EE152: Assignment #1

Shanghaitech University — March 19, 2020

1 Energy and Power Signals

(1) Determine the energy E_∞ and powerP_∞ of those signal, which are energy signals?

a. $x_1(t) = e^{j(2t+\pi/4)}$ (8)

Solution: Since $|e^{j\theta}| = \sqrt{\cos^2(\theta) + \sin^2(\theta)} = 1$, then

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |e^{j(2t+\pi/4)}|^2 dt = \int_{-\infty}^{\infty} |e^{j(2t+\pi/4)}|^2 dt = \int_{-\infty}^{\infty} 1 dt = \infty$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |e^{j(2t+\pi/4)}|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 1^2 dt = \lim_{T\to\infty} \frac{2T}{2T} = 1$$

It is a power signal.

b. $x_2(t) = \cos(t)$ (8)

Solution:

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |\cos(t)|^2 dt = \int_{-\infty}^{\infty} \frac{1+\cos(2t)}{2} dt = \infty$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |\cos(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{1+\cos(2t)}{2} dt = \lim_{T\to\infty} [\frac{1}{2} + \frac{\sin(2T)}{4T}] = \frac{1}{2}$$

It is a power signal.

(2) Determine the energy E_∞ and powerP_∞ of those signal, which are power signals?

a. $x_3[n] = e^{j(\pi/2n+\pi/8)}$ (8)

Solution:

$$E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} |e^{j(\pi/2n+\pi/8)}|^2 = \sum_{-\infty}^{\infty} 1 = \infty$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |e^{j(\pi/2n+\pi/8)}|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1 = \lim_{N\to\infty} \frac{2N+1}{2N+1} = 1$$

It is a power signal.

b. $x_4[n] = \cos(\frac{\pi}{4}n)$ (8)

Solution:

$$E_\infty = \lim_{N\to\infty}\sum_{n=-N}^{N}|\cos(\frac{\pi}{4}n)|^2 = \sum_{-\infty}^{\infty}\frac{1+\cos(\frac{\pi}{2}n)}{2} = \infty$$

$$\begin{aligned}
P_\infty &= \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}|\cos(\frac{\pi}{4}n)|^2 = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}\frac{1+\cos(\frac{\pi}{2}n)}{2}\\
&= \lim_{N\to\infty}\frac{1}{2N+1}[\frac{2N+1}{2}+\sum_{n=-N}^{N}\frac{\cos(\frac{\pi}{2}n)}{2}]\\
&= \frac{1}{2}+\lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}\frac{\cos(\frac{\pi}{2}n)}{2}\\
&= \frac{1}{2}
\end{aligned}$$

It is a power signal.

c. $x_5[n] = (\frac{2}{3})^{\frac{1}{2}n}u[n]$, where $u[n]$is the unit step function (8)

Solution:

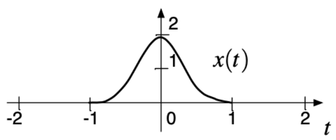
$$E_\infty = \lim_{N\to\infty}\sum_{n=-N}^{N}|(\frac{2}{3})^{\frac{1}{2}n}u[n]|^2 = \sum_{n=0}^{\infty}(\frac{2}{3})^n = \frac{1}{1-\frac{2}{3}} = 3$$

$$P_\infty = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}|(\frac{2}{3})^{\frac{1}{2}n}u[n]|^2 = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=0}^{N}(\frac{2}{3})^n = \lim_{N\to\infty}\frac{1}{2N+1}\frac{1-(2/3)^N}{1-2/3} = 0$$

It is an energy signal.

2 Time Scaling and Shifting

(1) Given the signal $x(t)$ shown below



draw the following signals and discuss their difference and similarity:

a. $x(2(t-1))$(10)

b. $x(2t-1)$(10)

Solution:

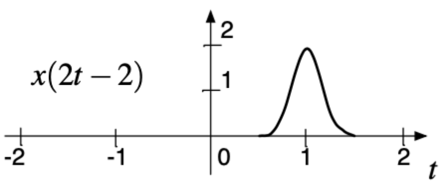


Figure 1: $x(2(t-1))$

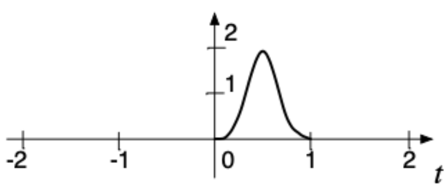
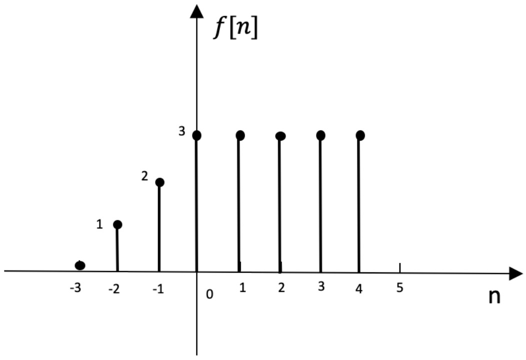


