

MATH 1401

Spring 2023 Chapter 4 Exam Review

Review Day: Tuesday, November 7<sup>th</sup>

Chapter 4 Exam: Thursday, November 9<sup>th</sup>

Instructor: Whitten

Instructions:

1. You will have a total of 1 hour and 15 minutes to complete this exam.
- 2. Calculators, electronic devices, scratch paper, and notecards are not allowed on this exam.**
3. Show ALL of your work. Partial credit can be awarded for work that is legible and mathematically correct.
4. Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%. A visible phone or electronics results in a zero.
5. We are not out to trick you!!! Make sure you understand the concepts on this review.
6. Take a deep breath, relax, and good luck!!

Find the critical value(s) for the following functions. Recall that a critical value is the x-coordinate of the critical point.

1.  $f(x) = ax^2 + bx + c$

2.  $f(x) = 2x^3 - 3x^2 - 36x + 12$

3.  $f(x) = 2x \cdot \ln x + 10$

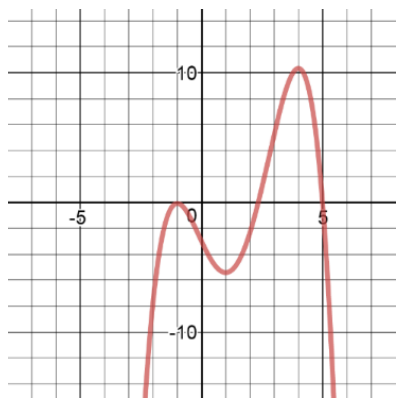
4.  $f(x) = \frac{x}{x^2 + 1}$

Find the absolute extrema of the following functions on the indicated closed interval:

7.  $f(x) = 2x^3 + 3x^2 - 12x + 1$  on  $[-1, 3]$

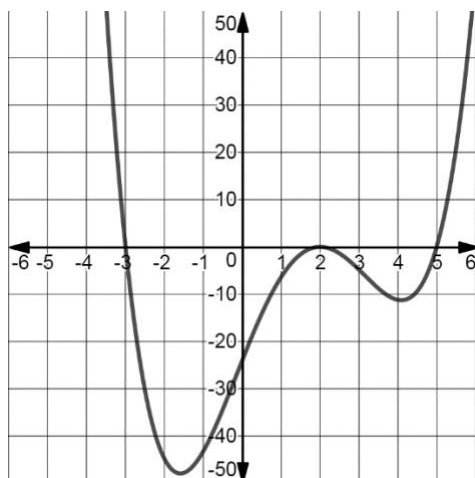
8.  $f(x) = \frac{x}{(x^2 + 3)^2}$  on  $[-2, 2]$

9. Below is the graph of a **function**  $f(x)$  with a domain of all real numbers. Find the following (if possible):



- a. The  $x$ -value(s) of all locations of all critical point(s) of  $f(x)$ : \_\_\_\_\_
- b. The  $x$ -value(s) where a local minimum occurs: \_\_\_\_\_
- c. The  $x$ -value(s) where a local maximum occurs: \_\_\_\_\_
- d. The  $x$ -value where the absolute minimum occurs: \_\_\_\_\_
- e. The  $x$  value where the absolute maximum occurs: \_\_\_\_\_

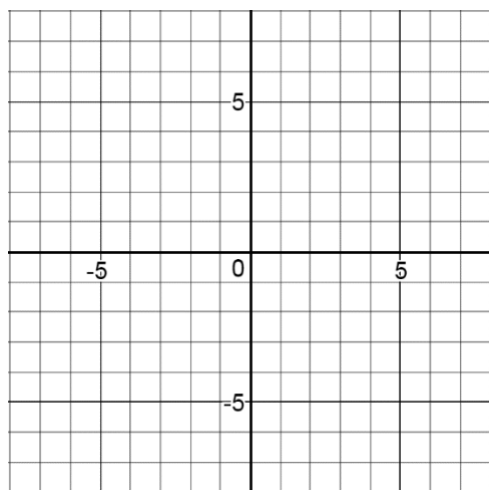
10. Below is the graph of the **derivative** of a function with a domain of all real numbers. Find the following (if possible):



