Quantum Measurements, Quantum Collapse and Density Operators

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Introduction

This is a tutorial on the use of Quantum *Mathematica* add-on to simulate measurements in a quantum state. Those measurements can be transformed to a density operator in Dirac notation.

Load the Package

First load the Quantum`Notation` package. Write:
Needs["Quantum`Notation`"]
then press at the same time the keys SHIFT-ENTER to evaluate. *Mathematica* will load the package:

```
Needs["Quantum`Notation`"]
```

```
Quantum`Notation` Version 2.2.0. (July 2010)
A Mathematica package for Quantum calculations in Dirac bra-ket notation
by José Luis Gómez-Muñoz

Execute SetQuantumAliases[] in order to use
  the keyboard to enter quantum objects in Dirac's notation
SetQuantumAliases[] must be executed again in each new
  notebook that is created, only one time per notebook.
```

In order to use the keyboard to enter quantum objects write:
SetQuantumAliases[ ];
then press at the same time the keys SHIFT-ENTER to evaluate. The semicolon prevents *Mathematica* from printing the help message. Remember that SetQuantumAliases[ ] must be evaluated again in each new notebook:

```
SetQuantumAliases[];
```

Eigenket Notation

The ket below represents an eigenstate of operators $\hat{p}$, $\hat{q}$, $\hat{r}$ with eigenvalues 10, 15 and 30. To enter the example press the keys:
[ESC]op[ESC] [ESC]on[ESC] [ESC]eeeket[ESC]
then press the [TAB] key several times to select the leftmost place-holder (empty square) and press:
q[TAB]10[TAB]p[TAB]15[TAB]q[TAB]30[TAB]r
then press at the same time [SHIFT]-[ENTER] at the same time in order to obtain the result of the calculation, which is the same ket mutiplied by the corresponding eigenvalue:

```
q̂ · |10_p̂, 15_q̂, 30_r̂⟩
```

```
15 |10_p̂, 15_q̂, 30_r̂⟩
```

Quantum Measurements

Measuring the operator $\hat{q}$ in one of its eigenstates will certainly (with probability one) return the corresponding eigenvalue:

```
QuantumMeasurement[ |10_p̂, 15_q̂, 30_r̂⟩, {q̂}]
```

Probability	Measurement	State
1	$\{\{15_{\hat{q}}\}\}$	$\vert 10_{\hat{p}}\rangle \otimes \vert 15_{\hat{q}}\rangle \otimes \vert 30_{\hat{r}}\rangle$
Probability	Measurement	State

The commmand **QuantumMeasurement** can be used to measure one or several operators. Notice (in the following example) that the command has two arguments separted by a comma; the first argument is a quantum state written as a linear superposition of kets (QuantumMeasurement internally normalizes the state); the second argument is a list of the operators that are going to be measured. The Eigenkets of this example can be obtained by pressing the keys [ESC]eeeket[ESC]; operator templates can be obtained by pressing the keys [ESC]op[ESC]:

```
QuantumMeasurement[3 |10_p̂, 15_q̂, 30_r̂⟩ + 7 |12_p̂, 20_q̂, 40_r̂⟩ + 5 |10_p̂, 20_q̂, 40_r̂⟩, {q̂, r̂}]
```

Probability	Measurement	State
$\frac{9}{83}$	$\{\{15_{\hat{q}}, 30_{\hat{r}}\}\}$	$\vert 10_{\hat{p}}\rangle \otimes \vert 15_{\hat{q}}\rangle \otimes \vert 30_{\hat{r}}\rangle$
$\frac{74}{83}$	$\{\{20_{\hat{q}}, 40_{\hat{r}}\}\}$	$(5 \vert 10_{\hat{p}}\rangle + 7 \vert 12_{\hat{p}}\rangle) \otimes \vert 20_{\hat{q}}\rangle \otimes \frac{\vert 40_{\hat{r}}\rangle}{\sqrt{74}}$
Probability	Measurement	State

Each row in the output represents a measurement outcome; first column is the probability of obtaining the outcome, second column is the measurement result, and third column it the collapsed state of the system after the measurement. Here is a different measurement in the same state as before:

```
QuantumMeasurement[3 |10_p̂, 15_q̂, 30_r̂⟩ + 7 |12_p̂, 20_q̂, 40_r̂⟩ + 5 |10_p̂, 20_q̂, 40_r̂⟩, {p̂}]
```