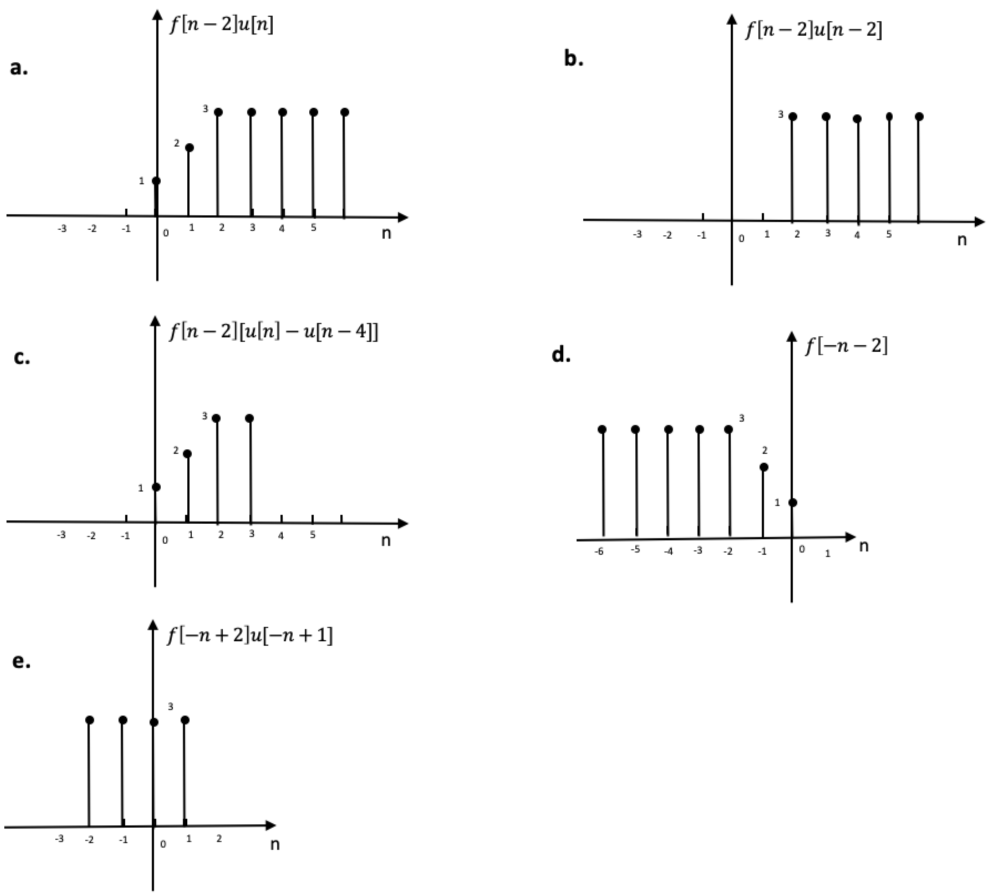
Figure 2: $x(2t-1)$

(2) Given the discrete signal $f[n]$ shown below

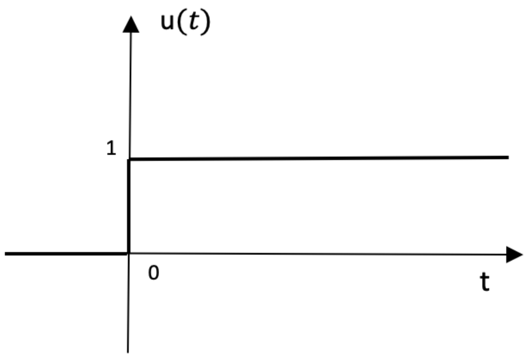


draw the following sequence waveforms:
a. $f[n-2]u[n]$ (4)
b. $f[n-2]u[n-2]$ (4)
c. $f[n-2][u[n]-u[n-4]]$ (4)
d. $f[-n-2]$ (4)
e. $f[-n+2]u[-n+1]$ (4)

Solution:



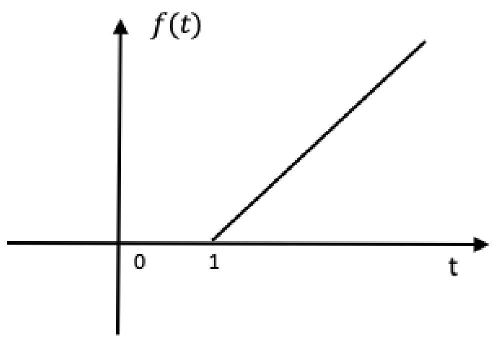
(3) Given the unit step function $u(t)$



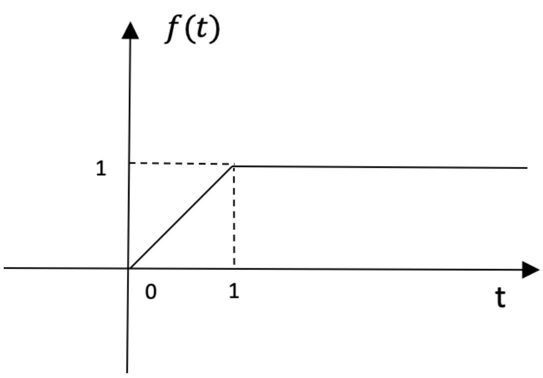
a. Draw the waveforms of the following signals:(10)

$$f(t) = (t-1)u(t-1)$$

Solution:



b. Write the function of the signal $f(t)$ in the figure(use $u(t)$ in your expression).(10)



Solution:

$$f(t) = tu(t) - (t-1)u(t-1)$$