Gallery of Quantum Gates

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Introduction

This is a tutorial on the use of Quantum`Computing` *Mathematica* add-on to use quantum gates in quantum circuits and algorithms

Load the Package

First load the Quantum`Computing` package. Write:
Needs["Quantum`Computing`"]
then press at the same time the keys SHIFT-ENTER to evaluate. *Mathematica* will load the package.

```
Needs["Quantum`Computing`"]
```

In order to use the keyboard to enter quantum objects write:
SetComputingAliases[ ];
then press at the same time the keys SHIFT-ENTER to evaluate. The semicolon prevents *Mathematica* from printing the help message. Remember that SetComputingAliases[ ] must be evaluated again in each new notebook:

```
SetComputingAliases[];
```

$\mathcal{X}_{\hat{q}}$ Gate

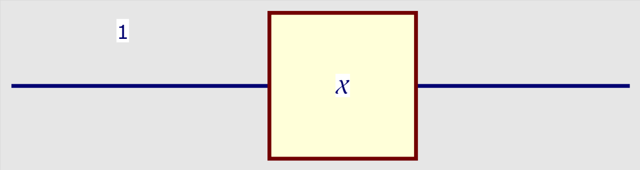
The $\mathcal{X}_{\hat{q}}$ gate is the first Pauli gate on qubit q. In order enter the template for this gate, press the keys:
[ESC]xg[ESC]
then press [TAB] to select the place-holder (empty square) and write the qubit number or label:

$\mathcal{X}_{\hat{1}}$

$\mathcal{X}_{\hat{1}}$

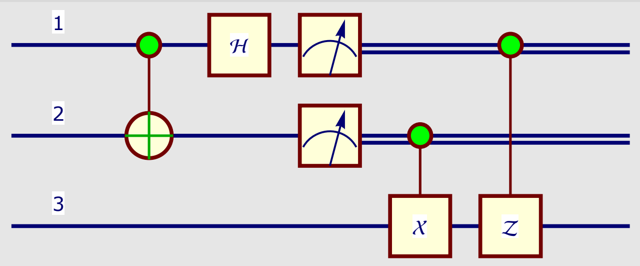
Use QuantumPlot to plot the gate in a quantum circuit:

```
QuantumPlot[𝒳_1̂]
```



This is a more interesting circuit (Teleportation) that includes this gate:

```
QuantumPlot[C^{1̂}[𝒵_3̂] · C^{2̂}[𝒳_3̂] · QubitMeasurement[ℋ_1̂ · C^{1̂}[𝒩𝒪𝒯_2̂], {1̂, 2̂}],
 QuantumGateShifting → False]
```



This is the Dirac representation of this gate

```
QuantumEvaluate[𝒳_1̂]
```

$| 1_{\hat{1}} \rangle \cdot \langle 0_{\hat{1}} | + | 0_{\hat{1}} \rangle \cdot \langle 1_{\hat{1}} |$

This is the action of this gate on a qubit on state $| 0 \rangle$

```
QuantumEvaluate[𝒳_1̂ · | 0_1̂⟩]
```

$| 1_{\hat{1}} \rangle$

This is the action of this gate on a qubit on state $| 1 \rangle$

```
QuantumEvaluate[𝒳_1̂ · | 1_1̂⟩]
```

$| 0_{\hat{1}} \rangle$

The $\mathcal{X}[\hat{q}]$ gate negates qubit q, therefore a controlled $\mathcal{X}[\hat{q}]$ gate is the same as a controlled not gate (notice the use of two equal symbols == in order to indicate comparison instead of assignment):

```
QuantumEvaluate[C^{1̂}[𝒳_2̂]] == QuantumEvaluate[C^{1̂}[𝒩𝒪𝒯_2̂]]
```

```
True
```

This is the matrix representation of this gate:

```
QuantumMatrixForm[𝒳_1̂]
```

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

This is the representation of this gate in terms of Pauli operators

```
PauliExpand[𝒳_1̂]
```

$\sigma_{\mathcal{X},\hat{1}}$

Products of Pauli gates are **not** simplified

$\mathcal{X}_{\hat{1}} \cdot \mathcal{Y}_{\hat{1}}$

$\mathcal{X}_{\hat{1}} \cdot \mathcal{Y}_{\hat{1}}$

On the other hand, products of Pauli operators **are** simplified

$\sigma_{\mathcal{X},\hat{1}} \cdot \sigma_{\mathcal{Y},\hat{1}}$

$i\, \sigma_{\mathcal{Z},\hat{1}}$

$\mathcal{Y}_{\hat{q}}$ Gate

The $\mathcal{Y}_{\hat{q}}$ gate is the second Pauli gate on qubit q. In order enter the template for this gate, press the keys:

[ESC]yg[ESC]

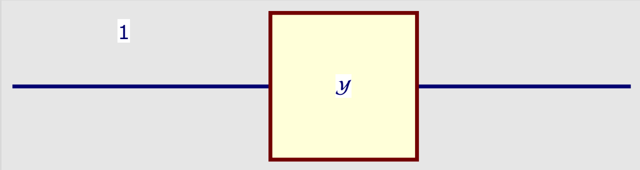
then press [TAB] to select the place-holder (empty square) and write the qubit number or label:

$\mathcal{Y}_{\hat{1}}$

$\mathcal{Y}_{\hat{1}}$

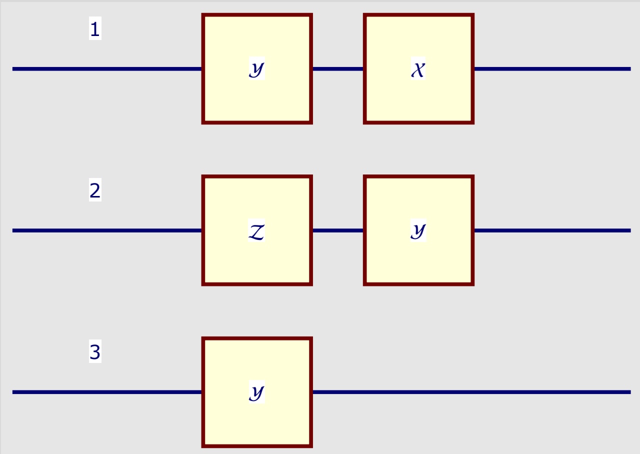
Use QuantumPlot to plot the gate in a quantum circuit:

QuantumPlot[$\mathcal{Y}_{\hat{1}}$]



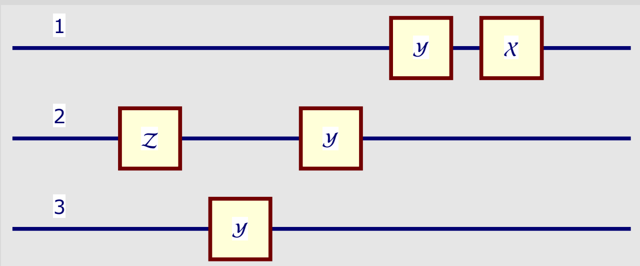
Notice the arragenment of gates in this quantum circuit: the $\mathcal{Y}$ gates are in different columns in the resulting plot:

QuantumPlot[$\mathcal{X}_{\hat{1}} \cdot \mathcal{Y}_{\hat{1}} \cdot \mathcal{Y}_{\hat{2}} \cdot \mathcal{Y}_{\hat{3}} \cdot \mathcal{Z}_{\hat{2}}$]



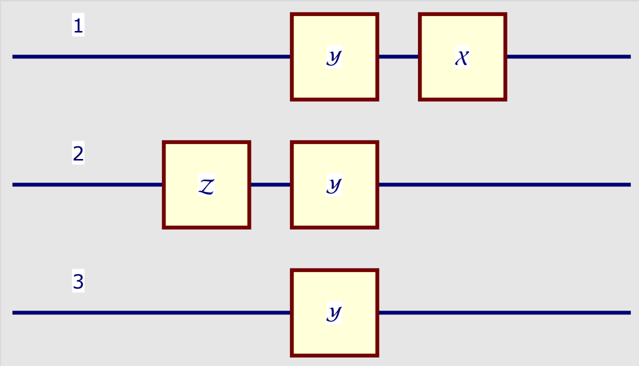
Even with the option QuantumGateShifting → False the $\mathcal{Y}$ gates are not in the same column

QuantumPlot[$\mathcal{X}_{\hat{1}} \cdot \mathcal{Y}_{\hat{1}} \cdot \mathcal{Y}_{\hat{2}} \cdot \mathcal{Y}_{\hat{3}} \cdot \mathcal{Z}_{\hat{2}}$, QuantumGateShifting → False]



There is a notation that allows to have all the $\mathcal{Y}$ gates in the same column.
Press the keys [ESC]yggg[ESC] for the template:

```
QuantumPlot[X_1̂ · Y_{1̂,2̂,3̂} · Z_2̂]
```



Both notations evaluate to the same Dirac expression:

```
QuantumEvaluate[Y_1̂ · Y_2̂ · Y_3̂] == QuantumEvaluate[Y_{1̂,2̂,3̂}]
```

```
True
```

This is the Dirac representation of this gate

```
QuantumEvaluate[Y_1̂]
```

$i\ |1_{\hat{1}}\rangle \cdot \langle 0_{\hat{1}}| - i\ |0_{\hat{1}}\rangle \cdot \langle 1_{\hat{1}}|$

This is the matrix representation of this gate:

```
QuantumMatrixForm[Y_1̂]
```

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

This is the representation of this gate in terms of Pauli operators

```
PauliExpand[Y_1̂]
```

$\sigma_{\mathcal{Y},\hat{1}}$

Products of Pauli gates are **not** simplified

$\mathcal{X}_{\hat{1}} \cdot \mathcal{Y}_{\hat{1}}$

$\mathcal{X}_{\hat{1}} \cdot \mathcal{Y}_{\hat{1}}$

On the other hand, products of Pauli operators **are** simplified

$$\sigma_{x,\hat{1}} \cdot \sigma_{y,\hat{1}}$$

$$i\, \sigma_{z,\hat{1}}$$

$\mathcal{Z}_{\hat{q}}$ Gate

The $\mathcal{Z}_{\hat{q}}$ gate is the third Pauli gate on qubit q. In order enter the template for this gate, press the keys:

[ESC]zg[ESC]

