Evolution of $\geq 6 M_\odot$ Stars

Carbon Ignition

The next fuel available is

$$^{12}C + {}^{12}C \to {}^{24}Mg$$

$$\hookrightarrow \alpha + {}^{20}Ne$$

$$\hookrightarrow p + {}^{23}Na$$

$$\hookrightarrow n + {}^{23}Mg$$

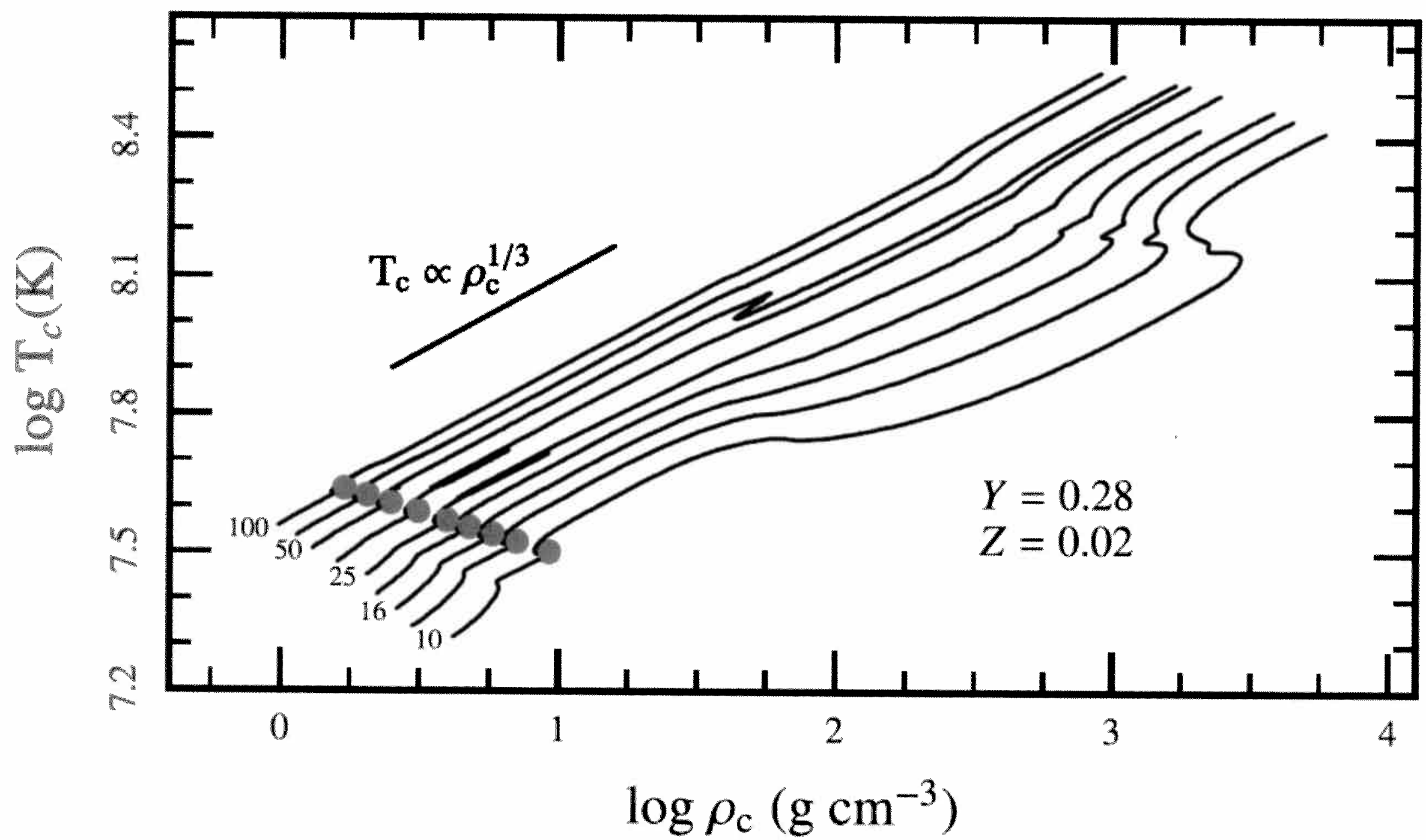
The large Coulomb barrier tells us that the star must be of order 10^9 K or very degenerate to undergo ignition. The competition at these T & ρ is with neutrino cooling.

