MATH 1401

Fall 2023 Chapter 3 Exam Review

Review Day: Tuesday, October 10th

Chapter 3 Exam: Thursday, October 12th

Instructor: Whitten

Instructions:

1. You will have a total of **1 hour and 15** minutes to complete this exam.
2. **Calculators, electronic devices, scratch paper, and notecards are not allowed on this exam.**
3. **Visible phones will result in a zero on the exam.**
4. Show ALL of your work. Partial credit can be awarded for work that is legible and mathematically correct.
5. Cheating of any kind on the exam will not be tolerated and will result in a grade of 0%.
6. We are not out to trick you!!! Make sure you understand the concepts on this review.
7. Take a deep breath, relax, and good luck!!
8. The following formulas will be provided on the exam:

$$\frac{d}{dx}sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}tan^{-1}x = \frac{1}{1+x^2} \qquad \frac{d}{dx}sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$$

MATH 1401 **Chapter 3 Exam Review** **NAME:_________________**

Using the definition of the derivative, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, find the slope of the tangent line: In order to receive full credit, you must use the definition and simplify accordingly.

1. $f(x) = 2x^2 - x$

2. $f(x) = \sqrt{x}$

3. $f(x) = \frac{2}{x}$

True or False:

4. A function that is differentiable everywhere must also be continuous everywhere.

5. A function that is continuous everywhere must also be differentiable everywhere.

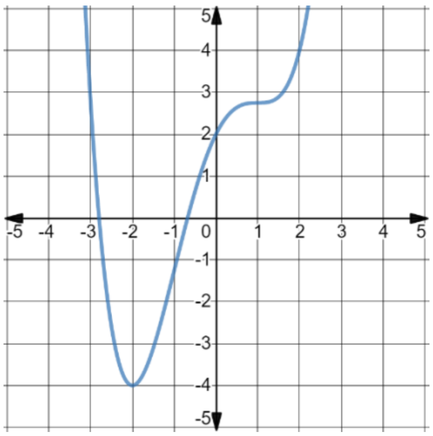
For questions 6-7, the derivative of a function is $f'(x) = \frac{(x+1)\cdot 2e^{2x} - e^{2x}}{(x+1)^2}$. Are the following simplifications correct or incorrect?

6. $f'(x) = \frac{(x+1)\cdot 2e^{2x} - e^{2x}}{(x+1)^2} = \frac{2e^{2x} - e^{2x}}{x+1} = \frac{e^{2x}}{x+1}$ Correct or Incorrect? Circle One

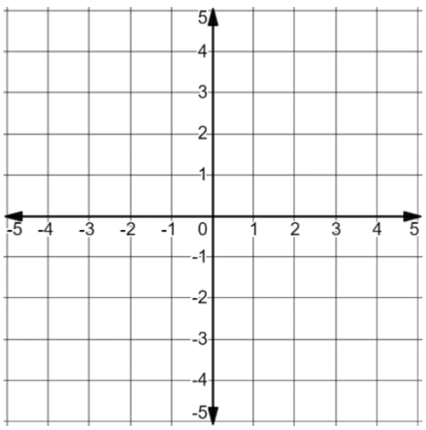
7. $f'(x) = \frac{(x+1)\cdot 2e^{2x} - e^{2x}}{(x+1)^2} = \frac{e^{2x}[2(x+1)-1]}{(x+1)^2} = \frac{e^{2x}(2x+1)}{(x+1)^2}$ Correct or Incorrect? Circle One

Given the graph of a function, sketch the graph of the derivative on the axes provided.

