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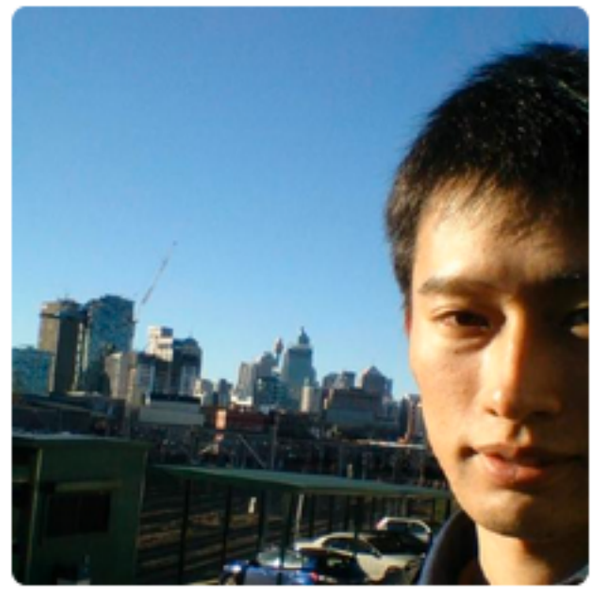
# AI for theorem proving in Isabelle/HOL

*This work was supported by the project AI&Reasoning (reg. no. CZ.02.1.01/0.0/0.0/15_003/0000466).*

Yutaka Nagashima
University of Innsbruck
Czech Technical University



**Yutaka Ng**
yutakang

Block or report user

CVUT, CTU, CIIRC

## Mathematics

Number Theory

Analysis

Algebra

Geometry

Probability Theory

etc.

## Physics

Acoustics

Astrophysics

Electromagnetism

Molecular Physics

Quantum Physics

etc.

## Informatics

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Architecture

Software Engineering

**Formal Method**
Computational Logic

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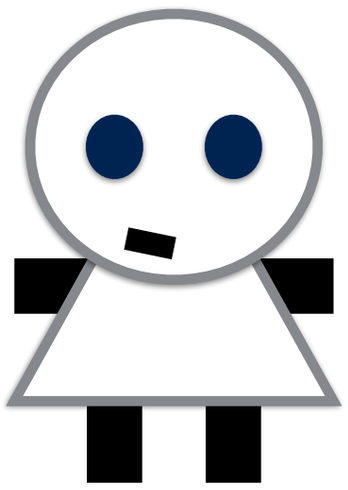

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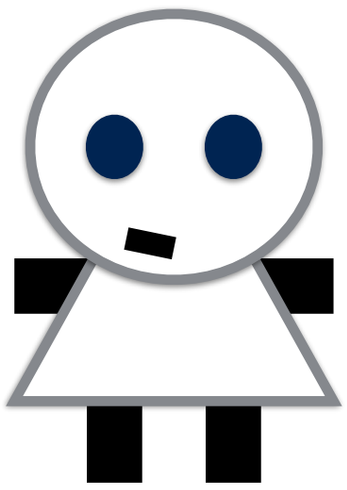

A tiny field inside Informatics. Who cares?

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Automate Logic using AI to Accelerate Science!

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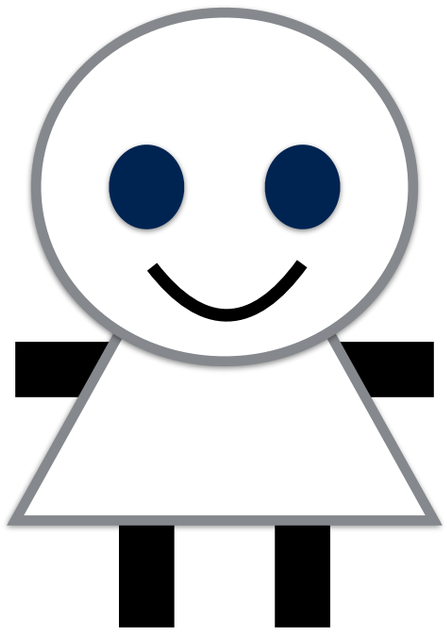

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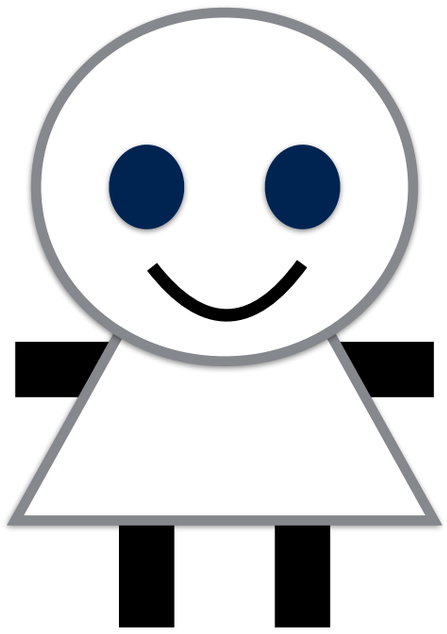
# Interactive theorem proving with Isabelle/HOL

**proof goal** **context**

**tactic / proof method**



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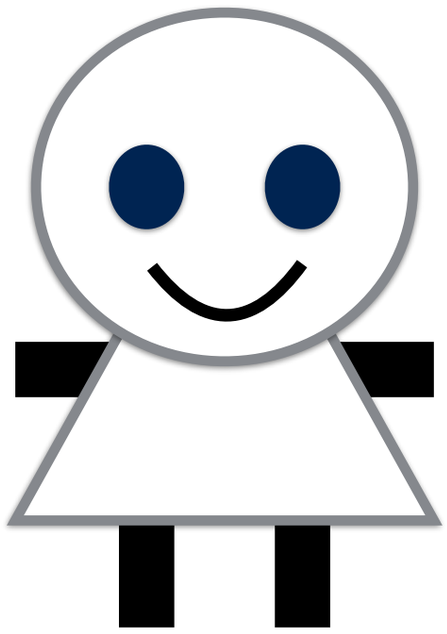


error-message

subgoals

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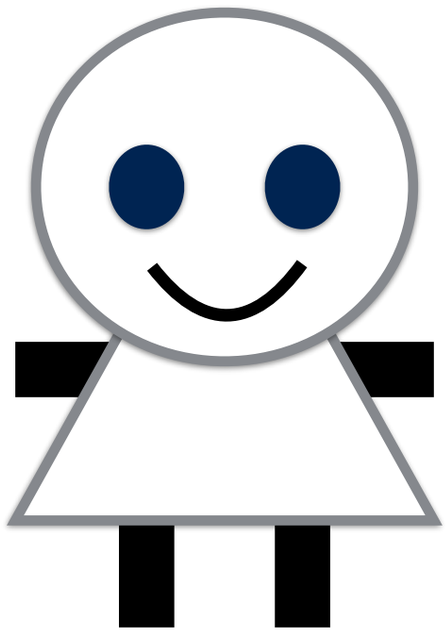


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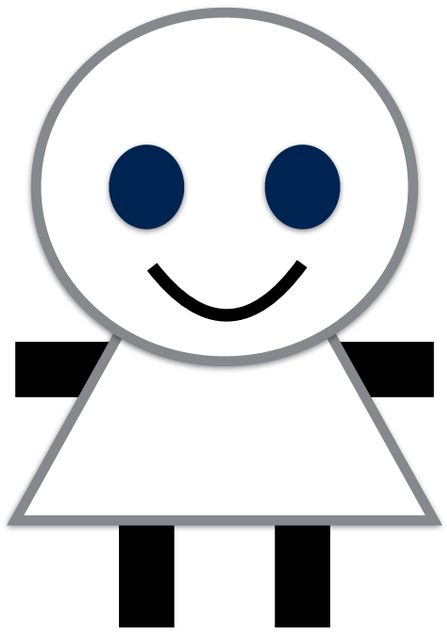