The neutrinos are very optically thin, so that they can escape freely from the star. In this case the evolution is very accelerated, as the ν's are setting the relevant timescale, not the ~~radi~~ conductive luminosity. In this case we have

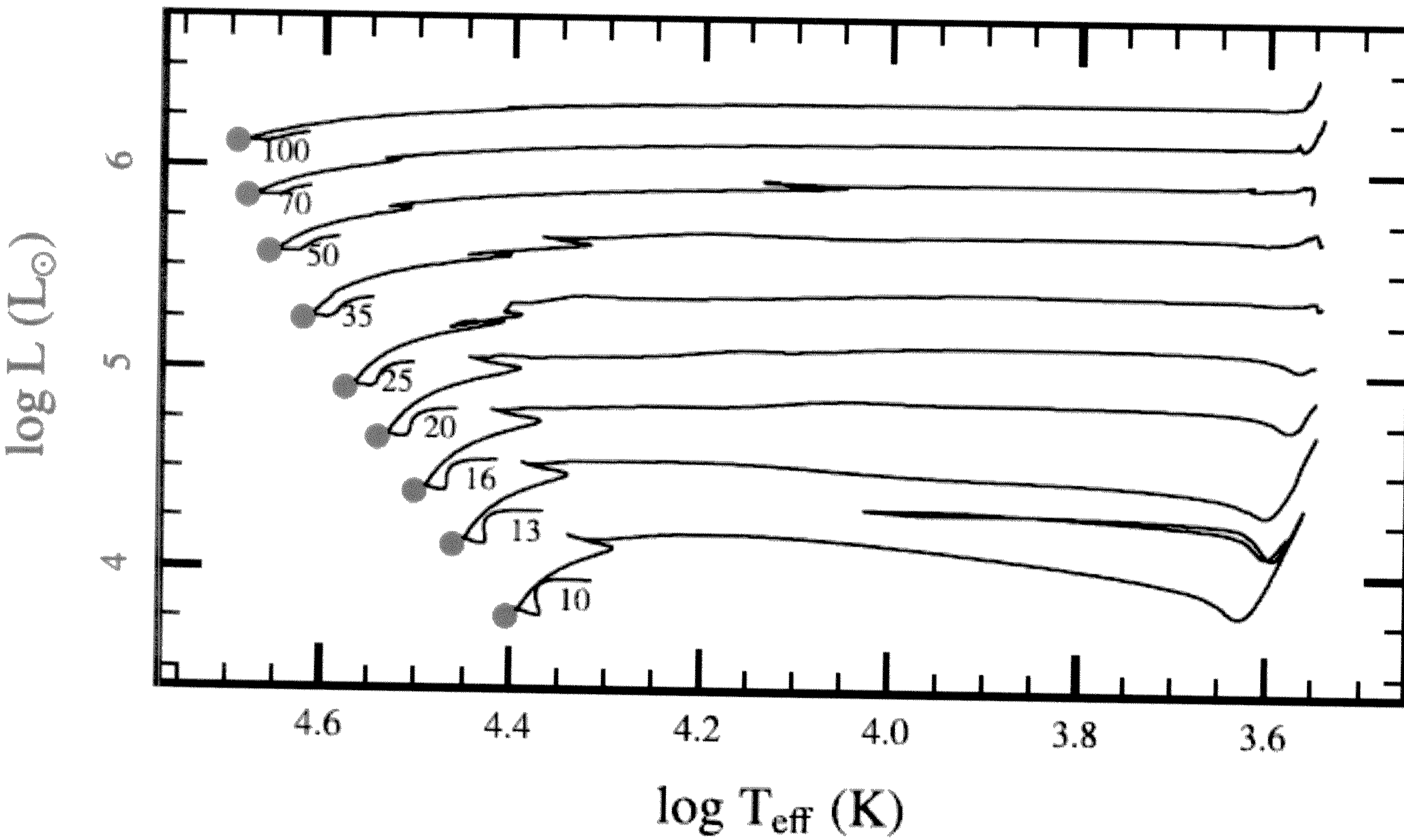
$$T\frac{ds}{dt} = -\varepsilon_\nu + \varepsilon_{cc} - \frac{1}{\rho}\underline{\nabla}\cdot F$$

