Stellar Evolution

Now I have explained how one finds the main sequence and understands that the stars live there for a time

$$t_{MS} \approx \frac{E_{nuc}}{L} = \left(\frac{E_{nuc}}{E_{th}}\right) t_{KH}$$

$\gg t_{KH}$ so that the stars on the main sequence are always in thermal equilibrium. This allows us to write

$$T\frac{ds}{dt} = 0 = \varepsilon_{nuc} - \frac{1}{\rho}\nabla\cdot F.$$

We will see that later in life this ~~approx~~ can go bad, especially when the stars cross the Hertzsprung Gap.

Main Sequence Evolution

Lets start by discussing what happens to the star during the Main Sequence in particular when there is still H in the center of the star.

MS Phase

$M > 1.5 M_\odot$: Convective cores and radiative envelopes, CNO burning:

Conv. mixing keeps μ uniform at a given instant of time. Burning reduces the # of particles in the core $\Rightarrow T_c \uparrow$ so as to keep same pressure. Now, in reality, things are quite complicated by the non-homologous evolution that actually takes place. I will neglect this & give you the simplest picture. Remember that

$$L = 4\pi R^2 \frac{c}{3\kappa\rho} \frac{1}{R} a T_c^4$$

and the Vir. Th. $\Rightarrow$ $k_B T = \frac{G M \mu m_p}{R}$

where remember

$$P = \frac{\rho k T}{\mu m_p} = n_i k T + n_e k T =$$

$$= (n_p + n_\alpha) kT + (n_p + 2n_\alpha) kT$$

$$= [2n_p + 3n_\alpha] kT = \frac{\rho k T}{m_p}\left[2 \frac{m_p n_p}{\rho} + \frac{3}{4}\frac{4 m_p n_\alpha}{\rho}\right]$$

$$= \frac{\rho k T}{m_p}\left[2X + \frac{3}{4} Y\right]$$

so we have $P = \rho \frac{kT}{\mu m_p}$ &

$$\frac{1}{\mu} = 2X + \frac{3}{4}Y \Rightarrow \mu = \frac{1}{2X + \frac{3}{4}Y}$$

so, ~~cosmic~~ if $X = 0.7$, $Y = 0.25$ $\mu = 0.63$, $X = 0$, $Y = 1$ $\mu = 4/3$

and $\kappa = \frac{cm^2}{gr} = 0.2(1+X)\,\frac{cm^2}{gr}$

so

$$L \propto R^2 \frac{R^3}{(1+X)M} \frac{1}{R} \frac{M^4}{R^4} \mu^4$$

$$L \propto M^3 \left[\frac{\mu^4}{1+X}\right]$$

$$\frac{\mu^4}{1+X} = \underset{\text{cosmic}}{7.9 \times 10^{-2}} \quad = \quad 3.16$$

$\rightarrow$ $\Delta L = 40$ if this simple!

In reality, the star is not homogeneous all the way through in which case its intrinsic luminosity does go up, but not by this much. Now as L is ind of R, we also need to otherwise find R.

You know the answer to this, as

$$L_{nuc} \propto T^{15-20},$$

so T is nearly constant, in which case

$$\frac{M\mu}{R} \approx \text{const} \qquad R \propto \mu$$

$$\cancel{L \propto R^2 \frac{R^3}{M} \frac{1}{R} \left(\frac{T_c}{T_0}\right)^4 \frac{M\mu_i}{R}} \qquad \cancel{\propto R^4}$$

Hence R can change by as much as

$$\frac{R_f}{R_i} \approx \frac{4/3}{0.63} = 2.116$$

a factor of ≈ 2 in radius.

The $15 M_\odot$ in the plot that follows goes from $3.24 \times 10^{11} \rightarrow 7.6 \times 10^{11}$ cm

≈ 2.38! Not bad!

The $5 M_\odot$ star goes from 1.6×10^{11} to 3×10^{11} cm or about a factor of 1.8!

So the massive $(> M_\odot)$ stars evolve on the nuclear timescale to change their radius. This is all happy until H depletes.

