If this is the case and κ = constant, then we just get.

$$L = \sigma T^4 4\pi R^2$$

$$T_{eff} \simeq 200\,K \left(\frac{M}{M_\odot}\right)^{3/5} \left(\frac{R_\odot}{R}\right)^{1/5}$$

which is still not good if we want electron scattering to be the major source of opacity.

It ends up that, in this case, the outer body sets the temperature.

And H^- which is a very loosely bound ion (0.75 eV) and contributes opacity as

$$\gamma + H^- \rightarrow e^- + H$$

More on this later.

Hansen & Kawaler give:

$$\boxed{\kappa_{H^-} = 2.5 \times 10^{-31} \rho^{1/2} T^9 \ \frac{cm^2}{gr}}$$

and

$$P = \frac{\rho k T}{\mu m_p} = \frac{g}{\kappa} \quad \text{at photosphe}$$

so

$$\rho = \frac{g}{\kappa} \frac{\mu m_p}{kT}$$

so

$$\rho = \frac{g \mu m_p}{kT \kappa_0 \rho^{1/2} T^9}$$

or

$$\rho^{3/2} = \frac{g \mu m_p}{k T^{10} \kappa_0} \Rightarrow \rho = \left(\frac{g \mu m_p}{k T^{10} \kappa_0}\right)^{2/3}$$

so

$$\kappa_{H^-} = \kappa_0 T^9 \left(\frac{g \mu m_p}{k_B T^{10} \kappa_0}\right)^{1/3}$$

$$\boxed{\kappa_{H^-} = \kappa_0^{2/3} T^{17/3} \left(\frac{g \mu m_p}{k_B}\right)^{1/3}}$$

$\frac{17}{3} \cdot \frac{2}{5} = \frac{34}{15}$ $\quad \frac{17}{3} \cdot \frac{2}{5} = \frac{34}{15}$

So

Skip $\frac{2}{5} \cdot \frac{2}{3} \quad \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{15}$

$$k_B T = 0.6 \frac{GM\mu m_p}{R}\left[\frac{R^2}{M\,\cancel{(K_0)^{1/3}}\,(g\mu m_p)^{1/3}}\right]^{2/15} \frac{1}{T}$$

$$k_B T = 0.6 \frac{GM\mu m_p}{R}\left(\frac{R^{4/15}}{M^{2/15}}\right)\frac{1}{K_0^{4/15}\,T^{34/15}}$$

$$* \left(\frac{k_B}{g\mu m_p}\right)^{2/15}$$

$$T^{\frac{49}{15}} = \frac{0.6\,GM\mu m_p}{R K_B}\,\frac{R^{4/15}}{M^{2/15}}\,\frac{1}{K_0^{4/15}}\left(\frac{R^2}{GM}\right)^{2/15}\left(\frac{k_B}{\mu m_p}\right)^{2/15}$$

$$T_{eff} \approx 2500\,K\left(\frac{M}{M_\odot}\right)^{1/7}\left(\frac{R}{R_\odot}\right)^{1/49}$$

