MATH 1401-001

Spring 2023 Chapter 2 Exam Review

Key

Chapter 2 Exam: Tuesday, February 14th

Review: Thursday, February 9th

Instructor: Whitten

Instructions:

1. You will have a total of 1 hour to complete this exam. The exam is worth 50 points.
2. **Calculators, electronic devices, scratch paper, and notecards are not allowed on this exam.**
3. Show ALL of your work. Partial credit can be awarded for work that is legible and mathematically correct.
4. Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%.
5. We are not out to trick you!!! Make sure you understand the concepts on this review.
6. Take a deep breath, relax, and good luck!!

MATH 1401 **Chapter 2 Exam Review** **NAME:**________________

1. Given the function $f(x) = 6\sin(x)$, find the average rate of change of $f(x)$ on the interval $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$. Your final answer must be simplified, which means no fractions in your fraction.

$$\frac{f(b)-f(a)}{b-a} = \frac{f(\frac{\pi}{2})-f(\frac{\pi}{6})}{\frac{\pi}{2}-\frac{\pi}{6}} = \frac{6\sin(\frac{\pi}{2})-6\sin(\frac{\pi}{6})}{\frac{3\pi}{6}-\frac{\pi}{6}} = \frac{6(1)-6(\frac{1}{2})}{\frac{2\pi}{6}}$$

$$= \frac{6-3}{\frac{\pi}{3}} = \frac{3}{\frac{\pi}{3}} = 3\cdot\left(\frac{3}{\pi}\right) = \frac{9}{\pi}$$

2. Given the function $f(x) = 25 - x^2$, find the slope of the secant line, m_{sec}, on the interval $[2, 3]$. $\frac{f(b)-f(a)}{b-a}$

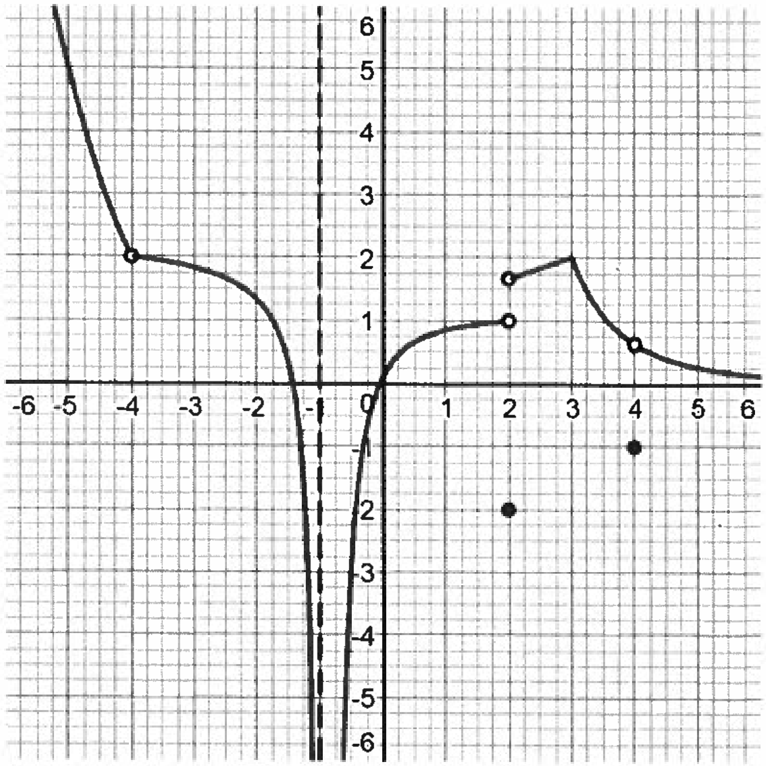
$$m_{sec} = \frac{f(3)-f(2)}{3-2} = \frac{25-3^2-(25-2^2)}{1} = 25-9-25+4 = -5$$

