Lecture

1

Wi '12

Magnitude Scale and Some Numbers

Optical astronomy has a very well defined, yet bewildering scale for measuring brightnesses, which is logarithmic. I am going to let you work through it on your own, as it is only bookkeeping. However, I do want you to know that it is all based on

Vega (8pc away).

$U = B = V = 0$ for Vega

$$\Rightarrow \quad f_V = 3.5 \times 10^{-20} \ \frac{\text{ergs}}{\text{cm}^2 \cdot \text{sec} \cdot \text{Hz}}$$

at 5500 Å or about

$$N_\lambda = 10^3 \ \frac{\gamma}{\text{cm}^2 \cdot \text{sec} \cdot \text{Å}} \quad \text{at } V$$

Band	λ_0 (Å)	FWHM (Å)	0 $\frac{\gamma}{\text{cm}^2 \cdot \text{s} \cdot \text{Å}}$
U	3650	680	780
B	4400	980	1450
V	5500	890	1000
R	7000	2200	610
I	9000	2400	380

Lets start with the Data from nearby stars on the HR Diagram: One can also view this plot as a Teff vs. L plot, in which case the T has only a factor of 10, whereas L ~~goes from~~ has 15 magnitudes and 5 magn = factor of 100, so we have a dynamic range in L of $\approx 10^6$. So we would like to understand & explain

① Why are the stars a so well defined sequence

② Why is the dynamic range in Teff so small when L is so large? What sets L, Teff & R?

③ Can we understand the # density of stars along the plot & the # of dwarfs?

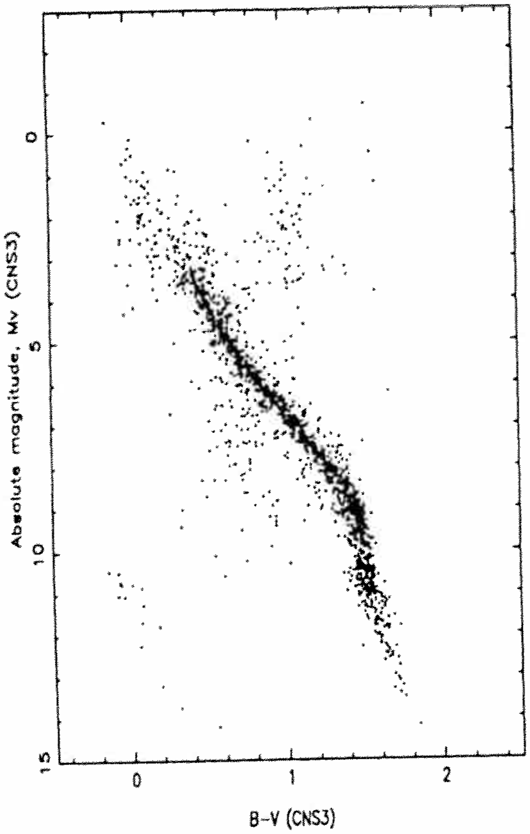
J $\sigma T^4 4\pi R^2 = 4\times10^{33}$ $T_{sun} \approx 5800K$

④ By the way the census gives all down to $M_V = 16$ or so

$$n_* = \frac{2241}{\frac{4\pi}{3} 22pc^3} \quad 0.05\ pc^{-3}$$

$r \approx 1.7pc$ is average separation

4



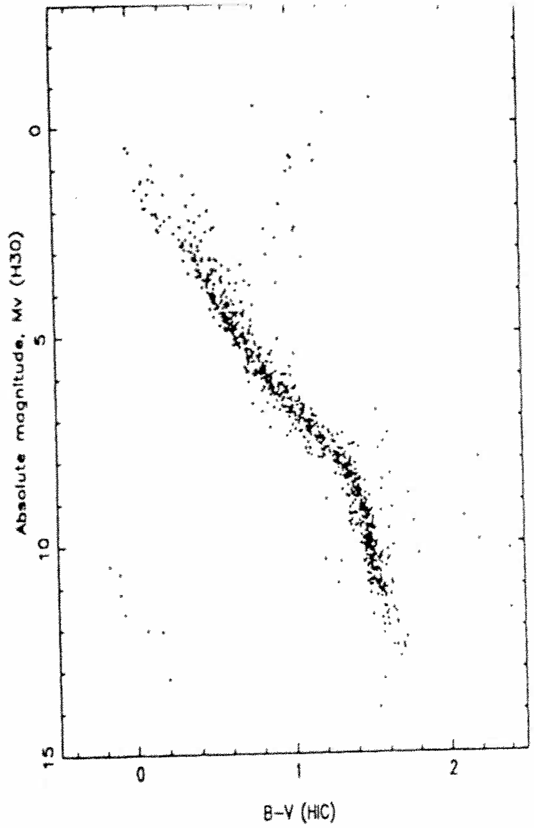
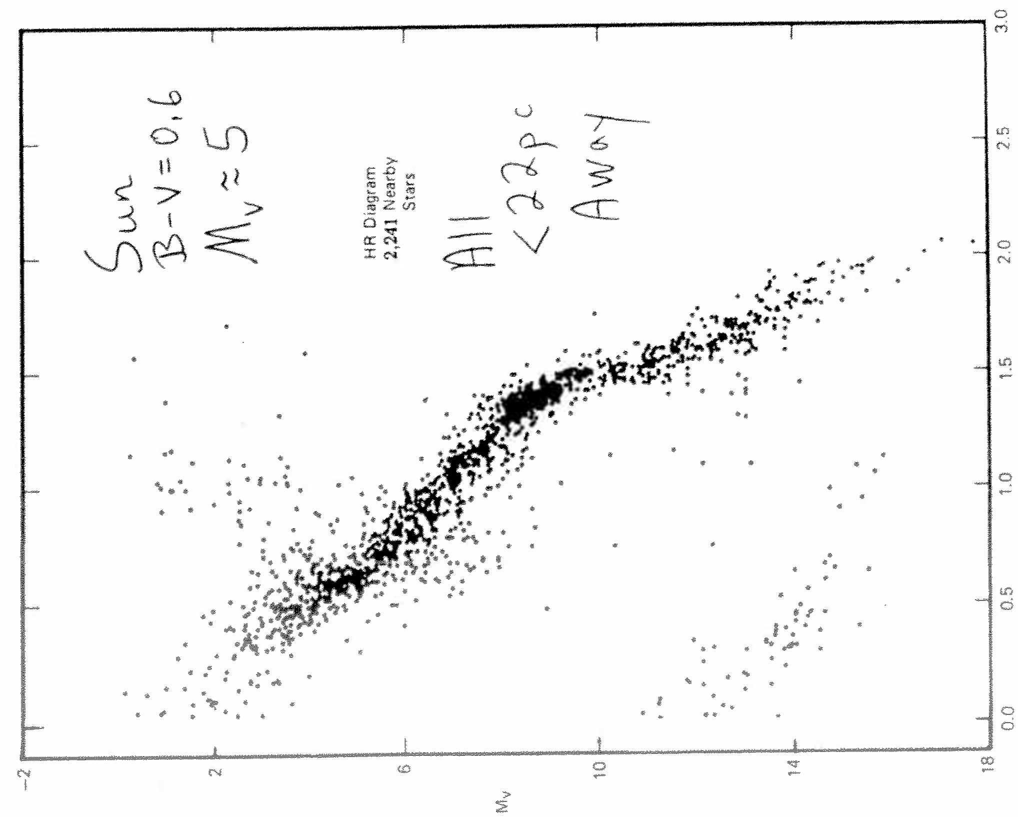


