Oxygen Burning

Coulomb barrier even higher now so we get to higher T's to do this.

$(S \sim 6 \times 10^{27}$ MeV - barns$)$ and

$$\tau = 136\, T_9^{-1/3}$$

so we have

$$\langle \sigma v \rangle \simeq 10^{11} \frac{1}{T_9} \exp\left(-\frac{136}{T_9^{1/3}}\right)$$

and makes

$$^{28}Si + \alpha$$

$$^{31}P + p$$

$$^{31}S + n$$

Where we must typically get to > few $\times 10^9$ K to beat the ν cooling

Ashes left are ^{28}Si, $^{30}Si + {}^{34}S + {}^{32}S$

The prior schematic steps were $4H \rightarrow {}^{4}He$, $3\,{}^{4}He \rightarrow {}^{12}C$

$2\,C \rightarrow {}^{24}Mg$, $2\,{}^{16}O \rightarrow {}^{32}S$.

The ashes from ^{16}O burning are primarily ^{28}Si, ^{30}Si, ... ^{32}S

Fusion of, say, ^{24}Mg or ^{28}Si with itself is very difficult, the Coulomb barrier requires, well lets check.

$^{28}Si + {}^{28}Si \rightarrow$

Presume non-resonant, then

$$E_G = (\pi \alpha Z_1 Z_2)^2\, 2 m_r c^2 \approx 500 \text{ GeV}$$

This has $(Z=12) = 267 / T_9^{1/3}$

$Z = 14 = 346 / T_9^{1/3}$

$$\cancel{\langle \sigma v \rangle = 10^{-} \left(\frac{{}^{-u}}{keV\text{-}b} \right) \tau^2 e^{-\tau}}$$

$$\cancel{\tau = 42.5 \left[\frac{Z_1^2 Z_2^2 A}{2 T_6} \right]^{1/3} = \frac{\quad}{T_9^{1/3}}}$$

$$\langle \sigma v \rangle = 4 \times 10^{-14} \frac{1}{T_9^{2/3}} \exp\left(\frac{-435}{T_9^{1/3}} \right)$$

Results for Each Stage

	$M = 15 M_\odot$		$M = 25 M_\odot$	
	$L_{ph} = 10^4 L_\odot$		$L_{ph} = 3\times10^5 L_\odot$	
Element	L_ν/L_{ph}	t (yr)	L_ν/L_{ph}	t (yr)
C	1	6000	83	170
Ne	2000	7	6.5×10^3	1.2
O	2×10^4	1.7	1.9×10^4	0.5
Si	9×10^5	0.017	3.2×10^6	0.004

~ a minute

These timescales are always longer than the dynamical time of

$$t_{dyn} \sim \frac{1}{\sqrt{G\rho}} \simeq 4\,\mathrm{sec}\,\frac{1}{\rho_6^{1/2}}$$

but are shorter than the charact. thermal time for the whole star. I have not discussed it all that much, but will later. The shell structure is building up in time & eventually we reach the final stage $\boxed{^{28}Si \text{ photodisint.}}$

Up to now we have mostly fused nuclei to get to higher Z. Now we will see an unusual chain of events in a pure Si gas due to other physics winning when the Coulomb barrier is too high

What begins to occur in earnest is nuclear photodisintegration, where

$$\gamma + {}^{28}Si \to {}^{24}Mg + {}^{4}He.$$

Remember that $kT = E_\gamma \approx 170$ keV at $T = 2\times10^9$

So, in the beginning, just the few out on the tail of the distribution matter, but they serve a great function, which is to bypass the large $28+28$ coulomb barrier by liberating low Z nuclei from the nuclei, so the chain is

$$\gamma + {}^{28}Si \to {}^{24}Mg + \alpha$$

followed immediately by

$$\alpha + {}^{28}Si \rightleftharpoons {}^{32}S + \gamma$$

$$\alpha + {}^{32}S \rightleftharpoons {}^{36}Ar + \gamma$$

$$\alpha + {}^{36}Ar \rightleftharpoons {}^{40}Ca + \gamma$$

.

.

.

$$\alpha + {}^{52}Fe \rightleftharpoons {}^{56}Ni + \gamma$$

which gets us all the way up to the iron group elements.

An impt. point to note is that when there is no time for β-decays one goes straight to α captures $\Rightarrow$ ^{56}Ni. However, if there is time allowed for some weak interaction, they tend to drive towards ^{56}Fe as the final element.

