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# The MatrixQuantum and QuantumMatrix Commands and the QubitList Option

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## Introduction

This is a tutorial on the use of MatrixQuantum and QuantumMatrix in order to transform between Dirac notation and matrix notation, and the use of the option QubitList in those two commands and also in QuantumTableForm (Truth tables) and QuantumPlot (circuits).

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## Load the Package

First load the Quantum`Computing` package. Write:

Needs["Quantum`Computing`"]

then press at the same time the keys **SHIFT-ENTER** to evaluate. *Mathematica* will load the package.

```
Needs["Quantum`Computing`"]
```

```
Quantum`Computing` Version 2.2.0. (July 2010)
A Mathematica package for Quantum Computing
  in Dirac bra-ket notation and plotting of quantum circuits
by José Luis Gómez-Muñoz

Execute SetComputingAliases[] in order to use
  the keyboard to enter quantum objects in Dirac's notation
SetComputingAliases[] must be executed again in each new notebook that is created
```

In order to use the keyboard to enter quantum objects write:

SetComputingAliases[ ];

then press at the same time the keys **SHIFT-ENTER** to evaluate. The semicolon prevents *Mathematica* from printing the help message. Remember that SetComputingAliases[ ] must be evaluated again in each new notebook:

```
SetComputingAliases[ ];
```

---

## MatrixQuantum and QuantumMatrix

A matrix can be entered in *Mathematica* by opening *Mathematica*'s menu **Insert** and selecting the option **Table/Matrix**, then selecting **New** and specifying the required parameters in the dialog window. Another way to enter a matrix is to use the **Basic Math Assistant** palette of *Mathematica*, in the **Basic Commands** section of the palette, in the tab with an icon similar to a matrix. Here is an example of a matrix that can be entered with either both procedures:

$$\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix}$$

```
{a, b, c, d}, {e, f, g, h}, {i, j, k, l}, {m, n, o, p}}
```

A matrix can also be entered also by using only the keyboard:

```
{{a, b, c, d}, {e, f, g, h}, {i, j, k, l}, {m, n, o, p}}
```

```
{a, b, c, d}, {e, f, g, h}, {i, j, k, l}, {m, n, o, p}}
```

This is the matrix transformed to a quantum operator in qubits  $\hat{1}$  and  $\hat{2}$ , where  $\hat{2}$  is the "less significant" qubit

$$\text{MatrixQuantum}\left[\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix}\right]$$

$$\begin{aligned} & a \mid 0_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 0_{\hat{2}} \mid + e \mid 0_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 0_{\hat{2}} \mid + \\ & i \mid 1_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 0_{\hat{2}} \mid + m \mid 1_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 0_{\hat{2}} \mid + \\ & b \mid 0_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 1_{\hat{2}} \mid + f \mid 0_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 1_{\hat{2}} \mid + j \mid 1_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 1_{\hat{2}} \mid + \\ & n \mid 1_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 1_{\hat{2}} \mid + c \mid 0_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 1_{\hat{1}}, 0_{\hat{2}} \mid + g \mid 0_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 1_{\hat{1}}, 0_{\hat{2}} \mid + \\ & k \mid 1_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 1_{\hat{1}}, 0_{\hat{2}} \mid + o \mid 1_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 1_{\hat{1}}, 0_{\hat{2}} \mid + d \mid 0_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 1_{\hat{1}}, 1_{\hat{2}} \mid + \\ & h \mid 0_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 1_{\hat{1}}, 1_{\hat{2}} \mid + l \mid 1_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 1_{\hat{1}}, 1_{\hat{2}} \mid + p \mid 1_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 1_{\hat{1}}, 1_{\hat{2}} \mid \end{aligned}$$

Copy-paste the result of the previous calculation as the argument of the QuantumMatrix command in order to recover the original matrix:

$$\begin{aligned} & \text{QuantumMatrix}[a \mid 0_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 0_{\hat{2}} \mid + \\ & e \mid 0_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 0_{\hat{2}} \mid + i \mid 1_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 0_{\hat{2}} \mid + m \mid 1_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 0_{\hat{2}} \mid + \\ & b \mid 0_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 1_{\hat{2}} \mid + f \mid 0_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 1_{\hat{2}} \mid + j \mid 1_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 1_{\hat{2}} \mid + \\ & n \mid 1_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 1_{\hat{2}} \mid + c \mid 0_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 1_{\hat{1}}, 0_{\hat{2}} \mid + g \mid 0_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 1_{\hat{1}}, 0_{\hat{2}} \mid + \\ & k \mid 1_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 1_{\hat{1}}, 0_{\hat{2}} \mid + o \mid 1_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 1_{\hat{1}}, 0_{\hat{2}} \mid + d \mid 0_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 1_{\hat{1}}, 1_{\hat{2}} \mid + \\ & h \mid 0_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 1_{\hat{1}}, 1_{\hat{2}} \mid + l \mid 1_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 1_{\hat{1}}, 1_{\hat{2}} \mid + p \mid 1_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 1_{\hat{1}}, 1_{\hat{2}} \mid] \end{aligned}$$

```
{a, b, c, d}, {e, f, g, h}, {i, j, k, l}, {m, n, o, p}}
```

QuantumMatrixForm gives the result with a nicer formatting:

```
QuantumMatrixForm[a | 01, 02⟩ · ⟨01, 02 | +
  e | 01, 12⟩ · ⟨01, 02 | + i | 11, 02⟩ · ⟨01, 02 | + m | 11, 12⟩ · ⟨01, 02 | +
  b | 01, 02⟩ · ⟨01, 12 | + f | 01, 12⟩ · ⟨01, 12 | + j | 11, 02⟩ · ⟨01, 12 | +
  n | 11, 12⟩ · ⟨01, 12 | + c | 01, 02⟩ · ⟨11, 02 | + g | 01, 12⟩ · ⟨11, 02 | +
  k | 11, 02⟩ · ⟨11, 02 | + o | 11, 12⟩ · ⟨11, 02 | + d | 01, 02⟩ · ⟨11, 12 | +
  h | 01, 12⟩ · ⟨11, 12 | + l | 11, 02⟩ · ⟨11, 12 | + p | 11, 12⟩ · ⟨11, 12 | ]
```

$$\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix}$$

## The Use of the QubitList Option in MatrixQuantum and QuantumMatrix

This is the matrix transformed to a quantum operator in qubits  $\hat{1}$  and  $\hat{2}$ , **where the option `QubitList`  $\rightarrow \{\hat{2}, \hat{1}\}$  specifies that  $\hat{1}$  is the "less significant" qubit.** Notice that the Dirac expression obtained below is different from the one obtained in the previous section of this document:

```
MatrixQuantum[ $\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix}$ , QubitList  $\rightarrow \{\hat{2}, \hat{1}\}$ ]
```

```
a | 01, 02⟩ · ⟨01, 02 | + i | 01, 12⟩ · ⟨01, 02 | +
  e | 11, 02⟩ · ⟨01, 02 | + m | 11, 12⟩ · ⟨01, 02 | +
  c | 01, 02⟩ · ⟨01, 12 | + k | 01, 12⟩ · ⟨01, 12 | + g | 11, 02⟩ · ⟨01, 12 | +
  o | 11, 12⟩ · ⟨01, 12 | + b | 01, 02⟩ · ⟨11, 02 | + j | 01, 12⟩ · ⟨11, 02 | +
  f | 11, 02⟩ · ⟨11, 02 | + n | 11, 12⟩ · ⟨11, 02 | + d | 01, 02⟩ · ⟨11, 12 | +
  l | 01, 12⟩ · ⟨11, 12 | + h | 11, 02⟩ · ⟨11, 12 | + p | 11, 12⟩ · ⟨11, 12 |
```

Copy-paste the result of the previous calculation as the argument of the `QuantumMatrixForm` command **with the option `QubitList`  $\rightarrow \{\hat{2}, \hat{1}\}$**  in order to recover the original matrix:

```
QuantumMatrixForm[a | 01, 02⟩ · ⟨01, 02 | + i | 01, 12⟩ · ⟨01, 02 | +
  e | 11, 02⟩ · ⟨01, 02 | + m | 11, 12⟩ · ⟨01, 02 | + c | 01, 02⟩ · ⟨01, 12 | +
  k | 01, 12⟩ · ⟨01, 12 | + g | 11, 02⟩ · ⟨01, 12 | + o | 11, 12⟩ · ⟨01, 12 | +
  b | 01, 02⟩ · ⟨11, 02 | + j | 01, 12⟩ · ⟨11, 02 | + f | 11, 02⟩ · ⟨11, 02 | +
  n | 11, 12⟩ · ⟨11, 02 | + d | 01, 02⟩ · ⟨11, 12 | + l | 01, 12⟩ · ⟨11, 12 | +
  h | 11, 02⟩ · ⟨11, 12 | + p | 11, 12⟩ · ⟨11, 12 | , QubitList  $\rightarrow \{\hat{2}, \hat{1}\}$ ]
```