Fig. 8. a The Hertzsprung-Russell diagram for objects listed in the CNS3 and observed by Hip
of M_V and $B - V$ given in that catalogue (i.e., for the 2095 entries observed by Hipparcos a
corresponding HR diagram for the 1052 stars from CNS3 for which $\pi > 40$ mas (i.e., $d < 25$
constructed on the basis of the V magnitudes given in the Hipparcos Input Catalogue and the Hip
is taken from the Hipparcos Input Catalogue. **c** as for **b**, but for the 718 stars from CNS3 for wh



Hertzsprung-Russell Diagram for Nearby Stars. This plot of the absolute visual magnitude, Mv, against color index, B-V, illustrates the heavily populated main sequence running from the hot, luminous upper left to the cooler, less-luminous lower right, as well as the faint white dwarf stars in the lower left. It has been compiled from data for 2,241 nearby stars given in the Appendix of this book.

A+A 304, 69

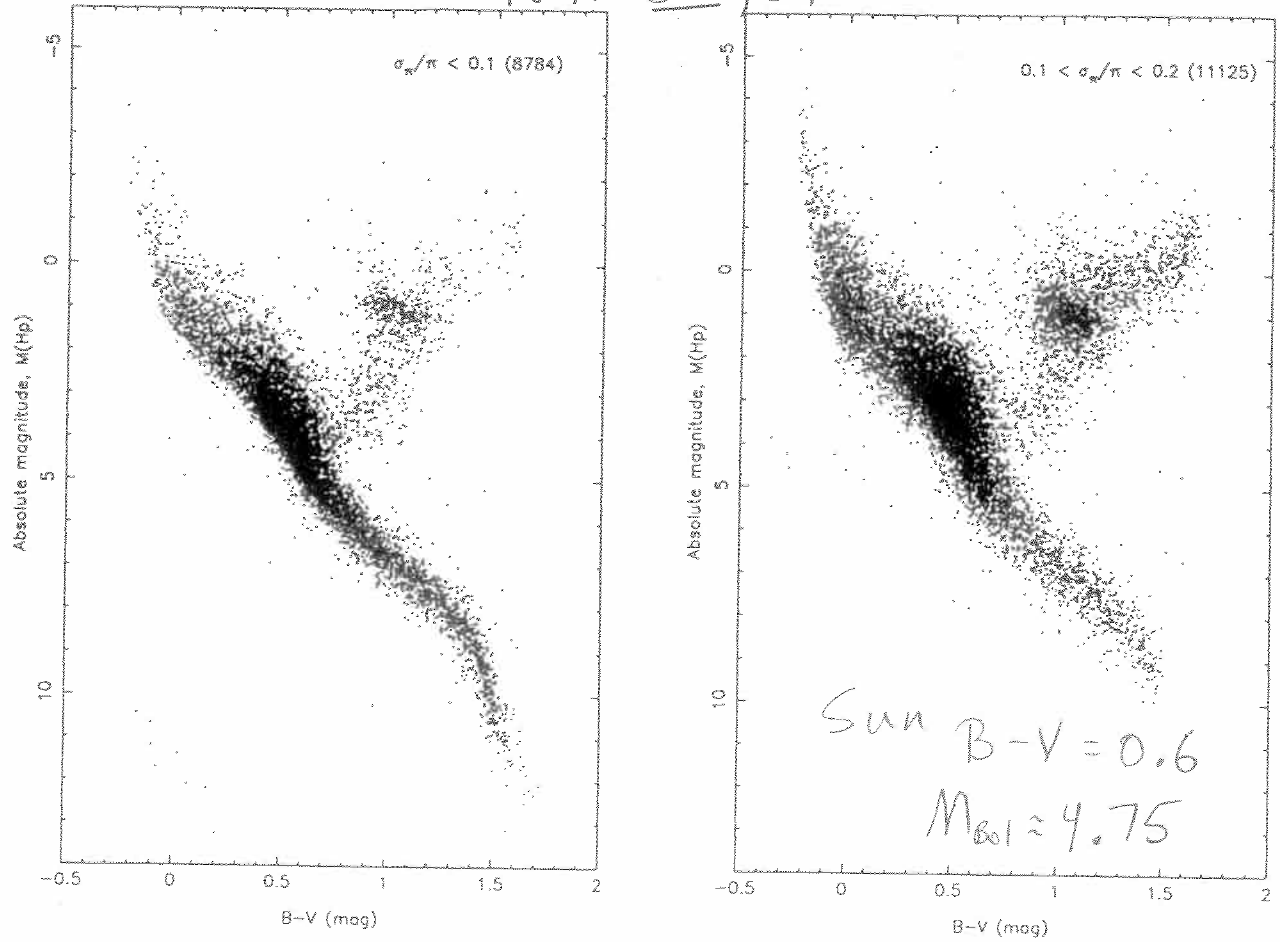


Fig. 6. a The observational HR diagram constructed from the preliminary Hipparcos catalogue H30, for the 8784 stars with $\sigma_\pi/\pi < 0.1$ and $\sigma_{B-V} < 0.025$ mag, and supplemented by six white dwarfs as described in the text. The ordinate gives the absolute magnitude, M_{Hp}, derived from the satellite-determined parallaxes and the median satellite-derived Hp magnitudes. The abscissa gives the colour index $(B-V)$, derived from the ground-based observations compiled in the Hipparcos Input Catalogue. **b** as for **a**, but based on the 11 125 stars from H30 satisfying $0.1 \leq \sigma_\pi/\pi < 0.2$ and $\sigma_{B-V} < 0.025$ mag.

