Markov Chain Monte Carlo Applied to the Study of Neuronal Interaction from Experimental Data

Pedro Brandimarte

November 30, 2017

frontiers IN NEUROSCIENCE

Novel experience induces persistent sleep-dependent plasticity in the cortex but not in the hippocampus

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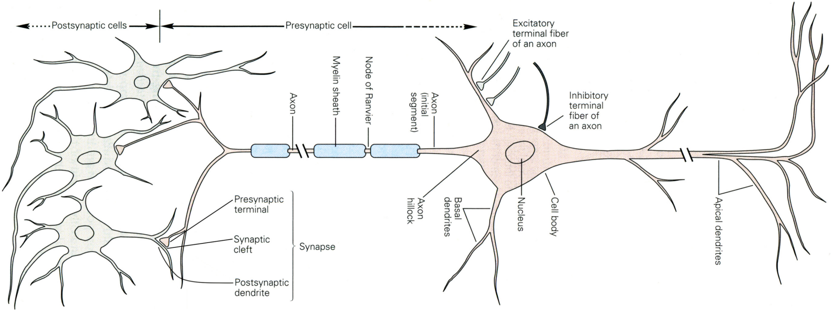
5. Laboratory of Neural Ensemble Technology, École Polytechnique Fédérale de Lausanne, Lausanne, Switzerland



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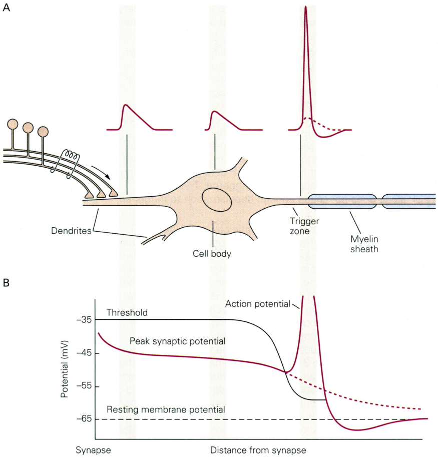

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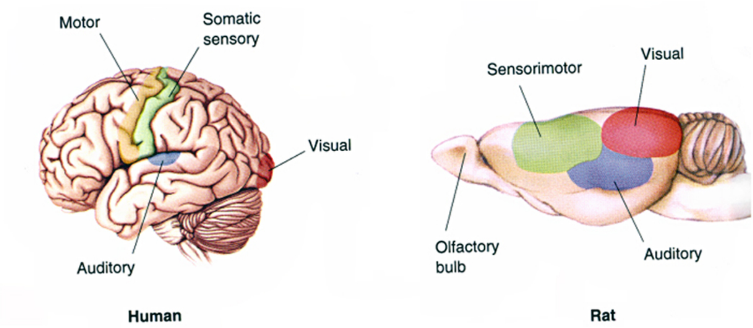
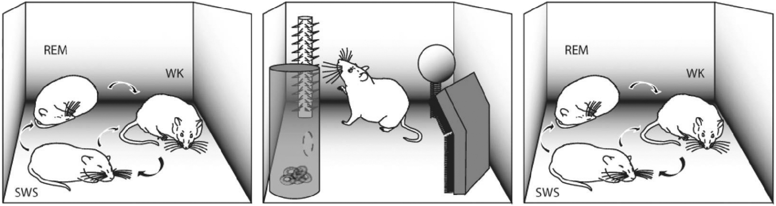
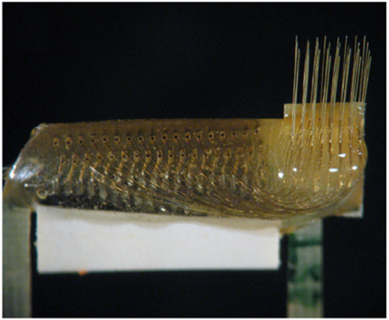


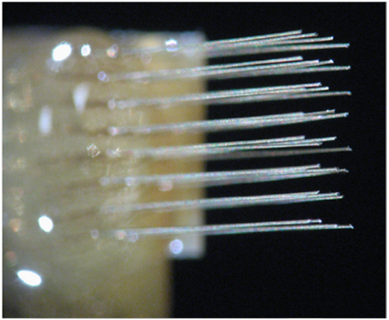
M. Bear, B. Connors and M. Paradiso. *Neuroscience - Exploring The Brain*. Lippincott W & W, $3^{\underline{a}}$ edition (2006).



