CoGrammar

PROBABILITY

SKILLS FOR LIFE
SKILLS BOOTCAMPS

Department for Education

Foundational Sessions Housekeeping

- The use of disrespectful language is prohibited in the questions, this is a supportive, learning environment for all - please engage accordingly. **(FBV: Mutual Respect.)**
- No question is daft or silly - **ask them!**
- There are **Q&A sessions** midway and at the end of the session, should you wish to ask any follow-up questions. Moderators are going to be answering questions as the session progresses as well.
- If you have any questions outside of this lecture, or that are not answered during this lecture, please do submit these for upcoming Open Classes. You can submit these questions here:
  SE Open Class Questions or DS Open Class Questions

Foundational Sessions Housekeeping cont.

- For all **non-academic questions**, please submit a query: **www.hyperiondev.com/support**
- Report a **safeguarding** incident: **www.hyperiondev.com/safeguardreporting**
- We would love your **feedback** on lectures: **Feedback on Lectures**

Reminders!

GLH requirements

Guided Learning Hours

By now, ideally you should have 7 GLHs per week accrued. Remember to attend any and all sessions for support, and to ensure you reach 112 GLHs by the close of your Skills Bootcamp.

Progression Criteria

✅ Criterion 1: Initial Requirements

- Complete **15 hours of Guided Learning Hours** and the **first four tasks** within **two weeks**.

✅ Criterion 2: Mid-Course Progress

- **Software Engineering:** Finish 14 tasks by week 8.
- **Data Science: Finish** 13 tasks by week 8.

✅ Criterion 3: Post-Course Progress

- Complete all mandatory tasks by **24th March 2024**.
- Record an Invitation to Interview within 4 weeks of course completion, or by 30th March 2024.
- Achieve 112 GLH by 24th March 2024.

✅ Criterion 4: Employability

- Record a Final Job Outcome within 12 weeks of graduation, or by 23rd September 2024.

What does the probability value of an event indicate?

A. The frequency of the event in a series of trials
B. The likelihood of the occurrence of the event
C. The duration of the event
D. The impact of the event

What is a 'sample space' in probability?

A. The space where experiments are conducted
B. The set of all possible outcomes of a probability experiment
C. A type of probability distribution
D. The outcome with the highest probability.

Two events are independent if:

A. The occurrence of one affects the probability of the occurrence of the other
B. They occur simultaneously
C. The occurrence of one does not affect the probability of the occurrence of the other
D. They are mutually exclusive events

Recap of Linear Algebra

CoGrammar

Vectors, Matrices, and Operations

Vector: quantities having both magnitude and direction, represented as an array of numbers.

- Example: $\vec{v} = [3, 4]$ represents movement 3 units to the right and 4 units up

Matrices: rectangular arrays of numbers or expressions, used to represent complex data structures or transformations.

- A 2 x 2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ could represent a linear Transformation in a plane

Scalar Operations: multiplying a vector by a scalar changes its magnitude but not direction.

Dot Product: a measure of the similarity of two vectors, calculated as the sum of the products of their corresponding entries.

Probability Topics

1. Sample Space and Events
2. Basic Probability Theory
3. Addition and Multiplication Rules
4. Conditional Probability and Independence
5. Common Distributions

CoGrammar

Predicting Customer Churn

Consider a telecommunications company that is experiencing a high rate of customer churn (customers leaving for competitors). We want to predict which customers are most likely to churn so the company can take action.

- How do we use Probability Theory to Predict Customer Churn?

Example: Coin Toss

- Sample Space: $S = \{Heads, Tails\}$.
- Probability of an Event: $P(E) = \frac{Number\ of\ favorable\ outcomes}{Total\ number\ of\ outcomes}$
- For a fair coin, $P(Heads) = \frac{1}{2}$

Sample Space and Events

- **Sample Space:** The set of all possible outcomes.
- **Events:** Specific outcomes or sets of outcomes from the sample space.

E.g. If $\{1, 2, 3, 4, 5, 6\}$ is the sample space,

then $\{2, 4, 6\}$ is one of the events.