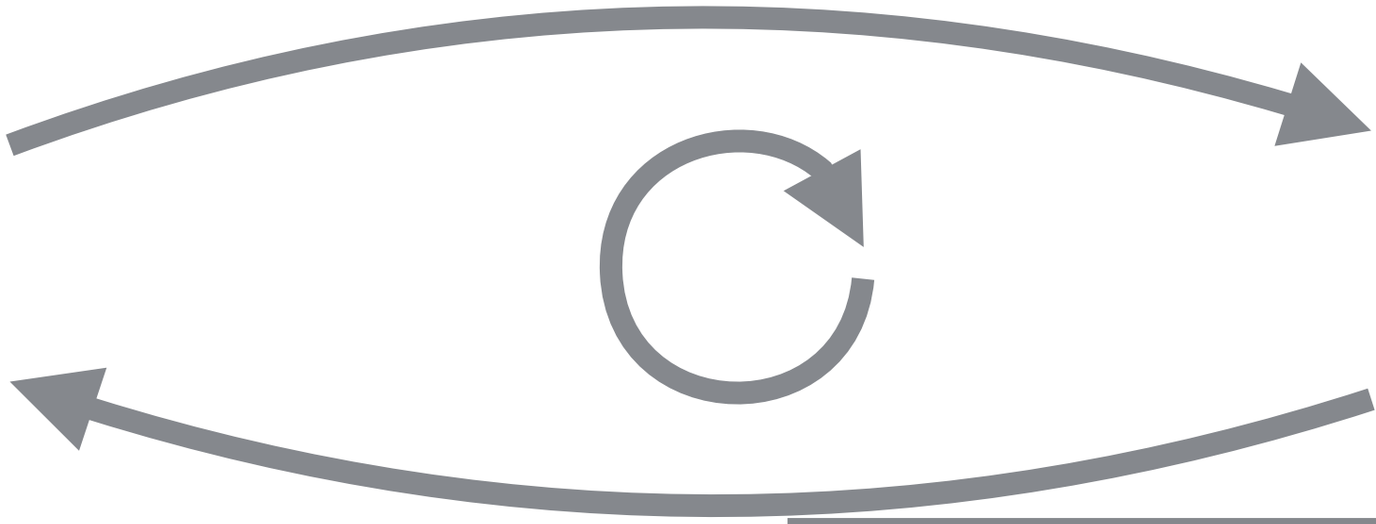


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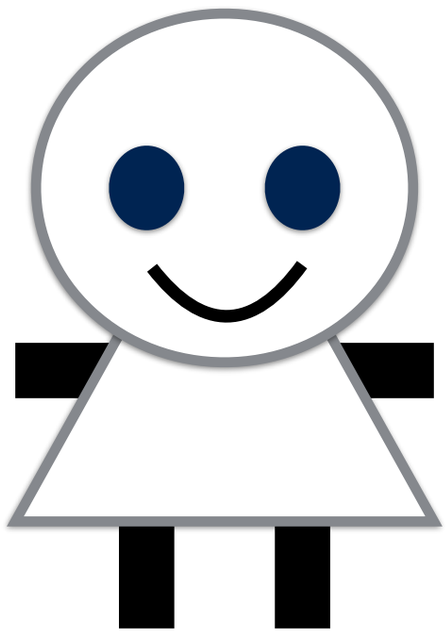


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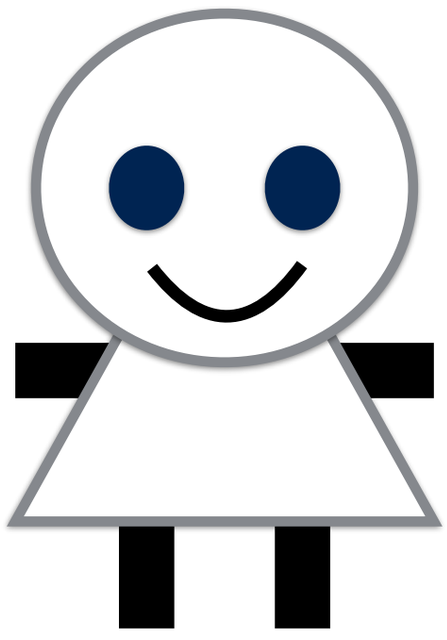


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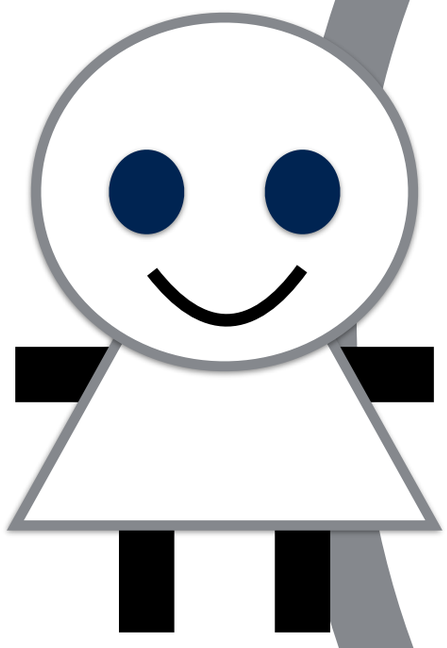

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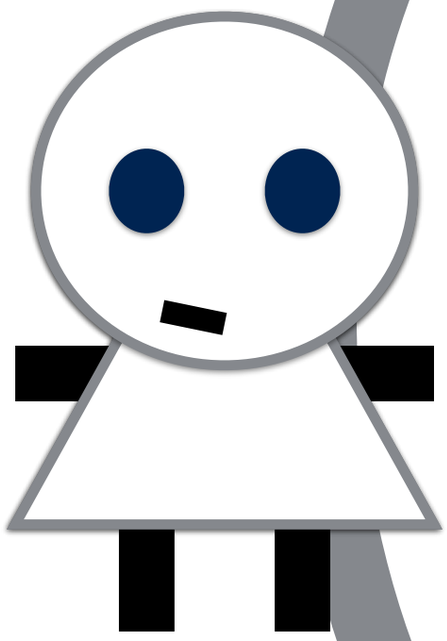

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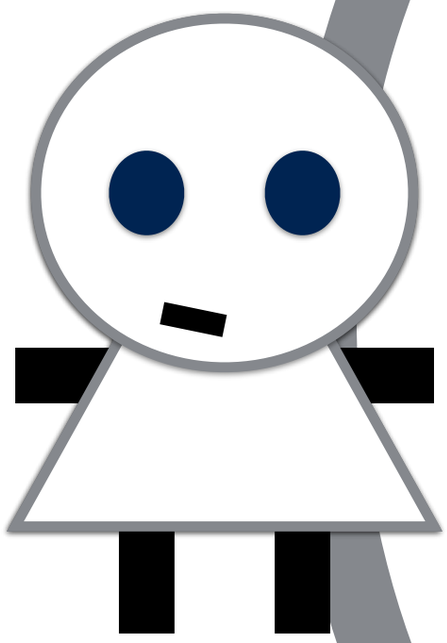


error-message

o-goal!

```
It's blatantly clear
You stupid machine, that what
I tell you is true
(Michael Norrish)
```

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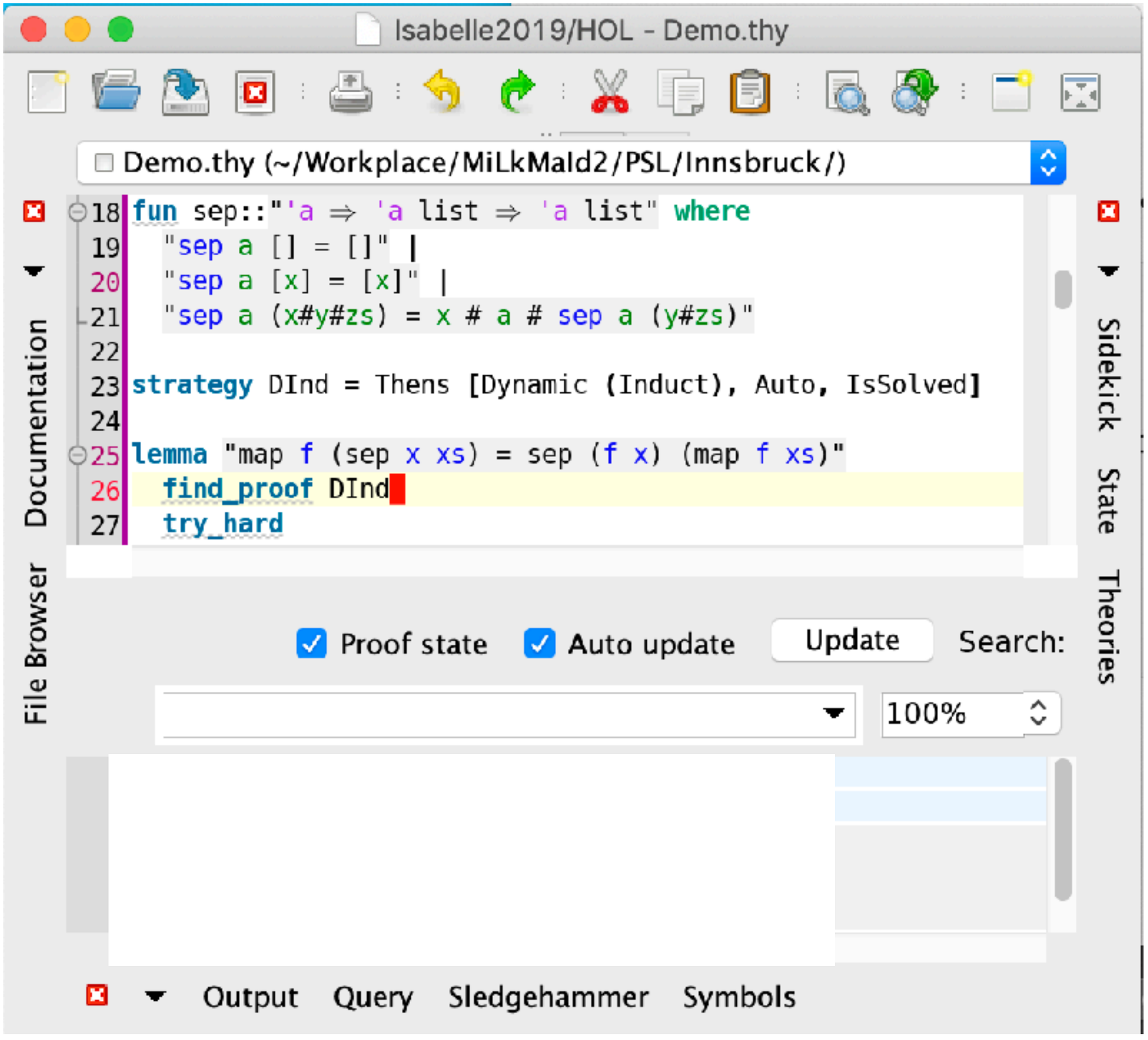