$\boxed{M < 1.5 M_\odot}$ Radiative core, pp chain

$$L_{nuc} \propto T^4$$

Since L_{nuc} is only weakly dependent on T the T must actually increase to match the enhanced L as the fuel burns & $\mu \uparrow$ ~~star expands~~

$\Rightarrow$ counteracts the expansion we would have otherwise obtained. Hence

$$kT = \frac{GMm_p \mu}{R}$$

in which case $T \uparrow$ by keeping R fixed

$\Rightarrow$ Evolution at the low mass end is nearly one of constant radius (i.e. almost $\parallel$ to the MS)

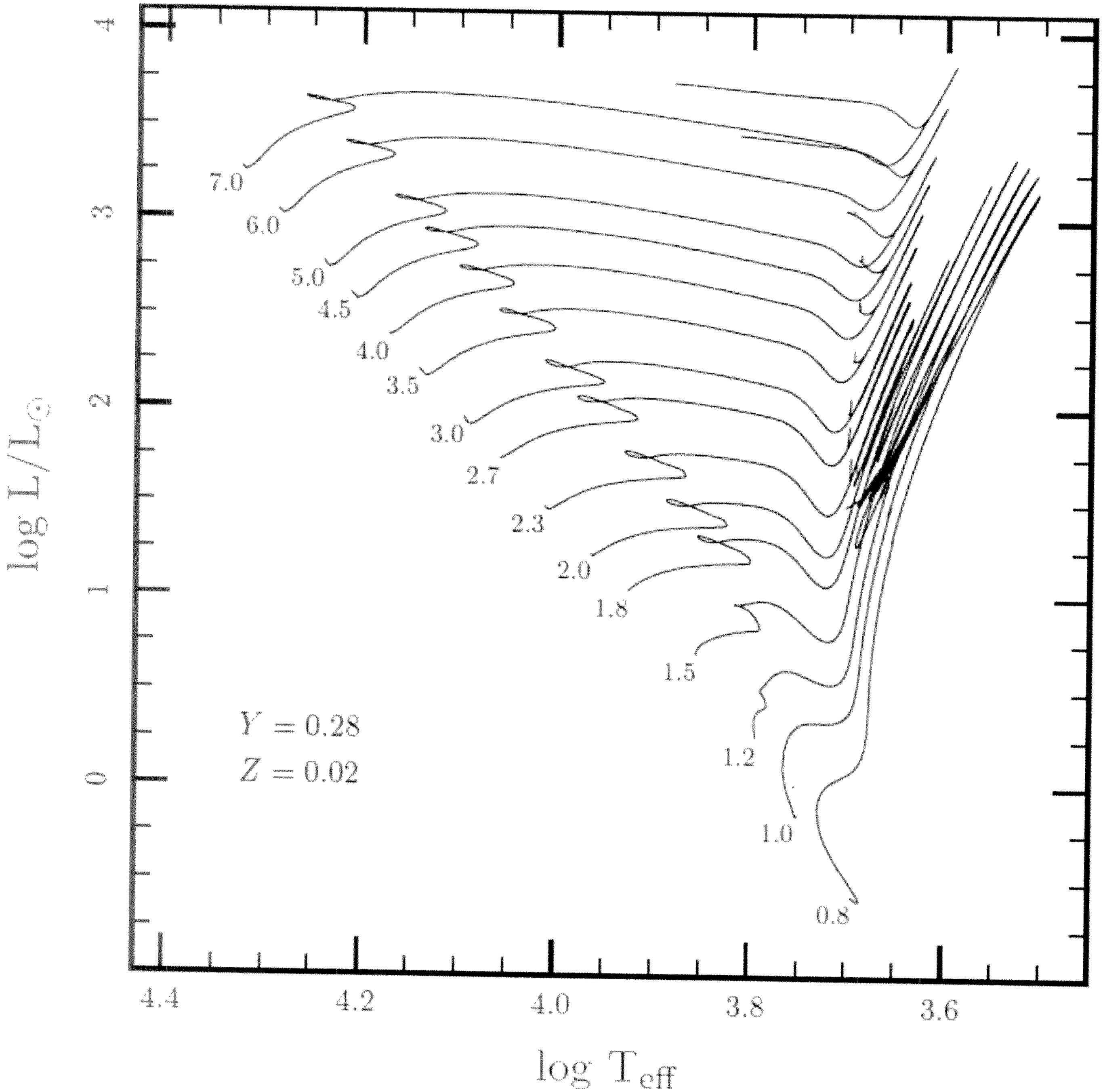
$\Rightarrow$ Sketch Isochrones

Due to mixing, it ends up that the massive stars actually consume a much larger fraction of their H than low mass stars.