$$\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix}$$

If you use `QuantumTensor` instead of `QuantumMatrix` (or `QuantumTensorForm` instead of `QuantumMatrixForm`) in order to get nested matrices showing the structure of the Hilber space:

```
QuantumTensorForm[a | 01, 02⟩ · ⟨01, 02 | + i | 01, 12⟩ · ⟨01, 02 | +
  e | 11, 02⟩ · ⟨01, 02 | + m | 11, 12⟩ · ⟨01, 02 | + c | 01, 02⟩ · ⟨01, 12 | +
  k | 01, 12⟩ · ⟨01, 12 | + g | 11, 02⟩ · ⟨01, 12 | + o | 11, 12⟩ · ⟨01, 12 | +
  b | 01, 02⟩ · ⟨11, 02 | + j | 01, 12⟩ · ⟨11, 02 | + f | 11, 02⟩ · ⟨11, 02 | +
  n | 11, 12⟩ · ⟨11, 02 | + d | 01, 02⟩ · ⟨11, 12 | + l | 01, 12⟩ · ⟨11, 12 | +
  h | 11, 02⟩ · ⟨11, 12 | + p | 11, 12⟩ · ⟨11, 12 |, QubitList → {2̂, 1̂}]
```

$$\begin{pmatrix} \begin{pmatrix} a & b \\ e & f \end{pmatrix} & \begin{pmatrix} c & d \\ g & h \end{pmatrix} \\ \begin{pmatrix} i & j \\ m & n \end{pmatrix} & \begin{pmatrix} k & l \\ o & p \end{pmatrix} \end{pmatrix}$$

## QubitList as an Option of QuantumTableForm

If QubitList is **not** specified, then QuantumTableForm works assuming that the qubits that are explicitly written in the expression are the only qubits that must be used to generate the truth table, and the order is the "canonical" order. For instance, if the expression only contains contains qubits  $\hat{1}, \hat{3}, \hat{7}$ , then the rows of the table will be  $0 \rightarrow |0_{\hat{1}}, 0_{\hat{3}}, 0_{\hat{7}}\rangle, 1 \rightarrow |0_{\hat{1}}, 0_{\hat{3}}, 1_{\hat{7}}\rangle, 2 \rightarrow |0_{\hat{1}}, 1_{\hat{3}}, 0_{\hat{7}}\rangle, 3 \rightarrow |0_{\hat{1}}, 1_{\hat{3}}, 1_{\hat{7}}\rangle$  etc, as shown in this example:

```
QuantumTableForm[SWAP7,3 · C(1)7 [NOT3] · H1]
```