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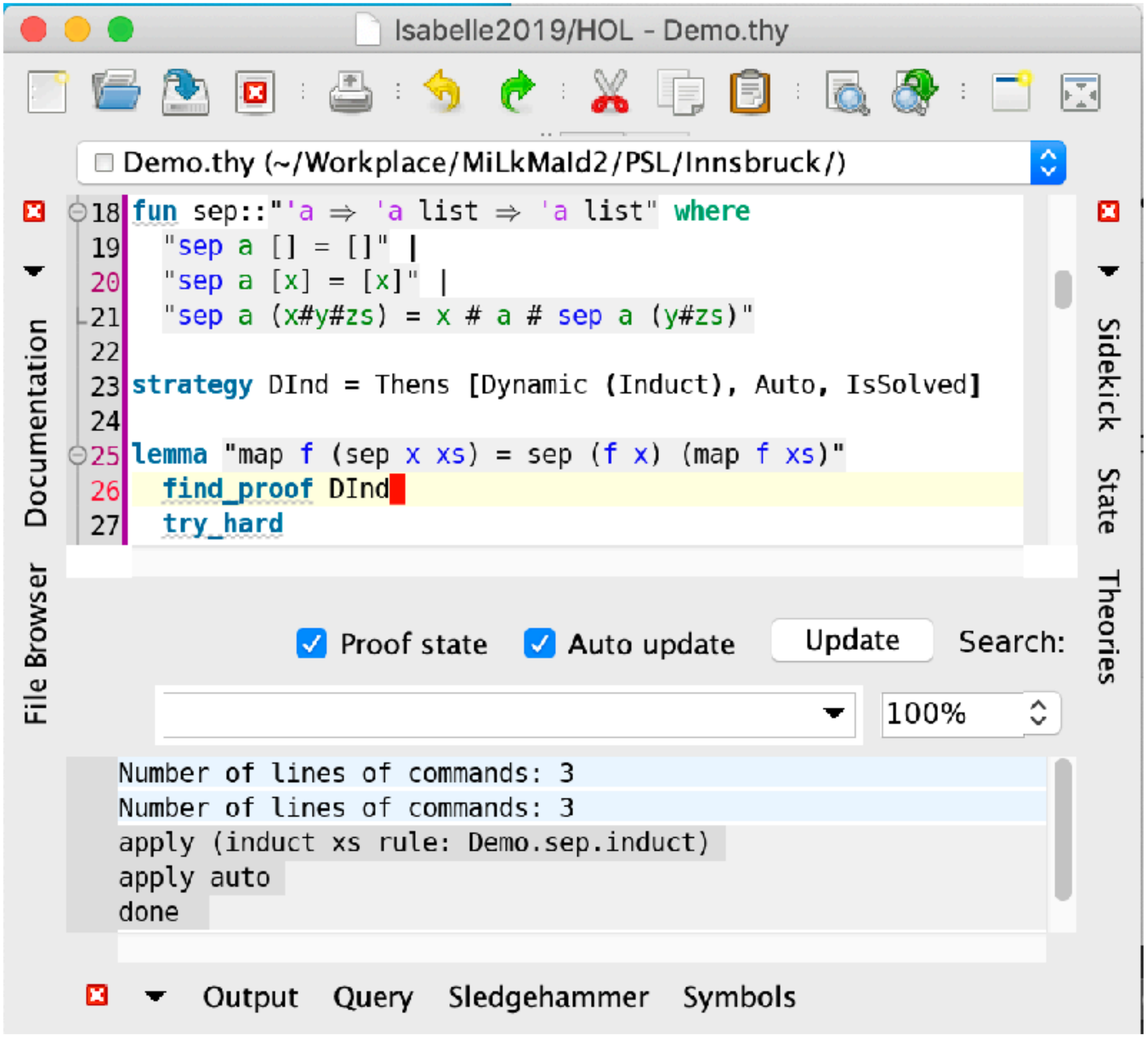

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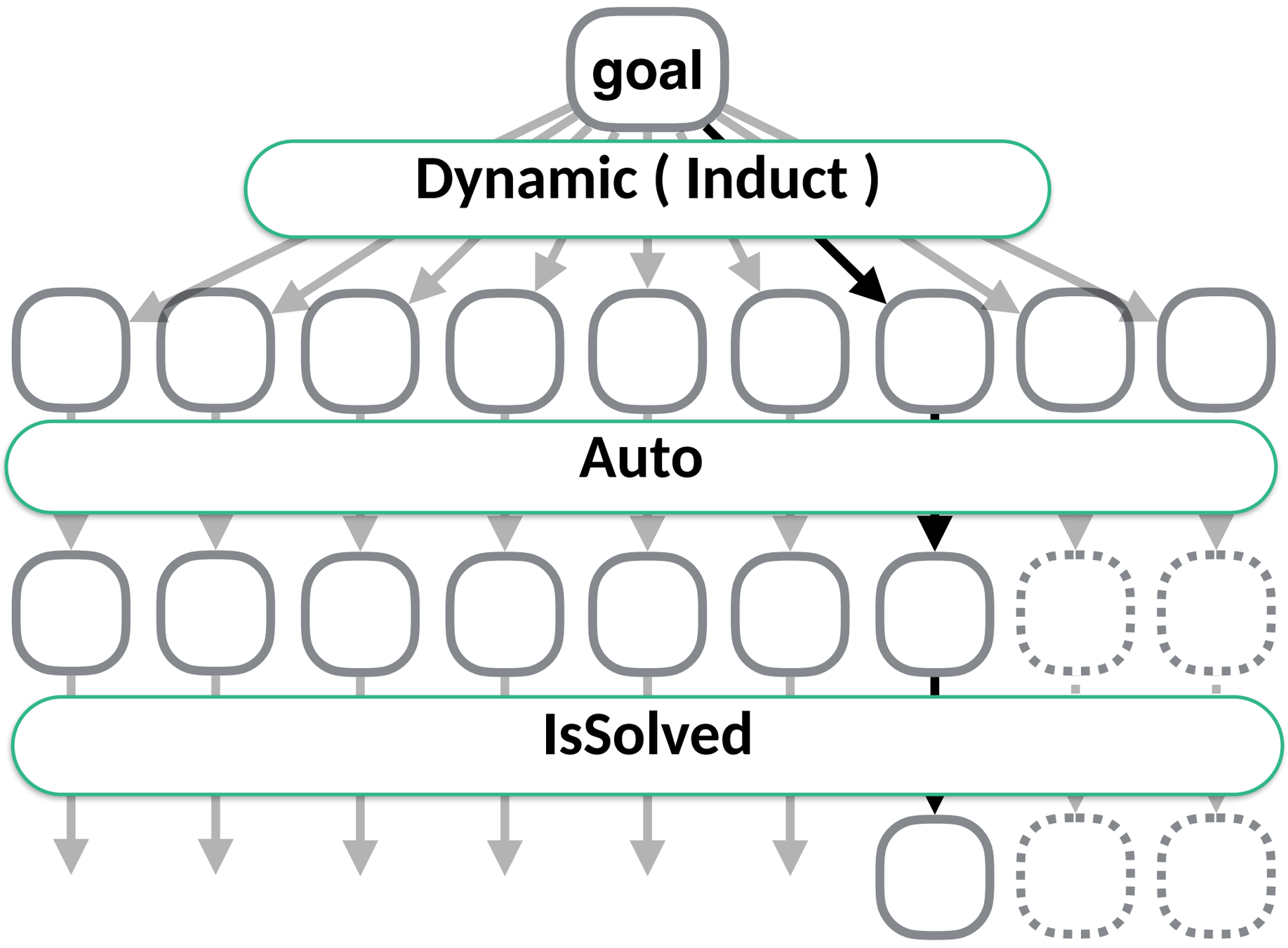

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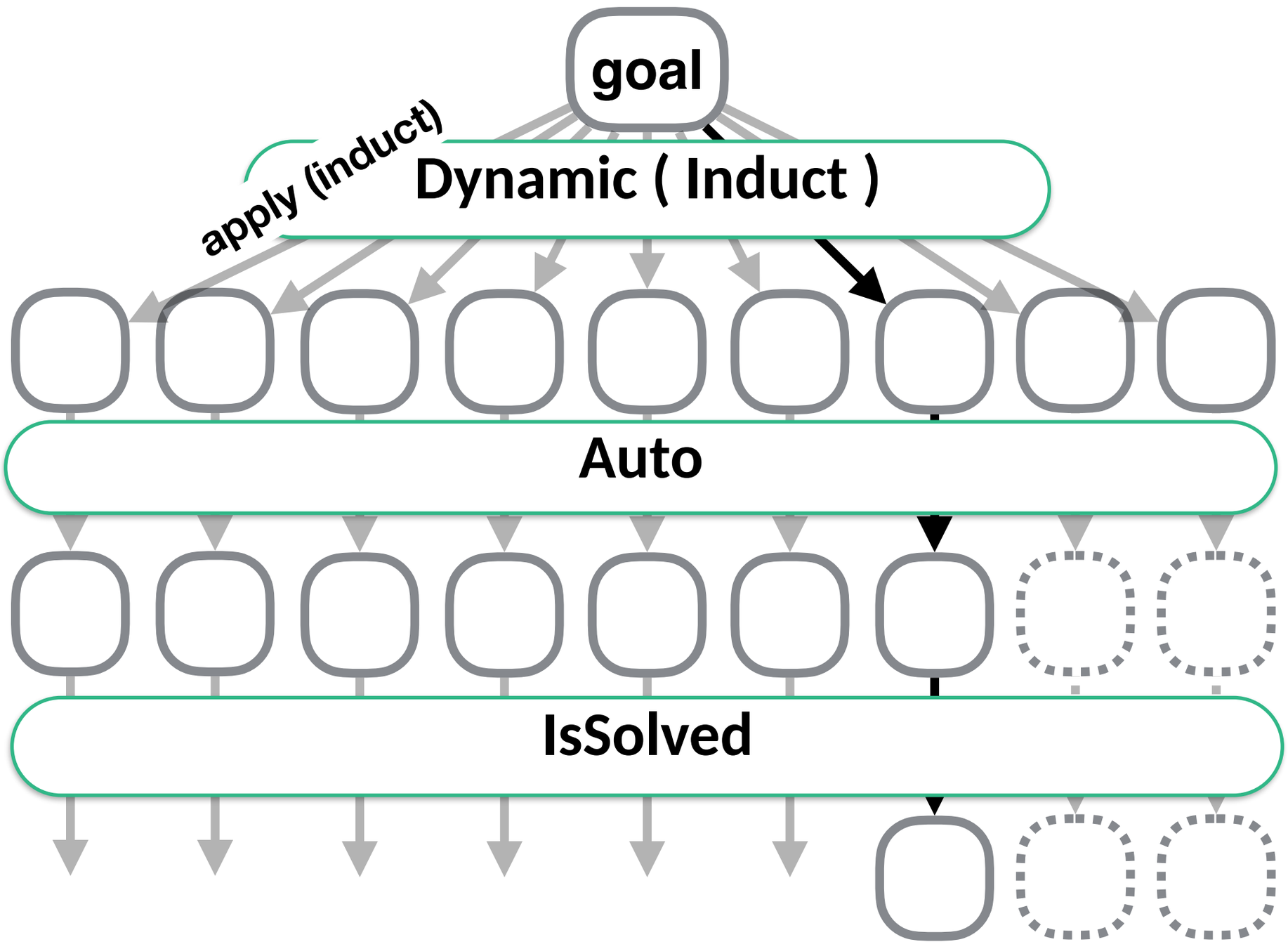

