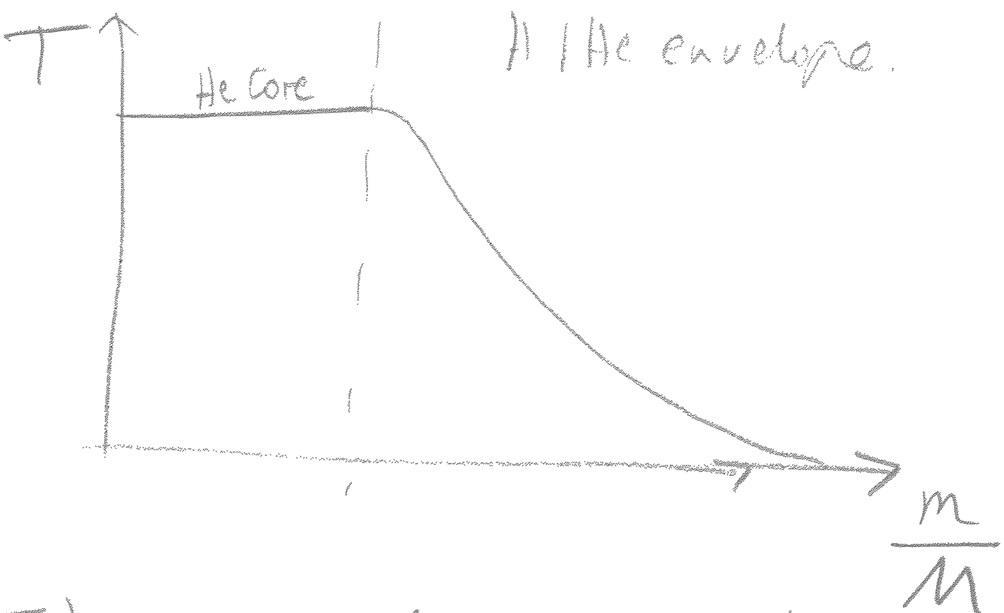
Schönberg-Chandra Limit

Consider a star after it is done burning and has a core of pure helium which is non-degen. ~~and because~~. As $t_{KH} \ll t_{life}$, the core will very nearly be in equilibrium, which, as there are no heat source, is then roughly isothermal:



The envelope exerts pressure on the core and we want to see if this is always ok or not. Go back to

$$\frac{dP}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$$

+ multiply by $4\pi r^3$ as before

then integrate over core

$$\underbrace{\int_0^{R_c} 4\pi r^3 \frac{dP}{dr}\, dr}_{= LHS} = -\underbrace{G\int_0^{R_c} \frac{4\pi r^2 m(r)\rho(r)\,dr}{r}}_{= RHS}$$

$dv = \frac{dP}{dr}\,dr \quad v = P$

$u = r^3 \quad du = 3r^2\,dr$

so start with the left-hand side.

$$\int_0^{R_c} 4\pi r^3 \frac{dP}{dr}\, dr = 4\pi r^3 P\Big|_0^{R_c} - 12\pi \int_0^{R_c} r^2 P(r)\,dr$$

$$= 4\pi R_c^3\, P_c(R_c) - 12\pi \int_0^{R_c} r^2 P(r)\,dr$$

whereas the RHS is roughly

$$= -\frac{G M_c^2}{R_c}$$

so if isothermal, + an ideal gas then (in core $\mu = \mu_c$)

$$+12\pi \int_0^{R_c} r^2 \frac{\rho k_B T}{\mu_c m_p}\,dr = 3\,\frac{k_B T}{\mu_c m_p} M_c$$