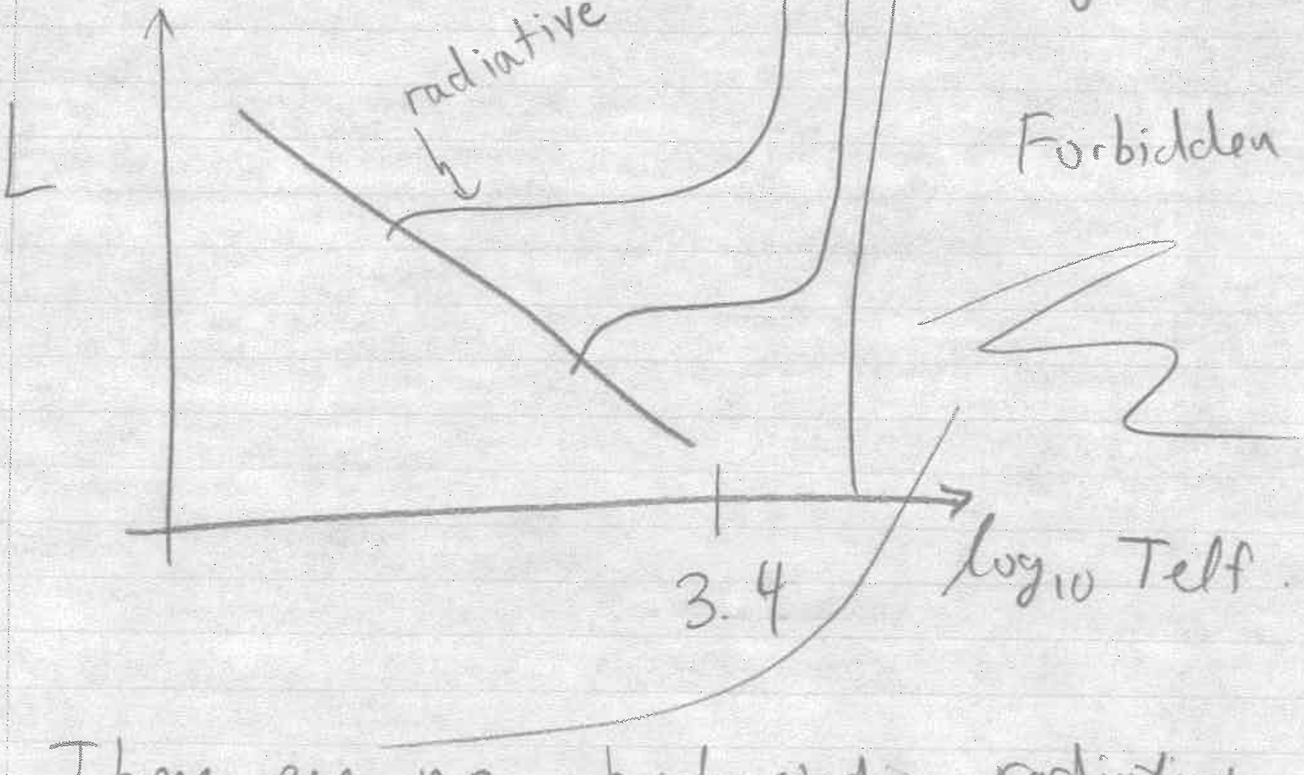
So the luminosity goes as

$$\frac{L}{L_\odot} = 0.034\left(\frac{M}{M_\odot}\right)^{4/7}\left(\frac{R}{R_\odot}\right)^{102/49}$$

If we choose to write in terms of L rather than R, then:

$$\boxed{T_{eff} \approx 2600\,K \left(\frac{L}{L_\odot}\right)^{\frac{1}{102}} \left(\frac{M}{M_\odot}\right)^{7/51}}$$

Or, as you may notice, basically independent of the Luminosity. This is then a straight vertical line in the HR diagram.



There are no hydrostatic radiating solns over here which have the same entropy as in the core.

The contraction down the track is relatively rapid at the high L end but changes as the star contracts.

So the initial evolution is down the "Hayashi" Track until a radiative zone exists, after which point the star evolves along a nearly constant L track.

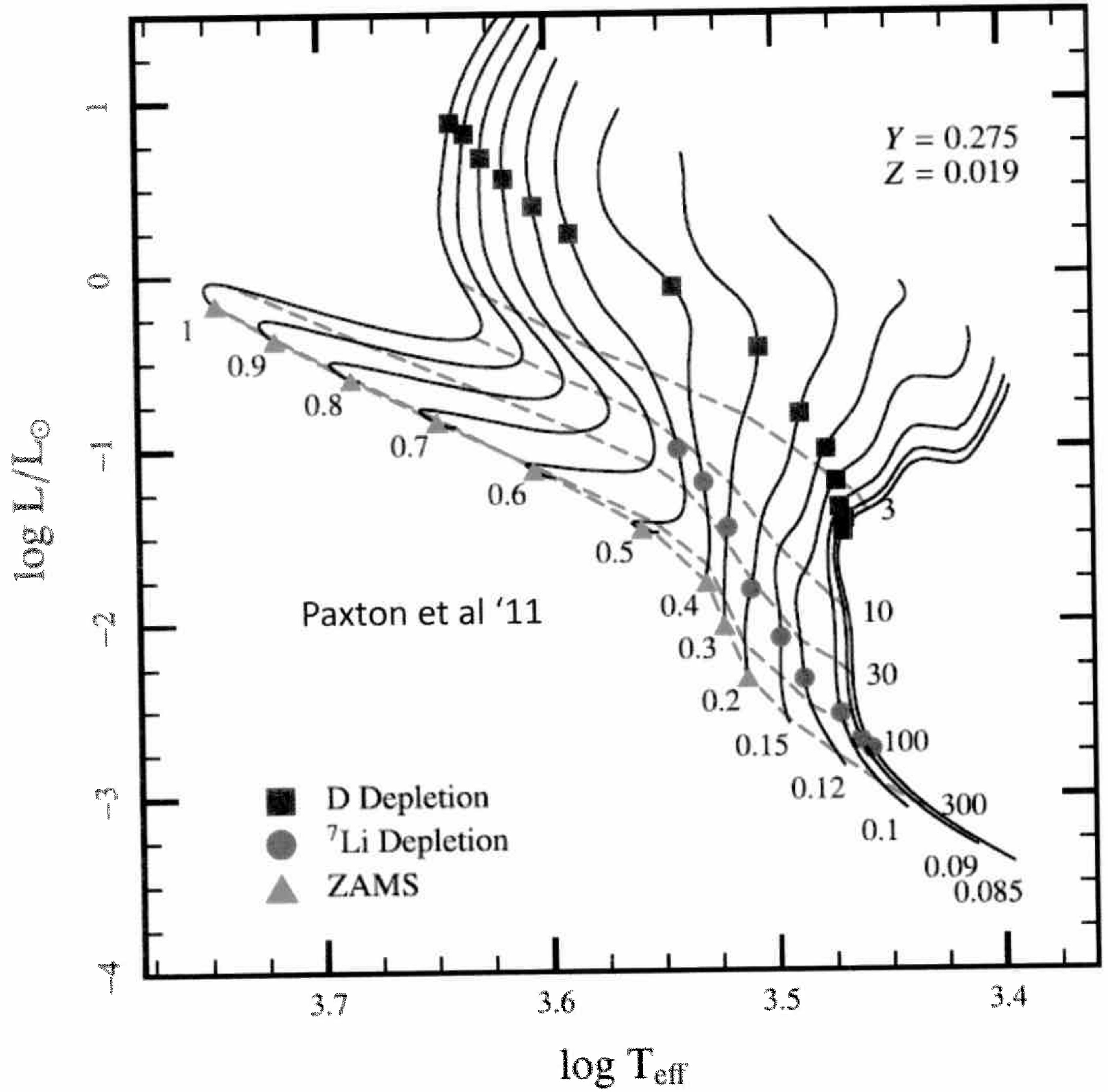


Figure 15. Location in the Hertzsprung-Russell (H-R) diagram for $0.085\, M_\odot < M < 1\, M_\odot$ stars as they arrive at the main sequence for $Y = 0.275$ and

