White Dwarfs + Chandra Mass

At late stages of stellar evolution, the electrons become more + more degenerate, eventually dominating the pressure. Lets work out the resulting M(R) relation in this limit. The EOS is just.

$$n_e = \frac{2}{h^3}\int_0^{p_f} 4\pi p^2 dp = \frac{8\pi}{3h^3} p_f^3$$

and the pressure is just (as derived earlier)

$$P_e = \frac{2}{5} n_e E_F$$

when things are non-relativistic, as we presume for now. So we get

$$P = \frac{2}{5} n_e \frac{1}{2m_e} p_f^2 = \frac{n_e}{5m_e}\left(\frac{3h^3 n_e}{8\pi}\right)^{2/3}$$

$$P = n_e^{5/3}\left(\frac{3h^3}{8\pi}\right)^{2/3}\frac{1}{5m_e}$$

We typically have $\rho = Am_p n_i$ and $n_e = Zn_i$ so

$$n_e = Z\frac{\rho}{Am_p} = \frac{\rho}{2m_p}$$ when $A = 2Z$ (He, C, O).

$$\Rightarrow P = \left(\frac{\rho}{2m_p}\right)^{5/3}\left(\frac{3h^3}{8\pi}\right)^{2/3}\frac{1}{5(m_e)}$$ – follow

Now, lets just ask what the rough properties are of an object supported by degenerate e's. Yet again, lets just write

$$P_c \Rightarrow \quad \frac{dP}{d\cancel{z}} = -\rho \frac{Gm}{r^2}$$

$$\Rightarrow \quad P \sim G \frac{M}{R^2} \frac{M}{R^2} = \frac{GM^2}{R^4}$$

but lets set $\rho R^3 = M \Rightarrow R = (M/\rho)^{1/3}$, so

$$P_c \sim \frac{GM^2}{M^{4/3}} \rho^{4/3} = \frac{1}{5 m_e} \left(\frac{\rho}{2m_p}\right)^{5/3} \left(\frac{3h^3}{8\pi}\right)^{2/3}$$

so we get

$$GM^{2/3} = \frac{1}{5 m_e} \frac{1}{(2m_p)^{5/3}} \left(\frac{3h^3}{8\pi}\right)^{2/3} \rho^{1/3}$$

$$\Rightarrow \rho \sim G^3 M^2 \left(\frac{(5m_e)^3 (2m_p)^5}{(3h^3/8\pi)^2} \right)$$

$$\rho \sim 4 \times 10^5 \frac{gr}{cm^3} \left(\frac{M}{0.1 M_\odot}\right)^2 \left(\frac{m_\alpha}{m_e}\right)^3$$

Now, this is the crude scaling at low masses and it is impt to ask what happens as the mass increases? $\rho \uparrow$, which implies the radius decreases. We can get the radius as well, since

$$\rho = \frac{M}{R^3} = 4\times10^5 \frac{gr}{cm^3}\left(\frac{M}{0.1M_\odot}\right)^2\left(\frac{m_x}{m_e}\right)^3$$

$$\Rightarrow \quad R \sim 8\times10^8\,cm\left(\frac{0.1M_\odot}{M}\right)^{1/3}\left(\frac{m_e}{m_x}\right)$$

so the star gets smaller as the mass increases.

[Insert true/proper formula here

$$R = 2\times10^9\,cm\left(\frac{0.1M_\odot}{M}\right)^{1/3}\left[\frac{m_e}{m_x}\right]$$

$\frac{1}{16}$ SO we get

$$R = 8\times10^8\,cm\ \frac{m_e}{m_x}$$

$$= 800\ \cancel{km}$$

$$\frac{2\times10^9}{2000} = 10^6\,cm$$

$$= 10\,km$$

when neutrons hold it up.

As $M\uparrow$, eventually the e^- start to become relativistic, in particular we can just ask when

$$P_f \approx m_e c, \quad \text{which}$$

is

$$m_e c = P_f = \left(\frac{3h^3 n_e}{8\pi}\right)^{1/3} = \left(\frac{3h^3 \rho}{2m_p 8\pi}\right)^{1/3}$$

$$\Rightarrow \quad \rho = (m_e c)^3 \frac{2m_p 8\pi}{3h^3} \approx \frac{16\pi}{3} m_p \left(\frac{m_e c}{h}\right)^3$$

This is a suggestive way to write things, or

$$\lambda_c = \frac{h}{m_e c}.$$

$$\Rightarrow \quad \boxed{\rho \approx 2\times 10^6 \ \mathrm{gr/cm^3}}$$

So, and these densities things get weird. The first imp't point is that the relativistic gas we know of will be unstable & thus it is impossible to build such an object. In the extreme relativistic limit, the Pe is

$$P = \frac{1}{4} n_e E_F = \frac{1}{4} n_e P_f c$$

$$= \frac{1}{4} n_e c \left(\frac{3h^3 n_e}{8\pi}\right)^{1/3} = n_e^{4/3} \frac{c}{4}\left(\frac{3h^3}{8\pi}\right)^{1/3}$$

And $P_c \sim G M^{2/3} \rho^{4/3} = \left(\frac{\rho}{2m_p}\right)^{4/3} \frac{c}{4} \left(\frac{3h^3}{8\pi}\right)^{1/3}$

and notice that the density cancels, giving an eqn for the mass!

$$\& \quad M^{2/3} = \frac{1}{G} \frac{c}{4} \frac{1}{(2m_p)^{4/3}} \left(\frac{3h^3}{8\pi}\right)^{1/3}$$

$$\Rightarrow \quad M = \frac{1}{G^{3/2}} \left(\frac{c}{4}\right)^{3/2} \frac{1}{(2m_p)^2} \left(\frac{3h^3}{8\pi}\right)^{1/2}$$

$$\approx 0.3 M_\odot$$ from the crude calculation.

[Fill in and provide more of the detailed stuff

Add in more discussion here of what happens during collapse.

The properly detailed calculation, using the polytropic relations puts in new factors giving the proper Chandrasekhar mass

$$M_{ch} = 1.456 \left(\frac{2}{\mu_e}\right)^2 M_\odot,$$

where $\mu_e = 1/e^-$ per baryon.

If the star gets heavier than this it undergoes catastrophic collapse, the outcome of which most likely depends on the content

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and M_{Ch} denote the radius and mass in the Chandrasekhar approximation for $\mu = A/Z = 2$. The other entries are results using our improved equation of state for He^4 ($\mu = 2.002$), C^{12} ($\mu = 2.001$), and Mg^{24} ($\mu = 1.999$). The entries marked with an asterisk have the highest central densities at which C^{12} or Mg^{24} is stable; later entries refer to models with a core of Ne^{24} and an outer zone of C^{12} and Mg^{24}, respectively. Table 2 gives the results for $_{26}Fe^{56}$ ($\mu = 2.152$), together with the Chandrasekhar ap-

TABLE 4*

ZERO-TEMPERATURE MODELS FOR EQUILIBRIUM COMPOSITION

	LOG ρ_c							
	8 627	8 920	9 147	9 361	9 692	10 28	11 28	11 53
$(Z; A)$	28; 62	28; 64	28; 64	28; 66	28; 66	30; 78	32; 90	38; 120
R.	0 400	0 343	0 300	0 267	0 216	0 157	0 080	0 074
M . . .	1 000	1 011	1 015	1 005	0 990	0 913	0 753	0 711

* $(Z; A)$ denotes the nuclear species at the center (ρ_c in gm/cc, R in units of 0 01$R_\odot$, M in units of $M_\odot$)

