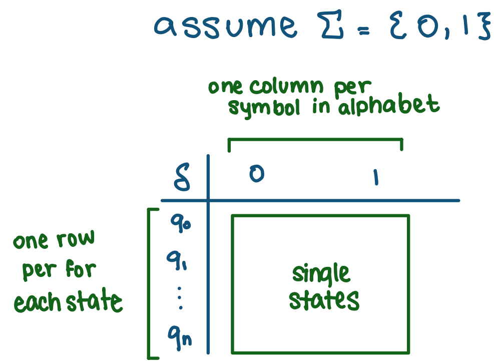
CSC 320 - Tutorial

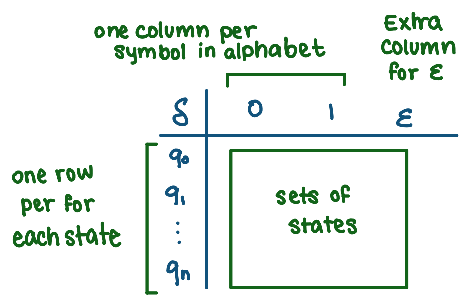
1. Deterministic Finite Automaton
2. Non-deterministic Finite Automaton



Deterministic Finite Automaton (DFA)

- Is expressed as a **5-tuple** (Q, Σ, δ, q_0, F)
  - Q: finite set of states
  - Σ: alphabet finite set
  - δ: transition function $(Q \times \Sigma) \rightarrow Q$
  - q_0: start state $q_0 \in Q$
  - F: sets of accept/final states $F \subseteq Q$
- The language L of a deterministic finite automata M, L(M) is exactly the set of all strings that M accepts ∴ M recognizes L(M)
- A given language L is **regular** iff it is recognized by some deterministic finite automaton

Non-deterministic Finite Automata (NFA)



- Is expressed as a **5-tuple** (Q, Σ, δ, q_0, F)
  a. Q: finite set of states
  b. Σ: alphabet
  c. δ: transition function $Q \times (\Sigma \cup \{\varepsilon\}) \rightarrow P(Q)$
  d. q_0: start state $q_0 \in Q$
  e. F: sets of accept/final states $F \subseteq Q$
- The language L of a non-deterministic finite automata N, L(N) is exactly the set of all strings that N accepts
- For every DFA M there exists an equivalent NFA N (ie. L(M) = L(N))
- For every NFA N there exists an equivalent DFA M

Questions 5-tuple

1. Give the formal specification of a DFA for the following languages:
    a. $L_1 = \{0\}^*$ over $\Sigma = \{0\}$ $L_1 = \{\varepsilon, 0, 00, 000, \dots\}$

$M = (Q, \Sigma, \delta, q^*, F)$

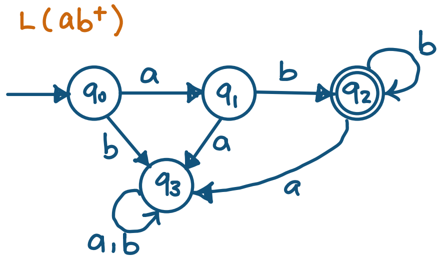
$M = (\{q_0\}, \{0\}, \delta, q_0, \{q_0\})$

δ	0
q_0	q_0

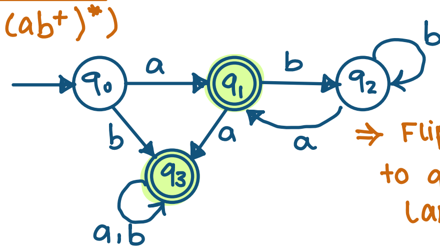
* always double check that every state in your DFA has one transition/symbol in alphabet

    b. $L_2 = \{w \in \{a, b\}^* \mid w$ is a string NOT in $L((ab^+)^*)\}$

1) $L(ab^+)$

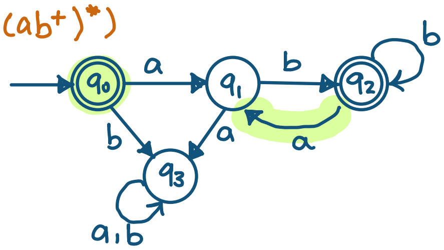


3) $\overline{L((ab^+)^*)}$



⇒ Flip accept states to get the complement language

2) $L((ab^+)^*)$



$M = (Q, \Sigma, \delta, q^*, F)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$

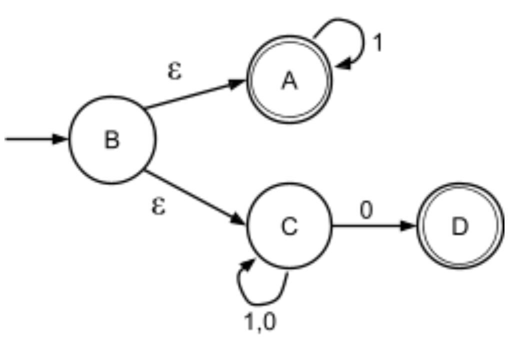
δ

$q^* = q_0$

$F = \{q_1, q_3\}$

δ	a	b
q_0	q_1	q_3
q_1	q_3	q_3
q_2	q_1	q_2
q_3	q_3	q_3

2. Consider the state diagram below:



    a. Is this state machine a DFA or an NFA? How can you tell?

NFA ⇒ we have ε-transitions

state A doesn't have a 0 transition

0011 is not accepted

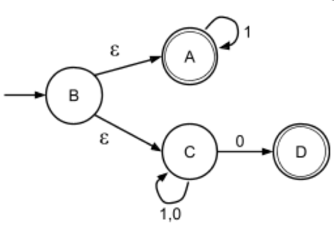
1100 is accepted ($B \to C \to C \to C \to D$)

$w = 110 \quad 0$

b. Is the string 0011 accepted by this state machine? What about 1100?