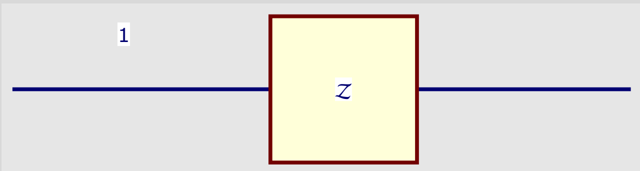
then press [TAB] to select the place-holder (empty square) and write the qubit number or label:

$$\mathcal{Z}_{\hat{1}}$$

$$\mathcal{Z}_{\hat{1}}$$

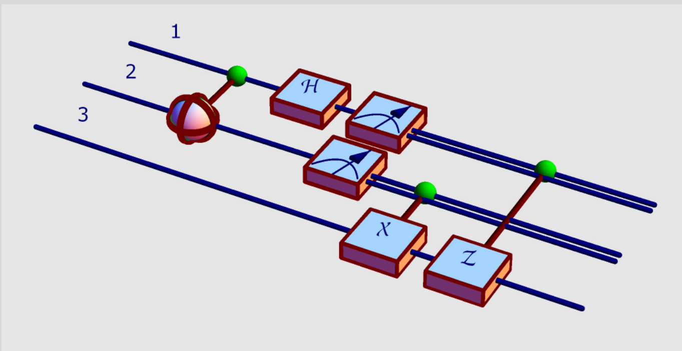
Use QuantumPlot to plot the gate in a quantum circuit:

$$\texttt{QuantumPlot}\left[\mathcal{Z}_{\hat{1}}\right]$$



This is a more interesting circuit (Teleportation) that includes this gate:

$$\texttt{QuantumPlot3D}\left[C^{\{\hat{1}\}}\left[\mathcal{Z}_{\hat{3}}\right] \cdot C^{\{\hat{2}\}}\left[\mathcal{X}_{\hat{3}}\right] \cdot \texttt{QubitMeasurement}\left[\mathcal{H}_{\hat{1}} \cdot C^{\{\hat{1}\}}\left[\mathcal{NOT}_{\hat{2}}\right], \left\{\hat{1}, \hat{2}\right\}\right], \texttt{QuantumGateShifting} \rightarrow \texttt{False}\right]$$



This is the Dirac representation of this gate

QuantumEvaluate$[\mathcal{Z}_{\hat{1}}]$

$| 0_{\hat{1}} \rangle \cdot \langle 0_{\hat{1}} | - | 1_{\hat{1}} \rangle \cdot \langle 1_{\hat{1}} |$

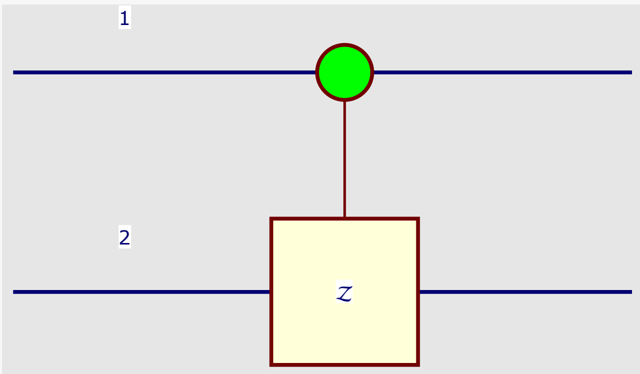
Controlled-$\mathcal{Z}[\hat{q}]$ gates are the same if the control qubit and the controlled qubit are exchanged (notice the use of two equal symbols == in order to indicate comparison instead of assignment):

QuantumEvaluate$\left[C^{\{\hat{1}\}}[\mathcal{Z}_{\hat{2}}]\right]$ == **QuantumEvaluate**$\left[C^{\{\hat{2}\}}[\mathcal{Z}_{\hat{1}}]\right]$

True

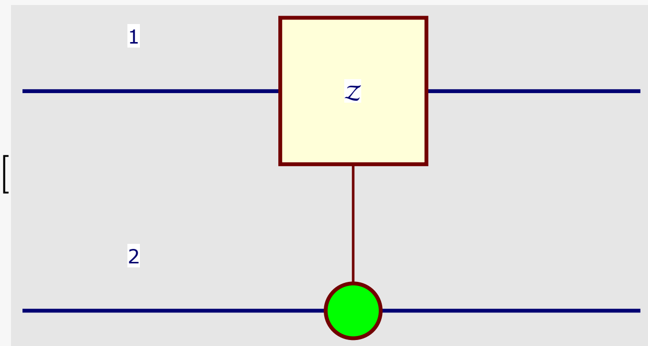
Controlled-$\mathcal{Z}[\hat{q}]$ gates are the same if the control qubit and the controlled qubit are exchanged (notice the use of two equal symbols == in order to indicate comparison instead of assignment):

QuantumEvaluate[



] ==

QuantumEvaluate[



]

True

This is the matrix representation of this gate:

QuantumMatrixForm$[\mathcal{Z}_{\hat{1}}]$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

This is the representation of this gate in terms of Pauli operators

```
PauliExpand[𝒵_1̂]
```

$\sigma_{\mathcal{Z},\hat{1}}$

This is the representation of a controlled-$\mathcal{Z}[\hat{q}]$ gate in terms of Pauli operators

```
PauliExpand[C^{1̂}[𝒵_2̂]]
```

$$\frac{1}{2}\sigma_{0,\hat{1}}\cdot\sigma_{0,\hat{2}}+\frac{1}{2}\sigma_{\mathcal{Z},\hat{1}}\cdot\sigma_{0,\hat{2}}+\frac{1}{2}\sigma_{0,\hat{1}}\cdot\sigma_{\mathcal{Z},\hat{2}}-\frac{1}{2}\sigma_{\mathcal{Z},\hat{1}}\cdot\sigma_{\mathcal{Z},\hat{2}}$$

Notice that the representation is the same if the qubits are exchanged. This is a property of controlled-$\mathcal{Z}[\hat{q}]$ gates:

```
PauliExpand[C^{2̂}[𝒵_1̂]]
```

$$\frac{1}{2}\sigma_{0,\hat{1}}\cdot\sigma_{0,\hat{2}}+\frac{1}{2}\sigma_{\mathcal{Z},\hat{1}}\cdot\sigma_{0,\hat{2}}+\frac{1}{2}\sigma_{0,\hat{1}}\cdot\sigma_{\mathcal{Z},\hat{2}}-\frac{1}{2}\sigma_{\mathcal{Z},\hat{1}}\cdot\sigma_{\mathcal{Z},\hat{2}}$$

Identity Gate $\mathcal{I}_{\hat{q}}$

The $\mathcal{I}_{\hat{q}}$ gate is the identity gate on qubit q. In order enter the template for this gate, press the keys:
[ESC]ig[ESC]
then press [TAB] to select the place-holder (empty square) and write the qubit number or label:

```
𝓘_1̂
```

$\mathcal{I}_{\hat{1}}$

The identity gate leaves the qubit without nay change

```
QuantumTableForm[𝓘_1̂]
```

	Input	Output
0	$\mid 0_{\hat{1}}\rangle$	$\mid 0_{\hat{1}}\rangle$
1	$\mid 1_{\hat{1}}\rangle$	$\mid 1_{\hat{1}}\rangle$

This is an application of the identity gate: we get the representation of the $\mathcal{Y}[\hat{2}]$, taking into account that qubit $\hat{1}$ also exists:

```
QuantumMatrixForm[𝓘_1̂ ⊗ 𝓨_2̂]
```

$$\begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}$$

The option QubitList can be used (instead of $\mathcal{I}[\hat{1}]$) in order to specify that that qubit $\hat{1}$ also exists:

```
QuantumMatrixForm[𝓨_2̂, QubitList → {1̂, 2̂}]
```

$$\begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}$$

If neither $\mathcal{I}[\hat{1}]$ nor QubitList->$\{\hat{1},\hat{2}\}$ are used, the QuantumMatrixForm assumes that $\hat{2}$ is the only qubit:

```
QuantumMatrixForm[𝓨_2̂]
```

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

This is an application of the identity gate: we get the representation of the $\mathcal{Y}[\hat{2}]$, taking into account that qubits $\hat{1}$ and $\hat{3}$ also exists:

```
QuantumMatrixForm[𝓘_1̂ ⊗ 𝓨_2̂ ⊗ 𝓘_3̂]
```

$$\begin{pmatrix} 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & i & 0 & 0 \end{pmatrix}$$

This is the representation of this gate in terms of Pauli operators

```
PauliExpand[𝓘_1̂]
```

$\sigma_{0,\hat{1}}$

Pauli gates are **not** simplified

```
𝓘_1̂ · 𝓨_1̂
```

$\mathcal{I}_{\hat{1}} \cdot \mathcal{Y}_{\hat{1}}$

On the other hand, Pauli operators **are** simplified

$\sigma_{0,\hat{1}} \cdot \sigma_{y,\hat{1}}$

$\sigma_{y,\hat{1}}$

Hadamard $\mathcal{H}_{\hat{q}}$ Gate

The $\mathcal{H}_{\hat{q}}$ gate is the Hadamard gate on qubit q. In order enter the template for this gate, press the keys:

[ESC]hg[ESC]

then press [TAB] to select the place-holder (empty square) and write the qubit number or label:

$\mathcal{H}_{\hat{1}}$

$\mathcal{H}_{\hat{1}}$

The matrix representation of the Hadamard gate:

```
QuantumMatrixForm[ℋ_1̂]
```

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

The bra-ket representation of the Hadamard gate:

```
QuantumEvaluate[ℋ_1̂]
```

$$\frac{|0_{\hat{1}}\rangle \cdot \langle 0_{\hat{1}}|}{\sqrt{2}} + \frac{|1_{\hat{1}}\rangle \cdot \langle 0_{\hat{1}}|}{\sqrt{2}} + \frac{|0_{\hat{1}}\rangle \cdot \langle 1_{\hat{1}}|}{\sqrt{2}} - \frac{|1_{\hat{1}}\rangle \cdot \langle 1_{\hat{1}}|}{\sqrt{2}}$$

This is the "truth table" for the Hadamard gate:

```
QuantumTableForm[ℋ_1̂]
```

	Input	Output
0	$\lvert 0_{\hat{1}}\rangle$	$\frac{\lvert 0_{\hat{1}}\rangle}{\sqrt{2}} + \frac{\lvert 1_{\hat{1}}\rangle}{\sqrt{2}}$
1	$\lvert 1_{\hat{1}}\rangle$	$\frac{\lvert 0_{\hat{1}}\rangle}{\sqrt{2}} - \frac{\lvert 1_{\hat{1}}\rangle}{\sqrt{2}}$

A Hadamard gate is usually combined with a controlled-not gate in order to generate Bell states:

```
QuantumTableForm[C^{1̂}[NOT_2̂] · H_1̂]
```

	Input	Output
0	$\mid 0_{\hat{1}}, 0_{\hat{2}}\rangle$	$\frac{\mid 0_{\hat{1}},0_{\hat{2}}\rangle}{\sqrt{2}} + \frac{\mid 1_{\hat{1}},1_{\hat{2}}\rangle}{\sqrt{2}}$
1	$\mid 0_{\hat{1}}, 1_{\hat{2}}\rangle$	$\frac{\mid 0_{\hat{1}},1_{\hat{2}}\rangle}{\sqrt{2}} + \frac{\mid 1_{\hat{1}},0_{\hat{2}}\rangle}{\sqrt{2}}$
2	$\mid 1_{\hat{1}}, 0_{\hat{2}}\rangle$	$\frac{\mid 0_{\hat{1}},0_{\hat{2}}\rangle}{\sqrt{2}} - \frac{\mid 1_{\hat{1}},1_{\hat{2}}\rangle}{\sqrt{2}}$
3	$\mid 1_{\hat{1}}, 1_{\hat{2}}\rangle$	$\frac{\mid 0_{\hat{1}},1_{\hat{2}}\rangle}{\sqrt{2}} - \frac{\mid 1_{\hat{1}},0_{\hat{2}}\rangle}{\sqrt{2}}$