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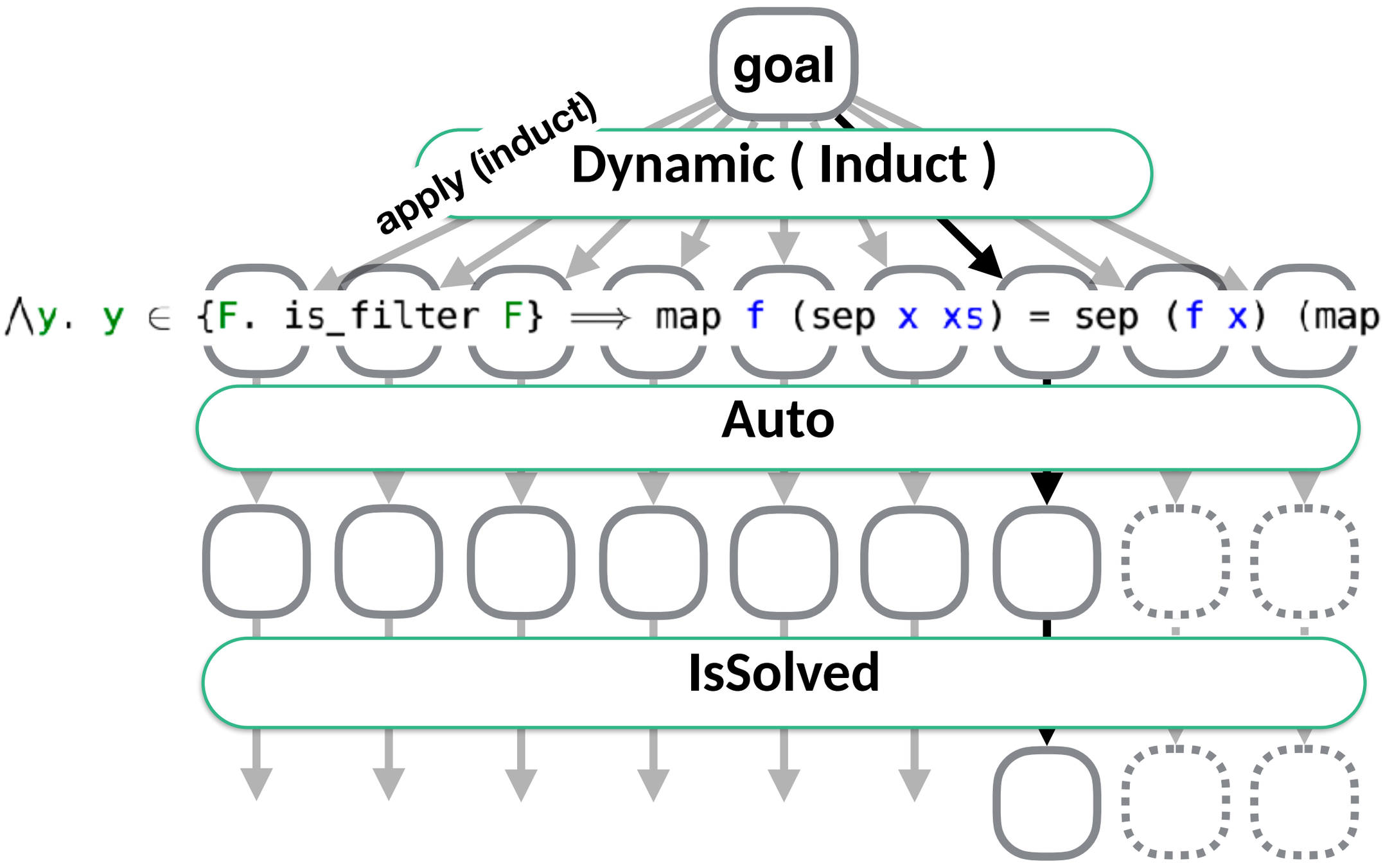

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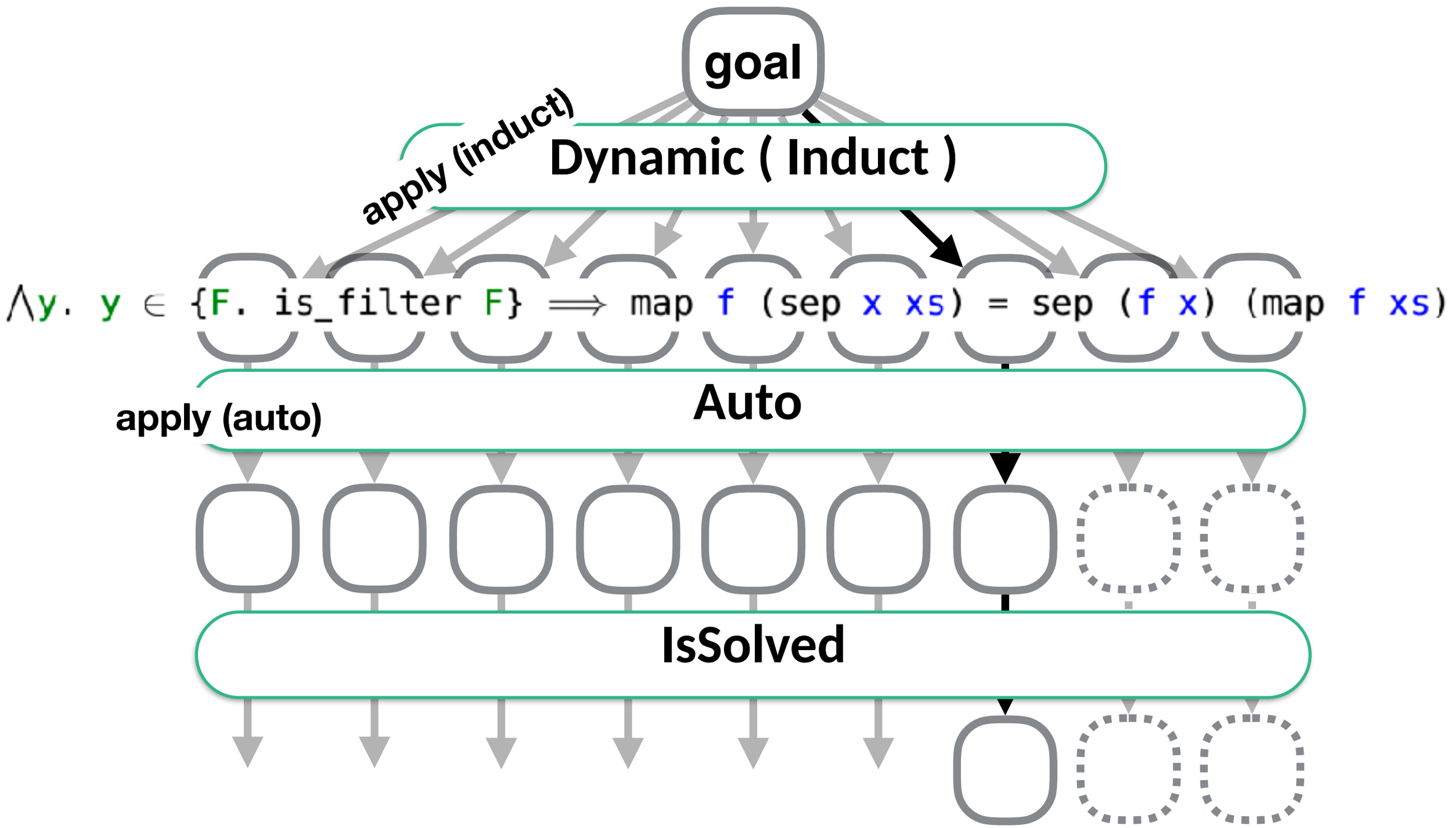

