# EE152: Assignment #2

## Shanghaitech University — March 18, 2020

**1.** For each of the following systems, determine whether the system is (1) stable, (2) causal,(3) linear, (4) time invariant.Give your reason.

**a.** $T(x[n]) = (\cos \pi n)x[n]$

**Solution:** Since $cos(\pi n)$ only takes on value of $+1$ or $-1$, this transformation outputs the current value of $x[n]$ multiplied by either $\pm 1$, $T(x[n]) = (-1)^n x[n]$.

- Hence, it is stable, because it doesn't change the magnitude of $x[n]$ and hence bounded in/bounded out
- It is causal, because each output depends only on the current value of $x[n]$.
- It is linear, Let $y_1[n] = T(x_1[n]) = \cos(\pi n)x_1[n]$, and $y_2[n] = T(x_2|n|) = cos(\pi n)x_2[n]$. Now

$$T(ax_1|n] + bx_2[n]) = \cos(\pi n)(ax_1[n] + bx_2[n]) = ay_1[n] + by_2[n] \tag{1}$$

- It is not time-invariant. If $y[n] = T(x[n]) = (-1)^n x[n]$, then $T(x[n-1]) = (-1)^n x[n-1] \neq y[n-1]$.

**b.** $T(x[n]) = x[n^2]$

**Solution:** This transformation simply "samples" at location which can be expressed as $k^2$.

- It is stable. Since if $x[n]$ is bounded, $x[n^2]$ is also bounded.
- It is not causal. For example, $Tx[4] = x[16]$.
- It is linear. Let $y_1[n] = T(x_1[n]) = x_1[n^2]$, and $y_2[n] = T(x_2|n|) = x_2[n^2]$. Now

$$T(ax_1[n] + bx_2[n]) = ax_1\left(n^2\right] + bx_2\left[n^2\right]) = ay_1[n] + by_2[n]$$

- It is not time-invariant. If $y[n] = T(x[n]) = x\left[n^2\right]$, then $T(x[n-1]) = x\left[n^2 - 1\right] \neq y[n-1]$

**2.** The following are the impulse responses of LTI systems. Determine whether each system is causal and/or stable. Give your reason.

