MATH 1401

Spring 2023 Chapter 3 Exam Review

Review Day: Tuesday, March 14th

Happy PI Day

Chapter 3 Exam: Thursday, March 16th

Instructor: Whitten

Instructions:

1. You will have a total of **1 hour and 15** minutes to complete this exam.
2. **Calculators, electronic devices, scratch paper, and notecards are not allowed on this exam.**
3. **Visible phones will result in a zero on the exam.**
4. Show ALL of your work. Partial credit can be awarded for work that is legible and mathematically correct.
5. Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%.
6. We are not out to trick you!!! Make sure you understand the concepts on this review.
7. Take a deep breath, relax, and good luck!!
8. The following formulas will be provided on the exam:

$$\frac{d}{dx}sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}tan^{-1}x = \frac{1}{1+x^2} \qquad \frac{d}{dx}sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$$

$\to$ only derivatives provided on exam

MATH 1401 **Chapter 3 Exam Review** **NAME:** Key

Using the definition of the derivative, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, find the slope of the tangent line: In order to receive full credit, you must use the definition and simplify accordingly.

1. $f(x) = 2x^2 - x$ $f(x+h) = 2(x+h)^2 - (x+h)$

$$\lim_{h \to 0} \frac{2(x+h)^2 - (x+h) - (2x^2 - x)}{h}$$

$$\lim_{h \to 0} \frac{2(x^2+2xh+h^2) - x - h - 2x^2 + x}{h}$$

$$\lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{x} - h - \cancel{2x^2} + \cancel{x}}{h}$$

$$\lim_{h \to 0} \frac{4xh + 2h^2 - h}{h} = \lim_{h \to 0} \frac{\cancel{h}(4x + 2\cancel{h} - 1)}{\cancel{h}} = \boxed{4x - 1 = f'(x)}$$

2. $f(x) = \sqrt{x}$ $f(x+h) = \sqrt{x+h}$

$$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$\lim_{h \to 0} \frac{x + h - x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}} = f'(x)}$$

3. $f(x) = \frac{2}{x}$ $f(x+h) = \frac{2}{x+h}$

$$\lim_{h \to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h}$$

$$\lim_{h \to 0} \frac{\frac{2}{x+h} \cdot \frac{x}{x} - \frac{2}{x} \cdot \frac{x+h}{x+h}}{h}$$

$$\lim_{h \to 0} \frac{\frac{2x}{x(x+h)} - \frac{2x+2h}{x(x+h)}}{h}$$

$$\lim_{h \to 0} \frac{\frac{2x - 2x - 2h}{x(x+h)}}{h}$$

$$\lim_{h \to 0} \frac{\frac{-2h}{x(x+h)}}{h}$$

$$\lim_{h \to 0} \frac{-2h}{x(x+h)} \cdot \frac{1}{h}$$

$$\lim_{h \to 0} \frac{-2}{x(x+h)} = \boxed{\frac{-2}{x^2} = f'(x)}$$

True or False:

4. A function that is differentiable everywhere must also be continuous everywhere. TRUE

5. A function that is continuous everywhere must also be differentiable everywhere. False

cusp

For questions 6-7, the derivative of a function is $f'(x) = \frac{(x+1) \cdot 2e^{2x} - e^{2x}}{(x+1)^2}$. Are the following simplifications correct or incorrect?

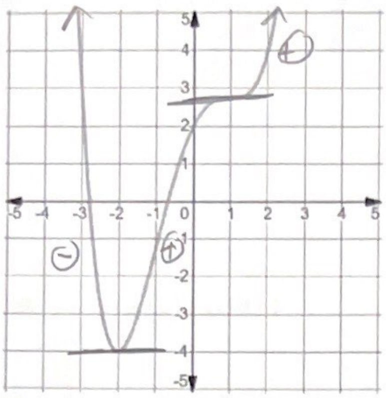
6. $f'(x) = \frac{(x+1) \cdot 2e^{2x} - e^{2x}}{(x+1)^2} = \frac{2e^{2x} - e^{2x}}{x+1} = \frac{e^{2x}}{x+1}$ Correct or [✓ Incorrect]? Circle One