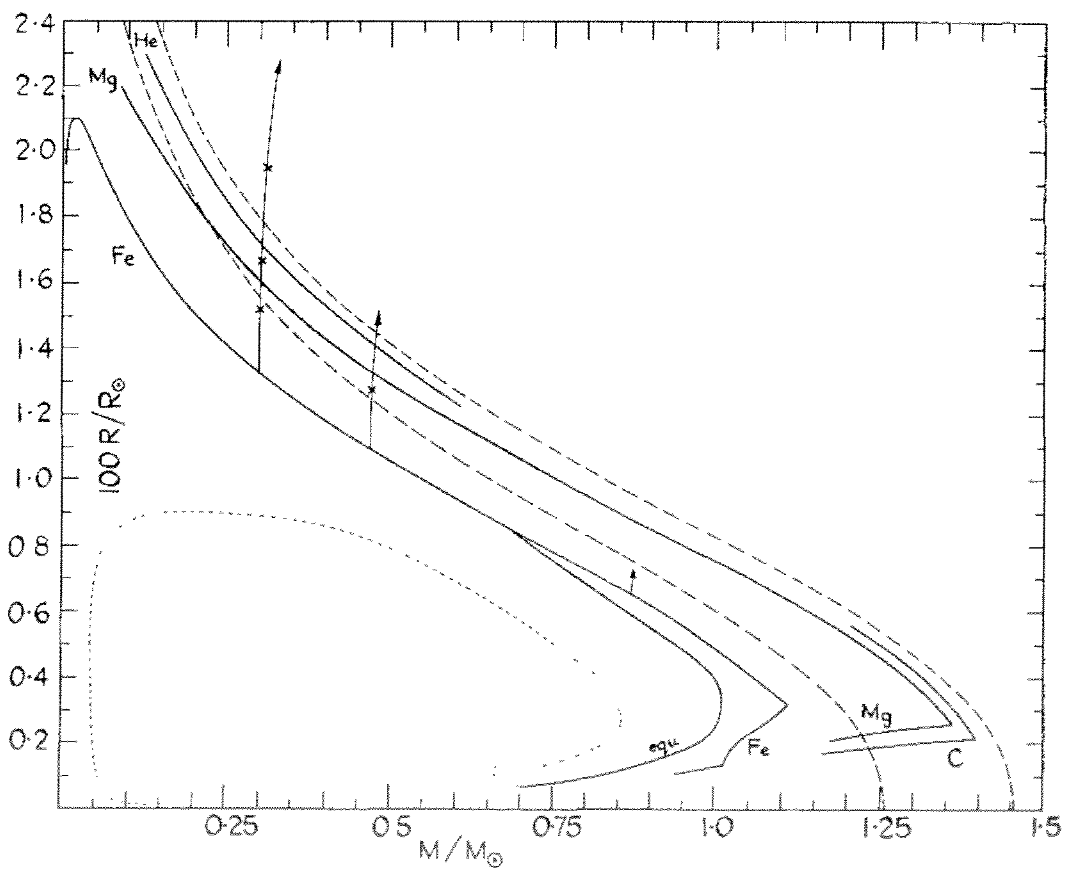

FIG. 1.—The relation between mass M and radius R for zero-temperature stars for He^4, C^{12}, Mg^{24}, and Fe^{56}. The curve marked *equ* denotes equilibrium composition at each density. The dashed curves denote the Chandrasekhar models, the upper one for $\mu = 2$ and the lower one for $\mu = 2.15$. The dotted curves denote neutron stars. The vertical arrows denote stars with H^1 in the outer layers.

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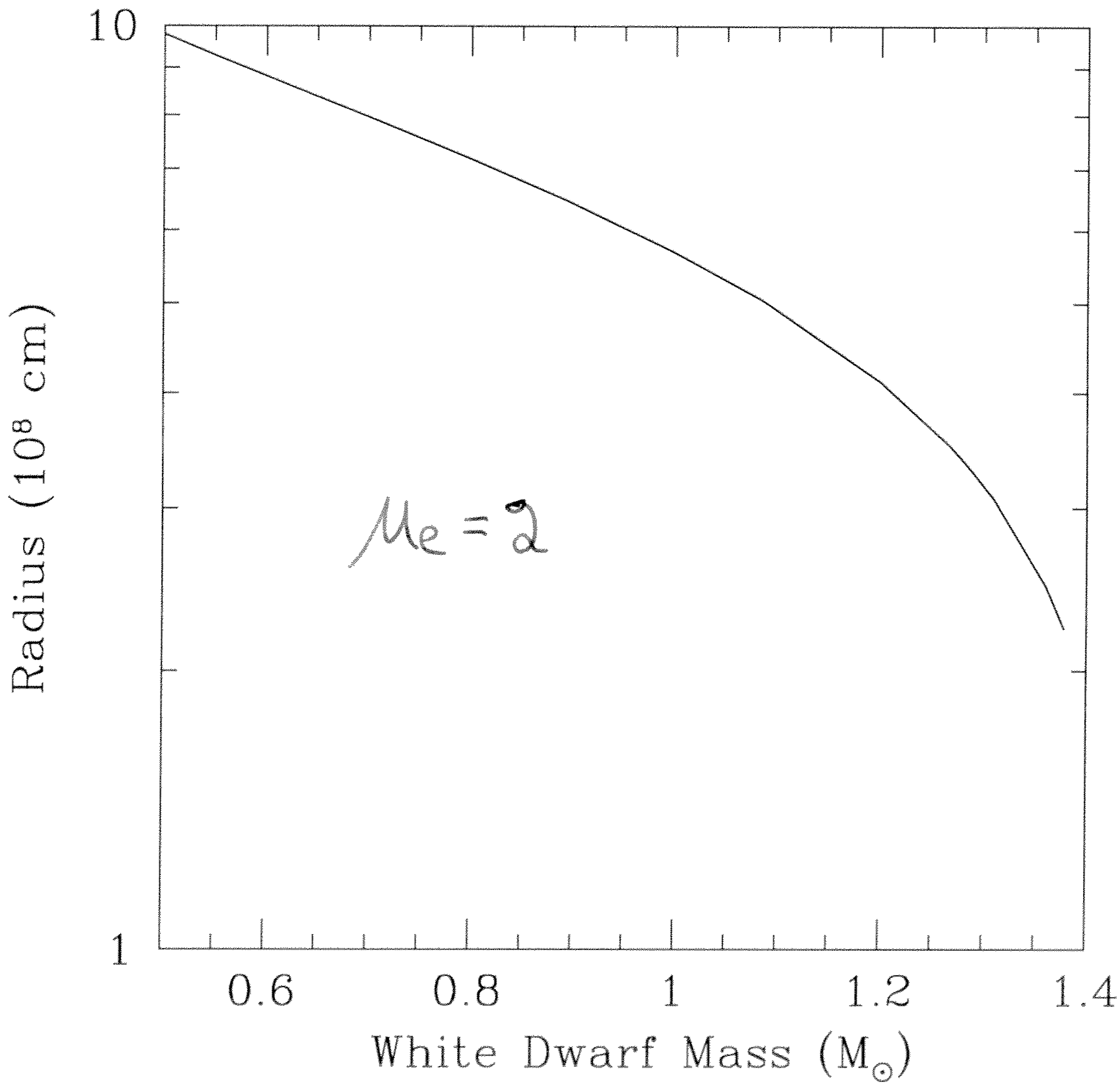

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Precise mass and radius values for the white dwarf and low mass M dwarf in the pre-cataclysmic binary NN Serpentis

