**MIT 9.520: Statistical Learning Theory** **Fall 2014**

# Problem Set 1

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**Problem 1**

**Problem 2** Please write your analysis on Problem 2 here

**Problem 3** Please write your analysis on Problem 3 here

**Problem 4** a) To check that the square loss function can be written as $\mathcal{L}(-yf(x))$ lets expand $\|f(x) - y\|^2$:

$$(y - f(x))^2 = (1 - 2yf(x) + f(x)^2)$$

but $y^2 = 1$ thus:

$$\mathcal{L}(-yf(x)) = (1 - 2yf(x) + (yf(x))^2)$$

To find the minimizer c(x) we need to minimize:

$$\mathbb{E}_{x,y}[(y - f(x))^2]$$

and specify the function that achieves this minimum. Lets find it by taking the derivative of the above wrt to f(x) and setting it to zero:

$$\frac{d}{df(x)}\mathbb{E}_x\mathbb{E}_{y|x}[(y - f(x))^2] = \mathbb{E}_x\frac{d}{df(x)}\mathbb{E}_{y|x}[(y - f(x))^2]$$

which can be minimized by finding the minimum of $\frac{d}{df(x)}\mathbb{E}_{y|x}[(y - f(x))^2]$:

$$\frac{d}{df(x)}\mathbb{E}_{y|x}[(y - f(x))^2] = \mathbb{E}_{y|x}\left[\frac{d}{df(x)}(y - f(x))^2\right] = 0$$

$$\mathbb{E}_{y|x}[2(y - f(x))] = 0$$

$$\mathbb{E}_{y|x}[y] = \mathbb{E}_{y|x}[f(x)]$$

$$\mathbb{E}_{y|x}[y] = f(x)\mathbb{E}_{y|x}[1]$$

$$\mathbb{E}_{y|x}[y] = f(x)$$

$$p_{y|x}(1|x) - p_{y|x}(-1|x) = f(x)\mathbb{E}_{y|x}[1]$$

Since $p_{y|x}(1|x) + p_{y|x}(-1|x) = 1$ then:

$$2p_{y|x}(1|x) - 1 = f(x)\mathbb{E}_{y|x}[1]$$

b) We want to solve:

$$f^*(x) = argmin_{f(x)}\mathbb{E}_{x,y}[e^{-yf(x)}]$$

$$\frac{d}{df(x)}\mathbb{E}_x\mathbb{E}_{y|x}[e^{-yf(x)}] = 0$$

Similar reasoning as the previous question we have:

$$\mathbb{E}_{y|x}\left[\frac{d}{df(x)}e^{-yf(x)}\right] = 0$$

$$\sum_{y\in\{1,-1\}} p_{y|x}(y|x)ye^{-yf(x)} = p_{y|x}(1|x)e^{-f(x)} - p_{y|x}(-1|x)e^{f(x)}$$

$$p_{y|x}(1|x) - p_{y|x}(-1|x)e^{2f(x)} = 0$$

$$p_{y|x}(1|x) = p_{y|x}(-1|x)e^{2f(x)}$$

$$\frac{p_{y|x}(1|x)}{p_{y|x}(-1|x)} = e^{2f(x)}$$

$$\frac{p_{y|x}(1|x)}{p_{y|x}(-1|x)} = e^{2f(x)}$$

$$\frac{1}{2}log\left(\frac{p_{y|x}(1|x)}{p_{y|x}(-1|x)}\right) = f(x)$$

or

$$\frac{1}{2}log\left(\frac{p_{y|x}(1|x)}{1 - p_{y|x}(1|x)}\right) = f(x)$$

c) When we apply a function that is monotonic to another function, then the value that minimizes it does not change. Said differently, if we have a function that preserves monotonicity (and thus preserves order), then the minimizer does not change. i.e. if $f(x) < f(y)$ and $g(x)$ is monotonic then $g(f(x)) < g(f(y))$ and because of that the value of x that minimized $f(x)$ also minimizes $g(f(x))$.

The function $g(x) = x + 1$ is clearly monotonic. So is the function $h(x) = log(x)$. Now consider the following function:

$$h(g(e^{-yf(x)})) = log(g(e^{-yf(x)})) = log(1 + e^{-yf(x)})$$

This time we are trying to minimize:

$$\mathcal{L}(-yf(x)) = (h(g(e^{-yf(x)}))$$

From part b) we notice that its just a composite function of the exponential loss function using two monotonic functions. So without the need of further calculations its clear that the minimizer is the same as part b:

$$\frac{1}{2}log\left(\frac{p_{y|x}(1|x)}{p_{y|x}(-1|x)}\right) = f(x)$$

or

$$\frac{1}{2}log\left(\frac{p_{y|x}(1|x)}{1 - p_{y|x}(1|x)}\right) = f(x)$$

**Problem 5 (MATLAB)** Please write your analysis on Problem 5 here