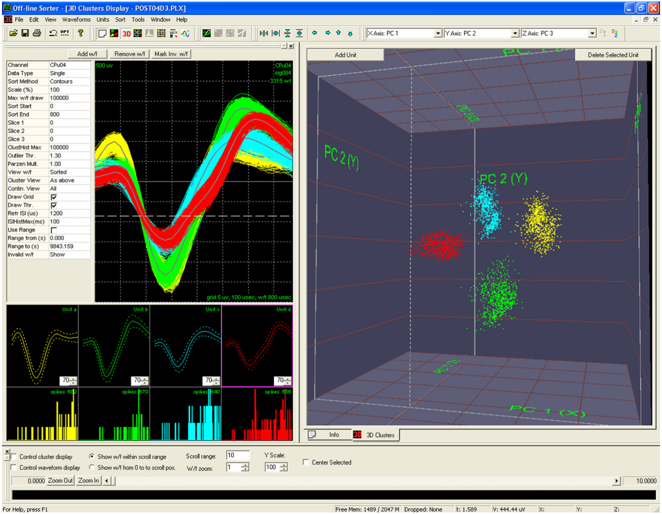
PRE-EXP (2 h) EXPERIENCE (20 min) POST-EXP (3 h)

S. Ribeiro *et al.* *Frontiers in Neuroscience* **1**(1), 43-55 (2007).





M. Freire *et al.* *PLoS ONE* **6**(11), e27554 (2011).



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Graph

Definition: *A* **graph** *is a set of* **vertices** V *and a set of* **edges** $\mathcal{A}$ *connecting the vertices.* $\square$

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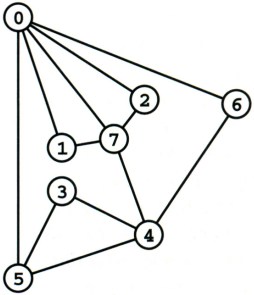

	0	1	2	3	4	5	6	7
0	0	1	1	0	0	1	1	1
1	1	0	0	0	0	0	0	1
2	1	0	0	0	0	0	0	1
3	0	0	1	0	1	0	0	0
4	0	0	0	1	0	1	1	0
5	1	0	0	1	1	0	0	0
6	1	0	0	0	1	0	0	0
7	1	1	1	0	1	0	0	0

R. Sedgewick. *Algorithms In C - Parts 1-4*. Addison Wesley, 3ª edition (1998).

Sample of N neurons $\mathbf{X} = (\mathbf{X}^{(1)}, \mathbf{X}^{(2)}, ..., \mathbf{X}^{(N)})$, where $\mathbf{X}^{(i)} = (x_0^{(i)}, x_1^{(i)}, ..., x_n^{(i)})$ are the recordings of firing from the i-th neuron at times $\{0, 1, 2, \ldots, n\}$

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$$\mathbb{P}(g|\mathbf{X}) = \frac{\exp\left[\sum_{(i,j)\in g} J_{ij}\left(\mathbf{X}^{(i)} \otimes \mathbf{X}^{(j)}\right)\right]}{\sum_{g'\in\mathcal{G}} \exp\left[\sum_{(i,j)\in g'} J_{ij}\left(\mathbf{X}^{(i)} \otimes \mathbf{X}^{(j)}\right)\right]} = \frac{\prod_{(i,j)\in g} e^{J_{ij}\left(\mathbf{X}^{(i)}\otimes\mathbf{X}^{(j)}\right)}}{\sum_{g'\in\mathcal{G}} \prod_{(i,j)\in g'} e^{J_{ij}\left(\mathbf{X}^{(i)}\otimes\mathbf{X}^{(j)}\right)}}$$

where $J_{ij} \in \mathbb{R}$ is the degree of interaction between neurons i - j.