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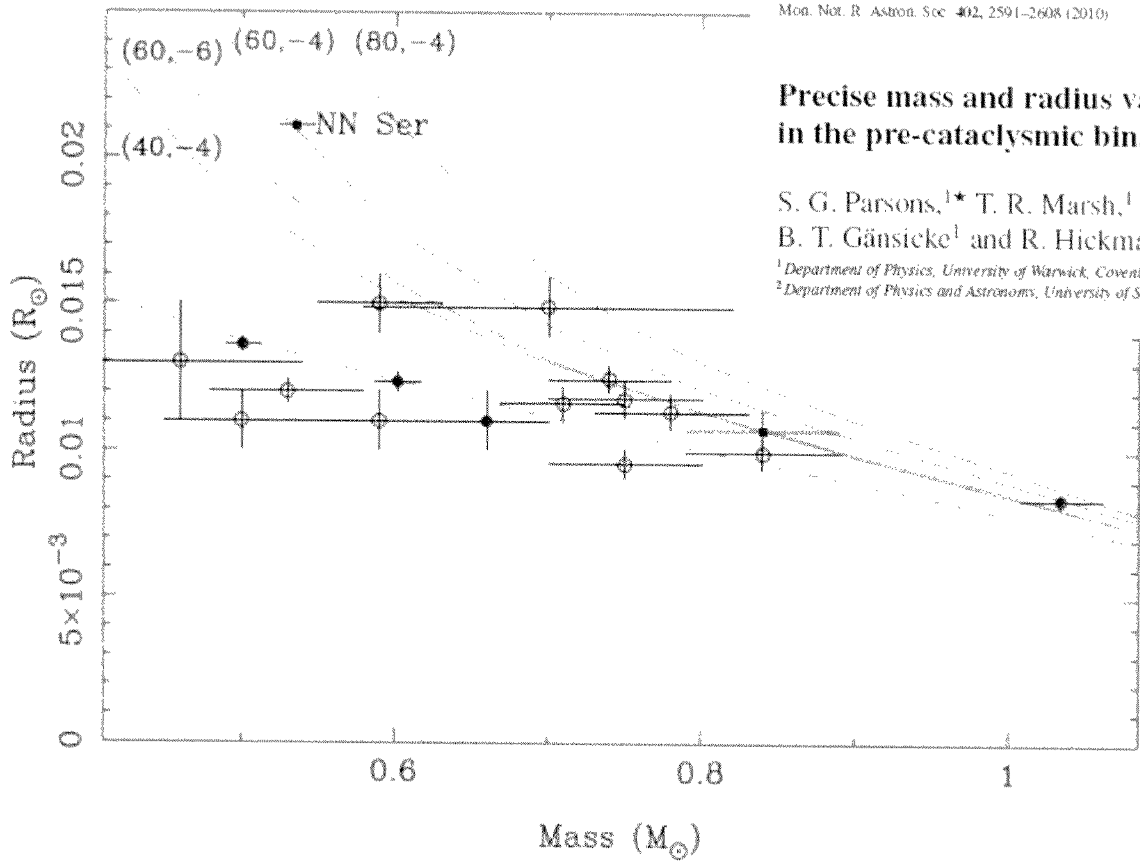


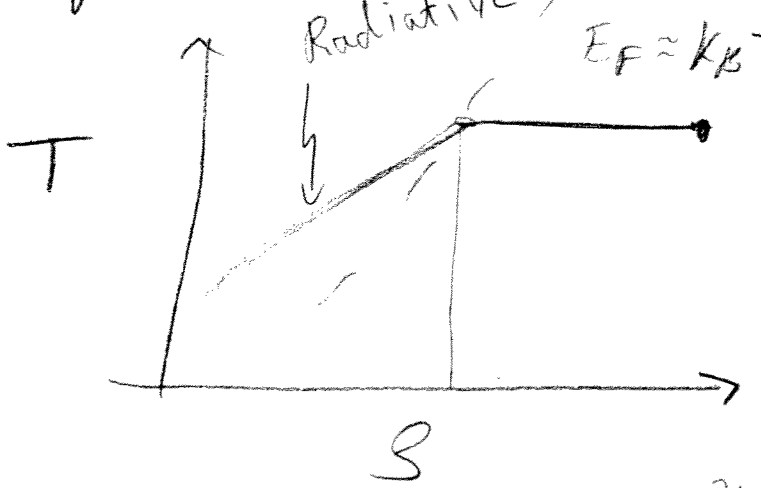
Figure 16. Mass–radius plot for white dwarfs measured independent of any mass–radius relations. Data from Provencal et al. (1998), Provencal et al. (2002) and Casewell et al. (2009) are plotted. The filled circles are visual binaries and the open circles are CPM systems. The solid lines correspond to different carbon–oxygen core pure hydrogen atmosphere models. The first number is the temperature, in thousands of degrees, the second number is the hydrogen layer thickness (i.e. lines labelled −4 have a thickness of $M_{\rm H}/M_{\rm WD} = 10^{-4}$) from Holberg & Bergeron (2006) and Benvenuto & Althaus (1999). The dashed line is the zero-temperature mass–radius relation of Eggleton from Verbunt & Rappaport (1988).

~~Lecture 22~~ 08.

$n_e = Z n_i = Z$ n_A

White Dwarf Cooling

The white dwarf is "born" hot, with $T \approx 2 \times 10^8$ K or so, now the interior is "completely" isothermal, as we have degenerate e^- carrying the heat. The picture is:



The transition is at

$$E_F = k_B T,$$

which gives a bottom boundary condition of

$$E_F = \frac{1}{2m_e}\left(\frac{3h^3 \rho}{8\pi 2 m_p}\right)^{2/3} = 16\,\text{eV}\,\rho^{2/3}$$

$$= k_B T \Rightarrow \quad 8625\, T_8 = 16 \rho^{2/3}$$

so

$$\boxed{\rho_{tr.} = 1.2 \times 10^4 \frac{\text{g}}{\text{cm}^3}\, T_8^{3/2}}$$

The mass of this outer shell is roughly just for g = constant where $R \approx 7 \times 10^8$ cm & $M = 0.6 M_\odot$

$$\Rightarrow \boxed{g = 1.6 \times 10^8\ \text{cm}/\text{s}^2}$$

so we know that $\left(n_i = \frac{\rho}{A m_p}\right)$

$$P = g y = \frac{(n_e + n_i) k_B T}{} = (Z+1) n_i k_B T$$

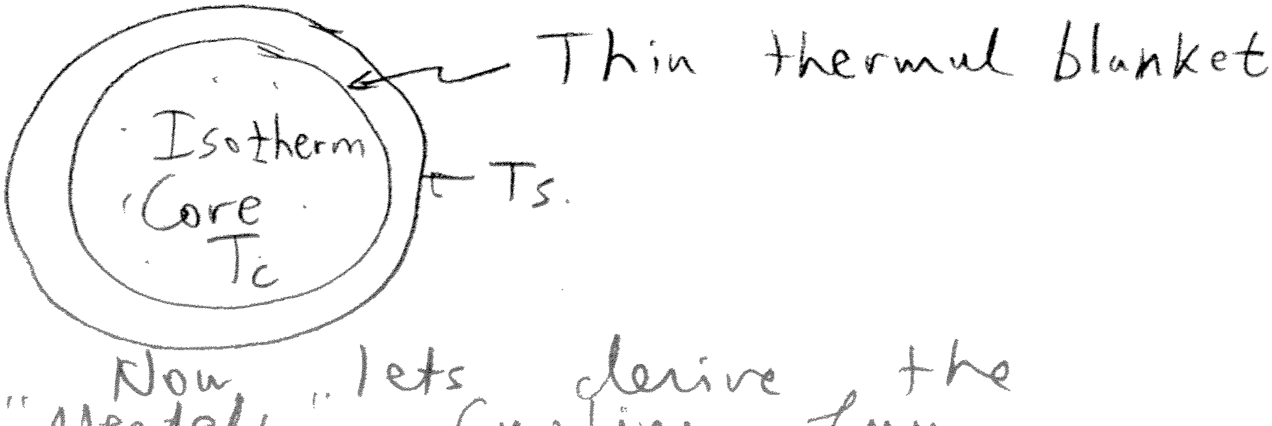
$$\Rightarrow \quad P = \left(\frac{Z+1}{A m_p}\right) \rho k_B T \approx \frac{1}{2} \frac{\rho k_B T}{m_p}$$

$$P_{tr} = 4.9 \times 10^{19} \frac{\text{erg}}{\text{cm}^3} T_8^{5/2}$$

$$y = \frac{P}{g} = 3.1 \times 10^{11} \ \text{gr}/\text{cm}^2,$$

so $\boxed{\Delta M = 4\pi R^2 y = 10^{-3} M_\odot \ T_8^{5/2}}$

So the outer mass shell is very light and decreases rapidly with time.