Basic Probability Theory

- Probability of an Event: $P(E) = \frac{Number\ of\ favorable\ outcomes}{Total\ number\ of\ outcomes}$

- If we roll a dice and want to know the probability of getting a 4, then $P(4) = \frac{1\ (since\ there\ is\ just\ one\ 4)}{6\ (since\ there\ are\ 6\ possible\ numbers)}$

Addition and Multiplication Rules

- **Addition Rule:** For mutually exclusive events A and B, $P(A \text{ or } B) = P(A) + P(B)$.

- For example, to find the probability of landing on a 4 or 5 with a fair dice: $P(4 \text{ or } 5) = P(4) + P(5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

- **Multiplication Rule:** For independent events A and B, $P(A \text{ and } B) = P(A) \times P(B)$. Try finding P(4 and 5).

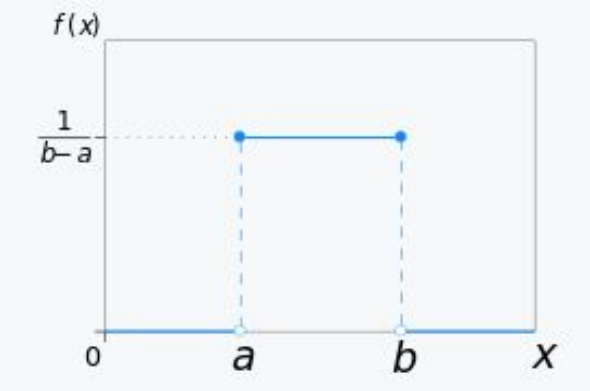
Conditional Probability and Independence

- **Condition Probability:** $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$ is the probability that A happened given that B already happened. E.g. $P(Heart|Red) = \frac{13}{26} = \frac{1}{2}$

- **Independence:** Events A and B are independent if $P(A|B) = P(A)$ And $P(B|A) = P(B)$.

Uniform Distribution

- In a uniform distribution all outcomes are equally likely.

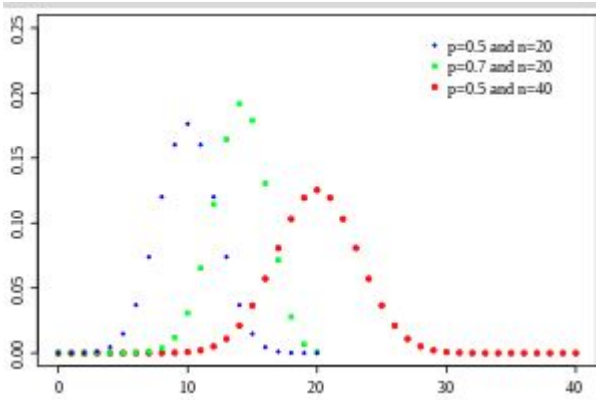


$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b, \\ 0 & \text{for } x < a \text{ or } x > b. \end{cases}$$

Source: https://en.wikipedia.org/wiki/Continuous_uniform_distribution

Binomial Distribution

- Number of success in a fixed number of trials.



$$f(k, n, p) = \Pr(k; n, p) = \Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

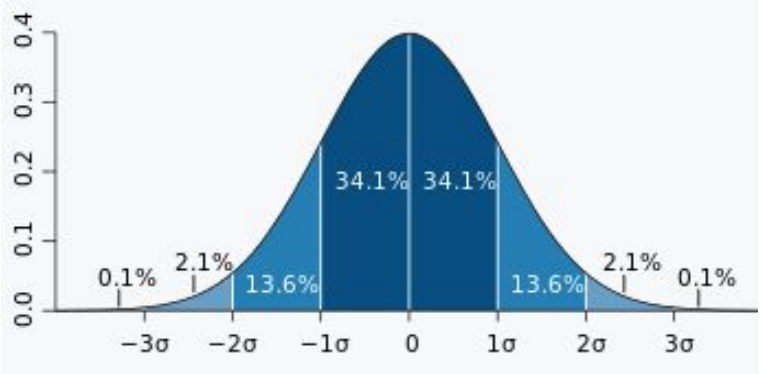
for $k = 0, 1, 2, \ldots, n$, where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Source: https://en.wikipedia.org/wiki/Binomial_distribution

Normal Distribution

- Describes data in clusters around a mean. It is the most common distribution in statistics since it tends to represent natural phenomena more accurately than most other distributions most of the time.