Probability	Measurement	State
$\frac{34}{83}$	$\{\{10_{\hat{p}}\}\}$	$\vert 10_{\hat{p}}\rangle \otimes \left(\frac{3 \vert 15_{\hat{q}}, 30_{\hat{r}}\rangle}{\sqrt{34}} + \frac{5 \vert 20_{\hat{q}}, 40_{\hat{r}}\rangle}{\sqrt{34}}\right)$
$\frac{49}{83}$	$\{\{12_{\hat{p}}\}\}$	$\vert 12_{\hat{p}}\rangle \otimes \vert 20_{\hat{q}}\rangle \otimes \vert 40_{\hat{r}}\rangle$
Probability	Measurement	State

Use the option **FactorKet → False** in order to have the output states in a nonfactorized form:

```
QuantumMeasurement[
 3 |10_p̂, 15_q̂, 30_r̂⟩ + 7 |12_p̂, 20_q̂, 40_r̂⟩ + 5 |10_p̂, 20_q̂, 40_r̂⟩, {p̂}, FactorKet → False]
```

Probability	Measurement	State
$\frac{34}{83}$	$\{\{10_{\hat{p}}\}\}$	$\frac{3\,\lvert 10_{\hat{p}},15_{\hat{q}},30_{\hat{r}}\rangle+5\,\lvert 10_{\hat{p}},20_{\hat{q}},40_{\hat{r}}\rangle}{\sqrt{34}}$
$\frac{49}{83}$	$\{\{12_{\hat{p}}\}\}$	$\lvert 12_{\hat{p}},20_{\hat{q}},40_{\hat{r}}\rangle$
Probability	Measurement	State

Measurements that require (generate) assumptions

When the eigenvalues are literals instead of numbers, **QuantumMeasurement** makes some assumptions in order to be able to give a simple answer. Those assumptions are reported at the bottom of the measurement results, **see the bottom of the output table**:

```
QuantumMeasurement[3 |a_p̂, 15_q̂, 30_r̂⟩ + 5 |b_p̂, 20_q̂, 40_r̂⟩ + 7 |c_p̂, 20_q̂, 40_r̂⟩, {p̂}]
```

Probability	Measurement	State
$\frac{9}{83}$	$\{\{a_{\hat{p}}\}\}$	$\lvert a_{\hat{p}}\rangle\otimes\lvert 15_{\hat{q}}\rangle\otimes\lvert 30_{\hat{r}}\rangle$
$\frac{25}{83}$	$\{\{b_{\hat{p}}\}\}$	$\lvert b_{\hat{p}}\rangle\otimes\lvert 20_{\hat{q}}\rangle\otimes\lvert 40_{\hat{r}}\rangle$
$\frac{49}{83}$	$\{\{c_{\hat{p}}\}\}$	$\lvert c_{\hat{p}}\rangle\otimes\lvert 20_{\hat{q}}\rangle\otimes\lvert 40_{\hat{r}}\rangle$
Assumptions → a ≠ b && a ≠ c && b ≠ c		

Specifying more assumptions

Next measurement gives a complex result because the denominators are actually piecewise-defined functions. This is necessary because if several literals are actually variables, when those variables have different values, two kets are different and orthogonal, on the other hand, if those variables have the same value, those two kets are actually the same ket, and the normalization denominators are different:

```
QuantumMeasurement[
 a |b_ĉ, d_ê, p_r̂⟩ + f |g_ĉ, h_ê, p_r̂⟩ + |m_ĉ, n_ê, x_r̂⟩, {r̂}, FactorKet → False]
```

