**MATH 1401 – Calculus I** **Name: ____________________**

**Fall 2023 Review for Final Exam**

Final Exam: **Saturday, December 9th, 2023, 9:00 – 11:00 am**

| **Section** | **Instructor** | **Final Exam Room Location** |
|---|---|---|
| 001 | Fahridzal Omar | Student Commons Building 2600 |
| 002 | Pamela Whitten | Student Commons Building 1500 |

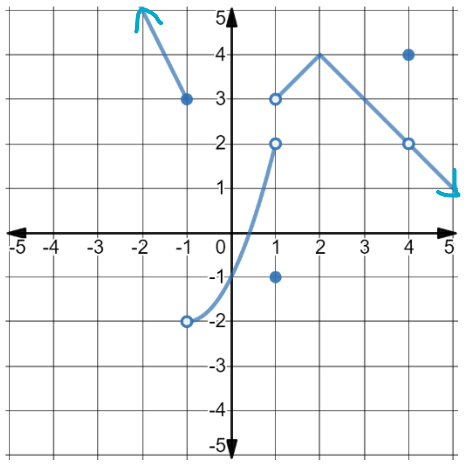
****Note**: This Review Assignment is provided to remind you of different types of mathematics problems that we have been working on throughout the semester. This review is NOT meant to provide you with an exact outline of the actual final exam. To accurately prepare I would recommend studying your old exams, old homework assignments, and completing the review assignment to ensure that you comprehend all the material covered this semester.**

Instructions for the Final Exam:

1. You will have a total of 2 hours to complete the final exam.
2. No calculators allowed.
3. All electronic devices must be turned off and put away.
4. No notecards will be allowed on the final, so prepare accordingly.
5. Partial credit can be earned on parts of the final exam when work is shown legibly.

**MATH 1401** **Final Exam Review**

1. Given the graph of $f(x)$, answer the following:




a. $\lim_{x \to -1^-} f(x)$

b. $\lim_{x \to -1^+} f(x)$

c. $\lim_{x \to -1} f(x)$

d. $\lim_{x \to 1^-} f(x)$

e. $\lim_{x \to 1^+} f(x)$

f. $\lim_{x \to 1} f(x)$

g. $\lim_{x \to 4} f(x)$

h. $\lim_{x \to -\infty} f(x)$

i. $\lim_{x \to \infty} f(x)$

j. $f(x)$ is discontinuous at $x =$ __________

k. $f(x)$ is non-differentiable at $x =$ __________

l. $\int_2^3 f(x)\, dx =$ _______________

m. Fill in the blanks with $f(3), f'(3), f''(3)$:

__________ < __________ < __________

2. Find $\lim_{x \to -2^-} f(x)$ if $f(x) = \begin{cases} 3x - x^2, & x < -2 \\ 4 + \ln(x+3), & x > -2 \end{cases}$

Evaluate the following limits:

3. $\lim_{x \to 3} \frac{x^2 - 3x}{x^2 + 4x - 21}$

4. $\lim_{x \to -2} \frac{x - 1}{(x + 2)^2}$

5. $\lim_{x \to 4} \frac{x^2 + 2}{x^2 + 4}$

6. $\lim_{x \to 9} \frac{x - 9}{\sqrt{x} - 3}$

7. $\lim_{x \to \infty} \frac{x^2 - x^3}{x^2 + x - 21}$

8. $\lim_{x \to 0} \frac{e^x - e^{-x}}{\sin x}$

9. $\lim_{x \to \infty} \frac{e^{2x}}{x^2}$

10. $\lim_{x \to 0^+} \left(x^2 \cdot \ln(x)\right)$

11. $\lim_{x \to 0^+} (1 + 2x)^{\frac{1}{3x}}$

12. Find all x-interval(s) where $f(x)=\sqrt{2x-1}$ is continuous:

Find the derivative of the following. You do not have to simplify your answers.

