How to implement a digital biquadratic notch filter (in software)

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# Background

As presented in the [complimentary white paper](https://github.com/branbick/oscar/blob/main/biquad_notch_filter/doc/damping_ratios.pdf), the continuous-time transfer function of a biquadratic—or “biquad”—notch filter is …

Where Laplace variable, damping ratio (—), center/notch frequency (rad/s), and the subscripts and designate “numerator” and “denominator,” respectively. is a design parameter, and the aforementioned white paper derives an algorithm for calculating and . That’s great, and can be practically utilized with the “help” of third-party software such as [MathWorks’ Control System Toolbox](https://www.mathworks.com/products/control.html). But what if the goal is to implement a notch filter in our *own* software—software running on an embedded system, for example? If that’s the case, then a *digital* filter is needed, and both its coefficients and corresponding difference equation must be determined.

# Derivation

The -transform is the discrete-time counterpart of the Laplace transform, where ...

And sampling period (s). The first-order, Taylor-series approximation of is . Therefore, …

The latter result is known as the bilinear transform: replacing with allows a continuous-time transfer function to be converted to discrete time. Unfortunately, that transformation causes a “warping” of the frequencies. For example, upon applying the bilinear transform to , a Bode plot of the *digital* (i.e., discrete-time) filter will show that —which was specified for the *analog* (i.e., continuous-time) filter—shifted. Obviously, that is problematic. Thankfully, it can be remedied via frequency prewarping.

TODO: Add more information about frequency prewarping and necessary equation

Resources:

* <https://www.mathworks.com/help/signal/ref/bilinear.html#mw_8d43fd06-fbf3-4108-be26-62642fa7af82> (accessed 4/22/22)
* <https://en.wikipedia.org/wiki/Bilinear_transform> (accessed 4/22/22)

TODO: Change notation of transfer function (in both white papers) so is not confused between transfer function and sampling period