Probability	Measurement	State
$\begin{cases} \frac{(a+f)\,(a^*+f^*)}{1+(a+f)\,a^*+(a+f)\,f^*} & b == g\ \&\&\ d == h \\ \frac{a\,a^*+f\,f^*}{1+a\,a^*+f\,f^*} & b \neq g\ \vert\vert\ d \neq h \\ \frac{a\,a^*+f\,f^*}{1+(a+f)\,a^*+(a+f)\,f^*} & \text{True} \end{cases}$	$\{\{p_{\hat{r}}\}\}$	$\frac{a\,\lvert b_{\hat{c}},d_{\hat{e}},p_{\hat{r}}\rangle+f\,\lvert g_{\hat{c}},h_{\hat{e}},p_{\hat{r}}\rangle}{\begin{cases} \sqrt{(a+f)\,(a^*+f^*)} & b==g\&\&d==h \\ \sqrt{a\,a^*+f\,f^*} & \text{True} \end{cases}}$
$\begin{cases} \frac{1}{1+a\,a^*+f\,f^*} & b \neq g\ \vert\vert\ d \neq h \\ \frac{1}{1+(a+f)\,a^*+(a+f)\,f^*} & \text{True} \end{cases}$	$\{\{x_{\hat{r}}\}\}$	$\lvert m_{\hat{c}},n_{\hat{e}},x_{\hat{r}}\rangle$
Assumptions → p ≠ x		

QuantumMeasurement accepts the option **Assumptions**. It has the same syntaxis as the option Assumptions of the other standard *Mathematica* commands, like Simplify and Integrate. Next we have the same example as above, but with the explicit assumption that b≠g. The symbol ≠ ("not-equal") can be entered by pressing the keys [ESC]!=[ESC]. Notice that at the bottom of the output table the assumptions used in the calculations are reported. These assumptions include user-specified assumptions (b≠g in this example) and *Mathematica*-generated assumptions (p≠x in this example):

```
QuantumMeasurement[a |b_ĉ, d_ê, p_r̂⟩ + f |g_ĉ, h_ê, p_r̂⟩ + |m_ĉ, n_ê, x_r̂⟩,
 {r̂}, FactorKet → False, Assumptions → b ≠ g]
```

Probability	Measurement	State
$\frac{a\,a^* + f\,f^*}{1 + a\,a^* + f\,f^*}$	$\{\{p_{\hat{r}}\}\}$	$\frac{a\,\vert b_{\hat{c}}, d_{\hat{e}}, p_{\hat{r}}\rangle + f\,\vert g_{\hat{c}}, h_{\hat{e}}, p_{\hat{r}}\rangle}{\sqrt{a\,a^* + f\,f^*}}$
$\frac{1}{1 + a\,a^* + f\,f^*}$	$\{\{x_{\hat{r}}\}\}$	$\vert m_{\hat{c}}, n_{\hat{e}}, x_{\hat{r}}\rangle$
Assumptions → b ≠ g && p ≠ x		

You can use the standard *Mathematica* command Assumig[ ] to specify assumptions. Remember that Assuming only affects the work of commands that accept the option Assumptions, like Integrate and Simplify. Other commands just ignorate the Assuming command:

```
Assuming[b ≠ g, QuantumMeasurement[
  a |b_ĉ, d_ê, p_r̂⟩ + f |g_ĉ, h_ê, p_r̂⟩ + |m_ĉ, n_ê, x_r̂⟩, {r̂}, FactorKet → False]]
```

Probability	Measurement	State
$\frac{a\,a^* + f\,f^*}{1 + a\,a^* + f\,f^*}$	$\{\{p_{\hat{r}}\}\}$	$\frac{a\,\vert b_{\hat{c}}, d_{\hat{e}}, p_{\hat{r}}\rangle + f\,\vert g_{\hat{c}}, h_{\hat{e}}, p_{\hat{r}}\rangle}{\sqrt{a\,a^* + f\,f^*}}$
$\frac{1}{1 + a\,a^* + f\,f^*}$	$\{\{x_{\hat{r}}\}\}$	$\vert m_{\hat{c}}, n_{\hat{e}}, x_{\hat{r}}\rangle$
Assumptions → b ≠ g && p ≠ x		

Extracting measurement results for further calculations

Here is a simple measurement:

```
QuantumMeasurement[a |g_ĉ, d_ê⟩ + f |g_ĉ, h_ê⟩, {ê}]
```

Probability	Measurement	State
$\frac{a\,a^*}{a\,a^* + f\,f^*}$	$\{\{d_{\hat{e}}\}\}$	$\vert g_{\hat{c}}\rangle \otimes \vert d_{\hat{e}}\rangle$
$\frac{f\,f^*}{a\,a^* + f\,f^*}$	$\{\{h_{\hat{e}}\}\}$	$\vert g_{\hat{c}}\rangle \otimes \vert h_{\hat{e}}\rangle$
Assumptions → d ≠ h		