47 TUCANAE 777

Hesser et al.

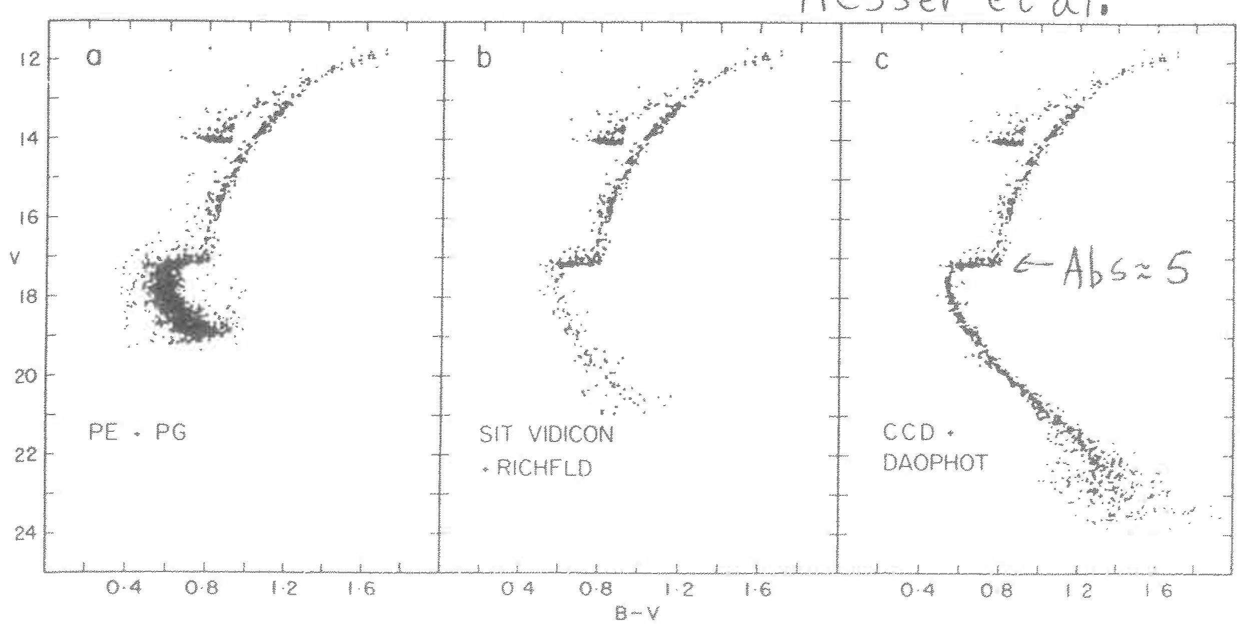


FIG. 9—CMDs for 47 Tuc derived from three independent studies: (a) photoelectrically calibrated photographic photometry (Hesser and Hartwick 1977), obtained with the CTIO 1.5-m telescope and traditional iris photometry; (b) photometry with a SIT Vidicon camera (Harris *et al.* 1983*a*,*b*), obtained with the CTIO 4-m and reduced through RICHFLD; (c) the present CCD+DAOPHOT photometry. In all three cases the giant and horizontal branches are the same (photographic) data, only the main-sequence data ($V > 17.2$) are different, to illustrate the progression in depth and internal precision that has been achieved.

19.9 MASS DENSITY IN THE SOLAR NEIGHBORHOOD [25–32]

Observed volume mass density

Interstellar matter (ISM)	0.04 ± 0.02	$\mathcal{M}_\odot$ pc^{-3}
Main Sequence Stars:		
$0.08 \leq \mathcal{M}/\mathcal{M}_\odot < 1.0$	0.036	$\mathcal{M}_\odot$ pc^{-3}
$1.0 \leq \mathcal{M}/\mathcal{M}_\odot < 100$	0.014	$\mathcal{M}_\odot$ pc^{-3}
Halo stars	0.0001	$\mathcal{M}_\odot$ pc^{-3}
Evolved stars:		
White dwarfs	0.005	$\mathcal{M}_\odot$ pc^{-3}
Dark extended halo, local density	0.01	$\mathcal{M}_\odot$ pc^{-3}
Total	0.10 ± 0.03	$\mathcal{M}_\odot$ pc^{-3}

Note that $0.01\mathcal{M}_\odot$ pc^{-3} is 0.3 Gev cm^{-3}.

Observed column mass densities, to $|z| = 1.1$ kpc

Neutral ISM	8	$\mathcal{M}_\odot$ pc^{-2}
Ionized ISM	2	$\mathcal{M}_\odot$ pc^{-2}
Molecular ISM	3	$\mathcal{M}_\odot$ pc^{-2}
ISM total	13 ± 3	$\mathcal{M}_\odot$ pc^{-2}
Stars:		
Disk main sequence	30	$\mathcal{M}_\odot$ pc^{-2}
Disk white dwarfs	3	$\mathcal{M}_\odot$ pc^{-2}
Thick disk	2	$\mathcal{M}_\odot$ pc^{-2}
Halo subdwarfs	< 1	$\mathcal{M}_\odot$ pc^{-2}
Stellar total	35 ± 5	$\mathcal{M}_\odot$ pc^{-2}
Observed total	48 ± 8	$\mathcal{M}_\odot$ pc^{-2}
Extended dark halo		
$\lvert z \rvert < 1.1$ kpc	23	$\mathcal{M}_\odot$ pc^{-2}
Total	71 ± 6	$\mathcal{M}_\odot$ pc^{-2}