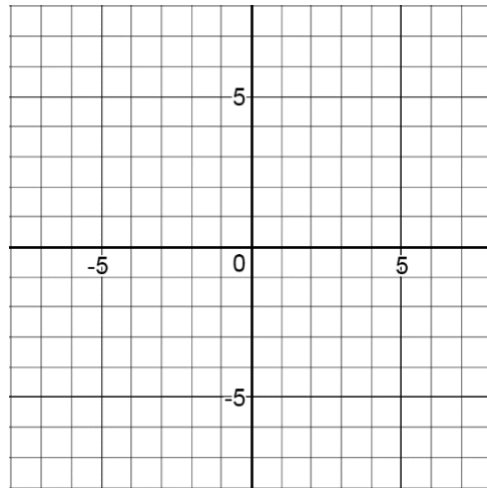
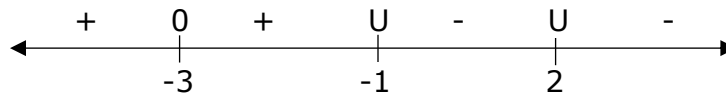
- The  $x$ -value(s) of all locations of all critical point(s) of  $f(x)$ : \_\_\_\_\_
- The  $x$ -value(s) where a local minimum occurs: \_\_\_\_\_
- The  $x$ -value(s) where a local maximum occurs: \_\_\_\_\_
- The  $x$ -value where the absolute minimum occurs: \_\_\_\_\_
- The  $x$  value where the absolute maximum occurs: \_\_\_\_\_

For questions 11 – 12, sketch the continuous function  $f(x)$  with the following properties:

11.  $f'(x) < 0$  on  $(-\infty, 2)$ ;  $f'(x) > 0$  on  $(2, 5)$ ;  $f'(x) < 0$  on  $(5, \infty)$ ,  $f'(2) = 0$ ,  $f'(5) = \text{undefined}$



12. Sign line of  $f'(x)$ :



For questions 13 – 14, given  $f(x)$ , find the following:

- The critical value(s) of  $f(x)$ .
- Use the First Derivative Test to determine if each critical value is the location of a local minimum, local maximum, or neither.
- State the x-interval(s) where  $f(x)$  is increasing and decreasing.

13.  $f(x) = 2x^5 - 5x^4 - 10x^3 + 4$

14.  $f(x) = e^x(x - 3)$

15. Use the Second Derivative Test to determine if each critical value is the location of a local minimum, local maximum, or neither for  $f(x) = x^2 \cdot e^{-x}$ .

Based upon the given information for questions 16 - 18, find the following:

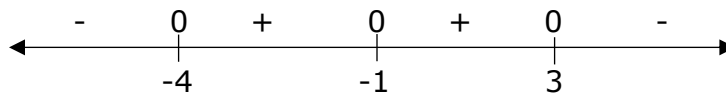
a. The x-interval(s) where  $f(x)$  is concave up and concave down.

b. The x-coordinate(s) of the inflection points of  $f(x)$ .

16.  $f(x) = 3x^5 - 20x^4 + 40x^3 - 7$

17.  $f(x) = \sqrt[3]{x-4}$

18. Sign line of  $f''(x)$ :



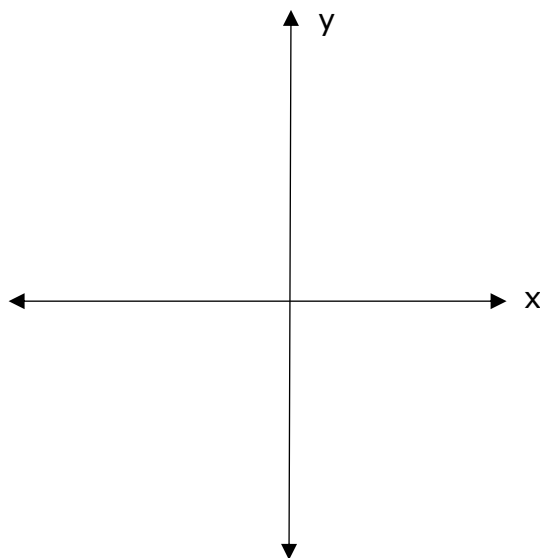
19. True or False?

