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Lect 13: Upper Main Sequence

The way this cycle succeeds is by making long-lived unstable nuclei that eventually β-decay on their own, then the rate limiting step is the $p + {}^{14}N$ reaction, with

$$E_G = 48.1 \text{ MeV}.$$

Again, if we presume this is it then $E = 28$ MeV released each

$$t = \frac{1}{n_p \langle \sigma v \rangle}$$

per seed nuclei so

$$\varepsilon = \left[\frac{1}{\rho}\right] \left(n_p n_s \langle \sigma v \rangle \right) 28 \text{ MeV}$$

$$\text{cm}^{-6} \quad \frac{\text{cm}^3}{s} = \frac{\text{cm}^3}{g}$$

where $S = 2.75$ keV Barn

$$\underset{\substack{p,\gamma \\ \text{on } {}^{14}N}}{\langle \sigma v \rangle} = 1.52 \times 10^{-15} \frac{1}{T_7^{2/3}} \exp\left(\frac{-72.19}{T_7^{1/3}} \right)$$

so $n_s \approx 10^{-3} n_p$

$$\varepsilon = 10^{-3} \frac{n_p^2}{\rho} \langle \sigma v \rangle 28 \text{ MeV}$$

$$= \frac{10^{-3}}{m_p} \frac{\rho}{m_p} \langle \sigma v \rangle 28 \text{ MeV}$$

$$= 2.5 \times 10^{25} \left(\frac{\text{cm}^{2}}{\text{gr s}} \right) S \frac{1}{T_7^{2/3}} \exp\left(\frac{-72.19}{T_7^{1/3}} \right)$$

$$\Rightarrow \quad \mathcal{E} = 2.5 \times 10^{25} \frac{\text{ergs}}{\text{gr} \cdot \text{sec}} \cdot \rho \frac{1}{T_7^{2/3}} \exp\left(\frac{-72.19}{T_7^{1/3}}\right)$$

So (really should integrate, but).

$$L_{nuc} \simeq M \mathcal{E} = \text{ instead}$$

This is became.

$$\exp\left(-\frac{72.19}{T_7^{1/3}}\right) \Rightarrow \nu = +\frac{72.19}{3\, T_7^{1/3}} = 24 / T_7^{1/3}$$

$$\Rightarrow \quad = \exp\left(-72.19\right) * \left(T_7\right)^{24}$$

and since $L_{nuc} \propto T_7^{24}$, we need not be terribly cautious in getting all the prefactors right if all we want is the central temperature so let's write M, R in solar units.

$$L_{nuc} \approx \left(\frac{M^2}{R^3}\right) 5 \times 10^{58} \frac{1}{T_7^{2/3}} \exp\left(-\frac{72.19}{T_7^{1/3}}\right)$$

