№1. $y_{k+2} + y_k = \sin\left(\frac{\pi k}{2}\right)$

$z = x + iy = r\cdot e^{i\varphi}$

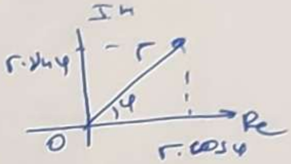
$r = |z| = \sqrt{x^2+y^2}$

$\operatorname{tg}\varphi = \frac{y}{x}$

$\cos\varphi = \frac{x}{|z|}$

$\sin\varphi = \frac{y}{|z|}$

$e^{i\varphi} = \cos\varphi + i\sin\varphi$



$\mu^2 + 1 = 0$

$\mu_{1,2} = \pm i$ $\quad r = 1 \quad \cos\varphi = 0, \ \sin\varphi = 1 \ \Big| \ e^{i\varphi}, \ \varphi = \frac{\pi}{2}$

$y_k = C_1\cdot i^k + C_2(-i)^k = C_1\cdot e^{i\frac{\pi}{2}k} + C_2\cdot k\cdot e^{-i\frac{\pi}{2}k} =$

$= C_1\left(\cos\frac{\pi k}{2} + i\sin\frac{\pi k}{2}\right) + C_2\left(\cos\frac{\pi k}{2} - i\sin\frac{\pi k}{2}\right) =$

$= \tilde{C}_1\cos\frac{\pi k}{2} + \tilde{C}_2\cdot\sin\frac{\pi k}{2}$ | однородное

неодн.: $\alpha = 1$

$1\cdot e^{\pm i\frac{\pi}{2}} = \pm i$ — корень? да, корень

$y_k^r = k\left(A\sin\frac{\pi k}{2} + B\cos\frac{\pi k}{2}\right) \ \Big| \ (k+2)\left(A\sin\frac{\pi(k+2)}{2} + B\cos\frac{\pi(k+2)}{2}\right) + k\left[A\sin\frac{\pi k}{2} + B\cos\frac{\pi k}{2}\right] = \sin\frac{\pi k}{2}$

$\cancel{y = k\left(A\sin\frac{\pi k}{2} + B\cos\frac{\pi k}{2}\right) + C\cos\frac{\pi k}{2} + D\sin\frac{\pi k}{2}}$

$\sin\frac{\pi k}{2}\left[A\overset{-1}{\cos\pi}(k+2) + kA\right] \mp \cancel{B\sin\frac{\pi k}{2}\cdot\overset{0}{\sin\pi}(k+2)} + \cos\frac{\pi k}{2}\left[\cancel{A\overset{0}{\sin\pi}(k+2)} + Bk + B\overset{-1}{\cos\pi}(k+2)\right] = \sin\frac{\pi k}{2}$

$-2A = 1 \qquad B = 0$ (вообще сразу было понятно $f_k = \sin\frac{\pi k}{2}$

$-2B = 0 \qquad A = -\frac{1}{2}$

$y = -\frac{k}{2}\sin\frac{\pi k}{2} + C_1\cos\frac{\pi k}{2} + C_2\sin\frac{\pi k}{2} = C_1\cos\frac{\pi k}{2} + \sin\frac{\pi k}{2}\left(C_2 - \frac{k}{2}\right)$

Проверка

$-C_1\cos\frac{\pi k}{2} + C_1\cos\frac{\pi k}{2} - \sin\frac{\pi k}{2}\left(C_2 - \frac{k}{2} - 1\right) + \sin\frac{\pi k}{2}\left(C_2 - \frac{k}{2}\right) = \sin\frac{\pi k}{2}$

$\boxed{\text{Ответ: } y = C_1\cdot\cos\frac{\pi k}{2} + \sin\frac{\pi k}{2}\left(C_2 - \frac{k}{2}\right).}$

№2

С.З.–?