7. $f'(x) = \frac{(x+1) \cdot 2e^{2x} - e^{2x}}{(x+1)^2} = \frac{e^{2x}[2(x+1)-1]}{(x+1)^2} = \frac{e^{2x}(2x+1)}{(x+1)^2}$ [✓ Correct] or Incorrect? Circle One

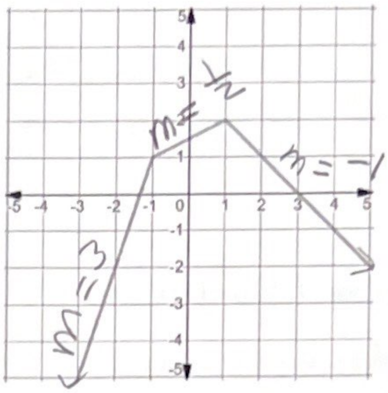
Given the graph of a function, sketch the graph of the derivative on the axes provided.

Domain of
g(x) + f(x)
$(-\infty, \infty)$

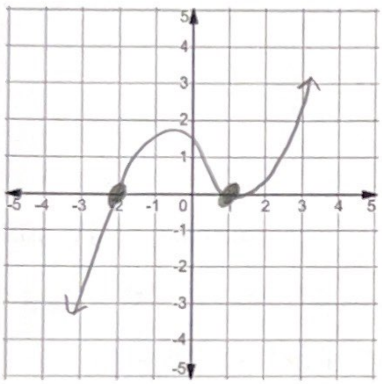
8. $f(x)$



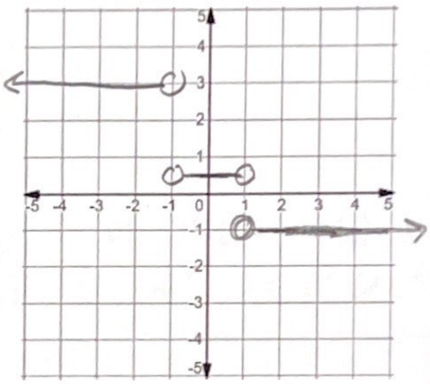
9. $g(x)$



$f'(x)$



$g'(x)$



Using the graph of $f(x)$ from question 8, answer the following:

10. True or False? $f(-1) > 0$.

false $f(-1) = -1$

not > 0

11. [✓ True] or False? $f'(-1) > 0$.

slope of tangent line at $x = -1$ is positive

Find the derivative of the following:

12. $f(x) = 2x^4 - x^3 + 3x^2 - x + 6$

$\boxed{f'(x) = 8x^3 - 3x^2 + 6x - 1}$

13. $f(x) = 3x^{-2}$

$f'(x) = -6x^{-3}$

OR $-\frac{6}{x^3}$

14. $f(x) = \frac{4}{x^5} = 4x^{-5}$

$f'(x) = -20x^{-6}$

OR $\frac{-20}{x^6}$

15. $f(x)=10e^{-5x}$

$f'(x) = 10e^{-5x}(-5)$

$f'(x) = -50e^{-5x}$

16. $f(x)=xe^{2x}$

$f'(x) = x(2e^{2x}) + e^{2x}(1)$

OR

$f'(x) = e^{2x}(2x+1)$

17. $f(x)=e^{5}$

$f'(x) = 0$

18. $y=\sin(x)+\cos(x)$

$y' = \cos x - \sin x$

19. $f(x)=\csc(x)-\sec(x)$

$f'(x) = -\csc x \cot x - \sec x \tan x$

20. $f(x)=4\sin(x)\cos(x)$

$f'(x) = 4\sin x(-\sin x) + \cos x(4\cos x)$

$= -4\sin^2 x + 4\cos^2 x$

OR $4(-\sin x + \cos^2 x)$

21. $f(x)=x^2\cdot\tan(x)$

$f'(x) = x^2\sec^2 x + \tan x(2x)$

22. $f(x)=\dfrac{x^2-8}{x+6}$

$$f'(x) = \frac{(x+6)2x - (x^2-8)(1)}{(x+6)^2}$$

$$= \frac{2x^2+12x-x^2+8}{(x+6)^2}$$

$$f'(x) = \frac{x^2+12x+8}{(x+6)^2}$$

23. $f(x)=\dfrac{e^{2x}}{x^2}$

$$f'(x) = \frac{x^2(2e^{2x}) - e^{2x}(2x)}{(x^2)^2}$$

$$= \frac{2x^2e^{2x} - 2xe^{2x}}{x^4}$$

OR $$= \frac{2xe^{2x}(x-1)}{x^4}$$

$$= \frac{2e^{2x}(x-1)}{x^3}$$

Find the value of the derivative at the value of $x=a$:

24. $f(x)=4x^3-2x^2+9x-17$ at $a=-1$.

$f'(x)=12x^2-4x+9$

$f'(-1)=12(-1)^2-4(-1)+9$

$f'(-1)=12+4+9=\boxed{25}$

25. Find $\frac{d^3y}{dx^3}$ if $y=e^{4x}$.

$\frac{dy}{dx}=4e^{4x}$

$\frac{d^2y}{dx^2}=16e^{4x}$

$\boxed{\frac{d^3y}{dx^3}=64e^{4x}}$

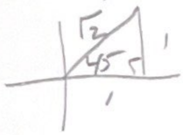
26. Given $f(x)=\tan(x)$, find the equation of the tangent line to $f(x)$ at $a=\frac{\pi}{4}$.

$f(\frac{\pi}{4})=\tan(\frac{\pi}{4})=1$

$\boxed{y-1=2(x-\frac{\pi}{4})}$

$f'(x)=\sec^2 x$

$f'(\frac{\pi}{4})=(\sec\frac{\pi}{4})^2$



$(\sqrt{2})^2=2$

27. Find the value(s) of x where $f(x)=x-\sin(x)$ will have a horizontal tangent line. deriv $=0$

$f'(x)=1-\cos x$

$0=1-\cos x$

$\cos x=1$ at $x=0, 2\pi, -2\pi, 4\pi, -4\pi, \ldots$

$x=2\pi n$

where n is an integer

28. The population (in thousands) of the state of Georgia from 1995 (t=0) is modeled by $p(t)=-0.27t^2+101t+7055$. (Calculator allowed for this question)

a. Find the average rate of change of the population from 1995 to 2005.

$AROC=\frac{P(10)-P(0)}{10-0}=\frac{-0.27(10)^2+101(10)+7055-(7055)}{10}$

$=98.3$ thousands of people/year

b. Find the instantaneous rate of change of the population in 2003.

$IROC=p'(t)=-0.54t+101$

$p'(8)=-0.54(8)+101=96.68$ thousands of people/year

29. A ball is thrown vertically upward from the edge of 80 foot cliff with an initial velocity of $32\frac{ft}{sec}$ on a planet where the acceleration due to gravity is $8\frac{ft}{sec^2}$. The height of the ball in feet at time t in seconds is $\overset{\text{Position}}{h(t)} = 80 + 32t - 4t^2$.

$v(t) = 32 - 8t$ $a(t) = -8$

a. What is the maximum height of the ball?

when $v(t) = 0$

$0 = 32 - 8t$

$8t = 32$

$t = 4$ sec

$h(4) = 80 + 32(4) - 4(4)^2$

$= \boxed{144 \text{ feet}}$

b. When does the ball hit the ground? when position is zero

$0 = 80 + 32t - 4t^2$

$0 = 4(20 + 8t - t^2)$

$0 = 4(10 - t)(2 + t)$

$t = 10$ $t = \cancel{-2}$

hits the ground after 10 seconds

or $t = 10$

c. What is the impact velocity of the ball?

