We left off last time having derived SAHA

$$\frac{n_p n_e}{n_H} = \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(\frac{-13.6\,eV}{kT}\right)$$

We first demand $n_p = n_e$ + lets find the 1/2 ionization place where $n_H = n_p = n_e$, then.

$$\frac{n_p^2}{n_H} = n_p = \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \exp\left(\quad\right)$$

$$n_p = 2.4\times10^{21}\,cm^{-3}\, T_4^{3/2} \exp\left(-\frac{15.76}{T_4}\right)$$

Set $n_p = 10^{16}$ so we get

$$\frac{n_p}{n_{H,e}} = \exp\left(\frac{-13.6}{kT}\right) \Rightarrow \text{as we expect.}$$

This gives, for $n_p = 10^{16}$

$$+12.38 = -\frac{3}{2}\ln T_4 + \frac{15.76}{T_4}$$

$$\frac{15.76}{T_4} = 12.38 + \frac{3}{2}\ln T_4$$

$$T_4 = \frac{15.76}{12.38 + \frac{3}{2}\ln T_4} \Rightarrow T = 12{,}400.$$

$$(kT = 1.1\,eV)$$

So the 1/2 ionization point happens about at 10^4 K for typical photospheric densities

[Note: It ends up that Saha is OK to use because collisional ionizations are not imp't in these photospheres].

Also remember that recombination occurs in the early universe when

$n \approx 3 \times 10^4$, $n_{Q,e} = 5 \times 10^{20}$

& $T \approx 4000$ K, $z \approx 1400$.

In the center of the sun the atoms begin to "overlap" as

$n \sim n_Q$ so we cannot fairly use Saha.

↳ Go into more detail on this.

Lecture 14

Atmospheres + Spectral Types + Lines:

We now "know" that most stars living on the Main sequence have $T_{eff} \sim 5000 \rightarrow 50,000$ K and I would like to talk today about how an observer finds these things out quantitiely.

Remember that

$$h \approx \frac{kT}{m_p g} << R \quad \text{so that}$$

all of the atmosphere problems are plane parallel. I discussed limb-darkening already, and so will not talk more about that here. The atmosphere has two imp't params

$g \downarrow$

$$g = \frac{GM}{R^2} = 2.7 \times 10^4 \frac{M}{R^2} \text{ in } \odot$$

and

$F = \sigma_{SB} T_{eff}^4$ serves to define

$$T_{eff} = \left(\frac{F}{\sigma_{SB}}\right)^{1/4}$$

If there was no lines etc etc then you would see a BB at

$$T_c = T_{eff}.$$

For a photosphere we take.

$$\frac{2}{3} = \int \kappa \rho \, dr = \kappa y \Rightarrow y_p = \frac{2}{3\kappa}$$

so

$$P_{ph} = g y = g \frac{2}{3\kappa} = 2 n k T$$

so

$$n_{ph} = \frac{g m_p}{3 \sigma_{Th} k_B T}$$

$$\simeq \left(10^{16} \text{ cm}^{-3}\right) \left(\frac{\kappa_{es}}{\kappa}\right) \left(\frac{10^4 \text{ K}}{T}\right) \left(\frac{g}{2.7 \times 10^4}\right)$$

Now, for the upper main sequence I showed that $R = M^{0.75}$ so

$$g = \frac{GM}{R^2} = \frac{GM}{M^{1.5}} = 2.7 \times 10^4 \left(\frac{M_\odot}{M}\right)^{1/2}$$

or decreasing at high M. At low masses . . . $\kappa \gg \kappa_{es}$ and so n_e can stay low.