- a. If  $f(x)$  has a relative max or min at  $x=c$ , then  $x=c$  is a critical point of  $f(x)$ .
- b. If  $x=c$  is a critical point of  $f(x)$ , then  $f(x)$  has a relative max or min at  $x=c$ .
- c. If  $f''(a)=0$ , then  $x=a$  is the location of an inflection point of  $f(x)$ .

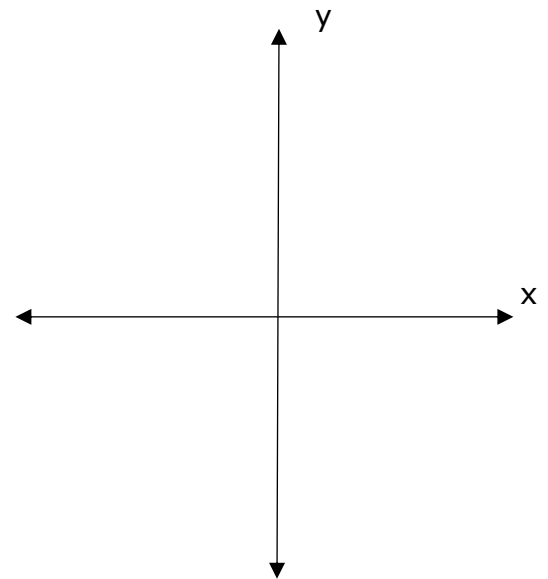
For questions 20 - 21, sketch a graph of the following functions. Be sure to find the following:

- a. The critical value(s) of  $f(x)$ .
- b. State the x-interval(s) where  $f(x)$  is increasing and decreasing.
- c. State the x-interval(s) where  $f(x)$  is concave up and concave down.
- d. Use the First or Second Derivative Test to determine if each critical value is the location of a local minimum, local maximum, or neither.
- e. State the x-coordinates of the inflection point(s) of  $f(x)$ .
- f. Find the important function values for your sketch.

20.  $f(x) = x^3 - 6x^2 + 9x$



21.  $f(x) = x \cdot \ln x$



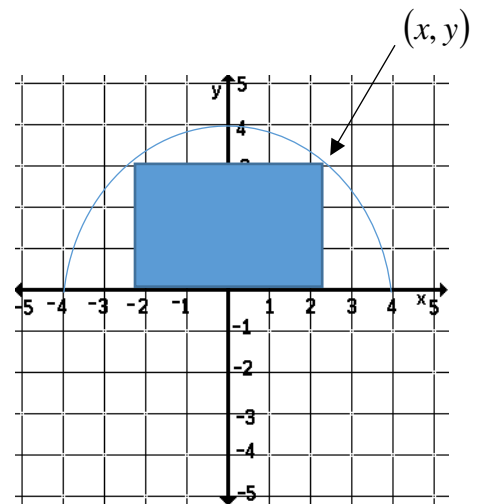
For questions 22 - 25, write the equation to be optimized, the constraining equation (if necessary), find the critical values of the function, and verify that your answer is the value that optimizes the function.

22. Maximize the product of 2 nonnegative numbers whose sum is 30.

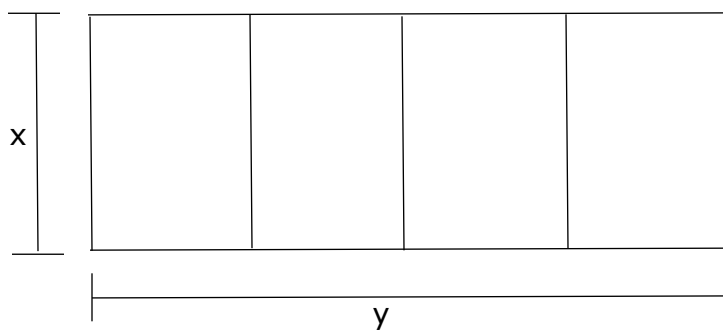


23. Of all boxes with a square base and a volume of 64 cubic centimeters, find the dimensions of the box with the minimum surface area.