Measurement results can be "extracted" using standard *Mathematica* notation. For example, this is probability of the second possible output, obtained with the standard *Mathematica* command **Part**:

```
Part[QuantumMeasurement[a |g_ĉ, d_ê⟩ + f |g_ĉ, h_ê⟩, {ê}], 1, 2, 1]
```

$$\frac{f\,f^*}{a\,a^* + f\,f^*}$$

The same value can be extracted using the **double-bracket notation** for extracting elements of arrays and lists. Notice there must **not** be any space between the brackets:

```
QuantumMeasurement[a |g_ĉ, d_ê⟩ + f |g_ĉ, h_ê⟩, {ê}][[1, 2, 1]]
```

$$\frac{f\,f^*}{a\,a^* + f\,f^*}$$

Next syntaxis should be easier to read, and it could be part of a larger calculation or program, because the expression for the probability is stored in the variable myprob, for further use in other calculations:

```
mymeasurement = QuantumMeasurement[a |g_ĉ, d_ê⟩ + f |g_ĉ, h_ê⟩, {ê}];
myprob = mymeasurement[[1, 2, 1]]
```

$$\frac{f\,f^*}{a\,a^* + f\,f^*}$$

You can extract all the measurement data as a list:

```
mymeasurement = QuantumMeasurement[a |g_ĉ, d_ê⟩ + f |g_ĉ, h_ê⟩, {ê}];
mylist = mymeasurement[[1]]
```

$$\left\{\left\{\frac{a\,a^*}{a\,a^* + f\,f^*}, \{\{d_{\hat{e}}\}\}, |g_{\hat{c}}, d_{\hat{e}}\rangle\right\}, \left\{\frac{f\,f^*}{a\,a^* + f\,f^*}, \{\{h_{\hat{e}}\}\}, |g_{\hat{c}}, h_{\hat{e}}\rangle\right\}\right\}$$

You can extract the assumptions made for producing the measurement results:

```
mymeasurement = QuantumMeasurement[a |g_ĉ, d_ê⟩ + f |g_ĉ, h_ê⟩, {ê}];
myassum = mymeasurement[[2]]
```

Assumptions → d ≠ h

Building a Density Operator from Measurement Results

Here is a simple measurement:

```
QuantumMeasurement[
 2 |1_p̂, a_q̂, 3.5_r̂⟩ + 3 |1_p̂, a_q̂, 4.2_r̂⟩ + 4 |3_p̂, a_q̂, 4.2_r̂⟩ + 5 |3_p̂, b_q̂, 3.5_r̂⟩, {q̂, p̂}]
```

Probability	Measurement	State
$\frac{13}{54}$	$\{\{1_{\hat{p}}, a_{\hat{q}}\}\}$	$\|1_{\hat{p}}\rangle \otimes \|a_{\hat{q}}\rangle \otimes \left(\frac{2\,\|3.5_{\hat{r}}\rangle}{\sqrt{13}} + \frac{3\,\|4.2_{\hat{r}}\rangle}{\sqrt{13}}\right)$
$\frac{8}{27}$	$\{\{3_{\hat{p}}, a_{\hat{q}}\}\}$	$\|3_{\hat{p}}\rangle \otimes \|a_{\hat{q}}\rangle \otimes \|4.2_{\hat{r}}\rangle$
$\frac{25}{54}$	$\{\{3_{\hat{p}}, b_{\hat{q}}\}\}$	$\|3_{\hat{p}}\rangle \otimes \|b_{\hat{q}}\rangle \otimes \|3.5_{\hat{r}}\rangle$
Assumptions → a ≠ b		