$$= L_\odot M^3$$

so

$$\frac{T_7^3}{M^3}\frac{M^2}{T_7^{2/3}}T_7^{19} = 3.2\times10^5 M^3$$

$$\Rightarrow \quad T_7^{21} \simeq 3.2\times10^5 M^4$$

$$\boxed{T_7 = 1.83\,M^{4/21} \approx 1.83\,M^{1/5}}$$

So $R \propto \frac{M}{T} \propto \frac{M}{M^{1/5}} \propto M^{0.8}$

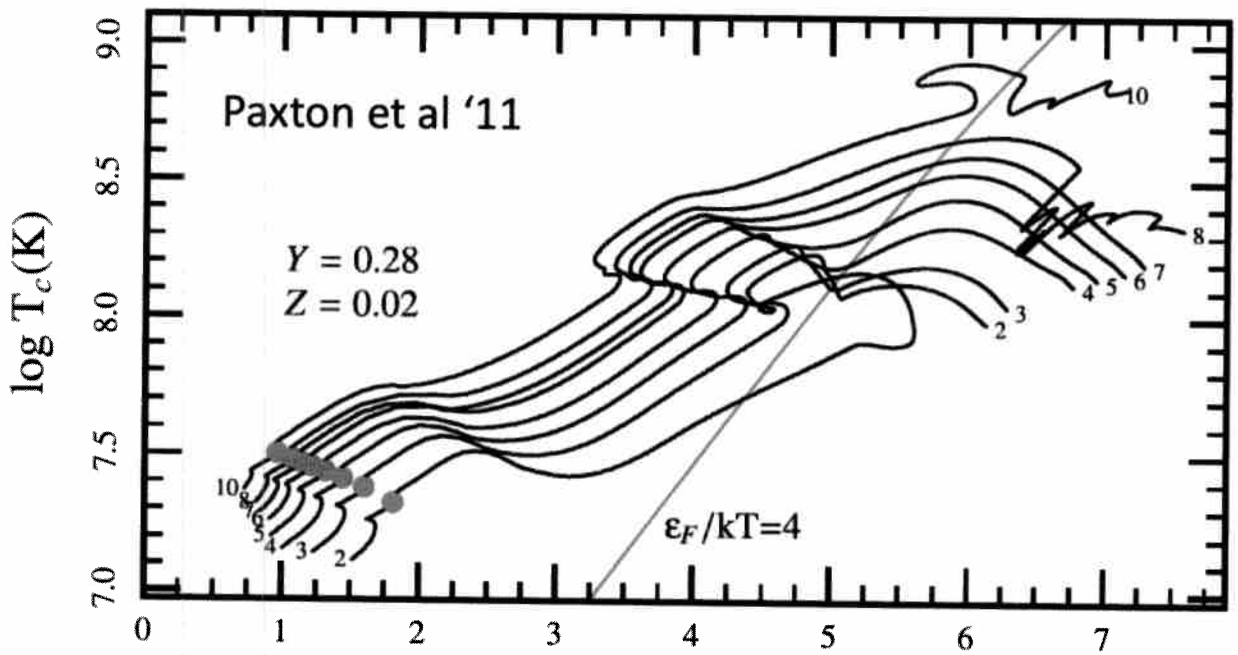
(you often see 0.75 quoted.

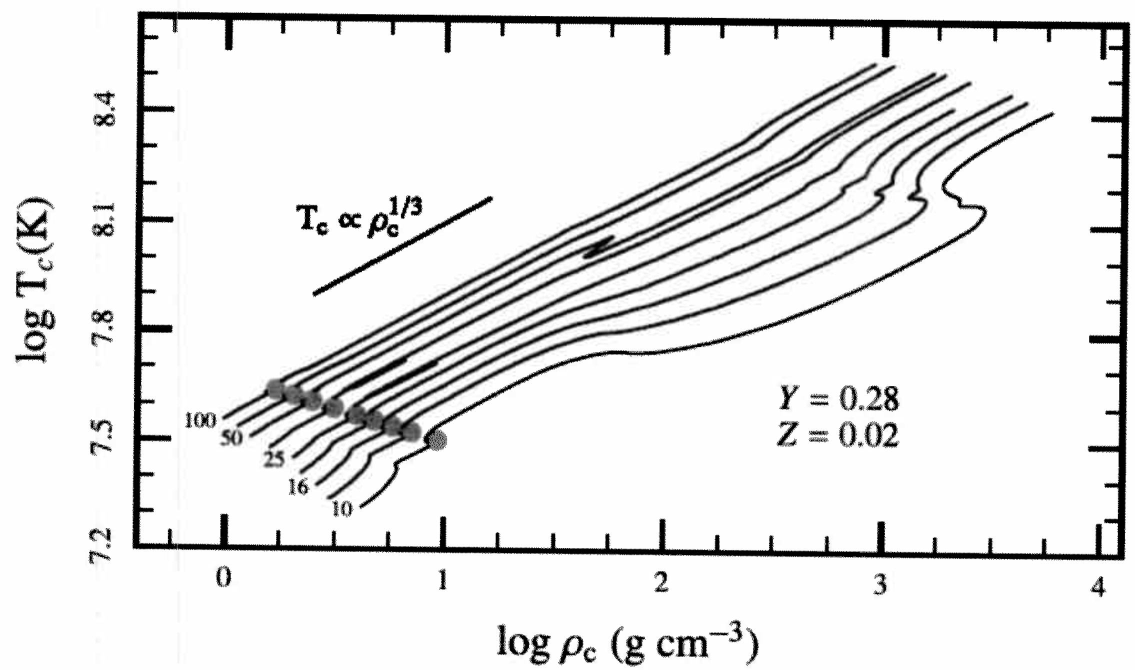
And since $L \propto M^3 \propto T_{eff}^4 R^2$
we get