**a.** $h(t) = e^{-6t}u(3-t)$

**b.** $h(t) = e^{-2t}u(t+50)$

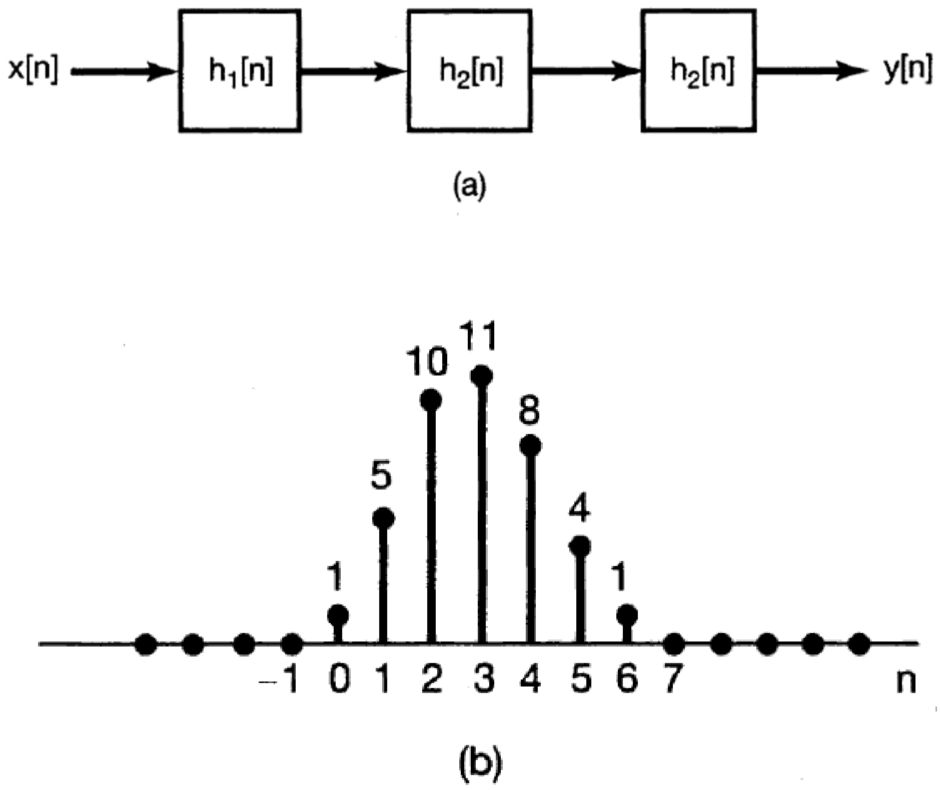
**c.** $h(t) = e^{-4t}u(t-2)$

**d.** $h[n] = n(\frac{1}{3})^n u[n-1]$

**e.** $h[n] = (\frac{1}{2})^n u[-n]$

**Solution:**
**a.** Not causal because $h(t) \neq 0$ for $t < 0$. Unstable because $\int_{-\infty}^{\infty} |h(t)| = \infty$
**b.** Not causal because $h(t) \neq 0$ for $t < 0$. a Stable because $\int_{-\infty}^{\infty} |h(t)| dt = e^{100}/2 < \infty$
**c.** Causal because $h(t) = 0$ for $t < 0$. Stable because $\int_{-\infty}^{\infty} |h(t)| dt = e^{-8}/4 < \infty$
**d.** Causal because $h[n] = 0$ for $n < 0$. Stable because $\sum_{n=-\infty}^{\infty} |h[n]| = 1 < \infty$
**e.** Not causal because $h[n] = 0$ for $n > 0$. Unstable because $\sum_{n=-\infty}^{0} (1/2)^n = \infty$

**3.** Consider the cascade interconnection of three causal LTI systems, illustrated in Fig(a), The impulse response $h_2[n] = u[n] - u[n-2]$, and the overall impulse response is as shown in Figure(b)




**a.** Find the impulse response $h_1[n]$, and draw it
**b.** Find the response of the overall system to the input

**Solution:**

**a.**

$h_2[n] = u[n] - u[n-2] = \delta[n] + \delta[n-1]$, therefore,

$$h_2[n] * h_2[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$$

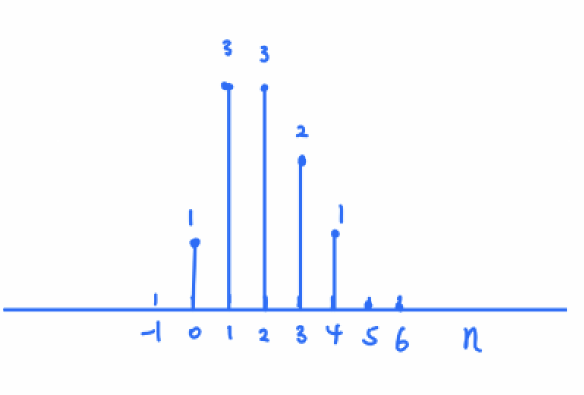
And:

$$\begin{aligned} h[n] &= h_1[n] * [h_2[n] * h_2[n]] \\ &= h_1[n] + 2h_1[n-1] + h_1[n-2] \end{aligned}$$

Therefore:

$$\begin{array}{rcl} h[0] = h_1[0] + 2h_1[-1] + h_1[-2] & \Rightarrow & h_1[0] = 1 \\ h[1] = h_1[1] + 2h_1[0] + h_1[-1] & \Rightarrow & h_1[1] = 3 \\ h[2] = h_1[2] + 2h_1[1] + h_1[0] & \Rightarrow & h_1[2] = 3 \\ h[3] = h_1[3] + 2h_1[2] + h_1[1] & \Rightarrow & h_1[3] = 2 \\ h[4] = h_1[4] + 2h_1[3] + h_1[2] & \Rightarrow & h_1[4] = 1 \\ h[5] = h_1[5] + 2h_1[4] + h_1[3] & \Rightarrow & h_1[5] = 0 \end{array}$$