so that $\varepsilon_\nu \gg \frac{1}{\rho}\underline{\nabla}\cdot \underline{F}$ and we learn that ignition is really defined by

$$\varepsilon_{cc} > \varepsilon_\nu .$$



$T_c \propto \rho_c^{1/3}$
100
50
25
16
10
$Y = 0.28$
$Z = 0.02$
log T_c(K)
7.2
7.5
7.8
8.1
8.4
0
1
2
3
4
log ρ_c (g cm^{-3})


100
70
50
35
25
20
16
13
10
6
5
4
log L ($L_\odot$)
4.6
4.4
4.2
4.0
3.8
3.6
log T_{eff} (K)

Neutrino Processes

The typical ν-cross section is

$$\sigma_\nu \sim 10^{-44} \left(\frac{E_\nu}{MeV}\right)^2 cm^2$$

and a star must have a radius

$$\frac{M}{m_p R^2}\sigma_\nu = 1$$

to be optically thick. This gives:

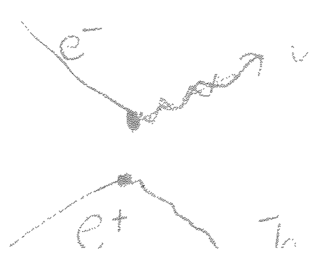
$$R \simeq 3 \times 10^6 \left(\frac{E_\nu}{MeV}\right) cm$$

or about 30 km when $E_\nu \sim$ MeV. As we will see much later, this is indeed relevant for collapsing NS's.

Production Processes

(1) $e^+ + e^- \to \nu_e + \bar{\nu}_e$

In gases with a high enough T to have a large equilibrium abundance of e^+e^- pairs



$$\frac{e^+ + e^- \to \nu + \bar{\nu}}{e^+ + e^- \to \gamma + \gamma} \simeq 10^{-19}$$

Th

At high T's there is an equilibrium abundance of $e^{\pm}$ pairs. Lets presume that production and annih. are in balance.

Prod is $\gamma \rightarrow e^+e^-$ via bremsst.

and $e^+e^- \rightarrow 2\gamma$ is annihilation. Then if they are non degenerate we have

$$\mu_{e^-} + \mu_{e^+} \rightleftharpoons \gamma \Rightarrow \mu_{e^-} = -\mu_{e^+}$$

or

and $$\mu_e = m_e c^2 - kT \ln\left[\frac{g_e n_{e,Q}}{n_e}\right]$$

$$\mu_{e^+} = m_e c^2 - kT \ln\left(\frac{g_e n_{e,Q}}{n_{e^+}}\right)$$

so $$n_{e,Q} = \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2}$$

$$2 m_e c^2 - kT \ln\left(\frac{g_e n_{e,Q}}{n_e}\right) = +kT \ln\left(\frac{g_e n_{e,Q}}{n_{e^+}}\right)$$

so $$\exp\left(\frac{2 m_e c^2}{kT}\right) = \frac{g_e^2 n_{e,Q}^2}{n_e n_{e^+}}$$

$$n_{e^+} = g_e^2 n_{e,Q}^2 \frac{1}{n_e} \exp\left[\frac{-2 m_e c^2}{kT}\right]$$

so

$$n_{e^+} = 4\left[\frac{2\pi m_e kT}{h^2}\right]^3 \frac{1}{n_e} \exp\left[\frac{-2m_e c^2}{kT}\right]$$

so we get that at $\rho_c = 10^4$, $n_e = 3\times10^{27}$ so we have:

$$n_{e^+} = 7.7\times10^{30}\left(\frac{10^4}{\rho}\right) T_9^3 \exp\left(-\frac{11.84}{T_9}\right)$$

so I get at $T = 10^9$ $n_{e^+} = n_e/100$, or just

T_9	n_{e^+}/n_e
1	0.02
2	>1

, so we must recalculate n_e as at very high T the e^+e^- pairs far outnumber those around due to the intrinsic density. In other words, lets find when $n_{e^+} = n_e$, this is just

