Quantum Teleportation

by José Luis Gómez-Muñoz
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Introduction

This is a tutorial on the use of Quantum`Computing` *Mathematica* add-on to simulate quantum teleportation.

Load the Package

First load the Quantum`Computing` package. Write:
Needs["Quantum`Computing`"];
then press at the same time the keys SHIFT-ENTER to evaluate. *Mathematica* will load the package:

```
Needs["Quantum`Computing`"]
```

```
Quantum`Computing` Version 2.2.0. (June 2010)
A Mathematica package for Quantum Computing
  in Dirac bra-ket notation and plotting of quantum circuits
by José Luis Gómez-Muñoz

Execute SetComputingAliases[] in order to use
  the keyboard to enter quantum objects in Dirac's notation
SetComputingAliases[] must be executed again in each new notebook that is created
```

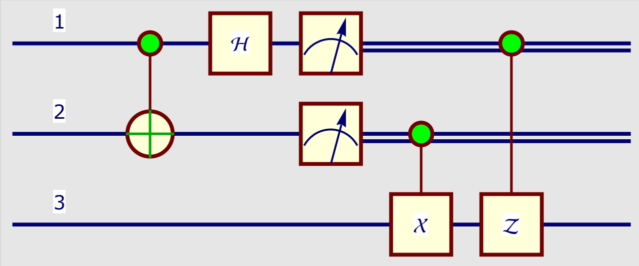
In order to use the keyboard to enter quantum objects write:
SetComputingAliases[ ];
then press at the same time the keys SHIFT-ENTER to evaluate. The semicolon prevents *Mathematica* from printing the help message. Remember that SetComputingAliases[ ] must be evaluated again in each new notebook:

```
SetComputingAliases[];
```

Quantum Teleportation

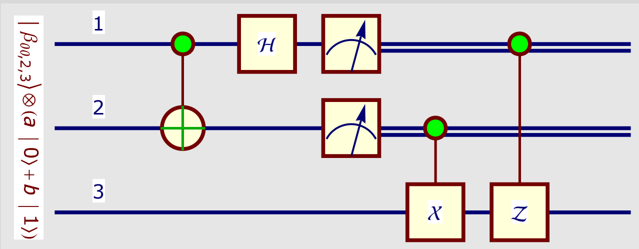
Below is the Quantum Teleportation circuit. Notice the syntaxis for specifying gates that are applied before the measurement and gates that are applied after the measurment.

$$\textbf{QuantumPlot}\left[\mathcal{C}^{\{\hat{1}\}}[\mathcal{Z}_{\hat{3}}]\cdot\mathcal{C}^{\{\hat{2}\}}[\mathcal{X}_{\hat{3}}]\cdot\textbf{QubitMeasurement}\left[\mathcal{H}_{\hat{1}}\cdot\mathcal{C}^{\{\hat{1}\}}[\mathcal{NOT}_{\hat{2}}],\ \{\hat{1},\ \hat{2}\}\right]\right]$$



Actually, Quantum Teleportation requires the second and third qubits to be on their maximum entanglement state $\left|\mathcal{B}_{00,\hat{2},\hat{3}}\right\rangle$:

$$\textbf{QuantumPlot}\left[\mathcal{C}^{\{\hat{1}\}}[\mathcal{Z}_{\hat{3}}]\cdot\mathcal{C}^{\{\hat{2}\}}[\mathcal{X}_{\hat{3}}]\cdot \textbf{QubitMeasurement}\left[\mathcal{H}_{\hat{1}}\cdot\mathcal{C}^{\{\hat{1}\}}[\mathcal{NOT}_{\hat{2}}]\cdot\left|\mathcal{B}_{00,\hat{2},\hat{3}}\right\rangle\otimes\left(\mathbf{a}\ \left|0_{\hat{1}}\right\rangle+\mathbf{b}\ \left|1_{\hat{1}}\right\rangle\right),\ \{\hat{1},\ \hat{2}\}\right]\right]$$



QuantumEvaluate shows that this circuit actually "teleports" (cuts and pastes) **a** and **b** from the **first qubit** in the initial state:

$\left(\mathbf{a}\ \left|0_{\hat{1}}\right\rangle+\mathbf{b}\ \left|1_{\hat{1}}\right\rangle\right)\otimes\left|\mathcal{B}_{00,\hat{2},\hat{3}}\right\rangle$

to the **third qubit** in the final state:

$\left|\psi_{\hat{1},\hat{2}}\right\rangle\otimes(\mathbf{a}\ \left|0_{\hat{3}}\right\rangle+\mathbf{b}\ \left|1_{\hat{3}}\right\rangle)$

(Below the final states include a normalization factor $\sqrt{a\,a^* + b\,b^*}$)

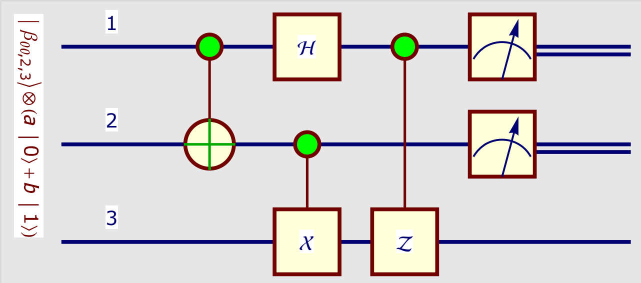
$$\textbf{QuantumEvaluate}\left[\mathcal{C}^{\{\hat{1}\}}[\mathcal{Z}_{\hat{3}}]\cdot\mathcal{C}^{\{\hat{2}\}}[\mathcal{X}_{\hat{3}}]\cdot \textbf{QubitMeasurement}\left[\mathcal{H}_{\hat{1}}\cdot\mathcal{C}^{\{\hat{1}\}}[\mathcal{NOT}_{\hat{2}}]\cdot\left|\mathcal{B}_{00,\hat{2},\hat{3}}\right\rangle\otimes\left(\mathbf{a}\ \left|0_{\hat{1}}\right\rangle+\mathbf{b}\ \left|1_{\hat{1}}\right\rangle\right),\ \{\hat{1},\ \hat{2}\}\right]\right]$$

Probability	Measurement	State
$\frac{1}{4}$	$\{\{0_{\hat{1}},\ 0_{\hat{2}}\}\}$	$\left|0_{\hat{1}}\right\rangle\otimes\left|0_{\hat{2}}\right\rangle\otimes\left(\frac{a\left|0_{\hat{3}}\right\rangle}{\sqrt{a\,a^*+b\,b^*}}+\frac{b\left|1_{\hat{3}}\right\rangle}{\sqrt{a\,a^*+b\,b^*}}\right)$
$\frac{1}{4}$	$\{\{0_{\hat{1}},\ 1_{\hat{2}}\}\}$	$\left|0_{\hat{1}}\right\rangle\otimes\left|1_{\hat{2}}\right\rangle\otimes\left(\frac{a\left|0_{\hat{3}}\right\rangle}{\sqrt{a\,a^*+b\,b^*}}+\frac{b\left|1_{\hat{3}}\right\rangle}{\sqrt{a\,a^*+b\,b^*}}\right)$
$\frac{1}{4}$	$\{\{1_{\hat{1}},\ 0_{\hat{2}}\}\}$	$\left|1_{\hat{1}}\right\rangle\otimes\left|0_{\hat{2}}\right\rangle\otimes\left(\frac{a\left|0_{\hat{3}}\right\rangle}{\sqrt{a\,a^*+b\,b^*}}+\frac{b\left|1_{\hat{3}}\right\rangle}{\sqrt{a\,a^*+b\,b^*}}\right)$
$\frac{1}{4}$	$\{\{1_{\hat{1}},\ 1_{\hat{2}}\}\}$	$\left|1_{\hat{1}}\right\rangle\otimes\left|1_{\hat{2}}\right\rangle\otimes\left(\frac{a\left|0_{\hat{3}}\right\rangle}{\sqrt{a\,a^*+b\,b^*}}+\frac{b\left|1_{\hat{3}}\right\rangle}{\sqrt{a\,a^*+b\,b^*}}\right)$
Probability	Measurement	State

Variation of Quantum Teleportation Circuit: Controlled Gates Commute with Measurements on the Control Qubits