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Source: https://en.wikipedia.org/wiki/Normal_distribution

Worked Example

A telecommunications company has identified that customers who exceed their data limit by more than 20% or contact customer service multiple times within a billing cycle have a higher chance of churn. The company offers two data plans: 5GB and 10GB monthly limits.

```python
import numpy as np

# Historical data indicates the following probabilities
p_exceed_5GB = 0.10  # Probability of exceeding 5GB limit by >20%
p_exceed_10GB = 0.05  # Probability of exceeding 10GB limit by >20%
p_contact_service = 0.15  # Probability of contacting service more than
twice

# Probability of churn given the customer has exceeded data limit and
# contacted customer service
p_churn_given_exceed_and_contact = (p_exceed_5GB * p_contact_service)

print(f"Probability of churn (5GB plan): {p_churn_given_exceed_and_contact:.2f}")

# Assuming independence between the events for the 10GB plan
p_churn_10GB_plan = p_exceed_10GB * p_contact_service
print(f"Probability of churn (10GB plan): {p_churn_10GB_plan:.2f}")
```

1. Calculate the probability of churn for customers on the 5GB and 10GB data plans.

2. Assess the impact of a promotional campaign that reduces the probability of contacting customer service to 0.10.
    a. Recalculate the churn probabilities for both plans with the new customer service contact probability.

Worked Example

A telecommunications company has identified that customers who exceed their data limit by more than 20% or contact customer service multiple times within a billing cycle have a higher chance of churn. The company offers two data plans: 5GB and 10GB monthly limits.

```python
import numpy as np

# Historical data indicates the following probabilities
p_exceed_5GB = 0.10  # Probability of exceeding 5GB limit by >20%
p_exceed_10GB = 0.05  # Probability of exceeding 10GB limit by >20%
p_contact_service = 0.15  # Probability of contacting service more than
twice

# Probability of churn given the customer has exceeded data limit and
# contacted customer service
p_churn_given_exceed_and_contact = (p_exceed_5GB * p_contact_service)

print(f"Probability of churn (5GB plan): {p_churn_given_exceed_and_contact:.2f}")

# Assuming independence between the events for the 10GB plan
p_churn_10GB_plan = p_exceed_10GB * p_contact_service
print(f"Probability of churn (10GB plan): {p_churn_10GB_plan:.2f}")
```

1. Calculate the probability of churn for customers on the 5GB and 10GB data plans.

   P(exceed 4GB) = 0.10, P(contact service) = 0.15,
   Then for both 5GB and 10GB the answers are P(churn) = P(exceed 10GB) x P(contact service) = 0.015. It does not matter if either 5GB or 10GB. Tricky! :)

2. Assess the impact of a promotional campaign that reduces the probability of contacting customer service to 0.10.

   Now it becomes 0.10 x 0.10 = 0.10.

   Feel free to check out the code, it abstracts all the complexity!

Summary

Sample Space and Events

- ★ The set of all possible outcomes of an experiment.
- ★ An event is a subset of the sample space that we are interested in.

Basic Probability

- ★ The likelihood of an event occurring, calculated as favorable outcomes divided by total outcomes.

Conditional Probability and Independence

- ★ Probability of an event given another has occurred.
- ★ Independence when one event does not influence another.

Summary

Probability Distributions

- ★ Uniform: Equal probability for all outcomes.
- ★ Binomial: Probability of 'success' in 'n' trials.
- ★ Normal: Bell-curved distribution, common in natural data.

What is conditional probability?

A. The probability of two events occurring together
B. The probability of an event, given that another event has occurred
C. The likelihood of an event occurring after a series of other events
D. The probability of an event occurring without any conditions.

Which statement is true about the normal distribution?

A. It is skewed to the left or right.
B. It is a distribution where all outcomes are equally likely.
C. It is symmetric around its mean.
D. It applies only to discrete random variables.

Questions and Answers

Questions around Probability