Now, lets derive the "Mestel" Cooling Law.

$$\frac{dP}{dz} = -\rho g$$

$$F = \text{const} = \frac{1}{3}\frac{c}{\kappa\rho}\frac{d}{dz}(aT^4)$$

where it is a constant since $\nabla \cdot F = \rho\varepsilon = 0$
Then we can always rewrite as

$$F = \frac{1}{3}\frac{c}{\kappa}\frac{d}{dy}(aT^4) \qquad \text{since } P = gy$$

we get (& Kramers $\kappa = \kappa_0 \rho T^{-3.5}$) so

$$\frac{3F\kappa}{c} = \frac{d}{dy}(aT^4)$$

$$\Rightarrow \frac{3F}{c}\kappa_0 \rho T^{-3.5} = \frac{d}{dy}(aT^4)$$

but in the outer layers we have that
$P = \rho kT/\mu m_p$ so

$$\rho = \frac{\mu m_p P}{kT} \qquad \text{so we get}$$

$$\frac{3F}{c}\kappa_0 \frac{\mu m_p P}{kT} T^{-3.5} = g\frac{d}{dP}(aT^4)$$

Putting all the constants together this gives

$$\left[\frac{3F}{4c}\kappa_0 \frac{\mu m_p}{k_B g}\frac{1}{a}\right] P\,dP = T^{7.5}\,dT$$

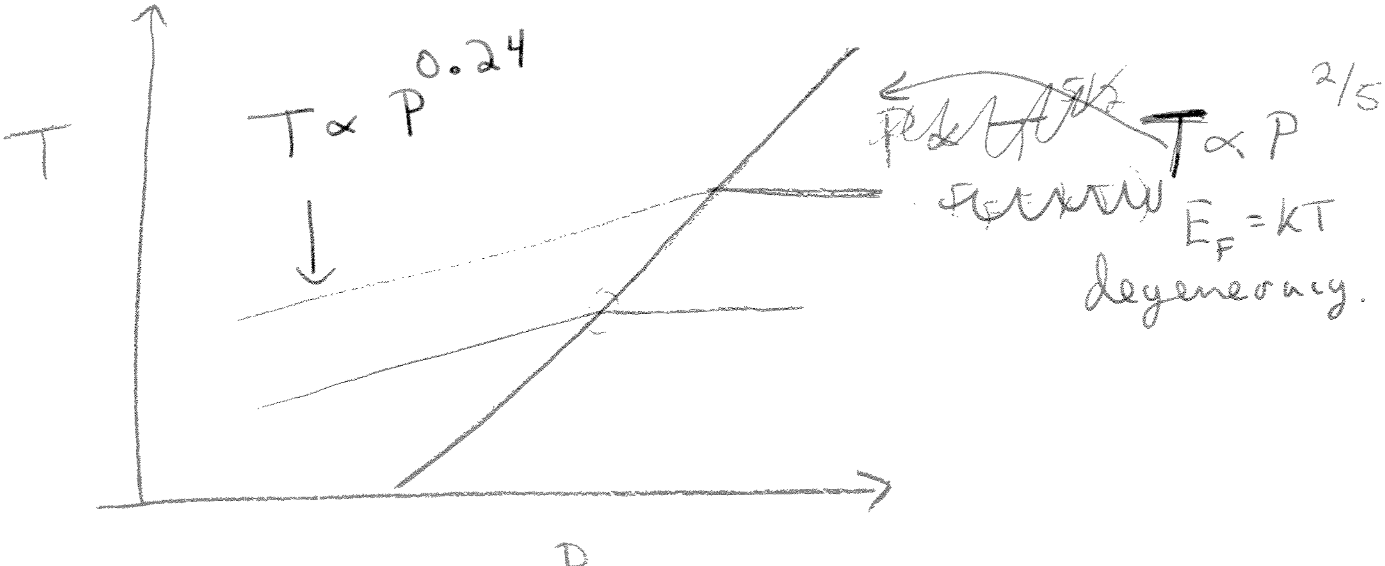
So if we now integrate this from
the center to the inner edge
& set $P = P_{ph}$ and $T = T_{eff}$

$T \propto P^{\dots}$

We get that $P^2 \propto T^{8.5}$ as the Outer body will <u>not</u> matter once we are far into the envelope. So I get

$$\frac{F}{g} \propto \left(\frac{3F}{4c} k_0 \frac{\mu m_p}{a k_B g}\right) \frac{1}{2} P^2 = \frac{1}{8.5} T^{8.5}$$

These solns thus look like



We then find the flux by taking this back to the degenerat point as

$$P_d = 5 \times 10^{19}\, T_8^{5/2}$$

$(k_0 = 1.4 \times 10^{23})$

$$\Rightarrow \frac{F}{g} = \frac{2T^{8.5}}{8.5\, P^2} \frac{4 c k_B a}{3 k_0 \mu m_p}$$

$$\left[\frac{L}{4\pi R^2} \frac{k_-}{c\, \mu}\right] = \frac{\quad}{\quad}$$

So we get

$$\frac{L}{4\pi G M} = 3\times10^{9}\,\frac{T_8^{8.5}}{T_8^{5}} = 3\times10^{9}\,T_8^{3.5}$$

$$\Rightarrow \boxed{L = 2.5\,L_\odot \left(\frac{M}{0.6\,M_\odot}\right)\left(\frac{T_c}{10^8\,K}\right)^{7/2}}$$

Mestel Cooling Law.

This is a very convenient form for analysis of the cooling. A ~~slightly~~ more accurate soln is

$$\frac{L}{L_\odot} = 2.5\left(\frac{M}{0.6\,M_\odot}\right)\left(\frac{T_c}{10^8}\right)^{7/2}$$

The total thermal energy content in the WD core is in the ions as the degenerate e^- contribute little or nothing to C_V so we get

$$E_{th} = \frac{3}{2} N_i k_B T = \frac{3}{2}\left(\frac{M}{12 m_p}\right) k_B T$$

$$= 1.25\times10^{48}\ \text{ergs}\left(\frac{M}{0.6\,M_\odot}\right) T_8$$

The equation of cooling is

$$10^{34}\,\frac{\text{erg}}{\text{sec}}\left(\frac{M}{0.6\,M_\odot}\right)\left(\frac{T_c}{10^8}\right)^{7/2} = -\frac{dT_8}{dt}\left(1.25\times10^{48}\right)\left(\frac{M}{0.6}\right.$$

and notice that the <u>mass</u> of the star cancels, so that all W-D's basically cool at the same rate. Now we get

$$\frac{T_8^{7/2}}{\tau} = -\frac{dT_8}{dt} \qquad \tau = 4\times 10^6 \text{ yrs}$$

and the core temperature follows

$$-\int_0^t \frac{dt}{\tau} = \int_{T_i}^{T_f} \frac{dT_8}{T_8^{7/2}}$$

$$\Rightarrow \quad \frac{-t}{\tau} = -\frac{2}{5}\left[\frac{1}{T_f^{5/2}} - \frac{1}{T_i^{5/2}}\right]$$

so after a short while the <u>initial</u> temp matters very little then we just get

$$T_8^{5/2} = \frac{2}{5}\frac{\tau}{t} \Rightarrow \boxed{T_8 = \left(\frac{2}{5}\frac{\tau}{t}\right)^{2/5}}$$

This is a simple/humble result first found by Mestel. so we get

$$T_8 = \left(\frac{1.6\times 10^6 \text{ yrs}}{t}\right)^{2/5}$$

So

$$\frac{L}{L_\odot} = 2.5 \left(\frac{M}{0.6 M_\odot}\right)\left(\frac{1.6\times10^6\,\mathrm{yrs}}{t}\right)^{7/5}$$

so at $t = 10^9$ yrs we get $3\times10^{-4} L_\odot$ and $T_c = 7.6\times10^6$ K and an effective T of around 7600 K.

