1 ✠ Expand the following functions about the given center x_0 You can type this up using LaTeX or other word processor.

1. $f(x) = \sin 2x$ and $x_0 = 0$
   $= 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 + O(x^7)$
   We can rewrite this as $\sum_{n=0}^{\infty} \frac{2^{2n+1}}{(2n+1)!}x^{2n+1}$ whose radius of convergence is does not exist (goes to ∞)

2. $f(x) = \ln 2x$ and $x_0 = 1$
   $= \ln 2 + x - 1 - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + O((x-1)^5)$

3. $f(x) = e^{2x}$ and $x_0 = 1$
   $= e^2 + e^2(x-1)$

4. $f(x) = 3x^2 - 2x + 5$ and $x_0 = 0$
   $= 3x^2 - 2x + 5$

5. $f(x) = 3x^2 - 2x + 5$ and $x_0 = 1$
   $= 6 + 4(x-1) + 3(x-1)^2$

6. $f(x) = (3x^2 - 2x + 5)^{-1}$ and $x_0 = 1$
   $= \frac{1}{6} - \frac{1}{9}x - 1 - \frac{1}{108}(x-1)^2 + \frac{5}{81}(x-1)^3 + O((x-1)^4)$

7. $f(x) = \cosh x - 3$ and $x_0 = 1$
   $= \frac{e^2+1}{2e} - 3 + \frac{e^2-1}{2e}(x-1) + \frac{e^2-1}{4e}(x-1)^2 + \frac{e^2-1}{12e}(x-1)^3 + O((x-1)^4)$

8. $f(x)$ and $x_0 = a$
   $f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2} + f'''(a)\frac{(x-a)^3}{6} + O(x^5)$

9. $f(a)$ and $x_0 = x$

   $f(x) + f'(x)(a-x) + f''(x)\frac{(a-x)^2}{2} + f'''(x)\frac{(a-x)^3}{6} + O(x^5)$

10. $f(a+h)$ and $x_0 = a$
    $f(a+h) + f'(a+h)(x-a-h) + f''(a+h)\frac{(x-a-h)^2}{2} + f'''(a+h)\frac{(x-a-h)^3}{6}$

    Find the radius of convergence of each of the series

2 ✠ Differentiate

1. $\int x \sin 2x \, dx$ (by parts)
   $\sin 2x + 2x \cos 2x + C$

2. $\int x e^{x^2} \, dx$ (by substitution)
   $= e^{x^2} + 2x^2 e^{x^2} + C$

3. $\int x e^x \, dx$ (by parts)
   $= e^x + x e^x + C$

4. $\int e^{x^2} \, dx$ (by integrand in a Taylor series)
   $2x e^{x^2} + C$

5. $\int x\sqrt{1+x} \, dx$
   $= \sqrt{1+x} + \frac{x}{2\sqrt{1+x}} + C$

6. $\int \sec\theta \, d\theta$
   $= \sec(x)\tan(x) + C$

7. $\int \sec^2\theta \, d\theta$
   $= 2\sec^2(x)\tan(x) + C$

8. $\int sech^2(\theta) = \int \frac{1}{\cosh^2(\theta)} \, d\theta$
   $= -\frac{1}{\cosh^2(\theta)} \tanh\theta + C$

9. $\int \frac{x^2+2}{7-x^2} \, dx$
   $= \frac{2x}{7-x^2} + \frac{2(x^2+2)}{(7-x^2)^2} + c$

10. $\int \frac{1}{ap-bp^2} \, dp$
    $= \frac{-1}{ap^2} + \frac{b^2}{-a(a-bp)^2}$

3 ✠ Compute the solutions for the following simple initial value problems:

1. $\frac{dx}{dt} = 3x$ and $x(0) = 1$
   $x = 3xt + 1$

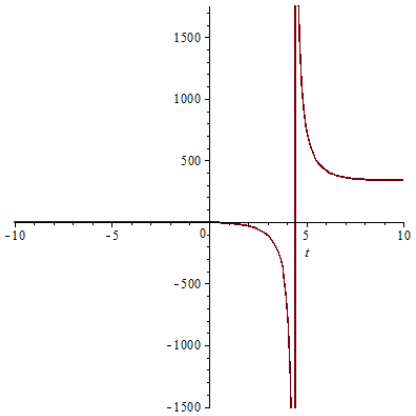
2. $\frac{dx}{dt} = 3tx$ and $x(0) = 4$
   $x = 4e^{\frac{3}{2}t^2}$

3. $\frac{dx}{dt} = 1x - .003x^2$ and $x(0) = 4$
   $x = \frac{1}{.003-.247e^{-t}} = \frac{e^t}{.003e^t-.247}$

4. $\frac{dx}{dt} = 1x - .003x^2$ and $x(0) = 400$
   $x = \frac{e^t}{.003e^t - .0005}$

4 ✠ Graph the solution of the last two differential equations in the Problem above. Just graph them.

1. $\frac{dx}{dt} = 1x - .003x^2$ and $x(0) = 4$



2. $\frac{dx}{dt} = 1x - .003x^2$ and $x(0) = 400$