The states generated by $C^{\{\hat{1}\}}[\mathcal{NOT}[\hat{2}]] \cdot \mathcal{H}[\hat{1}]$ are the Bell states:

```
Grid[QuantumEvaluate[{{ | B_{00,1̂,2̂}⟩}, { | B_{01,1̂,2̂}⟩}, { | B_{10,1̂,2̂}⟩}, { | B_{11,1̂,2̂}⟩}}],
 Dividers → All]
```

$\frac{\mid 0_{\hat{1}},0_{\hat{2}}\rangle}{\sqrt{2}} + \frac{\mid 1_{\hat{1}},1_{\hat{2}}\rangle}{\sqrt{2}}$
$\frac{\mid 0_{\hat{1}},1_{\hat{2}}\rangle}{\sqrt{2}} + \frac{\mid 1_{\hat{1}},0_{\hat{2}}\rangle}{\sqrt{2}}$
$\frac{\mid 0_{\hat{1}},0_{\hat{2}}\rangle}{\sqrt{2}} - \frac{\mid 1_{\hat{1}},1_{\hat{2}}\rangle}{\sqrt{2}}$
$\frac{\mid 0_{\hat{1}},1_{\hat{2}}\rangle}{\sqrt{2}} - \frac{\mid 1_{\hat{1}},0_{\hat{2}}\rangle}{\sqrt{2}}$

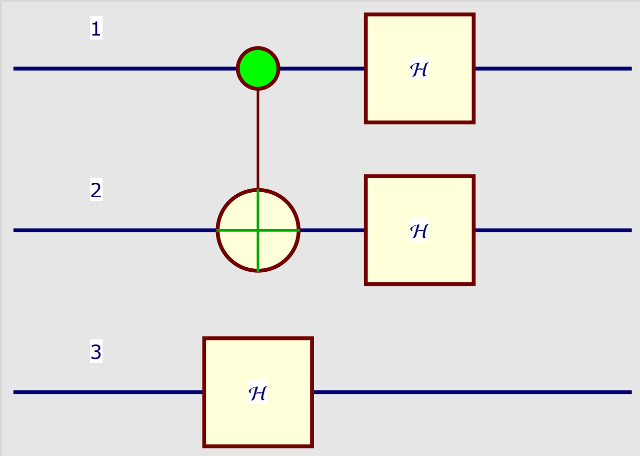
This is a "tensor power" of Hadamard gates:

$$(\mathcal{H}_{\hat{1}})^{\otimes 3}$$

$$\mathcal{H}_{\hat{1}} \cdot \mathcal{H}_{\hat{2}} \cdot \mathcal{H}_{\hat{3}}$$

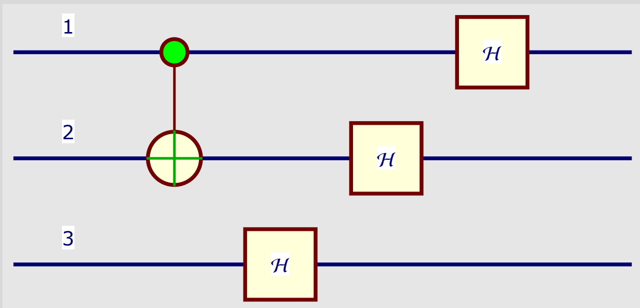
Notice that the Hadamard gates are not in a column:

```
QuantumPlot[(ℋ_1̂)^⊗3 · C^{1̂}[𝒩𝒪𝒯_2̂]]
```



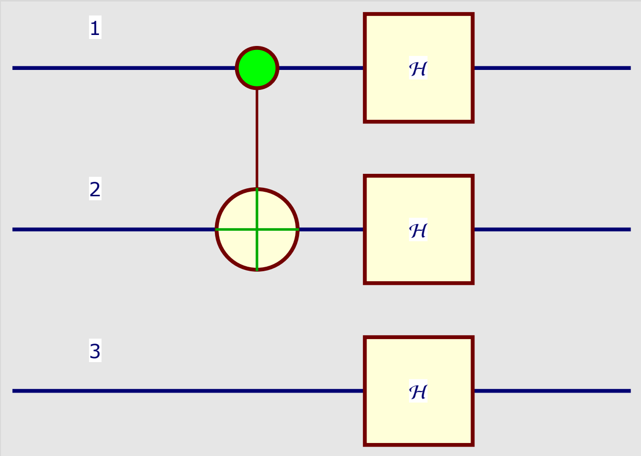
Notice that the Hadamard gates are not in a column:

```
QuantumPlot[(ℋ_1̂)^⊗3 · C^{1̂}[𝒩𝒪𝒯_2̂], QuantumGateShifting → False]
```



There is a notation that allows to have all the $\mathcal{H}$ gates in the same column.
Press the keys [ESC]hggg[ESC] for the template:

```
QuantumPlot[ℋ_{1̂,2̂,3̂} · C^{1̂}[NOT_2̂]]
```



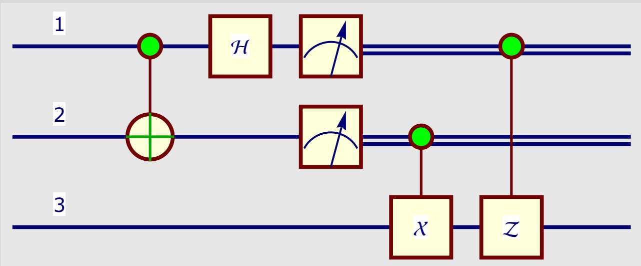
Both notations evaluate to the same Dirac expression:

```
QuantumEvaluate[(ℋ_1̂)^⊗3] == QuantumEvaluate[ℋ_{1̂,2̂,3̂}]
```

```
True
```

This is a more interesting circuit (Teleportation) that includes this gate:

```
QuantumPlot[C^{1̂}[Z_3̂] · C^{2̂}[X_3̂] · QubitMeasurement[ℋ_1̂ · C^{1̂}[NOT_2̂], {1̂, 2̂}],
 QuantumGateShifting → False]
```



This is the Hadamard gate in terms of Pauli operators:

```
PauliExpand[ℋ_1̂]
```

$$\frac{\sigma_{X,\hat{1}}}{\sqrt{2}} + \frac{\sigma_{Z,\hat{1}}}{\sqrt{2}}$$

Parametric Phase Gate $\mathcal{P}_{\hat{q}}[\alpha]$

The $\mathcal{P}_{\hat{\alpha}}$ gate is the parametric phase gate on qubit q. In order enter the template for this gate, press the keys:

[ESC]pg[ESC]

then press [TAB] to select the first place-holder (empty square) and write the qubit number or label. Press [TAB] again to select the remaining place-holder and write the parameter, for example t:

```
𝒫₁̂[t]
```

```
𝒫₁̂[t]
```

Matrix representation:

```
QuantumMatrixForm[𝒫₁̂[t]]
```

$$\begin{pmatrix} 1 & 0 \\ 0 & e^{i t} \end{pmatrix}$$

Dirac representation:

```
QuantumEvaluate[𝒫₁̂[t]]
```

$$|0_{\hat{1}}\rangle \cdot \langle 0_{\hat{1}}| + e^{i t}\ |1_{\hat{1}}\rangle \cdot \langle 1_{\hat{1}}|$$

A specific value for the parameter:

```
QuantumEvaluate[𝒫₁̂[π / 4]]
```

$$|0_{\hat{1}}\rangle \cdot \langle 0_{\hat{1}}| + \frac{(1 + i)\ |1_{\hat{1}}\rangle \cdot \langle 1_{\hat{1}}|}{\sqrt{2}}$$

The T-π/8 gate is a particular case of the parametric phase gate:

```
Simplify[𝒫₁̂[π / 4] == 𝒯₁̂]
```

```
True
```

The S-phase gate is another particular case of the parametric phase gate:

```
Simplify[𝒫₁̂[π / 2] == 𝒮₁̂]
```

```
True
```

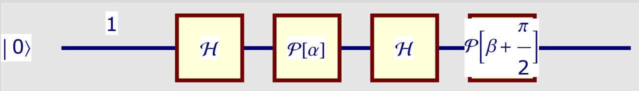
The Z-Pauli gate is another particular case of the parametric phase gate:

$$\text{Simplify}\left[\mathcal{P}_{\hat{1}}[\pi] == \mathcal{Z}_{\hat{1}}\right]$$

```
True
```

This simple quantum circuit can be used to generate an arbitrary superposition in a single qubit:

$$\text{QuantumPlot}\left[\mathcal{P}_{\hat{1}}[\beta + \pi / 2] \cdot \mathcal{H}_{\hat{1}} \cdot \mathcal{P}_{\hat{1}}[\alpha] \cdot \mathcal{H}_{\hat{1}} \cdot \;|0_{\hat{1}}\rangle\right]$$



Replace QuantumPlot with QuantumEvaluate in order to "simulate" the circuit operation:

$$\text{QuantumEvaluate}\left[\mathcal{P}_{\hat{1}}[\beta + \pi / 2] \cdot \mathcal{H}_{\hat{1}} \cdot \mathcal{P}_{\hat{1}}[\alpha] \cdot \mathcal{H}_{\hat{1}} \cdot \;|0_{\hat{1}}\rangle\right]$$

$$\left(\frac{1}{2} + \frac{e^{i\alpha}}{2}\right) \;|0_{\hat{1}}\rangle + \left(\frac{1}{2} e^{i\left(\frac{\pi}{2}+\beta\right)} - \frac{1}{2} e^{i\alpha + i\left(\frac{\pi}{2}+\beta\right)}\right) \;|1_{\hat{1}}\rangle$$

Press [ESC]pgg[ESC] to get the template that represents two phase gates with the same parameter:

$$\text{QuantumEvaluate}\left[\mathcal{P}_{\{\hat{1},\hat{2}\}}[\pi / 3]\right]$$

$$|0_{\hat{1}}, 0_{\hat{2}}\rangle \cdot \langle 0_{\hat{1}}, 0_{\hat{2}}| + \frac{1}{2}\;|0_{\hat{1}}, 1_{\hat{2}}\rangle \cdot \langle 0_{\hat{1}}, 1_{\hat{2}}| + \frac{1}{2}\, i\sqrt{3}\;|0_{\hat{1}}, 1_{\hat{2}}\rangle \cdot \langle 0_{\hat{1}}, 1_{\hat{2}}| + \frac{1}{2}\;|1_{\hat{1}}, 0_{\hat{2}}\rangle \cdot \langle 1_{\hat{1}}, 0_{\hat{2}}| + \frac{1}{2}\, i\sqrt{3}\;|1_{\hat{1}}, 0_{\hat{2}}\rangle \cdot \langle 1_{\hat{1}}, 0_{\hat{2}}| - \frac{1}{2}\;|1_{\hat{1}}, 1_{\hat{2}}\rangle \cdot \langle 1_{\hat{1}}, 1_{\hat{2}}| + \frac{1}{2}\, i\sqrt{3}\;|1_{\hat{1}}, 1_{\hat{2}}\rangle \cdot \langle 1_{\hat{1}}, 1_{\hat{2}}|$$