$$\begin{cases} \dfrac{y_{k+1} - 2y_k + y_{k-1}}{h^2} = -\lambda y_k \quad h = \dfrac{1}{N} \quad 1 \leq k \leq N-1 \\ \dfrac{2}{h^2}(y_1 - y_0) = -\lambda y_0 \\ -\dfrac{2}{h^2}(y_N - y_{N-1}) = -\lambda \cdot y_N \end{cases}$$

$Ax = \lambda x$

$|A - \lambda I| = 0$

$$\Updownarrow$$

$$\frac{1}{h^2}\begin{pmatrix} -2 & 1 & & 0 \\ 1 & -2 & 1 & \\ & 1 & \ddots & \\ & & \ddots & 1 \\ 0 & & 1 & -2 \end{pmatrix}\begin{pmatrix} y_1 \\ \vdots \\ y_{n-1} \end{pmatrix} = -\lambda \begin{pmatrix} y_1 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

$$y_{k+1} + y_{k-1} + y_k[\lambda \cdot h^2 - 2] = 0$$

$$\mu^2 + \mu[\lambda h^2 - 2] + 1 = 0$$

$$\mu_{1,2} = \frac{(\lambda h^2 - 2) \pm \sqrt{(\lambda h^2 - 2)^2 - 4}}{2} = \underbrace{\left[\frac{\lambda h^2}{2} - 1\right]}_{p} \pm \sqrt{\underbrace{\left[\frac{\lambda h^2}{2} - 1\right]}_{p}^2 - 1}$$

$$\mu_{1,2} = \left[\frac{\lambda h^2}{2} - 1\right] \pm \sqrt{\left[\frac{\lambda h^2}{2} - 1\right]^2 - 1} = p \pm \sqrt{p^2 - 1}$$

если $p^2 = 1 \Rightarrow p = \pm 1$

то $\mu_1 = \mu_2 = p$. Тогда $y_k = c_1 p^k + c_2 \cdot k \cdot p^k$

НУ:

I. $\frac{2}{h^2}(c_1 p + c_2 p - c_1 - 0) = -\lambda \cdot c_1$

II. $-\frac{2}{h^2}(c_1 p^N + c_2 N \cdot p^N - c_1 p^{N-1} - c_2 (N-1) p^{N-1} = -\lambda (c_1 p^N + c_2 N \cdot p^N)$

$p = 1$:
$$\begin{cases} \frac{2}{h^2} c_2 = -\lambda \cdot c_1 \\ -\frac{2}{h^2} \cancel{\ } \cdot c_2 = -\lambda \cdot (c_1 + c_2 \cdot N) \end{cases} \Rightarrow -\frac{2}{h^2} \cdot c_2 = \frac{2}{h^2} \cdot c_2 - \lambda \cdot c_2 \cdot N$$

$$c_2 \cdot \left[-\frac{4}{h^2} + \lambda \cdot N\right] = 0$$

т.к. $p = \frac{\lambda h^2}{2} - 1 = 1 \Rightarrow \frac{\lambda h^2}{2} = 2$

~~т.к.~~ $-1 + N = 0$

$N = 1$ (не м.б)

если $N \neq 1$, то $c_2 = 0$

$\Downarrow$

$c_1 = 0$,

$y_k = 0$

$p = -1$:
$$\begin{cases} \frac{2}{h^2}(-2c_1 - c_2) = -\lambda c_1 \\ -\frac{2}{h^2}(2c_1(-1)^N - 2c_2 N (-1)^N - c_2 (-1)^N) = -\lambda [c_1 (-1)^N + c_2 \cdot N (-1)^N] \end{cases} \quad \Big| \; c_2 = -c_1 \left(-\lambda + \frac{4}{h^2}\right) \cdot \frac{h^2}{2}$$

$\Downarrow$

$$-\frac{2}{h^2}\left(2c_1 + 2c_1 \cdot N\left(-\frac{\lambda \cdot h^2}{2} + 2\right) + c_1\left(-\lambda \frac{h^2}{2} + 2\right) = -\lambda \cdot \left[c_1 + N c_1 \left(-\lambda \cdot \frac{h^2}{2} + 2\right)\right]\right.$$

$$p = \frac{\lambda \cdot h^2}{2} - 1 = -1 \Rightarrow \lambda \frac{h^2}{2} = 0$$

$\Leftarrow$

$-(2c_1 + 2c_1 \cdot N \cdot 2 + c_1 \cdot 2) = 0$

$c_1 (2 + 2N) = 0 \quad | \; N = -1$ (не м.б.)