AJ, 121, 1040

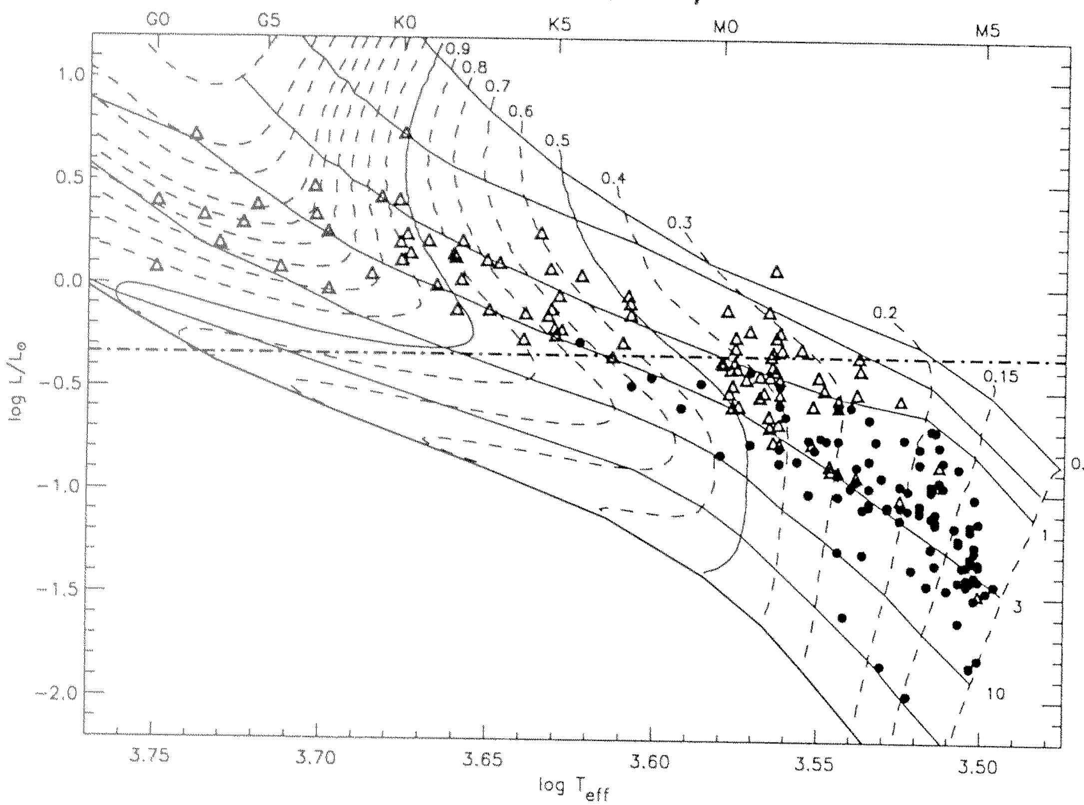


FIG. 8.—H-R diagram for the PMS stars in Upper Sco. The previously known PMS stars from PZ99 are shown as triangles, and the new PMS detected in this study are shown as circles. The dashed lines show the evolutionary tracks from D'Antona & Mazzitelli (1994) and are labeled by their masses in solar units. The 1 $M_\odot$ and 0.5 $M_\odot$ tracks are shown as solid lines. The other solid lines show the D'Antona & Mazzitelli (1994) isochrones for the ages of 0.1, 0.3, 1, 3, 10, and 30 Myr and finally the main sequence. The gray band shows the region in which we expect 90% of the PMS stars to lie, based on the assumption of a common age of 5 Myr for all stars and taking proper account of the uncertainties and the effects of unresolved binaries (see text for details). The dash-dotted line indicates the sensitivity limit of the X-ray observations that were the basis of the X-ray selected PMS sample (see PZ99).

AJ 120, 479

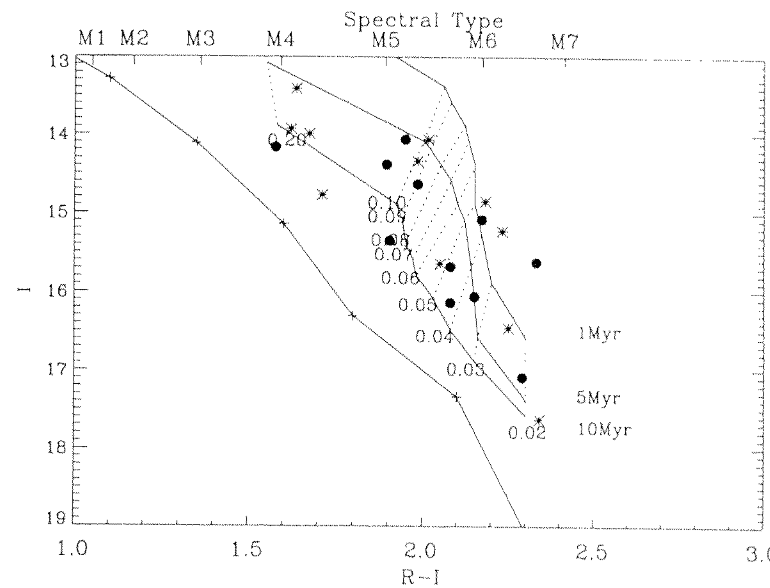


FIG. 6.—Color-magnitude diagram of objects observed spectroscopically, indicating objects for which the Hα equivalent width is above the field envelope (*circles*; see text) and objects for which the Hα equivalent width is below the field (*stars*). All objects have been dereddened. The upper axis shows the spectral types calculated using the color–spectral type calibration from Kirkpatrick & McCarthy (1994). Also shown are the evolutionary models by D'Antona & Mazzitelli (1994) for masses from 0.20 to 0.02 $M_\odot$ and isochrones from 1 to 10 Myr. Of those objects observed spectroscopically, only two (UScoCTIO 18 and UScoCTIO 132) lack Hα.

