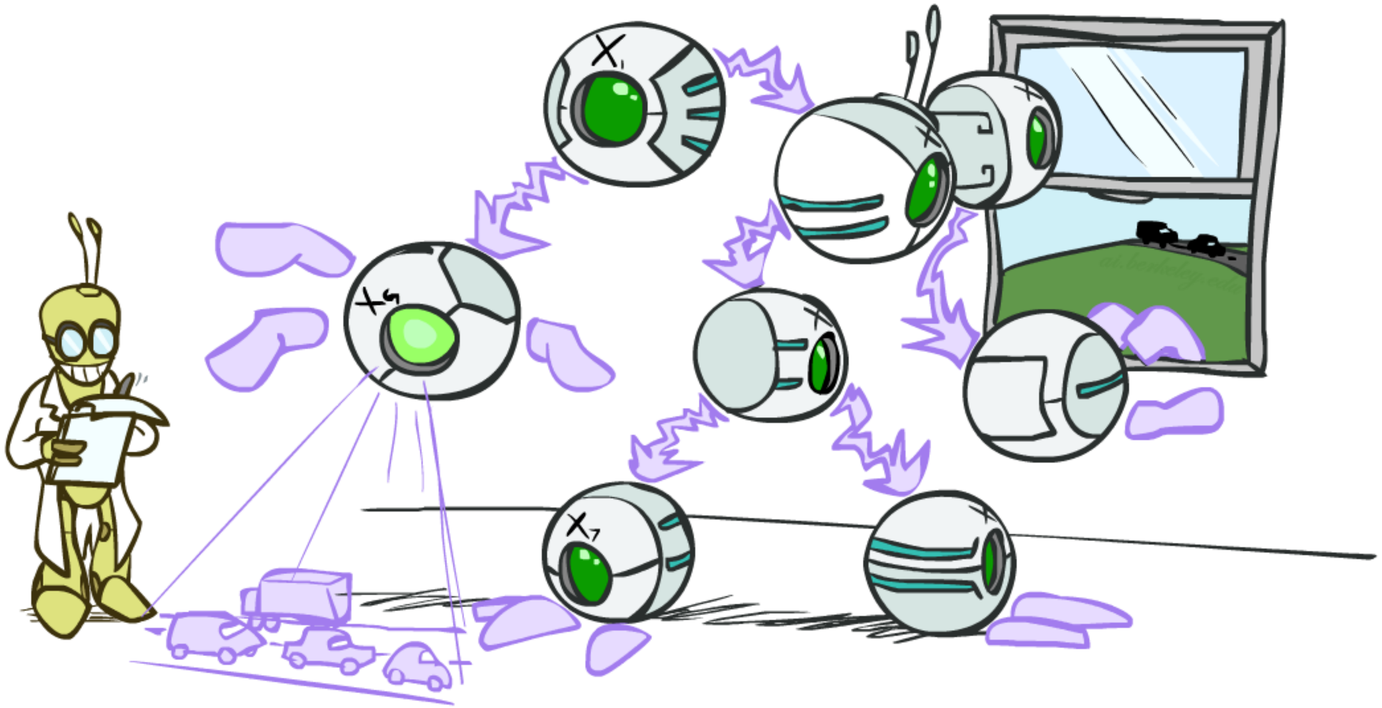
Announcement

- Project 2 due next Monday!
- No Class next Monday!
- Midterm: Nov. 2

Bayes Nets: Exact Inference



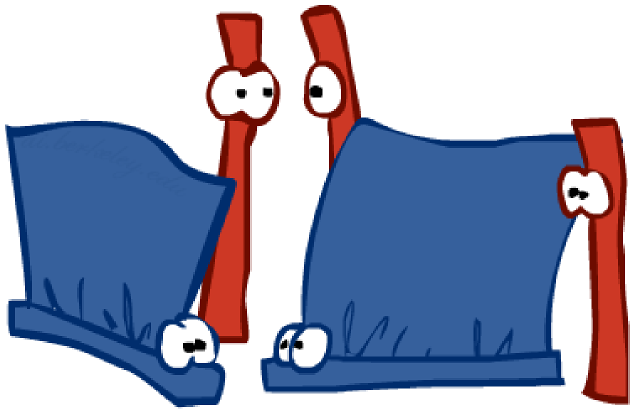
AIMA Chapter 14.4, PRML Chapter 8.4

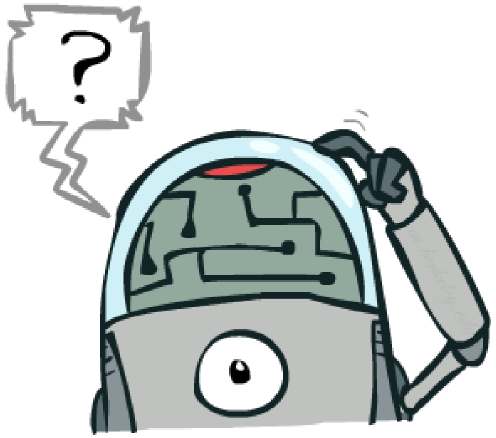
[Adapted from slides by Dan Klein and Pieter Abbeel at UC Berkeley]

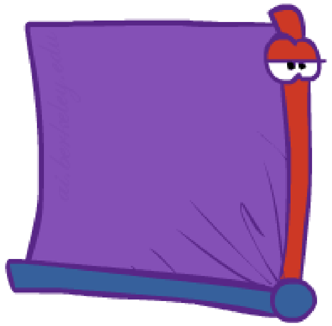
Inference

- Inference: calculating some useful quantity from a probability model (joint probability distribution)

- Examples:
  - Posterior marginal probability
    - $P(Q|e_1,..,e_k)$
    - E.g., what disease might I have?
  - Most likely explanation:
    - $\text{argmax}_q\ P(Q=q|e_1,..,e_k)$
    - E.g., what did he say?







Inference by Enumeration

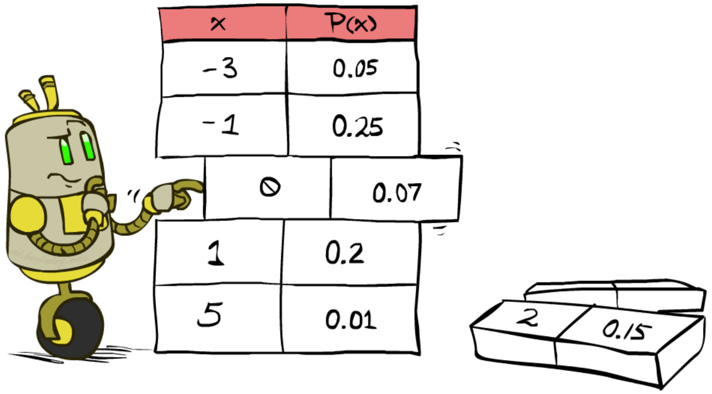
- General case:
  - Evidence variables: $E_1 \dots E_k = e_1 \dots e_k$
  - Query variable: Q
  - Hidden variables: $H_1 \dots H_r$

  $\left.\right\}$ $X_1, X_2, \dots X_n$ *All variables*

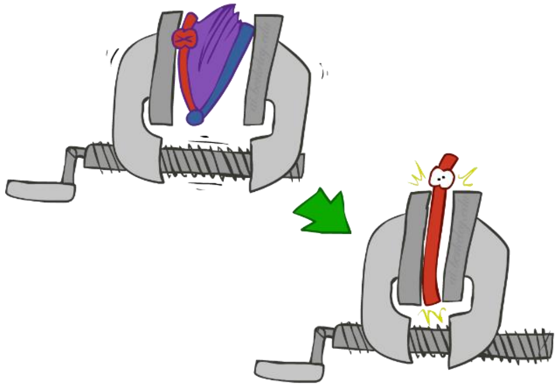
- We want:

$$P(Q|e_1 \dots e_k)$$

- Step 1: Select the entries consistent with the evidence



- Step 2: Sum out H to get joint of Query and evidence



$$P(Q, e_1 \dots e_k) = \sum_{h_1 \dots h_r} \underbrace{P(Q, h_1 \dots h_r, e_1 \dots e_k)}_{X_1, X_2, \dots X_n}$$