или $c_1 = 0 \Rightarrow c_2 = 0 \Rightarrow y_k = 0$.

итого: $\mu_1 = \mu_2 = p \quad y_k = 0$.

$\measuredangle \; \mu_1 \neq \mu_2 \qquad y_k = c_1 \mu_1^k + c_2 \mu_2^k$

НУ:
$$\begin{cases} \text{I.} \; \frac{2}{h^2}(c_1 \mu_1 + c_2 \mu_2 - c_1 - c_2) = -\lambda (c_1 + c_2) \\ \text{II.} \; -\frac{2}{h^2}(c_1 \mu_1^N + c_2 \mu_2^N - c_1 \mu_1^{N-1} - c_2 \mu_2^{N-1}) = -\lambda \cdot (c_1 \mu_1^N + c_2 \mu_2^N) \end{cases}$$

~~$\frac{2}{h^2}\left(p(c_1 + c_2) + \sqrt{p^2 - 1}(c_1 - c_2) - (c_1 + c_2)\right) = -\lambda (c_1 + c_2)$~~

~~$p(c_1 + c_2) + \sqrt{p^2 - 1}(c_1 - c_2) = (p)(c_1 + c_2)$~~

~~$\left(-\lambda \cdot \frac{h^2}{2} + 1\right)(c_1 + c_2)$~~

~~$2p(c_1 + c_2) = \sqrt{p^2 - 1}(c_2 - c_1)$~~ $\; | \; c_1 \cdot \mu_1 + c_2 \mu_2 = 0$

$-p$

$\frac{2}{h^2}(c_1 \mu_1^N + c$

I. $c_1\mu_1 + c_2\mu_2 = (c_1+c_2)\Big[\underbrace{-\frac{h^2\lambda}{2} + 1\Big)}_{-p = -\frac{\mu_1+\mu_2}{2}}$

$-(c_1\mu_1 + c_2\mu_2) = \frac{(c_1+c_2)(\mu_1+\mu_2)}{2}$

I. $-(c_1\mu_1 + c_2\mu_2) = c_1\frac{\mu_1}{2} + c_2\frac{\mu_2}{2} + c_1\frac{\mu_2}{2} + c_2\frac{\mu_1}{2}$

$c_1 = \frac{-c_2(3\mu_2+\mu_1)}{3\mu_1+\mu_2}$

II. $c_1\mu_1^N + c_2\mu_2^N - c_1\mu_1^{N-1} - c_2\mu_2^{N-1} = (p+1)(c_1\mu_1^N + c_2\mu_2^N)$

$p+1 = \frac{\mu_1+\mu_2}{2} + 1$

$-c_1\mu_1^{N-1} - c_2\mu_2^{N-1} = c_1\frac{\mu_1^N\cdot\mu_1}{2} + c_1\frac{\mu_2\mu_1^N}{2} + c_2\frac{\mu_2^N\cdot\mu_2}{2} + c_2\frac{\mu_2^N\mu_1}{2}$

$$\boxed{c_1 = \frac{\frac{3}{2}c_2\mu_2 + \frac{c_2\mu_1}{2}}{-\frac{3}{2}\mu_1 - \frac{\mu_2}{2}} = -c_2\frac{(3\mu_2+\mu_1)}{(3\mu_1+\mu_2)}}$$

$$\begin{cases} -c_1 \mu_1^{N-1} - c_2 \mu_2^{N-1} = \dfrac{(\mu_1+\mu_2)}{2}\left(c_1 \mu_1^{N} + c_2 \mu_2^{N}\right) \\ c_1 = -c_2 \dfrac{(3\mu_2+\mu_1)}{3\mu_1+\mu_2} \end{cases}$$