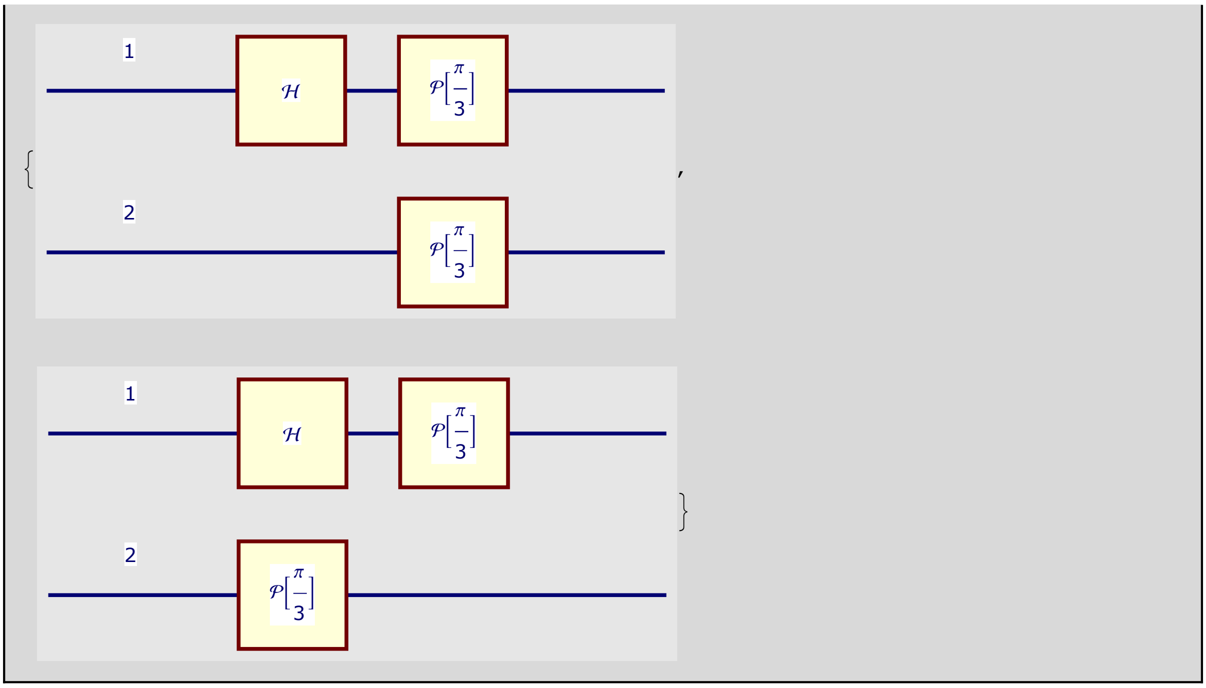
The two gates below, wich have the same parameter, represent the same as above:

$$\text{QuantumEvaluate}\left[\mathcal{P}_{\hat{1}}[\pi / 3] \cdot \mathcal{P}_{\hat{2}}[\pi / 3]\right]$$

$$|0_{\hat{1}}, 0_{\hat{2}}\rangle \cdot \langle 0_{\hat{1}}, 0_{\hat{2}}| + \frac{1}{2}\;|0_{\hat{1}}, 1_{\hat{2}}\rangle \cdot \langle 0_{\hat{1}}, 1_{\hat{2}}| + \frac{1}{2}\, i\sqrt{3}\;|0_{\hat{1}}, 1_{\hat{2}}\rangle \cdot \langle 0_{\hat{1}}, 1_{\hat{2}}| + \frac{1}{2}\;|1_{\hat{1}}, 0_{\hat{2}}\rangle \cdot \langle 1_{\hat{1}}, 0_{\hat{2}}| + \frac{1}{2}\, i\sqrt{3}\;|1_{\hat{1}}, 0_{\hat{2}}\rangle \cdot \langle 1_{\hat{1}}, 0_{\hat{2}}| - \frac{1}{2}\;|1_{\hat{1}}, 1_{\hat{2}}\rangle \cdot \langle 1_{\hat{1}}, 1_{\hat{2}}| + \frac{1}{2}\, i\sqrt{3}\;|1_{\hat{1}}, 1_{\hat{2}}\rangle \cdot \langle 1_{\hat{1}}, 1_{\hat{2}}|$$

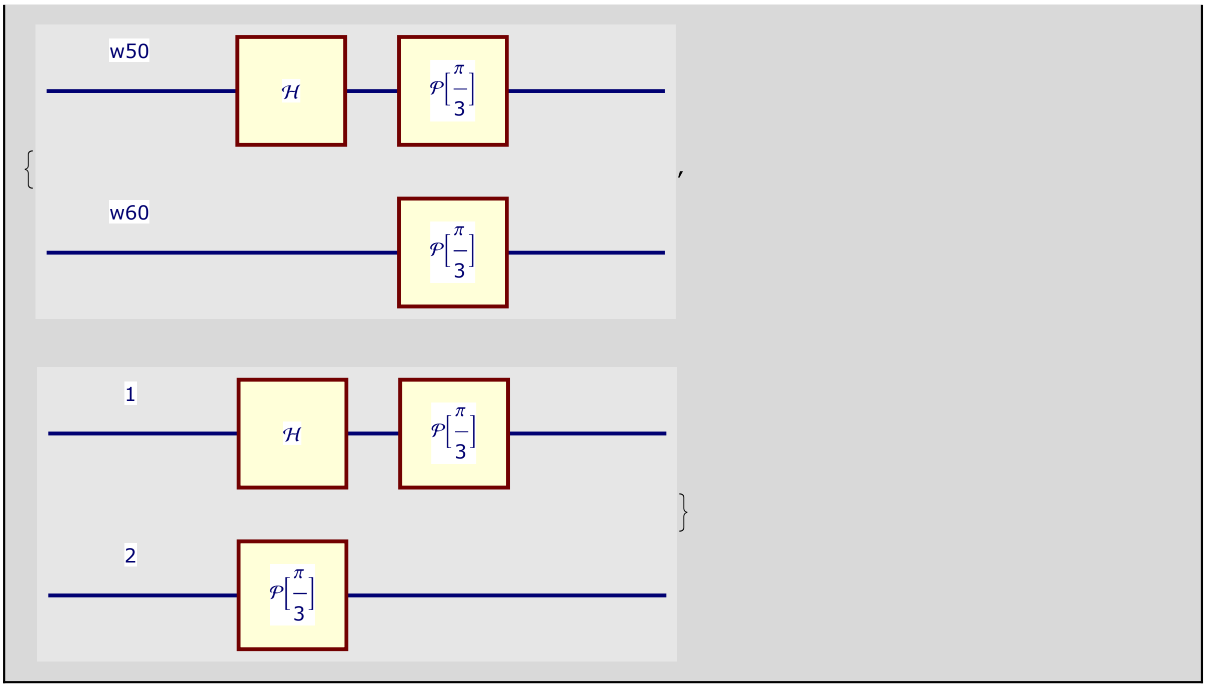
The main use of the [ESC]pgg[ESC] template is to create circuit plots where both pahse gates are placed in the same column:

$\{\mathtt{QuantumPlot}[\mathcal{P}_{\{\hat{1},\hat{2}\}}[\pi/3]\cdot\mathcal{H}_{\hat{1}}],\ \mathtt{QuantumPlot}[\mathcal{P}_{\hat{1}}[\pi/3]\cdot\mathcal{P}_{\hat{2}}[\pi/3]\cdot\mathcal{H}_{\hat{1}}]\}$



The "quantum register" (press [ESC]qr[ESC]) notation has the same effect, with some advantages in the flexibility to name the qubits that make the register:

$\{\mathtt{QuantumPlot}[\mathcal{P}_{\mathcal{R}egister[50,60,10,w]}[\pi/3]\cdot\mathcal{H}_{\widehat{w50}}],\ \mathtt{QuantumPlot}[\mathcal{P}_{\hat{1}}[\pi/3]\cdot\mathcal{P}_{\hat{2}}[\pi/3]\cdot\mathcal{H}_{\hat{1}}]\}$



$\mathcal{S}_{\hat{q}}$ Gate

The $\mathcal{S}_{\hat{q}}$ gate is the S phase gate on qubit q. In order enter the template for this gate, press the keys:

[ESC]sg[ESC]

then press [TAB] to select the place-holder (empty square) and write the qubit number or label:

$\mathcal{S}_{\hat{1}}$

$\mathcal{S}_{\hat{1}}$

Truth-table:

```
QuantumTableForm[S_1̂]
```

	Input	Output
0	$\vert 0_{\hat{1}}\rangle$	$\vert 0_{\hat{1}}\rangle$
1	$\vert 1_{\hat{1}}\rangle$	$i \vert 1_{\hat{1}}\rangle$

Matrix representation

```
QuantumMatrixForm[S_1̂]
```

$$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

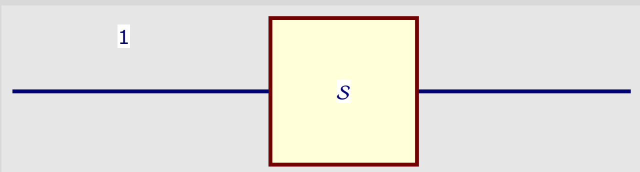
Bra-ket representation:

```
QuantumEvaluate[S_1̂]
```

$$\vert 0_{\hat{1}}\rangle \cdot \langle 0_{\hat{1}}\vert + i \vert 1_{\hat{1}}\rangle \cdot \langle 1_{\hat{1}}\vert$$

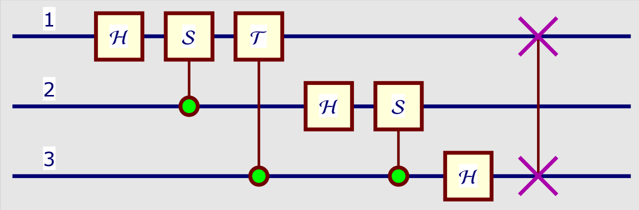
Gate in a circuit:

```
QuantumPlot[S_1̂]
```



This "Quantum Fourier Transform" for three qubits includes (a controlled) $\mathcal{S}$ gate:

QuantumPlot$\left[\mathcal{SWAP}_{\hat{1},\hat{3}} \cdot \mathcal{H}_{\hat{3}} \cdot C^{\{\hat{3}\}}\left[\mathcal{S}_{\hat{2}}\right] \cdot \mathcal{H}_{\hat{2}} \cdot C^{\{\hat{3}\}}\left[\mathcal{T}_{\hat{1}}\right] \cdot C^{\{\hat{2}\}}\left[\mathcal{S}_{\hat{1}}\right] \cdot \mathcal{H}_{\hat{1}}\right]$



The $\mathcal{S}_{\hat{1}}$ gate transformed to Pauli operators.