1096

109a

PASP
100, 1474
(1988)

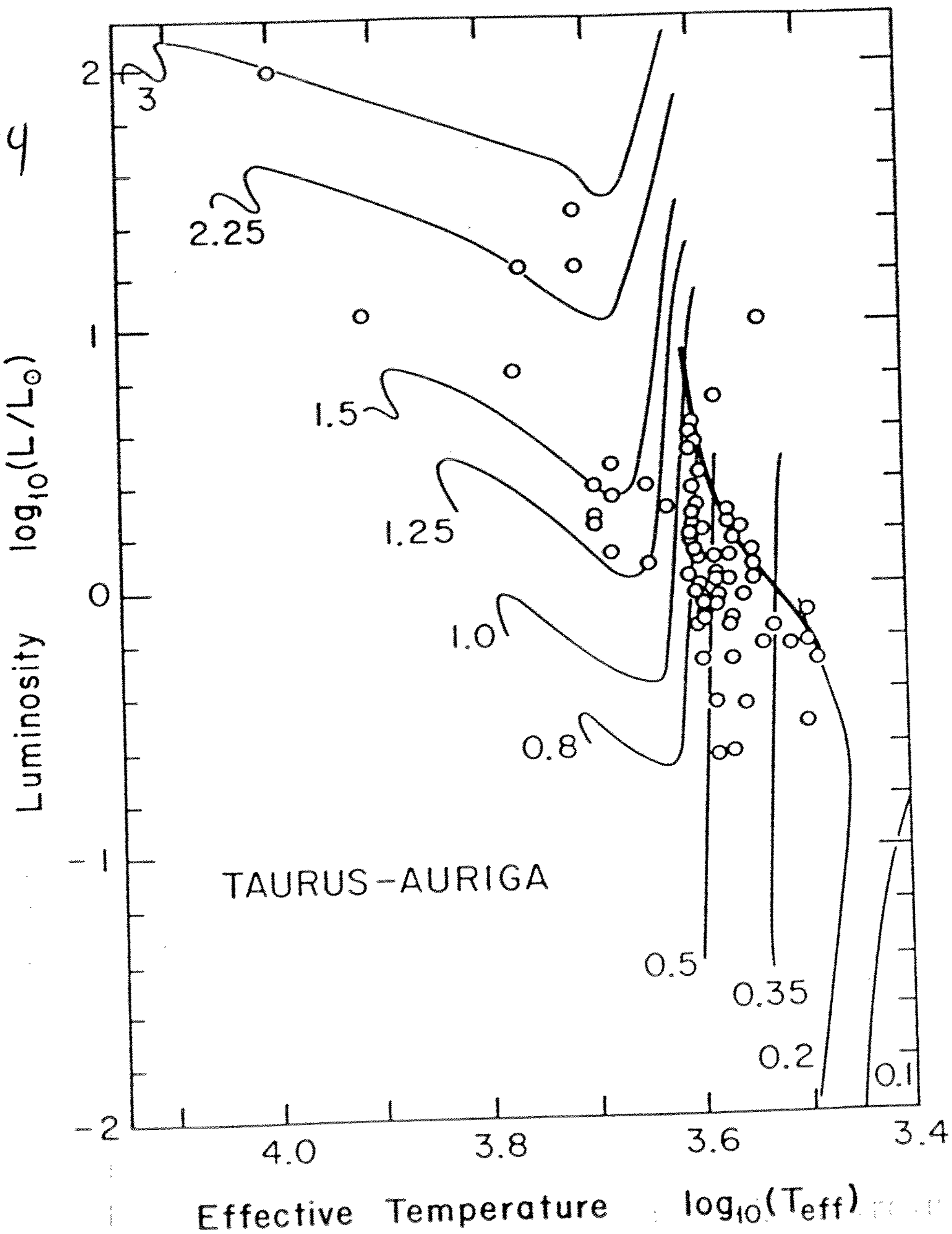


FIG. 4—Observational H-R diagram of the Taurus-Auriga molecular cloud complex. Open circles represent the T Tauri observations of Col Kuhi (1979). The light solid lines are the theoretical pre-main-sequence tracks of Iben (1965) and Grossman and Graboske (1971), with the appr masses (in solar units) labeled. The heavy solid line is the birthline of Stahler (1983).