$$3\times10^{27} = 7.7\times10^{30}\left(\frac{1}{\rho_4}\right)^2 T_9^3 \exp\left(\frac{-11.84}{T_9}\right)$$

or just

$$\rho_4^2 = 2566\, T_9^3 \exp\left(\frac{-11.84}{T_9}\right)$$

$$2\ln\rho_4 = 7.85 + 3\ln T_9 - \frac{11.84}{T_9}$$

So

$$\frac{11.84}{T_9} = 7.85 + 3\ln T_9 - 2\ln \xi_4$$

$$T_9 = \frac{11.84}{7.85 + 3\ln T_9 - 2\ln \xi_4}$$

$$\boxed{T_9 = \frac{1.51}{1 + 0.38\ln T_9 - 0.25\ln \xi_4}}$$

$\Rightarrow \quad T_9 = 1.35$

So above T_9 roughly, we expect a large population of e^+ around.

Once we have the pair density, we can calculate the volume rate for neutrino emission. Lets presume we are in the limit where e^- are dominated by pair production. Then

$$n_{e^+} n_{e^-} = 4\left[\frac{2\pi m_e k T}{h^2}\right]^3 \exp\left[\frac{-2 m_e c^2}{kT}\right]$$

And the rate of ν production is and energy loss is

$$\frac{dU_\nu}{dt} = n_{e^+} n_{e^-} \langle \sigma v \rangle \underset{\text{energy}}{\underset{\uparrow}{W}} = \mathrm{cm}^{-6}\,\frac{\mathrm{cm}^3}{\mathrm{s}}\,\mathrm{erg} = \mathrm{erg}/\mathrm{cm}^3\cdot\mathrm{sec}$$

The reaction cross section scales as

$$\sigma = 1.4\times10^{-45}\,\frac{c}{v}\left(w^2-1\right)\ \mathrm{cm}^2$$

$$\approx 10^{-21}\,\sigma_T$$

w = Ecom incl rest mass = 2 ⇒ 4 − 1 = 3

so

$$\langle\sigma v\rangle \approx 4.2\times10^{-45}\,c$$

and the energy released is $2m_ec^2$ so we get

$$\frac{du_\nu}{dt} = 4\left(\frac{2\pi m_e kT}{h^2}\right)^3\left(4.2\times10^{-45}c\right)\left(2m_ec^2\right)\exp\left(\ \right)$$

$$= 4.8\times10^{18}\ T_9^{\,3}\exp\left(-\frac{11.84}{T_9}\right)\ \frac{\mathrm{erg}}{\mathrm{cm}^3\cdot\mathrm{sec}}$$

Note that this is indep. of density, which is because the $n_{e^+}n_{e^-}$ product is only dep on T

Now lets get a timescale ~~to~~ cool! At a density of 10^4 gr/cm^3 we have

$$E_{th} \approx nKT = 7\times10^{19}\ \frac{\mathrm{erg}}{\mathrm{cm}^3}\ T_9\,S_4$$

so

$$t_c = \frac{7\times10^{19}\,T_9\,S_4}{5\times10^{18}\,T_9^{\,3}}\exp\left(\frac{11.84}{T_9}\right)$$

$$t_c = 14 \text{ sec } \frac{\beta_4}{T_9^2} \exp\left[\frac{+11.84}{T_9}\right]$$

or just

T_9	t_c
1	0.74 month
2	0.36 hrs
3	
4	
0.62	163 yrs

So the ν cooling rather soon becomes the primary way in which the star is cooling & contracting. This leads to a very rapid stage of stellar evolution.

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(2) There is one other imp't neutrino process, namely

"Plasmon Decay"

A photon in a plasma obeys a dif't dispersion relation, namely

$$w^2 = (ck)^2 + \omega_0^2$$

where

$$\omega_0^2 = \frac{4\pi n_e e^2}{m_e} = \text{plasma frequency.}$$

Now, at high densities, $\hbar\omega_0 \sim kT$ and photons in the gas become very modified by the plasma. In this limit we can view the photons as

$$\hbar^2 \omega^2 = \hbar^2 (ck)^2 + (\hbar\omega_0)^2$$

or

$$E^2 = p^2 c^2 + m^2 c^4$$

(i.e. like a particle with rest mass).

