CSC 320 - Tutorial

1. Regular expressions
2. NFA to regular expression conversion
3. Regular languages are closed under Kleene star proof

Questions

1. Design a regular expression for the following languages over $\Sigma = \{0, 1\}$
   a. $L_1 = \{ w \mid$ every odd position of w is a 1 $\}$

$L_1 = \{ 1010, 1, 11, 10, 111, 101, \ldots \}$

$1 \cup (1(1 \cup 0))^* \cup (1(1 \cup 0))^* 1$

   b. $L_2 = \{ w \mid w$ is string of length **at most** 5 $\}$

1
0
ε

$(1 \cup 0 \cup \varepsilon)^5$ OR $(1 \cup 0)^n$ where $n \leq 5$

   c. $L_3 = \{ w \mid w$ contains an even number of 0s **or** exactly two 1s $\}$

$R_1 \quad \cup \quad R_2$

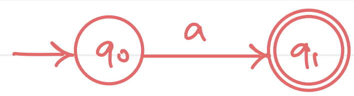
$R_2 = 0^* 1 0^* 1 0^*$ exactly two 1s

$R_1 = (1^* 0 1^* 0 1^*)^* \cup 1^*$

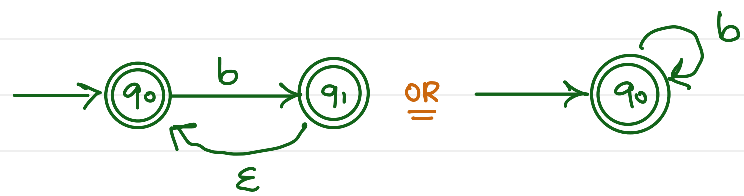
$\underbrace{1^* \cup (1^* 0 1^* 0 1^*)^*}_{R_1} \cup \underbrace{(0^* 1 0^* 1 0^*)}_{R_2}$

2. Convert the following regular expression to an NFA: $R = (a \cup b^*)\, a$

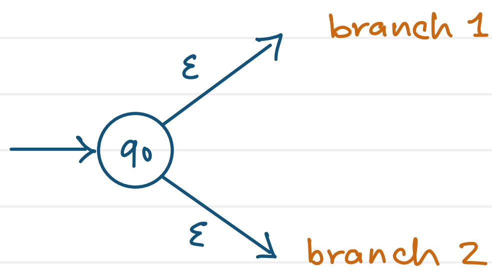
$R_1 = a$

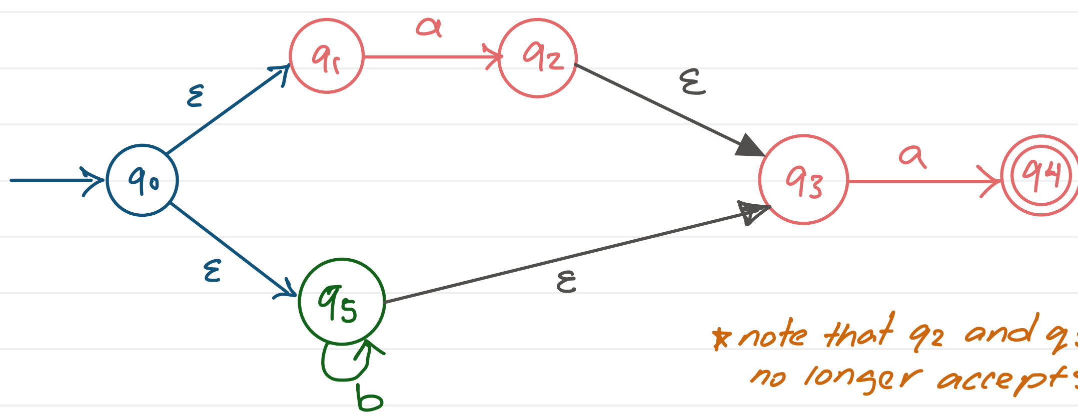


$R_2 = b^*$



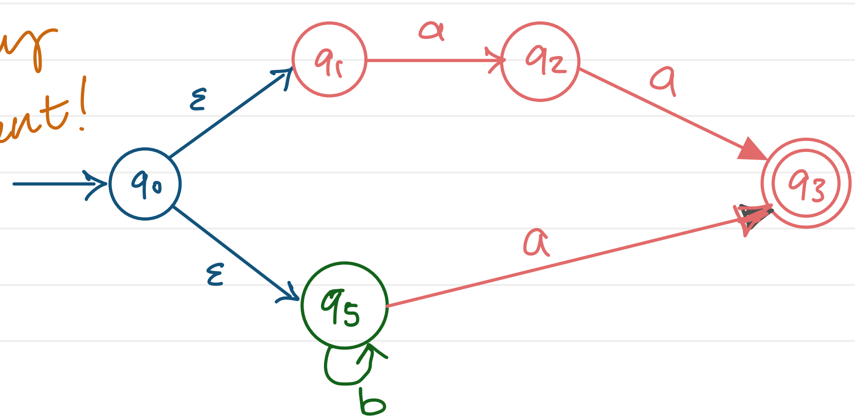
union $\cup$



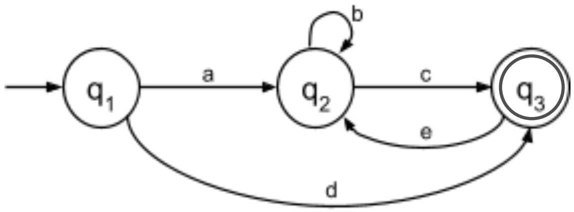


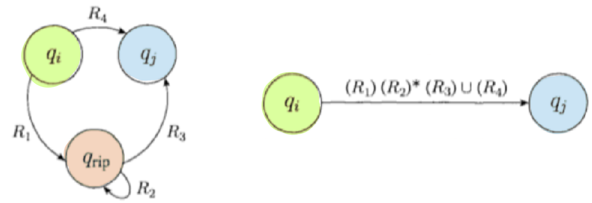
* note that q_2 and q_5 are no longer accept states

* alternatively this is equivalent!