Some have hoped that by finding the faintest W.D. they can date the galaxy. This is presently a tough challenge as those ~~W~~'s that old will have $L \sim 10^{-5} L_\odot$ or so and $T_{eff} \sim$ few 1000 K.

Eventually, as they cool, the ions in the center crystallize, this can hold phase separation which is thought to occur in the C/O W.D.'s as they cool. For a $0.6 M_\odot$ W.D. crystallization is most likely not all that impt.

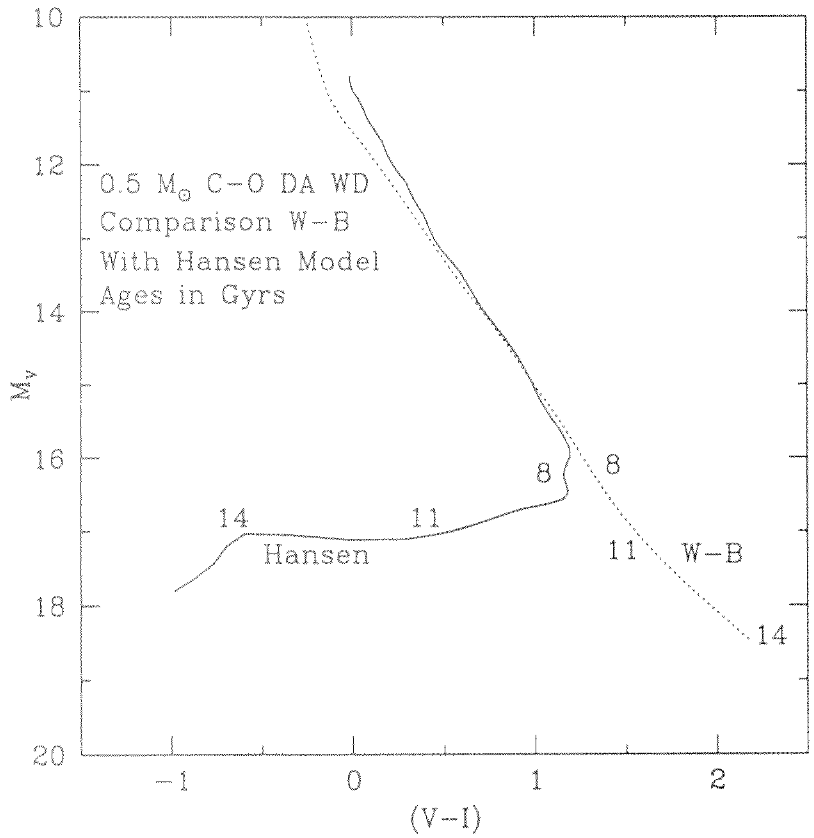


FIG. 1.—New 0.5 $M_\odot$ white dwarf cooling model of Hansen (1998, 1999) compared with a similar mass model constructed from the interiors of Wood (1992) and Bergeron et al. (1995) atmospheres (W-B). The main differences appeared around 8 Gyr, where the effects of atmospheric H_2 opacity become important.

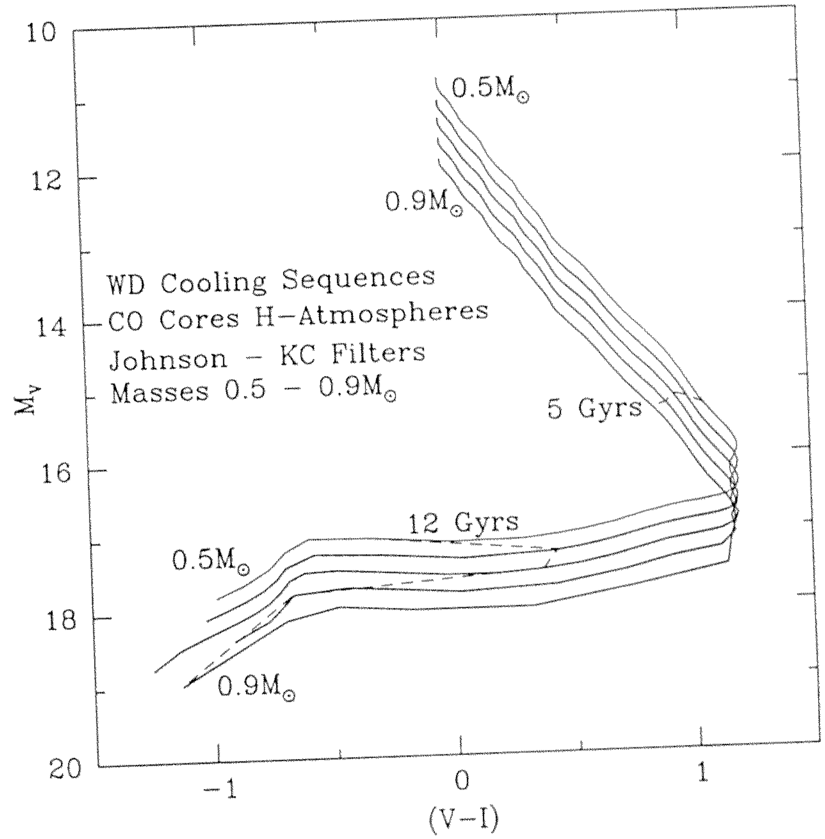


FIG. 2.—Cooling sequences for C-O core, hydrogen-rich white dwarfs of varying mass. The lines shown are for Johnson-Kron/Cousins filters. Constant ages of 5 and 12 Gyr are indicated on the diagram.

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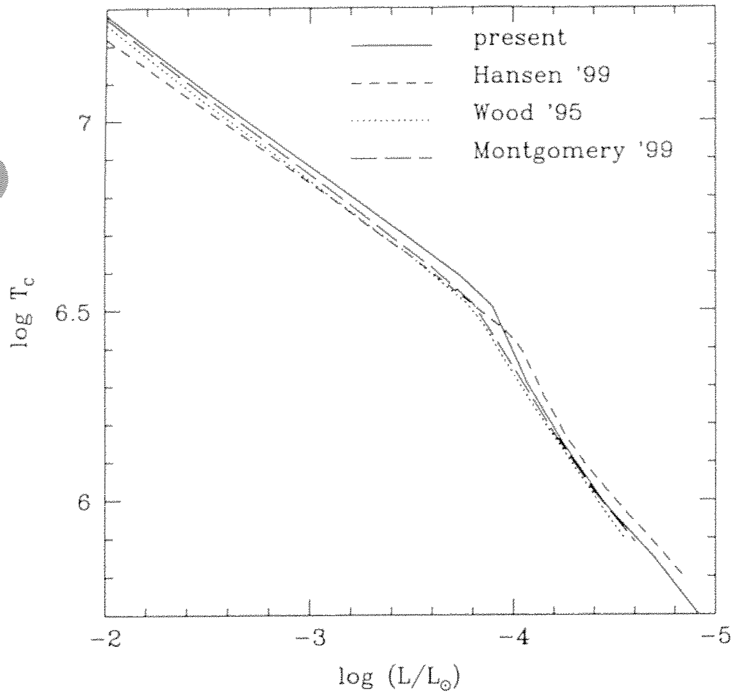


FIG. 2.—L-T_c relations from our calculations (*solid line*) and those of Hansen (1999) (*short-dashed line*), Wood (1995) (*dotted line*), and Montgomery et al. (1999) (*long-dashed line*) for a 0.6 $M_{\odot}$ WD with hydrogen and helium mass fractions $q(\mathrm{H}) = 10^{-4}$, $q(\mathrm{He}) = 10^{-2}$ and pure H atmosphere.