The product operation $\mathbf{X}^{(i)} \otimes \mathbf{X}^{(j)} = \sum_{t=0}^{n} x_t^{(i)} \cdot x_t^{(j)}$ defines the method used for calculating the posterior probability:

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Method 1:

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Method 2:

$$x_t^{(i)} \cdot x_t^{(j)} = \begin{cases} 1 & \text{if } x_t^{(i)} = 1 \text{ and } x_t^{(j)} = 1 \text{ or } x_t^{(i)} = 0 \text{ and } x_t^{(j)} = 0 \\ 0 & \text{otherwise} \end{cases}$$

Penalty measure related to the number of edges been considered:

$$\pi(g) = \frac{\exp\left[-cn\sum_{a\in\mathcal{A}}\mathbb{1}_{\{a\in g\}}\right]}{\sum_{g'\in\mathcal{G}}\exp\left[-cn\sum_{a\in\mathcal{A}}\mathbb{1}_{\{a\in g'\}}\right]}$$

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Thus, the posterior probability of a graph g given a sample $\mathbf{X}$, is:

$$\mathbb{P}(g|\mathbf{X}) = \frac{\prod_{(i,j)\in g} e^{J_{ij}\left(\mathbf{X}^{(i)}\otimes\mathbf{X}^{(j)}\right)-cn}}{\sum_{g'\in\mathcal{G}}\prod_{(i,j)\in g'} e^{J_{ij}\left(\mathbf{X}^{(i)}\otimes\mathbf{X}^{(j)}\right)-cn}}$$

Markov Chain Monte Carlo - Metropolis-Hastings

From an initial state g_i:

1. A candidate to next state g_j is proposed with transition probability Q_i^j.
2. The state g_j is accepted with acceptance probability $\alpha\left(g_i, g_j\right)$.
3. Otherwise, the state g_j is rejected and the chain remains on state g_i.
4. Go to step 1.

Considerations:

- Symmetric sampling kernel given by *Unif* $\Big[\,0, |\mathcal{A}|\,\Big]$, where $|\mathcal{A}|$ is the cardinality of the set of all edges.
- The candidate to next state g_j has **only one** edge different from g_i.

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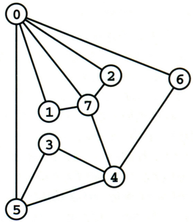
In this case, the acceptance probability can be simplified to:

$$\alpha\left(g_i, g_j\right) = \begin{cases} \min\left(1, e^{J_{uv}\left(\mathbf{X}^{(u)} \otimes \mathbf{X}^{(v)}\right) - cn}\right) & \text{if the edge } b = (u, v) \text{ is inserted} \\ \\ \min\left(1, e^{-J_{uv}\left(\mathbf{X}^{(u)} \otimes \mathbf{X}^{(v)}\right) + cn}\right) & \text{if the edge } b = (u, v) \text{ is removed} \end{cases}$$

Graph representation:



	0	1	2	3	4	5	6	7
0	0	1	1	0	0	1	1	1
1	1	0	0	0	0	0	0	1
2	1	0	0	0	0	0	0	1
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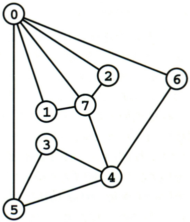

	0	1	2	3	4	5	6	7
0	0	1	1	0	0	1	1	1
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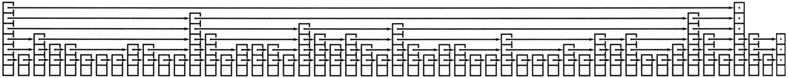
The element $[\,i, j\,]$ from the adjacency matrix can be recovered:

$$[\,i, j\,] \longrightarrow \begin{cases} \left[i + 1 + (j+1)\frac{j-2}{2} \right] & \text{if } i < j \\ \\ \left[j + 1 + (i+1)\frac{i-2}{2} \right] & \text{otherwise} \end{cases}$$

Skip List:



W. Pugh. *Skip Lists: A Probabilistic Alternative to Balanced Trees*. Comm. of the ACM **33**(6), 668-676 (1990).

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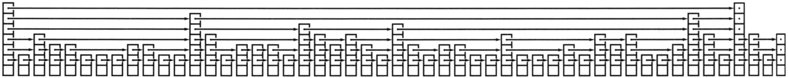

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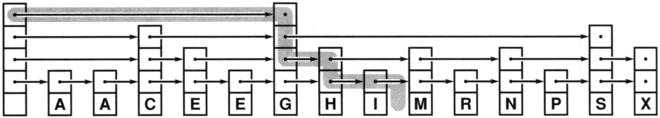
To insert a new item into a *skip list*, we generate a new j-link node with probability proportional to $1/t^j$, where t is a fixed parameter.