$v(10) = 32 - 8(10) = 32 - 80 = \boxed{-48 \text{ f/sec}}$

30. The position of an object (in meters) moving horizontally at time t (in seconds) is determined by the position function $x(t) = t^3 - 12t^2 + 36t - 5$, $t \geq 0$. Find the following:

a. The velocity function and the velocity of the object at $t = 1$ seconds.

$v(t) = 3t^2 - 24t + 36$

$v(1) = 3 - 24 + 36 = 15$ m/sec

b. The acceleration function and the acceleration of the object at $t = 1$ seconds.

$a(t) = 6t - 24$

$a(1) = 6 - 24 = -18$ m/s^2

c. The time(s) when the object is stopped. $v(t) = 0$

$0 = 3t^2 - 24t + 36$

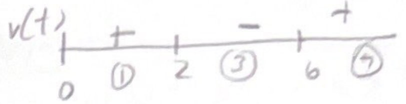
$0 = 3(t^2 - 8t + 12)$

$0 = 3(t - 6)(t - 2)$

$t = 6$ $t = 2$

at $t = 2, 6$ seconds

d. The time(s) when the object is changing direction.



$v(1) = + \cdot - \cdot - = +$

$v(3) = + \cdot - \cdot + = -$

$v(7) = + \cdot + \cdot + = +$

changes direction at $t = 2, 6$ seconds

e. The time interval(s) when the object is moving to the left. $v(t) < 0$

$(2, 6)$

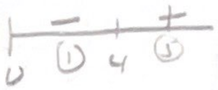
f. The time interval(s) when the object is speeding up and slowing down.

$a(t) = 0$

$0 = 6t - 24$

$24 = 6t$

$4 = t$



$a(t)$	−	−	+	+
$v(t)$	+	−	−	+
	0	2	4	6

speeding up

$(2,4), (6,\infty)$

slowing down

$(0,2), (4,6)$

Find the derivative:

31. $y = e^{\sqrt{x}}$ $\quad y = e^{x^{\frac{1}{2}}}$

$y' = e^{\sqrt{x}} \cdot \frac{1}{2} x^{-\frac{1}{2}}$

$\boxed{y' = \frac{e^{\sqrt{x}}}{2\sqrt{x}}}$

32. $f(x) = \tan(5x^2)$

$\boxed{f'(x) = \sec^2(5x^2) \cdot 10x}$

33. $g(x) = \sqrt{x^2+1} = (x^2+1)^{\frac{1}{2}}$

$g'(x) = \frac{1}{\cancel{2}}(x^2+1)^{-\frac{1}{2}}(\cancel{2}x)$

$\boxed{g'(x) = \frac{x}{\sqrt{x^2+1}}}$

34. $y = \frac{4}{(x^2+1)^2} = 4(x^2+1)^{-2}$

$y' = -8(x^2+1)^{-3}(2x)$

$y' = -16x(x^2+1)^{-3}$

$\boxed{y' = \frac{-16x}{(x^2+1)^3}}$

35. $f(x)=\cot^5 x \quad = (\cot x)^5$

$f'(x) = 5(\cot x)^4(-\csc^2 x)$

$$\boxed{f'(x) = -5\cot^4 x\csc^2 x}$$

36. $g(x)=\cot x^5$

$$\boxed{\begin{gathered} g'(x) = -\csc^2(x^5)\cdot 5x^4 \\ \text{OR } -5x^4\csc^2(x^5) \end{gathered}}$$

37. $f(x)=\left(\dfrac{3x}{4x+2}\right)^4$

$$f'(x) = 4\left(\frac{3x}{4x+2}\right)^3\left(\frac{(4x+2)(3)-3x(4)}{(4x+2)^2}\right)$$

$$= 4\,\frac{(3x)^3}{(4x+2)^3}\left(\frac{12x+6-12x}{(4x+2)^2}\right)$$

$$= \frac{4(27x^3)}{(4x+2)^3}\left(\frac{6}{(4x+2)^2}\right)$$

$$\boxed{f'(x) = \frac{648x^3}{(4x+2)^5}}$$

38. $y=\ln|\cos(3x)|$

$$y' = \frac{1}{\cos(3x)}(-\sin(3x))\cdot 3$$

$$\boxed{y' = -3\tan(3x)}$$

39. $f(x)=\sqrt[4]{x^5} \quad = x^{\frac{5}{4}}$

$$\boxed{\begin{gathered} f'(x) = \frac{5}{4}x^{\frac{1}{4}} \\ \text{OR } \frac{5}{4}\sqrt[4]{x} \end{gathered}}$$