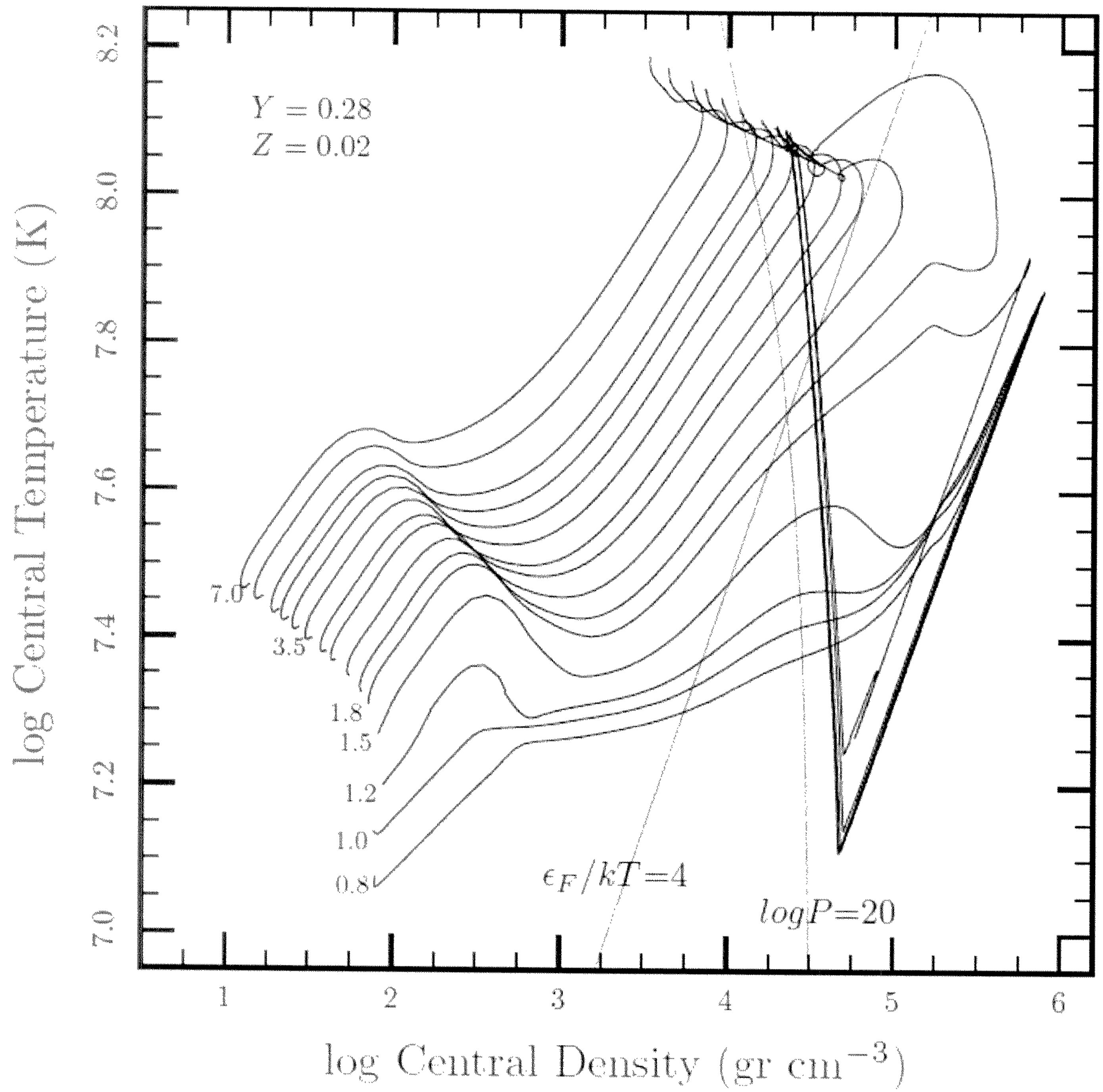
Central H Depletion

$\boxed{M > 1.5}$ $T \uparrow$ as $X \downarrow$ and eventually $X \to 0$ and the core cannot generate any heat, so the whole star contracts and we get the leftward $B \to C$ swing in about t_{KH}. Contraction halts once the H shell burning ignites

$\boxed{M < 1.5}$ As $T_c \uparrow$ during the MS phase, this system is not so far away from the shell burning, and grav. contraction is already present as a heat source. So the star gradually develops a shell source.



