Hw8

1.

We have

$$X(s) = \frac{\beta}{s+1}, Re\{s\} > -1.$$

Also,

$$G(s) = X(s) + \alpha X(-s), -1 < Re\{s\} < 1$$

Therefore.

$$G(s) = \beta\left[\frac{1 - s + \alpha s + \alpha}{1 - s^2}\right]$$

Comparing with the given equation for $G(s)$

$$\alpha = -1 \quad \beta = \frac{1}{2}$$

2.

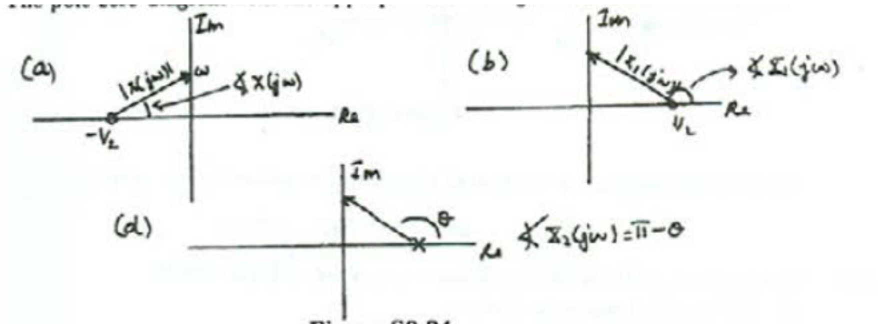


Figure S9.24

(a) The pole-zero diagram with the appropriate markings is shown in Figure S9.24

(b) By inspecting the pole-zero diagram of part (a), it is clear that the pole zero diagram shown in Figure S9.24 will also result in the same $X|(j\omega)|$. This would correspond to the Laplace transform

$$X_1(s) = s - \frac{1}{2} \quad Re\{s\} < \frac{1}{2}$$

(c)

$$\angle X(j\omega) = \pi - \angle X_1(j\omega)$$

(d) $X_2(s)$ with the pole-zero diagram shown below in Figure 9.24 would have the property that $\angle X_2(j\omega) = \angle X(j\omega)$. Here, $X_2(s) = \frac{-1}{s-1/2}$

(e) $|X_2(j\omega)| = 1/|X(j\omega)|$

(f) From the result of part (b), it is clear that $X_1(s)$ may be obtained by reflecting the poles and zeros in the right-half of the s-plane to the left-half of the s-plane. Therefore,

$$X_1(s) = \frac{s + 1/2}{s + 2}$$

From part (d), it is clear that$X_2(s)$ may be obtained by reflecting the poles (zeros)in the right-half of the *s*-plane to the left-half and simultaneously changing them to zeros (poles). Therefore,

$$X_2(s) = \frac{(s+1)^2}{(s+1/2)(s+2)}$$

3.

(a) Consider the signal $y(t) = x(t - t_0)$. Now,

$$Y(s) = \int_{-\infty}^{\infty} x(t - t_0)e^{-st}\,dt$$

Replacing $t - t_0$ by r, we get

$$Y(s) = \int_{-\infty}^{\infty} x(r)e^{-s(r+t_0)}\,dr$$

$$= e^{-st_0}\int_{-\infty}^{\infty} x(r)e^{-sr}\,dr$$

$$= e^{-st_0}X(s)$$

This obviously converges when $X(s)$ converges because e^{-st_0} has no poles. Therefore, the ROC of $Y(s)$ is the same as the ROC of $X(s)$.

(b) Consider the signal $y(t) = e^{s_0 t}x(t)$. Now,

$$y(t) = e^{s_0 t}x(t)$$

$$Y(s) = \int_{-\infty}^{\infty} x(t)e^{s_0 t}e^{-st}\,dt$$

$$= \int_{-\infty}^{\infty} x(t)e^{-(s-s_0)t}\,dt$$

$$= X(s - s_0)$$

If $X(s)$ converges in the range $a < Re\{s\} < b$, then $X(s - s_0)$ converges in the range $a + s_0 < s < b + s_0$. This is the ROC of $Y(s)$.