	Input	Output
0	$ 0_{\hat{1}}, 0_{\hat{3}}, 0_{\hat{7}}\rangle$	$\frac{ 0_{\hat{1}}, 0_{\hat{3}}, 0_{\hat{7}}\rangle}{\sqrt{2}} + \frac{ 1_{\hat{1}}, 0_{\hat{3}}, 1_{\hat{7}}\rangle}{\sqrt{2}}$
1	$ 0_{\hat{1}}, 0_{\hat{3}}, 1_{\hat{7}}\rangle$	$\frac{ 0_{\hat{1}}, 1_{\hat{3}}, 0_{\hat{7}}\rangle}{\sqrt{2}} + \frac{ 1_{\hat{1}}, 1_{\hat{3}}, 1_{\hat{7}}\rangle}{\sqrt{2}}$
2	$ 0_{\hat{1}}, 1_{\hat{3}}, 0_{\hat{7}}\rangle$	$\frac{ 0_{\hat{1}}, 0_{\hat{3}}, 1_{\hat{7}}\rangle}{\sqrt{2}} + \frac{ 1_{\hat{1}}, 0_{\hat{3}}, 0_{\hat{7}}\rangle}{\sqrt{2}}$
3	$ 0_{\hat{1}}, 1_{\hat{3}}, 1_{\hat{7}}\rangle$	$\frac{ 0_{\hat{1}}, 1_{\hat{3}}, 1_{\hat{7}}\rangle}{\sqrt{2}} + \frac{ 1_{\hat{1}}, 1_{\hat{3}}, 0_{\hat{7}}\rangle}{\sqrt{2}}$
4	$ 1_{\hat{1}}, 0_{\hat{3}}, 0_{\hat{7}}\rangle$	$\frac{ 0_{\hat{1}}, 0_{\hat{3}}, 0_{\hat{7}}\rangle}{\sqrt{2}} - \frac{ 1_{\hat{1}}, 0_{\hat{3}}, 1_{\hat{7}}\rangle}{\sqrt{2}}$
5	$ 1_{\hat{1}}, 0_{\hat{3}}, 1_{\hat{7}}\rangle$	$\frac{ 0_{\hat{1}}, 1_{\hat{3}}, 0_{\hat{7}}\rangle}{\sqrt{2}} - \frac{ 1_{\hat{1}}, 1_{\hat{3}}, 1_{\hat{7}}\rangle}{\sqrt{2}}$
6	$ 1_{\hat{1}}, 1_{\hat{3}}, 0_{\hat{7}}\rangle$	$\frac{ 0_{\hat{1}}, 0_{\hat{3}}, 1_{\hat{7}}\rangle}{\sqrt{2}} - \frac{ 1_{\hat{1}}, 0_{\hat{3}}, 0_{\hat{7}}\rangle}{\sqrt{2}}$
7	$ 1_{\hat{1}}, 1_{\hat{3}}, 1_{\hat{7}}\rangle$	$\frac{ 0_{\hat{1}}, 1_{\hat{3}}, 1_{\hat{7}}\rangle}{\sqrt{2}} - \frac{ 1_{\hat{1}}, 1_{\hat{3}}, 0_{\hat{7}}\rangle}{\sqrt{2}}$

QubitList can be used to specify another ordering. For example,  $\text{QubitList} \rightarrow \{\hat{7}, \hat{3}, \hat{1}\}$  means that  $0 \rightarrow$

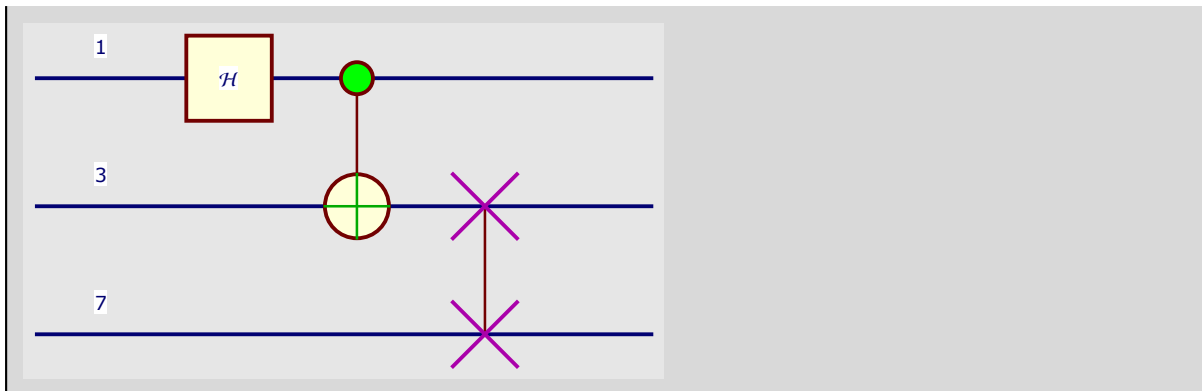
$|0_{\hat{1}}, 0_{\hat{3}}, 0_{\hat{7}}\rangle, 1 \rightarrow |1_{\hat{1}}, 0_{\hat{3}}, 0_{\hat{7}}\rangle, 2 \rightarrow |0_{\hat{1}}, 1_{\hat{3}}, 0_{\hat{7}}\rangle, 3 \rightarrow |1_{\hat{1}}, 1_{\hat{3}}, 0_{\hat{7}}\rangle$  etc, as shown below:

$$\text{QuantumTableForm}\left[\text{SWAP}_{\hat{7},\hat{3}} \cdot C^{\{\hat{1}\}}[\text{NOT}_{\hat{3}}] \cdot \mathcal{H}_{\hat{1}}, \text{QubitList} \rightarrow \{\hat{7}, \hat{3}, \hat{1}\}\right]$$