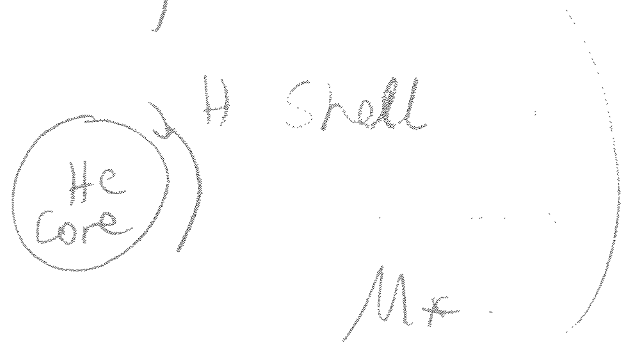
4
3
2
1
0
log L/L⊙
4.4
4.2
4.0
3.8
3.6
log T_eff
7.0
6.0
5.0
4.5
4.0
3.5
3.0
2.7
2.3
2.0
1.8
1.5
1.2
1.0
0.8
Y = 0.28
Z = 0.02



log Central Temperature (K)
8.2
8.0
7.8
7.6
7.4
7.2
7.0
log Central Density (gr cm^{-3})
1
2
3
4
5
6
$Y = 0.28$
$Z = 0.02$
7.0
3.5
1.8
1.5
1.2
1.0
0.8
$\epsilon_F/kT=4$
$logP=20$

H Shell Burning

The picture at late times is



H shell is the only nuclear source

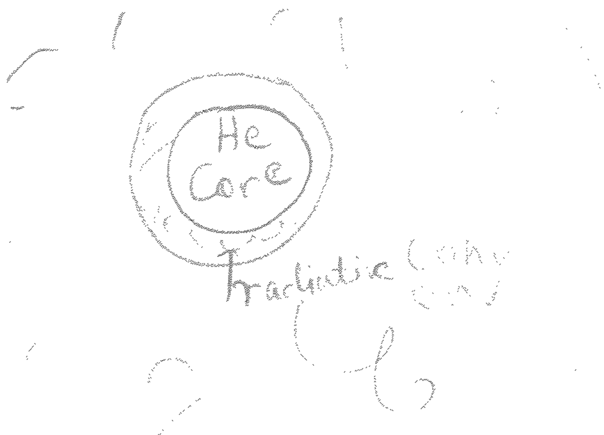
As shell $\rightarrow$ out, the radius of the core decreases while $R\uparrow$

[I will not explain this <u>very</u> non-homologous behavior]

Whether we can build a happy stable star as sketched above depends on an interesting limit which we calculate now.

H - Burning in the Shell

Once the star is far to the red, we have a very distinct core + envelope structure! For $M \leq 2$ the He core can be ~~mb~~ large, whereas for $M > 2$ it cannot exceed $0.2 M_\odot$ or $(0.1 M)$ the core contracts and ignites the helium (more on this later).



The H burning shell is constantly adding He to the core and, quite differently than before, sets the luminosity of the star.

$$kT_s \approx \frac{G M_c}{R_c} m_p \qquad (H \simeq R \text{ in the shell is big [illegible]})$$

Then [illegible] we [illegible] as usual write

$$L_{rad} = 4\pi R_c^2 \left[\frac{1}{3} \frac{c}{\kappa \rho} \frac{1}{R_c} a T_s^4 \right]$$

$$= \frac{4\pi}{3} \frac{R_c}{\kappa \rho} a c T_s^4$$

and we know κ, R_c, T_s, but not $\rho(R_c)$.

We in this case, find ρ by matching the shell $L_{nuc} = \varepsilon M_{shell}$.

$$\Rightarrow \frac{4\pi}{3} \frac{R_c}{\kappa\rho} a c T_s^4 = \varepsilon \, 4\pi R_c^3 \rho$$