+ Iben ApJ, 141, 993
+ Stahler ApJ, 274, 822
332, 804

Star Formation / Jean's Mass

Imagine a part of the ISM with density ρ & temperature T. The grav. E.F. is

$$E_{gr} \approx -\frac{GM^2}{R}$$

whereas the total K.E. content is

$$E_{th} = \frac{3}{2}\frac{M}{m_p}KT.$$

If we want collapse then

$$\frac{GM^2}{R} > \frac{3}{2}\frac{M}{m_p}KT$$

but we can say that $M = \frac{4\pi}{3}R^3\rho$

so
$$R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$$ so we want

$$\frac{GM^2}{(3M)^{1/3}}(4\pi\rho)^{1/3} > \frac{3}{2}\frac{M}{m_p}KT$$

$$\Rightarrow M^{2/3} > \frac{3KT}{2m_p}\,3^{1/3}\,\frac{1}{(4\pi\rho)^{1/3}}\,\frac{1}{G}$$

$$\Rightarrow \boxed{M > 500\, M_\odot \left(\frac{T}{10K}\right)^{3/2}\left(\frac{1}{n}\right)^{1/2}}$$

so if the cloud is more massive than this value, it will collapse and contract. Now the important question is can collapse continue indef. Well,

$$M_J \propto T^{3/2} \rho^{-1/2}$$

so, if say isothermal then we are OK as M_J will decrease, allowing for fragmentation. What if the collapse is adiabatic? Then

$$\frac{P}{\rho^{\gamma}} = \text{constant} \quad \text{so}$$

$$\Rightarrow \rho T \propto \rho^{\gamma} \qquad \gamma = 5/3 \text{ if ideal gas,}$$

$$\text{then} \quad \rho^{2/3} \propto T \quad \text{so}$$

$$M_J(\text{ad}) \propto \rho \rho^{-1/2} \propto \rho^{1/2}$$

or, increasing during collapse, in which case the collapse of the cloud is eventually halted.

So, much of the physics associated with fragmentation and collapse is in the isothermal adiabatic transition. The real question is whether the star can loose the internal energy ~~fast~~ it is gaining due to compression

Extra Comments on Protostars

Much more could be said about star formation, both from theory & observation. Since in this field observations are way ahead of theory let me just tell you ~~that~~ a few highlights.

① ~~Re~~ After ≈ 10 My it seems that accretion has halted!

② Disks have been seen and likely are present even at late times as planetary systems

③ The accretion of this disk might well set the initial stellar spin and jets are seen to emit!

[Get picture from Shu et al].

$\Rightarrow$ Eventually hydrostatic balance dominates.

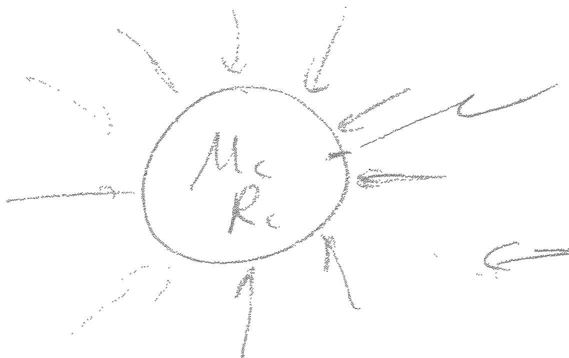
fast enough to continue collapse & fragmentation. In most cases the collapse is dynamical (strong cooling so that all pdV work is lost to radiation), then

$$t_{dyn} = \frac{1}{\sqrt{G\rho}} \approx \frac{10^7 \text{ yr}}{(n/100)^{1/2}}$$

is the timescale for collapse.

Now, much has occurred in protostar theory, starting with Larson's discovery that the collapse is not homologous, but rather we first find a hydrostatic core at the middle, which accretes matter.

$$L_* \approx L_{acc} = \frac{GM}{R}\dot{M}$$



optically thick to outgoing rad'n.

Quite often by the time the star reveals itself it is already able to be in a radiative state.

(Stahler, PASP, 100, 1474)

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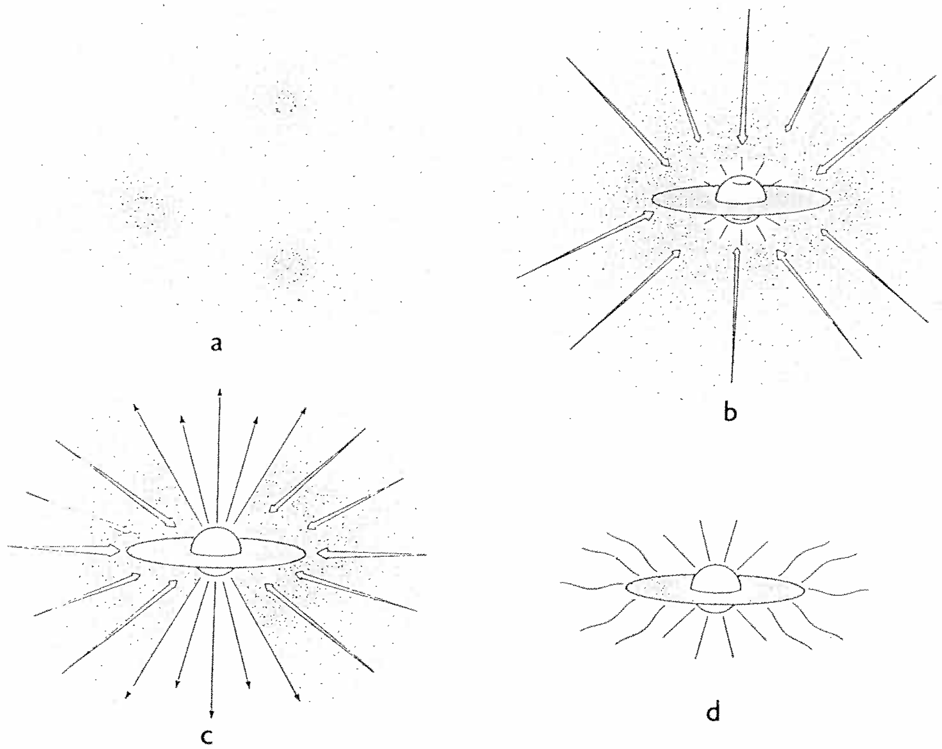