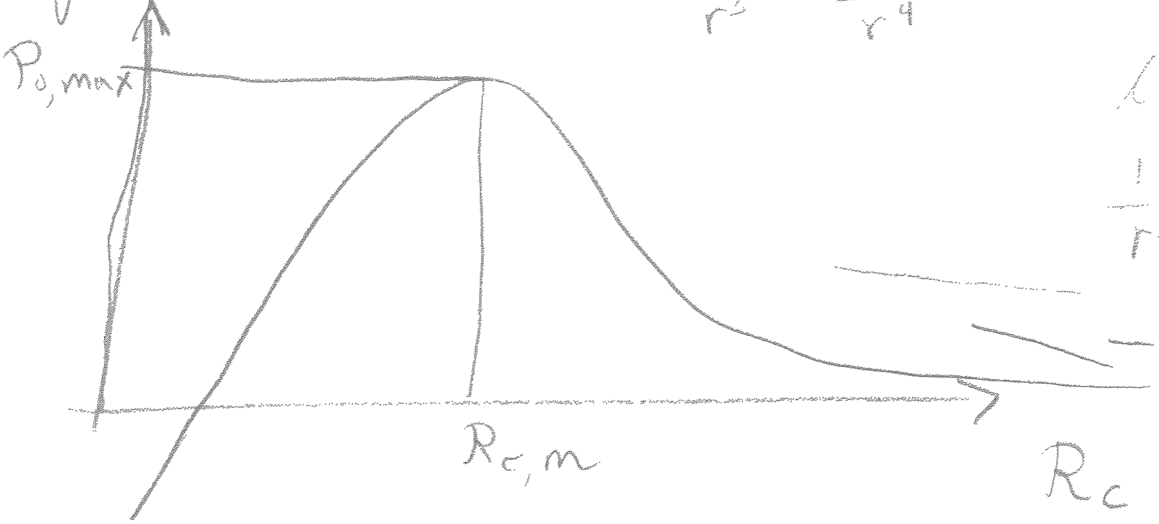
so we have:

$$4\pi R_c^3\, P_c(R_c) - 3\frac{k_B T}{\mu_c m_p} M_c = -\frac{G M_c^2}{R_c}$$

or just

$$\boxed{P_c(R_c) = \frac{3}{4\pi}\frac{k_B T_c}{\mu_c m_p}\frac{M_c}{R_c^3} - \frac{1}{4\pi}\frac{G M_c^2}{R_c^4}}$$

For a fixed mass & T_c, lets plot this: $\frac{a}{r^3} - \frac{b}{r^4}$



Large r

$\frac{1}{r^3}\left(1 - \frac{1}{r}\right)$

$r \to$

$r P = 1 - \frac{1}{r}$

$$P_c(R_c) = \frac{1}{4\pi R_c^3}\left[\frac{3 k_B T_c M_c}{\mu_c m_p} - \frac{G M_c^2}{R_c}\right]$$

$$\frac{\partial P_c}{\partial R_c} = \frac{-9}{4\pi}\frac{k_B T}{\mu_c m_p}\frac{M_c}{R_c^4} + \frac{1}{\pi}\frac{G M_c^{\cancel{2}}}{R_c^5}$$

$$\Rightarrow \boxed{R_c = \frac{G M_c \mu_c m_p}{k_B T}\frac{4}{9}}$$

$\frac{G M_c \mu_c m_p}{k_B T} \cdot \frac{9}{4}$

Giving:

$$P_{c,max} = \frac{1}{4\pi R_{c,m}^3}\left[\frac{3 k_B T_c M_c}{\mu_c m_p} - \frac{\cancel{G} M_c^{\cancel{2}} k_B T_c}{\cancel{G M_c} \mu m_p}\frac{9}{4}\right]$$

$$P_{c,max} = \frac{1}{4\pi R_{c,m}^3}\left(\frac{M_c k_B T}{\mu_c m_p}\right)\left(3 - \frac{9}{4}\right)$$

$$= \frac{3}{4}\frac{1}{4\pi R^3}\left(\frac{M_c}{\mu m_p}\right)(k_B T)$$

$\frac{12}{4} - \frac{9}{4} = 3/4$

so

$$P_{c,m} = \frac{3}{4}\frac{1}{4\pi}\frac{M_c}{\mu_c m_p} k_B T \left(\frac{9 k_B T}{4 G M \mu_c m_p}\right)^3$$

$$= \frac{3}{4}\frac{1}{4\pi}\frac{9^3}{4^3}\left[\frac{M_c (k_B T)^4}{(\mu_c m_p)^4 (G M_c)^3}\right]$$

$$\boxed{P_{c,m} = 0.68 \left(\frac{k_B T_c}{\mu_c m_p}\right)^4 \frac{1}{G^3 M_c^2}}$$

