Dirac Notation, Matrix Notation and Operator Diagonalization.

by José Luis Gómez-Muñoz
http://homepage.cem.itesm.mx/lgomez/quantum/
jose.luis.gomez@itesm.mx
With contributions by Francisco Delgado-Cepeda

Introduction

This is a tutorial on the use of Quantum *Mathematica* add-on to transform expressions in Dirac Notation to Matrix Notation and viceversa.

Load the Package

First load the Quantum`Notation` package. Write:
Needs["Quantum`Notation`"];
then press at the same time the keys SHIFT-ENTER to evaluate. *Mathematica* will load the package:

```
Needs["Quantum`Notation`"]
```

```
Quantum`Notation` Version 2.2.0. (July 2010)
A Mathematica package for Quantum calculations in Dirac bra-ket notation
by José Luis Gómez-Muñoz

Execute SetQuantumAliases[] in order to use
  the keyboard to enter quantum objects in Dirac's notation
SetQuantumAliases[] must be executed again in each new
  notebook that is created, only one time per notebook.
```

In order to use the keyboard to enter quantum objects write:
SetQuantumAliases[ ];
then press at the same time the keys SHIFT-ENTER to evaluate. The semicolon prevents *Mathematica* from printing the help message. Remember that SetQuantumAliases[ ] must be evaluated again in each new notebook:

```
SetQuantumAliases[];
```

From Matrix Notation to Dirac Notation

A matrix can be generated with the standard *Mathematica* command Table.

```
mymatrix = Table[k[i, j], {i, 0, 3}, {j, 0, 3}]
```

```
{{k[0, 0], k[0, 1], k[0, 2], k[0, 3]}, {k[1, 0], k[1, 1], k[1, 2], k[1, 3]},
 {k[2, 0], k[2, 1], k[2, 2], k[2, 3]}, {k[3, 0], k[3, 1], k[3, 2], k[3, 3]}}
```

The matrix can be visualized with the standard *Mathematica* command MatrixForm:

```
MatrixForm[mymatrix]
```

$$\begin{pmatrix} k[0,0] & k[0,1] & k[0,2] & k[0,3] \\ k[1,0] & k[1,1] & k[1,2] & k[1,3] \\ k[2,0] & k[2,1] & k[2,2] & k[2,3] \\ k[3,0] & k[3,1] & k[3,2] & k[3,3] \end{pmatrix}$$

The matrix can be transformed to Dirac Notation using the Quantum *Mathematica* command MatrixToDirac. Here it is specified that there is only one quantum number with 4 posible values (0,1,2,3):

```
MatrixToDirac[mymatrix, {4}]
```

$$\begin{aligned}
&k[0,0]\,|0_{\hat{1}}\rangle\cdot\langle 0_{\hat{1}}| + k[1,0]\,|1_{\hat{1}}\rangle\cdot\langle 0_{\hat{1}}| + k[2,0]\,|2_{\hat{1}}\rangle\cdot\langle 0_{\hat{1}}| + k[3,0]\,|3_{\hat{1}}\rangle\cdot\langle 0_{\hat{1}}| + \\
&k[0,1]\,|0_{\hat{1}}\rangle\cdot\langle 1_{\hat{1}}| + k[1,1]\,|1_{\hat{1}}\rangle\cdot\langle 1_{\hat{1}}| + k[2,1]\,|2_{\hat{1}}\rangle\cdot\langle 1_{\hat{1}}| + k[3,1]\,|3_{\hat{1}}\rangle\cdot\langle 1_{\hat{1}}| + \\
&k[0,2]\,|0_{\hat{1}}\rangle\cdot\langle 2_{\hat{1}}| + k[1,2]\,|1_{\hat{1}}\rangle\cdot\langle 2_{\hat{1}}| + k[2,2]\,|2_{\hat{1}}\rangle\cdot\langle 2_{\hat{1}}| + k[3,2]\,|3_{\hat{1}}\rangle\cdot\langle 2_{\hat{1}}| + \\
&k[0,3]\,|0_{\hat{1}}\rangle\cdot\langle 3_{\hat{1}}| + k[1,3]\,|1_{\hat{1}}\rangle\cdot\langle 3_{\hat{1}}| + k[2,3]\,|2_{\hat{1}}\rangle\cdot\langle 3_{\hat{1}}| + k[3,3]\,|3_{\hat{1}}\rangle\cdot\langle 3_{\hat{1}}|
\end{aligned}$$