Controlled gates commute with measurements on the control qubits. Therefore the Teleportation Circuit can have the measurements at the end:

$$\texttt{QuantumPlot}\Big[\texttt{QubitMeasurement}\Big[\mathcal{C}^{\{\hat{1}\}}[\mathcal{Z}_{\hat{3}}]\cdot\mathcal{C}^{\{\hat{2}\}}[\mathcal{X}_{\hat{3}}]\cdot\mathcal{H}_{\hat{1}}\cdot\mathcal{C}^{\{\hat{1}\}}[\mathcal{NOT}_{\hat{2}}]\cdot\ \left|\mathcal{B}_{00,\hat{2},\hat{3}}\right\rangle\otimes\left(\mathbf{a}\ \left|0_{\hat{1}}\right\rangle+\mathbf{b}\ \left|1_{\hat{1}}\right\rangle\right),\ \{\hat{1},\hat{2}\}\Big]\Big]$$



Again Teleportation calculations, but this time with the measurement at the end. This circuit actually "teleports" (cuts and pastes) **a** and **b** from the **first qubit** in the initial state:

$\left(\mathbf{a}\ \left|0_{\hat{1}}\right\rangle+\mathbf{b}\ \left|1_{\hat{1}}\right\rangle\right)\otimes\left|\mathcal{B}_{00,\hat{2},\hat{3}}\right\rangle$

to the **third qubit** in the final state:

$\left|\psi_{\hat{1},\hat{2}}\right\rangle\otimes(\mathbf{a}\left|0_{\hat{3}}\right\rangle+\mathbf{b}\left|1_{\hat{3}}\right\rangle)$

(Below the final states include a normalization factor $\sqrt{a\,a^{*}+b\,b^{*}}$)

$$\texttt{QuantumEvaluate}\Big[\texttt{QubitMeasurement}\Big[\mathcal{C}^{\{\hat{1}\}}[\mathcal{Z}_{\hat{3}}]\cdot\mathcal{C}^{\{\hat{2}\}}[\mathcal{X}_{\hat{3}}]\cdot\mathcal{H}_{\hat{1}}\cdot\mathcal{C}^{\{\hat{1}\}}[\mathcal{NOT}_{\hat{2}}]\cdot\ \left|\mathcal{B}_{00,\hat{2},\hat{3}}\right\rangle\otimes\left(\mathbf{a}\ \left|0_{\hat{1}}\right\rangle+\mathbf{b}\ \left|1_{\hat{1}}\right\rangle\right),\ \{\hat{1},\hat{2}\}\Big]\Big]$$

Probability	Measurement	State
$\frac{1}{4}$	$\{\{0_{\hat{1}},\ 0_{\hat{2}}\}\}$	$\left\|0_{\hat{1}}\right\rangle\otimes\left\|0_{\hat{2}}\right\rangle\otimes\left(\frac{a\left\|0_{\hat{3}}\right\rangle}{\sqrt{a\,a^{*}+b\,b^{*}}}+\frac{b\left\|1_{\hat{3}}\right\rangle}{\sqrt{a\,a^{*}+b\,b^{*}}}\right)$
$\frac{1}{4}$	$\{\{0_{\hat{1}},\ 1_{\hat{2}}\}\}$	$\left\|0_{\hat{1}}\right\rangle\otimes\left\|1_{\hat{2}}\right\rangle\otimes\left(\frac{a\left\|0_{\hat{3}}\right\rangle}{\sqrt{a\,a^{*}+b\,b^{*}}}+\frac{b\left\|1_{\hat{3}}\right\rangle}{\sqrt{a\,a^{*}+b\,b^{*}}}\right)$
$\frac{1}{4}$	$\{\{1_{\hat{1}},\ 0_{\hat{2}}\}\}$	$\left\|1_{\hat{1}}\right\rangle\otimes\left\|0_{\hat{2}}\right\rangle\otimes\left(\frac{a\left\|0_{\hat{3}}\right\rangle}{\sqrt{a\,a^{*}+b\,b^{*}}}+\frac{b\left\|1_{\hat{3}}\right\rangle}{\sqrt{a\,a^{*}+b\,b^{*}}}\right)$
$\frac{1}{4}$	$\{\{1_{\hat{1}},\ 1_{\hat{2}}\}\}$	$\left\|1_{\hat{1}}\right\rangle\otimes\left\|1_{\hat{2}}\right\rangle\otimes\left(\frac{a\left\|0_{\hat{3}}\right\rangle}{\sqrt{a\,a^{*}+b\,b^{*}}}+\frac{b\left\|1_{\hat{3}}\right\rangle}{\sqrt{a\,a^{*}+b\,b^{*}}}\right)$
Probability	Measurement	State

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