1.

（a） The signal cos(Πn/2) can be broken up into a sum of two exponentials $x_1[\mathrm{n}] = (1/2)e^{j\pi n/2}$ and $x_2[\mathrm{n}] = (1/2)e^{-j\pi n/2}$. From the given information, we know that $x_1[\mathrm{n}]$ passes through the given LTI system, it experiences a delay of 2 samples the system has a real impulse response, it has an even group delay function. Then the complex exponential $x_2[\mathrm{n}]$ with frequency $-w_0$ also experiences a group delay samples. The output $y[\mathrm{n}]$ of the LTI system when the input is $\mathrm{x[n]} = x_1[\mathrm{n}] + x_2[\mathrm{n}]$.

Therefore

$$\mathrm{y[n]} = 2x_1[\mathrm{n}-2]+2x_2[\mathrm{n}-2] = 2\cos\left(\frac{\pi}{2}(\mathrm{n}-2)\right) = 2\cos(\frac{\pi}{2}\mathrm{n}-\pi)$$

(b) The signal $x[\mathrm{n}] = \sin(\frac{7\pi}{2}\mathrm{n}+\frac{\pi}{4})$ is the same as $-\sin(\frac{\pi}{2}\mathrm{n}-\frac{\pi}{4})$. This signal may again be broken up into complex exponentials of frequency $\frac{\pi}{2}$ and $-\frac{\pi}{2}$. We then use an argument similar to the one used in part(a) to argue that the output is

$$\begin{aligned}\mathrm{y[n]} = 2x[\mathrm{n}-2] &= 2\sin\left(\frac{7\pi}{2}(\mathrm{n}-2)+\frac{\pi}{4}\right)\\ &= 2\sin\left(\frac{7\pi}{2}\mathrm{n}-7\pi+\frac{\pi}{4}\right)\\ &= 2\sin\left(\frac{7\pi}{2}\mathrm{n}-\pi+\frac{\pi}{4}\right)\\ &= 2\sin\left(\frac{7\pi}{2}\mathrm{n}-\frac{3\pi}{4}\right)\end{aligned}$$

2.

Taking the Fourier transform of both sides of the h[n] difference equation of and simplifying, we obtain the frequency response H(e^jw) of the first filter.

$$\mathrm{H}(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{\sum_{k=0}^{q} b_k e^{-jwk}}{1+\sum_{k=1}^{p} a_k e^{-jwk}}$$

Taking the Fourier transform of both sides of the g[n] difference equation and simplifying, we obtain the frequency response H(e^jw) of the first filter.

$$\mathrm{G}(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{\sum_{k=0}^{q}(-1)^k b_k e^{-jwk}}{1+\sum_{k=1}^{p}(-1)^k a_k e^{-jwk}}$$

This may be also be written as

$$\mathrm{G}(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{\sum_{k=0}^{q} b_k e^{-j(w-\pi)k}}{1+\sum_{k=1}^{p} a_k e^{-j(w-\pi)k}} = \mathrm{H}\ \ (e^{j(w-\pi)})$$

Therefore, the frequency response of the g(n) filter is obtained shifting the frequency response of the first filter by π.

So it's a high-pass filter

3.

(1)

Let us first find the differential equation governing the input and output of this circuit.

Current through resistor and inductor=Current through capacitor=C$\frac{dy(t)}{dt}$.

Voltage across resistor = RC$\frac{dy(t)}{dt}$.

Voltage across inductor = LC$\frac{d^2y(t)}{dt^2}$.

Total input voltage = Voltage across inductor + Voltage across resistor + Voltage across capacitor.

Therefore,

$$\mathrm{x(t)} = \mathrm{LC}\frac{d^2y(t)}{dt^2} + \mathrm{RC}\frac{dy(t)}{dt} + y(t).$$

The frequency response of this circuit is therefore

$$\mathrm{H(jw)} = \frac{1}{LC(jw)^2 + RCjw + 1}$$

We may rewrite this to be

$$\mathrm{H(jw)} = \frac{1}{(\frac{jw}{1/\sqrt{LC}})^2 + 2(R/2)\sqrt{C/L}\frac{jw}{1/\sqrt{LC}} + 1}$$

(2)

$x(t) = \delta(t)$, so y(t) = h(t).

$$\mathrm{H(jw)} = \frac{1/LC}{(jw)^2 + 2(R/2)\sqrt{C/L}\frac{jw}{\sqrt{LC}} + 1/LC}$$

Compare H(jw) to the fraction $H(j\omega) = \frac{{\omega_n}^2}{(j\omega-c_1)(j\omega-c_2)} = \frac{M_1}{(j\omega-c_1)} - \frac{M_2}{(j\omega-c_2)}$

(c_1, c_2: roots of $(j\omega)^2 + 2\zeta\omega_n(j\omega) + {\omega_n}^2 = 0$) :

$$\omega_n = 1/\sqrt{LC}$$
$$\zeta = (R/2)\sqrt{C/L}$$

If $\zeta \neq 1$:

$h(t) = M[e^{c_1 t} - e^{c_2 t}]u(t)$, with:

$$c_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1} = -\frac{R}{2L} + \frac{1}{2L}\sqrt{\frac{R^2C-4L}{C}}, \quad c_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1} = -\frac{R}{2L} - \frac{1}{2L}\sqrt{\frac{R^2C-4L}{C}}$$

$$M_1 = M_2 = M = \frac{\omega_n}{2\sqrt{\zeta^2 - 1}} = \frac{1}{\sqrt{R^2C^2 - 4LC}}$$

if $\zeta = 1$:

$$h(t) = {\omega_n}^2 t e^{-\omega_n t}u(t)$$

(3)

The damping constant $\zeta = (\mathrm{R}/2)\sqrt{C/L}$. In order for the step response to have no oscillations, we must have $\zeta \geq 1$. Therefore, we require

$$\mathrm{R} \geq 2\sqrt{\frac{L}{C}}$$

4.

(1)

Using the Bode magnitude plot specified in figure 2, we may obtain an expression for H(jw). The figure shows that H(jw) has the break frequencies w_1=0.2, w_2=10, w_3=50. We can conclude that

$$\mathrm{H(jw)} = \frac{A(jw + w_1)^2}{(jw + w_2)(jw + w_3)}$$

For w=0, we can calculate A=50000.Then

$$\mathrm{H(jw)} = \frac{50000(jw + 0.2)^2}{(jw + 50)(jw + 10)}$$

(2)