PauliExpand$\left[\mathcal{S}_{\hat{1}}\right]$

$$\left(\frac{1}{2} + \frac{i}{2}\right) \sigma_{0,\hat{1}} + \left(\frac{1}{2} - \frac{i}{2}\right) \sigma_{\mathcal{Z},\hat{1}}$$

$\mathcal{T}_{\hat{q}}$ Gate

The $\mathcal{T}_{\hat{q}}$ gate is the $\pi/8$ gate on qubit q. In order enter the template for this gate, press the keys:

[ESC]tg[ESC]

then press [TAB] to select the place-holder (empty square) and write the qubit number or label:

$\mathcal{T}_{\hat{1}}$

$\mathcal{T}_{\hat{1}}$

Truth-table:

QuantumTableForm$\left[\mathcal{T}_{\hat{1}}\right]$

	Input	Output
0	$\lvert 0_{\hat{1}}\rangle$	$\lvert 0_{\hat{1}}\rangle$
1	$\lvert 1_{\hat{1}}\rangle$	$\frac{(1+i)\,\lvert 1_{\hat{1}}\rangle}{\sqrt{2}}$

The fourth-power of the $\pi/8$ gate is the $\mathcal{Z}_{\hat{q}}$ gate:

$\{$**QuantumTableForm**$\left[\mathcal{T}_{\hat{1}}^{4}\right]$, **QuantumTableForm**$\left[\mathcal{Z}_{\hat{1}}\right]\}$

	Input	Output
0	$\lvert 0_{\hat{1}}\rangle$	$\lvert 0_{\hat{1}}\rangle$
1	$\lvert 1_{\hat{1}}\rangle$	$-\lvert 1_{\hat{1}}\rangle$

	Input	Output
0	$\lvert 0_{\hat{1}}\rangle$	$\lvert 0_{\hat{1}}\rangle$
1	$\lvert 1_{\hat{1}}\rangle$	$-\lvert 1_{\hat{1}}\rangle$

The fourth-power of the $\pi/8$ gate is the $\mathcal{Z}_{\hat{q}}$ gate:

PauliExpand$\left[\mathcal{T}_{\hat{1}}{}^{4}\right]$

$\sigma_{z,\hat{1}}$

Matrix representation:

QuantumMatrixForm$\left[\mathcal{T}_{\hat{1}}\right]$

$$\begin{pmatrix} 1 & 0 \\ 0 & \frac{1+i}{\sqrt{2}} \end{pmatrix}$$

Numerical matrix representation:

N$\left[\right.$**QuantumMatrixForm**$\left[\mathcal{T}_{\hat{1}}\right]\left.\right]$

$$\begin{pmatrix} 1. & 0. \\ 0. & 0.707107 + 0.707107\, i \end{pmatrix}$$

Bra-ket representation:

QuantumEvaluate$\left[\mathcal{T}_{\hat{1}}\right]$

$$\left| 0_{\hat{1}} \right\rangle \cdot \left\langle 0_{\hat{1}} \right| + \frac{(1+i)\ \left| 1_{\hat{1}} \right\rangle \cdot \left\langle 1_{\hat{1}} \right|}{\sqrt{2}}$$

This "Quantum Fourier Transform" for three qubits includes this gate. The evaluation of this command can take several seconds before showing the result:

QuantumTableForm$\left[\mathcal{SWAP}_{\hat{1},\hat{3}} \cdot \mathcal{H}_{\hat{3}} \cdot C^{\{\hat{3}\}}\left[\mathcal{S}_{\hat{2}}\right] \cdot \mathcal{H}_{\hat{2}} \cdot C^{\{\hat{3}\}}\left[\mathcal{T}_{\hat{1}}\right] \cdot C^{\{\hat{2}\}}\left[\mathcal{S}_{\hat{1}}\right] \cdot \mathcal{H}_{\hat{1}}\right]$

	Input	Output
0	$\lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}\rangle$	$\frac{\lvert 0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} + \frac{\lvert 0_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle}{2\sqrt{2}} + \frac{\lvert 0_{\hat{1}},1_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} + \frac{\lvert 0_{\hat{1}},1_{\hat{2}},1_{\hat{3}}\rangle}{2\sqrt{2}} + \frac{\lvert 1_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} + \frac{\lvert 1_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle}{2\sqrt{2}} + \frac{\lvert 1_{\hat{1}},1_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} + \lvert 1$
1	$\lvert 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}\rangle$	$\frac{\lvert 0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} + \left(\frac{1}{4}+\frac{i}{4}\right) \lvert 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}\rangle + \frac{i \lvert 0_{\hat{1}},1_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} - \left(\frac{1}{4}-\frac{i}{4}\right) \lvert 0_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}\rangle - \frac{\lvert 1_{\hat{1}},0_{\hat{2}},}{2\sqrt{2}}$
2	$\lvert 0_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}\rangle$	$\frac{\lvert 0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} + \frac{i \lvert 0_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle}{2\sqrt{2}} - \frac{\lvert 0_{\hat{1}},1_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} - \frac{i \lvert 0_{\hat{1}},1_{\hat{2}},1_{\hat{3}}\rangle}{2\sqrt{2}} + \frac{\lvert 1_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} + \frac{i \lvert 1_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle}{2\sqrt{2}} - \frac{\lvert 1_{\hat{1}},1_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}}$
3	$\lvert 0_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}\rangle$	$\frac{\lvert 0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} - \left(\frac{1}{4}-\frac{i}{4}\right) \lvert 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}\rangle - \frac{i \lvert 0_{\hat{1}},1_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} + \left(\frac{1}{4}+\frac{i}{4}\right) \lvert 0_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}\rangle - \frac{\lvert 1_{\hat{1}},0_{\hat{2}},}{2\sqrt{2}}$
4	$\lvert 1_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}\rangle$	$\frac{\lvert 0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} - \frac{\lvert 0_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle}{2\sqrt{2}} + \frac{\lvert 0_{\hat{1}},1_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} - \frac{\lvert 0_{\hat{1}},1_{\hat{2}},1_{\hat{3}}\rangle}{2\sqrt{2}} + \frac{\lvert 1_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} - \frac{\lvert 1_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle}{2\sqrt{2}} + \frac{\lvert 1_{\hat{1}},1_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} - \lvert 1$
5	$\lvert 1_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}\rangle$	$\frac{\lvert 0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} - \left(\frac{1}{4}+\frac{i}{4}\right) \lvert 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}\rangle + \frac{i \lvert 0_{\hat{1}},1_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} + \left(\frac{1}{4}-\frac{i}{4}\right) \lvert 0_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}\rangle - \frac{\lvert 1_{\hat{1}},0_{\hat{2}},}{2\sqrt{2}}$
6	$\lvert 1_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}\rangle$	$\frac{\lvert 0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} - \frac{i \lvert 0_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle}{2\sqrt{2}} - \frac{\lvert 0_{\hat{1}},1_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} + \frac{i \lvert 0_{\hat{1}},1_{\hat{2}},1_{\hat{3}}\rangle}{2\sqrt{2}} + \frac{\lvert 1_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} - \frac{i \lvert 1_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle}{2\sqrt{2}} - \frac{\lvert 1_{\hat{1}},1_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}}$
7	$\lvert 1_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}\rangle$	$\frac{\lvert 0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} + \left(\frac{1}{4}-\frac{i}{4}\right) \lvert 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}\rangle - \frac{i \lvert 0_{\hat{1}},1_{\hat{2}},0_{\hat{3}}\rangle}{2\sqrt{2}} - \left(\frac{1}{4}+\frac{i}{4}\right) \lvert 0_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}\rangle - \frac{\lvert 1_{\hat{1}},0_{\hat{2}},}{2\sqrt{2}}$

$\mathcal{SWAP}_{\hat{q1},\hat{q2}}$ Gate

The $\mathcal{SWAP}_{\hat{q1},\hat{q2}}$ gate is the swap gate on qubits q1, q2. In order enter the template for this gate, press the keys:
[ESC]swap[ESC]
then press [TAB] to select the place-holders (empty square) and write the qubit numbers or labels:

$\mathcal{SWAP}_{\hat{1},\hat{2}}$

$\mathcal{SWAP}_{\hat{1},\hat{2}}$

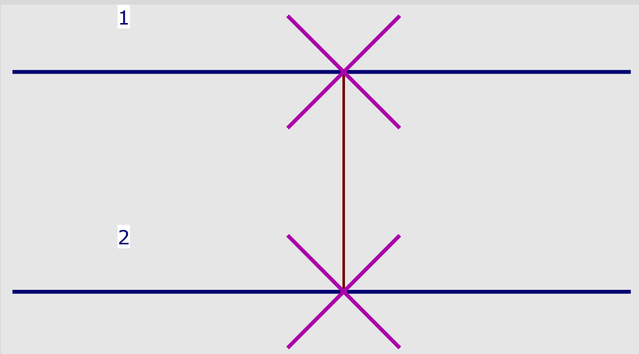
Truth table:

```
QuantumTableForm[SWAP_{1̂,2̂}]
```

	Input	Output
0	$\mid 0_{\hat{1}}, 0_{\hat{2}}\rangle$	$\mid 0_{\hat{1}}, 0_{\hat{2}}\rangle$
1	$\mid 0_{\hat{1}}, 1_{\hat{2}}\rangle$	$\mid 1_{\hat{1}}, 0_{\hat{2}}\rangle$
2	$\mid 1_{\hat{1}}, 0_{\hat{2}}\rangle$	$\mid 0_{\hat{1}}, 1_{\hat{2}}\rangle$
3	$\mid 1_{\hat{1}}, 1_{\hat{2}}\rangle$	$\mid 1_{\hat{1}}, 1_{\hat{2}}\rangle$

Plot of the swap gate

```
QuantumPlot[SWAP_{1̂,2̂}]
```



Bra-ket notation:

```
QuantumEvaluate[SWAP_{1̂,2̂}]
```

$$\mid 0_{\hat{1}}, 0_{\hat{2}}\rangle \cdot \langle 0_{\hat{1}}, 0_{\hat{2}} \mid + \mid 1_{\hat{1}}, 0_{\hat{2}}\rangle \cdot \langle 0_{\hat{1}}, 1_{\hat{2}} \mid + \mid 0_{\hat{1}}, 1_{\hat{2}}\rangle \cdot \langle 1_{\hat{1}}, 0_{\hat{2}} \mid + \mid 1_{\hat{1}}, 1_{\hat{2}}\rangle \cdot \langle 1_{\hat{1}}, 1_{\hat{2}} \mid$$

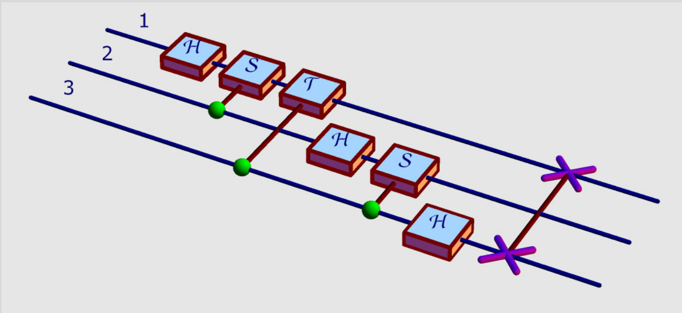
TraditionalForm Bra-ket notation:

```
TraditionalForm[QuantumEvaluate[𝒮𝒲𝒜𝒫_{1̂,2̂}]]
```