$h_1[n] = 0$ for $n < 0$ and $n \geq 5$




**b.**

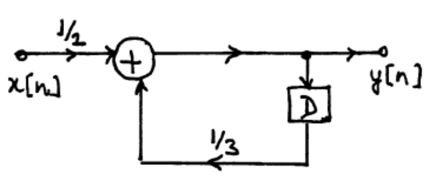
$y[n] = x[n] * h[n] = (\delta[n] - \delta[n-1]) * h[n] = (h[n] - h[n-1])$

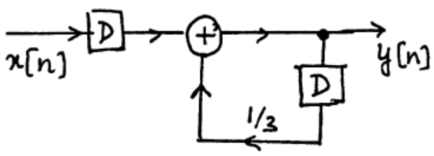
**4.** Draw block diagram representations for causal LTI systems described by the following difference equations:

**a.** $y[n] = \frac{1}{3}y[n-1] + \frac{1}{2}x[n]$

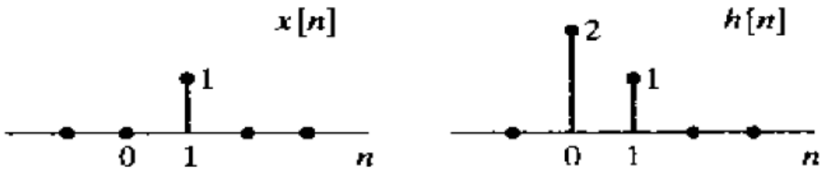
**b.** $y[n] = \frac{1}{3}y[n-1] + x[n-1]$

**Solution:**

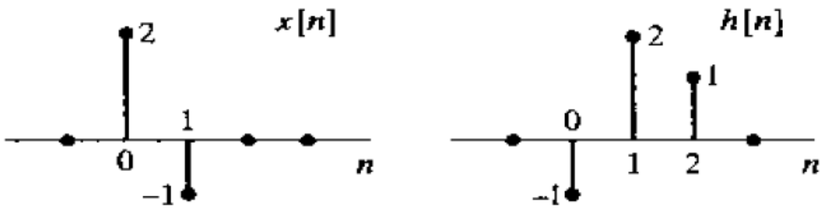


**5.** Compute the convolution $y[n] = x[n] * h[n]$ of the following pairs of signals




(a)




(b)

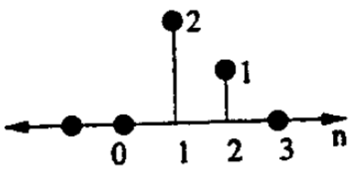
**Solution:**

We use the graphical approach to compute the convolution:

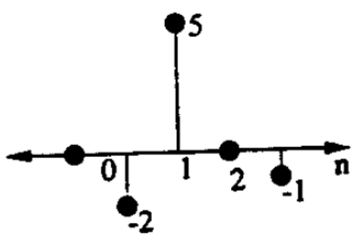
$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \end{aligned}$$

(a) $y[n] = x[n] * h[n]$

$$y[n] = \delta[n-1] * h[n] = h[n-1]$$



(b) $y[n] = x[n] * h[n]$



**6.** Consider a causal LTI system $S$ whose input $x[n]$ and output $y[n]$ are related by the difference equation

$$y[n] = -ay[n-1] + b_0x[n] + b_1x[n-1]$$

**a.** Verify that $S$ may be considered a cascade connection of two causal LTI systems $S_l$ and $S_2$ with the following input-output relationship:

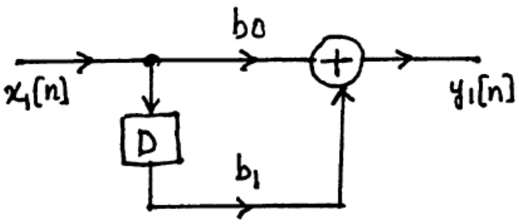
$$\begin{aligned} S_1 &: y_1[n] = b_0x_1[n] + b_1x_1[n-1] \\ S_2 &: y_2[n] = -ay_2[n-1] + x_2[n] \end{aligned}$$