0011 is not accepted

1100 is accepted ($B \xrightarrow{\varepsilon} C \xrightarrow{1} C \xrightarrow{1} C \xrightarrow{0} C \xrightarrow{0} D$)



c. What is the language of this machine?

$$L = \underbrace{\{1\}^*}_{\text{upper branch}} \cup \underbrace{\{w0 \mid w \in \{0,1\}^*\}}_{\text{lower branch}}$$

ex. string 1100

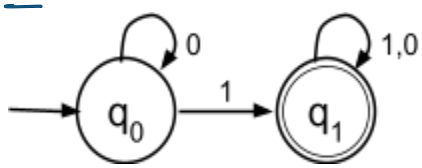
$w = 110$

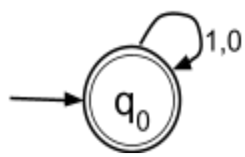
concatinated w/ 0

$\therefore w0 \to 1100$

3. Given an example of how regular languages L_1 and L_2 are closed under intersection using the DFAs M_1 and M_2 below (proof by construction) where $L_1 = L(M_1)$ and $L_2 = L(M_2)$

M_1 M_2





M_1 recognizes L_1 $\therefore$ L_1 is regular $\quad M_1 = (Q_1, \Sigma, \delta_1, q_1^*, F_1)$

M_2 recognizes L_2 $\therefore$ L_2 is regular $\quad M_2 = (Q_2, \Sigma, \delta_2, q_2^*, F_2)$

Construct a DFA M where $L(M) = L_1 \cap L_2$

$M = (Q, \Sigma, \delta, q^*, F)$

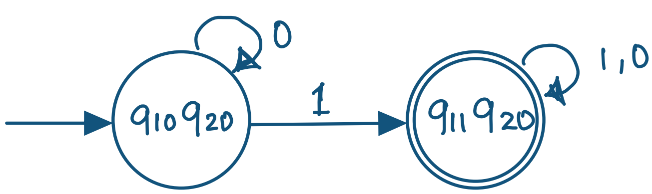
$Q = \{(r_1, r_2) \mid r_1 \in Q_1 \;\; r_2 \in Q_2\}$

$\Sigma = \Sigma$

$q^* = (q_1^*, q_2^*)$

$\delta((r_1, r_2), a) \to (\delta_1(r_1, a), \delta_2(r_2, a))$

$F = \{(r_1, r_2) \mid r_1 \in F_1 \text{ AND } r_2 \in F_2\}$



$Q = \{q_{10}q_{20}, q_{11}q_{20}\}$

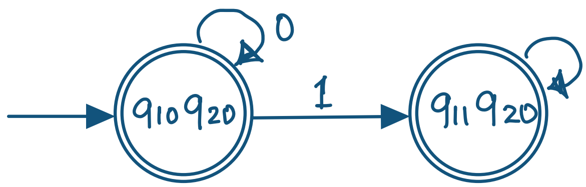
Σ = the same as M_1 and M_2

δ	1	0
$q_{10}q_{20}$	$q_{11}q_{20}$	$q_{10}q_{20}$
$q_{11}q_{20}$	$q_{11}q_{20}$	$q_{11}q_{20}$

$q^* = q_{10}q_{20}$

$F = \{q_{11}q_{20}\}$

6 a. How would the state machine change for the union? Ie. if the new DFA is to recognize $L_1 \cup L_2$ instead.



$F = \{ (r_1, r_2) \mid r_1 \in F_1 \text{ OR } r_2 \in F_2 \}$

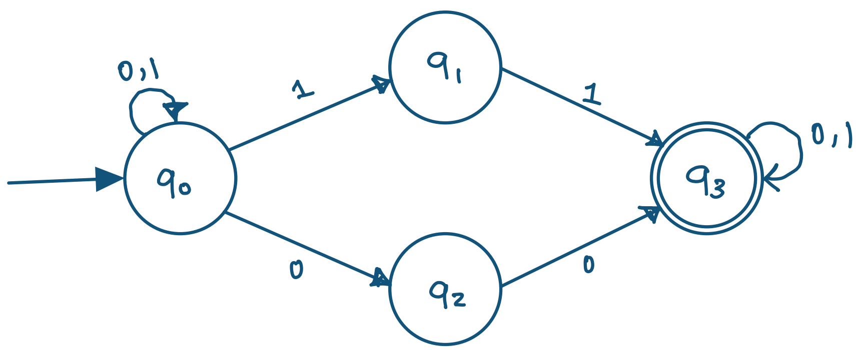
$F = \{ q_{10}q_{20} , q_{11}q_{20} \}$

any state containing a final state of M_1 or M_2 will be a final state in M

q_0 in M_2 and $q_0 \in F_1$

$\therefore q_{10}q_{20} \in F$

4. Design an NFA state diagram for the following language:
   a. $L = \{ w \in \{0, 1\}^* \mid w$ contains 00 or 11 as a substring $\}$



   b. Express the NFA as a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ and describe δ as a transition table

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{1, 0\}$

δ

$q^* = q_0$

$F = \{q_3\}$

don't forget column for ε

δ	0	1	ε
q_0	$\{q_0, q_2\}$	$\{q_0, q_1\}$	$\emptyset$
q_1	$\emptyset$	$\{q_3\}$	$\emptyset$
q_2	$\{q_3\}$	$\emptyset$	$\emptyset$
q_3	$\{q_3\}$	$\{q_3\}$	$\emptyset$