Here it is specified that there are two quantum numbers, with 2 posible values each one (0,1):

```
MatrixToDirac[mymatrix, {2, 2}]
```

$$\begin{aligned}
&k[0,0]\,|0_{\hat{1}}, 0_{\hat{2}}\rangle\cdot\langle 0_{\hat{1}}, 0_{\hat{2}}| + k[1,0]\,|0_{\hat{1}}, 1_{\hat{2}}\rangle\cdot\langle 0_{\hat{1}}, 0_{\hat{2}}| + \\
&k[2,0]\,|1_{\hat{1}}, 0_{\hat{2}}\rangle\cdot\langle 0_{\hat{1}}, 0_{\hat{2}}| + k[3,0]\,|1_{\hat{1}}, 1_{\hat{2}}\rangle\cdot\langle 0_{\hat{1}}, 0_{\hat{2}}| + \\
&k[0,1]\,|0_{\hat{1}}, 0_{\hat{2}}\rangle\cdot\langle 0_{\hat{1}}, 1_{\hat{2}}| + k[1,1]\,|0_{\hat{1}}, 1_{\hat{2}}\rangle\cdot\langle 0_{\hat{1}}, 1_{\hat{2}}| + k[2,1]\,|1_{\hat{1}}, 0_{\hat{2}}\rangle\cdot\langle 0_{\hat{1}}, 1_{\hat{2}}| + \\
&k[3,1]\,|1_{\hat{1}}, 1_{\hat{2}}\rangle\cdot\langle 0_{\hat{1}}, 1_{\hat{2}}| + k[0,2]\,|0_{\hat{1}}, 0_{\hat{2}}\rangle\cdot\langle 1_{\hat{1}}, 0_{\hat{2}}| + k[1,2]\,|0_{\hat{1}}, 1_{\hat{2}}\rangle\cdot\langle 1_{\hat{1}}, 0_{\hat{2}}| + \\
&k[2,2]\,|1_{\hat{1}}, 0_{\hat{2}}\rangle\cdot\langle 1_{\hat{1}}, 0_{\hat{2}}| + k[3,2]\,|1_{\hat{1}}, 1_{\hat{2}}\rangle\cdot\langle 1_{\hat{1}}, 0_{\hat{2}}| + k[0,3]\,|0_{\hat{1}}, 0_{\hat{2}}\rangle\cdot\langle 1_{\hat{1}}, 1_{\hat{2}}| + \\
&k[1,3]\,|0_{\hat{1}}, 1_{\hat{2}}\rangle\cdot\langle 1_{\hat{1}}, 1_{\hat{2}}| + k[2,3]\,|1_{\hat{1}}, 0_{\hat{2}}\rangle\cdot\langle 1_{\hat{1}}, 1_{\hat{2}}| + k[3,3]\,|1_{\hat{1}}, 1_{\hat{2}}\rangle\cdot\langle 1_{\hat{1}}, 1_{\hat{2}}|
\end{aligned}$$

Here it is specified that there are two quantum numbers, with 2 posible values each one, and explicit labels for each eigen-value and each operator are given:

$$\texttt{MatrixToDirac}\left[\texttt{mymatrix, \{2, 2\}, } \left\{0_{\hat{1}} \to a1_{\hat{a}},\ 1_{\hat{1}} \to a2_{\hat{a}},\ 0_{\hat{2}} \to b1_{\hat{b}},\ 1_{\hat{2}} \to b2_{\hat{b}}\right\}\right]$$

