Lecture 18

Helium Burning

3α- "Salpeter Process"

Lets consider $^4He + {}^4He \rightarrow {}^8Be\,\gamma$, now ^8Be is unstable, but it might be the case that there is some ^8Be around. The Gamow energy is

$$E_G = (\pi\alpha 4)^2\, 2\, m_r c^2$$

$$m_r = \frac{m_\alpha m_\alpha}{2 m_\alpha} = \frac{1}{2} m_\alpha \Rightarrow E_G = 31.4\ \text{MeV}$$

and at $T = 10^8$ $k_B T = 8.6$ keV, so as this is a purely strong reaction, lets guess that it has the standard form for a non-resonant reaction, namely a center energy

$$E_0 = \left(\frac{E_G (kT)^2}{4}\right)^{1/3} \simeq 83\,\text{keV}\left(\frac{T}{10^8}\right)^{2/3}$$

Now the ^8Be ground state is 91.8 keV above the $^4He + {}^4He$ mass and so, once $T \geq 10^8$ K or so, the fusion to form ^8Be can begin to take place. Now this element is unstable and decays with 2.6×10^{-16} sec.

~~Expand on this in the grad class~~

We won't go into detail here, but at high T & ρ the formation of ^8Be is fast enough, relative to decays, that we can presume a statistical equilibrium

$$^4He + {}^4He \rightleftharpoons {}^8Be.$$

$$\mu_4 + \mu_4 = \mu_8$$

$$\mu_4 = m_4 c^2 - kT \ln\left(\frac{g\, n_{Q,4}}{n_4}\right)$$

$$\mu_8 = m_8 c^2 - kT \ln\left(\frac{g\, n_{Q,8}}{n_8}\right).$$

$$2m_4 c^2 - 2kT\ln\left(\frac{g\, n_{Q,4}}{n_4}\right) = m_8 c^2 - kT\ln\left(\frac{g\, n_{Q,8}}{n_8}\right)$$

$$\frac{\left[2m_4 c^2 - m_8 c^2\right]}{kT} = 2\ln\left(\frac{g\, n_{Q,4}}{n_4}\right) - \ln\left(\frac{g\, n_{Q8}}{n_8}\right)$$

$$\exp\left[\frac{(2m_4 - m_8)c^2}{kT}\right] = \left(\frac{\cancel{g}\, n_{Q,4}}{n_4}\right)^2 \frac{n_8}{\cancel{g}\, n_{Q,8}}$$

$$n_Q = \left(\frac{2\pi m kT}{h^2}\right)^{3/2}$$

$g = 1$ as $J^\pi = 0$ (~~[illegible]~~ spin $= 0$).

$$\exp\left[\frac{(2m_4 - m_8)c^2}{kT}\right] = \frac{n_8}{n_4^2}\left(\frac{2\pi m_\alpha kT}{h^2}\right)^{3/2}\left(\frac{m_\alpha}{2m_\alpha}\right)^{3/2}$$

$$\Rightarrow \boxed{\frac{n_8}{n_4^2} = 2^{3/2}\left(\frac{h^2}{2\pi m_\alpha kT}\right)^{3/2} \exp\left(\frac{-(m_8 - 2m_4)c^2}{kT}\right)}$$

Now, if ~~T=2x10^8~~ $\rho = 10^4$ gr/cm³ of pure ^4He, then

$$n_4 = \frac{\rho}{4 m_p} = 1.5 \times 10^{27} \text{ cm}^{-3}$$

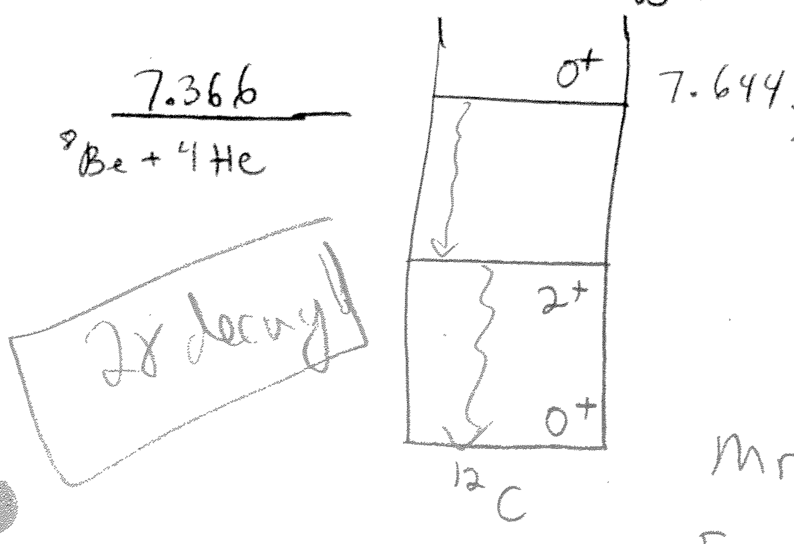
and

$$\frac{n_8}{n_4} = 2.8 \times 10^{-6} \left(\frac{10^8}{T}\right)^{3/2} \left(\frac{\rho}{10^4}\right) \exp\left[-\frac{91.8 \text{ keV}}{kT}\right]$$

$$\frac{n_8}{n_4} = 2.8 \times 10^{-6} \left(\frac{10^8}{T}\right)^{3/2} \left(\frac{\rho}{10^4}\right) \exp\left(-\frac{10.64}{T_8}\right)$$