40. $g(x)=\dfrac{12}{\sqrt[3]{x^2}} \quad = 12x^{-2/3}$

$$g'(x) = -\frac{2}{3}\cdot 12x^{-5/3}$$

$$= -8x^{-5/3}$$

$$\boxed{g'(x) = \frac{-8}{x^{5/3}} \quad \text{OR} \quad \frac{-8}{\sqrt[3]{x^5}}}$$

41. $h(x)=\left(\sqrt{2x^3+x^2}\right)^5 = \left((2x^3+x^2)^{\frac{1}{2}}\right)^5$

$h(x) = (2x^3+x^2)^{5/2}$

$$\boxed{\begin{gathered} h'(x) = \frac{5}{2}(2x^3+x^2)^{3/2}(6x^2+2x) \\ \text{OR } h'(x) = \frac{5}{2}(2x^3+x^2)^{3/2}(2x)(3x+1) \end{gathered}}$$

42. $y=7^{x^3}$

$$\boxed{y' = 7^{x^3}\cdot \ln 7\cdot 3x^2}$$

43. $y = \log_8(\csc x)$ $= \dfrac{\log \csc x}{\log 8}$

$y = \dfrac{1}{\log 8} \cdot \log \csc x$

$y' = \dfrac{1}{\log 8} \cdot \dfrac{1}{\cancel{\csc x}} \cdot -\cancel{\csc x} \cot x$

$y' = \dfrac{-\cot x}{\log 8}$

44. $y = \ln\left(\dfrac{x^2+1}{x-1}\right)$

$y' = \dfrac{1}{\left(\frac{x^2+1}{x-1}\right)} \cdot \dfrac{(x-1)(2x)-(x^2+1)(1)}{(x-1)^2}$

$y' = \dfrac{\cancel{x-1}}{x^2+1} \cdot \dfrac{2x^2-2x-x^2-1}{(x-1)^{\cancel{2}}}$

$$\boxed{y' = \dfrac{x^2-2x-1}{(x^2+1)(x-1)}}$$

45. $h(x) = 4^{-x} \cdot \sin x$

$$\boxed{\begin{array}{l} h'(x) = 4^{-x}\cos x + (\sin x)(4^{-x} \cdot \ln 4 \cdot (-1)) \\ \text{OR} \\ h'(x) = 4^{-x}(\cos x - (\sin x)\ln 4) \end{array}}$$

46. $y = x^3 \cdot \log_4(4x^2)$ $= x^3 \cdot \dfrac{\log(4x^2)}{\log 4}$

$$\boxed{\begin{array}{l} y'(x) = x^3 \cdot \dfrac{1}{\log 4} \cdot \dfrac{1}{4x^2} \cdot 8x + \dfrac{\log 4x^2}{\log 4} \cdot 3x^2 \\ \text{OR} \quad = \dfrac{2x^2}{\log 4} + \log_4(4x^2) \cdot 3x^2 \end{array}}$$

47. $g(x) = \sin^{-1}(3x)$ $\dfrac{d}{dx}\sin^{-1}(x) = \dfrac{1}{\sqrt{1-x^2}}$

$g'(x) = \dfrac{1}{\sqrt{1-(3x)^2}} \cdot 3$

$$\boxed{g'(x) = \dfrac{3}{\sqrt{1-9x^2}}}$$

48. $y = \tan^{-1}(e^x)$ $\dfrac{d}{dx}\tan^{-1}(x) = \dfrac{1}{1+x^2}$

$y' = \dfrac{1}{1+(e^x)^2} \cdot e^x$

$$\boxed{y' = \dfrac{e^x}{1+e^{2x}}}$$

$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}}$

49. $f(x)=\sec^{-1}\sqrt{x}, x\ge 1$

$f'(x) = \frac{1}{|\sqrt{x}|\sqrt{(\sqrt{x})^2-1}} \cdot \frac{1}{2}x^{-\frac{1}{2}}$