Just to see at what T these reaction occur, let's find

$$^{28}Si + \alpha \rightleftharpoons {}^{32}S + \gamma$$

$$\mu_{28} + \mu_4 = \mu_{32}$$

Just to see at what T these reactions occur, lets find

$$^{28}Si + \alpha \rightleftharpoons {}^{32}S + \gamma$$

$$\mu_{28} + \mu_4 = \mu_{32}$$

$$m_{28}c^2 + m_4 c^2 - kT \ln\left[\frac{n_{Q,28}}{n_{28}} \frac{n_{Q,4}}{n_4}\right]$$

$$= m_{32}c^2 - kT \ln\left(\frac{n_{Q,32}}{n_{32}}\right)$$

Define $Q = (m_{28} + m_4 - m_{32})c^2 = 6.95\,MeV$, then

$$\frac{n_{28}\, n_4}{n_{Q,28}\, n_{Q,4}} \cdot \frac{n_{Q,32}}{n_{32}} = \exp\left(\frac{-Q}{kT}\right)$$

$$\Rightarrow \frac{n_{28}\, n_4}{n_{32}} = \left[\frac{2\pi\, 4 m_p kT}{h^2}\right]^{3/2} \left[\frac{28 m_p}{32 m_p}\right]^{3/2} \exp\left(\frac{-Q}{kT}\right)$$

$$\frac{n_{28}}{n_{32}} = \frac{4.35\times10^{35}}{n_4} \exp\left(\frac{-Q}{kT}\right)$$

$$= \frac{4.35\times10^{35}}{n_4} \exp\left(\frac{-16}{(T/5\times10^9 K)}\right)$$

$\rho_{burn} = 3\times10^7\ g\,cm^{-3}$ $\quad n_4 = 4.5\times10^{30}\ cm^{-3}$.
Lets just see what would happen if $n_4 \approx 10^{28}$, then we get:

$$\frac{n_{28}}{n_{32}} \approx \frac{4\times10^{35}}{10^{28}} \exp\left(\frac{-16}{T/5\times10^9}\right) = 4.5 \quad \text{at } T = 5\times10^9$$

So, at Temperatures of this order even a very low α abundance can begin to meld the nuclei. The most bound nuclei are eventually reached but SAHA tells us that there will always be a range of nuclei around with a peak around ^{56}Fe. The burning has a long enough time to have β-decays act to get less ^{56}Fe.

We will find future that in more rapidly evolving systems there will be no β-decays and the system makes pure ^{56}Ni.

