

1 ♣ Expand the following functions about the given center x_0 . You can type this up using LaTeX or other word processor.

$$1. \quad f(x) = \sin 2x \text{ and } x_0 = 0 \\ = 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 + O(x^7)$$

$$2. \quad f(x) = \ln 2x \text{ and } x_0 = 1 \\ = \ln 2 + x - 1 - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + O((x-1)^5)$$

$$3. \quad f(x) = e^{2x} \text{ and } x_0 = 1 \\ = e^2 + e^2(x-1)$$

$$4. \quad f(x) = 3x^2 - 2x + 5 \text{ and } x_0 = 0 \\ = 3x^2 - 2x + 5$$

$$5. \quad f(x) = 3x^2 - 2x + 5 \text{ and } x_0 = 1 \\ = 6 + 4(x-1) + 3(x-1)^2$$

$$6. \quad f(x) = (3x^2 - 2x + 5)^{-1} \text{ and } x_0 = 1 \\ = \frac{1}{6} - \frac{1}{9}x - 1 - \frac{1}{108}(x-1)^2 + \frac{5}{81}(x-1)^3 + O((x-1)^4)$$

$$7. \quad f(x) = \cosh x - 3 \text{ and } x_0 = 1 \\ = \frac{e^2+1}{2e} - 3 + \frac{e^2-1}{2e}(x-1) + \frac{e^2-1}{4e}(x-1)^2 + \frac{e^2-1}{12e}(x-1)^3 + O((x-1)^4)$$

$$8. \quad f(x) \text{ and } x_0 = a \\ f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2} + f'''(a)\frac{(x-a)^3}{6} + O(x^5)$$

$$9. \quad f(a) \text{ and } x_0 = x \\ f(x) + f'(x)(a-x) + f''(x)\frac{(a-x)^2}{2} + f'''(x)\frac{(a-x)^3}{6} + O(x^5)$$

$$10. \quad f(a+h) \text{ and } x_0 = a \\ f(a+h) + f'(a+h)(x-a-h) + f''(a+h)\frac{(x-a-h)^2}{2} + f'''(a+h)\frac{(x-a-h)^3}{6}$$

2 ✚ Differentiate

1. $\int x \sin 2x \, dx$ (by parts)
 $\sin 2x + 2x \cos 2x + C$

2. $\int x e^{x^2} \, dx$ (by substitution)
 $= e^{x^2} + 2x^2 e^{x^2} + C$

3. $\int x e^x \, dx$ (by parts)
 $= e^x + x e^x + C$

4. $\int e^{x^2} \, dx$ (by integrand in a Taylor series)
 $2x e^{x^2} + C$

5. $\int x \sqrt{1+x} \, dx$
 $= \sqrt{1+x} + \frac{x}{2\sqrt{1+x}} + C$

6. $\int \sec \theta \, d\theta$
 $= \sec(x) \tan(x) + C$

7. $\int \sec^2 \theta \, d\theta$
 $= 2 \sec^2(x) \tan(x) + C$

8. $\int \operatorname{sech}^2(\theta) \, d\theta = \int \frac{1}{\cosh^2(\theta)} \, d\theta$
 $= -\frac{1}{\cosh^2(\theta)} \tanh \theta + C$

9. $\int \frac{x^2+2}{7-x^2} \, dx$
 $= \frac{2x}{7-x^2} + \frac{2(x^2+2)}{(7-x^2)^2} + c$

10. $\int \frac{1}{ap-bp^2} \, dp$
 $= \frac{-1}{ap^2} + \frac{b^2}{-a(a-bp)^2}$

3 ✚ Compute the solutions for the following simple initial value problems:

1. $\frac{dx}{dt} = 3x$ and $x(0) = 1$
 $x = 3xt + 1$

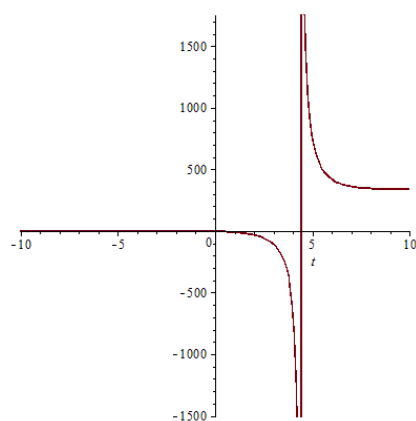
2. $\frac{dx}{dt} = 3tx$ and $x(0) = 4$
 $x = 4e^{\frac{3}{2}t^2}$

3. $\frac{dx}{dt} = 1x - .003x^2$ and $x(0) = 4$
 $x = \frac{1}{.003 - .247e^{-t}} = \frac{e^t}{.003e^t - .247}$

4. $\frac{dx}{dt} = 1x - .003x^2$ and $x(0) = 400$
 $x = \frac{e^t}{.003e^t - .0005}$

4 ✚ Graph the solution of the last two differential equations in the Problem above. Just graph them.

1. $\frac{dx}{dt} = 1x - .003x^2$ and $x(0) = 4$



2. $\frac{dx}{dt} = 1x - .003x^2$ and $x(0) = 400$