$$\begin{aligned}
&k[0,0]\,|a1_{\hat{a}}, b1_{\hat{b}}\rangle\cdot\langle a1_{\hat{a}}, b1_{\hat{b}}| + k[1,0]\,|a1_{\hat{a}}, b2_{\hat{b}}\rangle\cdot\langle a1_{\hat{a}}, b1_{\hat{b}}| + \\
&k[2,0]\,|a2_{\hat{a}}, b1_{\hat{b}}\rangle\cdot\langle a1_{\hat{a}}, b1_{\hat{b}}| + k[3,0]\,|a2_{\hat{a}}, b2_{\hat{b}}\rangle\cdot\langle a1_{\hat{a}}, b1_{\hat{b}}| + \\
&k[0,1]\,|a1_{\hat{a}}, b1_{\hat{b}}\rangle\cdot\langle a1_{\hat{a}}, b2_{\hat{b}}| + k[1,1]\,|a1_{\hat{a}}, b2_{\hat{b}}\rangle\cdot\langle a1_{\hat{a}}, b2_{\hat{b}}| + \\
&k[2,1]\,|a2_{\hat{a}}, b1_{\hat{b}}\rangle\cdot\langle a1_{\hat{a}}, b2_{\hat{b}}| + k[3,1]\,|a2_{\hat{a}}, b2_{\hat{b}}\rangle\cdot\langle a1_{\hat{a}}, b2_{\hat{b}}| + \\
&k[0,2]\,|a1_{\hat{a}}, b1_{\hat{b}}\rangle\cdot\langle a2_{\hat{a}}, b1_{\hat{b}}| + k[1,2]\,|a1_{\hat{a}}, b2_{\hat{b}}\rangle\cdot\langle a2_{\hat{a}}, b1_{\hat{b}}| + \\
&k[2,2]\,|a2_{\hat{a}}, b1_{\hat{b}}\rangle\cdot\langle a2_{\hat{a}}, b1_{\hat{b}}| + k[3,2]\,|a2_{\hat{a}}, b2_{\hat{b}}\rangle\cdot\langle a2_{\hat{a}}, b1_{\hat{b}}| + \\
&k[0,3]\,|a1_{\hat{a}}, b1_{\hat{b}}\rangle\cdot\langle a2_{\hat{a}}, b2_{\hat{b}}| + k[1,3]\,|a1_{\hat{a}}, b2_{\hat{b}}\rangle\cdot\langle a2_{\hat{a}}, b2_{\hat{b}}| + \\
&k[2,3]\,|a2_{\hat{a}}, b1_{\hat{b}}\rangle\cdot\langle a2_{\hat{a}}, b2_{\hat{b}}| + k[3,3]\,|a2_{\hat{a}}, b2_{\hat{b}}\rangle\cdot\langle a2_{\hat{a}}, b2_{\hat{b}}|
\end{aligned}$$

The command Table is used to generate a larger matrix:

```
mymatrix2 = Table[k[i, j], {i, 0, 5}, {j, 0, 5}]
```

```
{{k[0, 0], k[0, 1], k[0, 2], k[0, 3], k[0, 4], k[0, 5]},
 {k[1, 0], k[1, 1], k[1, 2], k[1, 3], k[1, 4], k[1, 5]},
 {k[2, 0], k[2, 1], k[2, 2], k[2, 3], k[2, 4], k[2, 5]},
 {k[3, 0], k[3, 1], k[3, 2], k[3, 3], k[3, 4], k[3, 5]},
 {k[4, 0], k[4, 1], k[4, 2], k[4, 3], k[4, 4], k[4, 5]},
 {k[5, 0], k[5, 1], k[5, 2], k[5, 3], k[5, 4], k[5, 5]}}
```

The larger matrix can be visualized with the standard *Mathematica* command MatrixForm:

```
MatrixForm[mymatrix2]
```

$$\begin{pmatrix} k[0,0] & k[0,1] & k[0,2] & k[0,3] & k[0,4] & k[0,5] \\ k[1,0] & k[1,1] & k[1,2] & k[1,3] & k[1,4] & k[1,5] \\ k[2,0] & k[2,1] & k[2,2] & k[2,3] & k[2,4] & k[2,5] \\ k[3,0] & k[3,1] & k[3,2] & k[3,3] & k[3,4] & k[3,5] \\ k[4,0] & k[4,1] & k[4,2] & k[4,3] & k[4,4] & k[4,5] \\ k[5,0] & k[5,1] & k[5,2] & k[5,3] & k[5,4] & k[5,5] \end{pmatrix}$$