13. $f(x)=x^3-2x+6-\dfrac{4}{x^3}+\dfrac{1}{\sqrt{x}}$

14. $f(x)=\sqrt[4]{x^3}$

15. $f(x)=\dfrac{x^3}{e^{3x}}$

16. $g(x)=x^2\cdot\ln|2x|$

17. $f(x)=\ln\left(\sqrt{x}\right)$

18. $g(x)=\sec\left(x^2+x\right)$

19. $y = \cos^2 x$

20. $f(x) = \log_4(\sin x)$

21. $g(x) = 3^{\left(-x^2\right)}$

22. $g(x) = \tan^{-1}\left(2x^3\right)$

23. If $g(x) = x^3 \cdot e^{2x+1}$, find $g'(1)$.

24. Given $f(x) = \dfrac{2x}{x-1}$, find the equation of the tangent line at $x = 3$.

25. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $y = \sin^2 x$.

26. The position of an object (in meters) moving horizontally at time t (in seconds) is determined by the position function $x(t) = t^3 - 9t^2 + 27t + 5$, $t \geq 0$. Find the following:

a. The time(s) when the object is stopped.

b. The time(s) when the object is changing direction.

c. The time interval(s) when the object is moving to the left.

d. The time interval(s) when the object is speeding up and slowing down.

27. A ball is thrown vertically upward from the edge of 48-foot cliff with an initial velocity of $20\frac{ft}{\sec}$ on the newly discovered planet X. The height of the ball in feet at time t in seconds is $h(t) = 48 + 20t - 2t^2$.

a. What is the maximum height of the ball?

b. What is the velocity of the ball when it hits the ground?

Evaluate the derivative using implicit differentiation:

28. $3x^3 + x^2y^2 = y^3 - 4$

29. $\cos(y^2) + x = e^y$.

30. Find $\frac{dy}{dx}$ and the equation of the tangent line for $y^2 - xy = 12x^2$ at $(1,4)$.

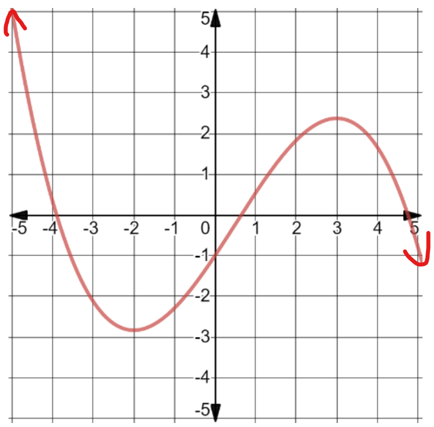
31. Find $\frac{dy}{dx}$, and $\frac{d^2y}{dx^2}$ for $2x^2 - y^2 = 4$.

Use logarithmic differentiation to find the derivative.

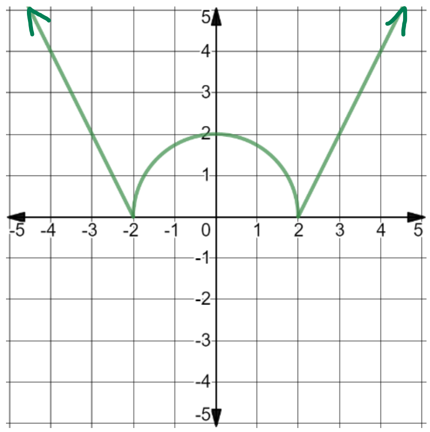
32. $y = (x+1)^x, \quad x > -1$

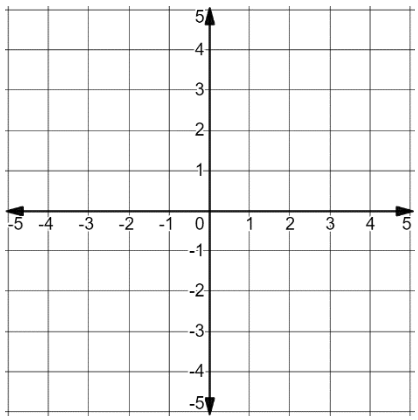
Given the graph of a function $f(x)$ shown below, sketch the graph of $f'(x)$.