Figure 7 The four stages of star formation. (*a*) Cores form within molecular clouds as magnetic and turbulent support is lost through ambipolar diffusion. (*b*) A protostar with a surrounding nebular disk forms at the center of a cloud core collapsing from inside-out. (*c*) A stellar wind breaks out along the rotational axis of the system, creating a bipolar flow. (*d*) The infall terminates, revealing a newly formed star with a circumstellar disk.

material passes through an accretion shock as it falls onto the central star and disk, which, along with accretion within the disk, produces the main contribution to the luminosity for low-mass protostars. The emergent spectral energy distributions of theoretical models in the infall stage are in close agreement with those of recently found infrared sources with negatively steep spectra in the near- and mid-infrared. Protostars of high mass, in a pure accretion phase, have yet to be found, although the source near the water masers in W3(OH) is probably close to being such an object.

As a protostar accretes matter, deuterium will eventually ignite in the central regions and drive the star nearly completely convective if its mass is less than about 2 $M_{\odot}$. If the convection and the differential rotation of the star combine to produce a dynamo, the star can naturally evolve toward a state with a stellar wind. However, at first the ram pressure from material falling directly onto the stellar surface suppresses breakout. Gradually, the "lid" of direct infall will weaken as the incoming material falls preferentially onto the disk rather than onto the star. The stellar wind then rushes through the channels of weakest resistance (the rotational

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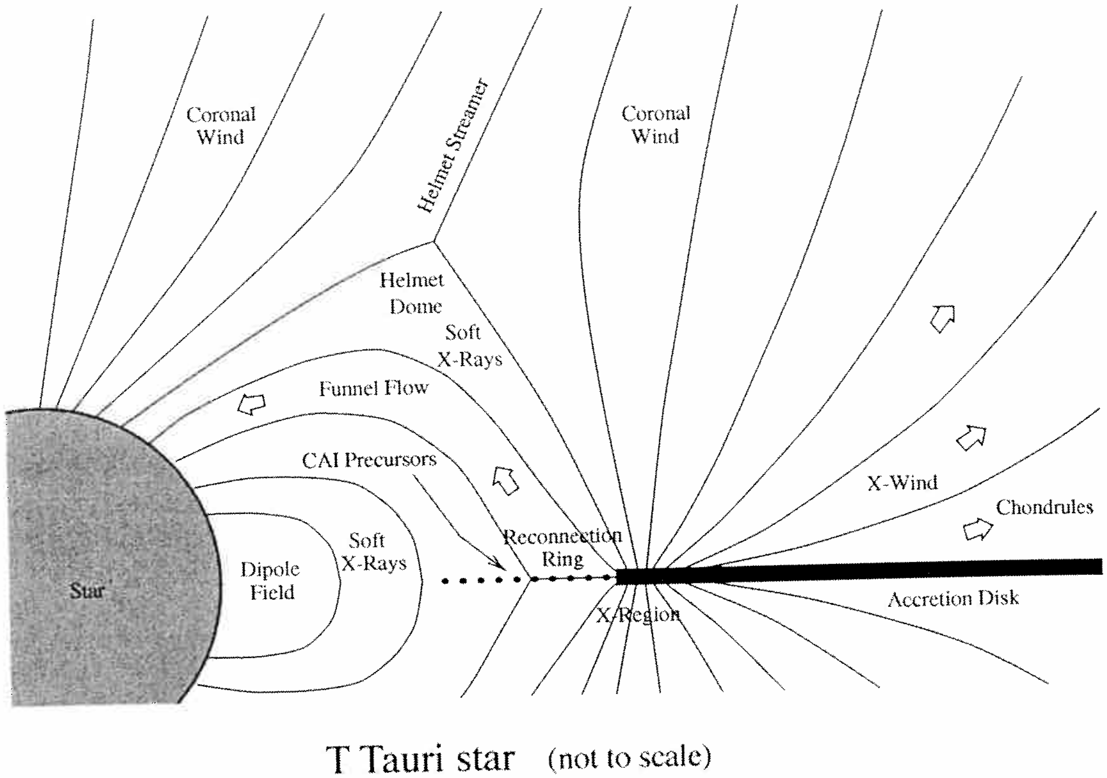

