CSC 320
Foundations of Computer Science

Lecture 11

Instructor: Dr. Ulrike Stege

Territory Acknowledgement

We acknowledge and respect the lək̓ʷəŋən peoples on whose traditional territory the university stands and the Songhees, Esquimalt and W̱SÁNEĆ peoples whose historical relationships with the land continue to this day.

This meeting will be recorded

"Please be aware our sessions are being screen-recorded to allow students who are not able to attend to watch later and will be posted in Brightspace."

Deadlines; Assessment

10%	25%	25%	40%
Quizzes	Assignments	Midterms	Final Exam
Quiz 1-8: 1% each Quiz 9: 2%	Assignment 1-5: 5% each	Midterm 1: 10% Midterm 2: 15%	

May

S	M	T	W	T	F	S
			3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31	1	2	3
4	5	6	7	8	9	10

June

S	M	T	W	T	F	S
28	29	30	31	1	2	[illegible]
4	5	6	7	8	9	[illegible]
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	1
2	3	4	5	6	7	8

July

S	M	T	W	T	F	S
25	26	27	28	29	30	1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

Timed quizzes (~30 min)
Review before starting quiz

Last time ....

- A language is context-free if and only if some PDA recognizes it
- Is $B = \{a^n b^n c^n \mid n \geq 0\}$ context-free?

Today

- How to prove that a language is regular
- How to prove that a language is nonregular
- How to prove that a language is not context-free

Showing that a language is regular
Example 1

- Midterm1 Question

2. (5 marks) Let $L_0 = \{aa, ab, bb, abc\}$. Give a regular expression R with $L(R) = L_0$.

$$R = aa \cup ab \cup bb \cup abc$$

How can we *prove* that $L(R) = L_0$?

$$\begin{aligned} L(R) &= L(aa \cup ab \cup bb \cup abc) \\ &= L(aa) \cup L(ab) \cup L(bb) \cup L(abc) \\ &= \{aa\} \cup \{ab\} \cup \{bb\} \cup \{abc\} \\ &= \{aa, ab, bb, abc\} \end{aligned}$$

Showing that a language is regular
Example 2

- Quiz 5

Q6

Consider the following language.

Let $\Sigma = \{a, b\}$, and let $L_1 = \{rtr | r, t \in \Sigma^*\}$.

- [x] $L_1 = L((a \cup b)^*)$
- [x] $L_1 = \Sigma^*$
- [x] If $r = \epsilon$ then $rtr \in L_1$ $\quad \epsilon t \epsilon$
- [ ] L_1 is the set of all strings with prefix x and suffix x, where $x \in \Sigma^*$, $x \neq \epsilon$
- [x] L_1 is the set of all strings with prefix x and suffix x, where $x = \epsilon$

Prove that $L_2 = \{0^m 1^m 0^m\}$is nonregular

Assume that $L_2 = \{0^m 1^m 0^m\}$ is regular

By PL: There exists integer $p > 0$ such that

For all $s \in L_2$, $|s| \geq p$, there exists strings x, y, z st

$s = xyz$

1. $xy^i z \in L_2$ for all $i \in \mathbb{N}$
2. $y \neq \epsilon$
3. $|xy| \leq p$

Let $s = 0^p 1^p 0^p$.

Show that no x, y, z exists such that $0^p 1^p 0^p = xyz$ satisfies all three properties

Prove that $L_2 = \{0^m 1^m 0^m\}$ is nonregular

1. $xy^iz \in L_2$ for all $i \in \mathbb{N}$
2. $y \neq \epsilon$
3. $|xy| \leq p$

Let $s = 0^p 1^p 0^p$.

Show that no x, y, z exists such that $0^p 1^p 0^p = xyz$ satisfies all three properties

We distinguish all possible cases for y

Because of 2. we know that y consist of at least one symbol, that is $i = 0^k, k > 0$

Because of 3. we know that xy is prefix of the leading p 0s, $i = 0^k, p \geq k > 0$

Properties 2. and 3. are satisfied for $s = 0^p 1^p 0^p$. Let's look at property 1.

Let $i = 0$. Then $xy^0z = xz = 0^{p-k} 1^p 0^p$

Since $xy^0z \notin L_2$, property 1 is not satisfied and therefore $s = 0^p 1^p 0^p$ is a counterexample

Therefore PL does not hold for L_2, which means that L_2 is nonregular

Next: Pumping Lemma for context-free languages

Pumping lemma for context-free languages

If L is a context-free language, then there is a number p (*pumping length*) such that: if s is any string in L of length at least p, then s may be divided into five pieces $s = uvxyz$ satisfying the conditions

1. for each $i \geq 0$, $uv^ixy^iz \in L$
2. $|vy| > 0$, and
3. $|vxy| \leq p$

Prove $B = \{a^n b^n c^n \mid n \geq 0\}$ is not context-free

Assume $B = \{a^n b^n c^n \mid n \geq 0\}$ is context-free.