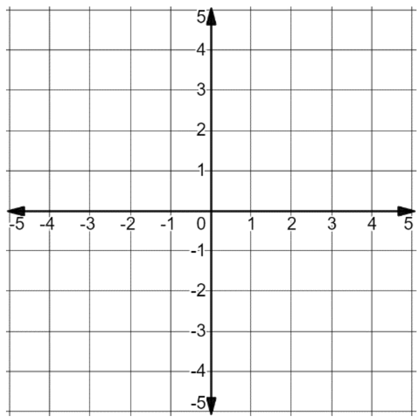
33.



34.







35. A 20-foot long ladder leans against the wall of a building. If the bottom of the ladder slides away from the wall at a constant rate of $2\,\frac{ft}{\sec}$, how fast is the top of the ladder sliding down the building when the top of the ladder is 16 feet above the ground?

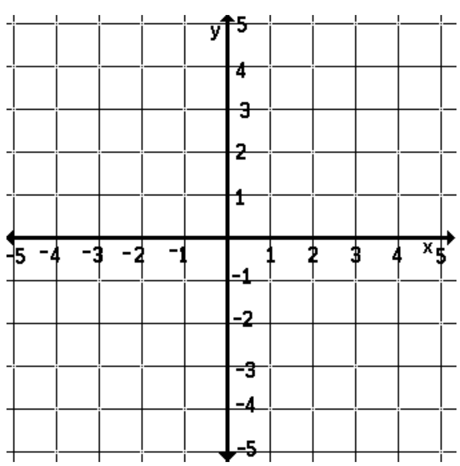
36. Air is leaking from a hole in a spherical air balloon at a constant rate of $12\pi \frac{cm^3}{\sec}$. How fast is the radius of the sphere changing when the volume of the sphere is $36\pi\ cm^3$?

37. Sketch a graph of a continuous function $f(x)$ with the following characteristics:

a. $x=-2$ and $x=3$ are critical points of $f(x)$

b. $f'(x)<0$ on $(-\infty,-2)$, $f'(x)<0$ on $(-2,3)$, $f'(x)>0$ on $(3,\infty)$.

c. $f'(-2)=0$, $f'(3)$ is undefined.



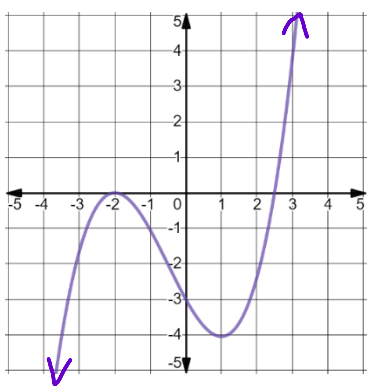
38. Use the definition of the derivative, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, to find the derivative of $f(x)=x^2-x$.

Simplify:

39. $\lim_{h\to 0}\frac{(x+h)^{10}-8(x+h)^6-x^{10}+8x^6}{h}$

40. $\lim_{x\to e}\frac{\ln x-1}{x-e}$

41. Below is the graph of the **derivative** of a function with a domain of all real numbers. Find the following (if possible):




a. The $x$-value(s) of all locations of all critical point(s) of $f(x)$: ______________

b. The $x$-value(s) where a local minimum occurs: ____________________

c. The $x$-value(s) where a local maximum occurs: ____________________

d. The $x$-value where the absolute minimum occurs: ____________________

e. The $x$-value where the absolute maximum occurs: ____________________

f. The $x$-interval(s) where $f(x)$ is concave down: ____________________

42. Find the absolute extrema of the function $f(x)=2x^3+3x^2-12x+1$ on $[-1,3]$.

43. Given $f(x)=6x^2-x^3$, find the following:

a. The critical point(s) of $f(x)$.

b. Use the Second Derivative Test to determine if each critical point is the location of a local minimum, local maximum, or inconclusive.