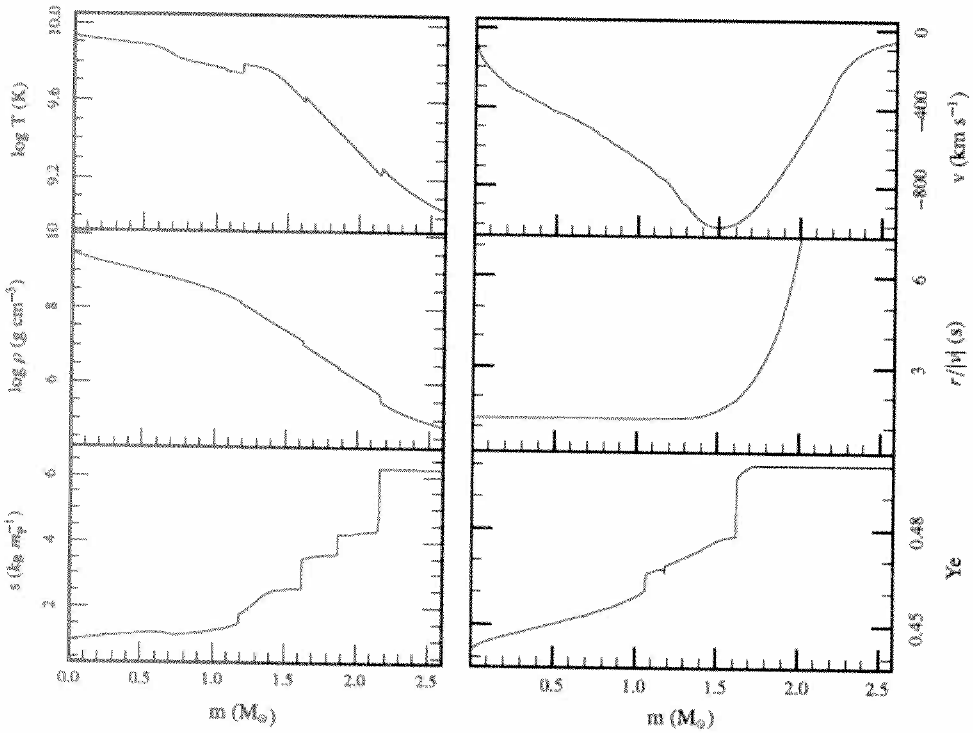
$\Downarrow$

Supernovae for example

This is the last stage of nuclear energy release, in which case the ν energy loss leads to further collapse of the core that cannot be halted by any intervening physics. However if the degenerate e^- were non-relativistic, we could form cores of degen. objects that would hold themselves up. Interes-

I showed a while back that a star in HB with relativistic particles had a $E_{tot} = 0$. This is what happens to the Fe cores, which is that the e^- become degenerate. Rather than carefully derive all of it, let's just walk through it:

Chandra Mass



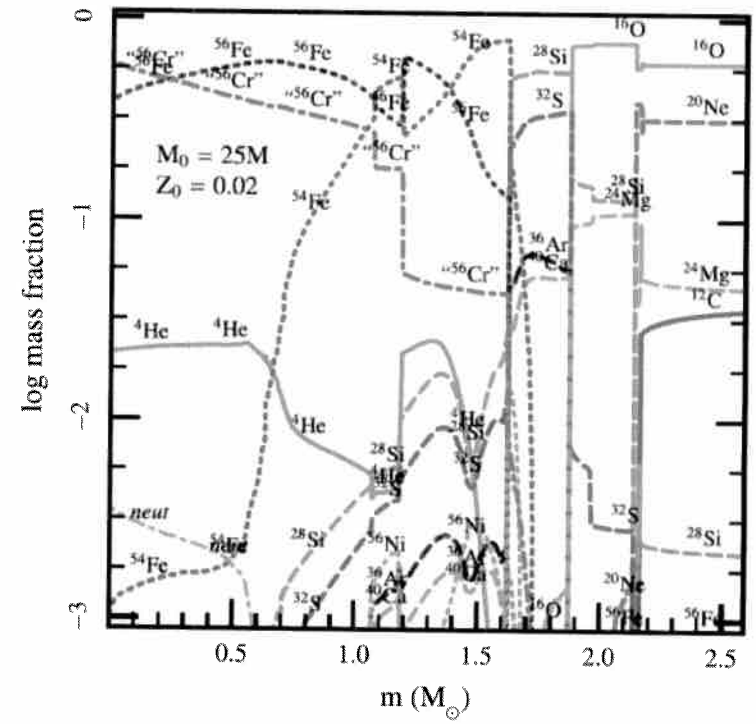


Figure 31. Mass fraction profiles of the inner 2.5 $M_\odot$ of the solar metallicity $M_i = 25\,M_\odot$ model at the onset of core collapse. The reaction network includes links between ^{54}Fe, ^{56}Cr, neutrons, and protons to model aspects of photodisintegration and neutronization.

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THE EVOLUTION AND EXPLOSION OF MASSIVE STARS. II. EXPLOSIVE HYDRODYNAMICS AND NUCLEOSYNTHESIS