Here is the density operator of the measurment

$$\text{QuantumDensityOperator}\left[\text{QuantumMeasurement}\left[2\left|1_{\hat{p}}, a_{\hat{q}}, 3.5_{\hat{r}}\right\rangle + 3\left|1_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right\rangle + 4\left|3_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right\rangle + 5\left|3_{\hat{p}}, b_{\hat{q}}, 3.5_{\hat{r}}\right\rangle, \{\hat{q}, \hat{p}\}\right]\right]$$

$$\frac{2}{27}\left|1_{\hat{p}}, a_{\hat{q}}, 3.5_{\hat{r}}\right\rangle \cdot \left\langle 1_{\hat{p}}, a_{\hat{q}}, 3.5_{\hat{r}}\right| + \frac{1}{9}\left|1_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right\rangle \cdot \left\langle 1_{\hat{p}}, a_{\hat{q}}, 3.5_{\hat{r}}\right| +$$
$$\frac{1}{9}\left|1_{\hat{p}}, a_{\hat{q}}, 3.5_{\hat{r}}\right\rangle \cdot \left\langle 1_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right| + \frac{1}{6}\left|1_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right\rangle \cdot \left\langle 1_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right| +$$
$$\frac{8}{27}\left|3_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right\rangle \cdot \left\langle 3_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right| + \frac{25}{54}\left|3_{\hat{p}}, b_{\hat{q}}, 3.5_{\hat{r}}\right\rangle \cdot \left\langle 3_{\hat{p}}, b_{\hat{q}}, 3.5_{\hat{r}}\right|$$

Next syntaxis should be easier to read, and it could be part of a larger calculation or program, because the density operator is stored in the **variable mydenop**, for further use in other calculations:

$$\text{myqm} = \text{QuantumMeasurement}\left[2\left|1_{\hat{p}}, a_{\hat{q}}, 3.5_{\hat{r}}\right\rangle + 3\left|1_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right\rangle + 4\left|3_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right\rangle + 5\left|3_{\hat{p}}, b_{\hat{q}}, 3.5_{\hat{r}}\right\rangle, \{\hat{q}, \hat{p}\}\right];$$
$$\text{mydenop} = \text{QuantumDensityOperator}[\text{myqm}]$$

$$\frac{2}{27}\left|1_{\hat{p}}, a_{\hat{q}}, 3.5_{\hat{r}}\right\rangle \cdot \left\langle 1_{\hat{p}}, a_{\hat{q}}, 3.5_{\hat{r}}\right| + \frac{1}{9}\left|1_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right\rangle \cdot \left\langle 1_{\hat{p}}, a_{\hat{q}}, 3.5_{\hat{r}}\right| +$$
$$\frac{1}{9}\left|1_{\hat{p}}, a_{\hat{q}}, 3.5_{\hat{r}}\right\rangle \cdot \left\langle 1_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right| + \frac{1}{6}\left|1_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right\rangle \cdot \left\langle 1_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right| +$$
$$\frac{8}{27}\left|3_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right\rangle \cdot \left\langle 3_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right| + \frac{25}{54}\left|3_{\hat{p}}, b_{\hat{q}}, 3.5_{\hat{r}}\right\rangle \cdot \left\langle 3_{\hat{p}}, b_{\hat{q}}, 3.5_{\hat{r}}\right|$$

Below we have a simple calculation with the density operator that was stored in the variable **mydenop** (see also calculation above)

$$\left\langle 1_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right| \cdot \text{mydenop} \cdot \left|1_{\hat{p}}, a_{\hat{q}}, 3.5_{\hat{r}}\right\rangle$$

$$\frac{1}{9}$$

Below we have a simple calculation with the density operator that was stored in the variable **mydenop** (see also calculation above)

$$\text{QuantumPartialTrace}[\text{mydenop}, \hat{q}]$$

$$\frac{2}{27}\left|1_{\hat{p}}, 3.5_{\hat{r}}\right\rangle \cdot \left\langle 1_{\hat{p}}, 3.5_{\hat{r}}\right| + \frac{1}{9}\left|1_{\hat{p}}, 4.2_{\hat{r}}\right\rangle \cdot \left\langle 1_{\hat{p}}, 3.5_{\hat{r}}\right| + \frac{1}{9}\left|1_{\hat{p}}, 3.5_{\hat{r}}\right\rangle \cdot \left\langle 1_{\hat{p}}, 4.2_{\hat{r}}\right| +$$
$$\frac{1}{6}\left|1_{\hat{p}}, 4.2_{\hat{r}}\right\rangle \cdot \left\langle 1_{\hat{p}}, 4.2_{\hat{r}}\right| + \frac{25}{54}\left|3_{\hat{p}}, 3.5_{\hat{r}}\right\rangle \cdot \left\langle 3_{\hat{p}}, 3.5_{\hat{r}}\right| + \frac{8}{27}\left|3_{\hat{p}}, 4.2_{\hat{r}}\right\rangle \cdot \left\langle 3_{\hat{p}}, 4.2_{\hat{r}}\right|$$