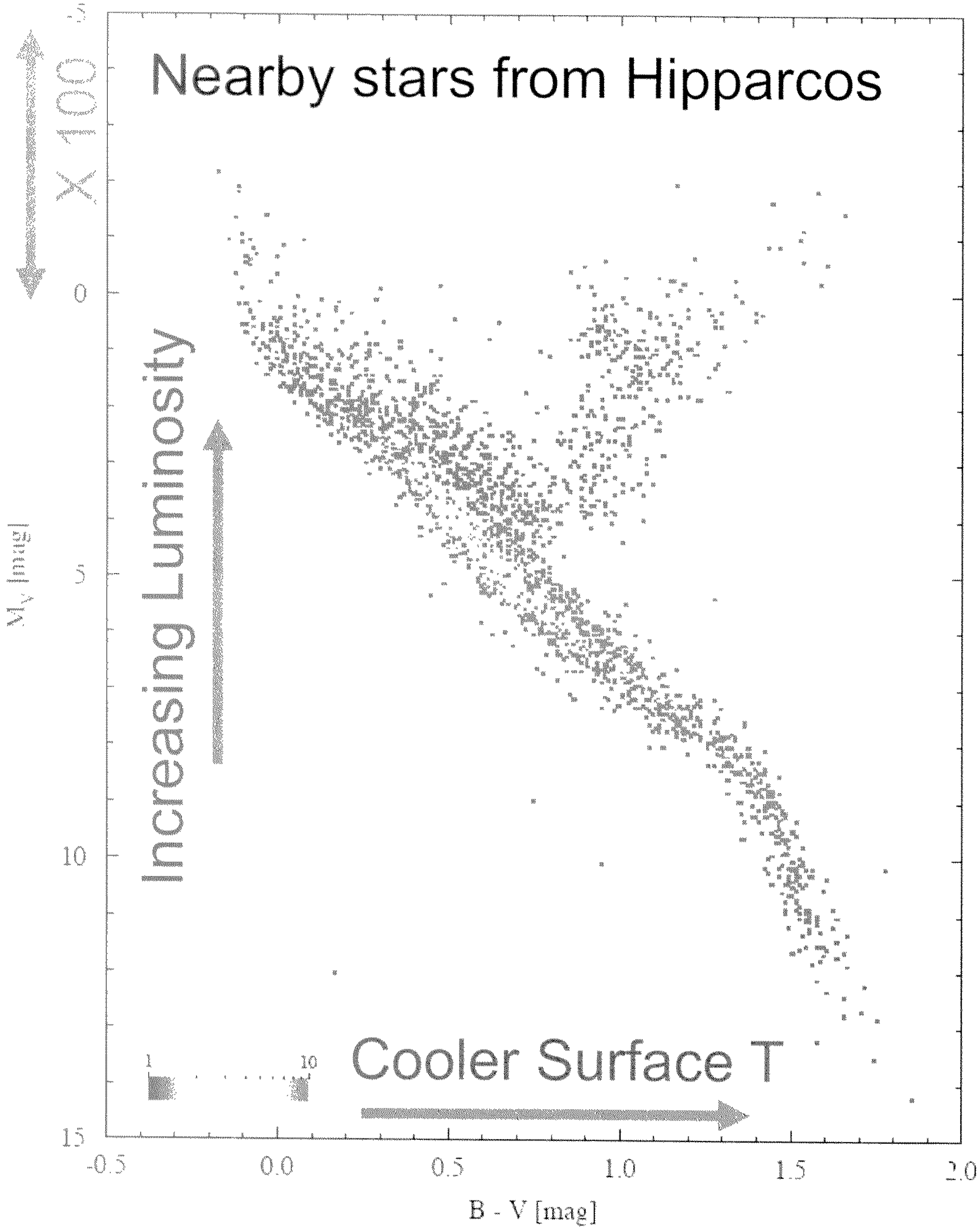
K dwarfs ($z \lesssim 160$ pc) $\rho_0 = 0.10 \pm 0.03\mathcal{M}_\odot$ pc^{-3}.

All determinations are consistent with each other and with zero local unidentified matter at the $\sim 1.5\,\sigma$ level.

Dynamical analysis of the column mass density, $\mathcal{M}_\odot$ pc^{-2}

K dwarfs ($300 \lesssim z \lesssim 2000$ pc)

$\sum_{\rm tot}(z \leq 1.1$ kpc$)$	$= 71 \pm 6\mathcal{M}_\odot$ pc^{-2},
$\sum_{\rm disk}$	$= 48 \pm 9\mathcal{M}_\odot$ pc^{-2},
$\sum_{\rm dark\ halo}$	$= 23\mathcal{M}_\odot$ pc^{-2},
Unidentified disk dark matter	$= 0 \pm 12\mathcal{M}_\odot$ pc^{-2}.



The outcome of star formation is construction of stars that only occupy certain regions of this diagram.

- Why?
- What is the controlling parameter?
- How do stars evolve in time?
- What do they become?

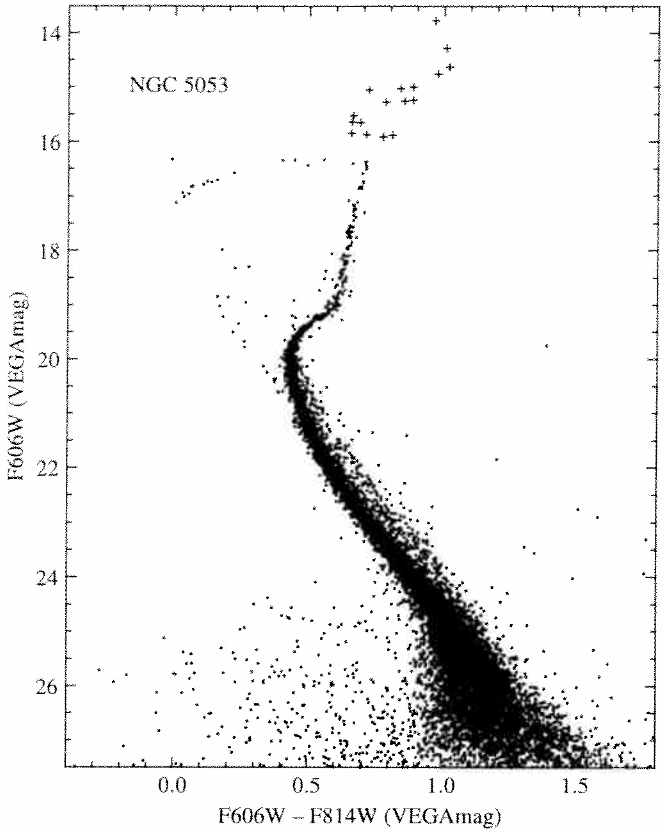


FIG. 3.—Same as Fig. 1, except that the CMD of NGC 5053 is shown, containing 15,618 stars and extending to about 30% of the tidal radius of 14′ (Harris 1996).