The "excess" energy, or m^2c^4 allows the plasmon to decay into two 2 ν's as now one can imagine "boosting" into the "plasmon" rest frame, where the excess energy goes into

$$\bar{\nu}_e \longleftarrow \bullet \longrightarrow \nu$$

the neutrino momentum.

$\Rightarrow$ Hence, the plasmon density is known from the B.F. statistics, and we only need to know the prob/ unit time for the decay process. This is known from weak interaction theory and I will not go into detail here.

However, it is imp't to note that this happens for all densities, but as the ν_e, $\bar{\nu}_e$ P d I and phase space as m_p are only imp't

Lecture 22

So, I left off last time by having shown that the onset of ν cooling greatly accelerates stellar evolution, and the characteristic timescale to cool the gas becomes

$$t_{cool} \approx \frac{\frac{3}{2} n k_B T}{d\varepsilon_\nu / dt} = 14 \sec \frac{S_4}{T_9^2} \exp\left[\frac{11.89}{T_9}\right]$$

or just a nuclear lifetime set by

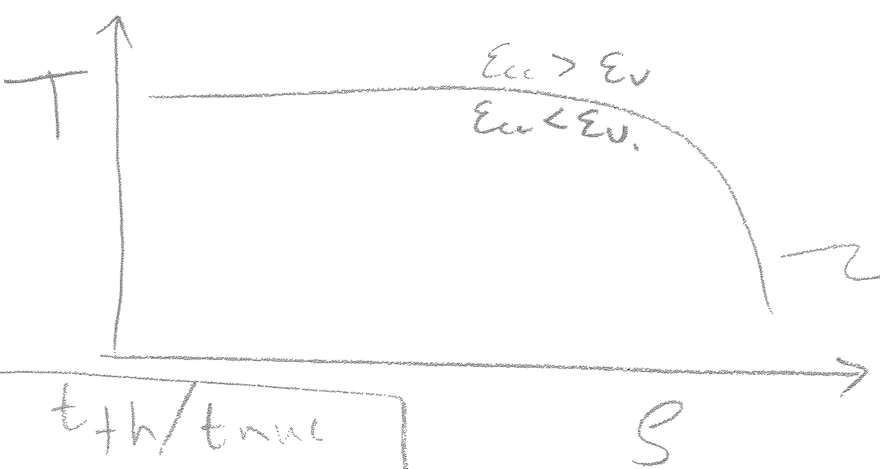
$$t_{nuc} = \left(\frac{E_{nuc}/\text{part}}{E_{th}/\text{part}}\right) t_{cool}$$

$$\approx \left(\frac{1\,\text{MeV}/m_p}{\frac{3}{2} k_B T / A m_p}\right) t_c = (7A)\, t_c$$

So the ^{12}C 5×10^{17} ergs/gr, $A = 12$

so $t_{nuc}(^{12}C) \sim 42\, t_{cool}$

and burning occurs at 7×10^8 or so, where we get $t_c \approx 20$ yrs and $t_{nu} \sim$ ~~400~~ 800 yrs, So, since ν cooling is the more important effect, we no longer need to worry about how the heat is transported out. The "ignition" is then a purely local question of when $\varepsilon_{cc} = \varepsilon_\nu$.



High density turnover not relevant for now.

Give t_{th}/t_{nuc} lifetime. . . .

lifetime.

Stages

(1) (i) ^{12}C Burning

$$^{12}C + {}^{12}C \rightarrow {}^{24}Mg$$

$$\hookrightarrow \alpha + {}^{20}Ne$$
$$p + {}^{23}Na$$
$$n + {}^{23}Mg$$

and the ashes from this are ^{20}Ne, ^{24}Mg, ^{23}Na.... plus the unburned ^{16}O!

$\Rightarrow$ Mention ONeMg WD's.

[example to clarify Coulomb Barrier.]