Below we have a simple calculation with the density operator that was stored in the variable **mydenop** (see also calculation above)

$$\mathbf{Expand}\left[\mathbf{mydenop}^2 - \mathbf{mydenop}\right]$$

$$-\frac{41}{729}\ \left|1_{\hat{p}}, a_{\hat{q}}, 3.5_{\hat{r}}\right\rangle \cdot \left\langle 1_{\hat{p}}, a_{\hat{q}}, 3.5_{\hat{r}}\right| - \frac{41}{486}\ \left|1_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right\rangle \cdot \left\langle 1_{\hat{p}}, a_{\hat{q}}, 3.5_{\hat{r}}\right| - \frac{41}{486}\ \left|1_{\hat{p}}, a_{\hat{q}}, 3.5_{\hat{r}}\right\rangle \cdot \left\langle 1_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right| - \frac{41}{324}\ \left|1_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right\rangle \cdot \left\langle 1_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right| - \frac{152}{729}\ \left|3_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right\rangle \cdot \left\langle 3_{\hat{p}}, a_{\hat{q}}, 4.2_{\hat{r}}\right| - \frac{725\ \left|3_{\hat{p}}, b_{\hat{q}}, 3.5_{\hat{r}}\right\rangle \cdot \left\langle 3_{\hat{p}}, b_{\hat{q}}, 3.5_{\hat{r}}\right|}{2916}$$

This is the matrix (in *Mathematica* list notation) that corresponds to the density operator that was stored in the variable **mydenop** (see also calculation above). Notice that it is specified that operator $\hat{p}$ has three possible eigenvalues, 1,2,3 eventhough in the operator only eigenvalues 1 and 3 appear:

$$\mathbf{DiracToMatrix}\left[\mathbf{mydenop}, \left\{\left\{1_{\hat{p}}, 2_{\hat{p}}, 3_{\hat{p}}\right\}, \left\{a_{\hat{q}}, b_{\hat{q}}\right\}, \left\{3.5_{\hat{r}}, 4.2_{\hat{r}}\right\}\right\}\right]$$

$$\left\{\left\{\frac{2}{27}, \frac{1}{9}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \left\{\frac{1}{9}, \frac{1}{6}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{8}{27}, 0, 0\right\}, \left\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{25}{54}, 0\right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\right\}$$

This is the matrix that corresponds to the density operator that was stored in the variable **mydenop** (see also calculation above). Notice that it is specified that operator $\hat{p}$ has three possible eigenvalues, 1,2,3 eventhough in the operator only eigenvalues 1 and 3 appear:

$$\mathbf{MatrixForm}\left[\mathbf{DiracToMatrix}\left[\mathbf{mydenop}, \left\{\left\{1_{\hat{p}}, 2_{\hat{p}}, 3_{\hat{p}}\right\}, \left\{a_{\hat{q}}, b_{\hat{q}}\right\}, \left\{3.5_{\hat{r}}, 4.2_{\hat{r}}\right\}\right\}\right]\right]$$

$$\begin{pmatrix}
\frac{2}{27} & \frac{1}{9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{9} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{8}{27} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{25}{54} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Same operator, but now it is specified that operator $\hat{p}$ has only two possible eigenvalues, 1 and 3, therefore the matrix is different:

```
MatrixForm[DiracToMatrix[mydenop, {{1_p̂, 3_p̂}, {a_q̂, b_q̂}, {3.5_r̂, 4.2_r̂}}]]
```

$$\begin{pmatrix} \frac{2}{27} & \frac{1}{9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{9} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{8}{27} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{25}{54} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The tensor representing the same operator as above:

```
MatrixForm[DiracToTensor[mydenop, {{1_p̂, 3_p̂}, {a_q̂, b_q̂}, {3.5_r̂, 4.2_r̂}}]]
```

$$\begin{pmatrix} \begin{pmatrix} \begin{pmatrix} \frac{2}{27} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{6} \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} & \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \\ \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} & \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & \frac{8}{27} \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} \frac{25}{54} & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \end{pmatrix}$$

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