- Step 3: Normalize

$$\times \frac{1}{Z}$$

$$Z = \sum_q P(Q, e_1 \cdots e_k)$$

$$P(Q|e_1 \cdots e_k) = \frac{1}{Z} P(Q, e_1 \cdots e_k)$$

Inference by Enumeration in Bayes Net

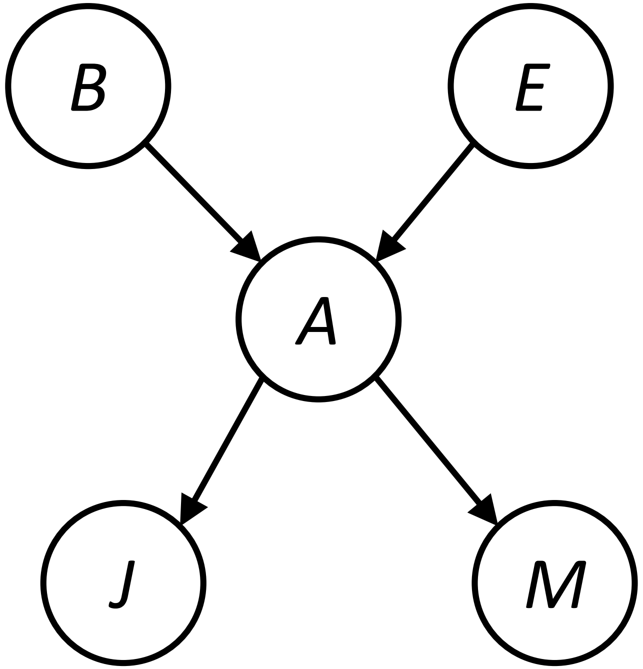
- The joint distribution can be computed from a BN by multiplying the conditional distributions
- Then we can do inference by enumeration

$$P(B \mid +j, +m) \;\; \propto_B P(B, +j, +m)$$

$$= \sum_{e,a} P(B, e, a, +j, +m)$$

$$= \sum_{e,a} P(B)P(e)P(a|B, e)P(+j|a)P(+m|a)$$

- Problem: sums of ***exponentially many*** products!



Inference by Enumeration in Bayes Net

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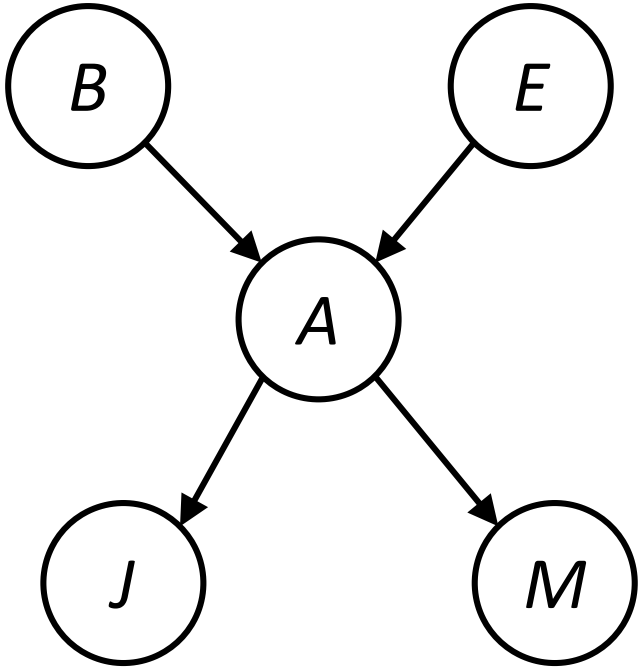

$$= P(B)P(+e)P(+a|B,+e)P(+j|+a)P(+m|+a) + P(B)P(+e)P(-a|B,+e)P(+j|-a)P(+m|-a)$$

$$P(B)P(-e)P(+a|B,-e)P(+j|+a)P(+m|+a) + P(B)P(-e)P(-a|B,-e)P(+j|-a)P(+m|-a)$$

Lots of repeated subexpressions!

Can we do better?

- Consider **uwy + uwz + uxy + uxz + vwy + vwz + vxy +vxz**
  - 16 multiplies, 7 adds
  - Lots of repeated subexpressions!
- Rewrite as **(u+v)(w+x)(y+z)**
  - 2 multiplies, 3 adds

Variable elimination: The basic ideas

- Move summations inwards as far as possible
  - $P(B \mid j, m) = \alpha \sum_{e,a} P(B)\, P(e)\, P(a|B,e)\, P(j|a)\, P(m|a)$
  - $= \alpha\, P(B) \sum_{e} P(e) \sum_{a} P(a|B,e)\, P(j|a)\, P(m|a)$
- Do the calculation from the inside out
  - I.e., sum over *a* first, the sum over *e*
  - Problem: $P(a|B,e)$ isn't a single number, it's a bunch of different numbers depending on the values of *B* and *e*
  - Solution: use arrays of numbers (of various dimensions) with appropriate operations on them; these are called ***factors***