	Input	Output
0	$ 0_{\hat{1}}, 0_{\hat{3}}, 0_{\hat{7}}\rangle$	$\frac{ 0_{\hat{1}}, 0_{\hat{3}}, 0_{\hat{7}}\rangle}{\sqrt{2}} + \frac{ 1_{\hat{1}}, 0_{\hat{3}}, 1_{\hat{7}}\rangle}{\sqrt{2}}$
1	$ 1_{\hat{1}}, 0_{\hat{3}}, 0_{\hat{7}}\rangle$	$\frac{ 0_{\hat{1}}, 0_{\hat{3}}, 0_{\hat{7}}\rangle}{\sqrt{2}} - \frac{ 1_{\hat{1}}, 0_{\hat{3}}, 1_{\hat{7}}\rangle}{\sqrt{2}}$
2	$ 0_{\hat{1}}, 1_{\hat{3}}, 0_{\hat{7}}\rangle$	$\frac{ 0_{\hat{1}}, 0_{\hat{3}}, 1_{\hat{7}}\rangle}{\sqrt{2}} + \frac{ 1_{\hat{1}}, 0_{\hat{3}}, 0_{\hat{7}}\rangle}{\sqrt{2}}$
3	$ 1_{\hat{1}}, 1_{\hat{3}}, 0_{\hat{7}}\rangle$	$\frac{ 0_{\hat{1}}, 0_{\hat{3}}, 1_{\hat{7}}\rangle}{\sqrt{2}} - \frac{ 1_{\hat{1}}, 0_{\hat{3}}, 0_{\hat{7}}\rangle}{\sqrt{2}}$
4	$ 0_{\hat{1}}, 0_{\hat{3}}, 1_{\hat{7}}\rangle$	$\frac{ 0_{\hat{1}}, 1_{\hat{3}}, 0_{\hat{7}}\rangle}{\sqrt{2}} + \frac{ 1_{\hat{1}}, 1_{\hat{3}}, 1_{\hat{7}}\rangle}{\sqrt{2}}$
5	$ 1_{\hat{1}}, 0_{\hat{3}}, 1_{\hat{7}}\rangle$	$\frac{ 0_{\hat{1}}, 1_{\hat{3}}, 0_{\hat{7}}\rangle}{\sqrt{2}} - \frac{ 1_{\hat{1}}, 1_{\hat{3}}, 1_{\hat{7}}\rangle}{\sqrt{2}}$
6	$ 0_{\hat{1}}, 1_{\hat{3}}, 1_{\hat{7}}\rangle$	$\frac{ 0_{\hat{1}}, 1_{\hat{3}}, 1_{\hat{7}}\rangle}{\sqrt{2}} + \frac{ 1_{\hat{1}}, 1_{\hat{3}}, 0_{\hat{7}}\rangle}{\sqrt{2}}$
7	$ 1_{\hat{1}}, 1_{\hat{3}}, 1_{\hat{7}}\rangle$	$\frac{ 0_{\hat{1}}, 1_{\hat{3}}, 1_{\hat{7}}\rangle}{\sqrt{2}} - \frac{ 1_{\hat{1}}, 1_{\hat{3}}, 0_{\hat{7}}\rangle}{\sqrt{2}}$

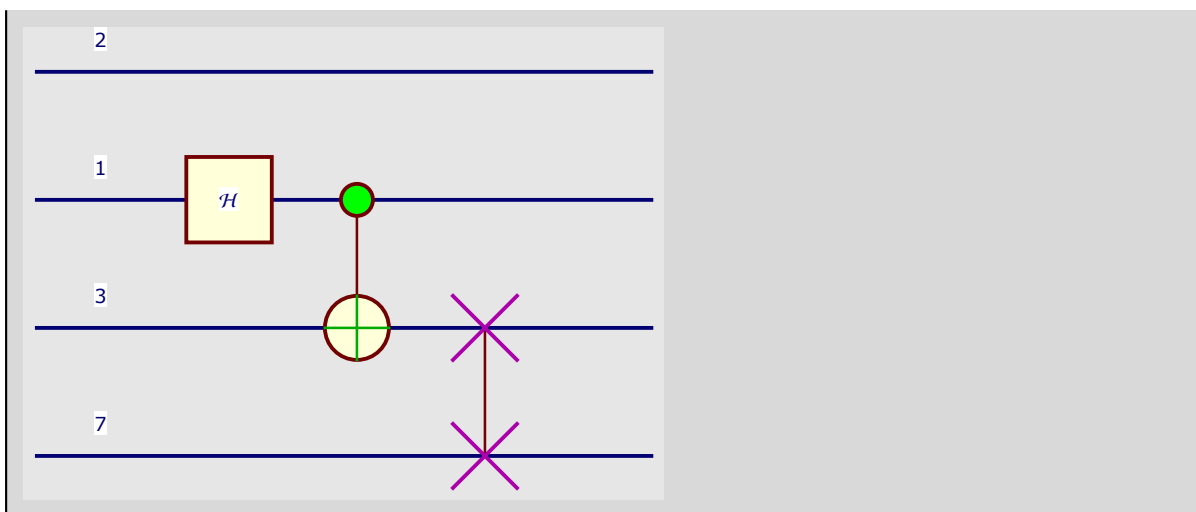
## QubitList as an Option of QuantumPlot