So the SAha eqn. tells us that the ionization temp for Hydrogen is about 10^4 K. The T's of the Main Sequence go as:

Sp Type	T	M
O3	52000	120
O5	44,500	60
O8	35,800	23
B0	30,000	17
B2	22,000	9
B5	15,400	6
A0	9,520	3
F0	7,200	1.6
G0	6030	1.05
K0	5250	0.79
M0	3850	0.51
M5	3240	0.21

Got to here

Lect 17

Ionization States

Now, most of the understanding of the presences or absences of lines in the atmosphere come from considering 3 types of elements

Metallic-Like (low FIP)
Li, Na, Mg, Al ≤ 5 eV

Middles: H, C, N, O, ... $10 < E_i < 20$

Noble: He + Ne > 20 eV

llow Philips and track

He & Na

24.6 5.14

eqn. tells us that
elate the g.s. population
# of ions

$$Na^+ + e^- \rightarrow Na + \gamma$$

$$\frac{n_+ n_e}{n_N} = \left(\frac{2\pi m_e k T}{h^2}\right)^{3/2} \exp\left[\frac{-E_i}{k_B T}\right]$$

so $$\boxed{\frac{n_+}{n_N} = \frac{2.4 \times 10^{15} T^{3/2}}{n_e} \exp\left[\frac{-E_i}{k_B T}\right]}$$

Now, lets compare Sodium to H, in which case

$$\frac{n_{Na^+}}{n_{Na}} = \exp\left[\frac{-(E_{Na} - E_H)}{K_B T}\right] \frac{n_p}{n_H} \simeq 10^7 \frac{n_p}{n_H}$$

at $T = 6000$ K. Whereas

$$\frac{n(He^+)}{n(He)} = \exp\left(-\frac{(E_{He} - E_H)}{K_B T}\right) \frac{n_p}{n_H} = 6 \times 10^{-10} \frac{n_p}{n_H}$$

for $T = 6000$K,

$1\,m^3 = 10^6\,cm^3$

so we really expect that the Alkali atoms ionize long before the H does. Now, there are even relatively rare, but can account for nearly all e^-s which are free. For example for the sun, $n_e \simeq 10^{13}\,cm^{-3}$, $T = 6000\,K$, then:

$$\frac{n_p}{n_H} = 4.3\times10^{-4}\ ;\ \frac{n_{He^+}}{n_{He}} = 2.5\times10^{-13}$$

$$\frac{n(Na^+)}{n(Na)} = 5.4\times10^{3}$$

So Na^+ and all other alkalis will be at least singly ionized in the sun, whereas H & He are neutral. Lets find the transition temps for a number of n_e's.

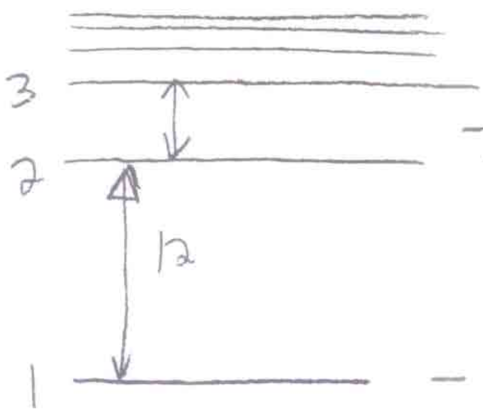
$n_e\,(cm^{-3})$	5.14 eV T_{Na}	13.6 eV T_H	24.6 T_{He}
10^{13}	3090	8000	14,500
10^{14}	3508	9082	16,500
10^{15}	4057	10500	19000
10^{16}	4810	12450	22570

$$1 = \frac{2.4\times10^{8}}{n_{13}}\exp\left(\frac{-E_i}{k_B T}\right) \Rightarrow k_B T = \frac{E_i}{19.3 - \ln n_{13}}$$

$$T = \frac{1.16\times10^4\,(E_i/eV)}{19.3 - \ln n_{13}}$$

H-lines: Balmer Series

Now, stars $> 10^4$ K will have ionized H completely & show no H lines.