Now lets just imagine a generic reaction between two nuclei of equal masses $A_1 = A_2$ and $Z = A/2$. The non-resonant rate is

$$\langle \sigma v \rangle = \frac{7\times10^{-19}\, S}{\mu Z_1 Z_2}\, e^{-\tau}\, \tau^2$$

where $\mu =$ reduced mass in mc^2 units
$= \frac{A}{2}$

and

$$\tau = \frac{3E_0}{KT} = \frac{4.25}{T_9^{1/3}}\left(Z_1^2 Z_2^2 \frac{A}{2}\right)^{1/3}$$

$$= \frac{84.2}{T_9^{1/3}}\left(\frac{Z}{6}\right)^{5/3}$$

and take $[S = 3\times10^{19}$ kev-barns for $^{12}C]$ (need to understand why so high?)

then

$$\langle \sigma v \rangle \simeq \left(7\times10^{-19}\right)\frac{3\times10^{19}}{6^3}\left(\frac{6}{Z}\right)^3 \frac{(84.2)^2}{T_9^{2/3}}\left(\frac{Z}{6}\right)^{10/3}$$

$$\times \exp\left(-\frac{84.2}{T_9^{1/3}}\left(\frac{Z}{6}\right)^{5/3}\right)$$

$$= 690\, \frac{1}{T_9^{2/3}}\, \exp\left(\frac{-84.2}{T_9^{1/3}}\left(\frac{Z}{6}\right)^{5/3}\right)$$

!!

In a gas at density ρ_{carbon}
$n = 3/Am_p = 5\times10^{28}\ \rho_6$ for
and the energy release is roughly

$$(2A)\,\mathrm{MeV}\ n_i^2 \langle\sigma v\rangle \qquad \frac{\mathrm{ergs}}{\mathrm{cm^3\cdot sec}}$$

$$\approx 3\times10^{55}\ \rho_6^2\ \frac{1}{T_9^{2/3}}\left[-\frac{84.2}{T_9^{1/3}}\left(\frac{z}{6}\right)^{5/3}\right]$$

whereas the cooling is

$$\frac{dw_\nu}{dt} = 5\times10^{18}\ T_9^3 \exp\left(\frac{-11.84}{T_9}\right)$$

so for $z = 6$ we get

$$\boxed{3\times10^{55}\ \frac{\rho_6^2}{T_9^{2/3}} \exp\left(\frac{-84.2}{T_9^{1/3}}\right) = 5\times10^{18}\ T_9^3 \exp\left(\frac{-11.84}{T_9}\right)}$$

lets check at $\rho_6 = 0.1$

$$80.08 - \frac{84.2}{T_9^{1/3}} = \frac{11}{3}\ln T_9 - \frac{11.84}{T_9}$$

$$1 - \frac{1.051}{T_9^{1/3}} = \frac{1}{21.8}\ln T_9 - \frac{1}{6.76\,T_9}$$

$$\Rightarrow T_9 \approx 0.6$$

or $T_{^{12}C} \gtrsim 6\times10^{8}$

and actually a bit higher when

(2) Neon Burning.

Why "Neon Burning" before ^{16}O?

Carbon burning ended up making a mixture of ^{20}Ne, and a bit of ^{23}Na + ^{24}Mg.

Then there is no ^{16}O produced, what happens next is

$$\gamma + {}^{20}Ne \rightarrow \alpha + {}^{16}O$$

with
$$\alpha + {}^{20}Ne \rightarrow \gamma + {}^{24}Mg$$
$$\alpha + {}^{24}Mg \rightarrow \gamma + {}^{28}Si$$

So after Neon burning, we have

$$^{16}O,\ ^{24}Mg\ \&\ ^{28}Si$$

Then later we start ^{16}O burning!

At a few $\times 10^9 K$, we can get $E_\gamma \sim$ few MeV & start photo-disintegration. More on this later.