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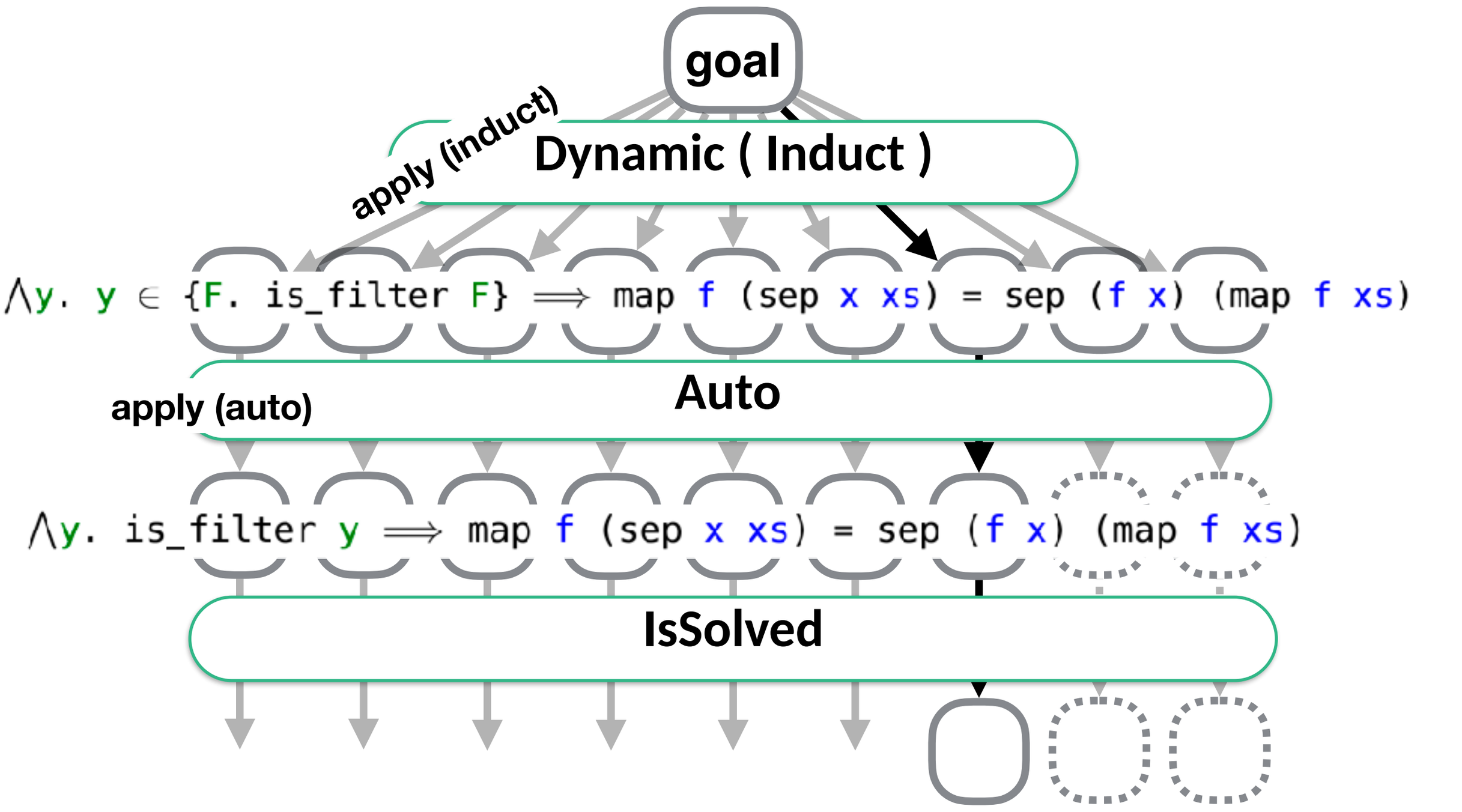

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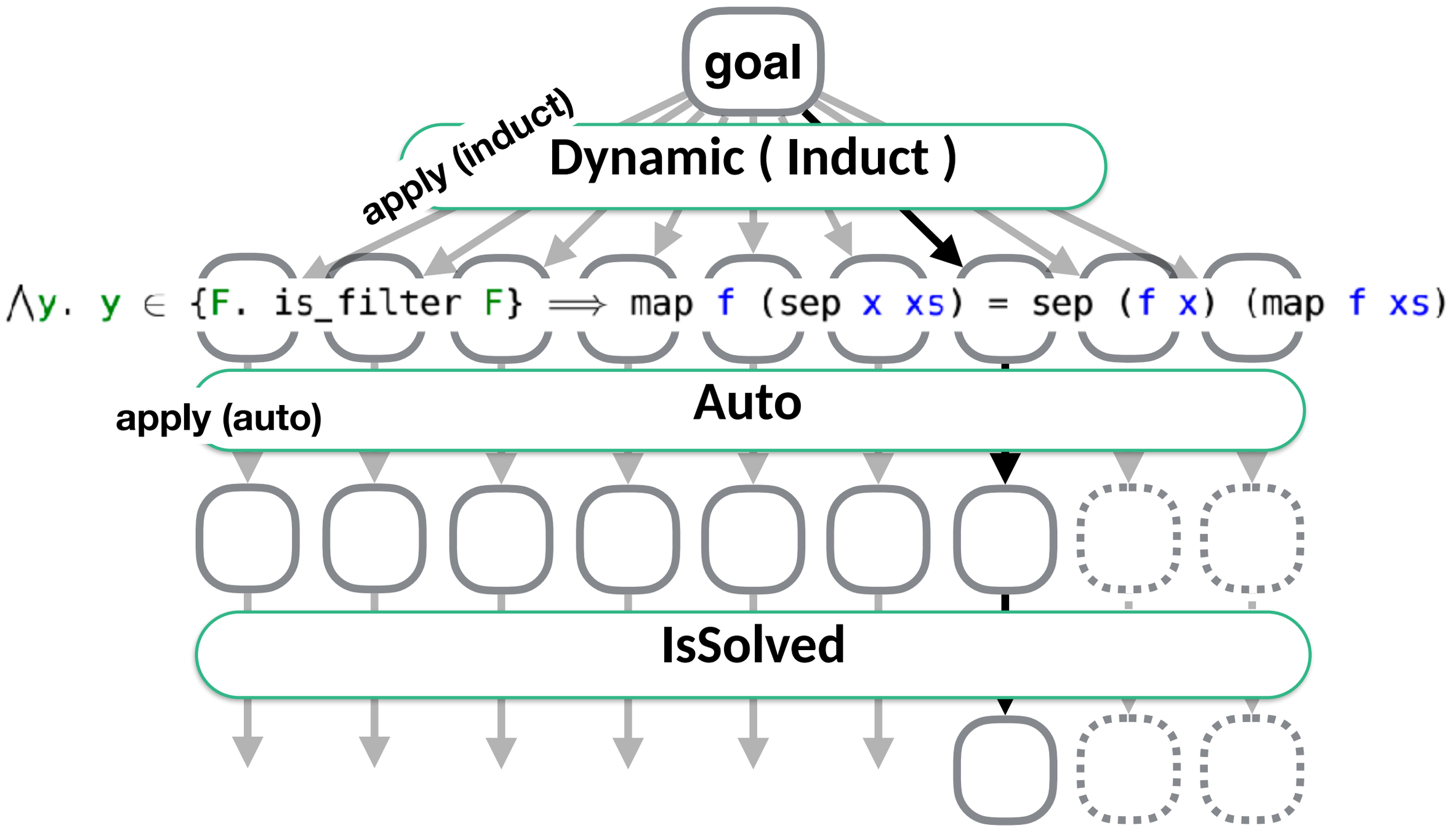

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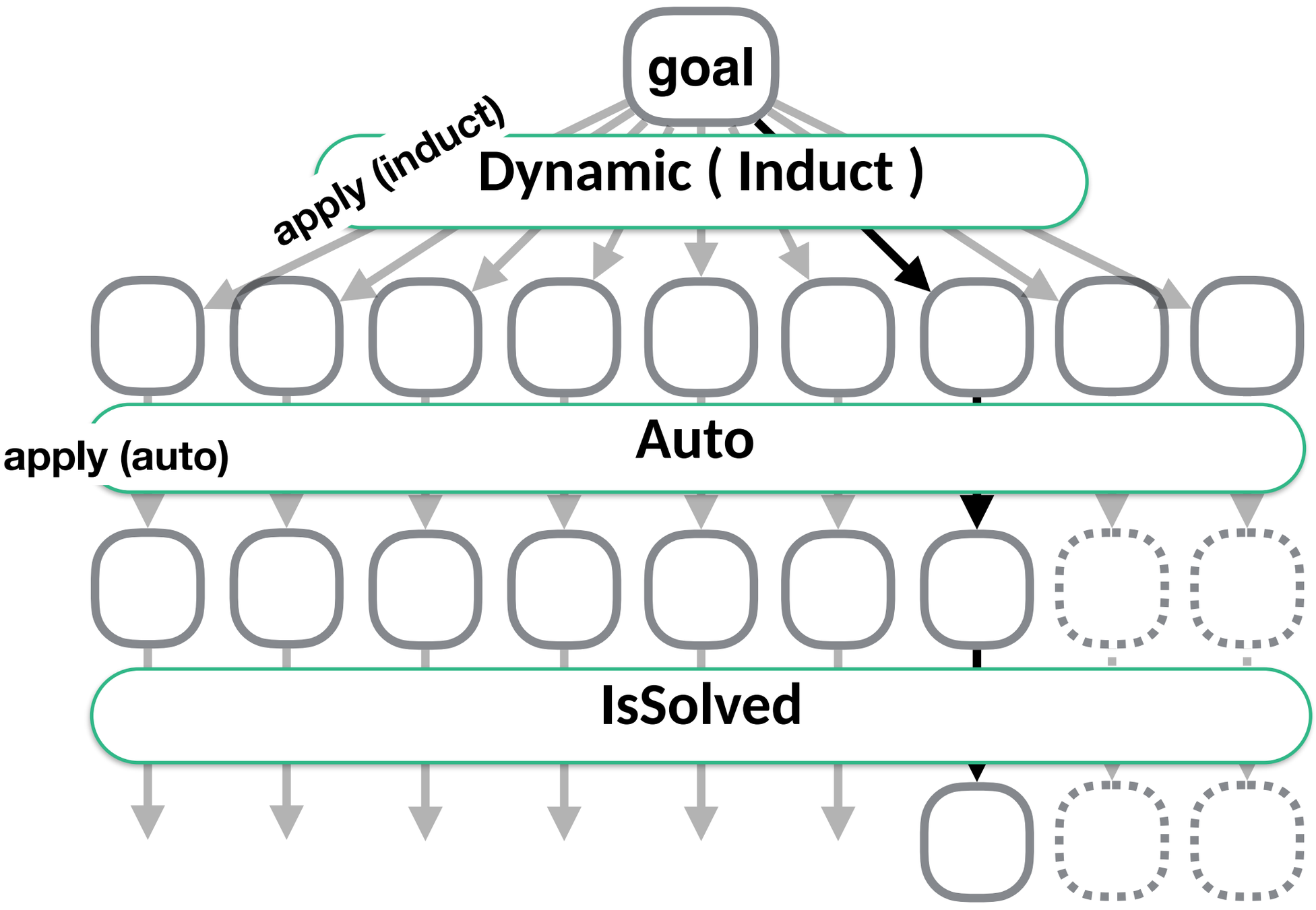