$1/3^2 = -1.51$

$\dfrac{-13.6\,eV}{2^2} = -3.4\,eV$ ⟩ $1.89\,eV$

$E = h\nu = \dfrac{hc}{\lambda}$

$\lambda = \dfrac{hc}{E}$

$-13.6\,eV$

Lymann Series $(m \to 1)$

m	Lyman	Balmer $\to 2$	Paschen $n=3$
2	1215		
3	1025	6562	
4	972	4861	18751
		4340	
↓		↓	↓
∞	911	3646	8203

So to see any of the Balmer lines we must have an appreciable # of H in the $n=2$ state

$$\Rightarrow \quad \frac{n_{n=2}}{n_{n=1}} = \exp\left(-\frac{10.20\,eV}{kT}\right) = 10^{-5} \text{ at } T = 10^4$$

which is large enough to give an appreciable absorption line, as

$\sigma_{line\text{-}center} \gg \sigma_{Th}$, and actually $\int \sigma \simeq \pi \lambda^2 \simeq 10^{-16}\ cm^2$, which is

$1\text{Å} = 10^{-8} \gg \sigma_{Th}$ so as long as this ratio is satisfied, which is about 10^{-8} so as long as

$$\exp\left(\frac{-10.20\,eV}{KT}\right) \frac{\sigma_{line}}{\sigma_{Th}} \frac{n_H}{n_p} > 1$$

we get Balmer lines. So when most H is neutral $(T < 10^4)$, then we just want

$$\exp\left(\frac{-10.20\,eV}{KT}\right) > \frac{\sigma_{Th}}{\sigma_{line}} \simeq 6 \times 10^{-9}$$

showplots. $\Rightarrow\ T > 6000\ K$.

So $A \rightarrow F$ stars have conspicuous H-Balmer lines. Hotter stars have all H ionized and, once $T > 15,000$ or so He is singly ionized and one sees lines from that

O-B Type: Absolutely all H is completely ionized. Helium is either neutral or partially ionized. At late B, start getting H Balmer.

A: 11,000 → 7500 : H + ionized Mg are prevalent. Few metal lines.

183a

note that there exists a way of converting the strength of absorption lines to abundances.

THE SPECTRAL TYPES

One of the earliest results of observational astronomy was the realization that there existed a correlation between properties of the stellar surface, such as its surface temperature, and the strength of specific absorption lines as seen in the spectrum of the star. In 1863 the Jesuit astronomer Angelo Secchi classified stars into four groups according to the prominent absorption lines in their spectra. An empirical classification scheme was subsequently developed in which stars were sorted into seven principal spectral types, each type being characterized by a certain range of surface temperature and the appearance of characteristic absorption lines. The detailed understanding of the correlation between surface temperatures and prominent absorption lines rests almost entirely in various applications of the Boltzmann and Saha formulas.

The principal classification groups of stars are traditionally labeled by the letters O, B, A, F, G, K, and M. The significances of the actual letters of the alphabet chosen to represent the spectral classes are mostly historical and are reminiscent of the analogous importance of the letters chosen to represent the various angular-momentum states in quantum mechanics. Each spectral class corresponds to a certain range in surface temperatures. Each of these major divisions or classes is further subdivided into 10 groups. For instance, the spectral type B is further subdivided into 10 subclasses labeled B0, B1, B2, . . . , B9, in order of decreasing temperature. A rough correlation of the surface temperatures of each spectral type with the prominent absorption lines appearing in the spectra of those stars is as follows:

Class O: Temperatures of 25,000°K and up. Lines of ionized helium are prominent. From the discussion of the Saha equation it is apparent that lines of ionized helium will appear only in such an extremely hot gas. Other atoms in high degrees of ionization are observed.

Class B: 25,000 to 11,000°K. The lines of hydrogen and neutral helium are conspicuous at class B0. Ionized oxygen and ionized carbon become strong at class B3. Neutral helium lines are strongest at class B5. Hydrogen lines become progressively stronger in the higher-numbered subdivisions of this class. By hydrogen lines we mean, of course, the Balmer series of hydrogen lines appearing in absorption. The intensity of such lines will, among other things, be proportional to the fraction of hydrogen atoms existing in the first excited state of hydrogen in thermal equilibrium. Thus, the strength of hydrogen lines is primarily determined by combined application of the Boltzmann and Saha formulas.