$|00\rangle\langle 00| + |10\rangle\langle 01| + |01\rangle\langle 10| + |11\rangle\langle 11|$

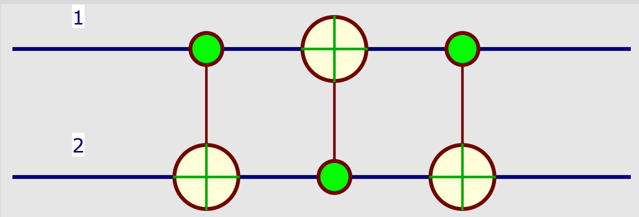
This "Quantum Fourier Transform" for three qubits includes the gate:

```
QuantumPlot3D[𝒮𝒲𝒜𝒫_{1̂,3̂} · ℋ_{3̂} · 𝒞^{3̂}[𝒮_{2̂}] · ℋ_{2̂} · 𝒞^{3̂}[𝒯_{1̂}] · 𝒞^{2̂}[𝒮_{1̂}] · ℋ_{1̂}]
```

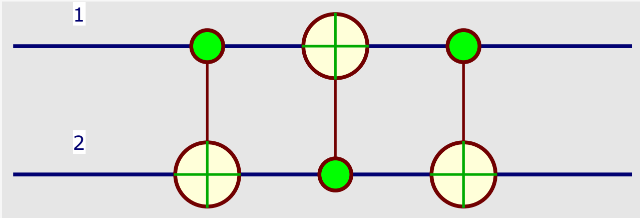


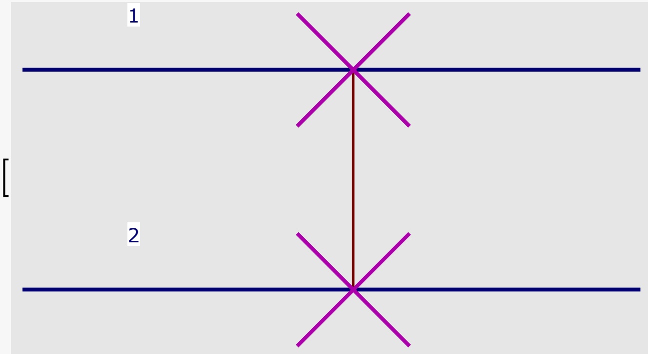
Next circuit is equivalent to a Swap gate:

```
QuantumPlot[𝒞^{1̂}[𝒩𝒪𝒯_{2̂}] · 𝒞^{2̂}[𝒩𝒪𝒯_{1̂}] · 𝒞^{1̂}[𝒩𝒪𝒯_{2̂}]]
```



Next circuit is equivalent to a Swap gate:

```
QuantumEvaluate[
```



```
] ==
QuantumEvaluate[
```



```
]
```

```
True
```

$C^{\{\hat{q1}\}}\left[\mathcal{NOT}_{\hat{q2}}\right]$ Gate

The $C^{\{\hat{q1}\}}\left[\mathcal{NOT}_{\hat{q2}}\right]$ gate is the control-not gate on qubits q1, q2. In order enter the template for this gate, press the keys:
[ESC]cnot[ESC]
then press [TAB] to select the place-holders (empty square) and write the qubit numbers or labels:

$C^{\{\hat{1}\}}\left[\mathcal{NOT}_{\hat{2}}\right]$

$C^{\{\hat{1}\}}\left[\mathcal{NOT}_{\hat{2}}\right]$

The matrix representation

QuantumMatrixForm$\left[C^{\{\hat{1}\}}\left[\mathcal{NOT}_{\hat{2}}\right]\right]$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

The tensor representation

```
QuantumTensorForm[C^{1̂}[NOT_2̂]]
```

$$\begin{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{pmatrix}$$

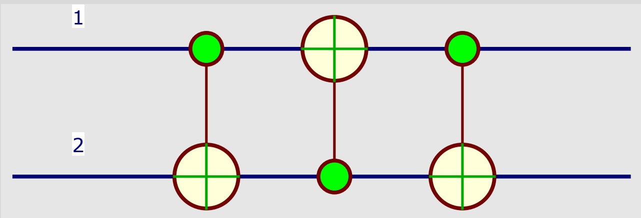
The truth table

```
QuantumTableForm[C^{1̂}[NOT_2̂]]
```

	Input	Output
0	$\lvert 0_{\hat{1}}, 0_{\hat{2}} \rangle$	$\lvert 0_{\hat{1}}, 0_{\hat{2}} \rangle$
1	$\lvert 0_{\hat{1}}, 1_{\hat{2}} \rangle$	$\lvert 0_{\hat{1}}, 1_{\hat{2}} \rangle$
2	$\lvert 1_{\hat{1}}, 0_{\hat{2}} \rangle$	$\lvert 1_{\hat{1}}, 1_{\hat{2}} \rangle$
3	$\lvert 1_{\hat{1}}, 1_{\hat{2}} \rangle$	$\lvert 1_{\hat{1}}, 0_{\hat{2}} \rangle$

Next circuit is equivalent to a Swap gate:

```
QuantumPlot[C^{1̂}[NOT_2̂] · C^{2̂}[NOT_1̂] · C^{1̂}[NOT_2̂]]
```



Next circuit is equivalent to a Swap gate:

```
QuantumTableForm[C^{1̂}[NOT_2̂] · C^{2̂}[NOT_1̂] · C^{1̂}[NOT_2̂]]
```

	Input	Output
0	$\lvert 0_{\hat{1}}, 0_{\hat{2}} \rangle$	$\lvert 0_{\hat{1}}, 0_{\hat{2}} \rangle$
1	$\lvert 0_{\hat{1}}, 1_{\hat{2}} \rangle$	$\lvert 1_{\hat{1}}, 0_{\hat{2}} \rangle$
2	$\lvert 1_{\hat{1}}, 0_{\hat{2}} \rangle$	$\lvert 0_{\hat{1}}, 1_{\hat{2}} \rangle$
3	$\lvert 1_{\hat{1}}, 1_{\hat{2}} \rangle$	$\lvert 1_{\hat{1}}, 1_{\hat{2}} \rangle$

$\mathcal{TOFFOLI}_{\hat{q1},\hat{q2},\hat{q3}}$ Gate

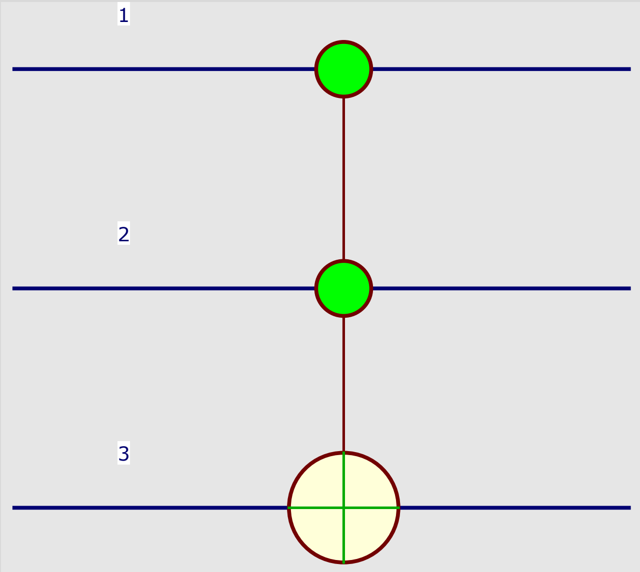
The $\mathcal{TOFFOLI}_{\hat{q1},\hat{q2},\hat{q3}}$ gate is the Toffoli gate (control-control-not) on qubits q1, q2, q3. In order enter the template for this gate, press the keys:
[ESC]toff[ESC]
then press [TAB] to select the place-holders (empty square) and write the qubit numbers or labels:

$\mathcal{TOFFOLI}_{\hat{1},\hat{2},\hat{3}}$

$C^{\{\hat{1},\hat{2}\}}[\mathcal{NOT}_{\hat{3}}]$

A Toffoli gate is actually a control-contol-not:

QuantumPlot$[\mathcal{TOFFOLI}_{\hat{1},\hat{2},\hat{3}}]$



Truth table:

QuantumTableForm$[\mathcal{TOFFOLI}_{\hat{1},\hat{2},\hat{3}}]$

	Input	Output
0	$\vert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}\rangle$	$\vert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}\rangle$
1	$\vert 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}\rangle$	$\vert 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}\rangle$
2	$\vert 0_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}\rangle$	$\vert 0_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}\rangle$
3	$\vert 0_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}\rangle$	$\vert 0_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}\rangle$
4	$\vert 1_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}\rangle$	$\vert 1_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}\rangle$
5	$\vert 1_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}\rangle$	$\vert 1_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}\rangle$
6	$\vert 1_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}\rangle$	$\vert 1_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}\rangle$
7	$\vert 1_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}\rangle$	$\vert 1_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}\rangle$

This is the Toffoli gate in terms of Pauli operators:

```
PauliExpand[TOFFOLI_{1̂,2̂,3̂}]
```

$$\frac{3}{4}\sigma_{0,\hat{1}}\cdot\sigma_{0,\hat{2}}\cdot\sigma_{0,\hat{3}}+\frac{1}{4}\sigma_{z,\hat{1}}\cdot\sigma_{0,\hat{2}}\cdot\sigma_{0,\hat{3}}+\frac{1}{4}\sigma_{0,\hat{1}}\cdot\sigma_{z,\hat{2}}\cdot\sigma_{0,\hat{3}}-\frac{1}{4}\sigma_{z,\hat{1}}\cdot\sigma_{z,\hat{2}}\cdot\sigma_{0,\hat{3}}+\frac{1}{4}\sigma_{0,\hat{1}}\cdot\sigma_{0,\hat{2}}\cdot\sigma_{x,\hat{3}}-\frac{1}{4}\sigma_{z,\hat{1}}\cdot\sigma_{0,\hat{2}}\cdot\sigma_{x,\hat{3}}-\frac{1}{4}\sigma_{0,\hat{1}}\cdot\sigma_{z,\hat{2}}\cdot\sigma_{x,\hat{3}}+\frac{1}{4}\sigma_{z,\hat{1}}\cdot\sigma_{z,\hat{2}}\cdot\sigma_{x,\hat{3}}$$