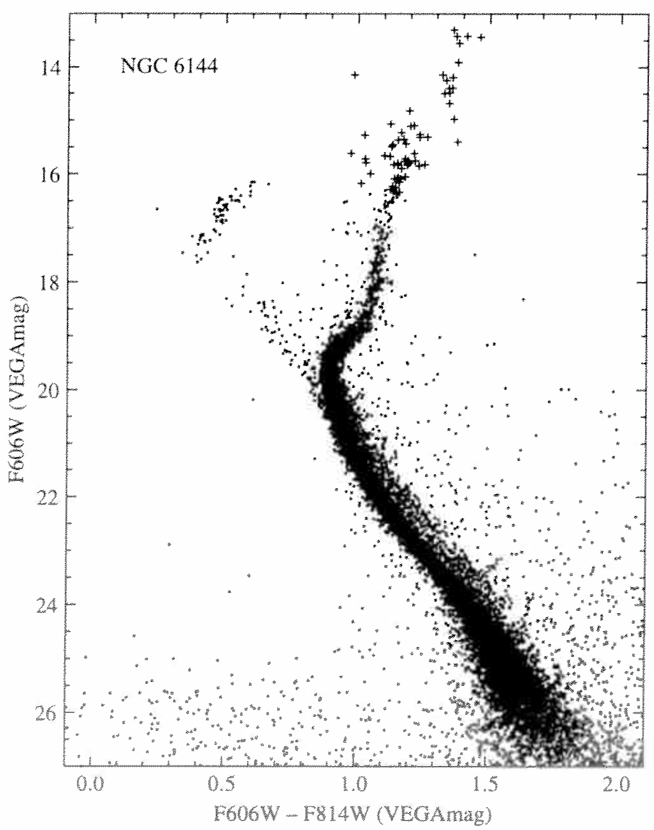


FIG. 4.—Same as Fig. 1, except that the CMD of NGC 6144 is shown, containing 19,442 stars and extending to about 13% of the tidal radius of 33′ (Harris 1996).

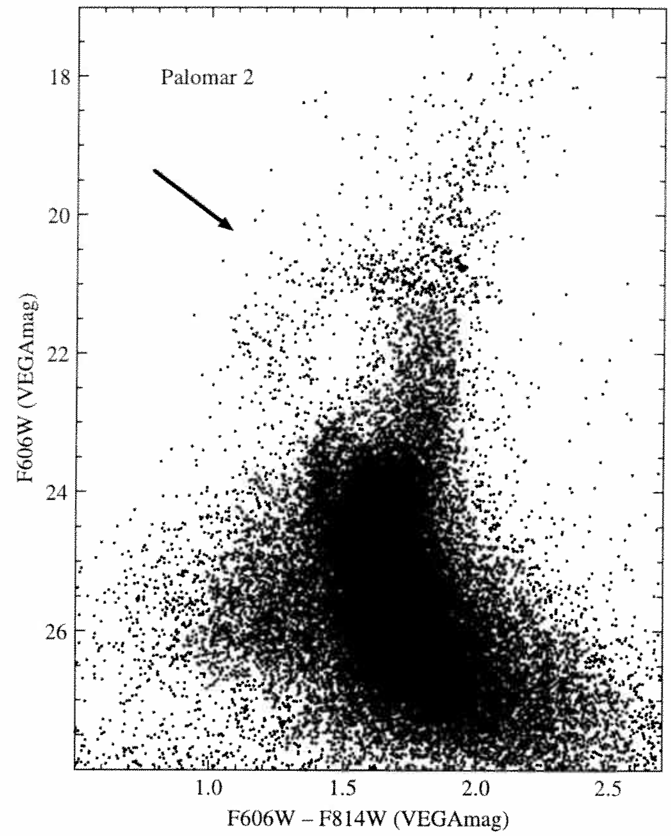


FIG. 5.—Same as Fig. 1, except that the CMD of Palomar 2 is shown, containing 43,242 stars and extending to about 60% of the tidal radius of 6.8′ (Harris 1996). The arrow is the reddening vector for E(F606W − F814W) = 0.3.