Class A: 11,000 to 7500°K. At class A0 hydrogen and ionized magnesium lines are strongest, whereas the helium and ionized oxygen lines have disappeared. Hydrogen lines weaken continuously in the higher subdivisions of this class, whereas ionized metals (Fe, Ti, Ca, etc.) strengthen. The hydrogen lines will

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continue to weaken as we progress toward cooler stars, because the temperature becomes less and less sufficient to maintain a significant fraction of hydrogen atoms in the first excited state. The lines of ionized metals are growing stronger because the relative number of metals in low degrees of ionization is increasing. In the hotter stars, the higher degree of ionization of the metals produces resonance lines that lie in the ultraviolet. The resonance lines of the slightly ionized metals, however, lie in the visible, and these grow stronger as the temperature cools.

Class F: 7500 to 6000°K. Class F0 is rich in lines of the ionized metals, the strongest being the *H* and *K* lines of singly ionized calcium. Metallic lines, particularly iron, strengthen and hydrogen lines weaken in the higher-numbered subdivisions of this class.

Class G: 6000 to 5000°K. In this class the lines of the neutral metals become strong, whereas the hydrogen lines continue to weaken. Lines of ionized calcium are very strong. Molecular bands of CN and CH appear. The sun belongs to the class G2.

Class K: 5000 to 3500°K. In general, molecular bands and lines of neutral metals become much stronger, whereas the lines of hydrogen and ionized metals continue to weaken. At K5 the lines of TiO are weakly visible.

Class M: 3500 to 2200°K. The characteristic feature of the spectrum of class M stars is the appearance of complex molecular oxide bands, of which TiO bands are strongest.

The majority of stars fall into one of these spectral classes. There are, however, some exceptions to this classification scheme: some stars have temperatures in a range parallel to one of the existing classes but show strikingly different spectral lines. Therefore, some additional spectral classes have been established which parallel the above classes in temperatures:

Class S: A low-temperature class parallel to class M. This class is still characterized by molecular oxide bands, but the most prominent feature is the ZrO bands. The elements Zr, Y, Ba, La, and Sr give strong atomic lines and oxide bands. Lines of neutral technetium are usually seen. It is believed that these abundances are enhanced because of nucleosynthesis within the interior which has been mixed to the surface.

Classes R and N (or Class C): Parallel in temperature to the ordinary classes K and M. The spectrum is characterized not by oxide bands but by molecular carbide bands, such as those of CN, C_2, and CH.

Class W: Extremely high-temperature type O objects, called *Wolf-Rayet stars*, with bright, broad emission lines of ionized helium and highly ionized carbon, oxygen, and nitrogen. Two sequences exist: (1) the WC stars have strong carbon lines and weak nitrogen lines, and (2) the WN stars have strong nitrogen lines and weak carbon lines. These stars are generally found to be emitting gas rapidly in space. The progression of spectral properties through the sequences

183c.

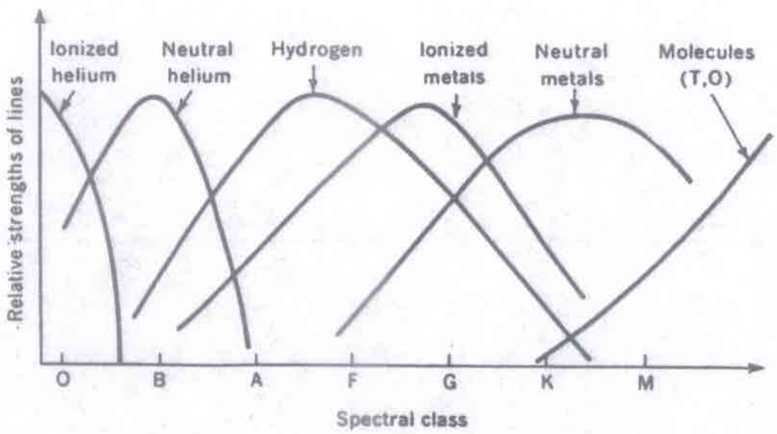


Fig. 1-11 The progression of selected spectral properties through the sequence of spectral classes. (*G. Abell, "Exploration of the Universe," Holt, Rinehart and Winston, Inc., New York,* 1964.)

of spectral classes and the correlation of color with surface temperature and spectral type are shown in Fig. 1-11.