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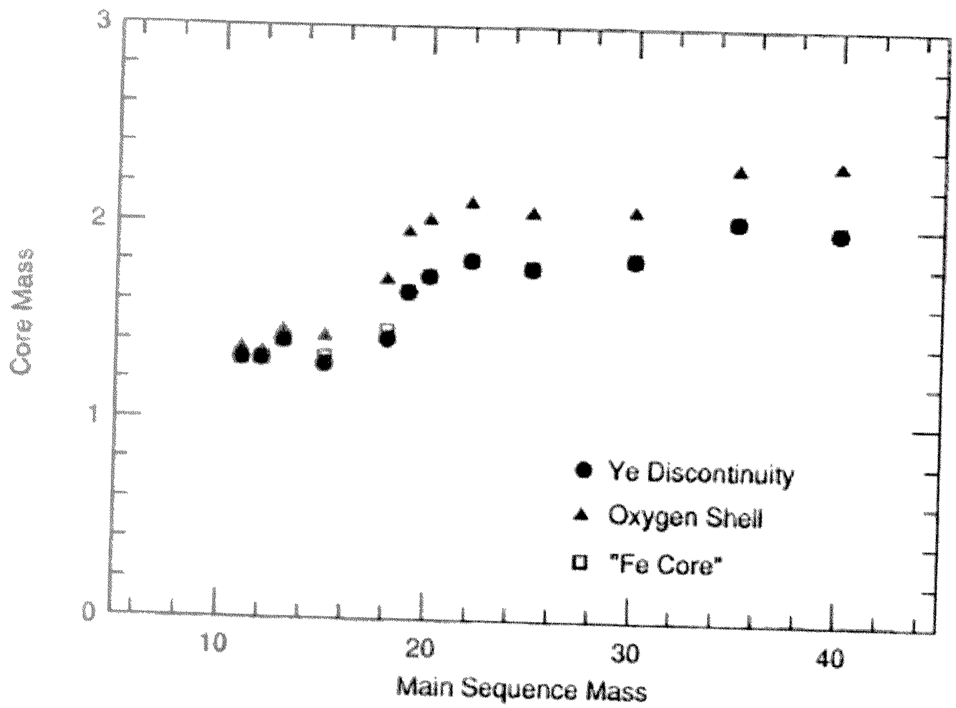
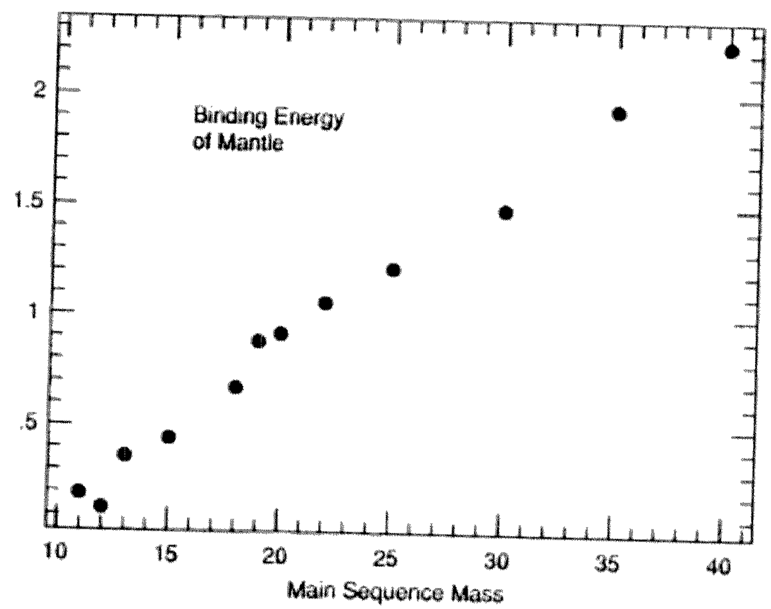


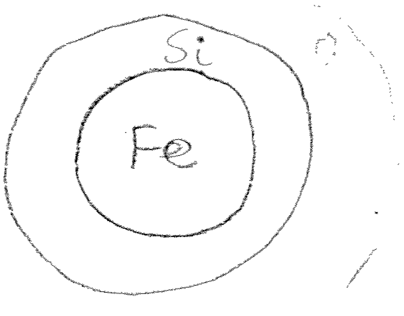


;. 5.—Net binding energy external to the piston mass point (Table
tars of solar metallity.

~~Lect 24~~ Lect 23 252 Notes 284 ~~248~~

Stellar Core Collapse

A massive star evolves to an onion shell. As time passes the mass of the Fe core grows until eventually it reaches the Chandrasekhar mass. There are slight corrections to M_{ch} ~~some because~~ due to Coulomb effects and high T. The collapse begins as the e^- become relativistic. Lets see what happens.