24. A rectangle is constructed so that its base is on the x-axis and two of its vertices are on the graph of the upper circle  $y = \sqrt{16 - x^2}$ . Find the dimensions of the rectangle that maximizes the area of this rectangle.



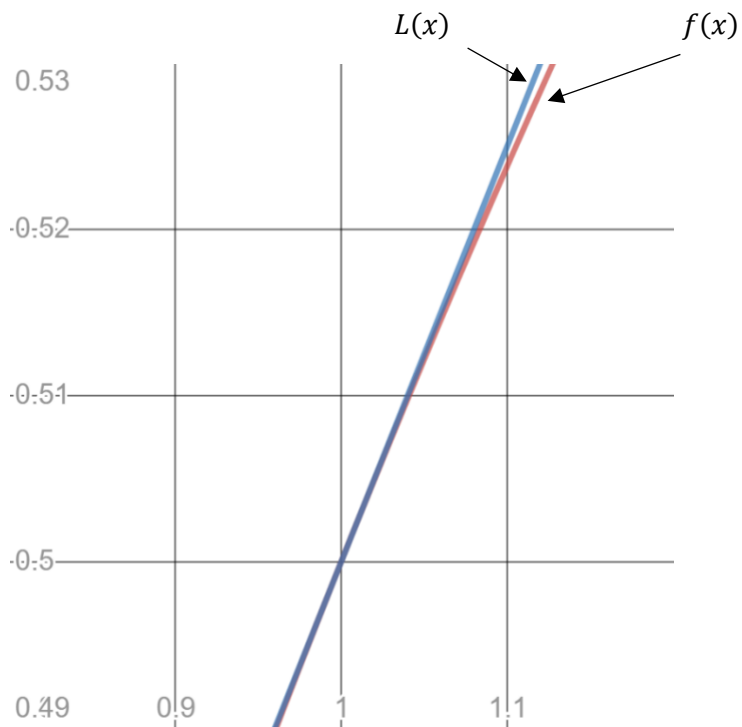
25. Farmer Sue needs to enclose a rectangular region on her farm. She has 1000 yards of fence and needs the region to have 4 equal subareas, as shown below. What dimensions maximize the area of the rectangular region?



26. Find the linear approximation of  $f(x) = (x^3 - 1)^3$  at  $x = 1.99$ .

27a. Find the linear approximation of  $f(x) = \frac{x}{x+1}$  at  $x = 1.1$ .

- b. Given the graph of  $f(x) = \frac{x}{x+1}$  below with the tangent line drawn at  $x = 1$ , label the location of the exact value of the function and the linear approximation at  $x = 1.1$ .