Problem 1-23: What is the spectral type of a normal star having a maximum in its continuous spectrum at H_α?
Ans: K3.

Problem 1-24: What is the spectral type of a star for which the number of hydrogen atoms in the first excited state exceeds the number in the second excited state by the ratio of 4:1?

Problem 1-25: Absorption lines of singly ionized helium cannot be seen if the temperature is too low, because the helium is mostly neutral and what little ionized helium there is lies in the ground level and cannot make visible absorption lines. On the other hand, the lines cannot be seen if the temperature is too high, because all the helium becomes doubly ionized. The helium absorption lines arising from a state in level n of singly ionized helium will be strongest when the largest possible fraction of the helium lies in that level, i.e., when $He_n^+/(He + He^+ + He^{++})$, the fraction of He in state n of He^+, is a maximum. Show that this condition occurs when the quantity

$$G(\mathrm{He}^+) \exp\left[\left(1 - \frac{1}{n^2}\right)\frac{\chi^+}{kT}\right] + G(\mathrm{He})\,\frac{n_e h^3}{2(2\pi mkT)^{3/2}} \exp\left\{\frac{\chi^0 + [(n^2 - 1)/n^2]\chi^+}{kT}\right\} + \frac{2(2\pi mkT)^{3/2}}{n_e h^3} \exp - \frac{\chi^+}{n^2 kT}$$

is a minimum. The ionization potential of neutral helium is $\chi^0 = 24.58$ ev, and χ^+ is the ionization potential of the He^+. Calculate for $n = 4$ and electron densities in the range $\log n_e \approx 19$ (the answer is not very sensitive to n_e) that temperature yielding the largest fraction of He^+ ions in the $n = 4$ level (assume $G = 1$). You may also notice that most of the helium is doubly ionized at this temperature. What is the spectral type?

Pressure Ionization

The Saha Equation goes bad when the density is high enough so that the e^- overlap:

or $\downarrow$ $\leftarrow$ do Back of Envelope Bohr Atom.

$$\rho = \frac{m_p}{a_0^3} \simeq 1 \ \text{gr/cm}^3.$$

F 7500-6000 : More Metals as things cool off.

GKM : Neutral Metals start appearing.

$\boxed{\text{One Pirttle } H^- \text{ ion}}$

For very cool stars, it was found that H^- is impt.

H^- = bound state of one $p + 2e^-$
2nd e^- barely bound with 0.75eV, so unlikely except at low T.

This is crucial, as at low T, one must identify a process to make γ's, in this case

$$e^- + H \rightleftharpoons \gamma + H^-$$

Now, it ends up that the metals make the free e^- at some density, thus we get

$$\overset{e+p}{\mu_e + \mu_H} \rightleftharpoons \overset{-}{\mu_{H^-}}$$

$$\Rightarrow \frac{n(H)}{n(H^-)} = \frac{n_Q}{n_e} \exp\left(-\frac{0.75\,eV}{kT}\right)$$