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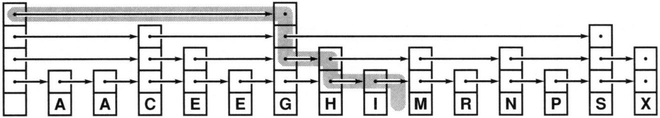

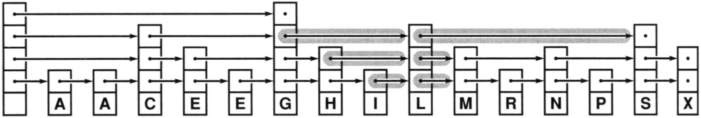

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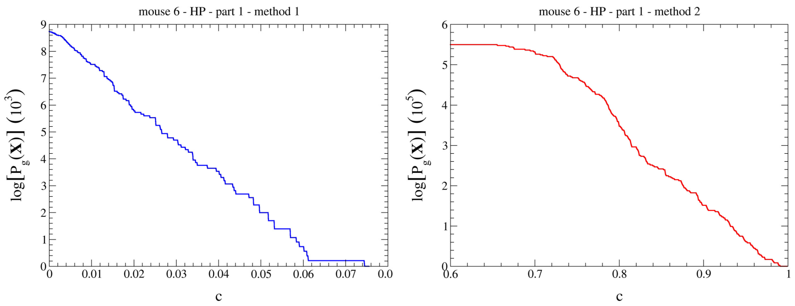
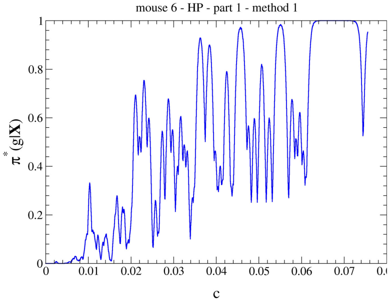
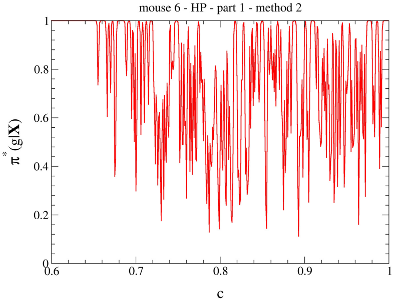
Likelihood of the most representative graph as a function of the penalty.

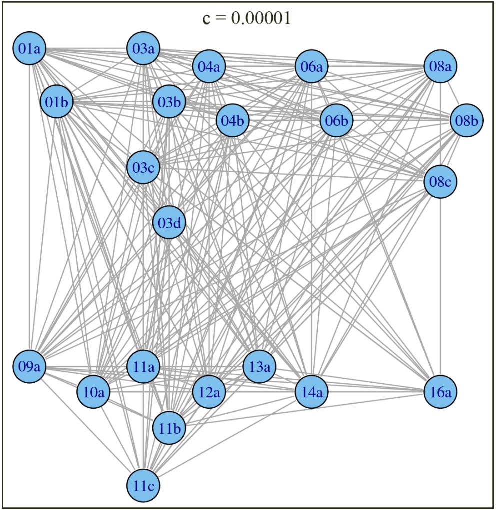


Empirical probabilities (frequency of the most representative graph divided by the number of MC steps) as a function of the penalty.





Method 1: $x_t^{(i)} = x_t^{(j)} = 1$



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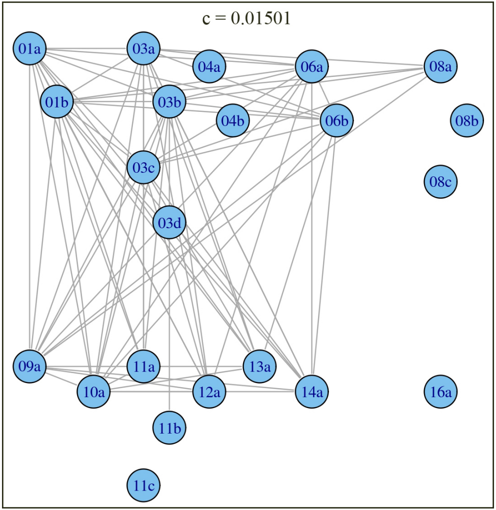