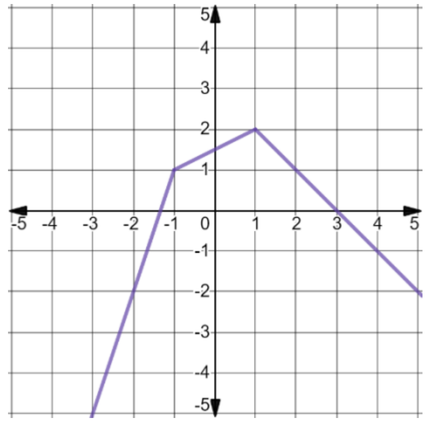
8. $f(x)$



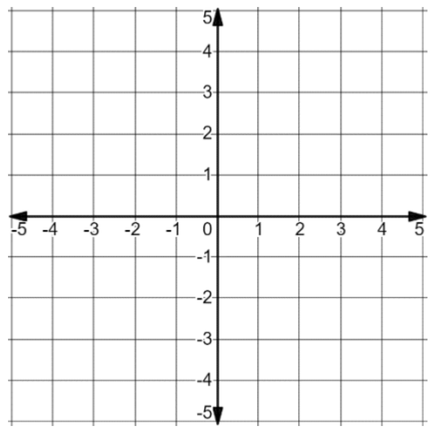
$f'(x)$



9. $g(x)$



$g'(x)$



Using the graph of $f(x)$ from question 8, answer the following:

10. True or False? $f(-1) > 0$.

11. True or False? $f'(-1) > 0$.

Find the derivative of the following:

12. $f(x) = 2x^4 - x^3 + 3x^2 - x + 6$

13. $f(x) = 3x^{-2}$

14. $f(x) = \dfrac{4}{x^5}$

15. $f(x) = 10e^{-5x}$

16. $f(x) = xe^{2x}$

17. $f(x) = e^5$

18. $y = \sin(x) + \cos(x)$

19. $f(x) = \csc(x) - \sec(x)$

20. $f(x) = 4\sin(x)\cos(x)$

21. $f(x) = x^2 \cdot \tan(x)$

22. $f(x) = \dfrac{x^2 - 8}{x + 6}$

23. $f(x) = \dfrac{e^{2x}}{x^2}$

Find the value of the derivative at the value of $x=a$:

24. $f(x)=4x^3-2x^2+9x-17$ at $a=-1$.

25. Find $\frac{d^3y}{dx^3}$ if $y=e^{4x}$.

26. Given $f(x)=\tan(x)$, find the equation of the tangent line to $f(x)$ at $a=\frac{\pi}{4}$.

27. Find the value(s) of x where $f(x)=x-\sin(x)$ will have a horizontal tangent line.

28. The population (in thousands) of the state of Georgia from 1995 (t=0) is modeled by $p(t)=-0.27t^2+101t+7055$. (Calculator allowed for this question)

a. Find the average rate of change of the population from 1995 to 2005.

b. Find the instantaneous rate of change of the population in 2003.

29. A ball is thrown vertically upward from the edge of 80 foot cliff with an initial velocity of $32\frac{ft}{\sec}$ on a planet where the acceleration due to gravity is $8\frac{ft}{\sec^2}$. The height of the ball in feet at time t in seconds is $h(t) = 80 + 32t - 4t^2$.

a. What is the maximum height of the ball?

b. When does the ball hit the ground?

c. What is the impact velocity of the ball?

30. The position of an object (in meters) moving horizontally at time t (in seconds) is determined by the position function $x(t) = t^3 - 12t^2 + 36t - 5$, $t \geq 0$. Find the following:

a. The velocity function and the velocity of the object at $t = 1$ seconds.

b. The acceleration function and the acceleration of the object at $t = 1$ seconds.

c. The time(s) when the object is stopped.

d. The time(s) when the object is changing direction.

e. The time interval(s) when the object is moving to the left.

f. The time interval(s) when the object is speeding up and slowing down.