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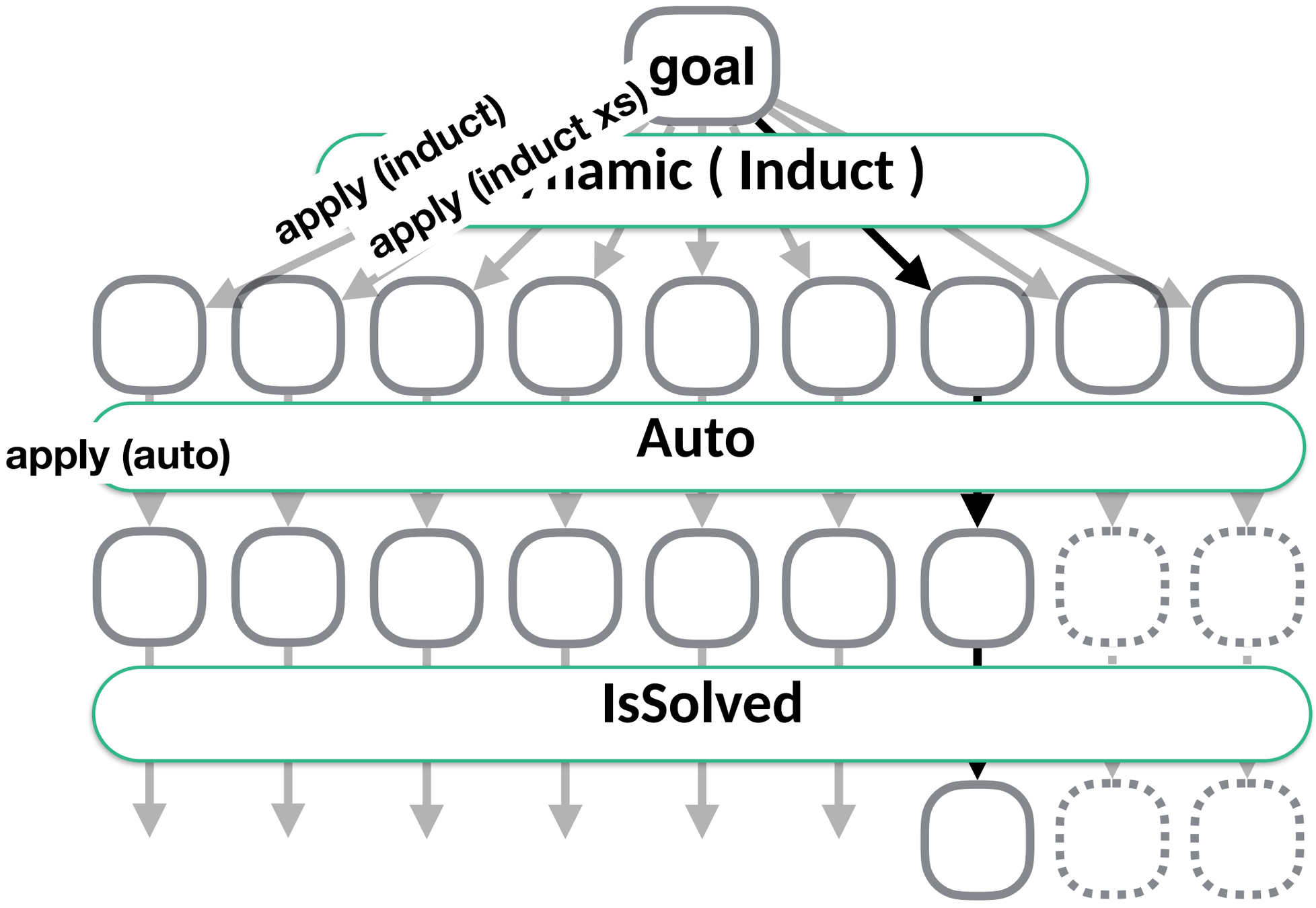

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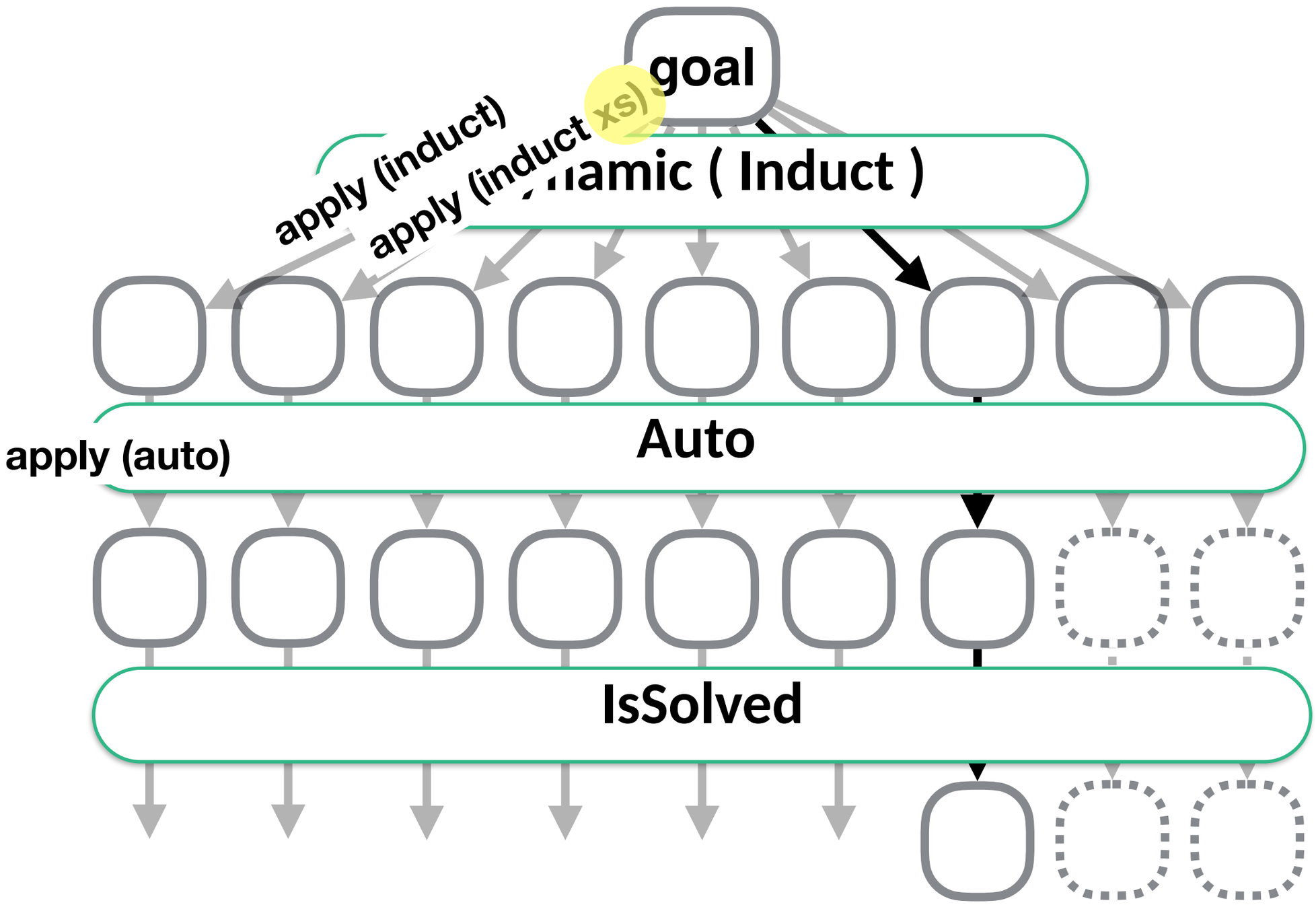

