Using the Keyboard to Enter Quantum Computing Notation in *Mathematica*

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Introduction

This is a tutorial on the use of Quantum`Computing` *Mathematica* add-on to enter Quantum Computing notation (kets, gates, quantum circuits, etc) in *Mathematica*.

Load the Package

First load the Quantum`Computing` package. Write:
Needs["Quantum`Computing`"];
then press at the same time the keys SHIFT-ENTER to evaluate. *Mathematica* will load the package and it will printa a welcome message:

```
Needs["Quantum`Computing`"]
```

```
Quantum`Computing` Version 2.2.0. (July 2010)
A Mathematica package for Quantum Computing
  in Dirac bra-ket notation and plotting of quantum circuits
by José Luis Gómez-Muñoz

Execute SetComputingAliases[] in order to use
  the keyboard to enter quantum objects in Dirac's notation
SetComputingAliases[] must be executed again in each new notebook that is created
```

In order to use the keyboard to enter quantum objects write:
SetComputingAliases[ ];
then press at the same time the keys SHIFT-ENTER to evaluate. *Mathematica* will print a message with all the new keyboard aliases. Remember that SetComputingAliases[ ] must be evaluated again in each new notebook:

```
SetComputingAliases[]
```

```
ALIASES:
[ESC]on[ESC]        Quantum concatenation symbol
  (operator application, inner product and outer product)
[ESC]qket0[ESC]     Ket of qubit 0 template
[ESC]qbra0[ESC]     Bra of qubit 0 template
[ESC]qket1[ESC]     Ket of qubit 1 template
[ESC]qbra1[ESC]     Bra of qubit 1 template
[ESC]qket[ESC]      Ket of qubit template
[ESC]qqket[ESC]     Ket of two qubits template
[ESC]qqqket[ESC]    Ket of three qubits template
[ESC]qbra[ESC]      Bra of qubit template
[ESC]qqbra[ESC]     Bra of two qubits template
[ESC]qqqbra[ESC]    Bra of three qubits template
[ESC]toqb[ESC]      Base-10 Integer to binary qubit template
[ESC]ket[ESC]       Ket template
[ESC]bra[ESC]       Bra template
[ESC]qb[ESC]        Qubit template
[ESC]qv[ESC]        Qubit-value template
[ESC]qketbra[ESC]   Element of a one-qubit operator template
[ESC]qqketbra[ESC]  Element of a two-qubits operator template
[ESC]qqqketbra[ESC] Element of a three-qubits operator template
[ESC]k+[ESC]        Plus ket (eigenstate of the first Pauli matrix)
[ESC]b+[ESC]        Plus bra
[ESC]k-[ESC]        Minus ket (eigenstate of the first Pauli matrix)
[ESC]b-[ESC]        Minus bra
[ESC]k00[ESC]       Ket of Bell State 00
[ESC]k01[ESC]       Ket of Bell State 01
[ESC]k10[ESC]       Ket of Bell State 10
[ESC]k11[ESC]       Ket of Bell State 11
[ESC]b00[ESC]       Bra of Bell State 00
[ESC]b01[ESC]       Bra of Bell State 01
[ESC]b10[ESC]       Bra of Bell State 10
[ESC]b11[ESC]       Bra of Bell State 11
[ESC]kphi+[ESC]     Ket of Bell State phi+
[ESC]kpsi+[ESC]     Ket of Bell State psi+
[ESC]kphi-[ESC]     Ket of Bell State phi-
[ESC]kpsi-[ESC]     Ket of Bell State psi-
[ESC]bphi+[ESC]     Bra of Bell State phi+
[ESC]bpsi+[ESC]     Bra of Bell State psi+
[ESC]bphi-[ESC]     Bra of Bell State phi-
[ESC]bpsi-[ESC]     Bra of Bell State psi-
[ESC]her[ESC]       Hermitian conjugate template
[ESC]con[ESC]       Complex conjugate template
[ESC]norm[ESC]      Quantum norm template
[ESC]trace[ESC]     Partial trace template
[ESC]tp[ESC]        Tensor-product symbol
[ESC]tprod[ESC]     Tensor-product template
[ESC]tprodqb[ESC]   Tensor-product of Qubit template
```

```
[ESC]tpow[ESC]       Tensor-power template
[ESC]tpowqb[ESC]     Tensor-power of Qubit template
[ESC]s0[ESC]         0th-Pauli operator (Identity) template
[ESC]s1[ESC]         1st-Pauli operator (X) template
[ESC]s2[ESC]         2nd-Pauli operator (Y) template
[ESC]s3[ESC]         3rd-Pauli operator (Z) template
[ESC]so[ESC]         0th-Pauli operator (Identity) template
[ESC]sx[ESC]         1st-Pauli operator (X) template
[ESC]sy[ESC]         2nd-Pauli operator (Y) template
[ESC]sz[ESC]         3rd-Pauli operator (Z) template
[ESC]sp[ESC]         General Pauli operator template
[ESC]ig[ESC]         Identity gate template
[ESC]xg[ESC]         Pauli-X gate
[ESC]yg[ESC]         Pauli-Y gate
[ESC]zg[ESC]         Pauli-Z gate
[ESC]hg[ESC]         Haddamard gate
[ESC]pg[ESC]         Parametric phase gate
[ESC]sg[ESC]         S Phase gate
[ESC]tg[ESC]         T π/8 gate
[ESC]swap[ESC]       Swap gate
[ESC]cgate[ESC]      Controlled-Gate template
[ESC]ccgate[ESC]     Controlled-controlled-Gate template
[ESC]cccgate[ESC]    Controlled-controlled-controlled-Gate template
[ESC]cnot[ESC]       Controlled-Not template
[ESC]ccnot[ESC]      Controlled-controlled-Not template
[ESC]cccnot[ESC]     Controlled-controlled-controlled-Not template
[ESC]toff[ESC]       Toffoli gate
[ESC]fred[ESC]       Fredkin gate
[ESC]qg[ESC]         Quantum gate of one argument
[ESC]qqg[ESC]        Quantum gate of one argument applied to two qubits
[ESC]qqqg[ESC]       Quantum gate of one argument applied to three qubits
[ESC]qgg[ESC]        Quantum gate of two arguments
[ESC]qggg[ESC]       Quantum gate of three arguments
[ESC]pqg[ESC]        Parametric quantum gate of one argument
[ESC]qr[ESC]         Quantum register template
[ESC]qrg[ESC]        Quantum-register gate template

SetComputingAliases[] must be executed again in
  each new notebook that is created, only one time per notebook.
```

Entering Quantum Computing Notation

In order to enter a logical zero in the first qubit, press the following keys (Warning: do not type the letter O instead of the number 0. Do not type the letter I nor the letter l instead of the number 1):
[ESC]qket0[ESC]
then press the [TAB] key in order to select the place-holder □ and press:
1
then press at the same time the keys SHIFT-ENTER to evaluate:

$|0_{\hat{1}}\rangle$

$|0_{\hat{1}}\rangle$

Quantum Computing kets can be easily entered. For example, press:
[ESC]k+[ESC][TAB]3
then press at the same time the keys SHIFT-ENTER to evaluate:

$| +_{\hat{3}} \rangle$

$| +_{\hat{3}} \rangle$

The command QuantumEvaluate gives the representation of this ket in terms of the computational basis:

```
QuantumEvaluate[ | +₃̂ ⟩]
```

$$\frac{| 0_{\hat{3}} \rangle}{\sqrt{2}} + \frac{| 1_{\hat{3}} \rangle}{\sqrt{2}}$$

In order to enter the ket of the first Bell state, press the keys:
[ESC]k00[ESC]
then press the [TAB] one or two times in order to select the first place-holder □ and press:
1[TAB]2
then press at the same time the keys SHIFT-ENTER to evaluate:

$| \mathcal{B}_{00,\hat{1},\hat{2}} \rangle$

$| \mathcal{B}_{00,\hat{1},\hat{2}} \rangle$

The command QuantumEvaluate gives the representation of this ket in terms of the computational basis:

```
QuantumEvaluate[ | ℬ₀₀,₁̂,₂̂ ⟩]
```

$$\frac{| 0_{\hat{1}}, 0_{\hat{2}} \rangle}{\sqrt{2}} + \frac{| 1_{\hat{1}}, 1_{\hat{2}} \rangle}{\sqrt{2}}$$

In order to enter the ket of the first Bell state in the Phi-Psi notation, press the keys:
[ESC]kphi+[ESC]
then press the [TAB] one or two times in order to select the first place-holder □ and press:
1[TAB]2
then press at the same time the keys SHIFT-ENTER to evaluate:

$| \Phi^{+}_{\hat{1},\hat{2}} \rangle$

$| \Phi^{+}_{\hat{1},\hat{2}} \rangle$

The command QuantumEvaluate gives the representation of this ket in terms of the computational basis:

```
QuantumEvaluate[ | Φ⁺_{1̂,2̂} ⟩ ]
```

$$\frac{|0_{\hat{1}}, 0_{\hat{2}}\rangle}{\sqrt{2}} + \frac{|1_{\hat{1}}, 1_{\hat{2}}\rangle}{\sqrt{2}}$$

In order to enter a tensor product of qubits, press the keys:
[ESC]qket0[ESC][ESC]tp[ESC][ESC]qket1[ESC]
then press the [TAB] key one or two times in order to select the first place-holder □ and press:
1[TAB]2
(Warning: Do not type the letter O instead of the number 0. Do not type the letter I nor the letter l instead of the number 1)
then press at the same time the keys SHIFT-ENTER to evaluate:

$$|0_{\hat{1}}\rangle \otimes |1_{\hat{2}}\rangle$$

$$|0_{\hat{1}}, 1_{\hat{2}}\rangle$$

In order to enter the ket of two qubits with logical zero, press the keys:
[ESC]qqket[ESC]
Then press the [TAB] key one or more times to select the first "place holder" □, and press the keys:
0[TAB]1[TAB]0[TAB]2
then press at the same time the keys SHIFT-ENTER to evaluate:

$$|0_{\hat{1}}, 0_{\hat{2}}\rangle$$

$$|0_{\hat{1}}, 0_{\hat{2}}\rangle$$

In order to enter an internal product, press the keys:
[ESC]qbra0[ESC][ESC]on[ESC][ESC]qket1[ESC]
Then press the [TAB] key one or more times to select the first "place holder" □, and press the keys:
4[TAB]4
(Notice that in the bra and in the ket the qubit number is the SAME)
then press at the same time the keys SHIFT-ENTER to evaluate.
The relative position of the bras, kets and operators determines without any ambiguity if [ESC]on[ESC] is an internal product, an external product or an operator application.

$$\langle 0_{\hat{4}} | \cdot | 1_{\hat{4}}\rangle$$

$$0$$

We can obtain specific information for Quantum *Mathematica* commands. For example, write:
? QuantumEvaluate
then press at the same time the keys SHIFT-ENTER

```
? QuantumEvaluate
```

QuantumEvaluate[expr] gives Dirac Kets and Bras for expr.
Notice that expr is made of quantum gates connected by the quantum product ·
In order to enter the quantum product · execute SetComputingAliases[]. Then press:
[ESC]on[ESC]   Quantum product template
SetComputingAliases[] must be executed again in each new notebook that is created, only one time per notebook.

Here we obtain a matrix element of the Hadamard-Gate operator:
QuantumEvaluate[[ESC]qbra1[ESC][ESC]on[ESC][ESC]hg[ESC][ESC]on[ESC][ESC]qket1[ESC]]
Then press the [TAB] key one or more times to select the first "place holder" □, and press the keys:
1[TAB]1[TAB]1
then press at the same time the keys SHIFT-ENTER to evaluate:

$$\text{QuantumEvaluate}\left[\langle 1_{\hat{1}} | \cdot \mathcal{H}_{\hat{1}} \cdot | 1_{\hat{1}} \rangle\right]$$

$$-\frac{1}{\sqrt{2}}$$

Here we can see the matrix that corresponds to the Hadamard-Gate.

$$\text{QuantumMatrixForm}\left[\mathcal{H}_{\hat{1}}\right]$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

QuantumMatrixForm[] and QuantumTensorForm[] give an output that is adequate for displaying purposes. On the other hand, for calculation purposes, is better to use QuantumMatrix[] and QuantumTensor[], which give a standard *Mathematica* matrix (list) as an output:

$$\text{QuantumMatrix}\left[\mathcal{H}_{\hat{1}}\right]$$

$$\left\{\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}\right\}$$

The Pauli gates can be entered:
QuantumEvaluate[[ESC]yg[ESC]] [TAB]3
then press at the same time the keys SHIFT-ENTER to evaluate:

$$\text{QuantumEvaluate}\left[\mathcal{Y}_{\hat{3}}\right]$$

$$i\ | 1_{\hat{3}} \rangle \cdot \langle 0_{\hat{3}} | - i\ | 0_{\hat{3}} \rangle \cdot \langle 1_{\hat{3}} |$$

This is the tensor form of the operator. Notice that it is assumed that there is only one qubit, eventhogh the label of that qubit is "3". Below it is explained how to indicate that more qubits also exist:

```
QuantumTensorForm[𝒴₃̂]
```

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Here we use identity gates to obtain a tensor form of the operator acting on the third qubit, taking into account that the first and second qubits also exist.
QuantumTensorForm[[ESC]ig[ESC] [ESC]tp[ESC] [ESC]ig[ESC] [ESC]tp[ESC] [ESC]yg[ESC]]
Then press the [TAB] key several times to select the first "place holder" □, and press the keys:
1 [TAB]2 [TAB]3
then press at the same time the keys SHIFT-ENTER to evaluate.
(NOTE: The template [ESC]tp[ESC] and the template [ESC]on[ESC] give exactly the same result in any calculation. Both of them mean internal product, external product, operator application, etc. Their precise meaning is given without ambiguity by the objects before and after them. So you can use [ESC]on[ESC] instead of [ESC]tp[ESC] and viceversa in any calculation)

```
QuantumTensorForm[ℐ₁̂ ⊗ ℐ₂̂ ⊗ 𝒴₃̂]
```

$$\begin{pmatrix} \begin{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \end{pmatrix} & \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \\ \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} & \begin{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \end{pmatrix} \end{pmatrix}$$

Another way to specify that qubits $\hat{1}$ and $\hat{2}$ exist is using the option QubitList. The arrow → can be entered pressing the keys [ESC]->[ESC]

```
QuantumTensorForm[𝒴₃̂, QubitList → {1, 2, 3}]
```

$$\begin{pmatrix} \begin{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \end{pmatrix} & \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \\ \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} & \begin{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \end{pmatrix} \end{pmatrix}$$

In order to print the Truth table for an operator, press the keys:
QuantumTableForm[[ESC]cnot[ESC][ESC]on[ESC][ESC]hg[ESC]]
Then press the [TAB] key several times to select the first "place holder" □, and press the keys:
a[TAB]b[TAB]a
then press at the same time the keys SHIFT-ENTER to evaluate.
(NOTE: The template [ESC]tp[ESC] and the template [ESC]on[ESC] give exactly the same result in any calculation. Both of them mean internal product, external product, operator application, etc. Their precise meaning is given without ambiguity by the objects before and after them. So you can use [ESC]on[ESC] instead of [ESC]tp[ESC] and viceversa in any calculation)

```
QuantumTableForm[C^{â}[NOT_b̂] · H_â]
```

	Input	Output
0	$\vert 0_{\hat{a}}, 0_{\hat{b}}\rangle$	$\frac{\vert 0_{\hat{a}}, 0_{\hat{b}}\rangle}{\sqrt{2}} + \frac{\vert 1_{\hat{a}}, 1_{\hat{b}}\rangle}{\sqrt{2}}$
1	$\vert 0_{\hat{a}}, 1_{\hat{b}}\rangle$	$\frac{\vert 0_{\hat{a}}, 1_{\hat{b}}\rangle}{\sqrt{2}} + \frac{\vert 1_{\hat{a}}, 0_{\hat{b}}\rangle}{\sqrt{2}}$
2	$\vert 1_{\hat{a}}, 0_{\hat{b}}\rangle$	$\frac{\vert 0_{\hat{a}}, 0_{\hat{b}}\rangle}{\sqrt{2}} - \frac{\vert 1_{\hat{a}}, 1_{\hat{b}}\rangle}{\sqrt{2}}$
3	$\vert 1_{\hat{a}}, 1_{\hat{b}}\rangle$	$\frac{\vert 0_{\hat{a}}, 1_{\hat{b}}\rangle}{\sqrt{2}} - \frac{\vert 1_{\hat{a}}, 0_{\hat{b}}\rangle}{\sqrt{2}}$

The TraditionalForm representation is closer to the notation used in textbooks and papers:

```
TraditionalForm[QuantumTableForm[ C^{â}[NOT_b̂] · H_â]]
```

