How to calculate the damping ratios of a biquadratic notch filter

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# Background

The standard, continuous-time transfer function of a second-order system is …

Where Laplace variable, natural frequency (rad/s), and damping ratio (—). A biquadratic—or “biquad”—transfer function is merely the ratio of two second-order transfer functions:

Regarding a notchfilter, the center/notch frequency, , is the defining frequency of the corresponding transfer function—as opposed to . Therefore, the transfer function of a biquad notchfilter is as follows—where the subscripts “2” and “1” are replaced by “num” (numerator) and “den” (denominator), respectively:

# Derivation

Ideally, a notch filter can be generated based on three design parameters:

1. (rad/s)
2. Bandwidth, (rad/s)
   1. Range of frequencies to attenuate
   2. Equal to the difference of the “-3 dB” frequencies right (minuend) and left (subtrahend) of —which are denoted and , respectively
3. Notch depth, (dB)
   1. Amount to attenuate the signal (at )
   2. Specified as a positive number

The question now is, “How are and calculated?” Well, …

In the frequency domain, is replaced by ; i.e., in the frequency domain, …

By definition, . Consequently, …

Unfortunately, it is not possible for and . [The solution to is trivial. Try it for yourself!] The “next best thing” is to let and (where constant)—remembering that, as previously stated, . As a result, …

Which is simply a quadratic equation whose solution is …

And—because 1) the radical term will always be greater than , and 2) both and must be positive—only the positive root is of (practical) interest. Finally, also by definition, . (Note that .) Thus, …

Plugging in for (earlier result) yields …

Where, again, only the positive root is of (practical) interest.

# Summary

The algorithm for calculating the damping ratios of a biquad notch filter is as follows: