MATH 1401

Spring 2023 Chapter 4 Exam Review

Key

Review Day: Tuesday, April 16^{th}

Chapter 4 Exam: Thursday, April 20^{th}

Instructor: Whitten

Instructions:

1. You will have a total of 1 hour and 15 minutes to complete this exam.
2. **Calculators, electronic devices, scratch paper, and notecards are not allowed on this exam.**
3. Show ALL of your work. Partial credit can be awarded for work that is legible and mathematically correct.
4. Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%.
5. We are not out to trick you!!! Make sure you understand the concepts on this review.
6. Take a deep breath, relax, and good luck!!

MATH 1401 **Chapter 4 Exam Review**

Find the critical value(s) for the following functions. Recall that a critical value is the x-coordinate of the critical point.

1. $f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b$

$0 = 2ax + b$

$-b = 2ax$

$\frac{-b}{2a} = x$

c.p. $x = -\frac{b}{2a}$

2. $f(x) = 2x^3 - 3x^2 - 36x + 12$

$f'(x) = 6x^2 - 6x - 36$

$0 = 6(x^2 - x - 6)$

$0 = 6(x-3)(x+2)$

$x = 3 \quad x = -2$

3. $f(x) = 2x \cdot \ln x + 10$

$f'(x) = 2x \cdot \frac{1}{x} + (\ln x)(2)$

$0 = 2 + 2\ln x$

$-2 = 2\ln x$

$-1 = \ln x$

$\boxed{e^{-1} = x \quad \text{OR } x = \frac{1}{e}}$

4. $f(x) = \frac{x}{x^2 + 1}$

$f'(x) = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2}$

$0 = \frac{x^2 + 1 - 2x^2}{(x^2+1)^2}$ ← always (+) so $f'(x)$ never undefined

$0 = 1 - x^2$

$x^2 = 1$

$x = \pm 1$

check endpoints

Find the absolute extrema of the following functions on the indicated closed interval:

7. $f(x)=2x^3+3x^2-12x+1$ on $[-1,3]$

$f(-1)=2(-1)^3+3(-1)^2-12(-1)+1=14$

$f(3)=2(3)^3+3(3)^2-12(3)+1=46$

	x	y
endpts	-1	14
	3	46
c.p.	1	-6

$f'(x)=6x^2+6x-12$

$0=6(x^2+x-2)$

$0=6(x+2)(x-1)$

$x=-2$ $x=1$

not on interval

$f(1)=2+3-12+1=-6$

$\boxed{\text{abs max of } 46 \text{ at } x=3 \quad \text{abs min of } -6 \text{ at } x=1}$

8. $f(x)=\dfrac{x}{(x^2+3)^2}$ on $[-2, 2]$

$f'(x)=\dfrac{(x^2+3)^2(1)-x(2(x^2+3))(2x)}{((x^2+3)^2)^2}$

$f'(x)=\dfrac{(x^2+3)[(x^2+3)-4x^2]}{(x^2+3)^4}$

$f'(x)=\dfrac{3-3x^2}{(x^2+3)^3}$ always (+) so f'(x) never undefined

$0=3-3x^2$

$3x^2=3$

$x^2=1$

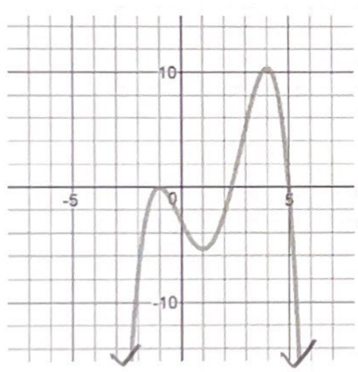
$x=\pm 1$

x	y
-2	$-\frac{2}{49}$
2	$\frac{2}{49}$
1	$\frac{1}{16}$
-1	$-\frac{1}{16}$

abs. max of $\frac{1}{16}$ at $x=1$

abs. min of $-\frac{1}{16}$ at $x=-1$

9. Below is the graph of a **function** $f(x)$ with a domain of all real numbers. Find the following (if possible):



a. The x-value(s) of all locations of all critical point(s) of $f(x)$: x= $-1, 1, 4$ $f'(x)=0$

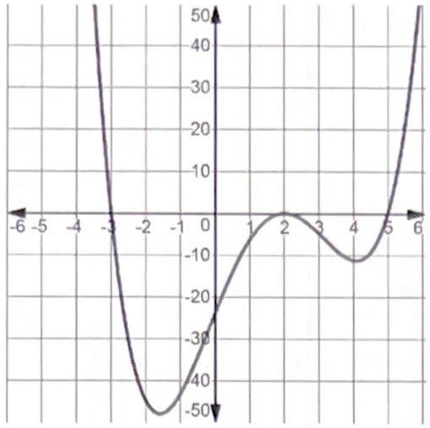
b. The x-value(s) where a local minimum occurs: $x=1$

c. The x-value(s) where a local maximum occurs: $x=-1, 4$ ← also absolute max

d. The x-value where the absolute minimum occurs: none ↙ ↓

e. The x value where the absolute maximum occurs: $x=4$

10. Below is the graph of the **derivative** of a function with a domain of all real numbers. Find the following (if possible):



$f'(x)$ + 0 − 0 − 0 +

−3 2 5

a. The x-value(s) of all locations of all critical point(s) of $f(x)$: $x=-3,2,5$ $f'(x)=0$

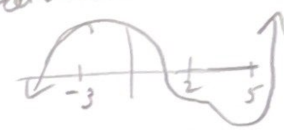
b. The x-value(s) where a local minimum occurs: $x=5$ $f'(x)$ changes $(-)$ to $(+)$

c. The x-value(s) where a local maximum occurs: $x=-3$ $f'(x)$ changes $(+)$ to $(-)$

d. The x-value where the absolute minimum occurs: none

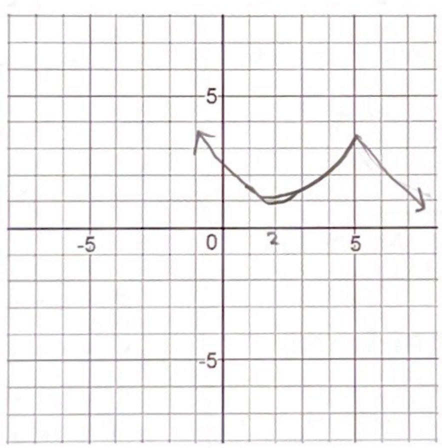
e. The x value where the absolute maximum occurs: none

sketch f(x)



For questions 11 – 12, sketch the continuous function $f(x)$ with the following properties:

11. $f'(x)<0$ on $(-\infty,2)$; $f'(x)>0$ on $(2,5)$; $f'(x)<0$ on $(5,\infty)$, $f'(2)=0$, $f'(5)=$ undefined



0 und

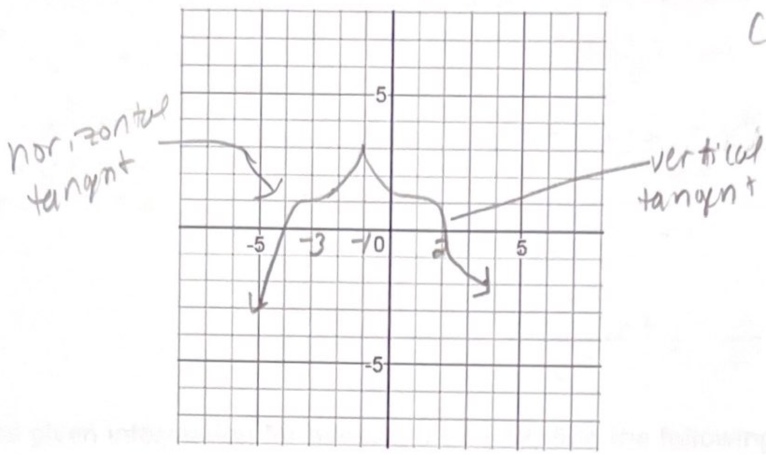
2 5

min

12. Sign line of $f'(x)$:

↑ ~ ↑ ↓ ↓

+	0	+	U	-	U	-
	-3		-1		2	



continuous
no holes or asymptotes

For questions 13 – 14, given $f(x)$, find the following:

a. The critical value(s) of $f(x)$.

b. Use the First Derivative Test to determine if each critical value is the location of a local minimum, local maximum, or neither.

c. State the x-interval(s) where $f(x)$ is increasing and decreasing.

13. $f(x)=2x^5-5x^4-10x^3+4$

$f'(x)=10x^4-20x^3-30x^2$

$0=10x^2(x^2-2x-3)$

$0=10x^2(x-3)(x+1)$

$x=0 \quad x=3 \quad x=-1$

a) c.p. $x=-1,0,3$

	f'	
$-\infty,-1$	-2	$+--=+$ ↑ max
$-1,0$	$-\frac{1}{2}$	$+-+=-$ ↓ neither
$0,3$	1	$+-+=-$ ↓ min
$3,\infty$	4	$+++=+$ ↑

b) local max $x=-1$
local min $x=3$
neither occurs at $x=0$

c) ↑ $(-\infty,-1)\cup(3,\infty)$
↓ $(-1,0)\cup(0,3)$

14. $f(x)=e^x(x-3)$

a) $f'(x)=e^x(1)+(x-3)e^x$

$0=e^x(1+x-3)$

$0=e^x(x-2)$

$e^x\neq 0 \quad x=2$ c.p

b)

	f'	
$-\infty,2$	0	$-$ ↓
$2,\infty$	3	$+$ ↑

local min at $x=2$

c) f ↓ $(-\infty,2)$
↑ $(2,\infty)$

15. Use the Second Derivative Test to determine if each critical value is the location of a local minimum, local maximum, or neither for $f(x) = x^2 \cdot e^{-x}$.

$f'(x) = x^2(-e^{-x}) + e^{-x}(2x)$

$0 = e^{-x}(-x^2 + 2x)$

$e^{-x} \neq 0 \qquad -x^2+2x=0$

$x(-x+2)=0$

$x=0 \quad x=2$

$f''(x) = e^{-x}(-2x+2) + (-x^2+2x)(-e^{-x})$

$0 = \dfrac{-2x+2}{e^x} - \dfrac{-x^2+2x}{e^x}$

$f''(x) = \dfrac{-2x+2+x^2-2x}{e^x} = \dfrac{x^2-4x+2}{e^x}$

$f''(0) = \dfrac{0^2-4(0)+2}{e^0} = \dfrac{+}{+} = +$ ∪ local min at $x=0$

$f''(2) = \dfrac{2^2-4(2)+2}{e^2} = \dfrac{-}{+} = -$ ∩ local max at $x=2$

Based upon the given information for questions 16 - 18, find the following:

a. The x-interval(s) where $f(x)$ is concave up and concave down.

b. The x-coordinate(s) of the inflection points of $f(x)$.

16. $f(x) = 3x^5 - 20x^4 + 40x^3 - 7$

a) $f'(x) = 15x^4 - 80x^3 + 120x^2$

$f''(x) = 60x^3 - 240x^2 + 240x$

$0 = 60x(x^2-4x+4)$

$0 = 60x(x-2)(x-2)$

$x=0 \quad x=2$

	f''	
$-\infty, 0$	-1	$- - - = -$ ∩
$0, 2$	1	$+ - - = +$ ∪
$2, \infty$	3	$+ + + = +$ ∪

∪ $(0,2), (2,\infty)$

∩ $(-\infty, 0)$

b) Point of inflection occurs at $x=0$

17. $f(x) = \sqrt[3]{x-4}$

$f'(x) = \frac{1}{3}(x-4)^{-2/3}$

$f''(x) = -\frac{2}{9}(x-4)^{-5/3}$

$0 = \dfrac{-2}{9\sqrt[3]{(x-4)^5}}$

$f''(x)$ undefined at $x=4$

$+ \quad \text{U} \quad -$

$-4 \quad 4 \quad 5$

f ∪ on $(-\infty, 4)$

f ∩ on $(4, \infty)$

b) inflection point occurs at $x=4$

18. Sign line of $f''(x)$:

∩ $-$ | 0 at -4 | ∪ $+$ | 0 at -1 | ∪ $+$ | 0 at 3 | ∩ $-$

a) f concave up $(-4,-1), (-1,3)$

f concave down $(-\infty,-4)\ (3,\infty)$

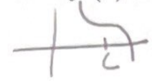
b) Inflection points at $x=-4, 3$

19. True or False?

a. If $f(x)$ has a relative max or min at $x=c$, then $x=c$ is a critical point of $f(x)$.

True

b. If $x=c$ is a critical point of $f(x)$, then $f(x)$ has a relative max or min at $x=c$.

False  horiz. tan. but not max/min

c. If $f''(a)=0$, then $x=a$ is the location of an inflection point of $f(x)$.

False f'' must change sign

For questions 20 - 21, sketch a graph of the following functions. Be sure to find the following:

a. The critical value(s) of $f(x)$.

b. State the x-interval(s) where $f(x)$ is increasing and decreasing.

c. State the x-interval(s) where $f(x)$ is concave up and concave down.

d. Use the First or Second Derivative Test to determine if each critical value is the location of a local minimum, local maximum, or neither.

e. State the x-coordinates of the inflection point(s) of $f(x)$.

f. Find the important function values for your sketch.

20. $f(x)=x^3-6x^2+9x$

a) $f'(x)=3x^2-12x+9$

$0=3(x^2-4x+3)$

$0=3(x-3)(x-1)$

C.P. $x=3,1$

b)

	f'	
$-\infty,1$	0	$+--=+$ ↑ max
$1,3$	2	$+-+=-$ ↓ min
$3,\infty$	4	$+++=+$ ↑

f ↑ $(-\infty,1),(3,\infty)$

f ↓ $(1,3)$

c) $f''(x)=6x-12$

$0=6x-12$

$12=6x$

$2=x$

	f''	
$-\infty,2$	0	$-$ ∩
$2,\infty$	3	$+$ ∪

f concave down $(-\infty,2)$, concave up $(2,\infty)$

d) $x=1$ local max

$x=3$ local min

e) POI at $x=2$

f) y int $x=0$ $(0,0)$

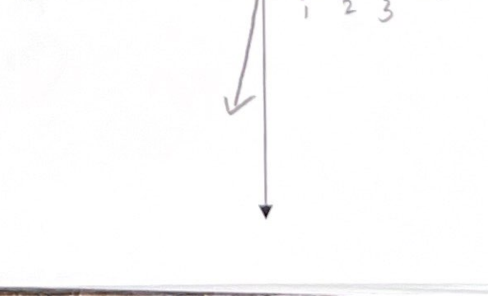
local max $x=1$ $(1,4)$

$f(1)=1-6+9=4$

local min $x=3$

$f(3)=3^3-6(3)^2+9(3)=0$

$f(2)=2$



21. $f(x)=x\cdot \ln x$ restricted domain $(0,\infty)$

a) $f'(x)= x\cdot\frac{1}{x}+\ln x(1)$

$0=1+\ln x$

$-1=\ln x$

$e^{-1}=x = \frac{1}{e}$ c.p.

$0,\frac{1}{e}$	$\frac{1}{4}$	$-$ ↓ min
$\frac{1}{e},\infty$	2	$+$ ↑

a) $x=\frac{1}{e}$ local min

b) ↓ $(0,\frac{1}{e})$

↑ $(\frac{1}{e},\infty)$

c) $f''(x)=\frac{1}{x}$

$0=\frac{1}{x}$ und at $x=0$

$(0,\infty)$ so all + after 0

$\frac{y\ \ +}{0}$ ∪ always $(0,\infty)$

∩ never

e) no point of inflection

f) min

$f(\frac{1}{e})=\frac{1}{e}\ln e^{-1}$

$\frac{1}{e}(-1)=-\frac{1}{e}$

$(\frac{1}{e},-\frac{1}{e})$

$\lim_{x\to 0^+} x\ln x$

$\lim_{x\to 0^+}\frac{\ln x}{x^{-1}}$

$\lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{1}{x^2}}$

$\lim_{x\to 0^+} -x=0$

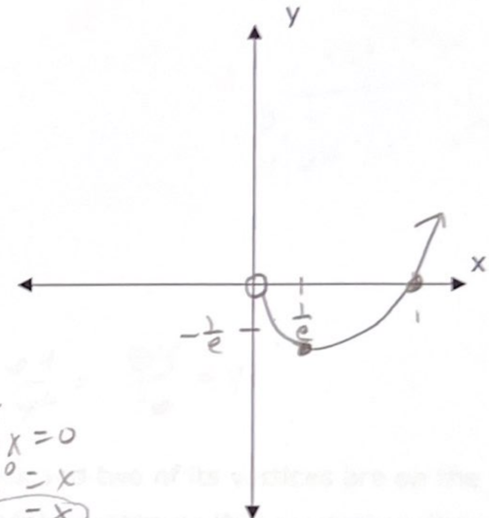
x int

$0=x\ln x$

$x\neq 0$ $\ln x=0$

$e^0=x$

$1=x$



For questions 22 - 25, write the equation to be optimized, the constraining equation (if necessary), find the critical values of the function, and verify that your answer is the value that optimizes the function.

22. Maximize the product of 2 nonnegative numbers whose sum is 30.

$x+y=30$ constraint

$y=30-x$

$xy=P$ optimize

$x(30-x)=P$

$30x-x^2=P$

$30-2x=P'$

$30-2x=0$

$30=2x$

$15=x$ ✓

abs max verify

$x=0$

$P(0)=0$

$x=15$

$P(15)=15(30-15)$

$=225$

$P(30)=30(30-30)$

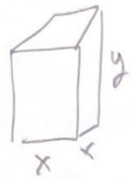
$=0$

max product is 225 when $x=15$, $y=15$

$y=30-15$

$y=15$

23. Of all boxes with a square base and a volume of 64 cubic centimeters, find the dimensions of the box with the minimum surface area.



$x^2 y = V$

Top/Bottom 4 sides

$S.A. = 2x^2 + 4xy$

$x^2 y = 64$ constraint

$y = \frac{64}{x^2}$

$A = 2x^2 + 4x\left(\frac{64}{x^2}\right) = 2x^2 + \frac{256}{x} = 2x^2 + 256x^{-1}$

$A' = 4x - 256x^{-2} = 4x - \frac{256}{x^2}$

$0 = 4x - \frac{256}{x^2}$

$\frac{256}{x^2} = 4x$

$256 = 4x^3$

$64 = x^3$

$4 = x$

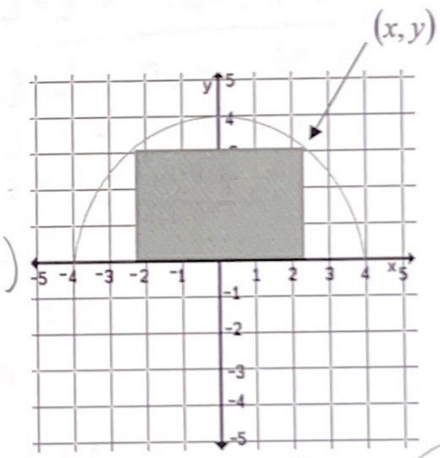
$y = \frac{64}{x^2} = \frac{64}{4^2} = \frac{64}{16} = 4$

verify

0, 4 | 1 − ↓
4, 64 | 5 + min

dimensions are 4 cm X 4 cm X 4 cm

24. A rectangle is constructed so that its base is on the x-axis and two of its vertices are on the graph of the upper circle $y = \sqrt{16 - x^2}$. Find the dimensions of the rectangle that maximizes the area of this rectangle.



$x^2 + y^2 = 4^2$

$y = \sqrt{16 - x^2}$ $A = 2xy$

$A = 2x(16 - x^2)^{\frac{1}{2}}$

$A' = 2x\left(\frac{1}{2}(16 - x^2)^{-\frac{1}{2}}(-2x)\right) + (16 - x^2)^{\frac{1}{2}}(2)$

$0 = \frac{-2x^2}{\sqrt{16 - x^2}} + 2\sqrt{16 - x^2}$

A' und at $x = \pm 4$

$\frac{2x^2}{\sqrt{16 - x^2}} = 2\sqrt{16 - x^2}$

$2x^2 = 2(16 - x^2)$

$2x^2 = 32 - 2x^2$

$4x^2 = 32$

$x^2 = 8$

$x = \pm\sqrt{8} = +2\sqrt{2}$

$-\sqrt{8}$ -4 not in domain

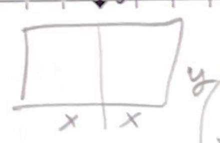
$x = 4$ endpoint

so verify

$A(0) = 0$

$A(2\sqrt{2}) = 16$

$A(4) = 0$



$y = \sqrt{16 - (\sqrt{8})^2}$

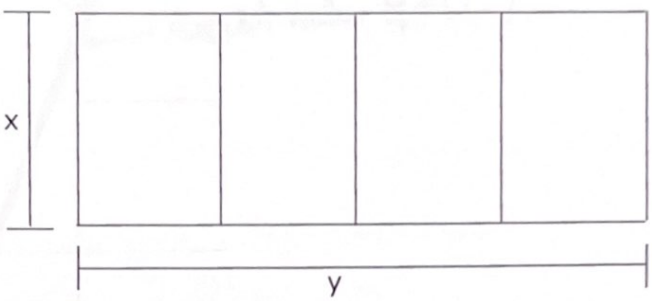
$= \sqrt{16 - 8}$

$= \sqrt{8}$

$= 2\sqrt{2}$

dimensions are $4\sqrt{2} \times 2\sqrt{2}$

25. Farmer Sue needs to enclose a rectangular region on her farm. She has 1000 yards of fence and needs the region to have 4 equal subareas, as shown below. What dimensions maximize the area of the rectangular region?



$5x+2y=1000$
constraint
$A = xy$

$2y = 1000 - 5x$
$y = 500 - \frac{5}{2}x$

$A = x(500 - \frac{5}{2}x)$
$A = 500x - \frac{5}{2}x^2$

$A' = 500 - 5x$
$0 = 500 - 5x$
$5x = 500$, $x = 100$

verify
$A(0) = 0$
$A(100) = 250,000$
$A(200) = 0$

$y = 500 - \frac{5}{2}(100)$
$y = 500 - 250$
$y = 250$

Dimensions $\boxed{100yd \times 250yd}$

26. Find the linear approximation of $f(x) = (x^3 - 1)^3$ at $x = 1.99$. $a = 2$

$f(2) = (2^3 - 1)^3 = 7^3 = 343$

$f'(x) = 3(x^3-1)^2(3x^2)$
$f'(2) = 3(2^3-1)^2(3(2)^2)$
$= 3(7)^2(12)$
$= 1764$

$y - 343 = 1764(x-2)$
$\boxed{L(x) = 343 + 1764(x-2)}$
$L(1.99) = 343 + 1764(1.99-2)$
$= 343 + 1764(-.01)$
$= 343 - 17.64$
$\boxed{f(1.99) = 325.36}$

27a. Find the linear approximation of $f(x) = \frac{x}{x+1}$ at $x = 1.1$. $a = 1$

$f(1) = \frac{1}{1+1} = \frac{1}{2}$

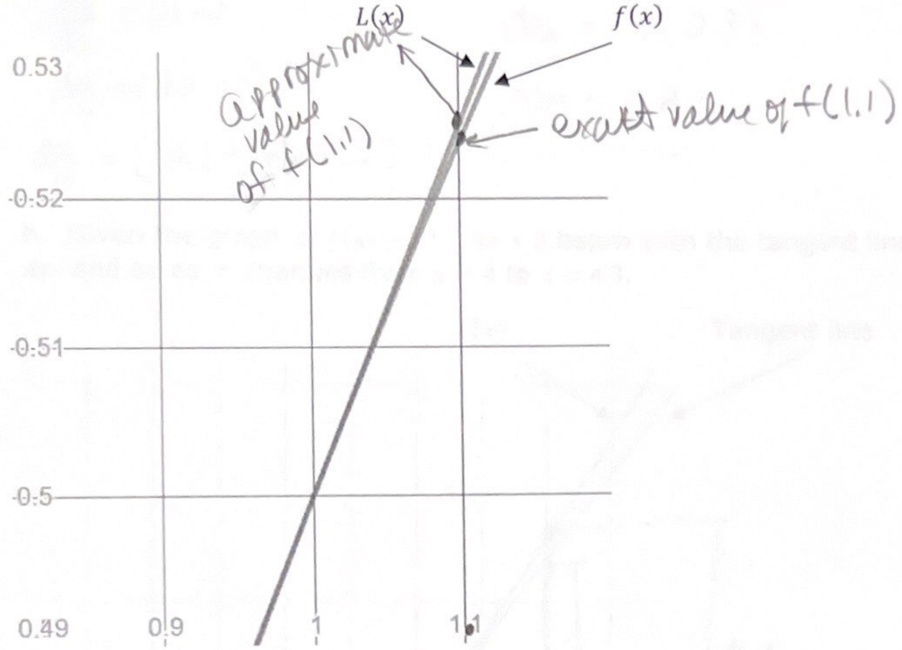
$f'(x) = \frac{(x+1)(1) - x(1)}{(x+1)^2}$

$f'(1) = \frac{(1+1) - 1}{(1+1)^2} = \frac{1}{4}$

$y - \frac{1}{2} = \frac{1}{4}(x-1)$

$L(x) = \frac{1}{2} + \frac{1}{4}(x-1)$
$L(1.1) = \frac{1}{2} + \frac{1}{4}(1.1-1)$
$= \frac{1}{2} + \frac{1}{4}(.1)$
$= \frac{1}{2} + \frac{1}{4}(\frac{1}{10})$
$= \frac{20}{40} + \frac{1}{40} = \boxed{\frac{21}{40}}$

b. Given the graph of $f(x) = \frac{x}{x+1}$ below with the tangent line drawn at $x = 1$, label the location of the exact value of the function and the linear approximation at $x = 1.1$.



28. Use the linear approximation method to estimate the value of $\sqrt[3]{28}$.

$f(x) = \sqrt[3]{x}$ $a = 27$

$f(27) = \sqrt[3]{27} = 3$

$y - 3 = \frac{1}{27}(x - 27)$

$f'(x) = \frac{1}{3}x^{-2/3}$

$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$

$f'(27) = \frac{1}{3\sqrt[3]{27^2}}$

$= \frac{1}{3 \cdot 3^2} = \frac{1}{27}$

$L(x) = 3 + \frac{1}{27}(x - 27)$

$L(28) = 3 + \frac{1}{27}(28 - 27)$

$= 3 + \frac{1}{27}$

$= \frac{81}{27} + \frac{1}{27}$

$= \frac{82}{27}$

Find the differential dy.

29. $f(x) = x^3 - \sin x + e^{x^2}$

$\frac{dy}{dx} = 3x^2 - \cos x + 2xe^{x^2}$

$\boxed{dy = (3x^2 - \cos x + 2xe^{x^2})dx}$

30. $h(t) = \ln(1 - 2t)$

$\frac{dh}{dt} = \frac{1}{1-2t} \cdot -2$

$dh = \left(\frac{-2}{1-2t}\right)dt$

31a. Use the differential dy to approximate the change in the function Δy for $f(x) = x^2 - 4x + 3$ as x changes from $x = 4$ to $x = 4.3$.

$\frac{dy}{dx} = 2x-4$

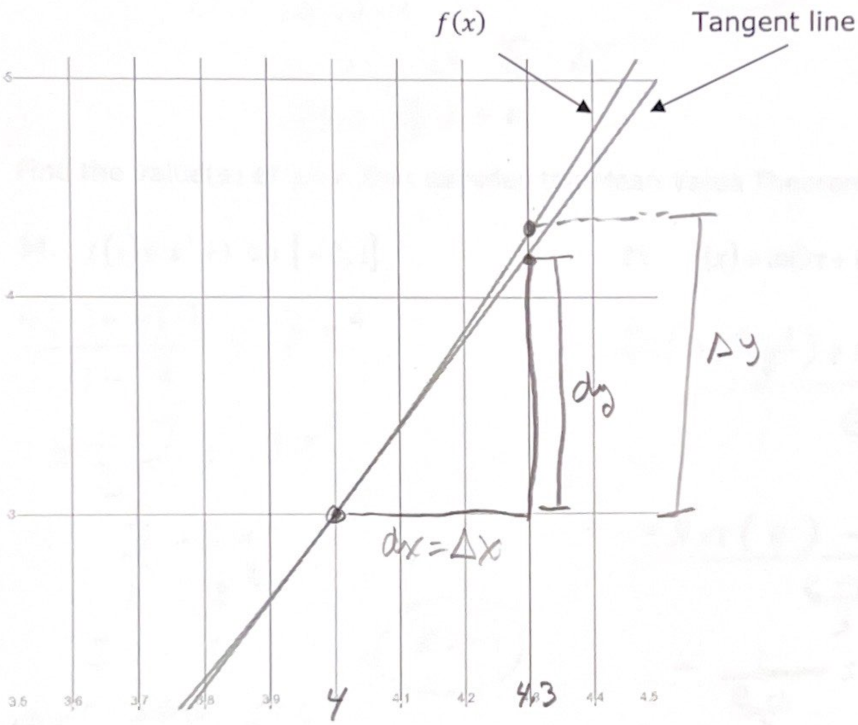
$dy = (2x-4)dx$

$dy = (2(4)-4)(.3)$

$dy = 4(0.3)$

$\boxed{dy = 1.2}$

b. Given the graph of $f(x) = x^2 - 4x + 3$ below with the tangent line drawn at $x = 4$, label dy, Δy, dx, and Δx as x changes from $x = 4$ to $x = 4.3$.



32. Use differentials to approximate the change in the volume of a right circular cone of fixed height 4 cm when the radius changes from 3 cm to 3.04 cm. The volume of a cone with fixed height 4 cm is $V = \frac{4}{3}\pi r^2$.

$\frac{dV}{dr} = \frac{8}{3}\pi r^2$

$r = 3$

$dr = 0.04$

$dV = \frac{8}{3}\pi r^2\, dr$

$dV = \frac{8}{3}\pi (3)^2 (0.04)$

$\boxed{dV = .32\pi}$

33. Use Rolle's Theorem to find the value of $x=c$ for $f(x)=\sin(2x)$ on $\left[0,\frac{\pi}{2}\right]$.

$f(0)=\sin(2\cdot 0)=0$ ✓

$f(\frac{\pi}{2})=\sin(2\cdot\frac{\pi}{2})=\sin\pi=0$ ✓

$f'(x)=2\cos(2x)$

$0=2\cos(2x)$

$0=\cos(2x)$

$\cos x=0$ at $\frac{\pi}{2},\frac{3\pi}{2}$

$2x=\frac{\pi}{2}+\pi k$

$x=\frac{\pi}{4}+\frac{\pi}{2}k$

$x=\frac{\pi}{4}$

$k=1$

$x=\frac{3\pi}{2}$ not in domain

Find the value(s) of $x=c$ that satisfies the Mean Value Theorem:

34. $f(x)=x^3+1$ on $[-2,1]$.

$\frac{f(1)-f(-2)}{1--2}=3x^2$

$\frac{2--7}{3}=3x^2$

$3=3x^2$

$1=x^2$

$\pm 1=x$

$x=-1$

MVT says $x=c$ on interval $(-2,1)$

Reject $x=1$

35. $f(x)=\ln(3x+1)$ on $\left[0,\frac{e-1}{3}\right]$

$\frac{\ln(3(\frac{e-1}{3})+1)-\ln(3(0)+1)}{\frac{e-1}{3}-0}=\frac{1}{3x+1}\cdot 3$

$=\frac{\ln(e)-\ln 1}{\frac{e-1}{3}}=\frac{3}{3x+1}$

$=\frac{1}{\frac{e-1}{3}}=\frac{3}{3x+1}$

$\frac{3}{e-1}=\frac{3}{3x+1}$

$3(3x+1)=3(e-1)$

$3x+1=e-1$

$3x=e-2$

$\boxed{x=\frac{e-2}{3}}$

Evaluate the following limits:

36. $\lim_{x\to 5^-}\frac{x^2+3}{x-5}$ VA $=\boxed{-\infty}$

Choose 4.9

$\lim_{x\to 5^-}\frac{(4.9)^2+3}{4.9-5}$ $\frac{+}{-}=-$

37. $\lim_{x\to 0}\frac{5\sin(8x)}{8\sin(5x)}$ %

$\lim_{x\to 0}\frac{5\cos(8x)\cdot 8}{8\cos(5x)\cdot 5}$

$=\frac{40}{40}=\boxed{1}$

38. $\lim_{x\to 3}\frac{x^2+x-12}{x^2-3x}$ %

$\lim_{x\to 3}\frac{2x+1}{2x-3}=\frac{2(3)+1}{2(3)-3}=\boxed{\frac{7}{3}}$

OR

factor + cancel

$\frac{0}{0}$

39. $\lim_{x\to 0} \frac{e^{2x}-1}{3x}$

$\lim_{x\to 0} \frac{2e^{2x}}{3} =$

$\frac{2e^{2(0)}}{3} = \boxed{\frac{2}{3}}$

$\frac{\infty}{\infty}$

40. $\lim_{x\to\infty} \frac{\ln(x^{100})}{\sqrt{x}}$

$\lim_{x\to\infty} \frac{\frac{1}{x^{100}} 100x^{99}}{\frac{1}{2}x^{-\frac{1}{2}}}$

$\lim_{x\to\infty} \frac{\frac{100}{x}}{\frac{1}{2\sqrt{x}}}$

$\lim_{x\to\infty} \frac{100}{x} \cdot \frac{2\sqrt{x}}{1}$

$\lim_{x\to\infty} 200x^{-\frac{1}{2}}$

$\lim_{x\to\infty} \frac{200}{\sqrt{x}} = \boxed{0}$

$0/0$

41. $\lim_{\theta\to 0} \frac{3\sin^2\theta}{\theta^2}$

$\lim_{\theta\to 0} \frac{6\sin\theta\cos\theta}{2\theta}$ $0/0$

$\lim_{\theta\to 0} \frac{3\sin\theta\cos\theta}{\theta}$

$\lim_{\theta\to 0} \frac{3\sin\theta(-\sin\theta) + \cos\theta(3\cos\theta)}{1}$

$\lim_{\theta\to 0} -3\sin^2\theta + 3\cos^2\theta = \boxed{3}$

42. $\lim_{x\to\infty} \frac{e^{2x}}{x}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{2e^{2x}}{1}$

$= \boxed{\infty}$

43. $\lim_{x\to-\infty} \frac{e^{2x}}{x}$

$\lim_{x\to-\infty} \frac{2e^{2x}}{1}$

$2e^{2(-\infty)}$

$= \frac{2}{e^{2(\infty)}} = \boxed{0}$

44. $\lim_{x\to\infty} x\cdot e^{-x}$ $\frac{x}{e^x}$ $\frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{1}{e^x} = \boxed{0}$

$\infty - \infty$

45. $\lim_{x\to 0^+} (\csc x - \cot x)$ need a fraction

$\lim_{x\to 0^+} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)$

$\lim_{x\to 0^+} \frac{1-\cos x}{\sin x}$ $0/0$

$\lim_{x\to 0^+} \frac{\sin x}{\cos x} = \frac{0}{1} = \boxed{0}$

46. $\lim_{x\to 0}(1+2x)^{\frac{5}{x}}$ $= \boxed{e^{10}}$

$y = \lim_{x\to 0}(1+2x)^{\frac{5}{x}}$

$\ln y = \lim_{x\to 0} \frac{5}{x}\ln(1+2x)$

$\ln y = \lim_{x\to 0} \frac{5\ln(1+2x)}{x}$ $0/0$

$\ln y = \lim_{x\to 0} \frac{\frac{5}{1+2x}\cdot 2}{1} = \frac{10}{1}$

$\ln y = 10$

$e^{10} = y$

47. $\lim_{x\to\infty} x^{\frac{1}{x}}$ $= \boxed{1}$

$y = \lim_{x\to\infty} x^{\frac{1}{x}}$

$\ln y = \lim_{x\to\infty} \frac{1}{x}\ln x$

$\ln y = \lim_{x\to\infty} \frac{\ln x}{x}$

$\ln y = \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$

$\ln y = 0$

$e^0 = y$

$1 = y$