So at 2×10^8 we get 10^{-8} of the nuclei are ^8Be. Now, we want to see if there is anything we can do with this. Indeed there is something, which is one more α capture, namely

$$\alpha + {}^8Be \rightarrow {}^{12}C + \gamma$$



> energy difference $\simeq 280$ keV.

Lets ask about

$$E_G = (\pi\alpha\, 2 \cdot 4)^2\, 2c^2 \tfrac{8}{3} m_p$$

$$m_r = \frac{m_4 m_8}{12 m_p} = \frac{32}{12} m_p = \frac{8}{3} m_p$$

$$E_G = 168 \text{ MeV}$$

$$\Rightarrow E_0 = 146 \text{ keV} \left(T/10^8\right)^{2/3} \approx 231 \text{ at}$$

Now, Hoyle made the brilliant suggestion that there must be an excited state in the ~~...~~ ^{12}C nucleus right around E_0, as the calc's for a non-resonant reaction gave rates way too slow. ~~Again~~, Such a state was found namely the one at 7.644 MeV. In this case the $^{12}C^*$ state can actually reach equilibrium so that we get SAHA abundance again,

$$^4He + {}^8Be \rightleftharpoons {}^{12}C^*$$

In which case, we could also just write down

$$^4He + {}^4He + {}^4He \rightleftharpoons {}^{12}C^*$$

$$3\mu_4 = \mu_{^{12}C^*}.$$

$$c^2 m_{12}^* = 3 m_4 c^2 + \cancel{380}\,\overset{379.5}{}\ \text{keV}$$

↑ excess energy.

Now, the $^{12}C^*$ nucleus almost always decays to $^4He + {}^8Be$ as that reaction is purely strong. However lets first find the abundance of $^{12}C^*$.

$$3m_4c^2 - kT\ln\left(\left(\frac{g\,n_{Q,4}}{n_4}\right)^3\right) = m_{12}^*c^2 - kT\ln\left(\frac{g\,n_{Q,12}}{n_{12}}\right)$$

$$\frac{3m_4c^2 - m_{12}c^2}{kT} = \ln\left[\frac{n_{12}}{n_{12,Q}}\,\frac{n_{4,Q}^3}{n_4^3}\right]$$

$$\exp\left(\frac{(3m_4 - m_{12}^*)c^2}{kT}\right) = \frac{n_{12}^*}{n_4^3}\,\frac{n_{4,Q}^3}{n_{12,Q}}$$

but

$$n_Q = \left(\frac{2\pi mkT}{h^2}\right)^{3/2}$$

$$\Rightarrow \frac{n_{12}^*}{n_4^3} = 3^{3/2}\left[\frac{h^2}{2\pi m_4 kT}\right]^3 \exp\left[\frac{(3m_4 - m_{12}^*)c^2}{kT}\right]$$

so

$$\frac{n_{12}^*}{n_4} = \left(\frac{\rho}{4m_p}\right)^2 3^{3/2}\left[\frac{h^2}{2\pi m_4 kT}\right]^3 \exp\left(-\frac{44}{T_8}\right)$$

$$\boxed{\frac{n_{12}^*}{n_4} = 5.2\times10^{-10}\left[\frac{\rho}{10^5}\right]^2\left(\frac{10^8}{T}\right)^3 \exp\left(\frac{-44}{T_8}\right)}$$

Now, this is the steady state population of $^{12}C^*$. Now, we are still not done, as

Phillips screwed up

$\Gamma_{rad} = 3.67\,\mathrm{meV}$ $\quad r = \frac{\Gamma}{\hbar} = 5.56\times10^{12}$