$$T_{eff}^4 \propto \frac{M^3}{R^2} \propto \frac{M^3}{M^{1.6}} \propto M^{1.4}$$

$$\Rightarrow \quad T_{eff} \propto M^{0.34}$$

so from $1 \to 10$ only factor of 2. difference in T_{eff}.


Paxton et al '11
Y = 0.28
Z = 0.02
εF/kT=4
log Tc(K)
0 1 2 3 4 5 6 7
7.0 7.5 8.0 8.5 9.0
10 8 6 5 4 3 2
2 3 4 5 6 7 8 10


Tc ∝ ρc^1/3
Y = 0.28
Z = 0.02
100 50 25 16 10
log Tc(K)
7.2 7.5 7.8 8.1 8.4
0 1 2 3 4
log ρc (g cm−3)

so for the Sun.

$$5\times 10^{58}\,\frac{1}{T_7^{2/3}}\exp\left(-\frac{72.19}{T_7^{1/3}}\right) = 4\times 10^{3}$$

$$57.8 = \frac{72.19}{T_7^{1/3}}$$

$$\Rightarrow \boxed{T_7 \approx 2.}$$

Ok so now go back and rewrite, as at $T_7 = 2$

~~$k_{\rm Rate} \propto T^{19}$~~ $\quad \nu \approx \frac{24}{T_7^{1/3}} = 19.$

so we get

$$\frac{M^2}{R^3}\left(5\times 10^{58}\right)\frac{1}{T_7^{2/3}}\left(1.3\times 10^{-25}\right)\left(\frac{T_7}{2}\right)^{19} = L_\odot M^3$$

$$\frac{M^2}{R^3}\frac{1}{T_7^{2/3}}T_7^{19} = 3.2\times 10^{5}\,M^3$$

$$\Rightarrow k_B T_c \approx \frac{GMm_p\mu}{R} \quad \propto \quad T_7 \sim \frac{M}{R}$$

$$\Rightarrow \quad R \sim \frac{M}{T_7}$$

So the upper main sequence is determined solely by the CNO cycle and has one property that is important to note, namely fully convective cores. Why is this?

The criteria for convection is

$$\frac{d\ln T}{d\ln P} > 1 - \frac{1}{\gamma} = \frac{2}{5} \text{ for an ideal gas with little rad. pressure}$$

What is $\frac{d\ln T}{d\ln r}$ at the center of the star? well:

$$-F = \frac{1}{3}\frac{c}{\kappa\rho}\frac{d}{dr}aT^4 = \frac{4}{3}\frac{ac}{\kappa\rho}T^3\frac{dT}{dr}$$