If QubitList is **not** specified, then QuantumPlot works assuming that the qubits that are explicitly written in the expression are the only qubits that must be plotted:

$$\text{QuantumPlot}\left[\text{SWAP}_{\hat{7},\hat{3}} \cdot C^{\{\hat{1}\}}[\text{NOT}_{\hat{3}}] \cdot \mathcal{H}_{\hat{1}}\right]$$


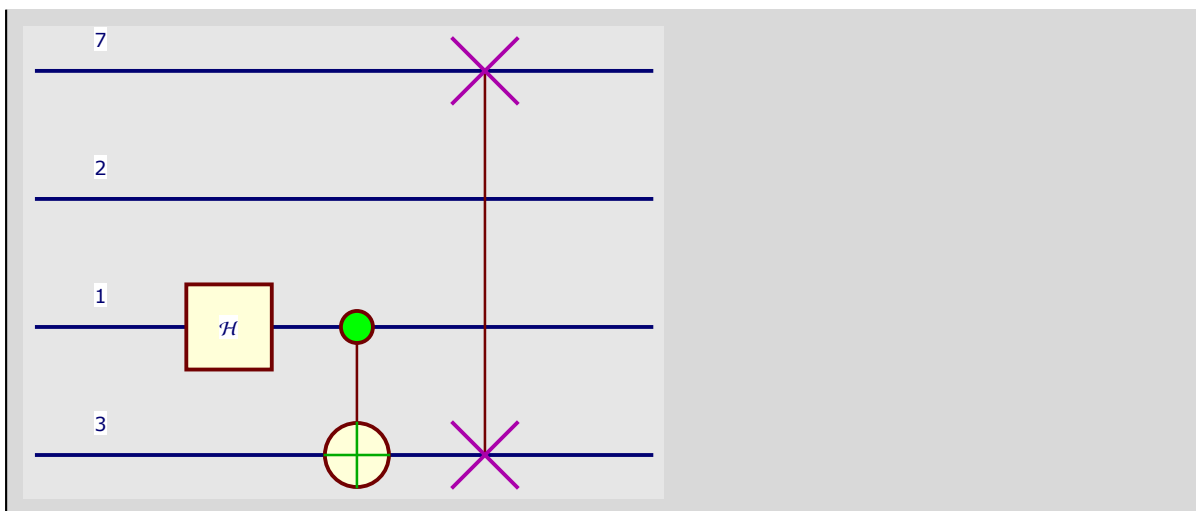
Here we specify that qubit  $\hat{2}$  must also be included in the plot. Notice that with this notation the added qubit is plotted at the top of the circuit:

```
QuantumPlot[SWAP $\hat{\gamma}, \hat{3}$  · C $^{(\hat{1})}$ [NOT $\hat{3}$ ] ·  $\mathcal{H}_{\hat{1}}$ , QubitList → { $\hat{2}$ }]
```



The option `QubitList` can also be used to specify in which order the qubits should be plotted. As an example, `QubitList → { $\hat{\gamma}$ ,  $\hat{2}$ }` means that qubit  $\hat{\gamma}$  must be plotted first, then qubit  $\hat{2}$ , and the rest of the qubits must be plotted in "canonical order":

```
QuantumPlot[SWAP $\hat{\gamma}, \hat{3}$  · C $^{(\hat{1})}$ [NOT $\hat{3}$ ] ·  $\mathcal{H}_{\hat{1}}$ , QubitList → { $\hat{\gamma}$ ,  $\hat{2}$ }]
```