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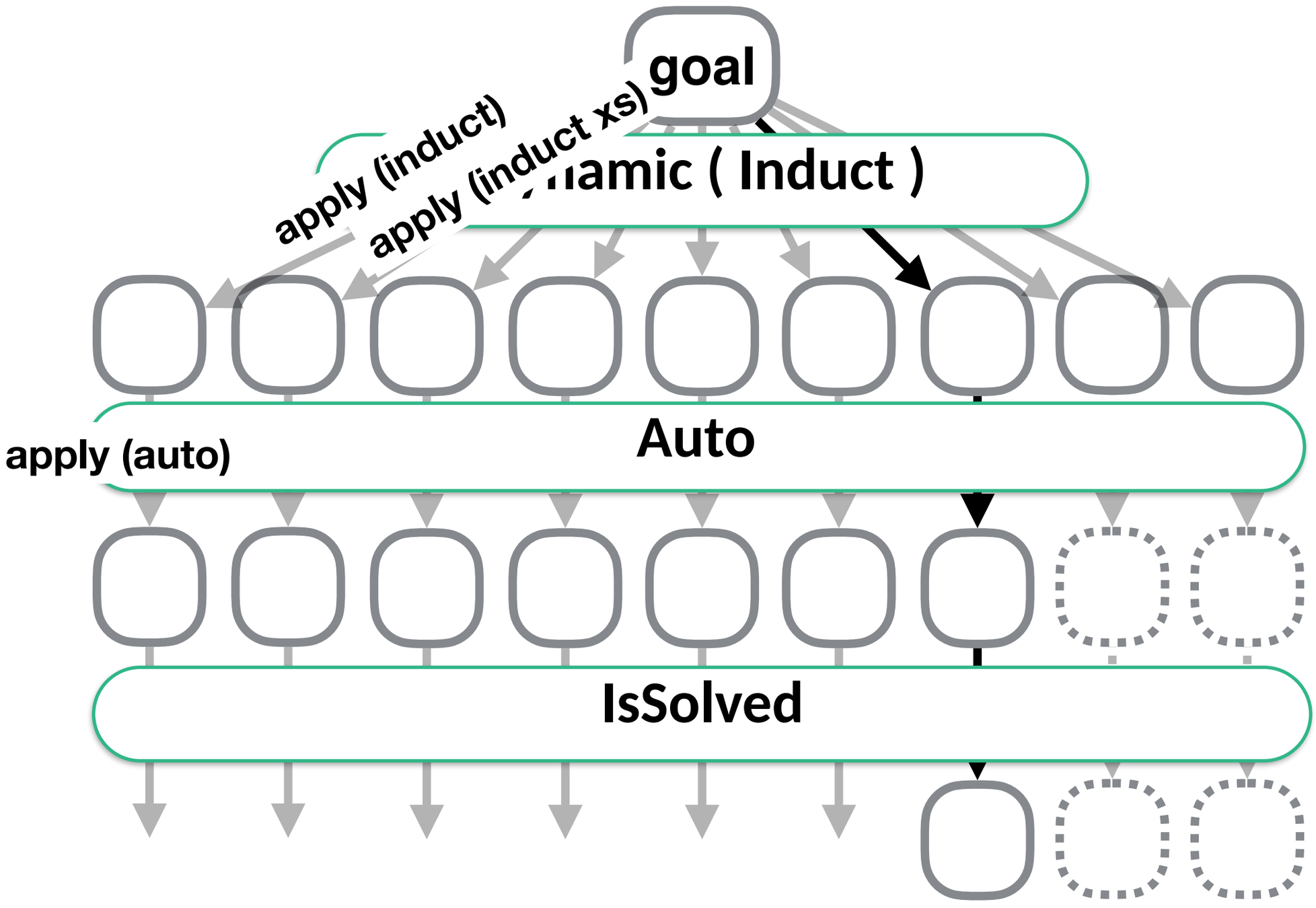

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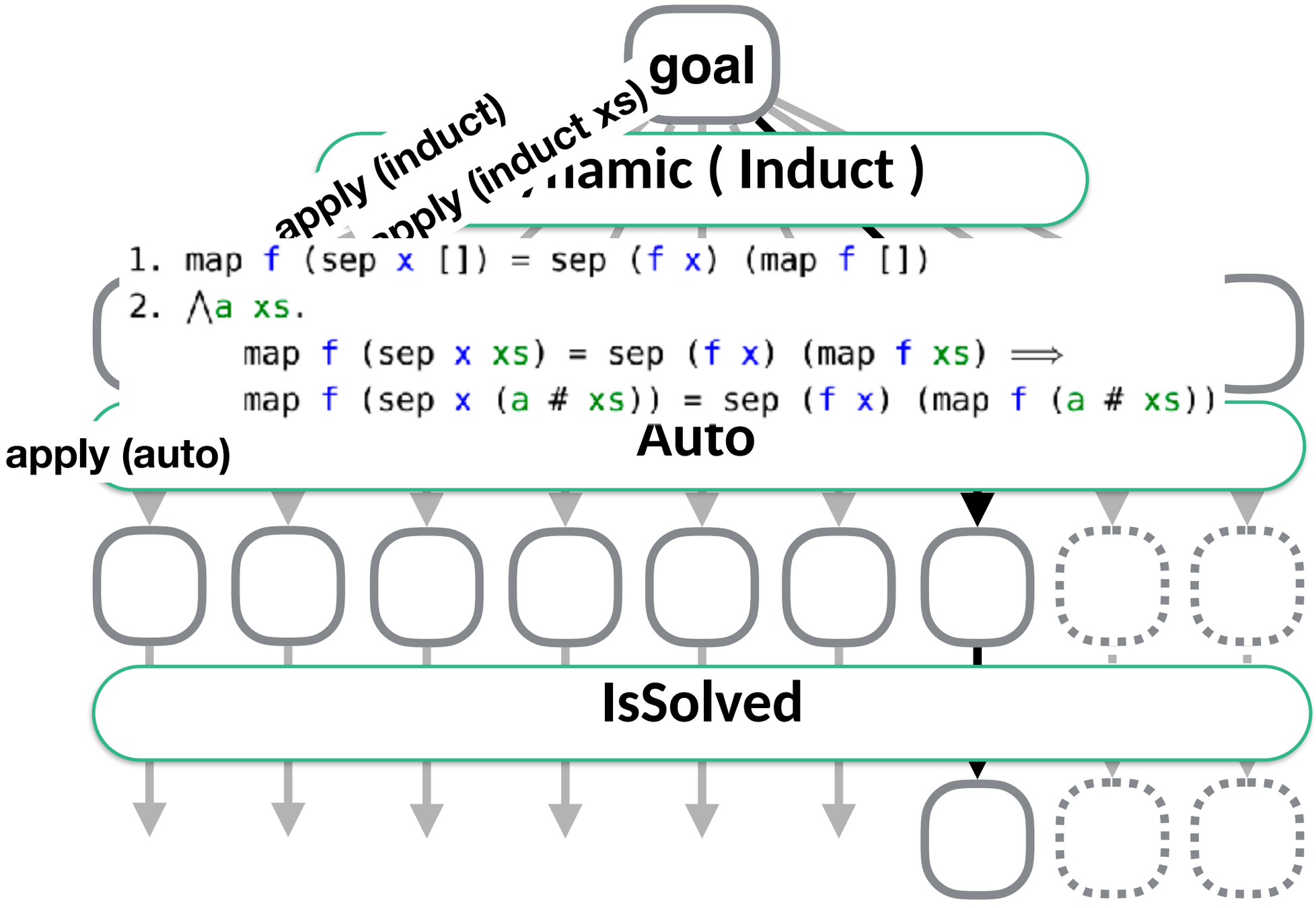

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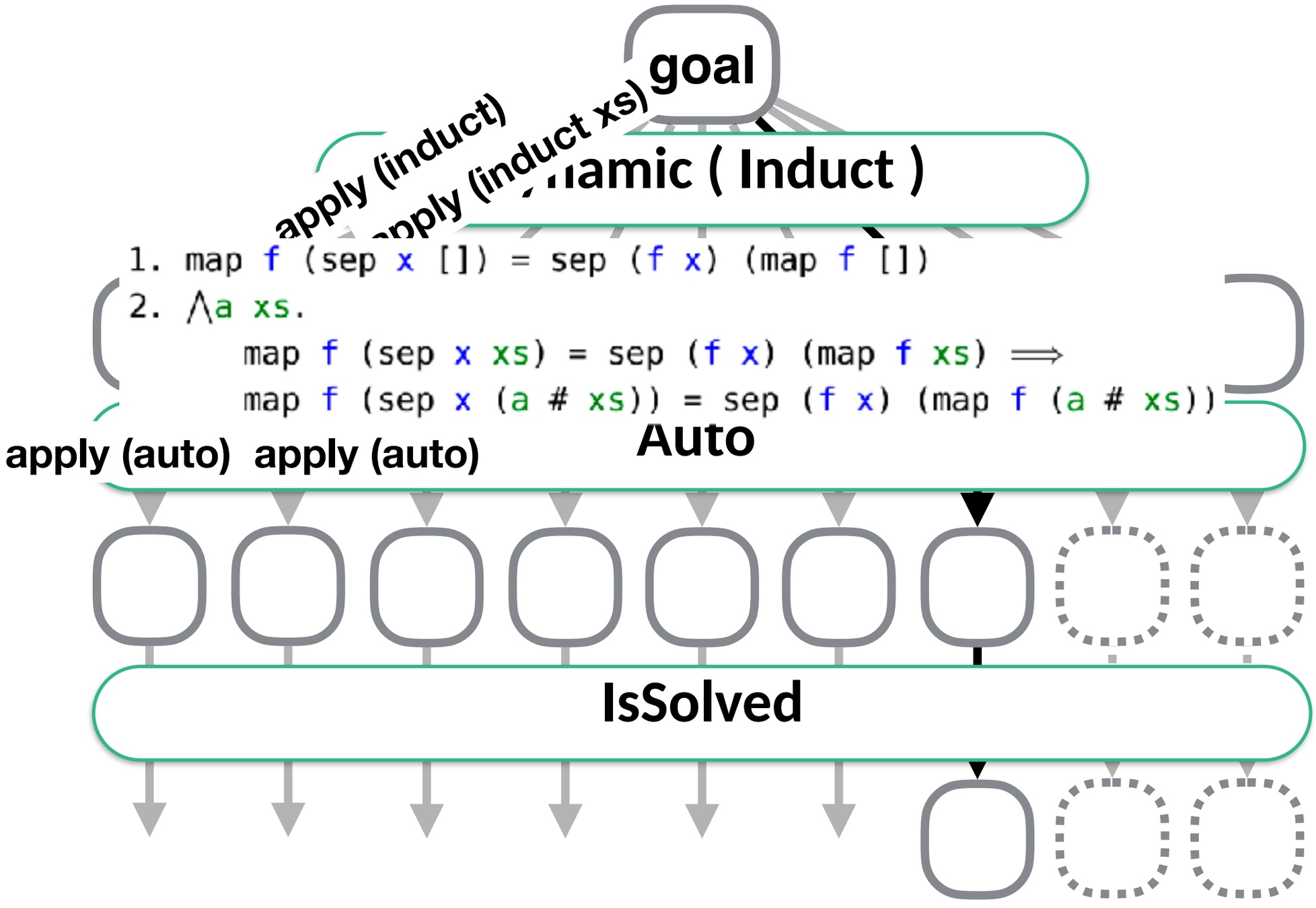