OK, now what if Pressure due to the overlying envelope is greater than $P_{c,m}$? Remember, envelope has $\mu = \mu_e$. and presume $M \gg M_c$ roughly

$$P_{base} \approx \frac{G M^2}{R^4} \qquad (M \gg M_c)$$

but the T is set by $k_B T_e \approx \frac{G M \mu_e m}{R}$

so

$$R = \frac{G M \mu_e m_p}{k_B T_e}, \quad \text{then}$$

$$P_{base} = G M^2 \frac{(k_B T_e)^4}{(G M \mu_e m_p)^4} = \left(\frac{k_B T_e}{\mu_e m_p}\right)^4 \frac{1}{G^3 M^2}$$

so $$0.68 \left(\frac{k_B T_c}{\mu_c m_p}\right)^4 \frac{1}{G^3 M_c^2} > \frac{1}{G^3 M^2}\left(\frac{k_B T_c}{\mu_e m_p}\right)^4$$

or

$$\frac{M^2}{M_c^2} > \left(\frac{\mu_c}{\mu_e}\right)^4$$

or

$$M_c < M\left(\frac{\mu_e}{\mu_c}\right)^2 \quad (\checkmark)^{\#}$$

now $\mu_e = 0.6$ &

$\mu_c =$ pure He $= 1.33$

so

$$\frac{\mu_e}{\mu_c} = 0.\cancel{4}5$$

Detailed Calcs give:

$$q_c = \frac{M_c}{M} = 0.37\left(\frac{\mu_e}{\mu_c}\right)^2 \simeq 0.08$$

So, if non-degenerate, then $q_c < 0.08$ for stability! Otherwise there is trouble

What if the core becomes degenerate. Well then we find

Begin Lect

$$4\pi R_c^3 P_c(R_c) - 12\pi \int_0^{R_c} r^2 P(r)\,dr = -\frac{GM_c^2}{R_c}$$

$$\int_0^{R_c} r^2 P(r)\,dr \propto R^3 \rho^{5/3}$$

or that

$$R_c^3 P_c(R_c) - 3\int_0^{R_c} r^2 P(r)\,dr = -\frac{GM_c^2}{R_c 4\pi}$$

but if the core is degenerate, then

$$P(r) = \frac{2}{5} n_e E_F \qquad E_F = \frac{p_f^2}{2m} = \frac{\hbar^2 k_F^2}{2m} \sim n_e^{2/3}$$

