How to perform system identification using the sine-sweep method

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# Background

In order to analyze and design control systems *in the frequency domain*, the ability to “identify”—i.e., compute the [frequency response](https://www.mathworks.com/discovery/frequency-response.html) of—the system of interest is necessary. For example, to effectively design a [notch filter](https://github.com/branbick/oscar/tree/main/biquad_notch_filter) for a “gimballed” sensor system with a resonance issue, the dynamics (frequency response) of that system must first be determined. Otherwise, how would we know the appropriate values to use for the center frequency, bandwidth, and notch depth of the aforementioned filter? “Okay; that makes sense. But how do we identify the system in the first place?” Excellent question!

# Derivation

Let be the (continuous-time) transfer function of a stable, linear and time-invariant (LTI), single-input and single-output (SISO) system; and let a signal of the form —where amplitude (e.g., deg), angular frequency (rad/s), and time (s)—be input into that system. Then, its output *at steady-state (i.e., after the transient response dies out)* is a signal of the form —where and —which can be expressed as a Fourier series, defined below for arbitrary periodic function :

period of (s)

Now replace with , which is centered about the time axis, meaning its average value over a multiple of is zero. Therefore, . Additionally, …

Applying the angle addition formula for results in …

The definite integral of the *first* term, utilizing the double-angle formula for , is …

And the definite integral of the *second* term, utilizing the power-reduction formula for , is …

Consequently, …

Going through the same process for yields …

So, …

and and

(Note that only the positive root is of interest because is a magnitude.) Furthermore, …

## Important notes

* Regarding , , where is a positive integer. And the same is true for : .
* Because ’s output must reach steady-state, which takes seconds, before the frequency response calculations can commence (assuming accurate calculations are desired), the limits of integration in the equations for and should actually be (lower) and (upper)—where is a positive integer *less than* , and . However, the arguments of and in the equations for and , respectively, must start at zero [as those coefficients practically represent how the respective trigonometric functions of frequency “align” with ]. Thus, the equations for the coefficients that constitute the aforementioned Fourier series expansion of are, more precisely, …
  + The “more precise” equations for and do not alter the previously derived equations for and . (Try rederiving them yourself!)
* and can be computed numerically (e.g., using the composite trapezoidal rule)
* and are estimates because only the Fourier series terms are employed [for the sake of simplicity and because is the sole frequency that comprises a nominal/non-noisy ]

# Summary

Let be the (continuous-time) transfer function of a stable, LTI, SISO system; and let a signal of the form —where amplitude (e.g., deg), angular frequency (rad/s), and time (s)—be input into that system. Then, its output *at steady-state* is a signal of the form —where and —which can be approximated as a Fourier series with just two coefficients:

Where period of (s); and are both positive integers; ; and is greater than or equal to the time it takes ’s output to reach steady-state. Ultimately, the procedure for using the sine-sweep method to perform system identification is as follows:

1. Numerically compute and (equations above)
2. Compute the magnitude and phase of at frequency , respectively
3. Change the value of and repeat steps 1 and 2 until magnitude and phase data for all the input frequencies of interest have been collected