MATH 1401 **Application Project** **NAMES:** Ashley, Brandon, Leah, Melanie, and Nicole

The maximum number of students to a group is 5. Only one project is to be submitted per group. This project must be uploaded to Canvas as a pdf no later than **November 30th at 11:59 pm. Do not approximate any values unless you are asked to do so.**

This is very important! If you have a group of 2-5 people, please send me an email with the names in your group by the end of the day on Friday, November 3rd and I will reply with your dimensions and unit costs for the project. If I don't hear from you by then, I will send out an email to random groups of four with your dimensions for the project. Please work together and check each other's work to minimize mistakes. You must get the dimensions and unit costs from me.

Building the most economical shed (or minimizing the cost of building a shed). A shed is to be constructed based upon the figure below. **The depth of the shed must be** 3 times the length. Find the minimum cost to build the shed and the values of x and y that minimize the cost.

The volume of your shed must be 570 ft^3. This is the constraint.

The unit cost to build each section is:

Base: $ 4.60 per square foot

Sides: $ 5.60 per square foot

Roof: $ 6.60 per square foot

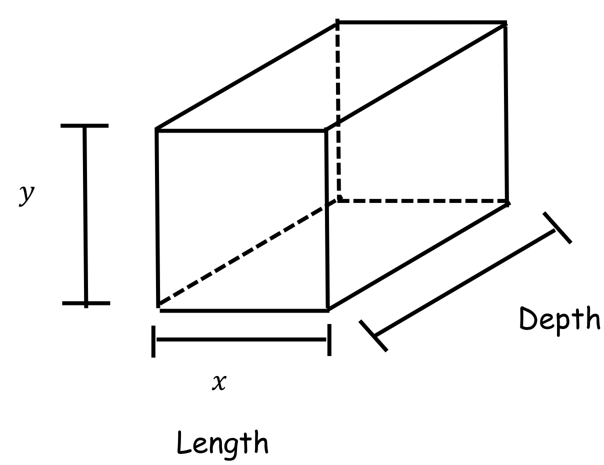
D = 3x

V = D * x * y = 570

x * 3x * y = 570

y = 570 / 3x^2

$$y = \frac{190}{x^2}$$



1. Creating the Objective Function:

The cost to build any section of the shed is the unit cost $\left(\frac{\$}{ft^2}\right)$ to build a section multiplied by the number of square feet of a section. We have the unit costs, so now we need to find the square footage of each section in terms of x and/or y.

a. The base:

Depth = 3x　　Length = x　　Area = 3x * x = 3x^2 sqft

Cost = 3x^2 * 4.60 = $13.80x^2

b. The sides:

1. Left side and right side. Since these sides have the same dimensions, find the area and cost of the left side and then double it.

Depth = 3x　　Height = y　　Area of both = 3xy sqft

Cost of both = 2 * 5.60 * 3xy = $33.60xy

2. Front side and back side. Since these sides have the same dimensions, find the area and cost of the front side and then double it.

Length = x　　Height = y　　Area of both = xy = xy sqft

Cost of both = 2 * 5.60 * xy = $11.20xy

c. The roof:

Depth = 3x　　Length = x　　Area = 3x * x = 3x^2 sqft

Cost = 6.60 * 3x^2 = $19.80x^2

d. Now we can write our cost equation in terms of x and y.

The objective function is the total cost to build the shed. Total cost is the cost to build the base and the sides and the roof. Since the cost to build the left and right sides are the same (and the front and back) combine those cost terms to get:

Total Cost = $13.80x^2 (cost of base) + $35.60xy (cost of left and right) + $11.20xy (cost of front and back) + $19.80x^2 (cost of roof)

After combining like terms: Total Cost = $33.60x^2 + $46.80xy

The constraint is that the volume of the shed must be 570 ft^3. The formula for the volume of our shed in terms of x and y is: (Recall volume of a box is length times width times height)

3x^2y = 570

y = 570 / 3x^2

y = 190 / x^2

570 = x * 3x * y = 3x^2y

(our volume) (length times width times height)

Solve this volume equation for y: $y =$ 190 / x^2

Replace y in your cost function to make it a function of x only. When you are done simplifying the cost equation you should get two terms: one with an x^2, one with an x in the denominator of a fraction. Depending upon your numbers, the coefficient of the quantity with x in the denominator might contain a decimal. DO NOT APPROXIMATE. I would use the MATH FRAC feature on a graphing calculator to convert this quantity to a fraction. Show your work below.