27 2a

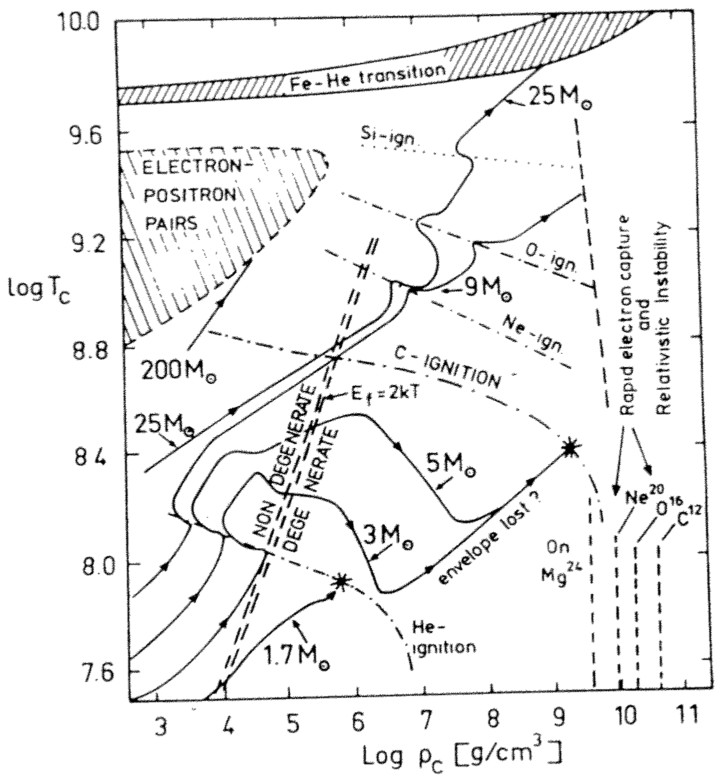


Table 9
Thermonuclear burning stages (after Arnett [16]) and timescales for a population I star with a mass of 25 $M_\odot$, after Weaver et al. [380,381]

Fuel	$T/10^9$ (K)	Ashes	E (erg/g fuel)	Cooling	Time (yr)
^{1}H	0.02	^{4}He, ^{14}N	(5–8) $\times 10^{18}$	photons	5×10^6
^{4}He	0.2	^{12}C, ^{16}O, ^{22}Ne	7×10^{17}	photons	5×10^5
^{12}C	0.8	^{20}Ne, ^{24}Mg, ^{16}O	5×10^{17}		
		^{23}Na, ^{25,26}Mg		neutrinos	60
	0.4	^{20}Ne, ^{23}Na	–		
^{20}Ne	1.5	^{16}O, ^{24}Mg, ^{28}Si	1.1×10^{17}	neutrinos	1
^{16}O	2	^{28}Si, ^{32}S	5×10^{17}	neutrinos	0.5
^{28}Si	3.5	^{56}Ni, $A \sim 56$ nuclei	(0–3) $\times 10^{17}$	neutrinos	0.01
^{56}Ni	6–10	n, ^{4}He, ^{1}H	-8×10^{18}	neutrinos	
$A \sim 56$ nuclei		(depends on photodisintegration and neutronization)			10^{-6}