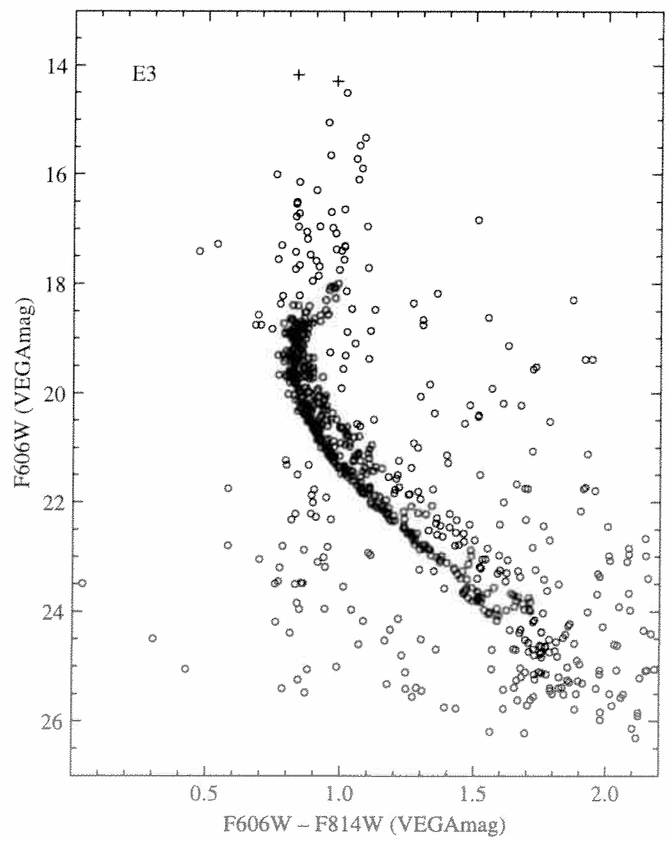
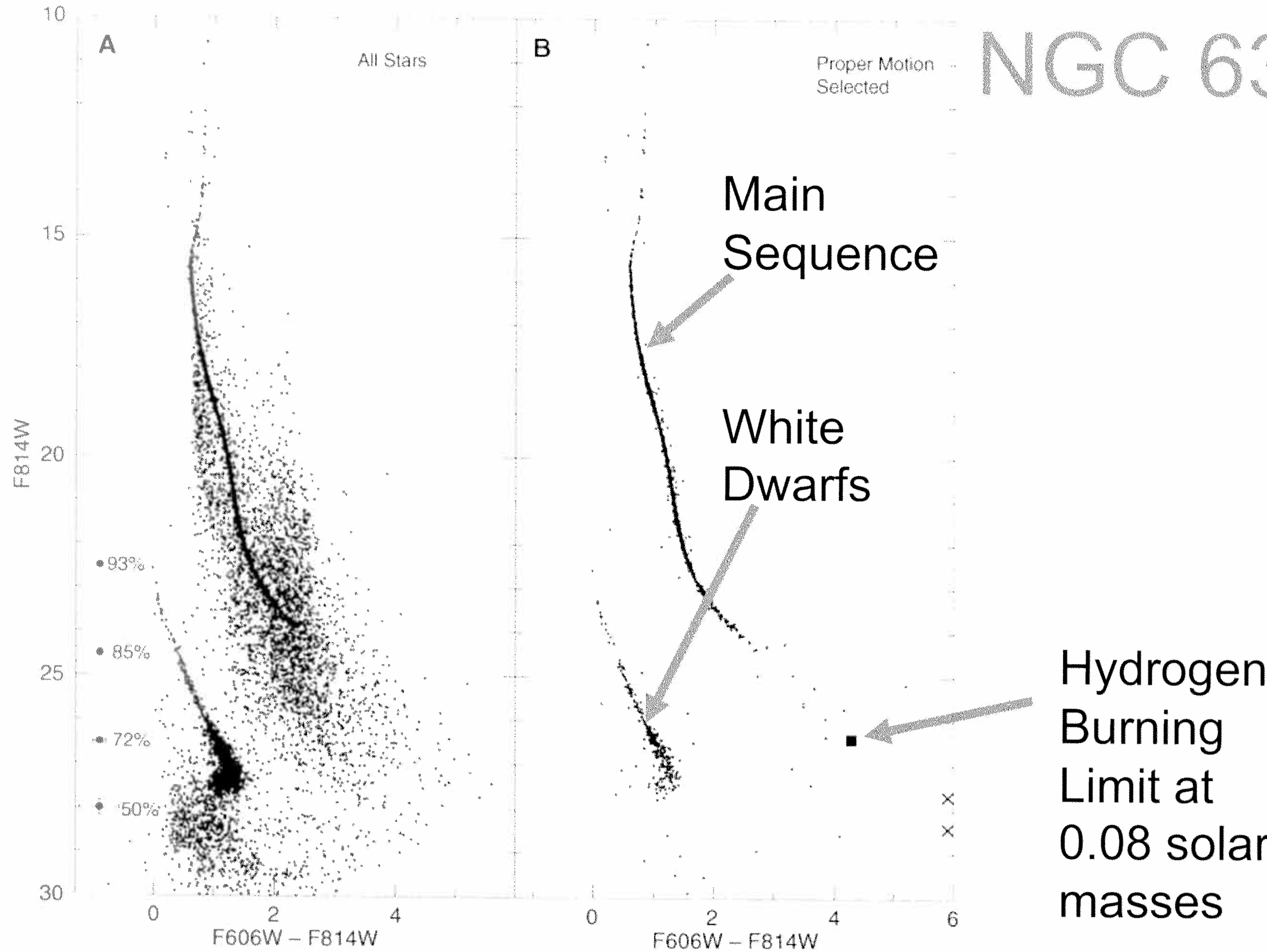


FIG. 6.—Same as Fig. 1, except that the CMD of E3 is shown, containing 852 stars and extending to about 40% of the tidal radius of 11′ (Harris 1996).

NGC 6397



Richer et al 2006, Science, 313, 936

19.9 MASS DENSITY IN THE SOLAR NEIGHBORHOOD [25–32]

Observed volume mass density

Interstellar matter (ISM)	0.04 ± 0.02	$\mathcal{M}_\odot$ pc^{-3}
Main Sequence Stars:		
$0.08 \le \mathcal{M}/\mathcal{M}_\odot < 1.0$	0.036	$\mathcal{M}_\odot$ pc^{-3}
$1.0 \le \mathcal{M}/\mathcal{M}_\odot < 100$	0.014	$\mathcal{M}_\odot$ pc^{-3}
Halo stars	0.0001	$\mathcal{M}_\odot$ pc^{-3}
Evolved stars:		
White dwarfs	0.005	$\mathcal{M}_\odot$ pc^{-3}
Dark extended halo, local density	0.01	$\mathcal{M}_\odot$ pc^{-3}
Total	0.10 ± 0.03	$\mathcal{M}_\odot$ pc^{-3}

Note that $0.01\mathcal{M}_\odot$ pc^{-3} is 0.3 Gev cm^{-3}.

Observed column mass densities, to $|z| = 1.1$ kpc

Neutral ISM	8	$\mathcal{M}_\odot$ pc^{-2}
Ionized ISM	2	$\mathcal{M}_\odot$ pc^{-2}
Molecular ISM	3	$\mathcal{M}_\odot$ pc^{-2}
ISM total	13 ± 3	$\mathcal{M}_\odot$ pc^{-2}
Stars:		
Disk main sequence	30	$\mathcal{M}_\odot$ pc^{-2}
Disk white dwarfs	3	$\mathcal{M}_\odot$ pc^{-2}
Thick disk	2	$\mathcal{M}_\odot$ pc^{-2}
Halo subdwarfs	< 1	$\mathcal{M}_\odot$ pc^{-2}
Stellar total	35 ± 5	$\mathcal{M}_\odot$ pc^{-2}
Observed total	48 ± 8	$\mathcal{M}_\odot$ pc^{-2}
Extended dark halo		
$\lvert z\rvert < 1.1$ kpc	23	$\mathcal{M}_\odot$ pc^{-2}
Total	71 ± 6	$\mathcal{M}_\odot$ pc^{-2}

K dwarfs ($z \lesssim 160$ pc) $\rho_0 = 0.10 \pm 0.03\mathcal{M}_\odot$ pc^{-3}.

All determinations are consistent with each other and with zero local unidentified matter at the $\sim 1.5\,\sigma$ level.