Then, because of the PL for context-free languages, there exists pumping length p st: if $s \in B,\ |s| \geq p$, then s may be divided into five pieces $s = uvxyz$ satisfying

1. for each $i \geq 0$, $uv^i xy^i z \in L$
2. $|vy| > 0$, and
3. $|vxy| \leq p$

We prove for $s = 0^p 1^p 0^p$, no such strings u, v, x, y, z exist

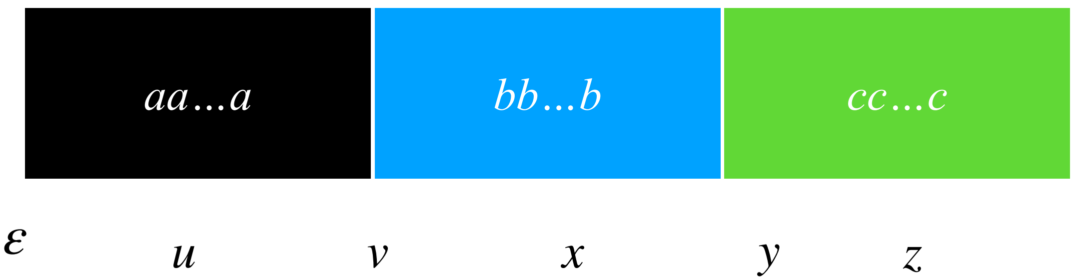
Show: $B = \{a^n b^n c^n \mid n \geq 0\}$ is not context-free

- $s = a^p b^p c^p$

- To show: s cannot be divided into five strings $s = uvxyz$ satisfying

1. for each $i \geq 0$, $uv^ixy^iz \in B$
2. $|vy| > 0$
3. $|vxy| \leq p$

- Let's think about dividing s into u, v, x, y, z



- Because of 2.: $vy \neq \varepsilon$, ie $v \neq \varepsilon$ or $y \neq \varepsilon$

- Because of 3., $|vxy| \leq p$, yielding these cases:

A. $vxy = a\ldots a \implies uv^2xy^2z = a^kb^pc^p$ with $k > p$

B. $vxy = a\ldots ab\ldots b \implies uv^2xy^2z = a^kb^\ell c^p$ with $k > p$ or $\ell > p$

C. $vxy = b\ldots b \implies uv^2xy^2z = a^pb^kc^p$ with $k > p$

D. $vxy = b\ldots bc\ldots c \implies uv^2xy^2z = a^pb^kc^\ell$ with $k > p$ or $\ell > p$

E. $vxy = c\ldots c \implies uv^2xy^2z = a^pb^pc^k$ with $k > p$

There is no rewriting of s into $s = uvxyz$ with $uv^2xy^2z \in L$

$L = \{ww \mid w \in \{0,1\}^*\}$ is not context-free

- Assume that L is context free

- p: pumping length for L

- Choose $s = 0^p1^p0^p1^p$

  - $0^p1^p0^p1^p \in L$ for $s = ww$ with $w = 0^p1^p$

1. for each $i \geq 0$, $uv^ixy^iz \in B$
2. $|vy| > 0$
3. $|vxy| \leq p$

- Show: there is no rewriting for s into $s = uvxyz$ such that PL conditions hold

- Because of 3.: vxy is of one of the following forms:

$00\ldots0$	$11\ldots1$	$00\ldots0$	$11\ldots1$

  - $vxy = 00\ldots0 \implies uv^2xy^2z = 0^k1^p0^\ell1^p$ with either $k > p$ or $\ell > p$

  - $vxy = 11\ldots1 \implies uv^2xy^2z = 0^p1^k0^p1^\ell$ with either $k > p$ or $\ell > p$

  - $vxy = 00\ldots011\ldots1 \implies$ either $uv^2xy^2z = 0^k1^\ell0^p1^p$ or $uv^2xy^2z = 0^p1^p0^k1^\ell$ with $k > p$ or $\ell > p$

  - $vxy = 11\ldots100\ldots0 \implies uv^2xy^2z = 0^p1^k0^\ell1^p$ with $k > p$ or $\ell > p$

- Conclusion: there is no rewriting for s into $s = uvxyz$ and $uv^2xy^2z \in L$