Energy Sinks

Eventually during the contraction the temp gets high enough for breaking up the ^{56}Fe. This will cost energy & thus continue to collapse. Lets imagine

$$\gamma + {}^{56}Fe \rightleftarrows 13\alpha + 4n$$

$$Q = (13 m_4 + 4 m_n - m_{56}) c^2 = 124.4 \text{ MeV}$$

$\Rightarrow$

$$\mu_{56} = 13\mu_4 + 4\mu_1$$

$$\frac{n_4^{13}\, n_1^{4}}{n_{56}} = \frac{g_4^{13}\, g_1^{4}}{g_{56}} \frac{n_{Q4}^{13}\, n_{Q1}^{4}}{n_{Q56}} \exp\left(\frac{-Q}{kT}\right)$$

but $g_4 = 1$, $g_1 = 2$, $g_{Fe} \approx 1.4$ (ground state has $J=0$ plus a bit of the low-lying states)
so

$$\frac{n_4^{13} n_1^4}{n_{56}} = \frac{16}{1.4} \frac{n_{Q4}^{13} n_{Q1}^4}{n_{Q,56}} \exp\left(\frac{-Q}{kT}\right)$$

Now lets find the place where 1/2 of the iron is compl. broken down, so

$$n_{56} = \frac{1}{2}\left(\frac{\rho}{56 m_p}\right) ; \quad n_4 = \frac{13}{2}\left(\frac{\rho}{56 m_p}\right)$$

$$n_1 = \frac{4}{2}\left(\frac{\rho}{56 m_p}\right)$$

so $\rho = \frac{1}{2}\rho + \frac{2 \cdot 13}{56}\rho + \frac{2}{56}\rho = \rho$.

So we get

$$\frac{n_4^{13} n_1^4}{n_{56}} = \frac{16}{1.4}\left[\frac{2\pi m_4 kT}{h^2}\right]^{13 \cdot \frac{3}{2}}\left[\frac{2\pi m_1 kT}{h^2}\right]^{\frac{3}{2} \cdot 4}\left[\frac{h^2}{2\pi m_{56} kT}\right]^{3/2} \exp\left(\frac{-144}{T_{10}}\right)$$

$$\frac{9.5\times10^{413}\rho_9^{13}\; 2.1\times10^{125}\rho_9^4}{5.4\times10^{30}\rho_9} = \frac{2.3\times10^{471}T_{10}^{39/2} \cdot 1.2\times10^{141}T_{10}^6}{7.9\times10^{37}T_{10}^{3/2}} \exp\left(\quad\right)$$

So we get

$$\rho_9^{16} = 9.5\times10^{-2}\, 10^{67}\, T_{10}^{24} \exp\left(-\frac{144}{T_{10}}\right)$$

or $$16 \ln \rho_9 = 151.9 + 24 \ln T_{10} - \frac{144}{T_{10}}$$

$$\frac{144}{T_{10}} = 151.9 + 24 \ln T_{10} - 16 \ln \rho_9$$

$$T_{10} = \frac{144}{151.9 + 24 \ln T_{10} - 16 \ln \rho_9}$$

$$T_{10} = \frac{9}{9.49 + 1.5 \ln T_{10} - \ln \rho_9}$$

ρ_9	T_{10}
0.1	0.78
1	0.95
10	1.20

So we get 1/2 ionization around a T of 10^{10} K. This then boils the Iron back down to α particles costing about 2 MeV/nucleon. Soon thereafter, the ^4He dissociate as well costing another 7 MeV / nucleon.

Cooling of this magnitude must come at the expense of gravitational contraction leading to collapse to even smaller radii.

This would be (at least, maybe more if $E_{tot} \simeq 0$)

$$\frac{GM}{R} \sim \frac{9\,\text{MeV}}{m_p}$$ or

$$R \sim \frac{GMm_p}{9\,\text{MeV}} \lesssim 200\ \text{km}$$

as if ~~degenerate~~ relativistic particles held it up then E_{tot} is $<< \frac{GM}{R}$ so small losses can lead ~~to~~ large contractions.

typically only need to break up ~$\frac{1}{4}$ of Iron to $T \to 0$

Collapse must occur as the processes of photodissociation cost the internal energy of the star.

The density at this point is

$$\rho \approx \frac{1.4 M_\odot}{\frac{4\pi}{3} R^3} \simeq 10^{11} \frac{g}{cm^3}$$

in which case the e^- Fermi energy is $>> m_e c^2$ and we start having one other event:

$\boxed{\text{Electron Capture}}$

$$e^- + p \to n + \nu_e$$

This really starts to cause trouble, especially once $E_F > (m_n - m_p)c^2 = 1.3\,\text{MeV}$

There are two imp't repercussions:

1) ν_e leaving are typically a few MeV → coolant

2) Electrons are what supplies pressure, so

$$M_{ch} = 1.46\, M_\odot \left[\frac{Y_e}{1/2}\right]^2$$

where Y_e = # e^- per baryon which for pure ^4He is

$\frac{1}{2}$ while for ^{56}Fe

it is $\frac{n_e}{n_b} = \frac{Z}{A} = \frac{26}{56} = 0.464$

or

$$M_{ch} \approx 1.26\, M_\odot \text{ for}$$

pure Iron

As e^- captures occur M_{ch} increases, so that eventually we get

$$M_{core} > M_{ch}$$

and then eventually we run into real trouble, and a dynamical collapse starts!