44. Given $f(x)=e^{2x}(x+4)$, find the following:

a. The critical point(s) of $f(x)$.

b. Use the First Derivative Test to determine if each critical point is the location of a local minimum, local maximum, or neither.

c. State the x-interval(s) where $f(x)$ is increasing and decreasing.

45. Sketch a graph of $f(x)=x^4-4x^3+1$. Be sure to find the following:

a. The critical value(s) of $f(x)$.

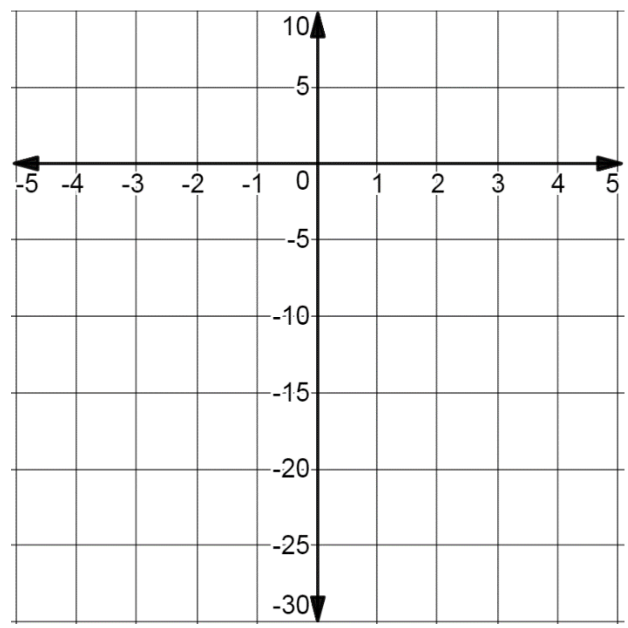
b. State the x-interval(s) where $f(x)$ is increasing and decreasing.

c. State the x-interval(s) where $f(x)$ is concave up and concave down.

d. Use the First or Second Derivative Test to determine if each critical value is the location of a local minimum, local maximum, or neither.

e. State the inflection point(s) of $f(x)$.

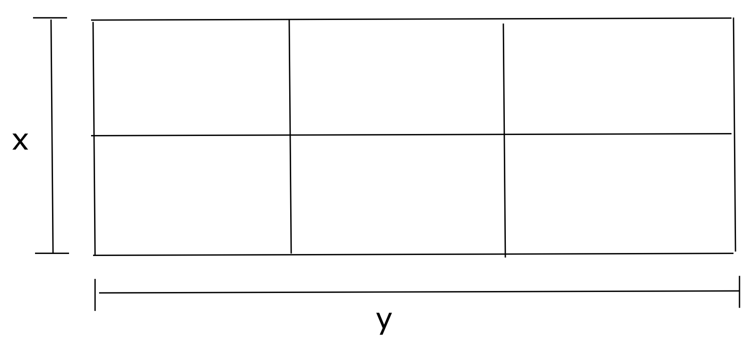
f. Find the important function values for your sketch.



For questions 46-47, write the equation to be optimized, the constraining equation (if necessary), find the critical values of the function, and verify that your answer is the value that optimizes the function.

46. Maximize the product of 2 nonnegative numbers whose sum is 36.

47. Farmer Sue needs to enclose a rectangular region on her farm. She has 1200 yards of fence and needs the region to have 6 equal subareas, as shown below. What dimensions maximize the area of the rectangular region?




48. Find the linear approximation of $f(x) = x^3 \cdot e^{2x}$ at $x = 1.02$.

49. Find the differential $dy$ if $f(t) = \sqrt[3]{t^2 + 6}$.

50. Use differentials to approximate the change in the volume of a cube when the length of the side changes from 5 cm to 4.97 cm.