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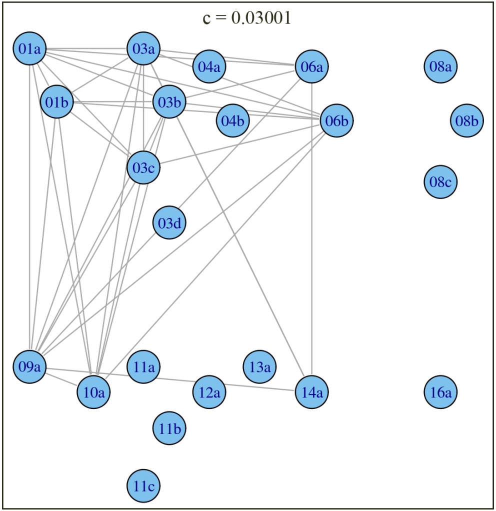

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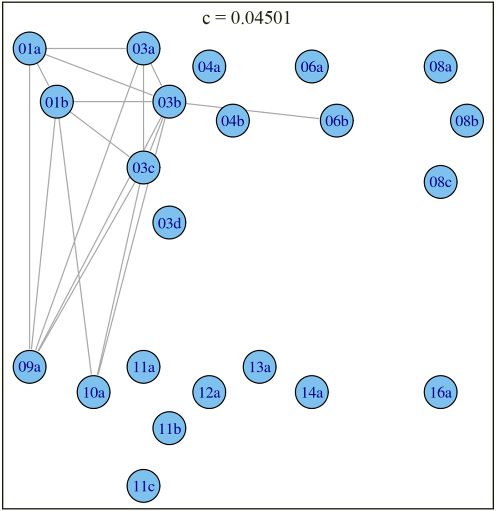
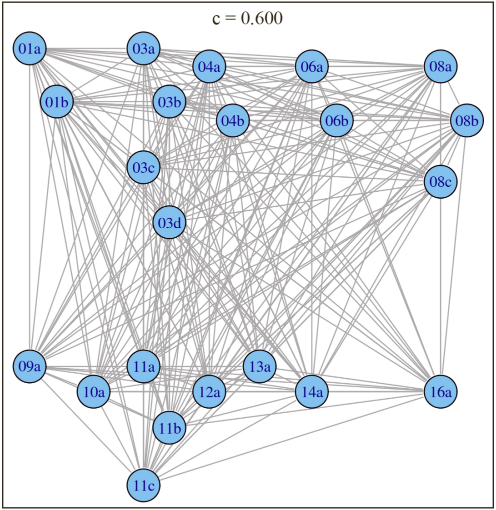


Method 2: $x_t^{(i)} = x_t^{(j)} = 1$ or $x_t^{(i)} = x_t^{(j)} = 0$



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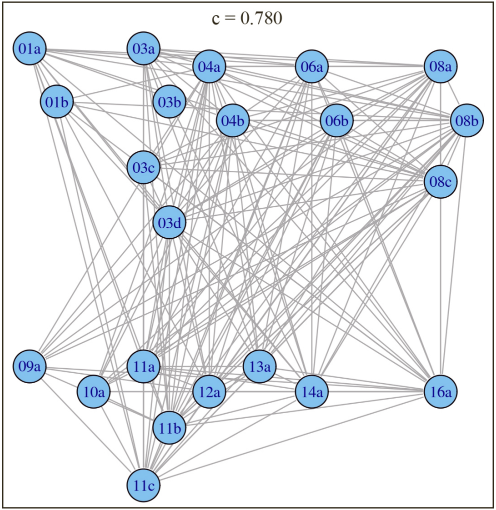

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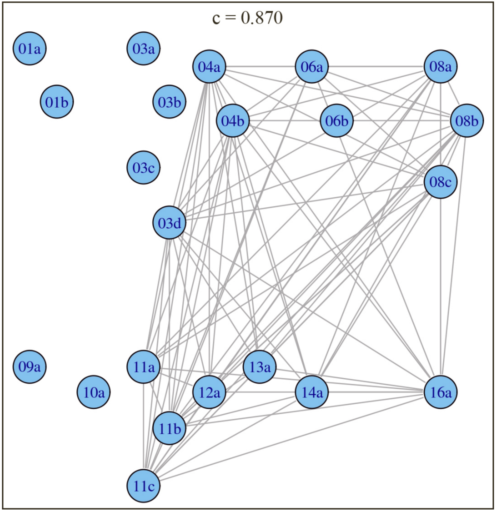

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