$$c_2\left[\frac{(3\mu_2+\mu_1)}{(3\mu_1+\mu_2)}\cdot\mu_1^{N-1} - \mu_2^{N-1}\right] + \frac{\mu_1^N(\mu_1+\mu_2)(3\mu_2+\mu_1)}{2(3\mu_1+\mu_2)} - \frac{\mu_2^N(\mu_1+\mu_2)}{2}\Bigg] = 0$$

$$2\mu_2(3\mu_2+\mu_1)\cdot\mu_1^N - 2(3\mu_1+\mu_2)\cdot\mu_1\cdot\mu_2^N + \mu_1\mu_2\cdot\mu_1^N\underbrace{(\mu_1+\mu_2)(3\mu_2+\mu_1)}_{\mu_1^2+3\mu_2^2+4\mu_1\mu_2} - \mu_2^N(\mu_1+\mu_2)\cdot(3\mu_1+\mu_2)\mu_1\mu_2 = 0$$

$$\left[6\mu_2^2+2\mu_2\mu_1\right]\mu_1^N + \mu_1^N\left[\mu_1^3\mu_2 + 3\mu_2^3\mu_1 + 4\mu_1^2\mu_2^2\right] - \mu_2^N\left[6\mu_1^2+2\mu_2\mu_1\right] - \mu_2^N\left[\mu_2^3\mu_1 + 3\mu_1^3\mu_2 + 4\mu_1^2\mu_2^2\right] = 0$$

Т. Виета $\mu_1\cdot\mu_2 = 1$

$$\left[6\mu_2^2+2\right]\mu_1^N + \mu_1^N\left[\mu_1^2+3\mu_2+4\right] - \mu_2^N\left[6\mu_1^2+2\right] - \mu_2^N\left[\mu_2^2+3\mu_1+4\right] = 0 \quad \Big| \cdot\frac{1}{\mu_1^2\mu_2^2} = 1$$

$$\cancel{\left[6\mu_1^{N-2}+2\right]\cdot\mu_1^N + \mu_1^{N+2} + 3\frac{\mu_1^{N-2}}{\mu_2} + 4\mu_1^N - 6\mu_2^{N-2} - 2\mu_2^N} - \mu_2^{N+2} - 3\cancel{\mu_2^{N-2}}\mu_1 - 4\mu_2^N = 0$$

$$\mu_1^N\left[\mu_1^2+3\mu_2+6+6\mu_2^2\right] - \mu_2^N\left[\mu_2^2+3\mu_1+6+6\mu_1^2\right] = 0 \quad \Big| : \frac{1}{\mu_1\mu_2}$$

$$\mu_1^2+3\mu_2+6\mu_2^2+6 = p^2+2\sqrt{\ } + p^2-1 + 6p^2 - 12\sqrt{\ } + 6(p^2-1) + 3p - 3\sqrt{\ } + 6 = 14p^2+3p-13\sqrt{p^2-1}$$

$$\mu_1^N\left(14p^2+3p\right) - \mu_2^N\left(14p^2+3p\right) + \left(\mu_1^N-\mu_2^N\right)\left(-13\sqrt{p^2-1}\right) = 0$$

$$\left(\mu_1^N+\mu_2^N\right)\left(14p^2+3p\right)$$

$$\left(\frac{\mu_1^N+\mu_2^N}{\mu_1^N-\mu_2^N}\right)\left(\frac{14p^2+3p}{+13\sqrt{p^2-1}}\right) = 1$$

$\mu_1+\mu_2 = 2p \qquad (\mu_1+\mu_2)\dfrac{3}{2}$

$$\mu_2^2+3\mu_1+6+6\mu_1^2 = p^2-2\sqrt{\ } + p^2-1 + 6p^2 + 12\sqrt{\ } + 6(p^2-1) + 6 + 3p + 3\sqrt{\ } = 14p^2+3p+13\sqrt{p^2-1}$$