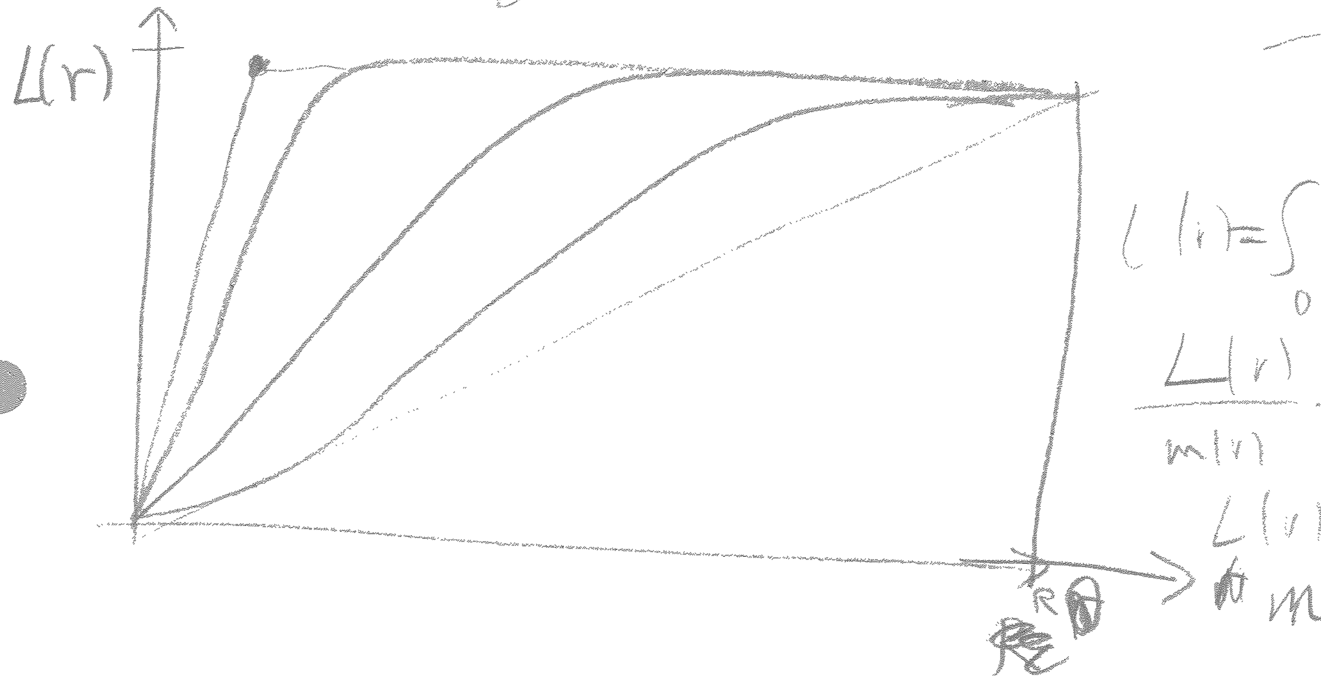
$$\frac{dP}{d\cancel{r}} = -\rho g \quad , \quad \text{so}$$

$$\frac{P}{T}\frac{dT}{dP} = \frac{F \cdot 3\kappa\cancel{\rho}}{4acT^{\cancel{3}}}\frac{1}{\cancel{\rho}g}\frac{P}{\cancel{T}}$$

$$\Rightarrow \frac{d\ln T}{d\ln P} = \left(\frac{P}{aT^4}\right)\frac{F \cdot 3\kappa}{4cg}$$

$$\frac{d\ln T}{d\ln P} = \left(\frac{P_{tot}}{aT^4}\right)\left(\frac{3}{4}\frac{\kappa}{c}\right)\frac{L(r)}{4\pi G m(r)}$$

$$= \left(\frac{P_{tot}}{aT^4}\right)\left[\frac{3}{16\pi}\frac{\sigma_T}{m_p c G}\frac{L(r)}{m(r)}\right]$$



$$L(r) = \int_0^m \varepsilon \, dm$$

$$\frac{L(r)}{m(r)} = c$$

$$L(r) = c\, m(r)$$

Thus the boundary of the core is when

$$\frac{P_{tot}}{aT^4}\left(\frac{3}{16\pi}\frac{\sigma_T}{m_p c G}\frac{L(r)}{m(r)}\right) < \frac{2}{5}$$

or $$q = \frac{m(r)}{M} = \frac{P_{tot}}{\frac{1}{3}aT^4}\,\frac{5}{32\pi}\,\frac{\sigma_T}{m_p c G}\,\frac{L(r)}{M}$$

Rather than find q lets first note the order of mag. involved. If all L is within r then we just have

$$\left(\frac{P_{tot}}{aT^4}\right)\left(\frac{3}{4}\ \frac{L}{L_{edd}}\ \frac{M}{m(r)}\right) < \frac{2}{5}$$