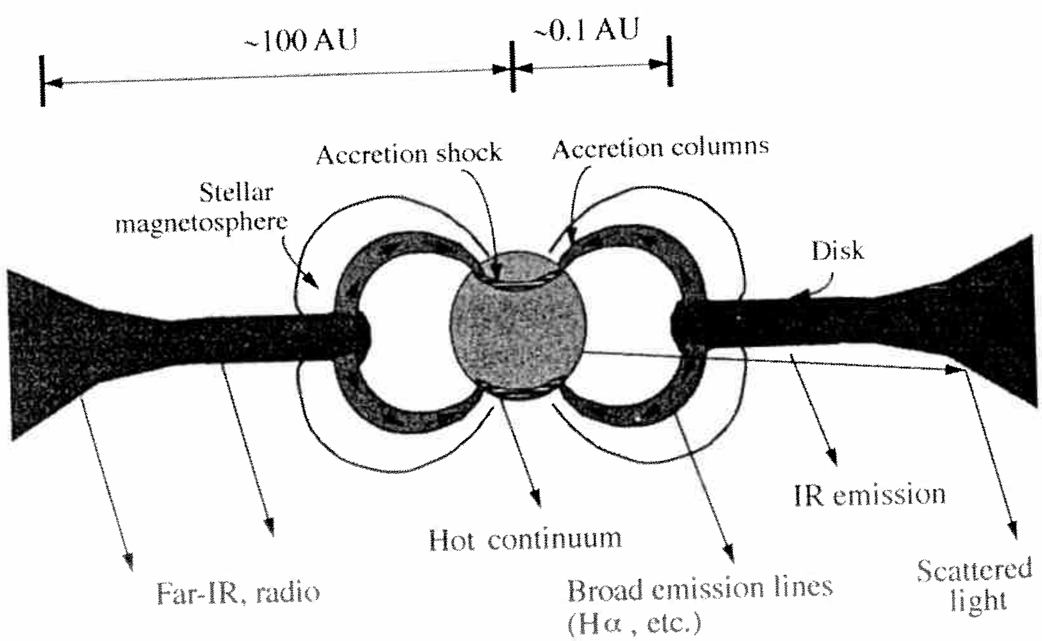

Figure 3 Two contemporary models for Class I–II YSOs, in which magnetic fields play crucial roles: (*top*) the x-wind model of YSOs showing magnetically collimated accretion and outflows with irradiated meteoritic solids (Shu et al 1997); (*bottom*) magnetically funneled accretion streams producing broadened emission lines (Hartmann 1998).

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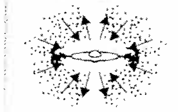
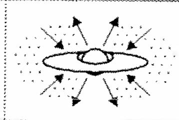
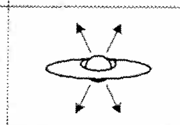
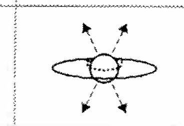
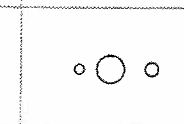
PROPERTIES	*Infalling Protostar*	*Evolved Protostar*	*Classical T Tauri Star*	*Weak-lined T Tauri Star*	*Main Sequence Star*
SKETCH					
AGE (YEARS)	10^4	10^5	$10^6 - 10^7$	$10^6 - 10^7$	$> 10^7$
mm/INFRARED CLASS	Class 0	Class I	Class II	Class III	(Class III)
DISK	Yes	Thick	Thick	Thin or Non-existent	Possible Planetary System
X-RAY	?	Yes	Strong	Strong	Weak
THERMAL RADIO	Yes	Yes	Yes	No	No
NON-THERMAL RADIO	No	Yes	No ?	Yes	Yes

Figure 1 The stages of low-mass young stellar evolution. This review chiefly addresses the bottom three rows of the chart. (Adapted from Carkner 1998.)

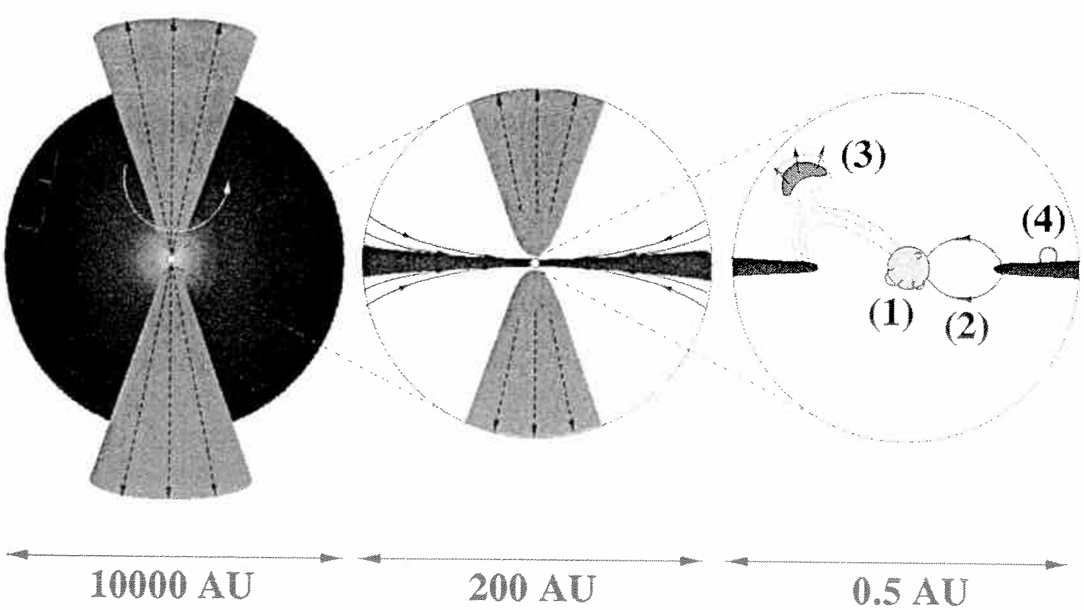


Figure 2 Four magnetic-field configurations that may be responsible for the magnetic activity of Class I protostars. The X rays come from the inner region of a complex structure comprising a collapsing extended envelope (*left*), an inner disk and outflow (*center*), and a star-disk magnetic-interaction region (*right*). (Courtesy of N. Grosso.)

Lect 9

Entropy Equation and the local description of Grav. Cnd.