Dynamical analysis of the column mass density, $\mathcal{M}_\odot$ pc^{-2}

K dwarfs ($300 \lesssim z \lesssim 2000$ pc)

$\sum_{\rm tot}(z \le 1.1$ kpc$)$	$= 71 \pm 6\mathcal{M}_\odot$ pc^{-2},
$\sum_{\rm disk}$	$= 48 \pm 9\mathcal{M}_\odot$ pc^{-2},
$\sum_{\rm dark\ halo}$	$= 23\mathcal{M}_\odot$ pc^{-2},
Unidentified disk dark matter	$= 0 \pm 12\mathcal{M}_\odot$ pc^{-2}.

Solar

RMP 67 781 1995

TABLE II. Element abundances.[a]

Species	Grevesse and Noels (1993a)	Anders and Grevesse (1989)	Grevesse (1984)	Ross and Aller (1976)	Lambert and Warner (1971)
C	8.55±0.05	8.56	8.69	8.62±0.12	8.55
N	7.97±0.07	8.05	7.99	7.94±0.15	7.93
O	8.87±0.07	8.93	8.92±0.02	8.84±0.07	8.77
Ne	8.08±0.06	8.09±0.10	8.08	7.57±0.12	7.88
Na	6.33±0.03	6.31±0.03	6.33±0.03	6.28±0.15	
Mg	7.58±0.05	7.58±0.02	7.58±0.05	7.60±0.15	7.48
Al	6.47±0.07	6.48±0.02	6.47	6.52±0.12	
Si	7.55±0.05	7.55±0.02	7.55±0.05	7.65±0.08	7.55
P	5.45±0.04	5.57±0.04	5.45	5.50±0.15	
S	7.21±0.06	7.27±0.05	7.21±0.06	7.2±0.15	7.28
Cl	5.5±0.3	5.27±0.06	5.5	...	
Ar	6.52±0.10	6.56±0.10	6.65	6.0	6.6
Ca	6.36±0.02	6.34±0.03	6.36±0.02	6.35±0.10	
Ti	5.02±0.06	4.93±0.02	5.02	5.05±0.12	
Cr	5.67±0.03	5.68±0.03	5.67±0.03	5.71±0.14	
Mn	5.39±0.03	5.53±0.04	5.53±0.04	5.42±0.16	
Fe	7.50±0.04	7.51±0.01	7.51±0.01	7.50±0.08	7.56
Ni	6.25±0.04	6.25±0.02	6.25±0.02	6.28	
Z/X	0.0245	0.0267	0.0277	0.0288	...

[a]The numerical entries are the logarithms of the number abundances, normalized to $\log N = 12$ for hydrogen.

For Sun $Y_S = 0.24$

Y(interior, Primordial) $= 0.278$ (Bahcall et al),

From BBN, expect

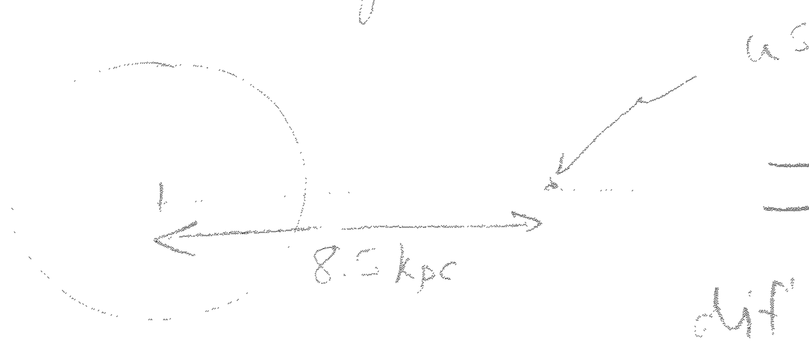
$Y \approx 0.23 \to 0.24$ (Olive et al ApJ 483, 788)

There are a few other things to note about the distribution of stars (and I refer you to Mihalas & Binney for more discussions).

Object	ρ $M_\odot / 10^3 pc^3$
O - B	0.9
A - F	4.
G - M	40.
WD's	20 ← WD's are about 1/3 of the space density.

We want to understand this

Crude Galactic Disk + Spheroid.



= 100 pc, but diff't for diff't ages.

Local ☆ Density

$$\approx 0.04 \frac{M_\odot}{pc^3}$$

Baade (1944).

Properties of the Disk Component (Pop I)

- In a thin region about the gal. plane all nearly circular orbits.
- Active Star Formation, Many Young Stars.
- Very Metal-Rich, Like $\odot$

  Z = by mass: few %

  Mostly $\frac{O}{H}, \frac{Ne}{H}, \frac{C}{H}, \frac{N}{H}$ where

  $\frac{O}{H} = 10^{-3}$, 5×10^{-4}, 4×10^{-4}, 10^{-4}

  and we will discuss this later.

  $\frac{Fe}{H} = 3\times10^{-5}$

  exponential in
  $r \approx$ few kpc
  $z \approx$ few 100 pc.

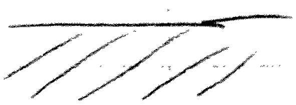
  Modest Abundance Gradient.

Properties of the Spheroidal Comp. (Pop II)

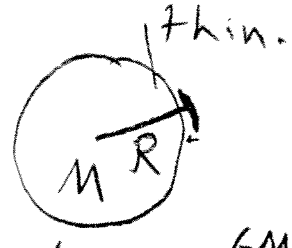
- Old population. No Yng. Stars
- Metal Poor $\rightarrow 10^{-2}$ or less of $\odot$
- kinematically have small net rotation and move on mostly radial orbits.
- Globular Clusters

Isothermal / Plane parallel atmosphere

Before launching into stars, I find it helpful to first derive the simplest possible model, that of a plane-parallel isothermal atmosphere. We start with hydrostatic balance.



$$\downarrow g \qquad \frac{dP}{dz} = -\rho g$$



and presume g = constant (in other words the atmosphere is thin $\ll R$) and T = constant. $g = \frac{GM}{R^2}$

Then

$$P = n\, k_B T = \frac{\rho}{m_\mu} k_B T$$

so

$$\frac{dP}{dz} = \frac{k_B T}{m}\frac{d\rho}{dz} = -\rho g$$

$$\Rightarrow \quad d\ln\rho = -dz\left(\frac{mg}{k_B T}\right)$$

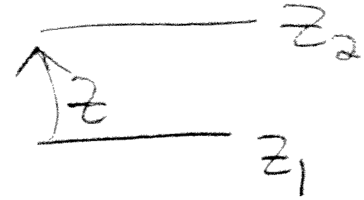
Lets define the "scale height" as this length scale

$$h = \frac{k_B T}{mg} = \text{constant for our case.}$$

11 [illegible]