But near the center

$$\frac{P_{tot}}{P_{rad}} \approx 2600\left(\frac{M_\odot}{M}\right)^2$$

and

$$\frac{L}{L_{edd}} = 4\times10^{-5}\left(\frac{M}{M_\odot}\right)^2$$

so

$$0.1\ \frac{M}{m(r)} < \frac{2}{5} \quad \text{— to be stable.}$$

$$\Rightarrow \boxed{m(r) > 0.25M}$$

So, as long as the energy generation is <u>not</u> too centrally concentrated, we are OK. However, things are <u>very</u> centrally concentrated as $\varepsilon \propto T^{15}$ at least. This means then that

$$L(r) = \int \varepsilon \, dM = \int \varepsilon_0 \left(\frac{T}{T_0}\right)^{15} dM$$

The sun just barely escapes this, but by $1.75\ M_\odot$, 10% of the core is conv. and by $30\ M_\odot$, ½ of the core is conv. The extent of the conv. zone $\left(q = \frac{M_c}{M}\right)$ grows as $M\uparrow$ mostly due to how β changes.

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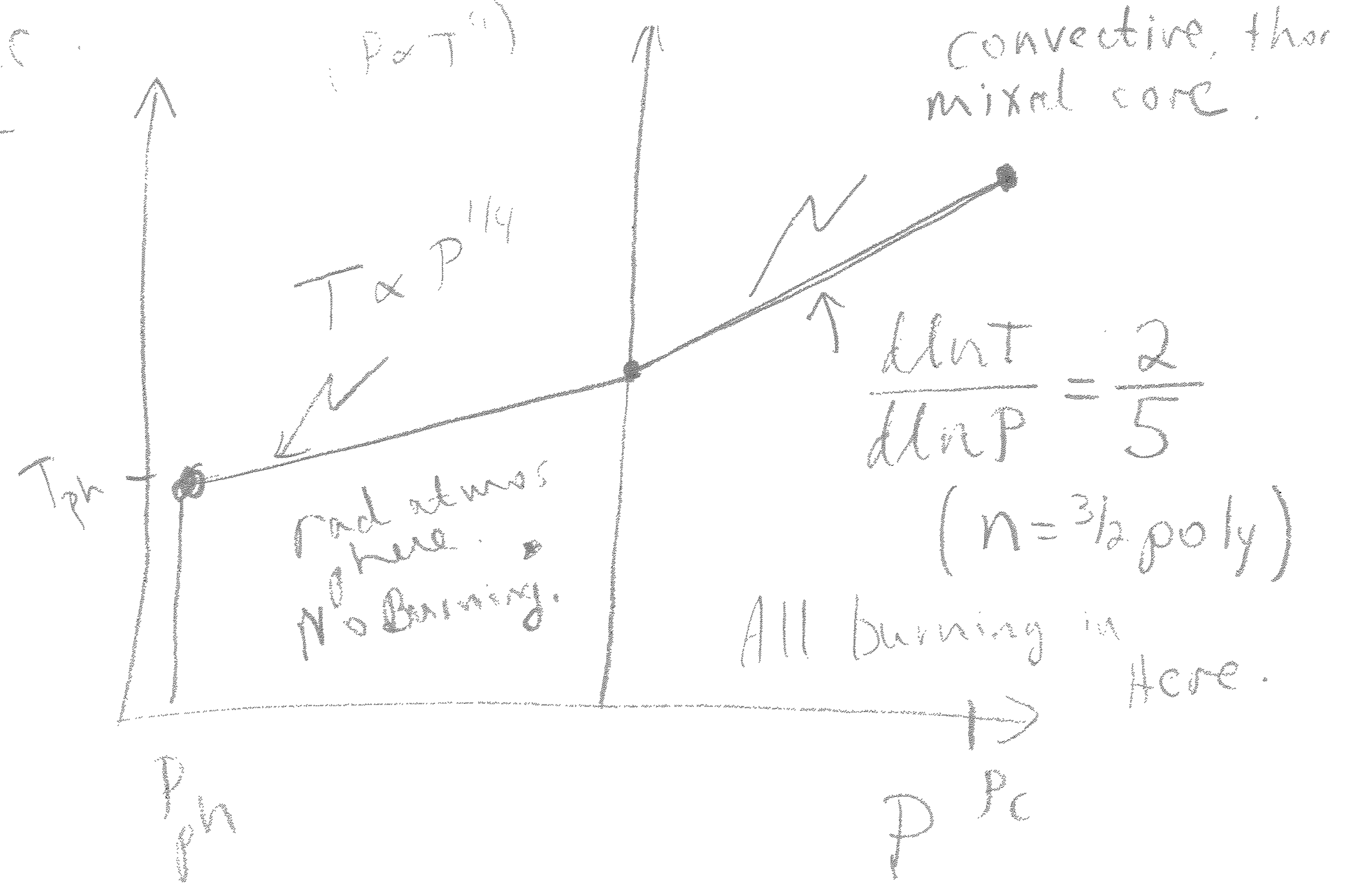
In reality we can get q if we presume <u>no</u> burning beyond the conv. core $L(r_c) = L$ and a radiative polytrope beyond then as