The matrix can be transformed to Dirac Notation using the Quantum *Mathematica* command MatrixToDirac. Here it is specified that there are two quantum numbers, the first one with 3 posible values (0,1,2) and the second one with 2 posible eigenvalues (0,1):

```
MatrixToDirac[mymatrix2, {3, 2}]
```

$$\begin{aligned}
&k[0,0]\ \left|0_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 0_{\hat{1}}, 0_{\hat{2}}\right| + k[1,0]\ \left|0_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 0_{\hat{1}}, 0_{\hat{2}}\right| + k[2,0]\ \left|1_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 0_{\hat{1}}, 0_{\hat{2}}\right| + \\
&k[3,0]\ \left|1_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 0_{\hat{1}}, 0_{\hat{2}}\right| + k[4,0]\ \left|2_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 0_{\hat{1}}, 0_{\hat{2}}\right| + k[5,0]\ \left|2_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 0_{\hat{1}}, 0_{\hat{2}}\right| + \\
&k[0,1]\ \left|0_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 0_{\hat{1}}, 1_{\hat{2}}\right| + k[1,1]\ \left|0_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 0_{\hat{1}}, 1_{\hat{2}}\right| + k[2,1]\ \left|1_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 0_{\hat{1}}, 1_{\hat{2}}\right| + \\
&k[3,1]\ \left|1_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 0_{\hat{1}}, 1_{\hat{2}}\right| + k[4,1]\ \left|2_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 0_{\hat{1}}, 1_{\hat{2}}\right| + k[5,1]\ \left|2_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 0_{\hat{1}}, 1_{\hat{2}}\right| + \\
&k[0,2]\ \left|0_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 1_{\hat{1}}, 0_{\hat{2}}\right| + k[1,2]\ \left|0_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 1_{\hat{1}}, 0_{\hat{2}}\right| + k[2,2]\ \left|1_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 1_{\hat{1}}, 0_{\hat{2}}\right| + \\
&k[3,2]\ \left|1_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 1_{\hat{1}}, 0_{\hat{2}}\right| + k[4,2]\ \left|2_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 1_{\hat{1}}, 0_{\hat{2}}\right| + k[5,2]\ \left|2_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 1_{\hat{1}}, 0_{\hat{2}}\right| + \\
&k[0,3]\ \left|0_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 1_{\hat{1}}, 1_{\hat{2}}\right| + k[1,3]\ \left|0_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 1_{\hat{1}}, 1_{\hat{2}}\right| + k[2,3]\ \left|1_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 1_{\hat{1}}, 1_{\hat{2}}\right| + \\
&k[3,3]\ \left|1_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 1_{\hat{1}}, 1_{\hat{2}}\right| + k[4,3]\ \left|2_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 1_{\hat{1}}, 1_{\hat{2}}\right| + k[5,3]\ \left|2_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 1_{\hat{1}}, 1_{\hat{2}}\right| + \\
&k[0,4]\ \left|0_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 2_{\hat{1}}, 0_{\hat{2}}\right| + k[1,4]\ \left|0_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 2_{\hat{1}}, 0_{\hat{2}}\right| + k[2,4]\ \left|1_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 2_{\hat{1}}, 0_{\hat{2}}\right| + \\
&k[3,4]\ \left|1_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 2_{\hat{1}}, 0_{\hat{2}}\right| + k[4,4]\ \left|2_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 2_{\hat{1}}, 0_{\hat{2}}\right| + k[5,4]\ \left|2_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 2_{\hat{1}}, 0_{\hat{2}}\right| + \\
&k[0,5]\ \left|0_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 2_{\hat{1}}, 1_{\hat{2}}\right| + k[1,5]\ \left|0_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 2_{\hat{1}}, 1_{\hat{2}}\right| + k[2,5]\ \left|1_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 2_{\hat{1}}, 1_{\hat{2}}\right| + \\
&k[3,5]\ \left|1_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 2_{\hat{1}}, 1_{\hat{2}}\right| + k[4,5]\ \left|2_{\hat{1}}, 0_{\hat{2}}\right\rangle\cdot\left\langle 2_{\hat{1}}, 1_{\hat{2}}\right| + k[5,5]\ \left|2_{\hat{1}}, 1_{\hat{2}}\right\rangle\cdot\left\langle 2_{\hat{1}}, 1_{\hat{2}}\right|
\end{aligned}$$