As CNO burning is actually the relevant rate, we write

$$\varepsilon_{CNO} \approx \varepsilon_0 \rho T^\nu$$

and remember $p + {}^{14}N$ is the slowest step, so the T dependence is

$$\nu = \left(\frac{E_G}{4kT}\right)^{1/3} = \frac{24}{T_7^{1/3}}$$

$$E_G = (\pi \alpha Z_1 Z_2)^2 \, 2 m_r c^2 = 47.1 \text{ MeV}$$

and $\varepsilon_{CNO} =$

$$\varepsilon_{CNO} = 6.6 \times 10^{24} \frac{\rho}{T_7^{2/3}} \cdot \exp\left[\frac{-70.68}{T_7^{1/3}}\right]$$

$$= \varepsilon_0 \rho T^\nu \quad \text{so}$$

$$\frac{1}{3} \frac{R_c}{\kappa} 4\sigma T_s^4 = \varepsilon_0 \rho^3 R_c^3 T_s^\nu$$

$$\Rightarrow \quad \rho = \left[\frac{4}{3} \frac{\sigma T_s^{4-\nu}}{\varepsilon_0 R_c^2 \kappa}\right]^{1/3}$$

So, we know ρ in the burning zone for any/all M_c R_c + T_s. We can now put this back in to find

$$L = \frac{4\pi R}{3\kappa} a c T^4 \left[\frac{3\varepsilon_0 R_c^2 \kappa}{4\sigma T_s^{4-\nu}}\right]^{1/3}$$

What are the #'s like. Well first lets scale as

$$L \propto R \; R^{2/3} \left[T^{12-4+\nu} \right]^{1/3} \propto R_c^{5/3} T^{\left(\frac{8+\nu}{3}\right)}$$

and $T \propto \frac{M_c}{R_c}$, so we get

$$L \propto R_c^{5/3} \frac{1}{R_c^{(8+\nu)/3}} M_c^{\frac{8+\nu}{3}} = \frac{1}{R_c^{1+\nu/3}} M_c^{\frac{8+\nu}{3}}$$

which w/ $\nu = 16$ at 3×10^7 K and $\nu = 11$ at 8×10^7 K gives a <u>very</u> steep dependence on M_c.

value at peak (β ...)

$$kT_s \approx \frac{4 G M_c m_p \mu}{9 R} \Rightarrow T_s \approx 1.2\times10^7 \left(\frac{M_c}{0.2 M_\odot}\right)\left(\frac{0.1 R_\odot}{R}\right)$$

or $$T_s \approx 10^7 \left(\frac{M_c}{0.2 M_\odot}\right)\left(\frac{0.1 R_\odot}{R}\right)$$

and at $T_7 = 1$ we get $\varepsilon_0 = 1.3\times10^{-6}$ $\nu = 23$

so

$$L = \frac{4\pi R}{3(0.4)} a c T_s^4 \left[\frac{3\varepsilon_0 R_c^2 (0.4)}{4\sigma T_s^{4-\nu}}\right]^{1/3}$$

[notice: Indep. of total mass M

$\varepsilon_0 =$

$L = 10 \; (R \ldots \; T_5)^{0.2} \quad L \left[\frac{\varepsilon_0 \, T^{\nu} \, R_c^2}{\sigma T_s^4} \right]^{1/3}$

$$\Rightarrow L = 5 \times 10^{31} \frac{erg}{sec} \left(\frac{R}{7 \times 10^9} \right)^{5/3} \left(\frac{T}{10^7} \right)^{(8+4)/3}$$

$$L = 5 \times 10^{31} \frac{erg}{sec} \left(\frac{R}{0.1 R_\odot} \right)^{5/3} \left(\frac{M_c}{0.2 M_\odot} \right)^{10} \left(\frac{0.1 R_\odot}{R} \right)^{1\ldots}$$

$$L = 5 \times 10^{31} \frac{erg}{sec} \left[\frac{0.1 R_\odot}{R_c} \right]^{25/3} \left[\frac{M_c}{0.2 M_\odot} \right]^{10}$$

now clearly we must be smaller than $0.1 R_\odot$ in order to make it rem. luminous. If we want to just get conv. going then we merely want $L > 10 \ldots (R/R_\odot)^{2} \approx 8 \times L_\odot$ at 2.

$$5 \times 10^{31} \left[\frac{0.1 R_\odot}{R_c} \right]^{25/3} \left(\quad \right) > 3.2 \times 10^{34}$$

$\Rightarrow \quad R_c < 0.046 R_\odot$

$R_c < 3 \times 10^9$ cm.

or $T > 2 \times 10^7$ K.

As you can see this is a very strong fn of mass. If we increase T_s the dependence gets weaker.

Here