I will not go into details of the collapse right now.

This can also just occur on the nuclei as well.

Collapse goes in 0.1 sec.

All of these are just details about how the collapse proceeds. All that is impt to remember is that there is no available fuel to give an explosion so that the collapse continues until

⇒ (1) Pressure support from a degenerate neutron gas becomes impt. What can we hold up with this?

$$P = \frac{2}{5} n_n E_F \quad \& \quad n_n = \frac{8\pi}{3h^3} P_f^3$$

$$P = \frac{2}{5} n_n \frac{P_f^2}{2m_n} = \frac{2}{5} n_n \frac{1}{2m_n}\left(\frac{3h^3 n_n}{8\pi}\right)^{2/3}$$

$$n_n = \rho/m_n$$

Now, at ~~???~~ $E_{F_n} = 0.3\,MeV \left(\frac{\rho}{10^{11}}\right)^{2/3}$

$$E_F = \frac{P_f^2}{2m_n} = \frac{1}{2m_n}\left(\frac{3h^3 n_n}{8\pi}\right)^{2/3} = 308\ keV \left(\frac{\rho}{10^{11}}\right)^{2/3}$$

for pure neutrons. So

$$P = 10^{28}\left(\frac{\rho}{10^{11}}\right)^{5/3} \frac{erg}{cm^3}$$

So ... usual, lets see what this can hold up. Again

$$P = \frac{GM^2}{}$$

So we get $\frac{dp}{dz} = \rho g \Rightarrow P = R\rho \frac{GM}{R^2}$

$= P = \rho \frac{GM}{R}$ but $\rho = \frac{3M}{4\pi R^3}$

$R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$

so $P = GM\rho\left(\frac{4\pi\rho}{3M}\right)^{1/3} \approx 8\times10^{29}\left(\frac{M}{M_\odot}\right)^{2/3}\left(\frac{\rho}{10^{11}}\right)^{4/3}$

equate with the neutron pressure

$$10^{28}\rho_{11}^{5/3} = 8\times10^{29} M^{2/3}\rho_{11}^{4/3}$$

$$\Rightarrow \rho_{11}^{1/3} = 80 M^{2/3}$$

Putting in all of the factors carefully gives.

$$\Rightarrow \boxed{\rho_c = 8\times10^{14}\frac{g}{cm^3}\left(\frac{M}{1.0M_\odot}\right)^2}$$

So, degenerate neutrons can hold it up, but at 10^{15} they have $E_F = 140$ MeV and are beginning to become relativistic. The Radius of these objects is

$$R = 14.6\,km\left(\frac{10^{15}}{\rho_c}\right)^{1/6}$$

$$= 14.6\,km\left(\frac{10^{15}}{8\times10^{14}}\right)^{1/6}\left(\frac{1.0M_\odot}{M}\right)^{1/3} = 15\,km\left(\frac{1M_\odot}{M}\right)^{1/3}$$

These densities also get up to nuclear values as the average neutron separation is.

$$n_n = \frac{3}{4\pi a^3} \Rightarrow a = \left(\frac{3}{4\pi n}\right)^{1/3}$$

$$\Rightarrow a \approx 7\times10^{-14}\,\text{cm}\left(\frac{10^{15}}{\rho}\right)^{1/3}$$ for pure neutrons.

$\boxed{\text{Supernovae}}$

Once at ρ_{nuc}, the core becomes tough to compress any further and thus "bounces". This has only been the inner 1.4 $M_\odot$ of the star, so the rest collapses down onto it. The NS is hot & dense enough so as to become optically thick to ν, when:

$$\tau = \frac{2\times10^{33}}{10^{-24}\cdot10^{12}} \cdot = \frac{2\times10^{33}}{10^{-12}} = 2\times10^{45}\cdot10^{-44}\left(\frac{E_\nu}{MeV}\right)$$

>1 when $R\sim10^6$ and E_ν is a few MeV. So the NS is emitting neutrinos as a Black body.