(c) Consider the signal $y(t) = x(at)$. Now,

$$Y(s) = \int_{-\infty}^{\infty} x(at)e^{-st}\,dt$$

Replacing at by r and assuming that $a > 1$, we get

$$Y(s) = (1/a)\int_{-\infty}^{\infty} x(r)e^{-s(r/a)}\,dr$$

$$= (1/a)X(s/a)$$

If $a < 0$, then

$$Y(s) = -(1/a)\int_{-\infty}^{\infty} x(r)e^{-s(r/a)}\,dr$$

$$= -(1/a)X(s/a)$$

Therefore,

$$Y(s) = \frac{1}{|a|}X(\frac{s}{a})$$

If $X(s)$ converges in tbe range $\alpha < Re\{s\} < \beta$, then $X(s/a)$ converges in the range $\alpha/a < s < \beta/\alpha$ when $a > 0$. When $a < 0$, then $X(s/a)$ converges in the range $\beta/a < s < \alpha/a$

(d) Consider the signal $y(t) = x(t) \cdot h(t)$. Now,

$$Y(s) = \int_{-\infty}^{\infty} [x(t) \cdot h(t)]e^{-st}\,dt$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(r)h(t - r)dr\, e^{-st}dt$$

$$= \int_{-\infty}^{\infty} x(r)\left[\int_{-\infty}^{\infty} h(t - r)e^{-st}dt\right]dr$$

Using the time-shifting property, we get

$$\begin{aligned} Y(s) &= \int_{-\infty}^{\infty} x(r)H(s)e^{-sr}\,dr \\ &= H(s)\int_{-\infty}^{\infty} x(r)e^{-sr}\,dr \\ &= H(s)X(s) \end{aligned}$$

4.

According to the clues given, we can get the following analysis:

$h(t)$ is real value function, we get that the poles and zeros occur in conjugate pairs, so the known poles are $-1 \pm j$, and the known zeros are $3 \pm j$.

As there are two infinite zeros, we know that there are at least two finite poles.

As $h(t)$ is stable and causal, we know all poles must lie in the left half s plane and the ROC must include jw axis and cover the right half plane.

True for (a). We get $e^{-3t}h(t) \iff X(s+3)$, as ROC of $X(s)$ includes the jw axis and is right half plane, we know the ROC of $X(s+3)$ must include jw axis, and (a) is true.

Insufficient information for (b). According to the analysis above, there may be other poles which lie in right side of $Re\{s\} = -1$, we cannot determine whether the ROC is $Re\{s\} > -1$

True for (c). Since H(s) is rational, H(s) can be expressed as a ratio of two polynomials in s. We get:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{P(s)}{Q(s)}$$

where $P(s)$, $Q(s)$ are the polynomials in s. We can get $Y(s)Q(s) = P(s)X(s)$, the differential equation can be derived from this. Since h(t) is real, we know that the coefficients of the polynomials are also real, so the coefficients of the differential equation are real.

False for (d). Since $H(s)$ has infinite zeros. we know $\lim_{s\to\infty} H(s) = 0$.

True for (e). According to the above analysis, we know that $H(s)$ has at least 4 poles.

False for (f). We know that e^{st} is the eigen function of LTI system, so output of input $x(t) = e^{st}$ is $H(s)e^{st}$. $e^{3t}sin(t)$ can be decomposed into $\frac{1}{2j}(e^{(3+j)t} - e^{(3-j)t})$. The s value $3 \pm j$ are the zeros of $H(s)$, so we know that the output is zero.

5.