Then we have

$$\int d\ln\rho = \int \frac{-dz}{h}$$



$$\ln\rho_2 - \ln\rho_1 = -\frac{1}{h}(z_2 - z_1)$$

$$\ln\frac{\rho_2}{\rho_1} = -\frac{1}{h}(z_2 - z_1)$$

$$\Rightarrow \boxed{\rho_2 = \rho_1 \exp\left[-\frac{(z_2 - z_1)}{h}\right]}$$

So lets just set $z_1 = 0$ arbitrary height, then.

$$\boxed{\rho(z) = \rho_0 \exp\left(\frac{-z}{h}\right)}$$

For our approximation to be valid, we must have $h \ll R$ so let's check what that means.

$$\frac{h}{R} = \frac{k_B T}{mg\,R} = \frac{\overbrace{k_B T}^{V_{th}^2}}{m_i \left(\frac{GM}{R}\right)}$$

$$\approx \frac{V_{th}^2}{V_{esc}^2} \ll 1 \Rightarrow$$ particles are very well bound and are deep in the potential well.

12 ~~[illegible]~~

The other interpretation of the scale height is that it is the distance the particle moves to gain (from gravity) the thermal energy as:

$$h * m_p g = \text{gravity} = k_B T$$

Alternatively, what if I ask a diff't question:

"For a gas at temp T what is the probability that a particle will be at height z?"

$$e^{-E_z/k_B T}$$

and $E_z = m_p g z \Rightarrow e^{-m_p g z / k_B T}$

$$\Rightarrow e^{-z/h}$$ (same as the statistical equil.)

The concept of a "scale height" is tremendously useful in this course and you should be sure you really understand it!

Now, the falling off of density means that ~~[illegible]~~ most of the pressure is determined locally...

The pressure at a given depth is also exponential since

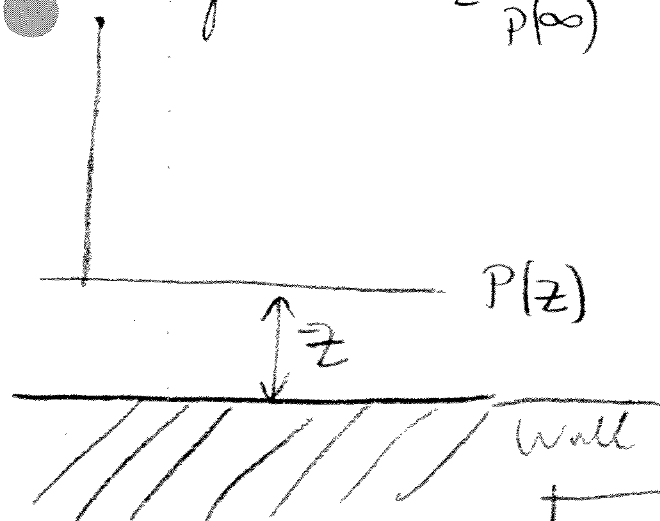
$$P = \frac{\rho}{m} k_B T$$

but we want to get a diff feel for what sets the pressure. Rewrite:

$$dP = -\rho g \, dz$$

for a real atmosphere. Now we want to integrate from $z = +\infty$ $P(\infty)$ to z to get

$$\int_z^{\infty} dP = -g \int_z^{\infty} \rho \, dz$$



but $\int_z^{\infty} dP = -P(z)$ as $P \to 0$ at ∞

so

$$\boxed{P(z) = g \int_z^{\infty} \rho \, dz'}$$

We want to define the "column density" $y(z) \equiv \int^{\infty} \rho \, dz'$ [just a definition!]

in which case

$$\boxed{P = g y}$$

The column density has units $\frac{gr}{cm^2}$ and you should think of it as the amount of stuff sitting on your head at a given altitude. How do we relate this

$$y(z) = \int_z^\infty \rho \, dz' = \int_z^\infty \rho_0 \, e^{-z'/h} \, dz'$$

$$y(z) = -\rho_0 h \left[e^{-z'/h} \Big|_z^\infty \right] = \rho_0 h e^{-z/h}$$

but

$\rho_0 e^{-z/h} = \rho(z)$ so we have:

$$y(z) = \rho(z) h$$

and hence the exponential falloff means that you get all of the column density in basically the first scale height so

$$P = g y(z) =$$

Mean Molecular Weights

$$P_{ion} = \sum n_i k_B T$$

but we write $n_i = \dfrac{X_i \rho}{A_i m_p}$

so X_i = mass fraction

$$P_{ion} = k_B T \sum \frac{X_i \rho}{A_i m_p}$$

$$= \frac{k_B T \rho}{m_p} \sum_i \frac{X_i}{A_i} = \frac{k_B T \rho}{m_p} \frac{1}{\mu_i}$$

$$P_{electron} = n_e k_B T = k_B T \left(\sum Z_i n_i \right)$$

Presume for now that all is ionized.

charge neutrality

$$P_e = k_B T \frac{\rho}{m_p} \sum_i \frac{Z_i X_i}{A_i} = \frac{k_B T \rho}{m_p} \sum \frac{Z_i X_i}{A_i}$$

$$= \frac{k_B T \rho}{m_p} \frac{1}{\mu_e}$$

so $P = \dfrac{\rho k T}{\mu m_p} = \dfrac{\rho k T}{m_p} \left(\dfrac{1}{\mu_i} + \dfrac{1}{\mu_e} \right)$

$$\frac{1}{\mu} = \sum \frac{X_i}{A_i} + \sum \frac{Z_i X_i}{A}$$

$$= \sum \frac{(1+Z_i)X_i}{A_i} = 2(0.7) + \frac{2}{4}(0.3)$$

$$\Rightarrow \mu \approx 0.64$$

You can think of this as you want to, but I like to consider it as the average weight of a particle that supplies pressure in the gas. Later, we will see that this quantity, and its evolution, plays a large and critical role in the nature of stellar evolution.

Since fusion tends to decrease the pressure support the star must continually readjust its structure so as ~~to~~ hold itself up.

Imp't!