or just

$$\frac{n(H^-)}{n(H)} = \frac{n_e}{n_Q} \exp\left(\frac{8700}{T}\right)$$

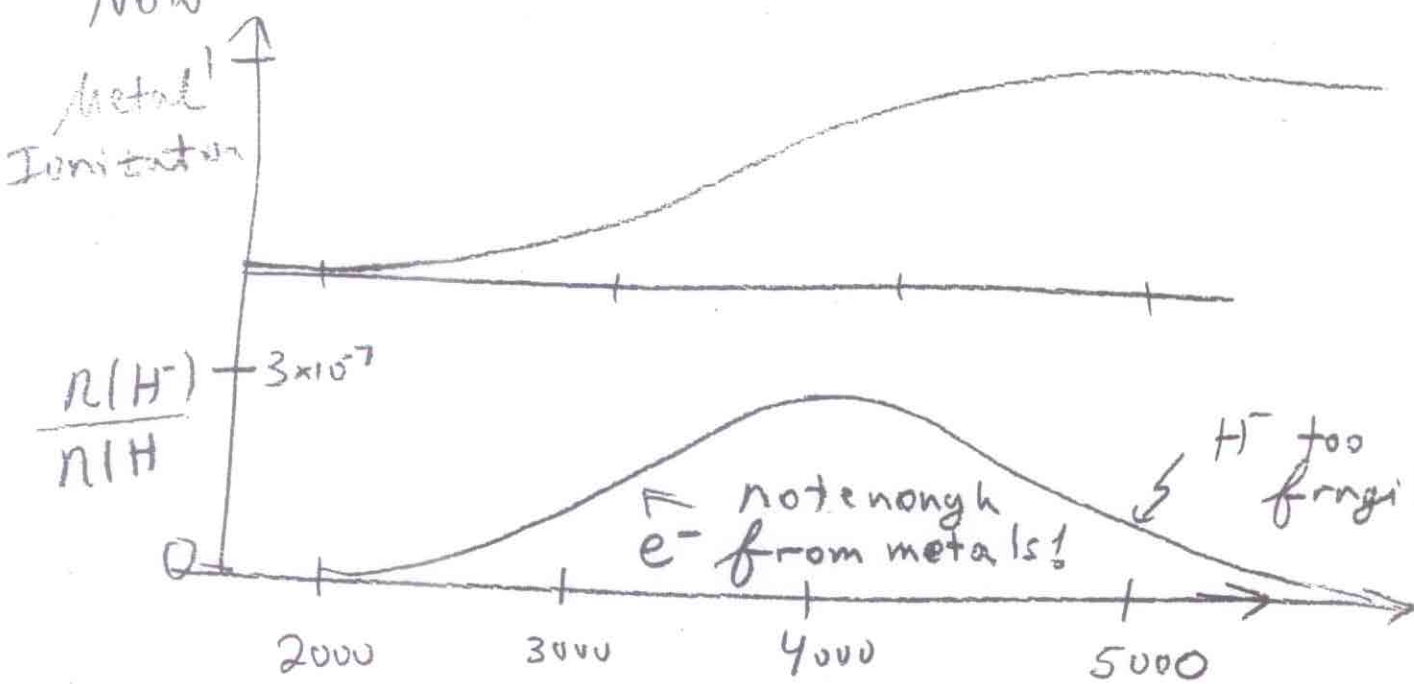
Take $n_e \simeq 10^{13}\,\text{cm}^{-3}$, then

$$\frac{n(H^-)}{n(H)} = \frac{\cancel{2.4\times10}\; 10^{13} \exp\left(8700/T\right)}{2.4\times10^{21}\left(T/10^4\text{K}\right)^{3/2}}$$

$$\simeq 10^{-8} \exp\left(\frac{8700}{T}\right)$$

Now

[H^- is the dominant opacity for Sun]



So, we find that at 3000

$\simeq 10^{-7}$ of H is in H^-

which is about the peak.

So, below 3000 K, there is a bit of a catastrophe, as most things are neutral. What we find there is that molecules begin to form and modify the opacity dramatically.

Now, that is about all we will do for the photosphere. The next question is more for the state of the interior of the star, where ionization will occur. For massive stars, nearly all elements are ionized at the photosphere, however ~~around the sets~~ in the sun, for example, the H is neutral at the surface and ionizes in the interior. These zones have in them rather abrupt changes in κ + γ $\Rightarrow$ convection quite often occurs.

Even deeper in the star, the SAHA eqn goes bad and we must worry about pressure ionization.