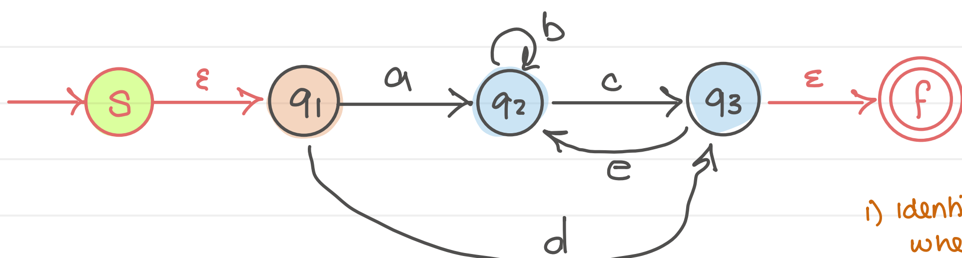


3. Write the regular expression that describes the language of the DFA below





1) Generalized NFA

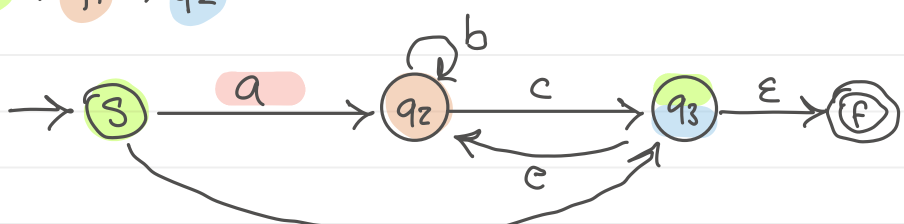


1) identify 3 state paths where the middle state is qrip
2) find regular expression $R_1 R_2^* R_3 \cup R_4$
3) construct new general NFA

qrip = q_1

S → q_1 → q_3 $d \cup \emptyset \Rightarrow d$

S → q_1 → q_2 a



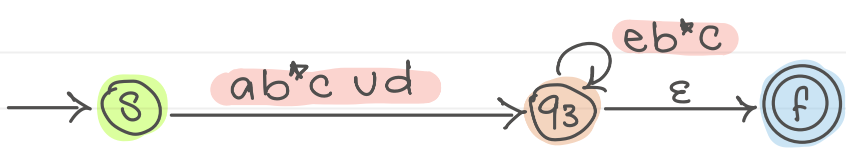
ab^*c s → q_3

OR   d   s → q_3

qrip = q_2

s → q_2 → q_3 $ab^*c \cup d$

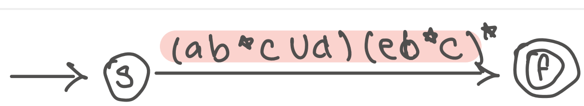
q_3 → q_2 → q_3 $eb^*c \cup \emptyset \Rightarrow eb^*c$



qrip = q_3

s → q_3 → f

$(ab^*c \cup d)(eb^*c)^*$



* only s and f are left ∴ we are done!

* don't forget proof of correctness

4. Prove that regular languages are closed under Kleene star

Recall that the Kleene star of a language L is the concatination of L w/ itself zero or more times.

$$L^* = \{\varepsilon\} \cup L \cup \underbrace{LL}_{\text{concatination}} \cup LLL \cup \ldots$$

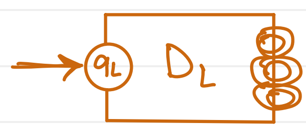
and we know regular languages are closed under concatination

Proof.

Let L be a regular language
there exists a DFA D_L that recognizes L
$D_L = (Q_L, \Sigma, \delta_L, q_L, F_L)$ and $L(D_L) = L$

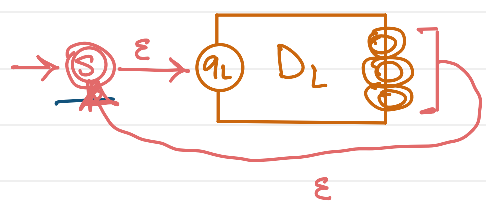
Construct an NFA $N = (Q, \Sigma, \delta, q, F)$
s.t. $L(N) = L^*$

$\varepsilon \in L^*$



$Q = Q_L \cup \{S\}$

Σ the same



$\delta = \delta_L \cup \{(S, \varepsilon) \to q_L\}$
$\cup \{(q_F, \varepsilon) \to S$ where $q_F \in F_L\}$

$q = S$

$F = F_L \cup \{S\}$

proof of Correctness $L(N) = L^*$

1) $L(N) \subseteq L^*$

$w \in L(N)$

$w = \varepsilon \Rightarrow w \in L^*$

when running w on NFA N

$\underbrace{s \to q_L \to \cdots \to q_f}_{w_1 \in L} \to \underbrace{s \to q_L \to \cdots \to q_f}_{w_2 \in L} \cdots$

we will start at s and eventually end on a state q_f where $q_f \in F_L$

then $w \in L^*$

2) $L^* \subseteq L(N)$

$w \in L^*$

1) $w = \varepsilon \Rightarrow \varepsilon \in L(N)$

2) $w = w_1 w_2 w_3 \ldots w_n$ where $w_i \in L$

for each w_i we start @ state s and eventually reach a state $q_f \in F_L$ where we can take an ε-transition back to s to process w_{i+1} $\therefore$ $w \in L(N)$