28. Use the linear approximation method to estimate the value of  $\sqrt[3]{28}$ .

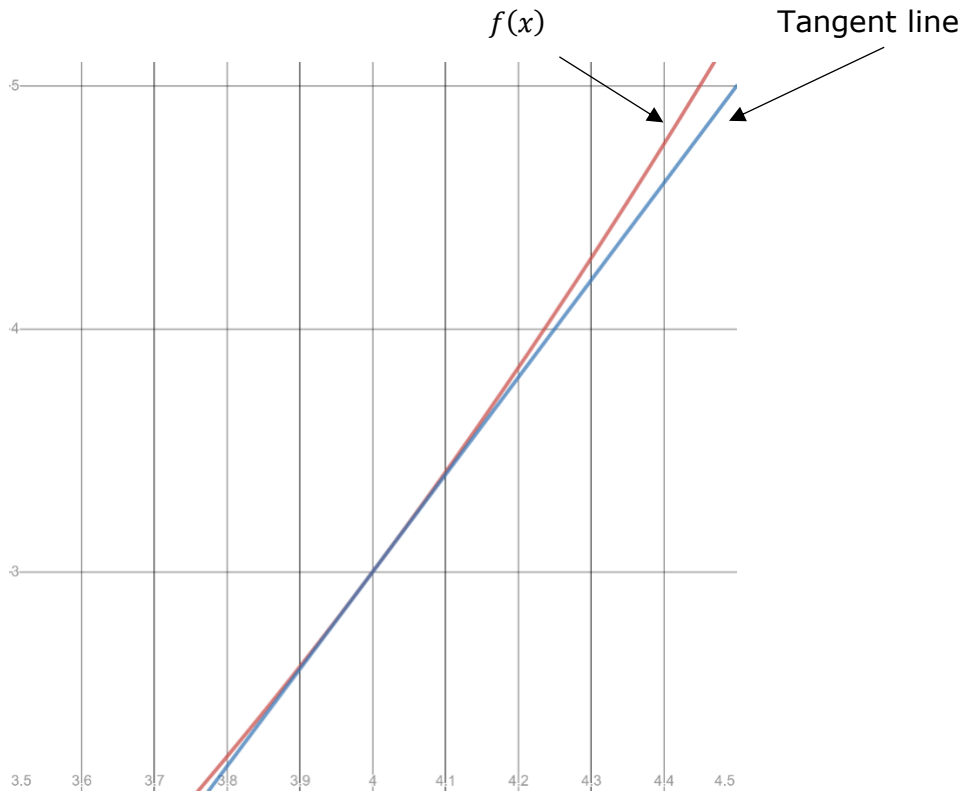
Find the differential  $dy$ .

29.  $f(x) = x^3 - \sin x + e^{x^2}$

30.  $h(t) = \ln(1 - 2t)$

31a. Use the differential  $dy$  to approximate the change in the function  $\Delta y$  for  $f(x) = x^2 - 4x + 3$  as  $x$  changes from  $x = 4$  to  $x = 4.3$ .

b. Given the graph of  $f(x) = x^2 - 4x + 3$  below with the tangent line drawn at  $x = 4$ , label  $dy$ ,  $\Delta y$ ,  $dx$ , and  $\Delta x$  as  $x$  changes from  $x = 4$  to  $x = 4.3$ .



32. Use differentials to approximate the change in the volume of a right circular cone of fixed height 4 cm when the radius changes from 3 cm to 3.04 cm. The volume of a cone with fixed height 4 cm is  $V = \frac{4}{3}\pi r^2$ .

33. Use Rolle's Theorem to find the value of  $x=c$  for  $f(x)=\sin(2x)$  on  $\left[0, \frac{\pi}{2}\right]$ .

Find the value(s) of  $x=c$  that satisfies the Mean Value Theorem:

34.  $f(x)=x^3+1$  on  $[-2, 1]$ .

35.  $f(x)=\ln(3x+1)$  on  $\left[0, \frac{e-1}{3}\right]$

Evaluate the following limits:

36.  $\lim_{x \rightarrow 5^-} \frac{x^2+3}{x-5}$

37.  $\lim_{x \rightarrow 0} \frac{5\sin(8x)}{8\sin(5x)}$

38.  $\lim_{x \rightarrow 3} \frac{x^2+x-12}{x^2-3x}$

$$39. \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{3x}$$

$$40. \lim_{x \rightarrow \infty} \frac{\ln(x^{100})}{\sqrt{x}}$$

$$41. \lim_{\theta \rightarrow 0} \frac{3 \sin^2 \theta}{\theta^2}$$

$$42. \lim_{x \rightarrow \infty} \frac{e^{2x}}{x}$$

$$43. \lim_{x \rightarrow -\infty} \frac{e^{2x}}{x}$$

$$44. \lim_{x \rightarrow \infty} x \cdot e^{-x}$$

$$45. \lim_{x \rightarrow 0^+} (\csc x - \cot x)$$

$$46. \lim_{x \rightarrow 0} (1 + 2x)^{\frac{5}{x}}$$

$$47. \lim_{x \rightarrow \infty} x^{\frac{1}{x}}$$