The $^{12}C^* \rightarrow {}^{12}C$ (g.s.) with a mean time of $\tau = \hbar/\Gamma_{rad} = 1.79\times10^{-13}\,s$ which is <u>much</u> slower than the decay of particles to $^4He + {}^8Be$. The rate of production of ^{12}C in g.s. is thus just

$$\frac{dn_{12}}{dt} = \frac{n_{12}^*}{\tau} = \frac{n_{12}^*\,\Gamma}{\hbar}$$

Which gives us an energy release of 7.65 MeV, so we get

$$\frac{\mathrm{ergs}}{\mathrm{cm^3\cdot sec}} = \frac{n_{12}^*\,E}{\tau}$$

or

$$\epsilon = \frac{1}{\rho}\,\frac{n_{12}^*\,E}{\tau} = \frac{n_{12}^*}{n_4}\left(\frac{E}{4m_p}\right)\frac{1}{\tau}$$

giving:

$$\boxed{\epsilon = 5.3\times10^{21}\,\frac{\mathrm{ergs}}{\mathrm{gr\cdot sec}}\left[\frac{\rho}{10^5}\right]^2\left(\frac{10^8}{T}\right)^3 \exp\left(-44/T_8\right)}$$

HW Problem

~~Now, the most dramatic thing to take note of is the extreme T sensitivity. Lets do a crude massive He star:~~

The only other competing reaction going on is

$$\alpha + {}^{12}C \rightarrow {}^{16}O + \gamma$$

And typically we find a little of both going on. The higher reaction get tougher. For example.

$\left(E_G = (\pi \alpha Z_1 Z_2)^2 2 m_r c^2 \right) = 982\,\mathrm{keV}\,(Z_1 Z_2)^2_{\mu}$

	E_G (MeV)	E_0 ($T = 2\times10^8$)	$\exp\left(-3\left(\frac{E_G}{4kT}\right)^{1/3}\right)$
$\alpha + {}^{12}C$	424.	315 keV	1.3×10^{-24}
$\alpha + {}^{16}O$	804.5	390	5.5×10^{-38}
$\alpha + {}^{20}Ne$	1310.0	460	1.5×10^{-44}

So the additional reactions are rather unlikely, except for the first one, which competes favorably with $3\alpha \rightarrow {}^{12}C$ leading to a typical final mix of

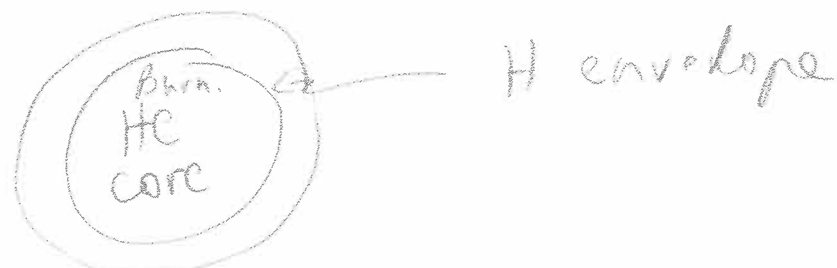
$${}^{12}C + {}^{16}O.$$

So the final products of Helium burning are ${}^{12}C$ & ${}^{16}O$ with the mix depending on where in (ρ, T) space things are happening.