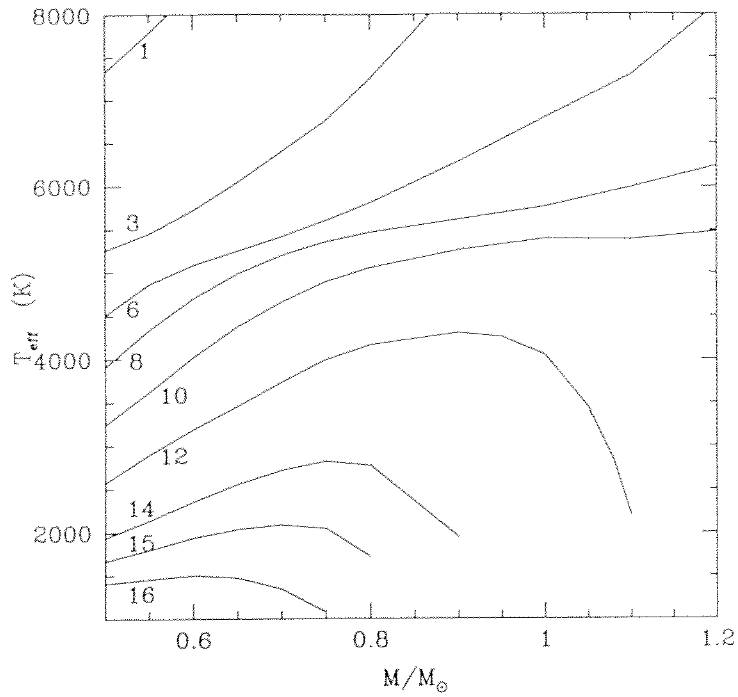


FIG. 5.—Mass-T_{eff} constant cooling times for H atmosphere WDs. Ages are indicated in Gyr for each curve.

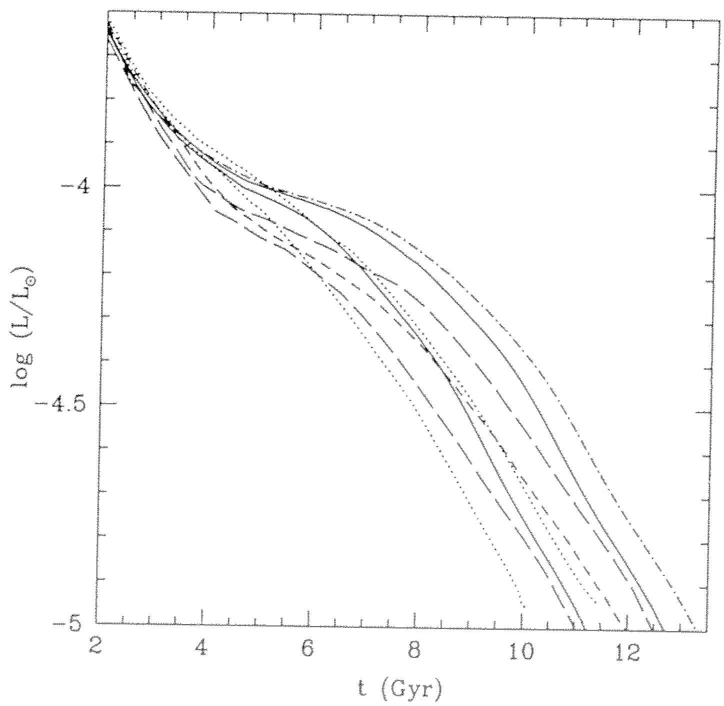
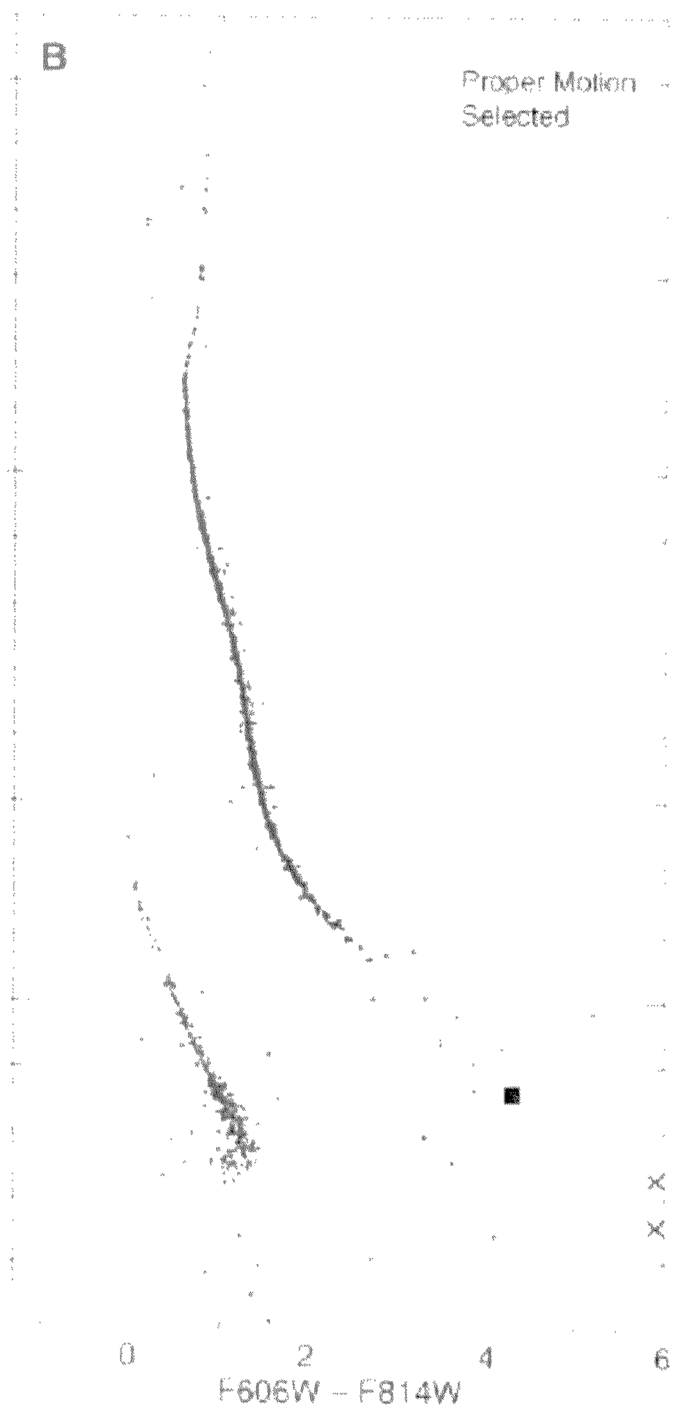


FIG. 3.—Cooling sequences with (*right curves*) and without (*left curves*) crystallization-induced fractionation for a 0.6 $M_{\odot}$ DA WD. *Solid curves*: calculations with our L-T_c relation; *long-dashed curves*: calculations with Hansen (1999) L-T_c relation; *dotted curves*: calculations with Wood (1995) L-T_c relation; *short-dashed curve*: Hansen (1999) cooling sequence; *dash-dotted curve*: present calculations with an initial profile obtained with a low $^{12}\mathrm{C}(\alpha, \gamma)^{16}\mathrm{O}$ reaction rate, with fractionation.



