

1 ♣ Expand the following functions about the given center x_0 . You can type this up using LaTeX or other word processor.

$$1. \quad f(x) = \sin 2x \text{ and } x_0 = 0 \\ = 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 + O(x^7)$$

$$2. \quad f(x) = \ln 2x \text{ and } x_0 = 1 \\ = \ln 2 + x - 1 - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + O((x-1)^5)$$

$$3. \quad f(x) = e^{2x} \text{ and } x_0 = 1 \\ = e^2 + e^2(x-1)$$

$$4. \quad f(x) = 3x^2 - 2x + 5 \text{ and } x_0 = 0 \\ = 3x^2 - 2x + 5$$

$$5. \quad f(x) = 3x^2 - 2x + 5 \text{ and } x_0 = 1 \\ = 6 + 4(x-1) + 3(x-1)^2$$

$$6. \quad f(x) = (3x^2 - 2x + 5)^{-1} \text{ and } x_0 = 1 \\ = \frac{1}{6} - \frac{1}{9}x - 1 - \frac{1}{108}(x-1)^2 + \frac{5}{81}(x-1)^3 + O((x-1)^4)$$

$$7. \quad f(x) = \cosh x - 3 \text{ and } x_0 = 1 \\ = \frac{e^2+1}{2e} - 3 + \frac{e^2-1}{2e}(x-1) + \frac{e^2-1}{4e}(x-1)^2 + \frac{e^2-1}{12e}(x-1)^3 + O((x-1)^4)$$

$$8. \quad f(x) \text{ and } x_0 = a \\ f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2} + f'''(a)\frac{(x-a)^3}{6} + O(x^5)$$

$$9. \quad f(a) \text{ and } x_0 = x \\ f(x) + f'(x)(a-x) + f''(x)\frac{(a-x)^2}{2} + f'''(x)\frac{(a-x)^3}{6} + O(x^5)$$

$$10. \quad f(a+h) \text{ and } x_0 = a \\ f(a+h) + f'(a+h)(x-a-h) + f''(a+h)\frac{(x-a-h)^2}{2} + f'''(a+h)\frac{(x-a-h)^3}{6}$$

2 ✚ Differentiate

$$1. \int x \sin 2x \, dx \text{ (by parts)} \\ = \frac{-x}{2} \cos 2x + \frac{1}{4} \sin 2x + C$$

$$2. \int x e^{x^2} \, dx \text{ (by substitution)} \\ = \frac{1}{2} e^{x^2} + C$$

$$3. \int x e^x \, dx \text{ (by parts)} \\ = x e^x - e^x$$

$$4. \int e^{x^2} \, dx \text{ (by integrand in a Taylor series)} = 1 + x^2 + \frac{1}{2} x^4 \mathcal{O}(x^6) \\ x + \frac{x^3}{3} + \frac{x^5}{10} + C + \mathcal{O}(x^7)$$

$$5. \int x \sqrt{1+x} \, dx \\ = \frac{2}{15} (1+x)^{\frac{3}{2}} (-2+3x) + C$$

$$6. \int \sec \theta \, d\theta \\ = \ln (\sec \theta + \tan \theta) + C$$

$$7. \int \sec^2 \theta \, d\theta \\ = 2 \frac{\sin \theta}{\cos \theta} + C$$

$$8. \int \operatorname{sech}^2(\theta) = \int \frac{1}{\cosh^2(\theta)} \, d\theta \\ = \frac{\sinh \theta}{\cosh \theta} + C$$

$$9. \int \frac{x^2+2}{7-x^2} \, dx \\ = -x + \frac{9\sqrt{7}}{7} \arctan\left(\frac{x\sqrt{7}}{7}\right) + c$$

$$10. \int \frac{1}{ap-bp^2} \, dp \\ = \frac{\ln|p|}{a} - \frac{\ln|bp-a|}{a}$$

3 ✚ Compute the solutions for the following simple initial value problems:

$$1. \frac{dx}{dt} = 3x \text{ and } x(0) = 1 \\ x = 3xt + 1$$

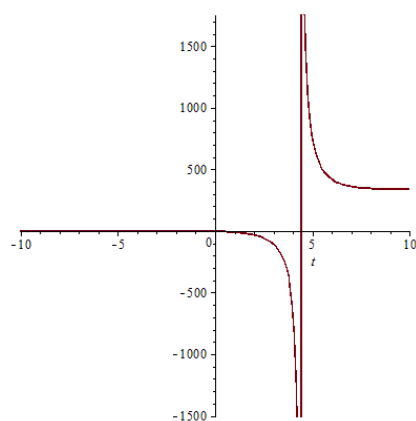
2. $\frac{dx}{dt} = 3tx$ and $x(0) = 4$
 $x = 4e^{\frac{3}{2}t^2}$

3. $\frac{dx}{dt} = 1x - .003x^2$ and $x(0) = 4$
 $x = \frac{1}{.003 - .247e^{-t}} = \frac{e^t}{.003e^t - .247}$

4. $\frac{dx}{dt} = 1x - .003x^2$ and $x(0) = 400$
 $x = \frac{e^t}{.003e^t - .0005}$

4 ✚ Graph the solution of the last two differential equations in the Problem above. Just graph them.

1. $\frac{dx}{dt} = 1x - .003x^2$ and $x(0) = 4$



2. $\frac{dx}{dt} = 1x - .003x^2$ and $x(0) = 400$