$\mathcal{FREDKIN}_{\hat{q1},\hat{q2},\hat{q3}}$ Gate

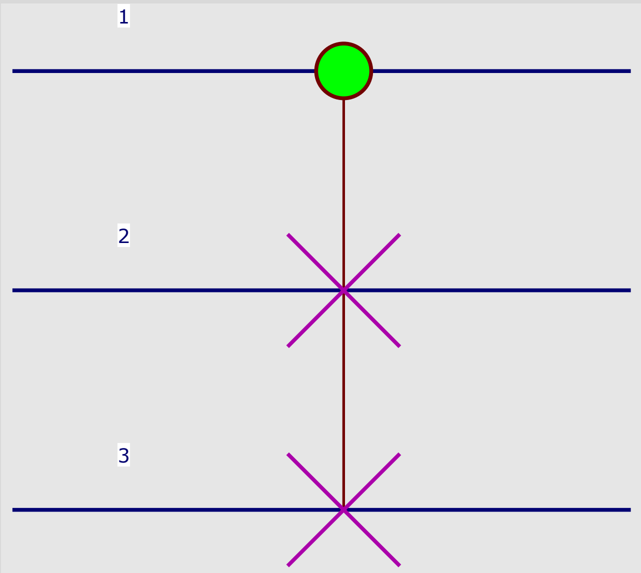
The $\mathcal{FREDKIN}_{\hat{q1},\hat{q2},\hat{q3}}$ gate is the Fredkin gate (control-swap) on qubits q1, q2, q3. In order enter the template for this gate, press the keys:
[ESC]fred[ESC]
then press [TAB] to select the place-holders (empty square) and write the qubit numbers or labels:

```
FREDKIN_{1̂,2̂,3̂}
```

$$C^{\{\hat{1}\}}\left[\mathcal{SWAP}_{\hat{2},\hat{3}}\right]$$

A Fredkin gate is actually a control-swap:

```
QuantumPlot[FREDKIN_{1̂,2̂,3̂}]
```



Truth table:

```
QuantumTableForm[𝓕𝓡𝓔𝓓𝓚𝓘𝓝_{1̂,2̂,3̂}]
```

	Input	Output
0	$\lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}\rangle$	$\lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}\rangle$
1	$\lvert 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}\rangle$	$\lvert 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}\rangle$
2	$\lvert 0_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}\rangle$	$\lvert 0_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}\rangle$
3	$\lvert 0_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}\rangle$	$\lvert 0_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}\rangle$
4	$\lvert 1_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}\rangle$	$\lvert 1_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}\rangle$
5	$\lvert 1_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}\rangle$	$\lvert 1_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}\rangle$
6	$\lvert 1_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}\rangle$	$\lvert 1_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}\rangle$
7	$\lvert 1_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}\rangle$	$\lvert 1_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}\rangle$

This is the Toffoli gate in terms of Pauli operators:

```
PauliExpand[𝓕𝓡𝓔𝓓𝓚𝓘𝓝_{1̂,2̂,3̂}]
```

$$\frac{3}{4}\sigma_{0,\hat{1}}\cdot\sigma_{0,\hat{2}}\cdot\sigma_{0,\hat{3}}+\frac{1}{4}\sigma_{z,\hat{1}}\cdot\sigma_{0,\hat{2}}\cdot\sigma_{0,\hat{3}}+\frac{1}{4}\sigma_{0,\hat{1}}\cdot\sigma_{x,\hat{2}}\cdot\sigma_{x,\hat{3}}-\frac{1}{4}\sigma_{z,\hat{1}}\cdot\sigma_{x,\hat{2}}\cdot\sigma_{x,\hat{3}}+\frac{1}{4}\sigma_{0,\hat{1}}\cdot\sigma_{y,\hat{2}}\cdot\sigma_{y,\hat{3}}-\frac{1}{4}\sigma_{z,\hat{1}}\cdot\sigma_{y,\hat{2}}\cdot\sigma_{y,\hat{3}}+\frac{1}{4}\sigma_{0,\hat{1}}\cdot\sigma_{z,\hat{2}}\cdot\sigma_{z,\hat{3}}-\frac{1}{4}\sigma_{z,\hat{1}}\cdot\sigma_{z,\hat{2}}\cdot\sigma_{z,\hat{3}}$$

TraditionalForm gives a format closer to the format used in papers and textbooks:

```
TraditionalForm[PauliExpand[𝓕𝓡𝓔𝓓𝓚𝓘𝓝_{1̂,2̂,3̂}]]
```

$$-\frac{1}{4}\sigma_1^z\sigma_2^x\sigma_3^x+\frac{1}{4}\sigma_1^0\sigma_2^x\sigma_3^x-\frac{1}{4}\sigma_1^z\sigma_2^y\sigma_3^y+\frac{1}{4}\sigma_1^0\sigma_2^y\sigma_3^y+\frac{1}{4}\sigma_1^z\sigma_2^0\sigma_3^0+\frac{1}{4}\sigma_1^0\sigma_2^z\sigma_3^z-\frac{1}{4}\sigma_1^z\sigma_2^z\sigma_3^z+\frac{3}{4}\sigma_1^0\sigma_2^0\sigma_3^0$$

Controlled Gates

Truth table for an arbitrary controlled-gate. Press [ESC]cgate[ESC] for the controlled-gate template $C^{\{\hat{\square}\}}[\square]$ and [ESC]qg[ESC] for the quantum gate template $\square_{\hat{\square}}$:

```
SetQuantumGate[mygate, 1];
QuantumTableForm[C^{1̂}[mygate_2̂]]
```

	Input	Output
0	$\lvert 0_{\hat{1}}, 0_{\hat{2}}\rangle$	$\lvert 0_{\hat{1}}, 0_{\hat{2}}\rangle$
1	$\lvert 0_{\hat{1}}, 1_{\hat{2}}\rangle$	$\lvert 0_{\hat{1}}, 1_{\hat{2}}\rangle$
2	$\lvert 1_{\hat{1}}, 0_{\hat{2}}\rangle$	$\lvert 1_{\hat{1}}\rangle\cdot \mathrm{mygate}_{\hat{2}}\cdot\lvert 0_{\hat{2}}\rangle$
3	$\lvert 1_{\hat{1}}, 1_{\hat{2}}\rangle$	$\lvert 1_{\hat{1}}\rangle\cdot \mathrm{mygate}_{\hat{2}}\cdot\lvert 1_{\hat{2}}\rangle$

Matrix representation for an arbitrary controlled-gate. Press [ESC]cgate[ESC] for the controlled-gate template:

```
SetQuantumGate[mygate, 1];
QuantumMatrixForm[C^{1̂}[mygate_2̂]]
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \langle 0_{\hat{2}} \mid \cdot\, \text{mygate}_{\hat{2}} \cdot \mid 0_{\hat{2}} \rangle & \langle 0_{\hat{2}} \mid \cdot\, \text{mygate}_{\hat{2}} \cdot \mid 1_{\hat{2}} \rangle \\ 0 & 0 & \langle 1_{\hat{2}} \mid \cdot\, \text{mygate}_{\hat{2}} \cdot \mid 0_{\hat{2}} \rangle & \langle 1_{\hat{2}} \mid \cdot\, \text{mygate}_{\hat{2}} \cdot \mid 1_{\hat{2}} \rangle \end{pmatrix}$$

Bra-ket representation for an arbitrary controlled-gate. Press [ESC]cgate[ESC] for the controlled-gate template:

```
SetQuantumGate[mygate, 1];
QuantumEvaluate[C^{1̂}[mygate_2̂]]
```

$$\mid 0_{\hat{1}}, 0_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 0_{\hat{2}} \mid + \mid 0_{\hat{1}}, 1_{\hat{2}} \rangle \cdot \langle 0_{\hat{1}}, 1_{\hat{2}} \mid + \mid 1_{\hat{1}} \rangle \cdot \text{mygate}_{\hat{2}} \cdot \langle 1_{\hat{1}} \mid$$

Quantum Fourier Transform $\mathcal{QFT}$

Press [ESC]qft[ESC] for the template of the one-qubit Quantum Fourier Transform

```
QFT_1̂
```

$\mathcal{QFT}_{\hat{1}}$

Its truth table:

```
QuantumTableForm[QFT_1̂]
```

	Input	Output
0	$\mid 0_{\hat{1}} \rangle$	$0.707107 \mid 0_{\hat{1}} \rangle + 0.707107 \mid 1_{\hat{1}} \rangle$
1	$\mid 1_{\hat{1}} \rangle$	$0.707107 \mid 0_{\hat{1}} \rangle - 0.707107 \mid 1_{\hat{1}} \rangle$

Press [ESC]qqft[ESC] for the template of the two-qubits Quantum Fourier Transform. **Scroll to the right in order to see the full answer:**