$f'(x) = \frac{1}{2\sqrt{x}\sqrt{x}\sqrt{x-1}}$

$\boxed{f'(x) = \frac{1}{2x\sqrt{x-1}}}$

50. $y=\tan^{-1}\left(\frac{1}{x}\right)$

$\frac{1}{x} = x^{-1}$

$y' = \frac{1}{1+\left(\frac{1}{x}\right)^2} \cdot -x^{-2}$

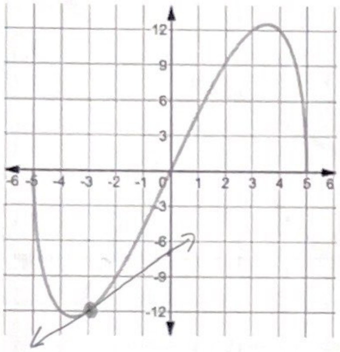
$y' = \frac{-1}{\left(1+\frac{1}{x^2}\right)x^2}$

$\boxed{y' = \frac{-1}{x^2+1}}$

51. Below is a graph of $f(x) = x\cdot\sqrt{25-x^2}$. Find the equation of the tangent line for $f(x)$ at $x=-3$. Draw the tangent line on the graph.

$f(-3) = -3\sqrt{25-(-3)^2} = -3\sqrt{25-9}$
$-3\sqrt{16} = -12$

$\boxed{y+12 = \frac{7}{4}(x+3)}$



$f'(x) = x\cdot\frac{1}{\cancel{2}}(25-x^2)^{-1/2}(-\cancel{2}x)+\sqrt{25-x^2}\,(1)$

$f'(x) = \frac{-x^2}{\sqrt{25-x^2}} + \sqrt{25-x^2}$

$f'(-3) = \frac{-(-3)^2}{\sqrt{25-(-3)^2}} + \sqrt{25-(-3)^2} = \frac{-9}{\sqrt{16}} + \sqrt{16}$

$f'(-3) = -\frac{9}{4} + 4 = -\frac{9}{4} + \frac{16}{4} = \frac{7}{4}$

$y+12 = \frac{7}{4}x + \frac{21}{4}$

$y = \frac{7}{4}x + \frac{21}{4} - \frac{48}{4} = \frac{7}{4}x - \frac{27}{4}$

Simplify: Hint: Think of the right-hand side of $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

52. $\lim_{h\to 0}\frac{\overset{f(x+h)}{\sin(x+h)}-\overset{f(x)}{\sin x}}{h}$

find $f(x) = \sin x$

$\boxed{f'(x) = \cos x}$

53. $\lim_{h\to 0}\frac{2(x+h)^3-(x+h)-\overset{-(2x^3-x)}{2x^3+x}}{h}$

$f(x) = 2x^3 - x$

$\boxed{f'(x) = 6x^2-1}$

Evaluate: Hint: Think of the right hand side of $f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

54. $\lim_{x\to 1}\frac{\overset{f(x)-f(a)}{x^{80}-1}}{x-1}$

$f(x) = x^{80}$ at $a = 1$

$f'(x) = 80x^{79}$

$f'(1) = 80(1)^{79} = \boxed{80}$

55. $\lim_{x\to 0}\frac{\overset{f(x)}{\cos(x)-1}}{x}$

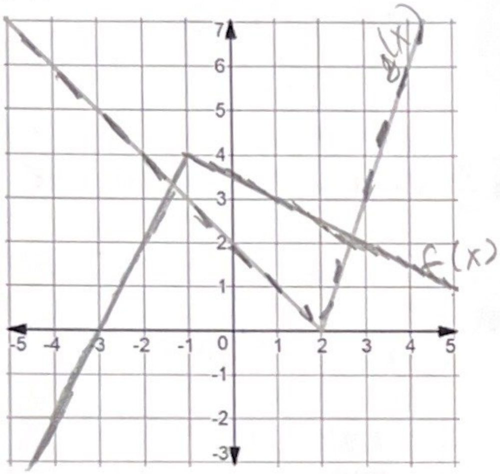
$f(x) = \cos x$

$f'(x) = -\sin x$

$f'(0) = -\sin(0) = \boxed{0}$

Use the graph of functions $f(x)$ and $g(x)$ to answer the following:

$g(x)$



$f(x)$

56. Let $Q(x)=\frac{g(x)}{f(x)}$. Find $Q'(1)$.

$Q'(x) = \frac{f(x)g'(x)-g(x)f'(x)}{(f(x))^2}$

$Q'(1) = \frac{f(1)g'(1)-g(1)f'(1)}{(f(1))^2}$

$= \frac{(3)(-1)-(1)(-\frac{1}{2})}{(3)^2}$

$= \frac{(-3+\frac{1}{2})}{(9)}\cdot\frac{2}{2} = \frac{-6+1}{18} = \boxed{\frac{-5}{18}}$

57. Let $h(x)=g(f(x))$. Find $h'(-3)$.

$h'(x) = g'(f(x))f'(x)$

$h'(-3) = g'(f(-3))f'(-3)$

$= g'(0)(2)$

$= (-1)(2)$

$= \boxed{-2}$

58. Use implicit differentiation to find $\frac{dy}{dx}$ for $3x - x^3\sqrt{y} = 4y^3$.

$$3 - \left(x^3 \cdot \frac{1}{2} y^{-\frac{1}{2}} \frac{dy}{dx} + y^{\frac{1}{2}}(3x^2)\right) = 12y^2\frac{dy}{dx}$$

$$3 - \frac{x^3}{2\sqrt{y}}\frac{dy}{dx} - 3x^2\sqrt{y} = 12y^2\frac{dy}{dx}$$

$$3 - 3x^2\sqrt{y} = 12y^2\frac{dy}{dx} + \frac{x^3}{2\sqrt{y}}\frac{dy}{dx}$$

$$3 - 3x^2\sqrt{y} = \frac{dy}{dx}\left(12y^2 + \frac{x^3}{2\sqrt{y}}\right)$$

$$\frac{\left(3 - 3x^2\sqrt{y}\right)}{\left(12y^2 + \frac{x^3}{2\sqrt{y}}\right)} \cdot \frac{(2\sqrt{y})}{(2\sqrt{y})} = \frac{dy}{dx}$$

$$\boxed{\frac{6\sqrt{y} - 6x^2y}{24y^{\frac{5}{2}} + x^3} = \frac{dy}{dx}}$$

59. Given $x^2 - 2y^2 = 24$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

$\frac{dy}{dx}$: $2x - 4y\frac{dy}{dx} = 0$

$$-4y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{-4y}$$

$$\boxed{\frac{dy}{dx} = \frac{x}{2y}}$$

$\frac{d^2y}{dx^2}$: $\frac{2y(1) - x\left(2\frac{dy}{dx}\right)}{(2y)^2}$

$$= \frac{2y - 2x\left(\frac{x}{2y}\right)}{4y^2}$$

$$= \frac{\left(2y - \frac{x^2}{y}\right)}{\left(4y^2\right)} \cdot \frac{(y)}{(y)}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{2y^2 - x^2}{4y^3}}$$

Use logarithmic differentiation to find the derivative.