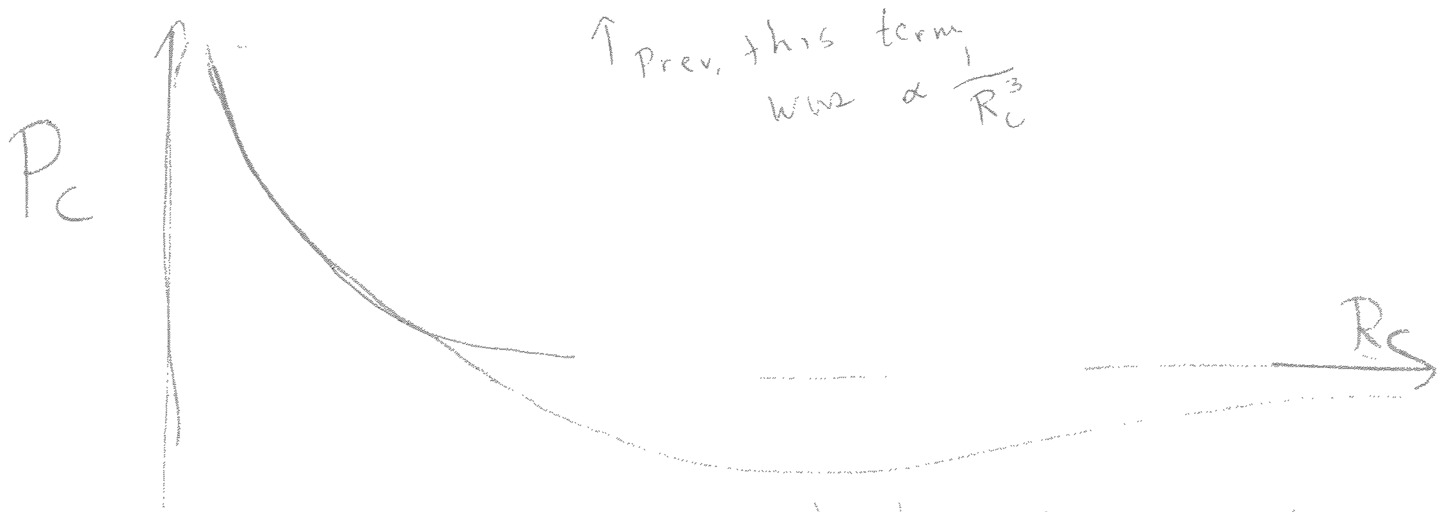
$\Rightarrow \quad P(r) = K\rho^{5/3}$, so we find:

$$R_c^3 P_c(R_c) = -\frac{GM_c^2}{4\pi R_c} + 3\int_0^{R_c} r^2 \rho^{5/3}\,dr$$

$$\simeq -\frac{GM_c^2}{4\pi R_c} + 3R_c^3 \frac{M^{5/3} K}{R_c^5}$$

$$\Rightarrow \quad P_c(R_c) = \frac{K M^{5/3}}{R_c^5} - \frac{1}{4\pi}\frac{GM_c^2}{R_c^4}$$

↑ Prev, this term was $\propto \frac{1}{R_c^3}$



So it can take on any value it needs to.

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Then

$$R_c^3 P_c(R_c) - 3\int_0^{R_c} r^2 P(r)\,dr = -\frac{GM^2}{R_c}$$

but

$$P(r) = \frac{2}{5} n_e E_F \propto \rho^{5/3} \quad (\text{NR } e^-)$$

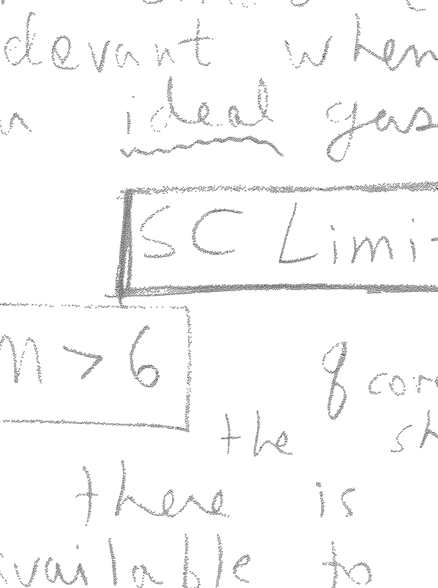
$$R_c^3 P_c - 3R^3 \rho^{5/3} = -\frac{GM^2}{R_c}$$

$$R_c^3 P_c = 3R^3 \frac{GM^{5/3}}{R^5} - \frac{GM^2}{R_c} \sim \frac{1}{R^2}$$

which can take on any value. So, the Schonb-Chandra limit is only relevant when the core consists of an *ideal* gas

$\boxed{\text{SC Limit's Role}}$ $\quad q_c = \frac{M_c}{M}$

$\boxed{M > 6}$ $q_{core} > 0.08$ always when the shell source ignites. Hence there is no isothermal core soln. available to these stars. The energy loss from the dT/dr must come from gravitational contraction of the core. This phase then lasts very briefly, of order



$$t = \frac{GM^2/R}{L} \approx \frac{(GM)(M/R)}{L_\odot (M/M_\odot)^3}$$

So we get

$$t \approx 1 \times 10^6 \, yr \left(\frac{6 M_\odot}{M}\right)\left(\frac{5 R_\odot}{R}\right)$$

or roughly 10^6 yr for $M = 6 M_\odot$ & $\approx 3 \times 10^4$ yr for $M = 30 M_\odot$. These are all such short times that it is very rare to catch a star in this state
$\Rightarrow$ "Hertzsprung Gap"