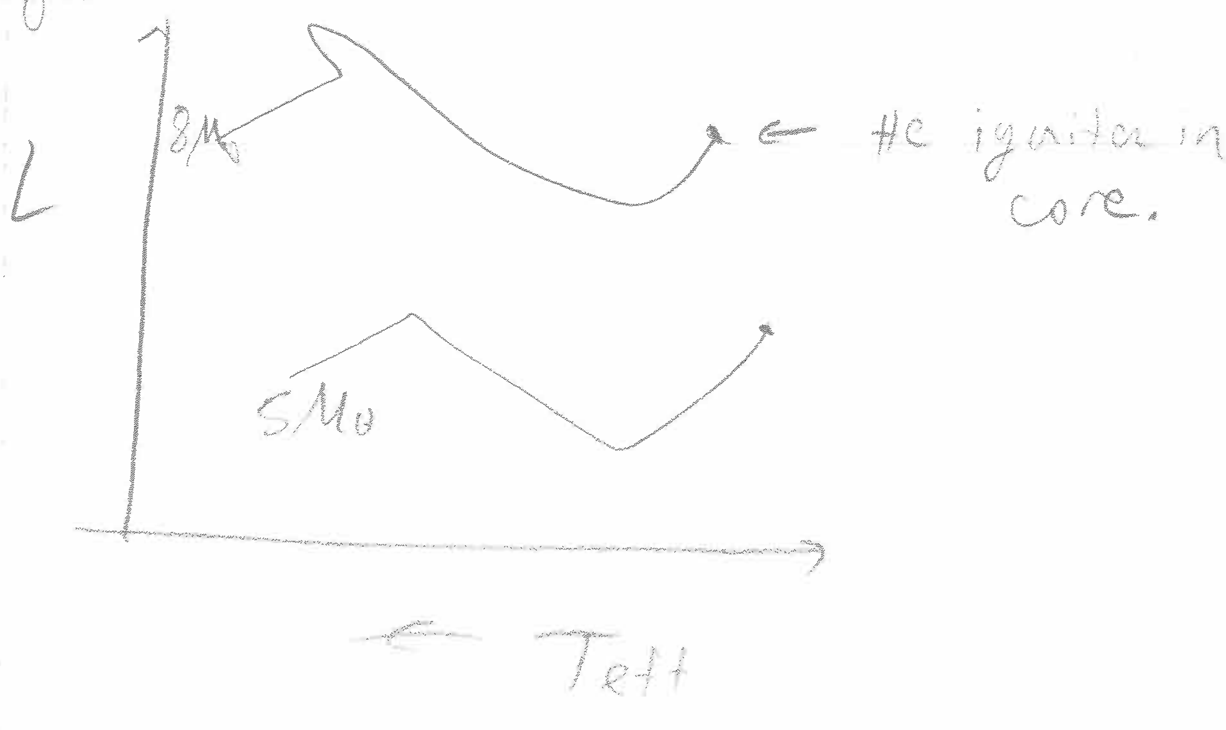
Helium Burning in Massive Stars

There zoom across during the KH contraction of the core until Helium ignites in a non-degenerate environment.

The picture for the massive stars is



There stars undergo multiple loops during the He burning phase and I will not go into that at all. Rather, I will just sketch:



While "He burning, the He core generates

$$L_{He} \simeq L_\odot \left(\frac{M_{He}}{M_\odot}\right)^3 \mu^4$$

but

$$\cancel{\frac{L_{He}}{L_H} \simeq 24 \quad 50}$$

$$\cancel{L = \left[24 L_\odot \left[\frac{q}{\;}\right]^3 + (\;)(1-q)^3\right] M^3}$$

$$\cancel{L \simeq \left[24 q^3 + (1-q)(1+q^2-2q)\right]}$$

$$\cancel{= (23 q^3 + 1 + q^2 - 2q - q}$$

q	L
1	24
.9	17
.8	2.3
0.	

which is nearly the 'Helium Main sequence' which would look like

$$L \propto M^3 \mu^4$$

so

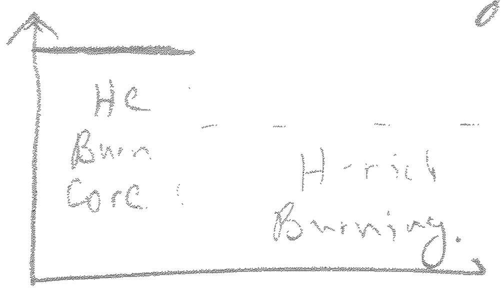
$$\frac{L_{He}}{L_H} = \left(\frac{\mu_{He}}{\mu_H}\right)^4 = \left(\frac{4/3}{0.6}\right)^4 = 24$$

or $\Delta \log L = 1.3$

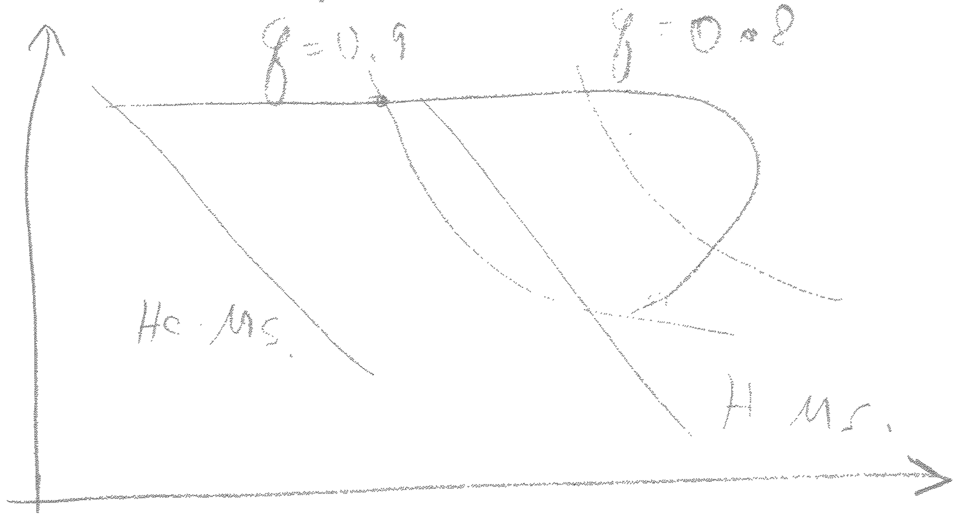
We can imagine constructing what is called a "general Main sequence" having

$M_{He} = qM \quad M_H = (1-q)M$

So something like:



I will just sketch this for now + not go into all that much detail



The stars live slightly longer while He burning than you would imagine. This is mostly due to the extensive shell burning of H.

The more massive stars undergo loops back + forth in the HR diagram. I will not go into this at all.

Reminder on

e^- deg EOS

$P \propto \rho^{5/3}$

$\hookrightarrow \quad R_c \propto \frac{1}{M_c^{1/3}}$