3. If the position of an object in feet at time t in seconds is defined by the function $s(t) = t^2 - 4t + 3$, find the average velocity of the object on the interval $[2, 5]$. Include units in your final answer. $\frac{f(b)-f(a)}{b-a}$

$$V_{avg} = \frac{s(5)-s(2)}{5-2} = \frac{5^2-4(5)+3-(2^2-4(2)+3)}{3} = \frac{25-20+3-(4-8+3)}{3}$$

$$= \frac{8-(-1)}{3} = \frac{9}{3} = 3 \text{ ft/sec}$$

4. Given the graph of $f(x)$, answer the following:



a. $\lim\limits_{x\to -4^-} f(x) = 2$

b. $\lim\limits_{x\to -4^+} f(x) = 2$

c. $\lim\limits_{x\to -4} f(x) = 2$

d. $\lim\limits_{x\to -1} f(x) = -\infty$

e. $\lim\limits_{x\to 2} f(x) = \text{DNE}$

$\lim\limits_{x\to 2^-} f(x) \neq \lim\limits_{x\to 2^+} f(x)$

f. $\lim\limits_{x\to 3} f(x) = 2$

g. $\lim\limits_{x\to -\infty} f(x) = \infty$

h. $\lim\limits_{x\to \infty} f(x) = 0$

i. $f(2) = -2$

j. The function is not continuous at $x =$: -4, -1, 2, 4

Evaluate the following limits:

$\frac{0}{0}$ hole, factor + cancel

5. $\lim_{x\to 1}\frac{x^2-x}{x^2+2x-3}$

$\lim_{x\to 1}\frac{x\cancel{(x-1)}}{(x+3)\cancel{(x-1)}} = \frac{1}{1+3}$

$= \frac{1}{4}$

6. $\lim_{x\to 2^+}\frac{1-x}{x-2}$ VA $\frac{-1}{0}$ $= -\infty$

choose 2.1

$\lim_{x\to 2^+}\frac{1-2.1}{2.1-2} = \frac{-}{+} = -$

7. $\lim_{x\to -3}\frac{x^2-9}{x^2+9} = \frac{(-3)^2-9}{(-3)^2+9} = \frac{0}{18}$

$= \boxed{0}$

$\frac{0}{0}$ mult by conj

8. $\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$

$\lim_{x\to 4}\frac{(x-4)(\sqrt{x}+2)}{x-4}$

$\lim_{x\to 4}\sqrt{x}+2 = \sqrt{4}+2$

$= 4$

9. $\lim_{x\to\infty}\frac{5-x}{x+8} = -1$

HA at $y=-1$

10. $\lim_{x\to 0}\frac{\frac{2}{x+7}-\frac{2}{7}}{x}$

$\lim_{x\to 0}\frac{\frac{2}{x+7}\cdot\frac{7}{7}-\frac{2}{7}\cdot\frac{x+7}{x+7}}{x}$

$\lim_{x\to 0}\frac{\frac{14}{7(x+7)}-\frac{2(x+7)}{7(x+7)}}{x}$

$\lim_{x\to 0}\frac{\frac{14-2x-14}{7(x+7)}}{x} = \lim_{x\to 0}\frac{\frac{-2x}{7(x+7)}}{x}$

$\lim_{x\to 0}\frac{-2\cancel{x}}{7(x+7)}\cdot\frac{1}{\cancel{x}} = \frac{-2}{7(7)} = \frac{-2}{49}$

11. $\lim_{x\to\infty}\frac{x+40000}{.01x^2+3x} = 0$

HA $y=0$

12. $\lim_{x\to -\infty}\frac{4x^5-x^2+5}{7-x^2} = \infty$

Power function $\frac{4x^5}{-x^2}$

$= -4x^3$

$\lim_{x\to -\infty} -4x^3 = -4(-\infty)^3$

-4 (−huge)

$+\infty$

13. $\lim_{x\to\infty}(3-2x^{-2})$

$\lim_{x\to\infty} 3 - \lim_{x\to\infty} 2x^{-2}$

$\lim_{x\to\infty} 3 - \lim_{x\to\infty}\frac{2}{x^2}\to 0$

$3 - 0$

3

14. $\lim_{x \to \infty} \frac{\cancel{\cos^2 x}}{\cancel{x^2}}$ 0

15. $\lim_{x \to 0} \frac{\sqrt{49-x}-7}{x} \cdot \frac{\sqrt{49-x}+7}{\sqrt{49-x}+7}$

$\lim_{x \to 0} \frac{49-x-49}{x(\sqrt{49-x}+7)}$

$\lim_{x \to 0} \frac{-x}{\cancel{x}(\sqrt{49-x}+7)}$

$\lim_{x \to 0} \frac{-1}{\sqrt{49-x}+7} = \frac{-1}{14}$

16. $\lim_{x \to 4} 7$ $= 7$

Find all x-interval(s) where $f(x)$ is continuous:

17. $f(x) = 4 - 2x + x^4$

$(-\infty, \infty)$

18. $f(x) = \sqrt{3-4x}$

$3 - 4x \geq 0$

$3 \geq 4x$

$\frac{3}{4} \geq x$

$(-\infty, \frac{3}{4}]$

19. $f(x) = \frac{2x-1}{x^2+4x}$

$x^2 + 4x \neq 0$

$x(x+4) \neq 0$

$x \neq 0 \quad x \neq -4$

$(-\infty, -4) \cup (-4, 0) \cup (0, \infty)$

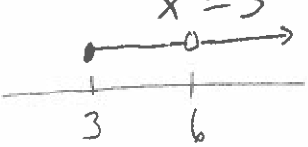
20. $f(x) = \frac{\sqrt{x-3}}{x-6}$

$x - 6 \neq 0$

$x \neq 6$

also $x - 3 \geq 0$

$x \geq 3$



$[3, 6) \cup (6, \infty)$

21. $f(x) = \sqrt{x^3 - 4x^2 - 21x}$

$x^3 - 4x^2 - 21x \geq 0$

$x(x^2 - 4x - 21) \geq 0$

$x(x-7)(x+3) \geq 0$

$x = 0 \quad x = 7 \quad x = -3$

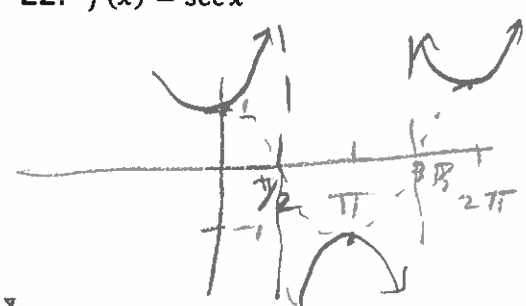
try -4 | try -1 | try 1 | try 8

$- \cdot - \cdot -$ $\quad -3 \quad$ $- \cdot - \cdot +$ $\quad 0 \quad$ $+ \cdot - \cdot +$ $\quad 7 \quad$ $+ \cdot + \cdot +$

$\ominus$ no $\quad \oplus$ yes $\quad \ominus$ no $\quad \oplus$ yes

$[-3, 0] \cup [7, \infty)$

22. $f(x) = \sec x$



x; $x \neq \frac{\pi}{2} + \pi K$

cannot be $\frac{\pi}{2}$ and every π later where K is any integer

23. Find all points of discontinuity, if any: $f(x)=\begin{cases} 3x-1, & x\le -2 \\ x^2-3, & -2<x<1 \\ 5\cos(x-1)-7, & x\ge 1 \end{cases}$

$\lim\limits_{x\to -2^-} 3x-1 = 3(-2)-1 = -6-1 = -7$

$\lim\limits_{x\to -2^+} x^2-3 = (-2)^2-3 = 4-3 = 1$ $\neq$ $\lim\limits_{x\to -2^-} f(x) \neq \lim\limits_{x\to -2^+} f(x)$

so discontinuous also at $x=-2$

$\lim\limits_{x\to 1^-} x^2-3 = (1)^2-3 = -2$ = so does $f(1) = -2$ also

$\lim\limits_{x\to 1^+} 5\cos(1-1)-7 = 5\cos(0)-7 = 5(1)-7 = -2$ ← yes

continuous at $x=1$

24. Find the value of k so that $g(x)$ is continuous everywhere:

$$g(x)=\begin{cases} x^2-2x+3\,, & x\le -2 \\ kx+1, & x>-2 \end{cases}$$

$\lim\limits_{x\to -2^-} x^2-2x+3 = (-2)^2-2(-2)+3 = 4+4+3 = 11$

$\lim\limits_{x\to -2^+} Kx+1 = K(-2)+1 = -2k+1$

so, $-2k+1 = 11$

$\quad -1 \quad -1$

$-2k = 10$

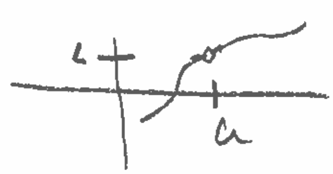
$\frac{-2k}{-2} = \frac{10}{-2}$

$K = -5$

25. True or False: $\lim\limits_{x\to a} x\cdot f(x) = x\cdot \lim\limits_{x\to a} f(x)$ false

$= \lim\limits_{x\to a} x \cdot \lim\limits_{x\to a} f(x)$ would be true

26. True or False: If $\lim\limits_{x\to a} f(x) = L$, then $f(x)$ is continuous at $x=a$ false



Evaluate:

27. $\lim\limits_{x \to \infty} \frac{\sqrt{4x^6+11x^2+1}}{7x^3-\sqrt{x^6+3x^4+x^2+3}}$

$\lim\limits_{x\to\infty} \frac{\sqrt{\frac{4x^6}{x^6}+\frac{11x^2}{x^6}+\frac{1}{x^6}}}{\frac{7x^3}{x^3}-\sqrt{\frac{x^6}{x^6}+\frac{3x^4}{x^6}+\frac{x^2}{x^6}+\frac{3}{x^6}}}$

$= \lim\limits_{x\to\infty} \frac{\sqrt{4+11/x^4+1/x^6}}{7-\sqrt{1+\frac{3}{x^2}+\frac{1}{x^4}+\frac{3}{x^6}}} = \frac{\sqrt{4}}{7-\sqrt{1}} = \frac{2}{6} = \frac{1}{3}$

28. $\lim\limits_{x \to -\infty} \frac{\sqrt{4x^6+11x^2+1}}{7x^3-\sqrt{x^6+3x^4+x^2+3}}$

Same except $\div -x^3$

$\lim\limits_{x\to-\infty} \frac{\sqrt{4+11/x^4+1/x^6}}{-7-\sqrt{1}} = \frac{\sqrt{4}}{-7-1} = \frac{2}{-8} = -\frac{1}{4}$

For Questions 29 – 36, let $f(x) = \frac{x^2+x-6}{x^2-4} = \frac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$ hole at $x=2$ VA at $x=-2$

29. $\lim\limits_{x\to 2} f(x) = \frac{2+3}{2+2} = \frac{5}{4}$

30. $\lim\limits_{x\to -2} f(x) =$ DNE

choose -2.1

$\lim\limits_{x\to -2^-} \frac{-2.1+3}{-2.1+2} = \frac{+}{-} = -\infty$

choose -1.9

$\lim\limits_{x\to -2^+} \frac{-1.9+3}{-1.9+2} = \frac{+}{+} = +\infty$

$\neq$

31. $\lim\limits_{x\to \infty} f(x) = 1$

H.A. at $y = \frac{1}{1}$

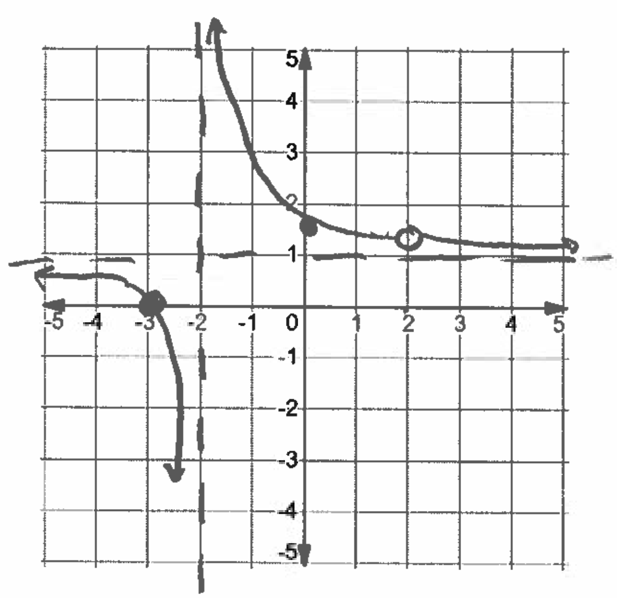
32. The **equation** of the horizontal asymptote of $f(x)$ is: $y = 1$

33. The **equation(s)** of the vertical asymptote(s) of $f(x)$ is/are: $x = -2$

34. Is there a hole in the graph of $f(x)$? yes If so, at $x =$ 2

35. Does the function have a removable discontinuity? If so, it occurs at $x =$ 2 (hole at $(2, \frac{5}{4})$)

36. Use this information (and find the x-intercept) to make a sketch of $f(x)$.



x-int $y=0$

$0 = \frac{x+3}{x+2}$

$0 = x+3$

$-3 = x$ $(-3, 0)$

y-int $x=0$

$\frac{0+3}{0+2} = \frac{3}{2}$ $(0, \frac{3}{2})$