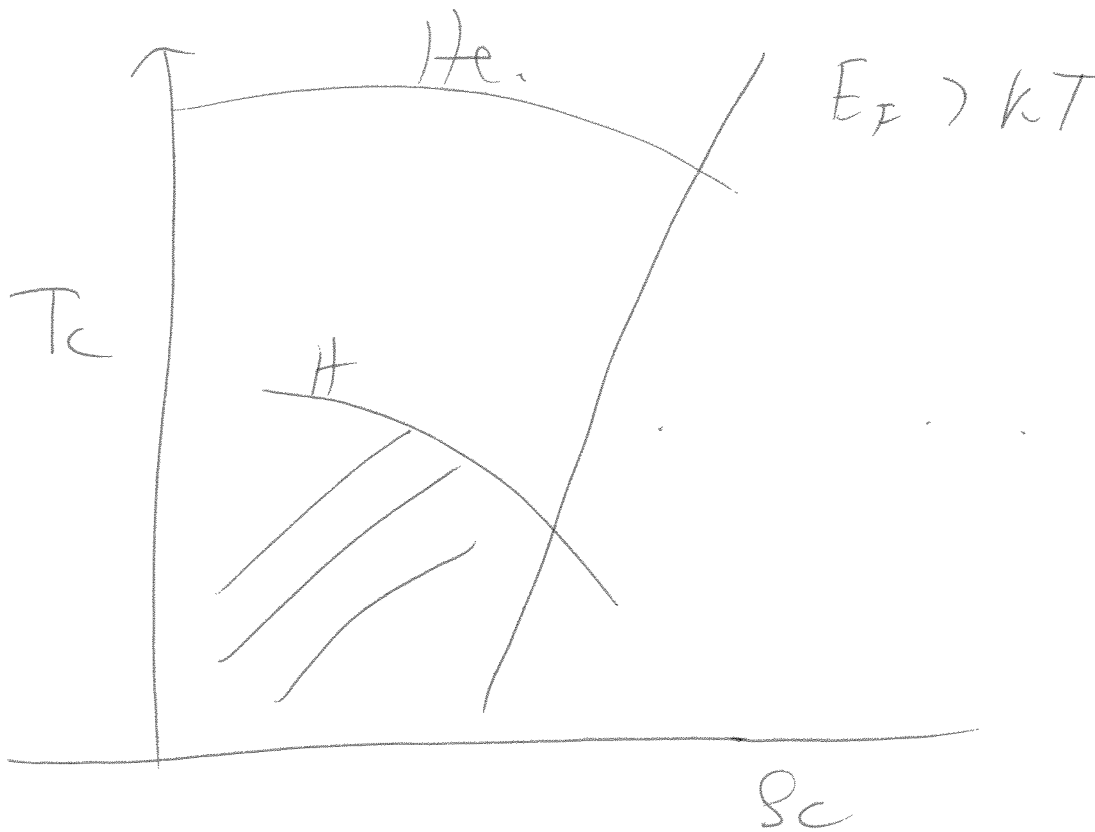
$\boxed{2 < M < 6}$ Initial value $q_c < 0.08$ when H burning is complete.