$$q = \frac{m(r)}{M} = \left(\frac{P_{tot}}{P_{rad}}\right)\frac{5}{8}\,\frac{L(r)}{L_{Edd}}$$

which for weak radn. pressure is simple, but otherwise not. Basically then, a massive stars looks like:



Main Sequence from Hansen and Kawaler

t_{ev}	$M/M_\odot$	$\log(L/L_\odot)$	$\log T_e$	$T_{c,6}$	q_c	q_{env}
	60	5.701	4.68	[illegible]	0.73	0
y.	30	5.06	4.61	36	0.56	0
	15	4.29	4.50	32.7	0.4	0
10^7 yrs.	10	3.77	4.41	30	0.33	0
	5	2.77	4.26	26.4	0.23	0
	2	1.26	4.0	21.1	0.13	–
	1.5	0.76	3.9	19	0.07	–
[illegible]	1.0	0	3.75	14	0	0.0035
$1\times10^{?}$ yr	0.6	−1.172	3.59	9.3	0	~1
	0.4	−1.723	3.54	8.1	0	1
	0.10	−3.023	3.48	4.51	0	1
too long	0.08			(3.30)		1

q_c = Mass fraction in convective core

q_{env} = " " " envelope

Now, between 0.08 & 60 $M_\odot$, many things change, the first one that we more or less know about was opacity. Now lets discuss two other imp't facts. First, since the CNO cycle is very T sensitive (higher Coulomb barrier) it dominates in the higher mass stars. This means:

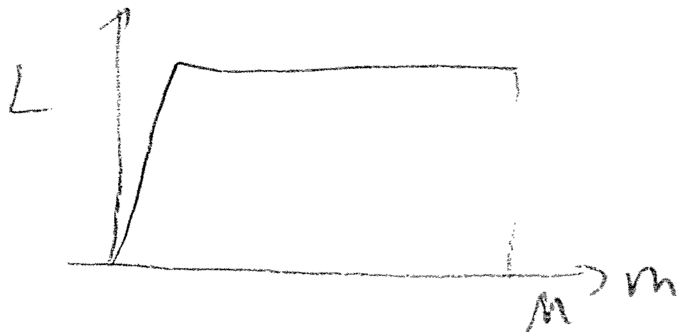
① PP cycle much less imp't at high masses.

② Get some sensitivity to Z

~~1660~~ 170 g.

~~[illegible]~~

③ The core become convective since most of the energy is released very near the core, in which case. all Luminosity is generated near the center and a high flux. so



$(1\text{-}2)M_\odot \lesssim M \lesssim 60$: Convective core + radiative envelope, all burning from CNO.

Then

$0.08 \leq M \lesssim M_\odot$

$\Rightarrow$ pp cycle takes over and ~~Dece~~ core becomes radiative Outer envelope becomes convective ~~and might eve~~ due to rapid changes in opacity in the outer envelope (H/He Ionization zones.).