	Input	Output
0	$\vert 00\rangle$	$\frac{\vert 00\rangle}{\sqrt{2}} + \frac{\vert 11\rangle}{\sqrt{2}}$
1	$\vert 01\rangle$	$\frac{\vert 01\rangle}{\sqrt{2}} + \frac{\vert 10\rangle}{\sqrt{2}}$
2	$\vert 10\rangle$	$\frac{\vert 00\rangle}{\sqrt{2}} - \frac{\vert 11\rangle}{\sqrt{2}}$
3	$\vert 11\rangle$	$\frac{\vert 01\rangle}{\sqrt{2}} - \frac{\vert 10\rangle}{\sqrt{2}}$

In order to obtain the operator in Dirac notation, press the keys:
QuantumEvaluate[[ESC]cnot[ESC][ESC]on[ESC][ESC]hg[ESC]]
Then press the [TAB] key several times to select the first "place holder" □, and press the keys:
a[TAB]b[TAB]a
then press at the same time the keys SHIFT-ENTER to evaluate:

```
QuantumEvaluate[C^{â}[NOT_b̂] · H_â]
```

$$\frac{\vert 0_{\hat{a}}, 0_{\hat{b}}\rangle \cdot \langle 0_{\hat{a}}, 0_{\hat{b}}\vert}{\sqrt{2}} + \frac{\vert 1_{\hat{a}}, 1_{\hat{b}}\rangle \cdot \langle 0_{\hat{a}}, 0_{\hat{b}}\vert}{\sqrt{2}} + \frac{\vert 0_{\hat{a}}, 1_{\hat{b}}\rangle \cdot \langle 0_{\hat{a}}, 1_{\hat{b}}\vert}{\sqrt{2}} + \frac{\vert 1_{\hat{a}}, 0_{\hat{b}}\rangle \cdot \langle 0_{\hat{a}}, 1_{\hat{b}}\vert}{\sqrt{2}} + \frac{\vert 0_{\hat{a}}, 0_{\hat{b}}\rangle \cdot \langle 1_{\hat{a}}, 0_{\hat{b}}\vert}{\sqrt{2}} - \frac{\vert 1_{\hat{a}}, 1_{\hat{b}}\rangle \cdot \langle 1_{\hat{a}}, 0_{\hat{b}}\vert}{\sqrt{2}} + \frac{\vert 0_{\hat{a}}, 1_{\hat{b}}\rangle \cdot \langle 1_{\hat{a}}, 1_{\hat{b}}\vert}{\sqrt{2}} - \frac{\vert 1_{\hat{a}}, 0_{\hat{b}}\rangle \cdot \langle 1_{\hat{a}}, 1_{\hat{b}}\vert}{\sqrt{2}}$$

The TraditionalForm representation is closer to the notation used in textbooks and papers:

```
TraditionalForm[QuantumEvaluate[C^{â}[NOT_b̂] · H_â]]
```

$$\frac{\vert 00\rangle\langle 00\vert}{\sqrt{2}} + \frac{\vert 11\rangle\langle 00\vert}{\sqrt{2}} + \frac{\vert 01\rangle\langle 01\vert}{\sqrt{2}} + \frac{\vert 10\rangle\langle 01\vert}{\sqrt{2}} + \frac{\vert 00\rangle\langle 10\vert}{\sqrt{2}} - \frac{\vert 11\rangle\langle 10\vert}{\sqrt{2}} + \frac{\vert 01\rangle\langle 11\vert}{\sqrt{2}} - \frac{\vert 10\rangle\langle 11\vert}{\sqrt{2}}$$

TeXForm produces a $\TeX$ output that can be copy-pasted to a $\TeX$ editor:

```
TeXForm[TraditionalForm[QuantumEvaluate[C^{â}[NOT[b̂]] · H[â]]]]
```

```
\frac{|00\rangle \langle 00|}{\sqrt{2}}+\frac{|11\rangle \langle
   00|}{\sqrt{2}}+\frac{|01\rangle \langle
   01|}{\sqrt{2}}+\frac{|10\rangle \langle
   01|}{\sqrt{2}}+\frac{|00\rangle \langle
   10|}{\sqrt{2}}-\frac{|11\rangle \langle
   10|}{\sqrt{2}}+\frac{|01\rangle \langle
   11|}{\sqrt{2}}-\frac{|10\rangle \langle 11|}{\sqrt{2}}
```

In order to obtain the operator in Pauli operators, press the keys:
PauliExpand[[ESC]cnot[ESC][ESC]on[ESC][ESC]hg[ESC]]
Then press the [TAB] key several times to select the first "place holder" □, and press the keys:
a[TAB]b[TAB]a
then press at the same time the keys SHIFT-ENTER to evaluate:

```
PauliExpand[C^{â}[NOT_b̂] · H_â]
```

$$\frac{\sigma_{0,\hat{a}}\cdot\sigma_{0,\hat{b}}}{2\sqrt{2}}+\frac{\sigma_{x,\hat{a}}\cdot\sigma_{0,\hat{b}}}{2\sqrt{2}}+\frac{i\,\sigma_{y,\hat{a}}\cdot\sigma_{0,\hat{b}}}{2\sqrt{2}}+\frac{\sigma_{z,\hat{a}}\cdot\sigma_{0,\hat{b}}}{2\sqrt{2}}-\frac{\sigma_{0,\hat{a}}\cdot\sigma_{x,\hat{b}}}{2\sqrt{2}}+\frac{\sigma_{x,\hat{a}}\cdot\sigma_{x,\hat{b}}}{2\sqrt{2}}-\frac{i\,\sigma_{y,\hat{a}}\cdot\sigma_{x,\hat{b}}}{2\sqrt{2}}+\frac{\sigma_{z,\hat{a}}\cdot\sigma_{x,\hat{b}}}{2\sqrt{2}}$$

The TraditionalForm representation is closer to the notation used in textbooks and papers:

```
TraditionalForm[PauliExpand[C^{â}[NOT_b̂] · H_â]]
```

$$-\frac{i\,\sigma_a^{\mathcal{Y}}\sigma_b^{\mathcal{X}}}{2\sqrt{2}}+\frac{\sigma_a^{\mathcal{Z}}\sigma_b^{\mathcal{X}}}{2\sqrt{2}}+\frac{\sigma_a^{\mathcal{X}}\sigma_b^{\mathit{0}}}{2\sqrt{2}}-\frac{\sigma_a^{\mathit{0}}\sigma_b^{\mathcal{X}}}{2\sqrt{2}}+\frac{\sigma_a^{\mathcal{X}}\sigma_b^{\mathcal{X}}}{2\sqrt{2}}+\frac{i\,\sigma_a^{\mathcal{Y}}\sigma_b^{\mathit{0}}}{2\sqrt{2}}+\frac{\sigma_a^{\mathcal{Z}}\sigma_b^{\mathit{0}}}{2\sqrt{2}}+\frac{\sigma_a^{\mathit{0}}\sigma_b^{\mathit{0}}}{2\sqrt{2}}$$

TeXForm produces a $\TeX$ output that can be copy-pasted to a $\TeX$ editor:

```
TeXForm[TraditionalForm[PauliExpand[C^{â}[NOT_b̂] · H_â]]]
```

```
-\frac{i \sigma _a^{\mathcal{Y}}\sigma _b^{\mathcal{X}}}{2
   \sqrt{2}}+\frac{\sigma _a^{\mathcal{Z}}\sigma
   _b^{\mathcal{X}}}{2 \sqrt{2}}+\frac{\sigma
   _a^{\mathcal{X}}\sigma _b^{\mathit{0}}}{2
   \sqrt{2}}-\frac{\sigma _a^{\mathit{0}}\sigma
   _b^{\mathcal{X}}}{2 \sqrt{2}}+\frac{\sigma
   _a^{\mathcal{X}}\sigma _b^{\mathcal{X}}}{2 \sqrt{2}}+\frac{i
   \sigma _a^{\mathcal{Y}}\sigma _b^{\mathit{0}}}{2
   \sqrt{2}}+\frac{\sigma _a^{\mathcal{Z}}\sigma
   _b^{\mathit{0}}}{2 \sqrt{2}}+\frac{\sigma _a^{\mathit{0}}\sigma
   _b^{\mathit{0}}}{2 \sqrt{2}}
```

In order to see the circuit operator in Tensor notation, press the keys:
QuantumTensorForm[[ESC]cnot[ESC][ESC]on[ESC][ESC]hg[ESC]]
Then press the [TAB] key several times to select the first "place holder" □, and press the keys:
a[TAB]b[TAB]a
then press at the same time the keys SHIFT-ENTER to evaluate:

```
QuantumTensorForm[C^{â}[NOT_b̂] · H_â]
```

$$\begin{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{pmatrix} & \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \\ \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 \end{pmatrix} & \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 \end{pmatrix} \end{pmatrix}$$

Remember that for actual *Mathematica* calculations QuantumTensor[] must be used instead of QuantumTensorForm[]:

```
QuantumTensor[C^{â}[NOT_b̂] · H_â]
```

$$\left\{\left\{\left\{\left\{\frac{1}{\sqrt{2}}, 0\right\}, \left\{0, \frac{1}{\sqrt{2}}\right\}\right\}, \left\{\left\{\frac{1}{\sqrt{2}}, 0\right\}, \left\{0, \frac{1}{\sqrt{2}}\right\}\right\}\right\}, \left\{\left\{\left\{0, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, 0\right\}\right\}, \left\{\left\{0, -\frac{1}{\sqrt{2}}\right\}, \left\{-\frac{1}{\sqrt{2}}, 0\right\}\right\}\right\}\right\}$$

In order to see the circuit operator in Matrix notation, press the keys:
QuantumMatrixForm[[ESC]cnot[ESC][ESC]on[ESC][ESC]hg[ESC]]
Then press the [TAB] key several times to select the first "place holder" □, and press the keys:
a[TAB]b[TAB]a
then press at the same time the keys SHIFT-ENTER to evaluate:

```
QuantumMatrixForm[C^{â}[NOT_b̂] · H_â]
```

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

Remember that for actual *Mathematica* calculations QuantumMatrix[] must be used instead of QuantumMatrixForm[]:

```
QuantumMatrix[C^{â}[NOT_b̂] · H_â]
```

$$\left\{\left\{\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0\right\}, \left\{0, \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right\}, \left\{0, \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}, 0\right\}\right\}$$

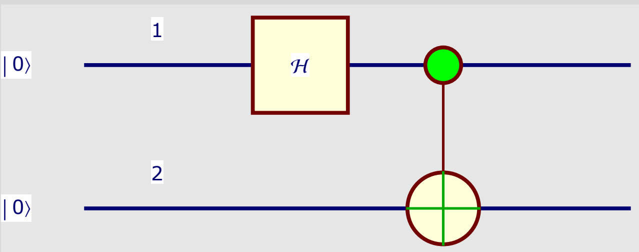
In order to generate a Bell state from a state of two qubits with logical zero, press the keys:
QuantumEvaluate[[ESC]cnot[ESC][ESC]on[ESC][ESC]hg[ESC][ESC]on[ESC][ESC]qqket[ESC]]
Then press the [TAB] key several times to select the first "place holder" □, and press the keys:
1[TAB]2[TAB]1[TAB]0[TAB]1[TAB]0[TAB]2
then press at the same time the keys SHIFT-ENTER to evaluate:

$$\texttt{QuantumEvaluate}\left[\mathcal{C}^{\{\hat{1}\}}\left[\mathcal{NOT}_{\hat{2}}\right]\cdot\mathcal{H}_{\hat{1}}\cdot\left|0_{\hat{1}},0_{\hat{2}}\right\rangle\right]$$

$$\frac{\left|0_{\hat{1}},0_{\hat{2}}\right\rangle}{\sqrt{2}}+\frac{\left|1_{\hat{1}},1_{\hat{2}}\right\rangle}{\sqrt{2}}$$

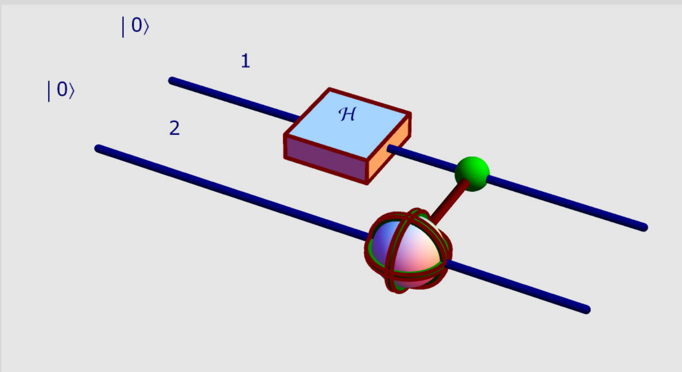
In order to plot the quantum circuit, press the keys:
QuantumPlot[[ESC]cnot[ESC][ESC]on[ESC][ESC]hg[ESC][ESC]on[ESC][ESC]qqket[ESC]]
Then press the [TAB] key several times to select the first "place holder" □, and press the keys:
1[TAB]2[TAB]1[TAB]0[TAB]1[TAB]0[TAB]2
then press at the same time the keys SHIFT-ENTER to evaluate:

$$\texttt{QuantumPlot}\left[\mathcal{C}^{\{\hat{1}\}}\left[\mathcal{NOT}_{\hat{2}}\right]\cdot\mathcal{H}_{\hat{1}}\cdot\left|0_{\hat{1}},0_{\hat{2}}\right\rangle\right]$$



In order to plot in **3D** the quantum circuit, press the keys:
QuantumPlot**3D**[[ESC]cnot[ESC][ESC]on[ESC][ESC]hg[ESC][ESC]on[ESC][ESC]qqket[ESC]]
Then press the [TAB] key several times to select the first "place holder" □, and press the keys:
1[TAB]2[TAB]1[TAB]0[TAB]1[TAB]0[TAB]2
then press at the same time the keys SHIFT-ENTER to evaluate:

$$\texttt{QuantumPlot3D}\left[\mathcal{C}^{\{\hat{1}\}}\left[\mathcal{NOT}_{\hat{2}}\right]\cdot\mathcal{H}_{\hat{1}}\cdot\left|0_{\hat{1}},0_{\hat{2}}\right\rangle\right]$$



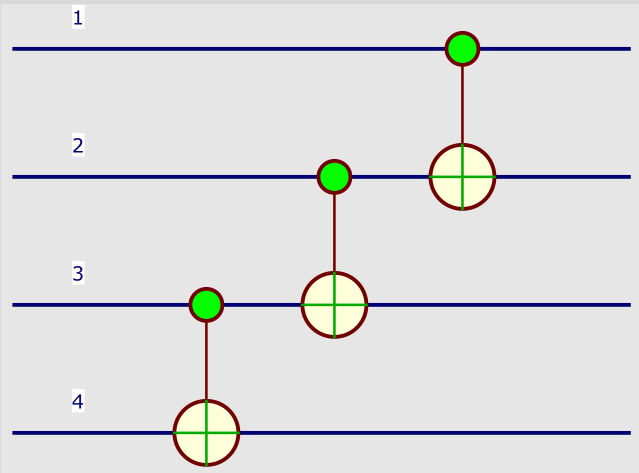
In order to enter the tensor product of a quantum expression, press the keys:
[ESC]tprod[ESC]
Then press the [TAB] key one or more times to select the first "place holder" □, and press the keys:
j[TAB]1[TAB]3[TAB]
Then press the keys:
[ESC]cnot[ESC]
Then press the [TAB] key one or more times to select the first "place holder" □, and press the keys:
j [TAB] j+1
then press at the same time the keys SHIFT-ENTER to evaluate:

$$\bigotimes_{j=1}^{3} \mathcal{C}^{\{\hat{j}\}}\left[\mathcal{NOT}_{\widehat{j+1}}\right]$$

$$\mathcal{C}^{\{\hat{1}\}}\left[\mathcal{NOT}_{\hat{2}}\right] \cdot \mathcal{C}^{\{\hat{2}\}}\left[\mathcal{NOT}_{\hat{3}}\right] \cdot \mathcal{C}^{\{\hat{3}\}}\left[\mathcal{NOT}_{\hat{4}}\right]$$

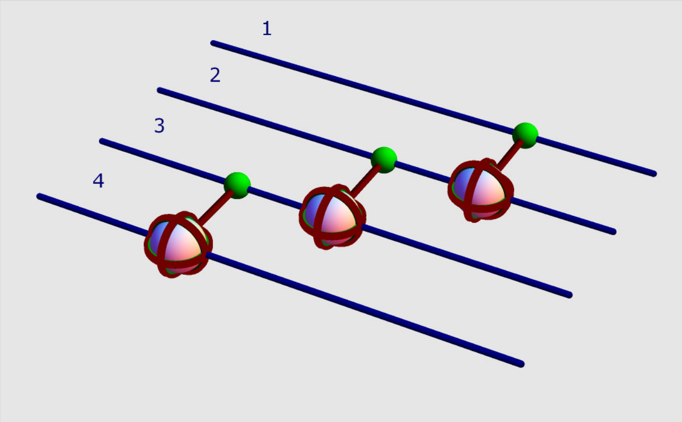
In order to plot the circuit of a tensor product of a quantum expression, press the keys:
QuantumPlot[[ESC]tprod[ESC]]
Then press the [TAB] key one or more times to select the first "place holder" □, and press the keys:
j[TAB]1[TAB]3[TAB]
Then press the keys:
[ESC]cnot[ESC]
Then press the [TAB] key one or more times to select the first "place holder" □, and press the keys:
j [TAB] j+1
then press at the same time the keys SHIFT-ENTER to evaluate:

QuantumPlot$\left[\bigotimes_{j=1}^{3} \mathcal{C}^{\{\hat{j}\}}\left[\mathcal{NOT}_{\widehat{j+1}}\right]\right]$



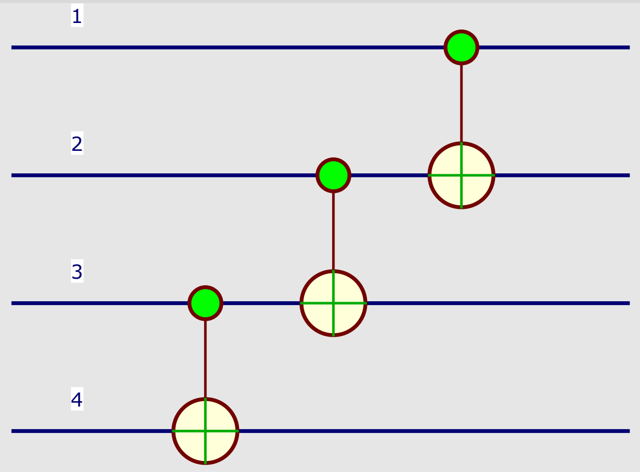
In order to plot in **3D** the circuit of a tensor product of a quantum expression, press the keys:
QuantumPlot**3D**[[ESC]tprod[ESC]]
Then press the [TAB] key one or more times to select the first "place holder" □, and press the keys:
j[TAB]1[TAB]3[TAB]
Then press the keys:
[ESC]cnot[ESC]
Then press the [TAB] key one or more times to select the first "place holder" □, and press the keys:
j [TAB] j+1
then press at the same time the keys SHIFT-ENTER to evaluate:

$$\texttt{QuantumPlot3D}\left[\bigotimes_{j=1}^{3} \mathcal{C}^{\{\hat{j}\}}\left[\mathcal{NOT}_{\widehat{j+1}}\right]\right]$$



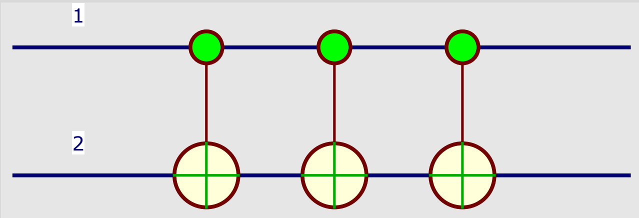
The same tensor product can be entered as a "tensor power". Press the keys:
QuantumPlot[[ESC]tpow[ESC]]
Then press the [TAB] key one or more times to select the first "place holder" □, and press the keys:
[ESC]cnot[ESC]
Then press the [TAB] key one or more times to select the first "place holder" □, and press the keys:
1[TAB]2[TAB]3
then press at the same time the keys SHIFT-ENTER to evaluate:

$$\texttt{QuantumPlot}\left[\left(\mathcal{C}^{\{\hat{1}\}}\left[\mathcal{NOT}_{\hat{2}}\right]\right)^{\otimes 3}\right]$$



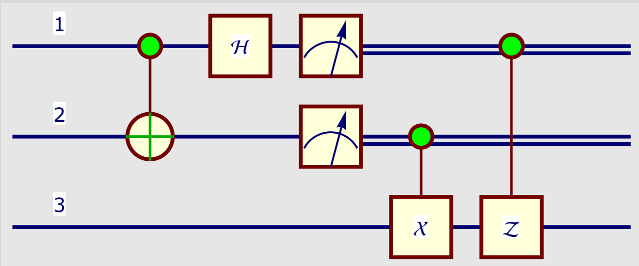
On the other hand, a "normal power" is very different from a "tensor power". Press the keys:
QuantumPlot[[ESC]po[ESC] , QuantumGatePowers[ESC]->[ESC]False]
Then press the [TAB] key one or more times to select the first "place holder" □, and press the keys:
[ESC]cnot[ESC]
Then press the [TAB] key one or more times to select the first "place holder" □, and press the keys:
1[TAB]2[TAB]3
then press at the same time the keys SHIFT-ENTER to evaluate:

QuantumPlot$\left[\left(C^{\{\hat{1}\}}\left[\mathcal{NOT}_{\hat{2}}\right]\right)^{3}$, **QuantumGatePowers → False**$\right]$



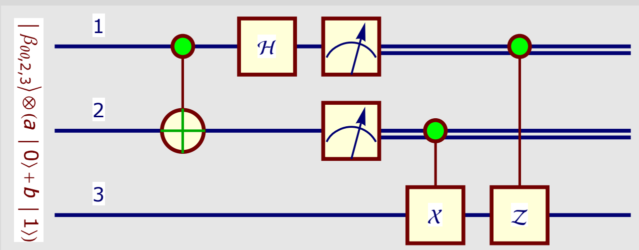
This is a more elaborated circuit. Notice how the syntaxis indicates if a gate is before or after the measuring meters. Press [ESC]cgate[ESC] for the controlled-gate template $C^{\{\hat{\square}\}}[\square]$; [ESC]cnot[ESC] for the control-not template $C^{\{\hat{\square}\}}\left[\mathcal{NOT}[\hat{\square}]\right]$; [ESC]xg[ESC] for the $\mathcal{X}[\hat{\square}]$ template, etc.

QuantumPlot$\left[C^{\{\hat{1}\}}\left[\mathcal{Z}_{\hat{3}}\right] \cdot C^{\{\hat{2}\}}\left[\mathcal{X}_{\hat{3}}\right] \cdot\right.$ **QubitMeasurement**$\left[\mathcal{H}_{\hat{1}} \cdot C^{\{\hat{1}\}}\left[\mathcal{NOT}_{\hat{2}}\right], \{\hat{1}, \hat{2}\}\right]\Big]$



Plot of the same circuit, now applied to the initial ket $\left|\mathcal{B}_{00,\hat{2},\hat{3}}\right\rangle \otimes \left(a \left|0_{\hat{1}}\right\rangle + b \left|1_{\hat{1}}\right\rangle\right)$

QuantumPlot$\left[C^{\{\hat{1}\}}\left[\mathcal{Z}_{\hat{3}}\right] \cdot C^{\{\hat{2}\}}\left[\mathcal{X}_{\hat{3}}\right] \cdot\right.$
QubitMeasurement$\left[\mathcal{H}_{\hat{1}} \cdot C^{\{\hat{1}\}}\left[\mathcal{NOT}_{\hat{2}}\right] \cdot \left|\mathcal{B}_{00,\hat{2},\hat{3}}\right\rangle \otimes \left(a \left|0_{\hat{1}}\right\rangle + b \left|1_{\hat{1}}\right\rangle\right), \{\hat{1}, \hat{2}\}\right]\Big]$



Evaluation of the same circuit, applied to the initial ket $\left|\mathcal{B}_{00,\hat{2},\hat{3}}\right\rangle \otimes \left(a \left|0_{\hat{1}}\right\rangle + b \left|1_{\hat{1}}\right\rangle\right)$. Notice that all possible outputs have the 3rd qubit in the combination $a \left|0_{\hat{3}}\right\rangle + b \left|1_{\hat{3}}\right\rangle$, therefore this circuit "teleports" (cuts and pastes) the initial state from qubit $\hat{1}$ (which initially is $a \left|0_{\hat{1}}\right\rangle + b \left|1_{\hat{1}}\right\rangle$) to the qubit $\hat{3}$ (which finally becomes $a \left|0_{\hat{3}}\right\rangle + b \left|1_{\hat{3}}\right\rangle$)

$$\texttt{QuantumEvaluate}\Big[C^{\{\hat{1}\}}[\mathcal{Z}_{\hat{3}}] \cdot C^{\{\hat{2}\}}[\mathcal{X}_{\hat{3}}] \cdot \texttt{QubitMeasurement}\Big[\mathcal{H}_{\hat{1}} \cdot C^{\{\hat{1}\}}[\mathcal{NOT}_{\hat{2}}] \cdot \left|\mathcal{B}_{00,\hat{2},\hat{3}}\right\rangle \otimes \left(a \left|0_{\hat{1}}\right\rangle + b \left|1_{\hat{1}}\right\rangle\right), \{\hat{1}, \hat{2}\}\Big]\Big]$$

Probability	Measurement	State
$\frac{1}{4}$	$\{\{0_{\hat{1}}, 0_{\hat{2}}\}\}$	$\left|0_{\hat{1}}\right\rangle \otimes \left|0_{\hat{2}}\right\rangle \otimes \left(\frac{a\left|0_{\hat{3}}\right\rangle}{\sqrt{a a^* + b b^*}} + \frac{b\left|1_{\hat{3}}\right\rangle}{\sqrt{a a^* + b b^*}}\right)$
$\frac{1}{4}$	$\{\{0_{\hat{1}}, 1_{\hat{2}}\}\}$	$\left|0_{\hat{1}}\right\rangle \otimes \left|1_{\hat{2}}\right\rangle \otimes \left(\frac{a\left|0_{\hat{3}}\right\rangle}{\sqrt{a a^* + b b^*}} + \frac{b\left|1_{\hat{3}}\right\rangle}{\sqrt{a a^* + b b^*}}\right)$
$\frac{1}{4}$	$\{\{1_{\hat{1}}, 0_{\hat{2}}\}\}$	$\left|1_{\hat{1}}\right\rangle \otimes \left|0_{\hat{2}}\right\rangle \otimes \left(\frac{a\left|0_{\hat{3}}\right\rangle}{\sqrt{a a^* + b b^*}} + \frac{b\left|1_{\hat{3}}\right\rangle}{\sqrt{a a^* + b b^*}}\right)$
$\frac{1}{4}$	$\{\{1_{\hat{1}}, 1_{\hat{2}}\}\}$	$\left|1_{\hat{1}}\right\rangle \otimes \left|1_{\hat{2}}\right\rangle \otimes \left(\frac{a\left|0_{\hat{3}}\right\rangle}{\sqrt{a a^* + b b^*}} + \frac{b\left|1_{\hat{3}}\right\rangle}{\sqrt{a a^* + b b^*}}\right)$
Probability	Measurement	State

In order to represent a 9 as a binary number made of 5 qubits, press the keys:
[ESC]toqb[ESC] [TAB] 9 [TAB] 5
then press at the same time the keys SHIFT-ENTER to evaluate:

$$\left|9\right\rangle_5$$

$$\left|0_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}, 1_{\hat{5}}\right\rangle$$

This is a normalized, equally-weighted, linear combination of the computational basis kets for three qubits. Press [ESC]si[ESC] for the sigma notation template $\sum_{\square}^{\square} \square$

$$\frac{1}{\sqrt{8}} \sum_{j=0}^{7} \left|j\right\rangle_3$$

$$\frac{1}{2\sqrt{2}}\left(\left|0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}\right\rangle + \left|0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}\right\rangle + \left|0_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}\right\rangle + \left|0_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}\right\rangle + \left|1_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}\right\rangle + \left|1_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}\right\rangle + \left|1_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}\right\rangle + \left|1_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}\right\rangle\right)$$

Here we calculate the norm. Press [ESC]norm[ESC] for the norm template $\|\square\|$

$$\left\|\frac{1}{\sqrt{8}} \sum_{j=0}^{7} \left|j\right\rangle_3\right\|$$

$$1$$

This is a normalized, equally-weighted, linear combination of the computational basis kets for four qubits:

```
n = 2^4;
```

$$\frac{1}{\sqrt{n}}\sum_{j=0}^{n-1} | j\rangle_{\text{Log}[2,n]}$$

$$\frac{1}{4}\left(| 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle + | 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle + | 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}, 0_{\hat{4}}\rangle + | 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}, 1_{\hat{4}}\rangle + | 0_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle + | 0_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle + | 0_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}, 0_{\hat{4}}\rangle + | 0_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}, 1_{\hat{4}}\rangle + | 1_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle + | 1_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle + | 1_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}, 0_{\hat{4}}\rangle + | 1_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}, 1_{\hat{4}}\rangle + | 1_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle + | 1_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle + | 1_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}, 0_{\hat{4}}\rangle + | 1_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}, 1_{\hat{4}}\rangle \right)$$

In order to save the definition of a ket, press the keys:
[ESC]ket[ESC] = a [ESC]qket[ESC] +b [ESC]qket[ESC]
Then press the [TAB] key one or more times to select the first "place holder" □, and press the keys:
[ESC]psi[ESC][TAB]0[TAB]1[TAB]1[TAB]1
then press at the same time the keys SHIFT-ENTER to evaluate:

$$| \psi\rangle = a\ | 0_{\hat{1}}\rangle + b\ | 1_{\hat{1}}\rangle$$

$$a\ | 0_{\hat{1}}\rangle + b\ | 1_{\hat{1}}\rangle$$

The correspondig bra can be used:
[ESC]bra[ESC] [TAB] [ESC]psi[ESC]
then press at the same time the keys SHIFT-ENTER to evaluate:

$$\langle \psi |$$

$$a^* \langle 0_{\hat{1}} | + b^* \langle 1_{\hat{1}} |$$

An external product can be calculated from the ket that was defined. Press the keys:
Expand[[ESC]ket[ESC] [ESC]on[ESC] [ESC]bra[ESC]]
then press the keys:
[TAB] [ESC]psi[ESC] [TAB] [ESC]psi[ESC]
then press at the same time the keys SHIFT-ENTER to evaluate:

$$\text{Expand}[\ | \psi\rangle \cdot \langle \psi |\ ]$$

$$a\, a^*\ | 0_{\hat{1}}\rangle \cdot \langle 0_{\hat{1}} | + b\, a^*\ | 1_{\hat{1}}\rangle \cdot \langle 0_{\hat{1}} | + a\, b^*\ | 0_{\hat{1}}\rangle \cdot \langle 1_{\hat{1}} | + b\, b^*\ | 1_{\hat{1}}\rangle \cdot \langle 1_{\hat{1}} |$$

An internal product can be calculated from the ket that was defined. Press the keys:
[ESC]bra[ESC] [ESC]on[ESC] [ESC]ket[ESC] [TAB] [ESC]psi[ESC] [TAB] [ESC]psi[ESC]
then press at the same time the keys SHIFT-ENTER to evaluate:

$$\langle \psi | \cdot | \psi\rangle$$

$$a\, a^* + b\, b^*$$

The norm of a quantum expression can be calculated. Press the keys:
[ESC]norm[ESC] [TAB] [ESC]ket[ESC] [TAB] [ESC]psi[ESC]
then press at the same time the keys SHIFT-ENTER to evaluate:

```
‖ | ψ〉‖
```

$\sqrt{a\, a^* + b\, b^*}$

Advanced *Mathematica* technical info: The definition is stored as an "upvalue" of psi ψ
? [ESC]psi[ESC]
then press at the same time the keys SHIFT-ENTER to evaluate:

```
? ψ
```

```
Global`ψ
```

$| \psi\rangle$ ^= $a\ | 0_{\hat{1}}\rangle + b\ | 1_{\hat{1}}\rangle$

In order to save the definition of another ket, press the keys:
[ESC]ket[ESC] = x [ESC]qket[ESC] +y [ESC]qket[ESC]
Then press the [TAB] key one or more times to select the first "place holder" □, and press the keys:
[ESC]x[ESC][TAB]0[TAB]1[TAB]1[TAB]1
then press at the same time the keys SHIFT-ENTER to evaluate:

$| \xi\rangle = x\ | 0_{\hat{1}}\rangle + y\ | 1_{\hat{1}}\rangle$

$x\ | 0_{\hat{1}}\rangle + y\ | 1_{\hat{1}}\rangle$

Different operations can be performed on the kets that were defined, and the results (output) of those operations are valid *Mathematica* expressions that can be copy-pasted and used as part of *Mathematica* input in other commands:

$\langle \xi | \cdot | \psi\rangle$

$a\, x^* + b\, y^*$

Different operations can be performed on the kets that were defined, and the results (output) of those operations are valid *Mathematica* expressions that can be copy-pasted and used as part of *Mathematica* input in other commands:

Expand[$| \xi\rangle \cdot \langle \psi |$]

$x\, a^*\ | 0_{\hat{1}}\rangle \cdot \langle 0_{\hat{1}} | + y\, a^*\ | 1_{\hat{1}}\rangle \cdot \langle 0_{\hat{1}} | + x\, b^*\ | 0_{\hat{1}}\rangle \cdot \langle 1_{\hat{1}} | + y\, b^*\ | 1_{\hat{1}}\rangle \cdot \langle 1_{\hat{1}} |$

A "Power" can be entered pressing [ESC]po[ESC]

```
Expand[( | ξ〉 · 〈ψ |)^2]
```

$$x^2 (a^*)^2\ |0_{\hat{1}}\rangle\cdot\langle 0_{\hat{1}}| + x\,y\,a^*\,b^*\ |0_{\hat{1}}\rangle\cdot\langle 0_{\hat{1}}| + x\,y\,(a^*)^2\ |1_{\hat{1}}\rangle\cdot\langle 0_{\hat{1}}| + y^2\,a^*\,b^*\ |1_{\hat{1}}\rangle\cdot\langle 0_{\hat{1}}| + x^2\,a^*\,b^*\ |0_{\hat{1}}\rangle\cdot\langle 1_{\hat{1}}| + x\,y\,(b^*)^2\ |0_{\hat{1}}\rangle\cdot\langle 1_{\hat{1}}| + x\,y\,a^*\,b^*\ |1_{\hat{1}}\rangle\cdot\langle 1_{\hat{1}}| + y^2\,(b^*)^2\ |1_{\hat{1}}\rangle\cdot\langle 1_{\hat{1}}|$$

A "Tensor Power" $(\square)^{\otimes\square}$ is very different from a "Power" $(\square)^{\square}$
In a TensorPower, the same expression is evaluated at **different qubits**
The "Tensor Power" can be entered by pressing [ESC]tpow[ESC]

```
Expand[( | ξ〉 · 〈ψ |)⊗2]
```

$$x^2 (a^*)^2\ |0_{\hat{1}}, 0_{\hat{2}}\rangle\cdot\langle 0_{\hat{1}}, 0_{\hat{2}}| + x\,y\,(a^*)^2\ |0_{\hat{1}}, 1_{\hat{2}}\rangle\cdot\langle 0_{\hat{1}}, 0_{\hat{2}}| + x\,y\,(a^*)^2\ |1_{\hat{1}}, 0_{\hat{2}}\rangle\cdot\langle 0_{\hat{1}}, 0_{\hat{2}}| + y^2\,(a^*)^2\ |1_{\hat{1}}, 1_{\hat{2}}\rangle\cdot\langle 0_{\hat{1}}, 0_{\hat{2}}| + x^2\,a^*\,b^*\ |0_{\hat{1}}, 0_{\hat{2}}\rangle\cdot\langle 0_{\hat{1}}, 1_{\hat{2}}| + x\,y\,a^*\,b^*\ |0_{\hat{1}}, 1_{\hat{2}}\rangle\cdot\langle 0_{\hat{1}}, 1_{\hat{2}}| + x\,y\,a^*\,b^*\ |1_{\hat{1}}, 0_{\hat{2}}\rangle\cdot\langle 0_{\hat{1}}, 1_{\hat{2}}| + y^2\,a^*\,b^*\ |1_{\hat{1}}, 1_{\hat{2}}\rangle\cdot\langle 0_{\hat{1}}, 1_{\hat{2}}| + x^2\,a^*\,b^*\ |0_{\hat{1}}, 0_{\hat{2}}\rangle\cdot\langle 1_{\hat{1}}, 0_{\hat{2}}| + x\,y\,a^*\,b^*\ |0_{\hat{1}}, 1_{\hat{2}}\rangle\cdot\langle 1_{\hat{1}}, 0_{\hat{2}}| + x\,y\,a^*\,b^*\ |1_{\hat{1}}, 0_{\hat{2}}\rangle\cdot\langle 1_{\hat{1}}, 0_{\hat{2}}| + y^2\,a^*\,b^*\ |1_{\hat{1}}, 1_{\hat{2}}\rangle\cdot\langle 1_{\hat{1}}, 0_{\hat{2}}| + x^2\,(b^*)^2\ |0_{\hat{1}}, 0_{\hat{2}}\rangle\cdot\langle 1_{\hat{1}}, 1_{\hat{2}}| + x\,y\,(b^*)^2\ |0_{\hat{1}}, 1_{\hat{2}}\rangle\cdot\langle 1_{\hat{1}}, 1_{\hat{2}}| + x\,y\,(b^*)^2\ |1_{\hat{1}}, 0_{\hat{2}}\rangle\cdot\langle 1_{\hat{1}}, 1_{\hat{2}}| + y^2\,(b^*)^2\ |1_{\hat{1}}, 1_{\hat{2}}\rangle\cdot\langle 1_{\hat{1}}, 1_{\hat{2}}|$$

Partial trace template: [ESC]trace[ESC]
Tensor power template: [ESC]tpow[ESC]

```
Tr_2̂[( | ξ〉 · 〈ψ |)⊗2]
```

$$x\left(x\,(a^*)^2\ |0_{\hat{1}}\rangle\cdot\langle 0_{\hat{1}}| + y\,(a^*)^2\ |1_{\hat{1}}\rangle\cdot\langle 0_{\hat{1}}| + x\,a^*\,b^*\ |0_{\hat{1}}\rangle\cdot\langle 1_{\hat{1}}| + y\,a^*\,b^*\ |1_{\hat{1}}\rangle\cdot\langle 1_{\hat{1}}|\right) + y\left(x\,a^*\,b^*\ |0_{\hat{1}}\rangle\cdot\langle 0_{\hat{1}}| + y\,a^*\,b^*\ |1_{\hat{1}}\rangle\cdot\langle 0_{\hat{1}}| + x\,(b^*)^2\ |0_{\hat{1}}\rangle\cdot\langle 1_{\hat{1}}| + y\,(b^*)^2\ |1_{\hat{1}}\rangle\cdot\langle 1_{\hat{1}}|\right)$$

Partial trace template: [ESC]trace[ESC]
Tensor power template: [ESC]tpow[ESC]

```
Expand[Tr_2̂[( | ξ〉 · 〈ψ |)⊗2]]
```

$$x^2 (a^*)^2\ |0_{\hat{1}}\rangle\cdot\langle 0_{\hat{1}}| + x\,y\,a^*\,b^*\ |0_{\hat{1}}\rangle\cdot\langle 0_{\hat{1}}| + x\,y\,(a^*)^2\ |1_{\hat{1}}\rangle\cdot\langle 0_{\hat{1}}| + y^2\,a^*\,b^*\ |1_{\hat{1}}\rangle\cdot\langle 0_{\hat{1}}| + x^2\,a^*\,b^*\ |0_{\hat{1}}\rangle\cdot\langle 1_{\hat{1}}| + x\,y\,(b^*)^2\ |0_{\hat{1}}\rangle\cdot\langle 1_{\hat{1}}| + x\,y\,a^*\,b^*\ |1_{\hat{1}}\rangle\cdot\langle 1_{\hat{1}}| + y^2\,(b^*)^2\ |1_{\hat{1}}\rangle\cdot\langle 1_{\hat{1}}|$$

Partial trace template: [ESC]trace[ESC]
Tensor power template: [ESC]tpow[ESC]

```
TraditionalForm[Expand[Tr_2̂[( | ξ〉 · 〈ψ |)⊗2]]]
```

$$x^2\,a^*\,b^*\ |0\rangle\langle 1| + x\,y\,a^*\,b^*\ |0\rangle\langle 0| + x\,y\,a^*\,b^*\ |1\rangle\langle 1| + y^2\,a^*\,b^*\ |1\rangle\langle 0| + x^2\left(a^*\right)^2\ |0\rangle\langle 0| + x\,y\left(a^*\right)^2\ |1\rangle\langle 0| + x\,y\left(b^*\right)^2\ |0\rangle\langle 1| + y^2\left(b^*\right)^2\ |1\rangle\langle 1|$$

Partial trace template: [ESC]trace[ESC]
Tensor power template: [ESC]tpow[ESC]

```
Tr_2̂[( | ξ⟩ · ⟨ψ |)^⊗2] · | 0_1̂, 0_3̂⟩
```

$x\left(x\,(a^*)^2\;|0_{\hat{1}},0_{\hat{3}}\rangle + y\,(a^*)^2\;|1_{\hat{1}},0_{\hat{3}}\rangle\right) + y\left(x\,a^*\,b^*\;|0_{\hat{1}},0_{\hat{3}}\rangle + y\,a^*\,b^*\;|1_{\hat{1}},0_{\hat{3}}\rangle\right)$

The standard *Mathematica* command Simplify[] can be used:

```
Simplify[Tr_2̂[( | ξ⟩ · ⟨ψ |)^⊗2] · | 0_1̂, 0_3̂⟩]
```

$a^*\,(x\,a^* + y\,b^*)\,\left(x\;|0_{\hat{1}},0_{\hat{3}}\rangle + y\;|1_{\hat{1}},0_{\hat{3}}\rangle\right)$

Press [ESC]her[ESC] for the Hermitian conjugate template $(\square)^\dagger$

```
( | ψ⟩)^†
```

$a^*\,\langle 0_{\hat{1}}| + b^*\,\langle 1_{\hat{1}}|$

Press [ESC]her[ESC] for the Hermitian conjugate template $(\square)^\dagger$ and [ESC]tg[ESC] for the $\mathcal{T}_{\hat{\square}}$ template

```
(𝒯_1̂)^†
```

$\mathcal{T}_{\hat{1}}^{\dagger}$

Press [ESC]her[ESC] for the Hermitian conjugate template $(\square)^\dagger$ and [ESC]tg[ESC] for the $\mathcal{T}_{\hat{\square}}$ template

```
PauliExpand[(𝒯_1̂)^†]
```

$\frac{1}{4}\left(2 + (1 - i)\sqrt{2}\right)\sigma_{0,\hat{1}} + \frac{1}{4}\left(2 - (1 - i)\sqrt{2}\right)\sigma_{z,\hat{1}}$

TraditionalForm:

```
TraditionalForm[PauliExpand[(𝒯_1̂)^†]]
```

$$\frac{1}{4}\left(2-(1-i)\sqrt{2}\right)\sigma_{1}^{Z}+\frac{1}{4}\left(2+(1-i)\sqrt{2}\right)\sigma_{1}^{0}$$

Press [ESC]qqft[ESC] for the template of the two-qubits Quantum Fourier Transform $\mathcal{QFT}_{\hat{\square},\hat{\square}}$

```
QuantumTableForm[𝒬ℱ𝒯_{1̂,2̂}]
```

	Input	Output
0	$\vert 0_{\hat{1}}, 0_{\hat{2}}\rangle$	$0.5\ \vert 0_{\hat{1}}, 0_{\hat{2}}\rangle + 0.5\ \vert 0_{\hat{1}}, 1_{\hat{2}}\rangle + 0.5\ \vert 1_{\hat{1}}, 0_{\hat{2}}\rangle + 0.5\ \vert 1_{\hat{1}}, 1_{\hat{2}}\rangle$
1	$\vert 0_{\hat{1}}, 1_{\hat{2}}\rangle$	$0.5\ \vert 0_{\hat{1}}, 0_{\hat{2}}\rangle + 0.5\,i\ \vert 0_{\hat{1}}, 1_{\hat{2}}\rangle - 0.5\ \vert 1_{\hat{1}}, 0_{\hat{2}}\rangle - 0.5\,i\ \vert 1_{\hat{1}}, 1_{\hat{2}}\rangle$
2	$\vert 1_{\hat{1}}, 0_{\hat{2}}\rangle$	$0.5\ \vert 0_{\hat{1}}, 0_{\hat{2}}\rangle - 0.5\ \vert 0_{\hat{1}}, 1_{\hat{2}}\rangle + 0.5\ \vert 1_{\hat{1}}, 0_{\hat{2}}\rangle - 0.5\ \vert 1_{\hat{1}}, 1_{\hat{2}}\rangle$
3	$\vert 1_{\hat{1}}, 1_{\hat{2}}\rangle$	$0.5\ \vert 0_{\hat{1}}, 0_{\hat{2}}\rangle - 0.5\,i\ \vert 0_{\hat{1}}, 1_{\hat{2}}\rangle - 0.5\ \vert 1_{\hat{1}}, 0_{\hat{2}}\rangle + 0.5\,i\ \vert 1_{\hat{1}}, 1_{\hat{2}}\rangle$

Press [ESC]qqqft[ESC] for the template of the three-qubits Quantum Fourier Transform $\mathcal{QFT}_{\hat{\square},\hat{\square},\hat{\square}}$

```
QuantumEvaluate[𝒬ℱ𝒯_{â,b̂,ĉ} · |1_â, 0_b̂, 1_ĉ⟩]
```

$0.353553\ \vert 0_{\hat{a}}, 0_{\hat{b}}, 0_{\hat{c}}\rangle - (0.25 + 0.25\,i)\ \vert 0_{\hat{a}}, 0_{\hat{b}}, 1_{\hat{c}}\rangle +$
$0.353553\,i\ \vert 0_{\hat{a}}, 1_{\hat{b}}, 0_{\hat{c}}\rangle + (0.25 - 0.25\,i)\ \vert 0_{\hat{a}}, 1_{\hat{b}}, 1_{\hat{c}}\rangle - 0.353553\ \vert 1_{\hat{a}}, 0_{\hat{b}}, 0_{\hat{c}}\rangle +$
$(0.25 + 0.25\,i)\ \vert 1_{\hat{a}}, 0_{\hat{b}}, 1_{\hat{c}}\rangle - 0.353553\,i\ \vert 1_{\hat{a}}, 1_{\hat{b}}, 0_{\hat{c}}\rangle - (0.25 - 0.25\,i)\ \vert 1_{\hat{a}}, 1_{\hat{b}}, 1_{\hat{c}}\rangle$

Press [ESC]qqqft[ESC] for the template of the three-qubits Quantum Fourier Transform $\mathcal{QFT}_{\hat{\square},\hat{\square},\hat{\square}}$

Scroll to the right in order to see the complete answer

```
TraditionalForm[QuantumTableForm[𝒬ℱ𝒯_{â,b̂,ĉ}]]
```

	Input	Output
0	$\vert 000\rangle$	$0.353553\ \vert 000\rangle + 0.353553\ \vert 001\rangle + 0.353553\ \vert 010\rangle + 0.353553\ \vert 011\rangle + 0.353553\ \vert 100\rangle + 0.353553\ \vert 1$
1	$\vert 001\rangle$	$0.353553\ \vert 000\rangle + (0.25 + 0.25\,i)\ \vert 001\rangle + 0.353553\,i\ \vert 010\rangle - (0.25 - 0.25\,i)\ \vert 011\rangle - 0.353553\ \vert 100\rangle - ($
2	$\vert 010\rangle$	$0.353553\ \vert 000\rangle + 0.353553\,i\ \vert 001\rangle - 0.353553\ \vert 010\rangle - 0.353553\,i\ \vert 011\rangle + 0.353553\ \vert 100\rangle + 0.353553\,i$
3	$\vert 011\rangle$	$0.353553\ \vert 000\rangle - (0.25 - 0.25\,i)\ \vert 001\rangle - 0.353553\,i\ \vert 010\rangle + (0.25 + 0.25\,i)\ \vert 011\rangle - 0.353553\ \vert 100\rangle + ($
4	$\vert 100\rangle$	$0.353553\ \vert 000\rangle - 0.353553\ \vert 001\rangle + 0.353553\ \vert 010\rangle - 0.353553\ \vert 011\rangle + 0.353553\ \vert 100\rangle - 0.353553\ \vert 1$
5	$\vert 101\rangle$	$0.353553\ \vert 000\rangle - (0.25 + 0.25\,i)\ \vert 001\rangle + 0.353553\,i\ \vert 010\rangle + (0.25 - 0.25\,i)\ \vert 011\rangle - 0.353553\ \vert 100\rangle + ($
6	$\vert 110\rangle$	$0.353553\ \vert 000\rangle - 0.353553\,i\ \vert 001\rangle - 0.353553\ \vert 010\rangle + 0.353553\,i\ \vert 011\rangle + 0.353553\ \vert 100\rangle - 0.353553\,i$
7	$\vert 111\rangle$	$0.353553\ \vert 000\rangle + (0.25 - 0.25\,i)\ \vert 001\rangle - 0.353553\,i\ \vert 010\rangle - (0.25 + 0.25\,i)\ \vert 011\rangle - 0.353553\ \vert 100\rangle - ($

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