Notes on reconstruction of optical spectra from 64-bit dFT spectrometer

Derek M. Kita[1] and Brando Miranda[2]

[1]Department of Materials Science & Engineering, Massachusetts Institute of Technology
[2]Center for Brains, Minds, and Machines, Massachusetts Institute of Technology

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Problem Statement & Objective

Given the measured interferogram y ($N \times 1$) and a calibration matrix A ($N \times D$), accurately reconstruct the input optical signal x that obeys:

$$y = Ax \tag{1}$$

where $D \gg N$, and in our case $D = 801$, $N = 64$. For our application, there are two types of signals of interest: (1) laser lines that produce sparse spectra, and (2) light sources (like an EDFA) with a broad spectrum (not-sparse).

L1 and L2 minimization

Since the problem we are solving is underdefined, there are an infinite number of solutions x that solve Eq. 1. However, with prior knowledge of the size of the correct solution's L1- and L2-norm, we can obtain better estimates of x by minimizing:

$$\min_{x} \left\{ ||y - Ax||^2 + \alpha_1 ||x||_1 + \alpha_2 ||x||_2^2 \right\} \tag{2}$$

Solving the above corresponds to the "elastic net" regularized regression method (removing only the L1 norm corresponds to "ridge regression" and removing only the L2 norm corresponds to "LASSO").

Radial Basis Function (RBF) Network

For some application, such as broad input sources, our prior information can consist of how "smooth" the spectrum is. In this case, we can construct a function h that represents our spectrum and we seek to minimize the following for appropriate α, and c_k.

$$\min_{x} \left\{ ||y - Ah(\lambda)||^2 + \alpha \sum_{k} c_k \int \frac{\partial^k h(\lambda)}{\partial \lambda^k} d\lambda \right\} \tag{3}$$

The solution to the above minimization problem (that puts constraints on the solution's smoothness), is a RBF Network, which approximates h with radial basis functions:

$$h_c(\lambda) = \sum_{d=1}^{D} c_d e^{-\beta|\lambda - \lambda_d|^2} \tag{4}$$

We can then use an appropriate algorithm like stochastic gradient descent (SGD) to solve the following, simpler minimization problem:

$$\min_{c} \left\{ ||y - Ah_c(\lambda)||^2 \right\} \tag{5}$$