The contraction of an incompressible core results in an outgoing shock wave through the mantle of the rest of the star, basically blowing it away & unbinding the outer parts of the star.

The collapsed core event. stiffens and halts, at which point the infalling matter strikes it & moves back out. The gravitational binding energy

$$E_{Gr} = \frac{GM^2}{R} \simeq 10^{53} \text{ ergs}$$

is mostly released in the optic. thick $\bar{\nu}$ cooling phase that lasts for 10's of seconds.

The observed K.E. of the ~~The~~ supernova is 10^{51} ergs ($\sim 10^{49}$ in light)

Shock Mechanisms

The outward moving shock has $\simeq 10^{52}$ ergs of K.E. and as it moves outward is "slowed" by the losses associated with dissociating iron still left around.

Whether or not the shock stalls has been the arbiter of what one can expect.

Collapses to NS and BH

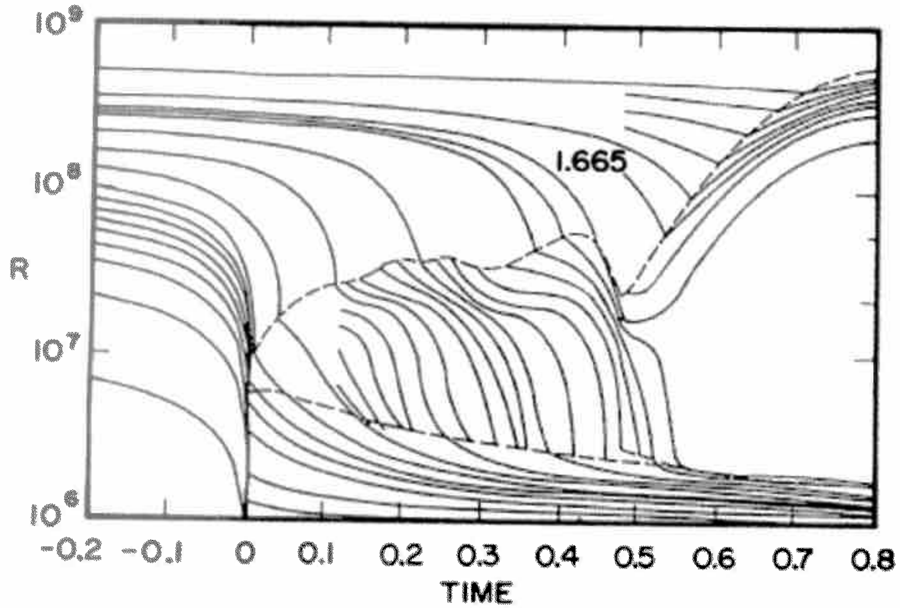


FIG. 14. Trajectories of various mass points, in an early calcu lation of J. R. Wilson (1985). Time after bounce is in seconds Mass 1.665 $M_\odot$ is the first mass point which is propelled out ward by the second shock, which is due to neutrino heating The empty region on the right is the "bubble," filled by elec tromagnetic radiation. The upper dashed curve is the shock the lower one the neutrino sphere.