$33.60x^2 + $46.80xy

33.60x^2 + 46.80x * (190/x^2)

33.60x^2 + 46.80 * 190 / x

$33.60x^2 + $8892/x

$C(x) =$ $ 33.60 x^2 + $ 8892/x

2. The domain of the Objective Function:

The domain in this problem represents the possible lengths of the base of our shed that can created with our fixed volume. Since our shed can be either very wide (large value of x) and short or very thin (small value of x) and tall, the domain of x is $(0, \infty)$.

3. The Critical Values of the Objective Function:

a. Now we need to find our derivative, $C'(x)$. Again, depending upon your numbers, a fractional coefficient might contain a decimal. DO NOT APPROXIMATE. I would use the MATH FRAC feature on a graphing calculator to convert this quantity to a fraction.

$ 33.60 x^2 + $ 8892/x
67.20x - 8892/x^2

$C'(x) =$ $67.20x - $8892 / x^2 ____________________

b. and then find any critical values on our domain. Show the work below to obtain your critical value. State the exact value and if you need to approximate, do so to **4 decimal places.**

Critical Points are when the derivative equals 0. They can represent a local maximum or minimum

67.20x - 8892/x^2 = 0

67.2x^3 - 8892 = 0

67.2x^3 = 8892

x^3 = 8892 / 67.2

x = (8892 / 67.2) ^ 1/3

x = 5.0958

Critical value (exact): $x =$ (8892 / 67.2) ^ 1/3

Critical value (approximate to 4 decimal places): $x =$ 5.0958

4. Verification That Our Critical Value is the Location of the Absolute Minimum Cost:

Verification is a required part of all optimization problems. Recall that the First and Second Derivative Tests are a test for local extrema, so we have to do a little work to make sure that our critical value is the location of the absolute minimum cost. Since our domain is the open interval $(0, \infty)$, you have to do the following:

a. Use a Derivative Test to verify that our critical value is the location of a local minimum.

First Derivative Test should be negative on the left and positive on the right of the given critical point

Left: x = 5	Right: x = 6
67.20(5) - 8892/(5)^2	67.2(6) - 8892/(6)^2
336 - 8892/25	407.2 - 8892/36
336 - 355.68 = Negative	407.2 - 247 = Positive

The critical point is confirmed to be a local minimum!

b. Is the critical value the only critical value on the domain? yes________ If so, then we know that if a function has only one critical value on the domain and it is the location of a local minimum, then it is also the location of the absolute minimum.

other critical point is negative and thus out of the domain.

5. Answer the Following Questions:

If you need to approximate, do so to **4 decimal places.** Include units.

a. What are the values of x and y that minimize the cost?

$x =$ 5.0958

$y =$ 190/(5.0958) ^ 2 = 7.3169

b. What are the dimensions of the shed?

Length = 5.0958

Depth = 15.2874

Height = 7.3169

c. What is the minimum cost to build the shed? (Round to 2 decimal places for dollars and cents)

Cost = 33.6(5.0958)^2 + 8892/5.0958 = $ 2617.46

d. Calculate the second derivative, $C''(x)$.

C' = 67.20x - 8892/x^2
C" = 67.2 + 17784/x^3

$C''(x) =$ 67.2 + 17784/x^3

This problem can be solved without calculus using some sort of graphing technology. We will use DESMOS. Graph the cost function, the derivative of the cost function, and the second derivative of the cost function on the same set of axes. **Be sure that you pick a reasonable set of x-values and a range of y-values that allows me to see all the exciting information from your graphs.**

Answer the following:

e. What x-value did you calculate to provide the minimum cost? x = 5.0958

f. What do you notice about the graph of the cost function at this x-value? C reached a minimum at this point

g. What do you notice about the graph of the derivative at this x-value? The graph of the derivative hits 0

h. What do you notice about the sign of the second derivative from the graph at this x-value?

Positive or Negative (Circle one)

i. Explain how the graph of the derivative supports the conclusion of the location of a local minimum using the First Derivative Test.

