

1 ♣ Expand the following functions about the given center x_0 . You can type this up using LaTeX or other word processor.

1. $f(x) = \sin 2x$ and $x_0 = 0$

$$= 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 + O(x^7)$$

We can rewrite this as $\sum_{n=0}^{\infty} \frac{2^{2n+1}}{(2n+1)!} x^{2n+1}$ whose radius of convergence is does not exist (goes to ∞)

2. $f(x) = \ln 2x$ and $x_0 = 1$

$$= \ln 2 + x - 1 - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + O((x-1)^5)$$

3. $f(x) = e^{2x}$ and $x_0 = 1$

$$= e^2 + e^2(x-1)$$

4. $f(x) = 3x^2 - 2x + 5$ and $x_0 = 0$

$$= 3x^2 - 2x + 5$$

5. $f(x) = 3x^2 - 2x + 5$ and $x_0 = 1$

$$= 6 + 4(x-1) + 3(x-1)^2$$

6. $f(x) = (3x^2 - 2x + 5)^{-1}$ and $x_0 = 1$

$$= \frac{1}{6} - \frac{1}{9}x - 1 - \frac{1}{108}(x-1)^2 + \frac{5}{81}(x-1)^3 + O((x-1)^4)$$

7. $f(x) = \cosh x - 3$ and $x_0 = 1$

$$= \frac{e^2+1}{2e} - 3 + \frac{e^2-1}{2e}(x-1) + \frac{e^2-1}{4e}(x-1)^2 + \frac{e^2-1}{12e}(x-1)^3 + O((x-1)^4)$$

8. $f(x)$ and $x_0 = a$

$$f(a) + f'(a)(x - a) + f''(a)\frac{(x-a)^2}{2} + f'''(a)\frac{(x-a)^3}{6} + O(x^5)$$

9. $f(a)$ and $x_0 = x$

$$f(x) + f'(x)(a - x) + f''(x)\frac{(a-x)^2}{2} + f'''(x)\frac{(a-x)^3}{6} + O(x^5)$$

10. $f(a + h)$ and $x_0 = a$

$$f(a + h) + f'(a + h)(x - a - h) + f''(a + h)\frac{(x-a-h)^2}{2} + f'''(a + h)\frac{(x-a-h)^3}{6}$$

Find the radius of convergence of each of the series

2 ✚ Differentiate

1. $\int x \sin 2x \, dx$ (by parts)
 $\sin 2x + 2x \cos 2x + C$

2. $\int x e^{x^2} \, dx$ (by substitution)
 $= e^{x^2} + 2x^2 e^{x^2} + C$

3. $\int x e^x \, dx$ (by parts)
 $= e^x + x e^x + C$

4. $\int e^{x^2} \, dx$ (by integrand in a Taylor series)
 $2x e^{x^2} + C$

5. $\int x \sqrt{1+x} \, dx$
 $= \sqrt{1+x} + \frac{x}{2\sqrt{1+x}} + C$

$$6. \int \sec \theta d\theta \\ = \sec(x) \tan(x) + C$$

$$7. \int \sec^2 \theta d\theta \\ = 2 \sec^2(x) \tan(x) + C$$

$$8. \int \operatorname{sech}^2(\theta) = \int \frac{1}{\cosh^2(\theta)} d\theta \\ = -\frac{1}{\cosh^2(\theta)} \tanh \theta + C$$

$$9. \int \frac{x^2+2}{7-x^2} dx \\ = \frac{2x}{7-x^2} + \frac{2(x^2+2)}{(7-x^2)^2} + c$$

$$10. \int \frac{1}{ap-bp^2} dp \\ = \frac{-1}{ap^2} + \frac{b^2}{-a(a-bp)^2}$$

3 ✚ Compute the solutions for the following simple initial value problems:

$$1. \quad \frac{dx}{dt} = 3x \text{ and } x(0) = 1 \\ x = 3xt + 1$$

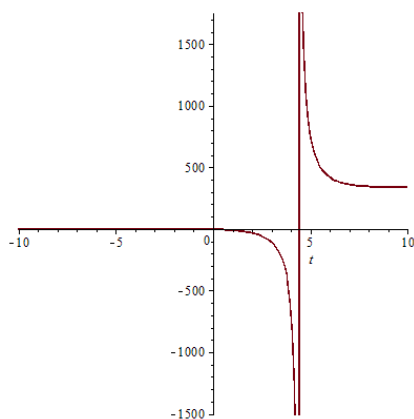
$$2. \quad \frac{dx}{dt} = 3tx \text{ and } x(0) = 4 \\ x = 4e^{\frac{3}{2}t^2}$$

$$3. \quad \frac{dx}{dt} = 1x - .003x^2 \text{ and } x(0) = 4 \\ x = \frac{1}{.003 - .247e^{-t}} = \frac{e^t}{.003e^t - .247}$$

4. $\frac{dx}{dt} = 1x - .003x^2$ and $x(0) = 400$
 $x = \frac{e^t}{.003e^t - .0005}$

4 ✚ Graph the solution of the last two differential equations in the Problem above. Just graph them.

1. $\frac{dx}{dt} = 1x - .003x^2$ and $x(0) = 4$



2. $\frac{dx}{dt} = 1x - .003x^2$ and $x(0) = 400$