51. Use the Mean Value Theorem for Derivatives to find the value of $x = c$ for $f(x) = 2x^3 - x$ on $[-2, 1]$.

Evaluate the following:

52. $\int 4x^3 - 10x + 6 - \frac{2}{x} - \frac{11}{x^3}\ dx$

53. $\int 8 \cdot \sqrt[3]{x^2} - \frac{8}{\sqrt[3]{x}}\ dx$

54. $\int \frac{t + 2}{t^2}\ dt$

55. $\int \csc x \cdot \cot x \ dx$

56. $\int (x^2+3)^2 \ dx$

57. $\int e^{-3x} \ dx$

58. If $f'(x) = 4x + 9\sqrt{x}$ and $f(4) = 30$, find the particular solution $f(x)$.

59. If the acceleration of an object moving along a line is $a(t) = 6t + 2$, $v(0) = 4$, $x(0) = -8$, find the velocity and position functions.

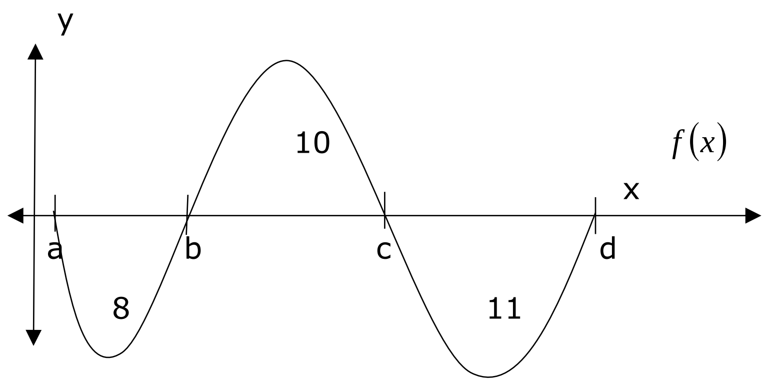
For question 60, calculate the Riemann Sum for the following functions over the indicated interval.

60. $f(x) = 81 - x^2$ on [-5,3] using n=4 rectangles and a midpoint approximation.

61. Use the graph of the integrand to evaluate the definite integral:

$$\int_0^5 \sqrt{25 - x^2}\ dx$$

Use the graph of the function $f(x)$ below with the areas of the regions bounded by the graph and the x-axis given to answer the following.




62. $\int_a^c f(x)dx$

63. $\int_b^a f(x)dx$

64. $\int_d^b f(x)dx$

Simplify:

65. $\frac{d}{dx}\int_{-3}^{x} 2t^3 - 4t + 6\,dt$

66. $\frac{d}{dx}\int_{x}^{2} \frac{t}{1+t^2}\,dt$

67. $\frac{d}{dx}\int_{7}^{4x^3} \frac{e^{3t}}{t}\,dt$

Evaluate:

68. $\int_{0}^{\frac{\pi}{3}} \sec x \tan x\,dx$

69. $\int_{0}^{2} x^2 - 1\,dx$

70. $\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \sin\theta\,d\theta$

71. $\int_{8}^{27} \frac{dx}{\sqrt[3]{x^2}}$

72. $\int_{-5}^{5} x^7 - 14x^3\,dx$

73. $\int_{-2}^{2} 10x^4 - 12x^2 - 3\,dx$

74. Find the average value of the function $f(x) = 25 - x^2$ on $[0,4]$.

Evaluate:

75. $\int 4x\sqrt{x^2+3}\,dx$ Let $u = x^2 + 3$

76. $\int \frac{4x^3+6x}{x^4+3x^2}\,dx$ Let $u = x^4 + 3x^2$

77. $\int e^{\cos x} \sin x \, dx$ Let $u = \cos x$

78. $\int \tan^2 x \cdot \sec^2 x \, dx$ Let $u = \tan x$

79. $\int_0^{\ln 4} \frac{e^x}{3+2e^x} dx$ Let $u = 3 + 2e^x$

80. $\int_{\sqrt{3}}^{2\sqrt{2}} \frac{x}{\sqrt{x^2+1}} dx$ Let $u = ???$