**b.** Draw a block diagram representation of $S_1$.
**c.** Draw a block diagram representation of $S_2$.
**d.** Draw a block diagram representation of S as a cascade connection of the block diagram representation of $S_1$ followed by the block diagram representation of $S_2$.
**f.** Show that the two unit-delay elements in the block diagram representation of $S$ obtained in part (e) may be collapsed into one unit-delay element. The resulting block diagram is referred to as a Direct Form II realization of S, while the block diagrams obtained in parts (d) and (e) are referred to as Direct Form I realizations of $S$.
**e.** Draw a block diagram representation of $S$ as a cascade connection of the block diagram representation of $S_2$ followed by the block diagram representation of $S_1$

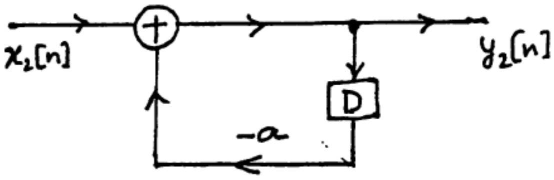
**Solution:** **a.** Because cascade connection, we get $x_2[n] = y_1[n]$. Then, the output

$$\begin{aligned} y[n] = y_2[n] &= -ay_2[n-1] + x_2[n] \\ &= -ay_2[n-1] + y_1[n] \\ &= -ay_2[n-1] + b_0x_1[n] + b_1x_1[n-1] \end{aligned}$$

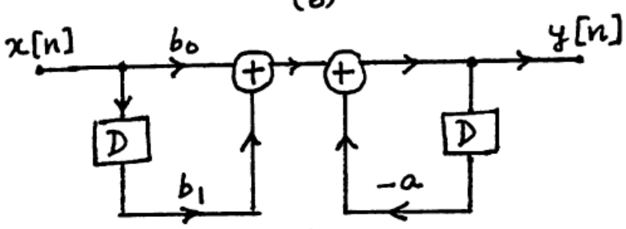
This is the same as the overall difference equation.



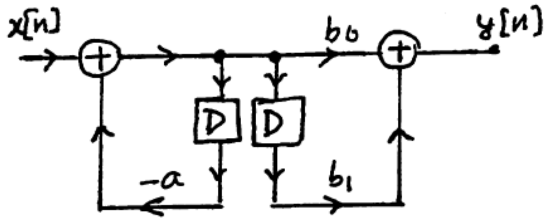

(b)

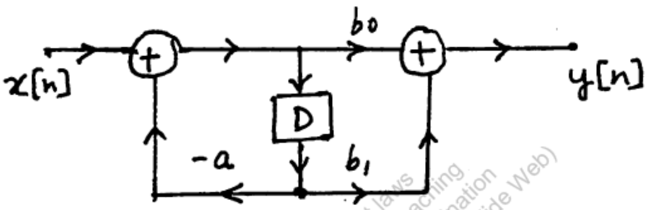



(c)




(d)




(f)

**7.** Consider the first-order difference equation

$$y[n] + 2y[n-1] = x[n]$$

Assuming the condition of initial rest find the impulse response of a system whose input and output are related by this difference equation. You may solve the problem by rearranging the difference equation so as to express $y[n]$ in terms of $y[n-1]$ and $x[n]$ and generating the values of $y[O], y[+1], y[+2]$, .. . in that order.

**Solution:**
$x[n] = \delta[n]$
$y[n] = x[n] - 2y[n-1]$

$$\begin{aligned} y[0] &= x[0] - 2y[-1] = 1 - 0 = 1 \\ y[1] &= x[1] - 2y[0] = 0 - 2 = -2 \\ y[2] &= x[2] - 2y[1] = 0 + 4 = 4 \\ y[3] &= x[3] - 2y[2] = 0 - 8 = -8 \end{aligned}$$

So, $h[n] = y[n] = (-2)^n u[n]$