EE152: Assignment #3

Deadline — 4 April

1.Suppose we are given the following information about the signal $x[n]$:

a. $x[n]$is a real and even signal

b. $x[n]$has period N=10 and Fourier coefficients a_k

c. a_{11}=5

d. $\frac{1}{10}\sum_{k=0}^{9}[x[n]]^2$=50

Show that $x[n]=Acos(Bn+C)$ and specify numerical value of A B C

Solution: Since the fourier series repeat every N=10,we have $a_1=a_{11}$=5.Further more since $x[n]$ is real and even,so a_k is also real and even.Therefore $a_1=a_{-1}$=5. We are given that:

$$\frac{1}{10}\sum_{n=0}^{9}[x[n]]^2=50$$

Using Parseval's relation:

$$\sum_{k=-1}^{8} a_k^2=50$$

Since $a_1=a_{-1}$=5,we can easily derive that:

$$\mathrm{a}_0=a_2=......=a_8=0$$

So $x[n] = \sum_k a_k e^{jw_0kn} = a_1e^{jw_0n} + a_{-1}e^{-jw_0n} = 5e^{j\frac{\pi}{5}n} + 5e^{-j\frac{\pi}{5}n} = 10cos(\frac{\pi}{5}n)$

2. Let

$$x(t) = \begin{cases} t, & \text{if } 0<=t<=1 \\ 2-t, & \text{if } 1<=t<=2 \end{cases}$$

be a periodic signal with fundamental period T=2 and Fourier coefficients a_k

a.Determine the value of a_0.

b.Determine the Fourier series representation of $\frac{dx(t)}{dt}$.

c.Use the result of part(b) and the differentiation property of the continuous-time Fourier series to help determine the Fourier series coefficients of $x(t)$.

Solution:

(a) We have

$$a_0 = \frac{1}{2}\int_0^1 t\,dt + \frac{1}{2}\int_1^2 (2-t)\,dt = 1/2.$$

(b) The signal $g(t) = dx(t)/dt$ is as shown in Figure S3.24.

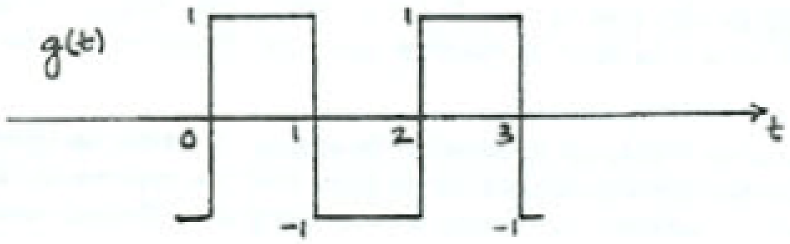

Figure S3.24

The FS coefficients b_k of $g(t)$ may be found as follows:

$$b_0 = \frac{1}{2}\int_0^1 dt - \frac{1}{2}\int_1^2 dt = 0$$

and

$$\begin{aligned} b_k &= \frac{1}{2}\int_0^1 e^{-j\pi kt}dt - \frac{1}{2}\int_1^2 e^{-j\pi kt}dt \\ &= \frac{1}{j\pi k}[1 - e^{-j\pi k}]. \end{aligned}$$

(c) Note that

$$g(t) = \frac{dx(t)}{dt} \xleftrightarrow{FS} b_k = jk\pi a_k.$$

Therefore,

$$a_k = \frac{1}{jk\pi}b_k = -\frac{1}{\pi^2 k^2}\{1 - e^{-j\pi k}\}.$$

3. Let $x(t)$ be a periodic signal with fundamental period T and Fourier coefficients a_k.Derive the Fourier series coefficients of each of the following signals in terms of a_k:

a. $x(t-t_0)+x(t+t_0)$.

b. Even part of $x(t)$.

Solution:

a.Using the time-shifting properties, $x(t-t_0)+x(t+t_0) \Longleftrightarrow e^{-jkw_0t}a_k + e^{jkw_0t}a_k = 2\cos(kw_0t)a_k$

b.Even part of $x(t)$ is $\frac{x(t)+x(-t)}{2}$. Since $x(-t) \Longleftrightarrow a_{-k}$, $\frac{x(t)+x(-t)}{2} \Longleftrightarrow \frac{a_k+a_{-k}}{2}$.

4. Let $x(t)$ be a real value signal with fundamental period T and Fourier coefficients a_k. Show that the Fourier coefficients of the even part of $x(t)$ are equal to $Re(a_k)$.

Solution:

Noted that $x(t)$ is a real signal, we can derive that:

$$x(t) = x^*(t)$$

Then:

$$a_k^* = a_{-k}$$

So:

$$Ev(\text{x(t)}) \Longleftrightarrow \frac{a_k+a_{-k}}{2} = \frac{a_k+a_k^*}{2} = Re(a_k)$$