Probing the Faintest Stars in a Globular Star Cluster

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NGC 6397 is the second closest globular star cluster to the Sun. Using 5 days of time on the Hubble Space Telescope, we have constructed an ultradeep color-magnitude diagram for this cluster. We see a clear truncation in each of its two major stellar sequences. Faint red main-sequence stars run out well above our observational limit and near to the theoretical prediction for the lowest mass stars capable of stable hydrogen burning in their cores. We also see a truncation in the number counts of faint blue stars, namely white dwarfs. This reflects the limit to which the bulk of the white dwarfs can cool over the lifetime of the cluster. There is also a turn toward bluer colors in the least luminous of these objects. This was predicted for the very coolest white dwarfs with hydrogen-rich atmospheres as the formation of H_2 and the resultant collision-induced absorption cause their atmospheres to become largely opaque to infrared radiation.

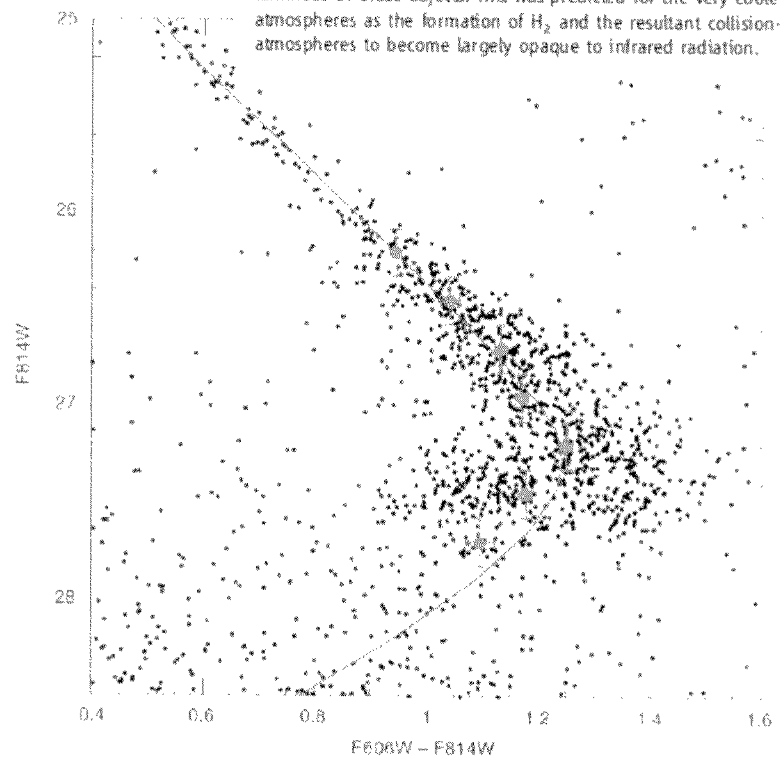

Fig. 4. The white dwarf region of the full CMD (i.e., not proper motion–cleaned) overlaid with the empirical cooling sequence (red dots with 1σ error bars) derived as described in the text. We use the full CMD here in lieu of the proper motion–cleaned one to avoid artificial truncation of the cooling sequence from losses of stars from the shorter exposure, earlier epoch data. The solid blue curve is a theoretical cooling sequence using the atmospheric models of Bergeron *et al.* (*7*) and a cooling model for 0.5 solar mass white dwarfs of Hansen (*6*).

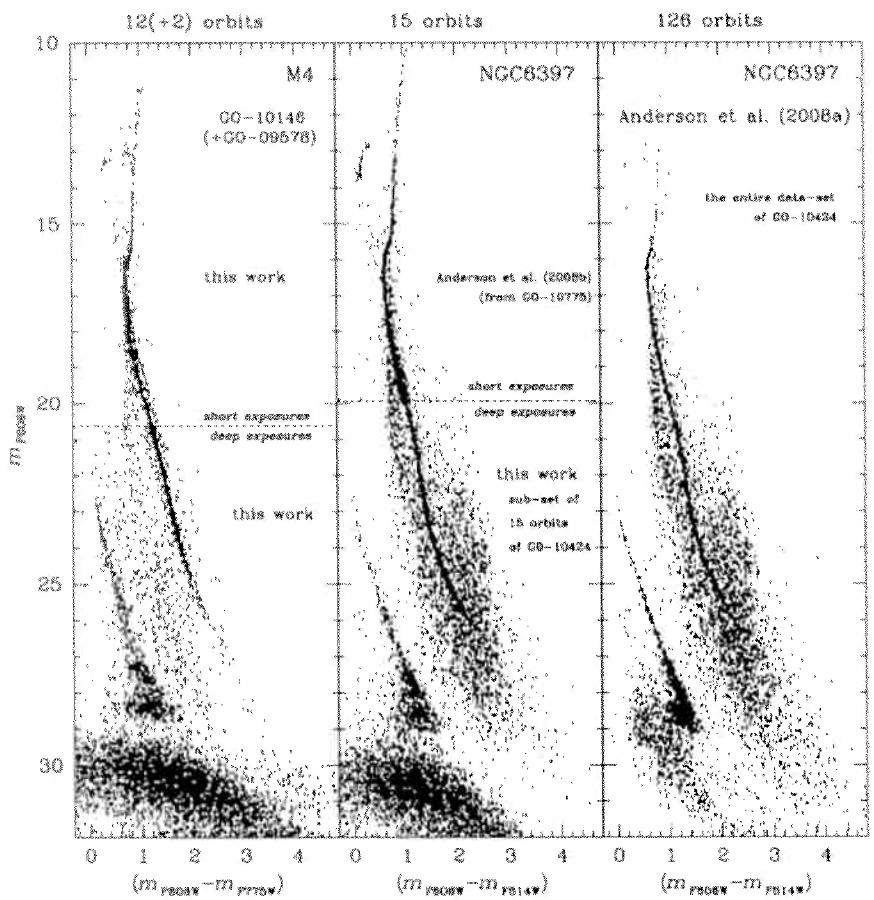

Figure 12. Left: CMD for M4 obtained combining our 12 orbit program with two orbits from the archive. Dotted lines indicate the onset of saturation for deep exposures. Middle: CMD for NGC 6397 using a subset of the GO-10424 images having the same total amount of exposure time for each filter of our M4 program. Photometry above saturation comes from a central field analyzed by Anderson et al. (2008a). Right: CMD for NGC 6397 obtained by Anderson et al. (2008b) using the entire 126 orbit data set.

The Astrophysical Journal, 697:965–979, 2009 June 1 doi:10.1088/0004-637X/697/2/965

THE END OF THE WHITE DWARF COOLING SEQUENCE IN M4: AN EFFICIENT APPROACH*

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ABSTRACT

We use 14 orbits of Advanced Camera for Surveys observations to reach the end of the white dwarf cooling sequence in the globular cluster M4. Our photometry and completeness tests show that the end is located at magnitude $m_{F606W} = 28.5 \pm 0.1$, which implies an age of 11.6 ± 0.6 Gyr (internal errors only). This is consistent with the age from fits to the main-sequence turnoff (12.0 ± 1.4 Gyr).

White Dwarf Types

Two Main Type

DA: Balmer Lines Only: no He I or metals present

(~ 80%)

DB: He I Lines: no H or metals

(~ 20%)

DC: Continuous spectrum with <u>no</u> lines

DO: He II strong He + H present

~~DQ: Carbon of any kind.~~

Metals are almost completely absent, which is considered to be due to settling effects.

The slight amount of metals left is more prevalent at high T_{eff} where radiative acceleration of the ions can keep them up.

Super-Cool WD's

At late times, a pure H atmosphere in a WD gets cold enough that the H → H_2 completely. The only real opacity source is then

$H_2 + H_2 \rightarrow$

the ~~so~~ absorption of a photon by a collisionally induced dipole moment.

This ends up making a WD rather blue as the long λ are absorbed out.

This discovery in ~~late~~ '99 drove observers to make much better color selections than previously done (all looked in the red) and so now some very faint WD's have been discovered.