60. $y=\dfrac{\left(3x^2-x+7\right)^4}{\sqrt{x^4+8x^2+5}}$

$$\ln y=\ln\left(\frac{(3x^2-x+7)^4}{\sqrt{x^4+8x^2+5}}\right)$$

$$\ln y=\ln(3x^2-x+7)^4-\ln(x^4+8x^2+5)^{\frac{1}{2}}$$

$$\ln y=4\ln(3x^2-x+7)-\tfrac{1}{2}\ln(x^4+8x^2+5)$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{4}{3x^2-x+7}(6x-1)-\frac{1}{2(x^4+8x^2+5)}\cdot(4x^3+16x)$$

$$\frac{dy}{dx}=y\left[\frac{4(6x-1)}{3x^2-x+7}-\frac{4x(x^2+4)}{2(x^4+8x^2+5)}\right]$$

$$\boxed{\frac{dy}{dx}=\frac{(3x^2-x+7)^4}{\sqrt{x^4+8x^2+5}}\left[\frac{4(6x-1)}{3x^2-x+7}-\frac{2x(x^2+4)}{x^4+8x^2+5}\right]}$$

61. $y=(\sin x)^{\tan x}$

$$\ln y=\ln(\sin x)^{\tan x}$$

$$\ln y=\tan x\ln\sin x$$

$$\frac{1}{y}\frac{dy}{dx}=\tan x\cdot\frac{1}{\sin x}\cos x+(\ln\sin x)\sec^2x$$

$$\frac{dy}{dx}=y\left[\tan x\cdot\cot x+\sec^2x\ln\sin x\right]$$

$$\boxed{\frac{dy}{dx}=(\sin x)^{\tan x}\left[1+\sec^2x\ln\sin x\right]}$$

62. Find the derivative of the inverse of $g(x)=2x^3+x+2$. let $f(x)$ be inverse of $g(x)$

want $f'(x)$ switch x and y find deriv.

$$x=2y^3+y+2$$

$$1=6y^2\frac{dy}{dx}+\frac{dy}{dx}$$

$$1=\frac{dy}{dx}(6y^2+1)$$

$$\frac{1}{6y^2+1}=\frac{dy}{dx}$$

$$\boxed{f'(x)=\frac{1}{6y^2+1}}$$

63. Find the derivative of the inverse of $f(x)=x^2+1$, $x>0$ at the point $(5,2)$, which is a point on the graph of the inverse. let $g(x)$ be inverse of $f(x)$

on inverse so $g(5)=2$

$$f(g(x))=x$$

$$f'(g(x))g'(x)=1$$

$$g'(x)=\frac{1}{f'(g(x))}$$

$$g'(5)=\frac{1}{f'(g(5))}=\frac{1}{f'(2)}=\boxed{\frac{1}{4}}$$

$$f'(x)=2x$$

$$f'(2)=4$$

Include units in your final answers for questions 64 and 65.

64. The volume of a spherical hot air balloon is decreasing at a constant rate of $9\pi \frac{m^3}{\text{min}}$ while maintaining its spherical shape. Find the rate of change of the radius of the balloon when the radius is 3 meters. The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.

Know

$\frac{dv}{dt} = -9\pi\, m^3/min$

want

$\frac{dr}{dt}$ when r = 3meters

$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-9\pi = 4\pi (3)^2 \frac{dr}{dt}$

$-9\pi = 36\pi \frac{dr}{dt}$

$\frac{-9\pi}{36\pi} = \frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = -\frac{1}{4}\ m/min}$

65. Sand is being dumped from an overhead bin at a constant rate of $120 \frac{ft^3}{\text{min}}$ and it forms a pile that is in the shape of a cone where the radius of the pile is always 3 times the height. How fast is the height of the pile changing when the pile is $10\ ft$ high? The volume of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.

Know

$\frac{dv}{dt} = 120\, f^3/min$

went

$\frac{dh}{dt}$ wlen h = 10ft

r = 3h

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (3h)^2 h$

$V = \frac{1}{3}\pi 9h^3$

$V = 3\pi h^3$

$\frac{dV}{dt} = 9\pi h^2 \frac{dh}{dt}$

$120 = 9\pi (10)^2 \frac{dh}{dt}$

$120 = 900\pi \frac{dh}{dt}$

$\frac{12\cancel{0}}{90\cancel{0}\pi} = \frac{dh}{dt}$

$\boxed{\frac{2}{15\pi}\ f/min = \frac{dh}{dt}}$