You are all familiar with the relation

$$dQ = TdS = dE + pdV$$

or just
$$T\frac{dS}{dt} = \frac{dE}{dt} + p\frac{dV}{dt}$$

Now what we want to first find and discuss are sources & sinks of energy. We prefer to work in units of energy/gram so

$$dQ = dE + p\,d\left(\frac{1}{\rho}\right) = dE - \frac{p}{\rho^2}d\rho$$

and $E_{int,id} = \frac{3}{2}k_B T \frac{1}{\mu m_p}$, so

$$\frac{3k_B}{2\mu m_p}dT + \frac{\rho k_B T}{\mu m_p}\frac{-1}{\rho^2}d\rho = 0$$

$$\Rightarrow \frac{3}{2}\frac{dT}{T} = \frac{d\rho}{\rho} \Rightarrow \frac{d\ln T}{d\ln \rho} = \frac{2}{3}$$

so $T \propto \rho^{2/3}$ [Compare to what you know & love $PV^\gamma = C \Rightarrow \frac{\rho T}{\rho^\gamma} = C$

$$T \propto \rho^{\gamma-1} = \rho^{\frac{5}{3}-1} = \rho^{2/3}\Big]$$

Then, for a given fluid element, the eqn of interest is

$$T\frac{ds}{dt} = \frac{dQ}{dt} = \text{energy lost or gained by fluid. (co-moving)}$$

What changes entropy.

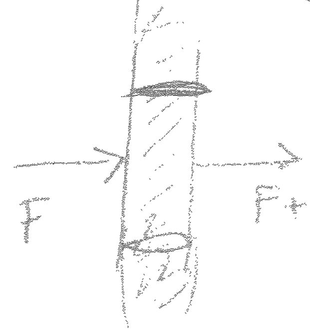
(1) Heat gained by nuclear reactions which we write as

$$\varepsilon = \frac{\text{ergs}}{\text{gr}\cdot\text{sec}}$$

(2) Heat gained (or lost) by transport processes:

$$-\frac{1}{\rho}\,\underset{\sim}{\nabla}\cdot\underset{\sim}{F}$$

$$F = \frac{\text{ergs}}{\text{cm}^2\cdot\text{sec}}$$



so

$$\varepsilon = FA - (F+dF)A = \frac{\text{ergs}}{\text{sec}}$$

divide by mass $= \rho A dr$ so

$$\varepsilon = \frac{-dF}{dr}\frac{1}{\rho}$$

You could equally well consider a "blob" where the ergs/sec is

$$\int \underset{\sim}{F}\cdot d\underset{\sim}{A}$$

Gauss Thm. allows us to rewrite as

$$\int dV \, \nabla \cdot F$$

so the local rate is divided by $\int \rho \, dV$

$$\Rightarrow \quad \varepsilon_h = -\frac{\nabla \cdot F}{\rho}$$

The Entropy equation is thus:

$$\boxed{T \frac{ds}{dt} = \varepsilon_{nuc} - \frac{\nabla \cdot F}{\rho}}$$

Lets first rewrite ($F = F_r \hat{r}$)

$$\frac{\nabla \cdot F}{\rho} = \frac{1}{\rho} \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 F_r \right)$$

but

$$L_r = 4\pi r^2 F_r \quad \text{so}$$

$$\boxed{\frac{\nabla \cdot F}{\rho} = \frac{1}{\rho \, 4\pi r^2} \frac{\partial}{\partial r} \left(L_r \right) = \frac{\partial L_r}{\partial m_r}}$$

Lets first imagine the case where there is no nuclear sources or others and we are therefore not in a steady state:

$$T \frac{ds}{dt} = -\frac{\partial L_r}{\partial m_r}$$

Begin lecture 7.

but we can relate $T\,dS$ to dE + $P\,dV$

$$T\frac{ds}{dt} = \frac{dE}{dt} - \frac{P}{\rho^2}\frac{d}{dt}\rho = \cancel{\Delta E_{grav}}$$

E = internal energy per gram

where we are now writing a Lagrangian formulation. This basically says how the deviation from adiabatic evolution is what generates luminosity. Imagine an ideal gas:

$$E = \frac{3}{2}\frac{kT}{\mu m_p} = \frac{3}{2}\frac{P}{\rho} \quad \text{so we get}$$

$$\frac{3}{2}\frac{d}{dt}\frac{P}{\rho} = \frac{3}{2}\frac{1}{\rho}\frac{dP}{dt} - \frac{3}{2\rho^2}P\frac{d\rho}{dt}$$

so the full piece is

$$T\frac{ds}{dt} = \frac{3}{2}\frac{1}{\rho}\frac{dP}{dt} - \frac{5}{2}\frac{P}{\rho^2}\frac{d\rho}{dt}$$

$$= \frac{P}{\rho}\left[\frac{3}{2}\frac{d\ln P}{dt} - \frac{5}{2}\frac{d\ln\rho}{dt}\right] = \frac{3}{2}\frac{P}{\rho}\left[\frac{d\ln P}{dt} - \frac{5}{3}\frac{d\ln\rho}{dt}\right]$$

$$T\frac{ds}{dt} = \frac{3}{2}\frac{P}{\rho}\frac{d}{dt}\boxed{\ln\left(\frac{P}{\rho^{5/3}}\right)}$$

so

$$\boxed{T\frac{ds}{dt} = +\frac{3}{2}\frac{P}{\rho}\frac{d}{dt}\ln\left(\frac{P}{\rho^{5/3}}\right)}$$

Now we often call $T\frac{ds}{dt} = -\varepsilon_{grav}$ ← sign same as

so that $\frac{\partial L}{\partial ...} = \varepsilon ... + \varepsilon$ ...