Find the derivative:

31. $y = e^{\sqrt{x}}$

32. $f(x) = \tan(5x^2)$

33. $g(x) = \sqrt{x^2 + 1}$

34. $y = \dfrac{4}{(x^2 + 1)^2}$

35. $f(x) = \cot^5 x$

36. $g(x) = \cot x^5$

37. $f(x) = \left(\dfrac{3x}{4x+2}\right)^4$

38. $y = \ln\left|\cos(3x)\right|$

39. $f(x) = \sqrt[4]{x^5}$

40. $g(x) = \dfrac{12}{\sqrt[3]{x^2}}$

41. $h(x) = \left(\sqrt{2x^3 + x^2}\right)^5$

42. $y = 7^{x^3}$

43. $y = \log_8(\csc x)$

44. $y = \ln\left(\dfrac{x^2+1}{x-1}\right)$

45. $h(x) = 4^{-x} \cdot \sin x$

46. $y = x^3 \cdot \log_4(4x^2)$

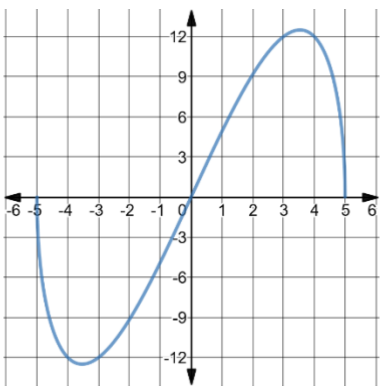
47. $g(x) = \sin^{-1}(3x)$

48. $y = \tan^{-1}(e^x)$

49. $f(x) = \sec^{-1}\sqrt{x},\ x \geq 1$

50. $y = \tan^{-1}\left(\frac{1}{x}\right)$

51. Below is a graph of $f(x) = x \cdot \sqrt{25 - x^2}$. Find the equation of the tangent line for $f(x)$ at $x = -3$. Draw the tangent line on the graph.



Simplify: Hint: Think of the right-hand side of $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

52. $\lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$

53. $\lim_{h \to 0} \frac{2(x+h)^3 - (x+h) - 2x^3 + x}{h}$

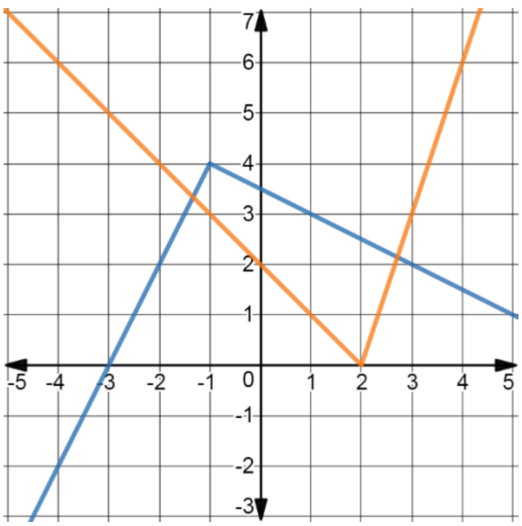
Evaluate: Hint: Think of the right hand side of $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

54. $\lim_{x \to 1} \frac{x^{80} - 1}{x - 1}$

55. $\lim_{x \to 0} \frac{\cos(x) - 1}{x}$

Use the graph of functions $f(x)$ and $g(x)$ to answer the following:

$g(x)$



$f(x)$

56. Let $Q(x) = \frac{g(x)}{f(x)}$. Find $Q'(1)$.

57. Let $h(x) = g(f(x))$. Find $h'(-3)$.

58. Use implicit differentiation to find $\frac{dy}{dx}$ for $3x - x^3\sqrt{y} = 4y^3$.

59. Given $x^2 - 2y^2 = 24$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

Use logarithmic differentiation to find the derivative.

60. $y = \dfrac{\left(3x^2 - x + 7\right)^4}{\sqrt{x^4 + 8x^2 + 5}}$

61. $y = (\sin x)^{\tan x}$

62. Find the derivative of the inverse of $g(x) = 2x^3 + x + 2$.

63. Find the derivative of the inverse of $f(x) = x^2 + 1$, $x > 0$ at the point $(5,2)$, which is a point on the graph of the inverse.