The graph of the derivative supports the conclusion because it is a critical point (reaches 0 at the given x value) and is a

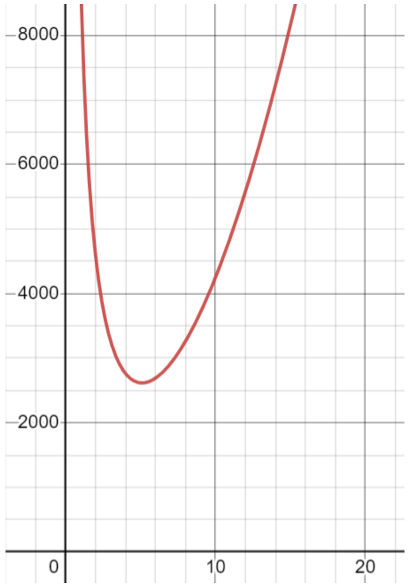
local minimum (goes from negative to positive accordingly from the left to the right of our critical point).

j. Explain how the graph of the second derivative supports the conclusion of the location of a local minimum using the Second Derivative Test.

The graph of the Second Derivative supports the location of a local minimum since the graph is positive throughout the domain

proving concavity is facing up. A local minimum has upward facing concavity.

k. Submit your graphs to your instructor by sending the url via email.

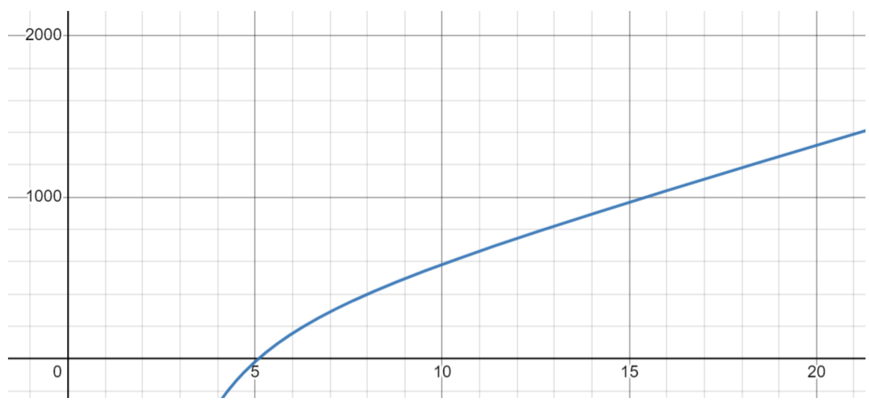


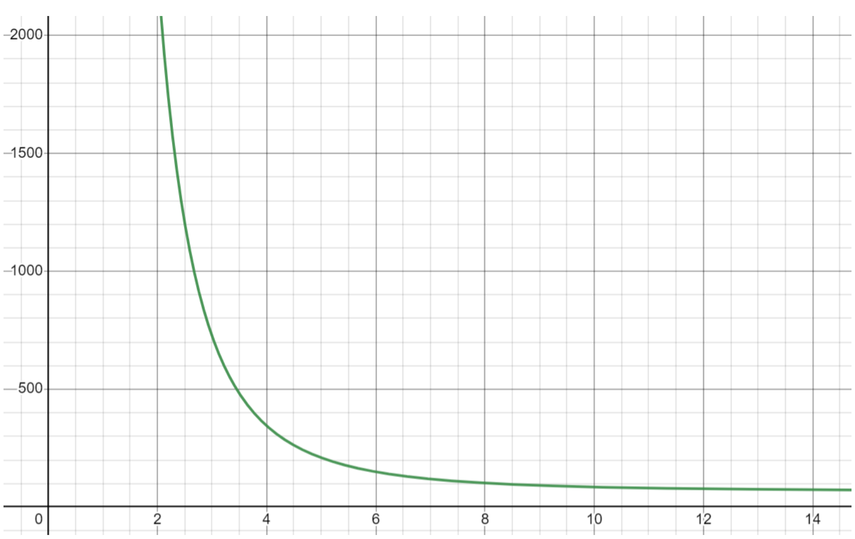
C = 33.6x^2 + 8892/x

original cost function

C' = 67.2x 8892/x^2
first derivative of the cost function

x = 0 at 5.0958





C'' = 67.2 + 17784/x^3
second derivative of the cost function

always positive proves the entire cost function is concave up and proves that our critical point is a local minimum