State of the ^{12}C / ^{16}O

The central density in the 0.6 $M_\odot$ W.D. is about

$\rho_c = 10^6$, so $n_{ion} = \frac{\rho}{A m_p}$

$\Rightarrow \quad n_{ion} = 5 \times 10^{28} = \frac{3}{4\pi a^3}$

so $\quad a^3 = \left(\frac{3}{4\pi}\right) \frac{1}{5 \times 10^{28} \rho_6}$

$$a = 1.7 \times 10^{-10} \rho_6^{-1/3}$$

$$\Gamma = \frac{Z^2 e^2}{a} \frac{1}{k_B T} = 3.57 \left(\frac{Z}{6}\right)^2 \rho_6^{1/3} \frac{1}{T_8}$$

so we are immediately in the liquid state and when

$$\Gamma = 170$$

become a crystal. This is typically when

$\frac{3.6}{T_8} = 170 \qquad T \simeq 2 \times 10^6$

or when

For low density W.D's we have

$$\left(\frac{M}{0.49}\right)^2 10^6 = \rho_c$$

or just $\rho_c = 10^6 \frac{gr}{cm^3} \left(\frac{M}{0.49}\right)^2$

so 1.5×10^6 for $0.6 M_\odot$ W.D.'s.

This will x-tallize when

$$\Gamma = \frac{Z^2 e^2}{a k_B T} = 170 = \frac{Z^2 e^2}{k_B T a}$$

$$n_i = \frac{3}{4\pi a^3} \qquad a^3 = \frac{3}{4\pi n_i}$$

$$a = \left(\frac{3 A m_p}{4\pi \rho}\right)^{1/3} = 1.7 \times 10^{-10} \left(\frac{1}{\rho_6^{1/3}}\right)$$

$$k_B T = \frac{Z^2 e^2}{170 a} =$$

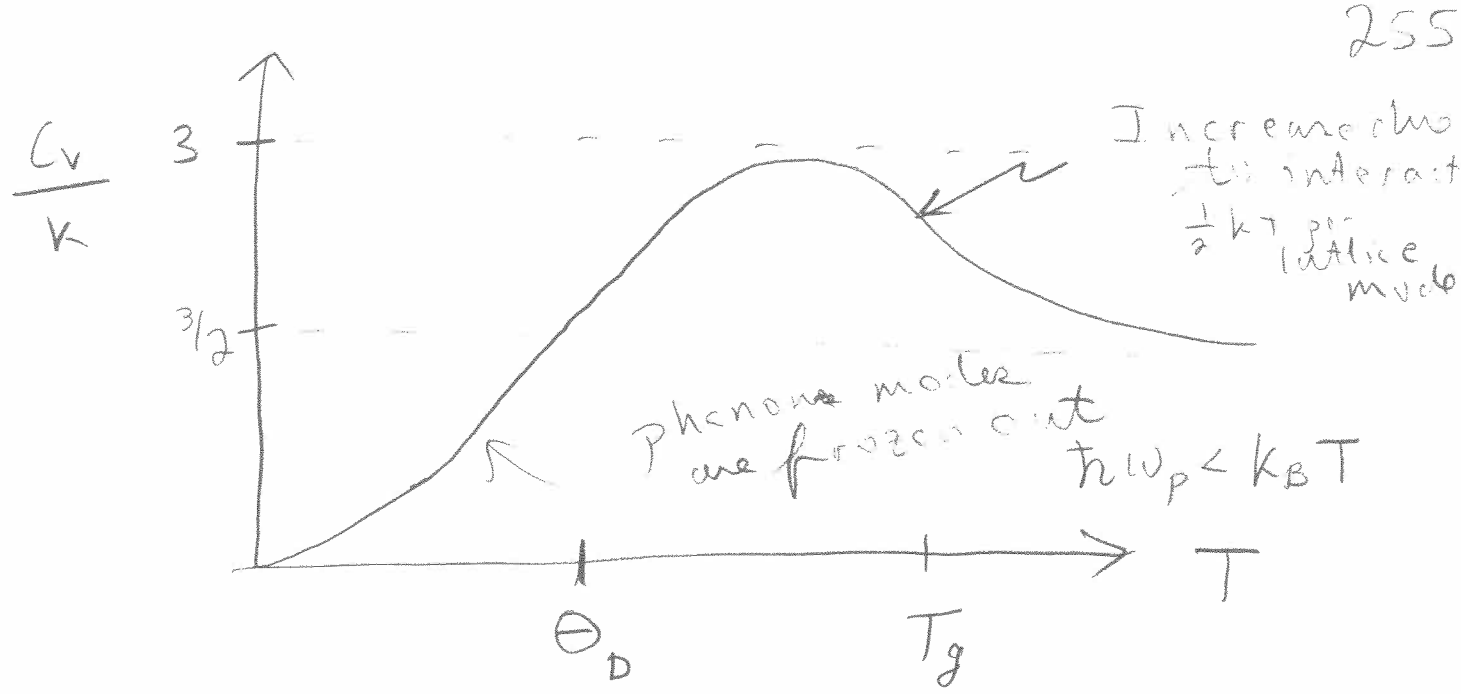
$$\Rightarrow T_c \approx 2 \times 10^6 \, K \, \rho_6^{1/3} \quad \text{for } {}^{12}C$$

or since $T_8 = \left(\frac{1.6 \times 10^6 yr}{t}\right)^{2/5}$, when it is

$$\boxed{t \approx 3 \times 10^{10} \, yrs \, \rho_{6,c}^{-5/6}}$$

or about when

$$t = 30 \, Gyr \left(\frac{M}{0.49 M_\odot}\right)^{-5/3} \approx 3 \Rightarrow$$ really is more important [illegible]



But what is more important is the onset of liquid state.

From March + Tosi "Atomic Dynamics of Liquids"

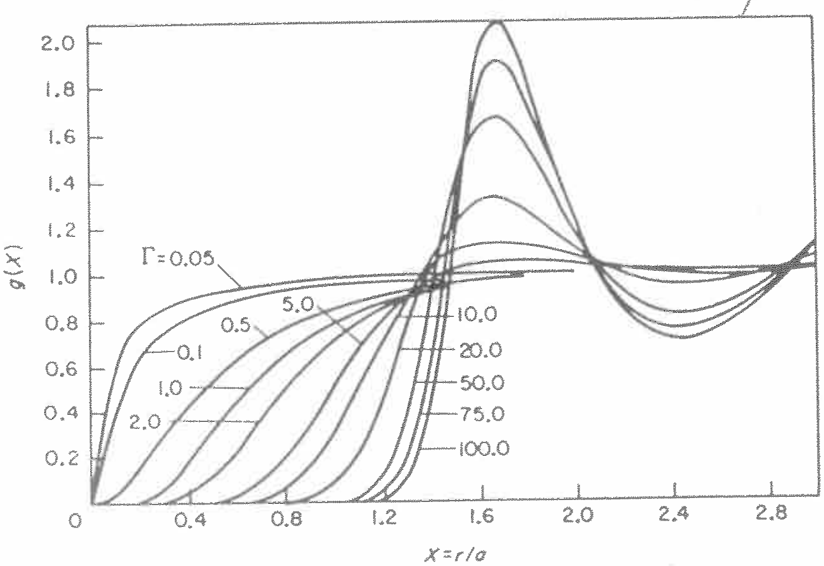


Figure 7.1 Radial distribution function of the classical one-component plasma as a function of distance (in units of $a=(4\pi\rho/3)^{-1/3}$) for a series of values of the plasma parameter $\Gamma = e^2/(k_B T a)$ (from Brush, Sahlin and Teller, 1966)

$g(x)$ defined as

$$dN = \frac{N}{V} 4\pi r^2 dr\, g(r)$$

= # in a shell at $r, r+dr$ about a fixed particle