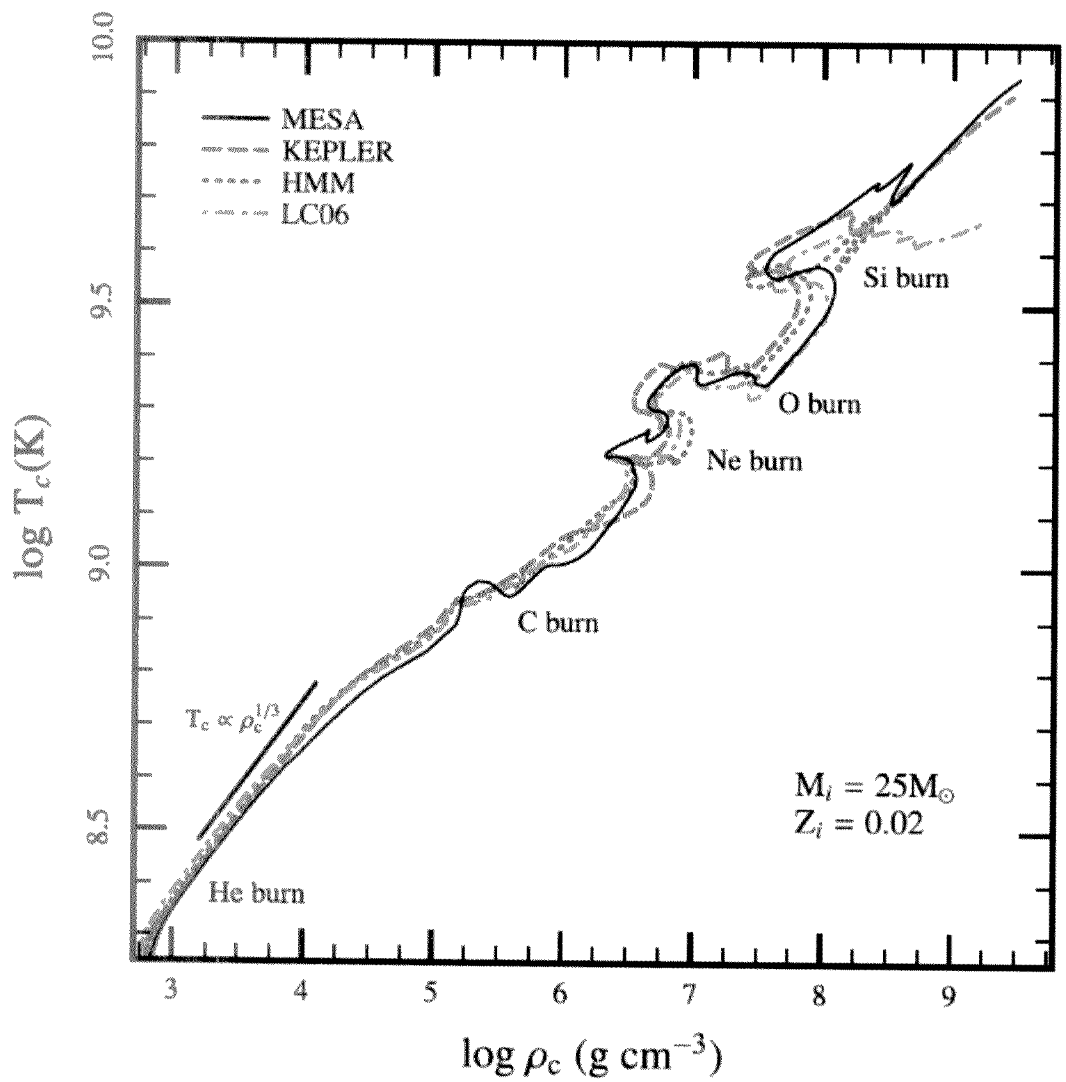


Figure 30. Evolution of the central temperature and central density in solar metallicity $M_i = 25\,M_\odot$ models from different stellar evolution codes. The locations of core helium, carbon, neon, oxygen, and silicon burning are labeled, as is the relation $T_c \propto \rho_c^{1/3}$.

Table 12
Massive Star Core Burning Lifetime Comparison

Core Burning Element	Lifetime (years) HMM	WHW	LSC	MESA	
		$M_i = 15\,M_\odot$			
H	1.13	1.11	1.07	1.14	$\times 10^7$
He	1.34	1.97	1.4	1.25	$\times 10^6$
C	3.92	2.03	2.6	4.23	$\times 10^3$
Ne	3.08	0.732	2.00	3.61	
O	2.43	2.58	2.43	4.10	
Si	2.14	5.01	2.14	0.810	$\times 10^{-2}$
		$M_i = 20\,M_\odot$			
H	7.95	8.13	7.48	8.01	$\times 10^6$
He	8.75	11.7	9.3	8.10	$\times 10^5$
C	9.56	9.76	14.5	13.5	$\times 10^3$
Ne	0.193	0.599	1.46	0.916	
O	0.476	1.25	0.72	0.751	
Si	9.52	31.5	3.50	3.32	$\times 10^{-3}$
		$M_i = 25\,M_\odot$			
H	6.55	6.706	5.936	6.38	$\times 10^6$
He	6.85	8.395	6.85	6.30	$\times 10^5$
C	3.17	5.222	9.72	9.07	$\times 10^2$
Ne	0.882	0.891	0.77	0.202	
O	0.318	0.402	0.33	0.402	
Si	3.34	2.01	3.41	3.10	$\times 10^{-3}$

References. HMM: Hirschi et al. (2004); WHW: Woosley et al. (2002); LSC: Limongi et al. (2000); MESA: this paper.