given in Fig. 5. As can be seen, after the fault is detected, the quadrotor is still able to follow the position and altitude command. This shows the satisfactory performance of the AFTC system.

## VI. CONCLUSIONS

This paper proposes a complete AFTC system for quadrotors which are subjected to a complete rotor failure. A novel FDI system is proposed which can efficiently detect and isolate the total failure of a rotor. Two controllers are

designed using an INDI approach: a nominal controller and a fault-tolerant controller. The performance as well as the robustness of the approaches are tested. The results show that the fault can be timely detected and a safe flight can still be maintained even after the complete loss of one rotor. An experiment validation of the proposed approach would be the future work.

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