O'CONNOR & OTT

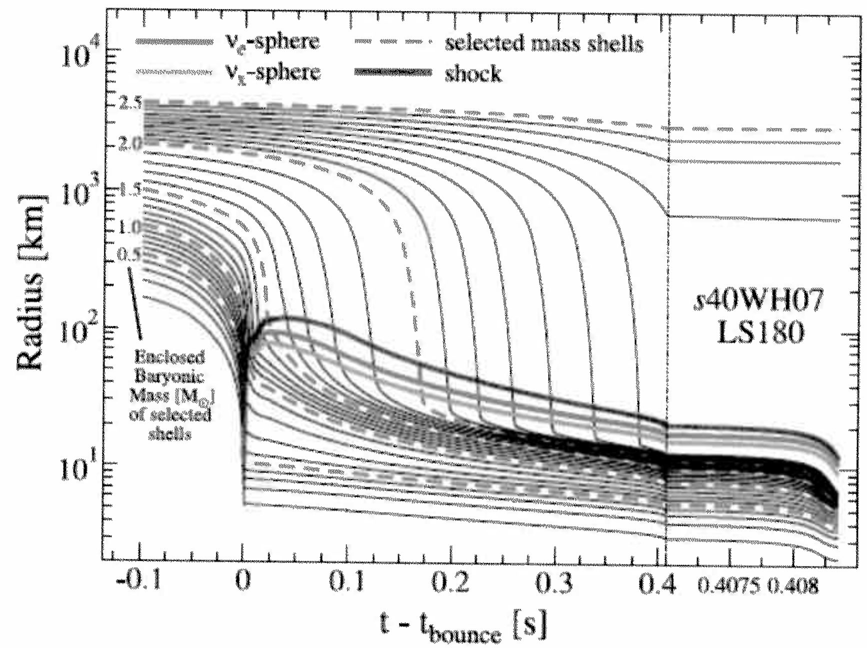


Figure 3. Evolution of baryonic mass shells in the nonrotating model *s*40WH07 evolved with the LS180 EOS. We also include the shock location and the radii of the ν_e and ν_x neutrinospheres. The $\bar{\nu}_e$-sphere (not shown) is inside, but very close to the ν_e-sphere. The vertical dotted line denotes a change of timescale in the plot, highlighting the final ~1 ms of evolution before the central density reaches ~4.2×10^{15} g cm^{-3} and the simulation halts. We specifically highlight the 0.5, 1.0, 1.5, 2.0, and 2.5 $M_\odot$ baryonic mass shells with dashed lines. With solid lines, for $M < 2\,M_\odot$, we plot every 0.1 $M_\odot$ mass shell. Above 2 $M_\odot$, we plot mass shells with a spacing of 0.05 $M_\odot$.

[I recommend Bethe 1990, RMP 62, 801]

1) Prompt (Direct) Shock:

This scenario is that the shock which starts in the collapsing iron core will move all the way through the star, having enough energy to diss. whatever is left of Iron

If $K.E.|_{shock} = 10^{52}$,

then $\left(\frac{9\,MeV}{m_p}\right) M_{Fe} = K.E.$

$$\Rightarrow M_{Fe} \sim 0.6\, M_\odot$$

$\rightarrow p, n$

so the "prompt" mechanism is most likely ~~to~~ work at low initial Fe core masses. For large ($> 1.33\, M_\odot$) cores the shock stalls.

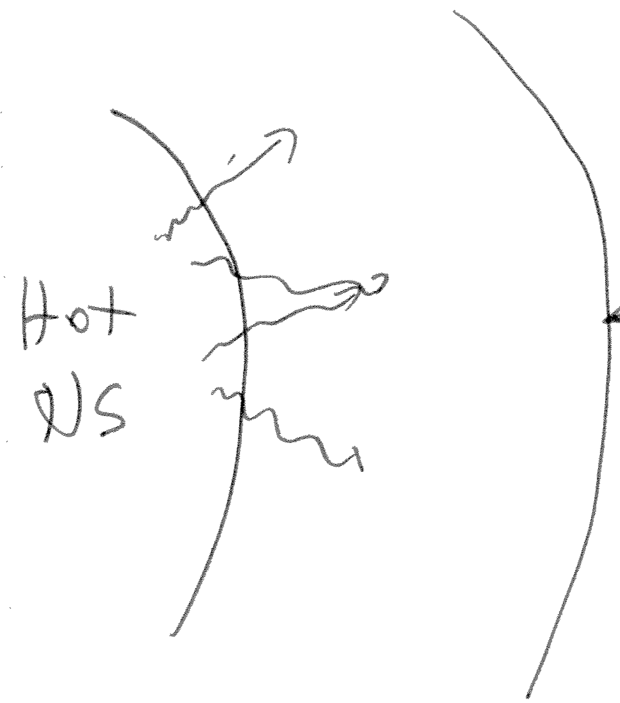
2) Delayed or ν - Heated.

The shock stalls (for massive Fe cores for sure) at a radius around

$\sim (2-8) \times 10^7$ cm due to

dis. losses. However the other idea was to "revive" the shock by ν interaction from the Hot proto-neutron star.

Most older 1D models had a tough time reviving the shock with ν inter- though



stalled Shock

One needs to keep in mind that all we need ~ 1/100 of the ν-energy.

As usual, most recent progress has been in relaxing the 1D assumption and allowing for convection. This gives a simple picture of heating via ν's the bubble at the bottom and allowing it to buoyantly rise upward, trans.