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CAAM 586
HW 4

Problem 1

1.

a.
$$Q = \begin{array}{c} \\ S \end{array} \overset{S}{\begin{pmatrix} -\lambda & +\lambda & & & \\ \mu & -(\mu+\lambda) & \lambda & & 0 \\ & \mu & -\mu+\lambda & \lambda & \\ 0 & & \ddots & \ddots & \ddots \\ & & & +\mu & -\mu \end{pmatrix}} = \begin{array}{c|cccccc} & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & -3 & 3 & & & & \\ 1 & 2 & -5 & 3 & & 0 & \\ 2 & & 4 & -7 & 3 & & \\ 3 & & & 4 & -7 & 3 & \\ 4 & & 0 & & 4 & -7 & 3 \\ 5 & & & & & 4 & -4 \end{array}$$

since $\lambda = 3$, $\mu = 2$

DTMC Method IDC: 0.0054231130085212855
Uniformization Method IDC: 0.0023830598614701308

Problem 2
IDC: 0.03766

Problem 3
E[X_40]: 20.194048369165383
Var[X_40]: 13.434169436576234

Problem 4

8.4

a. B/c $\frac{\sigma^2}{n} < \alpha$ is "true" variance we want

to generate $n > \frac{\sigma^2}{\alpha} = \frac{1}{.01} = 100$ samples generated

b.

c. See code

d.

e. They were not as more often than not only 100 samples were need until α was met. Further $\bar{\sigma}, \bar{\mu}$ are close to 1, 0 as expected.

Number generated: 100
Sample mean: -0.009157738667097496
Sample var: 0.9824323963802654

Problem 5

5.

a. First note $P(m > n) = P(U_1 \leq U_2 \leq \cdots \leq U_{n-1} \leq U_n)$

Since $P(U_n \geq U_{n-1}) = \frac{1}{2}$ $\forall n$ and $P(U_1 < U_2 < \cdots < U_{n-1} < U_n) = \frac{1}{(n-1)!}$

then clearly $P(m > n) = \frac{1}{n!}$.

b. $E[m] = \sum_{n=0}^{\infty} P(m > n) = \sum_{n=0}^{\infty} \frac{1}{n!} = e$

95% CI: [2.7179826765192265 , 2.818017323480773]

Problem 6
I couldn't get my code to work properly

7.

$p =$ prob. fail, $l_1 =$ nonfail cycle time length, $l_2 =$ time to failure of failing cycle

a. First note, $P(1^{st}$ failure on k^{th} cycle$) = (1-p)^{k-1} p$

"Time" to reach first fail on k^{th} cycle =

= "time" previous k-1 cycles fail + "time" of k^{th} cycle fail

$= l_1(k-1) + l_2$

thus $l = \sum_{k=1}^{\infty} (1-p)^{k-1} p[(k-1)l_1 + l_2]$

$= l_2 + pl_1 \sum_{k=1}^{\infty} (1-p)^{k-1} p$

and $E[l] = l_2 + \frac{1-p}{p} l_1$

$var(l) = var(l_2) + \left(\frac{1-p}{p}\right)^2 var(l_1)$ since iid cycles

so $\theta = E[l] + \frac{var(l)\, t_{\alpha/2}}{\sqrt{n}}$

b. $E[l_1] = 20.2$ $Var[l_1] = 18.6$ $p = \frac{87}{1000}$

$E[l_2] = 54$ $var[l_2] = 3.1$ $n = 1000$

$E[l] = 5.4 + 20.2\left(\frac{1-.087}{.087}\right)$ $var(l) = 3.1 + \left(\frac{1-.087}{.087}\right)^2 (18.6)$

$= 2051.51$

$= 217.34$

$\theta = 217.34 \pm 64.87\, t_{\alpha/2}$