1. Note that $h(t) = h_1(t-1)$ where $h_1(t) = \frac{\sin 4t}{\pi t}$. it is clear that $h_1(t)$ is the impulse response of an ideal lowpass filter whose passband is in the range $|\omega| < 4$. Therefore, $h(t)$ is the impulse response for an ideal lowpass filter shifted by 1 to the right. Using the shift property:

$$H(j\omega) = \begin{cases} e^{-j\omega} & |\omega| < 4 \\ 0 & otherwise \end{cases}$$

(a) We have:

$$X_1(j\omega) = \pi e^{\frac{j\pi}{12}} [\delta(\omega - 6)\delta(\omega + 6)]$$

$$Y_1(j\omega) = X_1(j\omega)H(j\omega) = 0 \rightarrow y_1(t) = 0$$

(b) We have:

$$X_2(j\omega) = \frac{\pi}{j} \sum_{k=0}^{\infty} \left[\left(\frac{1}{2}\right)^k \{\delta(\omega - 3k) - \delta(\omega + 3k)\} \right]$$

$$Y_2(j\omega) = X_2(j\omega)H(j\omega) = \frac{\pi}{j} \left[\left(\frac{1}{2}\right) \{\delta(\omega - 3) - \delta(\omega + 3)\} e^{-j\omega} \right]$$

$$y_2 = \frac{1}{2} \sin(3(t-1))$$

(c) We have:

$$X_3(j\omega) = \begin{cases} e^{j\omega} & |\omega| < 4 \\ 0 & otherwise \end{cases}$$

$$Y_3(j\omega) = X_3(j\omega)H(j\omega) = X_3(j\omega)e^{-j\omega}$$

$$y_3(t) = x_3(t-1) = \frac{\sin(4t)}{\pi t}$$

2. (a) The frequency response is:

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{3(3+j\omega)}{(4+j\omega)(2+j\omega)}$$

(b) Finding the partial fraction of answer in part (a) and taking its inverse Fourier Transform:

$$h(t) = \frac{3}{2}[e^{-4t} + e^{-2t}]u(t)$$

(c) We have

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{3(3+j\omega)}{(4+j\omega)(2+j\omega)} = \frac{9+3j\omega}{8+6j\omega-\omega^2}$$

Cross-multiplying and taking its inverse Fourier Transform:

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 3\frac{dx(t)}{dt} + 9x(t)$$

3. (a) The Fourier Transform of $X(j\omega)$ is:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{1}^{2} e^{-j\omega t}dt - \int_{2}^{3} e^{-j\omega t}dt = 2\frac{\sin\left(\frac{\omega}{2}\right)}{\omega}\left(1 - e^{-j\omega}\right)e^{-\frac{3j\omega}{2}}$$

(b) The Fourier series coefficients a_k are:

$$a_k = \frac{1}{T}\int_{T} \tilde{x}(t)e^{-jk\frac{2\pi}{T}t}\,dt = \frac{1}{2}\left(\int_{1}^{2} e^{-jk\frac{2\pi}{T}t}\,dt - \int_{2}^{3} e^{-jk\frac{2\pi}{T}t}\,dt\right) = \frac{\sin\left(\frac{k\pi}{2}\right)}{k\pi}\left(1 - e^{-jk\pi}\right)e^{-\frac{3jk\pi}{2}}$$

(c) Compare (a) and (b) it is clear that when T=2:

$$a_k = \frac{1}{T}X\left(j\frac{2\pi k}{T}\right)$$

4. (a) $\phi_{xy}(t) = \int_{-\infty}^{\infty} x(t+\tau)y(\tau)d\tau$. Let $r = t + \tau$, then $\tau = r - t$

$$\phi_{xy}(t) = \int_{-\infty}^{\infty} x(r)y(r-t)d(r-t) = \int_{-\infty}^{\infty} x(r)y(r-t)dr = \int_{-\infty}^{\infty} x(\tau)y(\tau-t)d\tau$$

$$= x(t) * y(-t) \ \therefore\ \phi_{yx}(t) = x(-t) * y(t); \quad \therefore \phi_{xy}(t) = \phi_{yx}(-t)$$

(b) $\phi_{xy}(t) = x(t) * y(-t) \quad = \therefore \phi_{xy}(j\omega) = X(j\omega)Y(-j\omega) = X(j\omega)Y^*(j\omega)$

(c) $\phi_{xx}(j\omega) = X(j\omega)X(-j\omega)$

$$\phi_{xy}(j\omega) = X(j\omega)Y(-j\omega) = X(j\omega)X^*(j\omega)H^*(j\omega) = \phi_{xx}(j\omega)H^*(j\omega)$$

$$\phi_{yy}(j\omega) = Y(j\omega)Y(-j\omega) = X(j\omega)H(j\omega)X(-j\omega)H(-j\omega) = X(j\omega)H(j\omega)X^*(j\omega)H^*(j\omega)$$

$$= \phi_{xx}(j\omega)|H(j\omega)|^2$$