

2D Poisson

Ramya Rao Basava
Department of Computer Science
University of British Columbia, Vancouver

1 Problem Statement

Consider the 2D Poisson problem with essential boundary conditions as given below:

$$\begin{aligned}\Delta u(x, y) &= (x^2 + y^2)e^{xy} & \text{in } \Omega &= \{(x, y) | 0 < x < 1, 0 < y < 1\} \\ u(x, y) &= e^{xy} & \text{on } \partial\Omega &= \{(x, y) | x = 0, 1 \text{ and } y = 0, 1\}\end{aligned}$$

The analytical solution to the problem is $u(x, y) = e^{xy}$.

2 Numerical Analysis

The domain is discretized using 10×10 source points and 40×40 collocation points which do not overlap as shown in Figure 1.

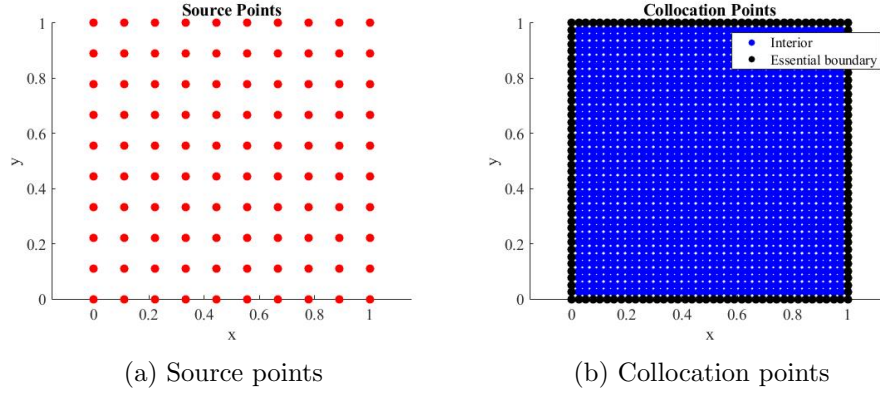


Figure 1: Domain discretization

The weight on the essential boundary is taken to be $\sqrt{\alpha^g} = N_S$, where N_S is the number of source points. The numerical and analytical solutions in the 2D domain are compared in Figure 2.

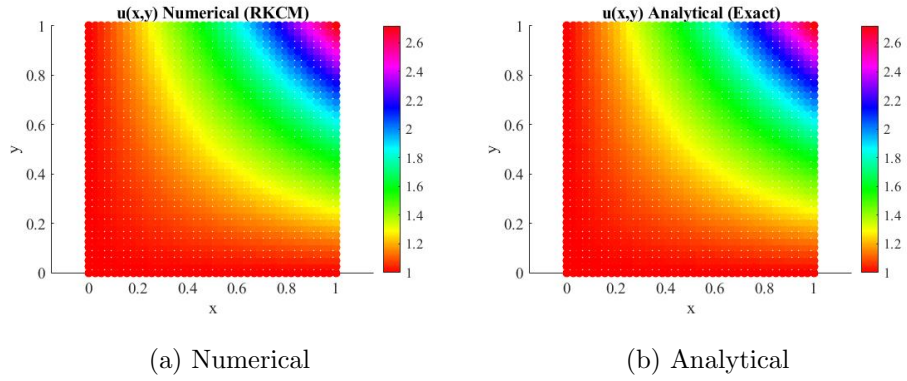


Figure 2: Comparison of numerical and analytical solution in 2D domain

Additionally, the numerical solution using RKCM obtained along the diagonal line passing through the points $(0,0)$ and $(1,1)$ is plotted in Figure 3 and compared with the analytical solution. The error between the two solutions along this diagonal line is also plotted.

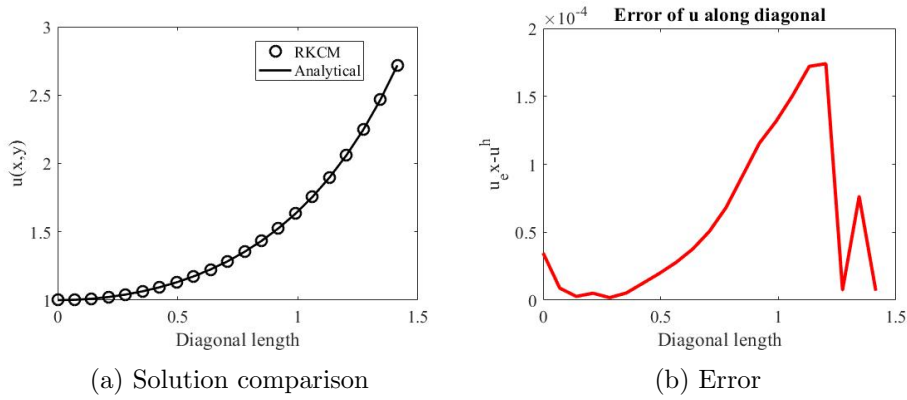


Figure 3: Comparison of numerical and analytical solution along the diagonal line

It can be seen that the numerical RKCM result is very close to the analytical solution.