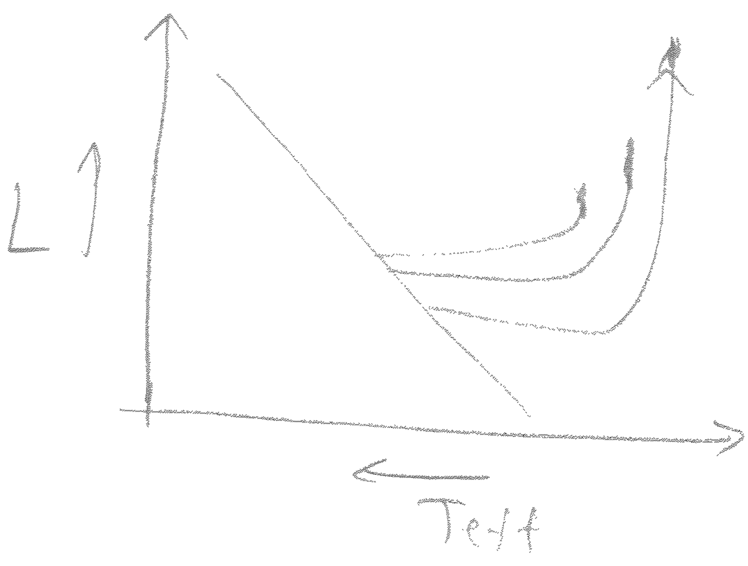
$\Rightarrow$ Core can be isothermal during H shell burning in which case the system lives for a while on H shell burning. Eventually $q_c > 0.08$ and we get collapse. This occurs <u>before</u> core becomes degenerate so that the core is <u>not</u> all that massive at collapse.

$\boxed{M < 2:}$ Core becomes degenerate before q_c exceed 0.08 and so can build up to arbitrarily large values.

Rhon

Inserted a summary discussion of





~~$\cancel{\frac{L}{L_\odot} \approx A\left(\frac{M_c}{0.16 M_\odot}\right)^2}$ is a reasonably good fit to the case where the core is degenerate.~~

$\boxed{M > 2.2 M_\odot}$ Don't get all that high up on the branch before core collapses ~~and~~ when $\rho > \rho_{sc}$ and ^4He ignite

$\boxed{M < 2.2 M_\odot}$ Core becomes degenerate before $\rho > \rho_{sc}$ so that we find "core" convergence, as all time end up only knowing about $M_c \Rightarrow$ all low-mass stars show the same. The degen core has:

$$R_c = 2\times10^9\,\text{cm}\left(\frac{0.1 M_\odot}{M}\right)^{1/3}$$

so we get

$$T_s = 2\times10^7\,K\left(\frac{M}{0.1 M_\odot}\right)^{4/3}$$

So at $T_s \sim 2\times10^7$ $\nu = 18$ $\varepsilon_0 \sim 1.8$ and we get:

$$L = L_\odot\left(\frac{0.1 M_\odot}{M_c}\right)^{7/9}\left(\frac{M_c}{0.1 M_\odot}\right)^{11.56} \approx L_\odot\left(\frac{M_c}{0.1 M_\odot}\right)^{11}$$

Better fit is $L = L_\odot \left(\frac{M_c}{0.16 M_\odot}\right)^{7.3}$ 7.23

This sets the lifetime as the system moves up the giant branch as $L(M_c)$ & $M_c \uparrow$ as He is added to the core, so

$$\dot{M}_c = \frac{gr}{sec} = \frac{L}{(E_{nuc}/m_p)} \approx 6 \times 10^{14} \frac{gr}{sec} \left(\frac{M_c}{0.16 M_\odot}\right)^{6.3}$$

So the stars spends less & less time at a given luminosity. The core is thus growing on a timescale:

$$t = \frac{M_c}{\dot{M}_c} \approx 1.7 \times 10^{10} \text{ yrs} \left(\frac{0.16}{M_c}\right)^{6.3}$$

!!

He core flash.

fill in #'s or if we put

M_c	t
0.16	1.7×10^{10}
0.25	10^9
0.40	5.3×10^7

spends less & less time as it moves up the RGB. When does it end? Well what is happening ~~as star~~ is that the He core is getting denser & hotter in which case the ^4He finally ignites. This happens at $M_c = 0.45 M_\odot$ when

$$L = 10^{3.2 \to 3.4} L_\odot$$

~~Talk about subtleties of the core flash.~~