From Dirac Notation to Matrix Notation

Here is a simple Dirac expression which will be used to illustrate the opposite procedure:

$$\mathbf{mydirac = k1}\ \left|a1_{\hat{a}}, b2_{\hat{b}}\right\rangle\cdot\left\langle a1_{\hat{a}}, b1_{\hat{b}}\right| + \mathbf{k2}\ \left|a1_{\hat{a}}, b1_{\hat{b}}\right\rangle\cdot\left\langle a2_{\hat{a}}, b1_{\hat{b}}\right|$$

$$k1\ \left|a1_{\hat{a}}, b2_{\hat{b}}\right\rangle\cdot\left\langle a1_{\hat{a}}, b1_{\hat{b}}\right| + k2\ \left|a1_{\hat{a}}, b1_{\hat{b}}\right\rangle\cdot\left\langle a2_{\hat{a}}, b1_{\hat{b}}\right|$$

The Dirac expression can be transformed to a matrix using the Quantum *Mathematica* command DiracToMatrix. It is specified that each operator has two possible eigenvalues:

```
m1 = DiracToMatrix[mydirac, {{a1â, a2â}, {b1b̂, b2b̂}}]
```

```
{{0, 0, k2, 0}, {k1, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

The calculated matrix can be visualized with the standard *Mathematica* command MatrixForm:

```
MatrixForm[m1]
```

$$\begin{pmatrix} 0 & 0 & k2 & 0 \\ k1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The Dirac expression can be transformed to a tensor (matrix of matrices) using the Quantum *Mathematica* command DiracToTensor. It is specified that each operator has two possible eigenvalues:

```
t1 = DiracToTensor[mydirac, {{a1â, a2â}, {b1b̂, b2b̂}}]
```

```
{{{{0, 0}, {k1, 0}}, {{k2, 0}, {0, 0}}}, {{{0, 0}, {0, 0}}, {{0, 0}, {0, 0}}}}
```

The calculated tensor can be visualized with the standard *Mathematica* command MatrixForm:

```
MatrixForm[t1]
```

$$\begin{pmatrix} \begin{pmatrix} 0 & 0 \\ k1 & 0 \end{pmatrix} & \begin{pmatrix} k2 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}$$

Operator Diagonalization

The command DiracEigensystem gives eigenvalues and eigenstates of operators. The input syntax is the same as the syntax of DiracToMatrix and DiracToTensor, and the output has the same format as the output of the standard *Mathematica* command Eigensystem: first the list of eigenvalues, then the list of the corresponding eigenvectors:

```
DiracEigensystem[
  | 0â⟩ · ⟨0â | + ⅈ | 1â⟩ · ⟨1â | + | 3â⟩ · ⟨2â | + | 2â⟩ · ⟨3â |,
  {{0â, 1â, 2â, 3â}}]
```

$$\left\{\{-1, i, 1, 1\}, \left\{-\frac{|2_{\hat{a}}\rangle}{\sqrt{2}} + \frac{|3_{\hat{a}}\rangle}{\sqrt{2}}, |1_{\hat{a}}\rangle, \frac{|2_{\hat{a}}\rangle}{\sqrt{2}} + \frac{|3_{\hat{a}}\rangle}{\sqrt{2}}, |0_{\hat{a}}\rangle\right\}\right\}$$

Here we use Transpose and Grid to give a nicer formating for the output:

```
Grid[Transpose[
  DiracEigensystem[
    | 0â〉 · 〈0â | + ⅈ | 1â〉 · 〈1â | + | 3â〉 · 〈2â | + | 2â〉 · 〈3â |,
   {{0â, 1â, 2â, 3â}}]
 ], Dividers → All]
```

-1	$-\frac{\vert 2_{\hat{a}}\rangle}{\sqrt{2}} + \frac{\vert 3_{\hat{a}}\rangle}{\sqrt{2}}$
ⅈ	$\vert 1_{\hat{a}}\rangle$
1	$\frac{\vert 2_{\hat{a}}\rangle}{\sqrt{2}} + \frac{\vert 3_{\hat{a}}\rangle}{\sqrt{2}}$
1	$\vert 0_{\hat{a}}\rangle$

From Symbolic Operators to Matrix Notation

Here is the definition of a simple operator (named "myop") which will be used to illustrate the procedure in the operators created with the Quantum *Mathematica* command DefineOperatorOnKets:

```
DefineOperatorOnKets[myop, { | 0î〉 :→ a | 0î〉 + b | 1î〉,  | 1î〉 :→ c | 0î〉 + d | 1î〉}]
```

```
| 0î〉 :→ a | 0î〉 + b | 1î〉
| 1î〉 :→ c | 0î〉 + d | 1î〉
```

Here is the simple operator applied to a linear combination of kets:

```
myop · (e | 0î〉 + g | 1î〉)
```

```
e (a | 0î〉 + b | 1î〉) + g (c | 0î〉 + d | 1î〉)
```

Here is the simple operator applied to a linear combination of kets and expanded:

```
Expand[myop · (e | 0î〉 + g | 1î〉)]
```

```
a e | 0î〉 + c g | 0î〉 + b e | 1î〉 + d g | 1î〉
```

Here is the simple operator applied to a linear combination of kets and kets are "collected":

```
CollectKet[myop · (e | 0î〉 + g | 1î〉)]
```

```
(a e + c g) | 0î〉 + (b e + d g) | 1î〉
```

The operator can be transformed to a matrix using the Quantum *Mathematica* command DiracToMatrix. It is specified that the operator has two possible eigenvalues:

```
matrixop = DiracToMatrix[myop, {{0_î, 1_î}}]
```

$$\{\{a, c\}, \{b, d\}\}$$

The calculated matrix can be visualized with the standard *Mathematica* command MatrixForm:

```
MatrixForm[matrixop]
```

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

We can also diagonalize this operator. Notice that *Mathematica* maninpulates a,b,c,d as **complex numbers**:

```
DiracEigensystem[myop, {{0_î, 1_î}}]
```

$$\left\{\left\{\frac{1}{2}\left(a+d-\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right), \frac{1}{2}\left(a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)\right\},\right.$$

$$\left\{\frac{\left(a-d-\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)\,|0_{\hat{1}}\rangle}{b\sqrt{4+\mathrm{Abs}\left[\frac{-a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}}{b}\right]^2}}+\frac{2\,|1_{\hat{1}}\rangle}{\sqrt{4+\mathrm{Abs}\left[\frac{-a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}}{b}\right]^2}},\right.$$

$$\left.\left.\frac{\left(a-d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)\,|0_{\hat{1}}\rangle}{b\sqrt{4+\mathrm{Abs}\left[\frac{a-d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}}{b}\right]^2}}+\frac{2\,|1_{\hat{1}}\rangle}{\sqrt{4+\mathrm{Abs}\left[\frac{a-d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}}{b}\right]^2}}\right\}\right\}$$

Here we use Transpose and Grid to give a nicer formating for the output:

```
Grid[Transpose[
  DiracEigensystem[myop, {{0_î, 1_î}}]
 ], Dividers → All]
```

$\frac{1}{2}\left(a+d-\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)$	$\frac{\left(a-d-\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)\,\vert 0_{\hat{1}}\rangle}{b\sqrt{4+\mathrm{Abs}\left[\frac{-a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}}{b}\right]^2}}+\frac{2\,\vert 1_{\hat{1}}\rangle}{\sqrt{4+\mathrm{Abs}\left[\frac{-a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}}{b}\right]^2}}$
$\frac{1}{2}\left(a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)$	$\frac{\left(a-d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)\,\vert 0_{\hat{1}}\rangle}{b\sqrt{4+\mathrm{Abs}\left[\frac{a-d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}}{b}\right]^2}}+\frac{2\,\vert 1_{\hat{1}}\rangle}{\sqrt{4+\mathrm{Abs}\left[\frac{a-d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}}{b}\right]^2}}$

by José Luis Gómez-Muñoz
http://homepage.cem.itesm.mx/lgomez/quantum/
jose.luis.gomez@itesm.mx