```
QuantumTableForm[QFT_1̂,2̂]
```

	Input	Output
0	$\mid 0_{\hat{1}}, 0_{\hat{2}} \rangle$	$0.5 \mid 0_{\hat{1}}, 0_{\hat{2}} \rangle + 0.5 \mid 0_{\hat{1}}, 1_{\hat{2}} \rangle + 0.5 \mid 1_{\hat{1}}, 0_{\hat{2}} \rangle + 0.5 \mid 1_{\hat{1}}, 1_{\hat{2}} \rangle$
1	$\mid 0_{\hat{1}}, 1_{\hat{2}} \rangle$	$0.5 \mid 0_{\hat{1}}, 0_{\hat{2}} \rangle + 0.5\, i \mid 0_{\hat{1}}, 1_{\hat{2}} \rangle - 0.5 \mid 1_{\hat{1}}, 0_{\hat{2}} \rangle - 0.5\, i \mid 1_{\hat{1}}, 1_{\hat{2}} \rangle$
2	$\mid 1_{\hat{1}}, 0_{\hat{2}} \rangle$	$0.5 \mid 0_{\hat{1}}, 0_{\hat{2}} \rangle - 0.5 \mid 0_{\hat{1}}, 1_{\hat{2}} \rangle + 0.5 \mid 1_{\hat{1}}, 0_{\hat{2}} \rangle - 0.5 \mid 1_{\hat{1}}, 1_{\hat{2}} \rangle$
3	$\mid 1_{\hat{1}}, 1_{\hat{2}} \rangle$	$0.5 \mid 0_{\hat{1}}, 0_{\hat{2}} \rangle - 0.5\, i \mid 0_{\hat{1}}, 1_{\hat{2}} \rangle - 0.5 \mid 1_{\hat{1}}, 0_{\hat{2}} \rangle + 0.5\, i \mid 1_{\hat{1}}, 1_{\hat{2}} \rangle$

The Hermitian Conjugate of any gate is its inverse, because by definition all gates are unitary (they preserve the norm of the state kets). Therefore, the Hermitian of the Quantum Fourier Transform is the inverserse transform, as you can see by comparing the result of the calculation below with the last row of the table above. Press [ESC]her[ESC] for the Hermitian Conjugate template $(\square)^{\dagger}$, and press [ESC]qqft[ESC] for the template of the two-qubits Quantum Fourier Transform $\mathcal{QFT}_{\hat{\square},\hat{\square}}$:

QuantumEvaluate$\left[\left(\mathcal{QFT}_{\hat{1},\hat{2}}\right)^{\dagger}\cdot\left(\frac{1}{2}\ |0_{\hat{1}},0_{\hat{2}}\rangle-\frac{1}{2}\,i\ |0_{\hat{1}},1_{\hat{2}}\rangle-\frac{1}{2}\ |1_{\hat{1}},0_{\hat{2}}\rangle+\frac{1}{2}\,i\ |1_{\hat{1}},1_{\hat{2}}\rangle\right)\right]$

$(0.+0.\,i)\ |0_{\hat{1}},0_{\hat{2}}\rangle+0.\ |0_{\hat{1}},1_{\hat{2}}\rangle+(0.+0.\,i)\ |1_{\hat{1}},0_{\hat{2}}\rangle+1.\ |1_{\hat{1}},1_{\hat{2}}\rangle$

Chop replaces approximate real numbers in expr that are close to zero by the exact integer 0:

Chop$\left[\textbf{QuantumEvaluate}\left[\left(\mathcal{QFT}_{\hat{1},\hat{2}}\right)^{\dagger}\cdot\left(\frac{1}{2}\ |0_{\hat{1}},0_{\hat{2}}\rangle-\frac{1}{2}\,i\ |0_{\hat{1}},1_{\hat{2}}\rangle-\frac{1}{2}\ |1_{\hat{1}},0_{\hat{2}}\rangle+\frac{1}{2}\,i\ |1_{\hat{1}},1_{\hat{2}}\rangle\right)\right]\right]$

$1.\ |1_{\hat{1}},1_{\hat{2}}\rangle$

Press [ESC]qqqft[ESC] for the template of the three-qubits Quantum Fourier Transform. **Scroll to the right in order to see the full answer:**

QuantumTableForm$\left[\mathcal{QFT}_{\hat{1},\hat{2},\hat{3}}\right]$

	Input	Output
0	$\|0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle$	$0.353553\ \|0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle+0.353553\ \|0_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle+0.353553\ \|0_{\hat{1}},1_{\hat{2}},0_{\hat{3}}\rangle$ ·
1	$\|0_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle$	$0.353553\ \|0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle+(0.25+0.25\,i)\ \|0_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle+0.353553\,i\ \|0_{\hat{1}},$
2	$\|0_{\hat{1}},1_{\hat{2}},0_{\hat{3}}\rangle$	$0.353553\ \|0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle+0.353553\,i\ \|0_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle-0.353553\ \|0_{\hat{1}},1_{\hat{2}},0_{\hat{3}}$
3	$\|0_{\hat{1}},1_{\hat{2}},1_{\hat{3}}\rangle$	$0.353553\ \|0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle-(0.25-0.25\,i)\ \|0_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle-0.353553\,i\ \|0_{\hat{1}},$
4	$\|1_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle$	$0.353553\ \|0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle-0.353553\ \|0_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle+0.353553\ \|0_{\hat{1}},1_{\hat{2}},0_{\hat{3}}\rangle$ ·
5	$\|1_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle$	$0.353553\ \|0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle-(0.25+0.25\,i)\ \|0_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle+0.353553\,i\ \|0_{\hat{1}},$
6	$\|1_{\hat{1}},1_{\hat{2}},0_{\hat{3}}\rangle$	$0.353553\ \|0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle-0.353553\,i\ \|0_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle-0.353553\ \|0_{\hat{1}},1_{\hat{2}},0_{\hat{3}}$
7	$\|1_{\hat{1}},1_{\hat{2}},1_{\hat{3}}\rangle$	$0.353553\ \|0_{\hat{1}},0_{\hat{2}},0_{\hat{3}}\rangle+(0.25-0.25\,i)\ \|0_{\hat{1}},0_{\hat{2}},1_{\hat{3}}\rangle-0.353553\,i\ \|0_{\hat{1}},$

Press [ESC]qqqqft[ESC] for the template of the four-qubits Quantum Fourier Transform. **Scroll to the right in order to see the full answer:**

QuantumTableForm[$\mathcal{QFT}_{\hat{1},\hat{2},\hat{3},\hat{4}}$]

	Input	Output
0	$\lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle$	$0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle + 0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle + 0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}, 0$
1	$\lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle$	$0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle + (0.23097 + 0.0956709\, i)\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle + ($
2	$\lvert 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}, 0_{\hat{4}}\rangle$	$0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle + (0.176777 + 0.176777\, i)\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle + 0$
3	$\lvert 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}, 1_{\hat{4}}\rangle$	$0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle + (0.0956709 + 0.23097\, i)\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle - ($
4	$\lvert 0_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle$	$0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle + 0.25\, i\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle - 0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}},$
5	$\lvert 0_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle$	$0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle - (0.0956709 - 0.23097\, i)\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle - ($
6	$\lvert 0_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}, 0_{\hat{4}}\rangle$	$0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle - (0.176777 - 0.176777\, i)\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle - 0$
7	$\lvert 0_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}, 1_{\hat{4}}\rangle$	$0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle - (0.23097 - 0.0956709\, i)\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle + ($
8	$\lvert 1_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle$	$0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle - 0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle + 0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}, 0$
9	$\lvert 1_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle$	$0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle - (0.23097 + 0.0956709\, i)\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle + ($
10	$\lvert 1_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}, 0_{\hat{4}}\rangle$	$0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle - (0.176777 + 0.176777\, i)\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle + 0$
11	$\lvert 1_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}}, 1_{\hat{4}}\rangle$	$0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle - (0.0956709 + 0.23097\, i)\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle - ($
12	$\lvert 1_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle$	$0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle - 0.25\, i\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle - 0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 1_{\hat{3}},$
13	$\lvert 1_{\hat{1}}, 1_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle$	$0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle + (0.0956709 - 0.23097\, i)\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle - ($
14	$\lvert 1_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}, 0_{\hat{4}}\rangle$	$0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle + (0.176777 - 0.176777\, i)\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle - 0$
15	$\lvert 1_{\hat{1}}, 1_{\hat{2}}, 1_{\hat{3}}, 1_{\hat{4}}\rangle$	$0.25\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 0_{\hat{4}}\rangle + (0.23097 - 0.0956709\, i)\ \lvert 0_{\hat{1}}, 0_{\hat{2}}, 0_{\hat{3}}, 1_{\hat{4}}\rangle + ($

TraditionalForm gives a format closer to the format used in papers and textbooks. **Scroll to the right in order to see the full answer:**

TraditionalForm[**QuantumTableForm**[$\mathcal{QFT}_{\hat{1},\hat{2},\hat{3},\hat{4}}$]]

	Input	Output
0	$\lvert 0000\rangle$	$0.25\ \lvert 0000\rangle + 0.25\ \lvert 0001\rangle + 0.25\ \lvert 0010\rangle + 0.25\ \lvert 0011\rangle + 0.25\ \lvert 0100\rangle + 0.25\ \lvert 0101\rangle + 0.25\ \lvert 0110\rangle +$
1	$\lvert 0001\rangle$	$0.25\ \lvert 0000\rangle + (0.23097 + 0.0956709\, i)\ \lvert 0001\rangle + (0.176777 + 0.176777\, i)\ \lvert 0010\rangle + (0.0956709 + 0.230$
2	$\lvert 0010\rangle$	$0.25\ \lvert 0000\rangle + (0.176777 + 0.176777\, i)\ \lvert 0001\rangle + 0.25\, i\ \lvert 0010\rangle - (0.176777 - 0.176777\, i)\ \lvert 0011\rangle - 0.2$
3	$\lvert 0011\rangle$	$0.25\ \lvert 0000\rangle + (0.0956709 + 0.23097\, i)\ \lvert 0001\rangle - (0.176777 - 0.176777\, i)\ \lvert 0010\rangle - (0.23097 + 0.09567$
4	$\lvert 0100\rangle$	$0.25\ \lvert 0000\rangle + 0.25\, i\ \lvert 0001\rangle - 0.25\ \lvert 0010\rangle - 0.25\, i\ \lvert 0011\rangle + 0.25\ \lvert 0100\rangle + 0.25\, i\ \lvert 0101\rangle - 0.25\ \lvert 011$
5	$\lvert 0101\rangle$	$0.25\ \lvert 0000\rangle - (0.0956709 - 0.23097\, i)\ \lvert 0001\rangle - (0.176777 + 0.176777\, i)\ \lvert 0010\rangle + (0.23097 - 0.09567$
6	$\lvert 0110\rangle$	$0.25\ \lvert 0000\rangle - (0.176777 - 0.176777\, i)\ \lvert 0001\rangle - 0.25\, i\ \lvert 0010\rangle + (0.176777 + 0.176777\, i)\ \lvert 0011\rangle - 0.2$
7	$\lvert 0111\rangle$	$0.25\ \lvert 0000\rangle - (0.23097 - 0.0956709\, i)\ \lvert 0001\rangle + (0.176777 - 0.176777\, i)\ \lvert 0010\rangle - (0.0956709 - 0.230$
8	$\lvert 1000\rangle$	$0.25\ \lvert 0000\rangle - 0.25\ \lvert 0001\rangle + 0.25\ \lvert 0010\rangle - 0.25\ \lvert 0011\rangle + 0.25\ \lvert 0100\rangle - 0.25\ \lvert 0101\rangle + 0.25\ \lvert 0110\rangle -$
9	$\lvert 1001\rangle$	$0.25\ \lvert 0000\rangle - (0.23097 + 0.0956709\, i)\ \lvert 0001\rangle + (0.176777 + 0.176777\, i)\ \lvert 0010\rangle - (0.0956709 + 0.230$
10	$\lvert 1010\rangle$	$0.25\ \lvert 0000\rangle - (0.176777 + 0.176777\, i)\ \lvert 0001\rangle + 0.25\, i\ \lvert 0010\rangle + (0.176777 - 0.176777\, i)\ \lvert 0011\rangle - 0.2$
11	$\lvert 1011\rangle$	$0.25\ \lvert 0000\rangle - (0.0956709 + 0.23097\, i)\ \lvert 0001\rangle - (0.176777 - 0.176777\, i)\ \lvert 0010\rangle + (0.23097 + 0.09567$
12	$\lvert 1100\rangle$	$0.25\ \lvert 0000\rangle - 0.25\, i\ \lvert 0001\rangle - 0.25\ \lvert 0010\rangle + 0.25\, i\ \lvert 0011\rangle + 0.25\ \lvert 0100\rangle - 0.25\, i\ \lvert 0101\rangle - 0.25\ \lvert 011$
13	$\lvert 1101\rangle$	$0.25\ \lvert 0000\rangle + (0.0956709 - 0.23097\, i)\ \lvert 0001\rangle - (0.176777 + 0.176777\, i)\ \lvert 0010\rangle - (0.23097 - 0.09567$
14	$\lvert 1110\rangle$	$0.25\ \lvert 0000\rangle + (0.176777 - 0.176777\, i)\ \lvert 0001\rangle - 0.25\, i\ \lvert 0010\rangle - (0.176777 + 0.176777\, i)\ \lvert 0011\rangle - 0.2$
15	$\lvert 1111\rangle$	$0.25\ \lvert 0000\rangle + (0.23097 - 0.0956709\, i)\ \lvert 0001\rangle + (0.176777 - 0.176777\, i)\ \lvert 0010\rangle + (0.0956709 - 0.230$

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