## CSC 320 - Tutorial 5

1. Context Free Grammars
2. Context Free Languages

### Context Free Grammar

A context free grammar is a 4-tuple $(V,\ \Sigma,\ R,\ S)$

- V: is a finite set of **variables**
- Σ: is a finite set of **terminals** - disjoint from V
- R: finite set of **rules**
- $S \in V$ the **start variable**

$uAv$ **yields** $uwv$, written as $uAv \Rightarrow uwv$

Means you can get from $uAv$ to $uwv$ in one "step" by applying a rule on A

$u$ **derives** $v$**,** written as $u \overset{*}{\Rightarrow} v$

Means either $u = v$

Or starting at $u$ then applying a series of rules, you can get to $v$

(ie. there exists a sequence $u_1, u_2, u_3, \ldots, u_k$ for k ≥ 0 such that

$u \Rightarrow u_1 \Rightarrow u_2 \Rightarrow u_3 \Rightarrow \ldots \Rightarrow u_k \Rightarrow v$ )

**Questions**

1. For each of the following languages over alphabet $\Sigma = \{0, 1\}$ define the CFG (4-tuple) that recognizes the language

context free Grammar

   a. $L_1 = \{w \mid w$ starts and ends with the same symbol $\}$

consider some strings in the language:

$L(0(0 \cup 1)^* 0$ or $1(0 \cup 1)^* 1$ or $1$ or $0)$

Rules:

$S \rightarrow 0A0 \mid 1A1 \mid 1 \mid 0 \mid \varepsilon$

$A \rightarrow 1A \mid 0A \mid \varepsilon$

4-tuple:

$V = \{S, A\}$

$\Sigma = \{0, 1\}$ * $\varepsilon$ is never in $\Sigma$

$R$

$S = S$

   b. $L_2 = \varnothing$

starting at the start variable we never derive a string

Rule:

$S \rightarrow S$ * infinite loop $\therefore$ never derives a string

4-tuple:

$V = \{S\}$

$\Sigma = \varnothing$

$R$

$S = S$

c. $L_3 = \{0^n 1^m \mid m, n \geq 0 \text{ and } n \leq m\}$

Consider some strings in the language:

$L_3 = \{ \varepsilon, 01, 1, 0011, 00111, \dots \}$

$(n = m = 0)$

if add a zero $\Rightarrow$ need to add a 1

can add as many 1s as I want

Rules:

$S \rightarrow 0S1 \mid S1 \mid \varepsilon$

4-tuple:

$V = \{S\}$

$\Sigma = \{0, 1\}$

$R$

$S = S$

d. $L_4 = \{0^n 1^m \mid 2n \leq m \leq 3n\}$

* similar to 1c

$n = 0 \Rightarrow m = 0$

$n = 1 \Rightarrow 2 \leq m \leq 3$ $\therefore$ if I add a zero $\Rightarrow$ need to add two or three 1s

Rules:

$S \rightarrow 0S11 \mid 0S111 \mid \varepsilon$

4-tuple:

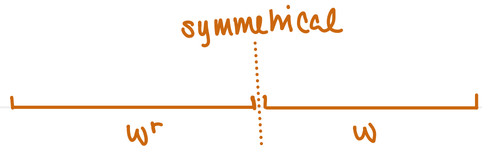
$V = \{S\}$

$\Sigma = \{0, 1\}$

$R$

$S = S$

e. $L_5 = \{w^r w\}$




Rules :

$S \rightarrow 0S0 \mid 1S1 \mid \varepsilon$

4-tuple:

$V = \{S\}$

$\Sigma = \{0, 1\}$

$R$

$S = S$

2. Derive/generate the string "**aabaa**" for the following grammar:

$S \rightarrow aAS \mid aSS \mid \varepsilon$

$A \rightarrow SbA \mid ba$

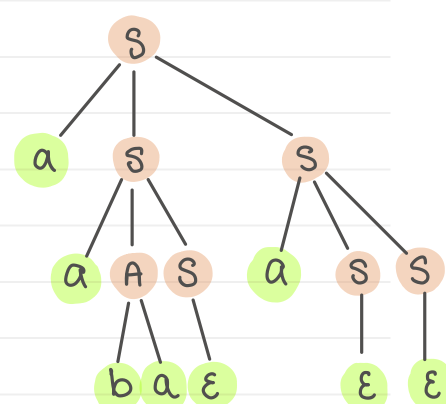
Start @ the start variable
(if not explicitly told the variable on LHS of 1st rule)

$S \rightarrow a S S \rightarrow a \underline{a A S} S$

$\rightarrow a a \underline{ba} S S \rightarrow a a b a \_ S$

$\rightarrow a a b a \underline{a S S} \rightarrow a a b a a \_ S$

$\rightarrow a a b a a \_$




is the string above derived ambiguously?

3. Complete the state diagram by adding transitions so that the constructed PDA recognizes the language L

(read, pop → push)

L = $\{a^m b^n \mid m, n \geq 0$ **and** ( **either** $m = n$ **or** $m = n + 2$ )

branches

case: m = n

Case: m = n+2