Taking the Laplace transform of both sides of the equation, we get:

$$(s^3Y(s)+6s^2+11s+6)Y(s)-(s^2+6s+11)y(0^-)-(s+6)y'(0^-)-y''(0^-) = X(s)$$

Set $y(0^-)$, $y^{I}(0^-)$, $y^{II}(0^-)$ all to zeros, we get:

$$Y(s) = \frac{1}{6}\left(\frac{1}{s+1} - 3\frac{1}{s+2} + 3\frac{1}{s+3} - \frac{1}{s+4}\right)$$

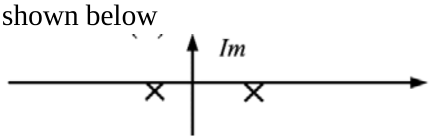
Then take the inverse transform, we get the zero state response:

$$y(t) = \frac{1}{6}(e^{-t} - 3e^{-2t} + 3e^{-3t} - e^{-4t})u(t)$$

6. (a) Taking the Laplace transform of both sides of the given differential equation and simplifying, we obtain

$$\mathrm{H(s)} = \frac{Y(s)}{X(s)} = \frac{1}{s^2 - s - 2} = \frac{1}{(s-2)(s+1)}$$

The pole-zero plot for H(s) is as shown below



(b) The partial fraction expansion of H(s) is

$$\mathrm{H(s)} = \frac{1}{(s-2)(s+1)} = \frac{1}{3(s-2)} - \frac{1}{3(s+1)}$$

1. If the system is stable, the ROC for H(s) has to be $-1 < \Re e\{s\} < 2$. Therefore

$$\mathrm{h(t)} = -\frac{e^{2\mathrm{t}}u(-t)}{3} - \frac{e^{-\mathrm{t}}u(t)}{3}$$

2. If the system is causal, the ROC for H(s) has to be $\Re e\{s\} > 2$. Therefore

$$\mathrm{h(t)} = \frac{e^{2\mathrm{t}}u(t)}{3} - \frac{e^{-\mathrm{t}}u(t)}{3}$$

3. If the system is neither stable nor causal, the ROC for H(s) has to be $\Re e\{s\} < -1$. Therefore

$$\mathrm{h(t)} = -\frac{e^{2\mathrm{t}}u(-t)}{3} + \frac{e^{-\mathrm{t}}u(-t)}{3}$$

7. (a)

$$v_i(\mathrm{t}) = \mathrm{RC}\frac{dv_0(t)}{\mathrm{dt}} + \mathrm{L}\frac{d\{C\frac{dv_0(t)}{\mathrm{dt}}\}}{\mathrm{dt}} + v_0(t)$$

$$v_i(\mathrm{t}) = \mathrm{RC}\frac{dv_0(t)}{\mathrm{dt}} + \mathrm{LC}\frac{d^2v_0(t)}{\mathrm{d}t^2} + v_0(t)$$

$$v_i(\mathrm{t}) = \frac{3dv_0(t)}{2\mathrm{dt}} + \frac{d^2v_0(t)}{2\mathrm{d}t^2} + v_0(t)$$

(b)

$$V_i(\mathrm{s}) = \mathrm{RC}(\mathrm{s}V_0(\mathrm{s}) - V_0(0_+)) + \mathrm{LC}(s^2V_0(\mathrm{s}) - sV_0(0_+) - \frac{dv_0(0_+)}{\mathrm{dt}}) + V_0(s)$$

$$V_i(\mathrm{s}) = \frac{3}{2}(\mathrm{s}V_0(\mathrm{s}) - 1) + \frac{1}{2}(s^2V_0(\mathrm{s}) - s - 2) + V_0(s)$$

$$\frac{1}{\mathrm{s+3}} = (\frac{1}{2}s^2 + \frac{3}{2}\mathrm{s} + 1)V_0(\mathrm{s}) - \frac{1}{2}s - \frac{5}{2}$$

$$V_0(\mathrm{s}) = \frac{s^2+8s+17}{(\mathrm{s+1})(\mathrm{s+2})(\mathrm{s+3})} = \frac{5}{\mathrm{s+1}} - \frac{5}{\mathrm{s+2}} + \frac{1}{\mathrm{s+3}}$$

Therefore,

$$v_0(\mathrm{t}) = (5e^{-\mathrm{t}} - 5e^{-2\mathrm{t}} + e^{-3\mathrm{t}})u(t)$$