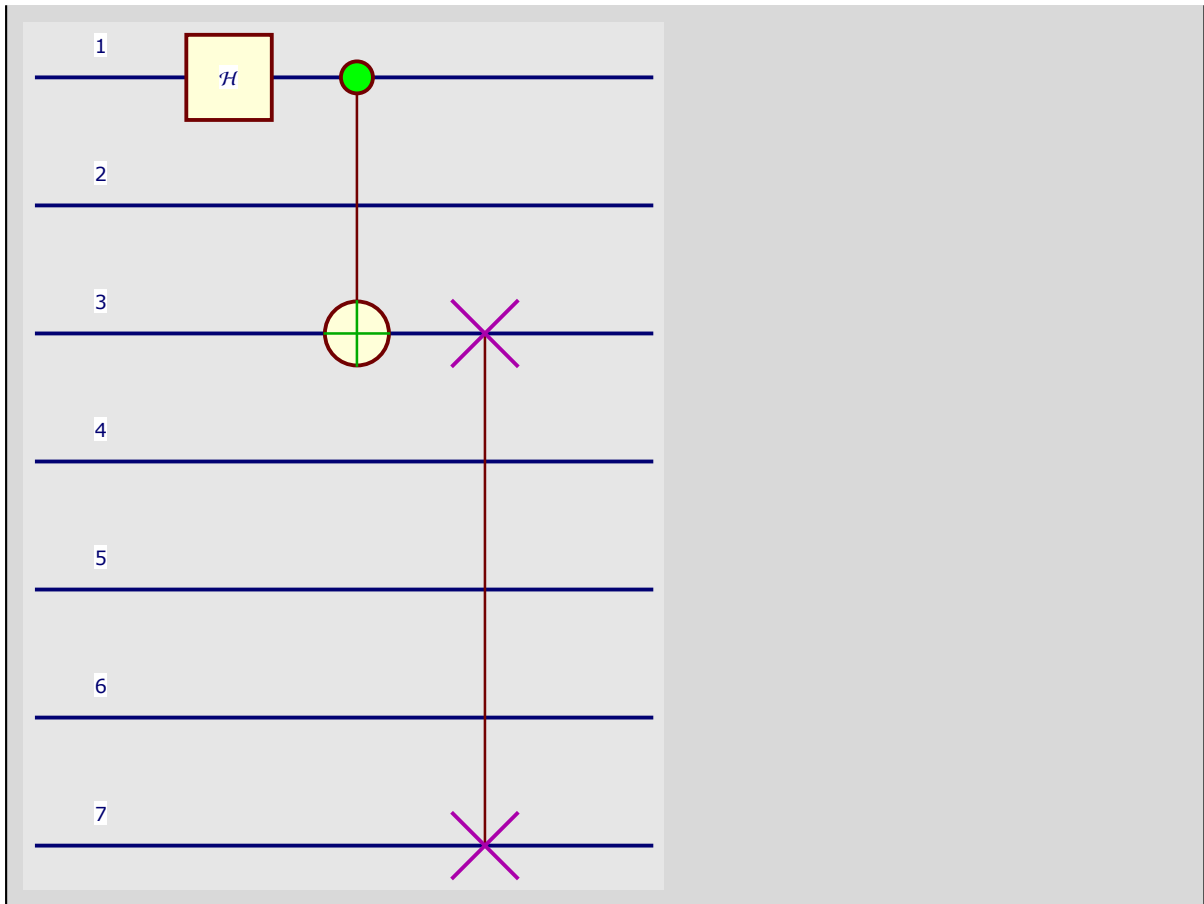
If `QubitList` is set to be a positive integer  $n$ , then it is transformed to the list of all the qubits  $\hat{1}, \hat{2}, \dots, \hat{n}$

```
QubitList → 7
```

```
QubitList → { $\hat{1}$ ,  $\hat{2}$ ,  $\hat{3}$ ,  $\hat{4}$ ,  $\hat{5}$ ,  $\hat{6}$ ,  $\hat{7}$ }
```

If `QubitList` is set to be a positive integer  $n$ , then it is transformed to the list of all the qubits  $\hat{1}, \hat{2}, \dots, \hat{n}$

```
QuantumPlot[SWAP1,3 · C(1)3[NOT3] · H1, QubitList → 7]
```




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