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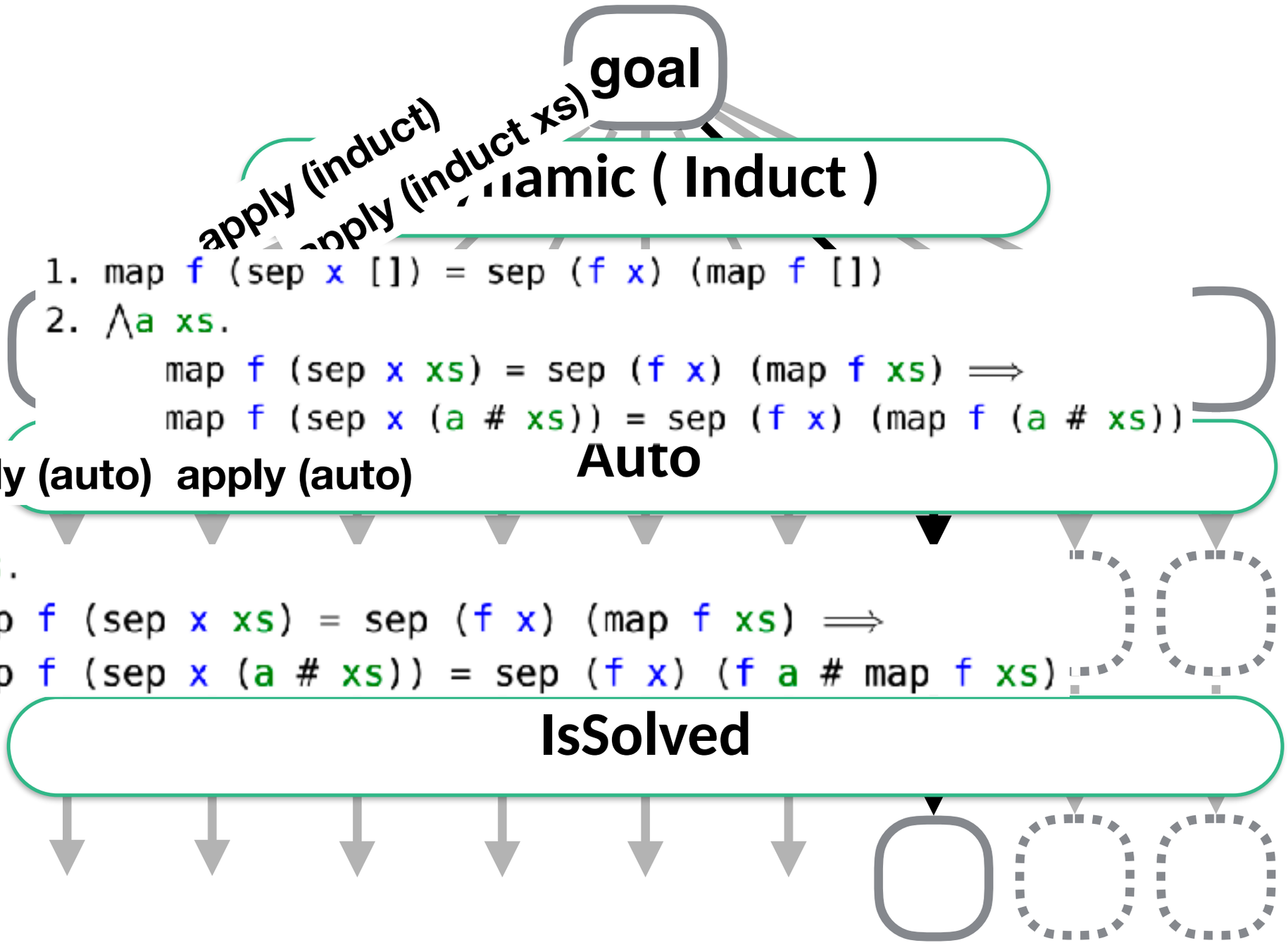

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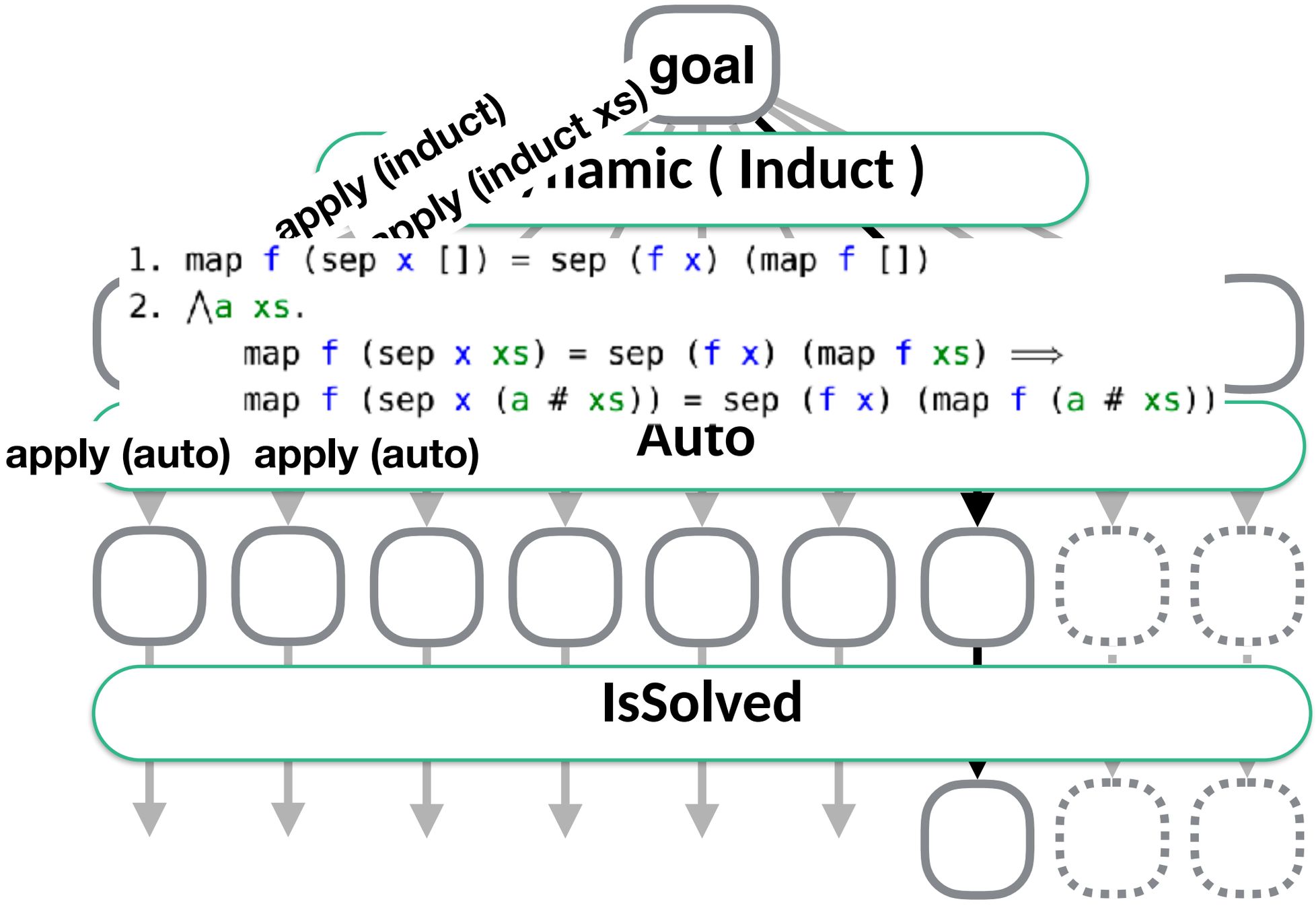

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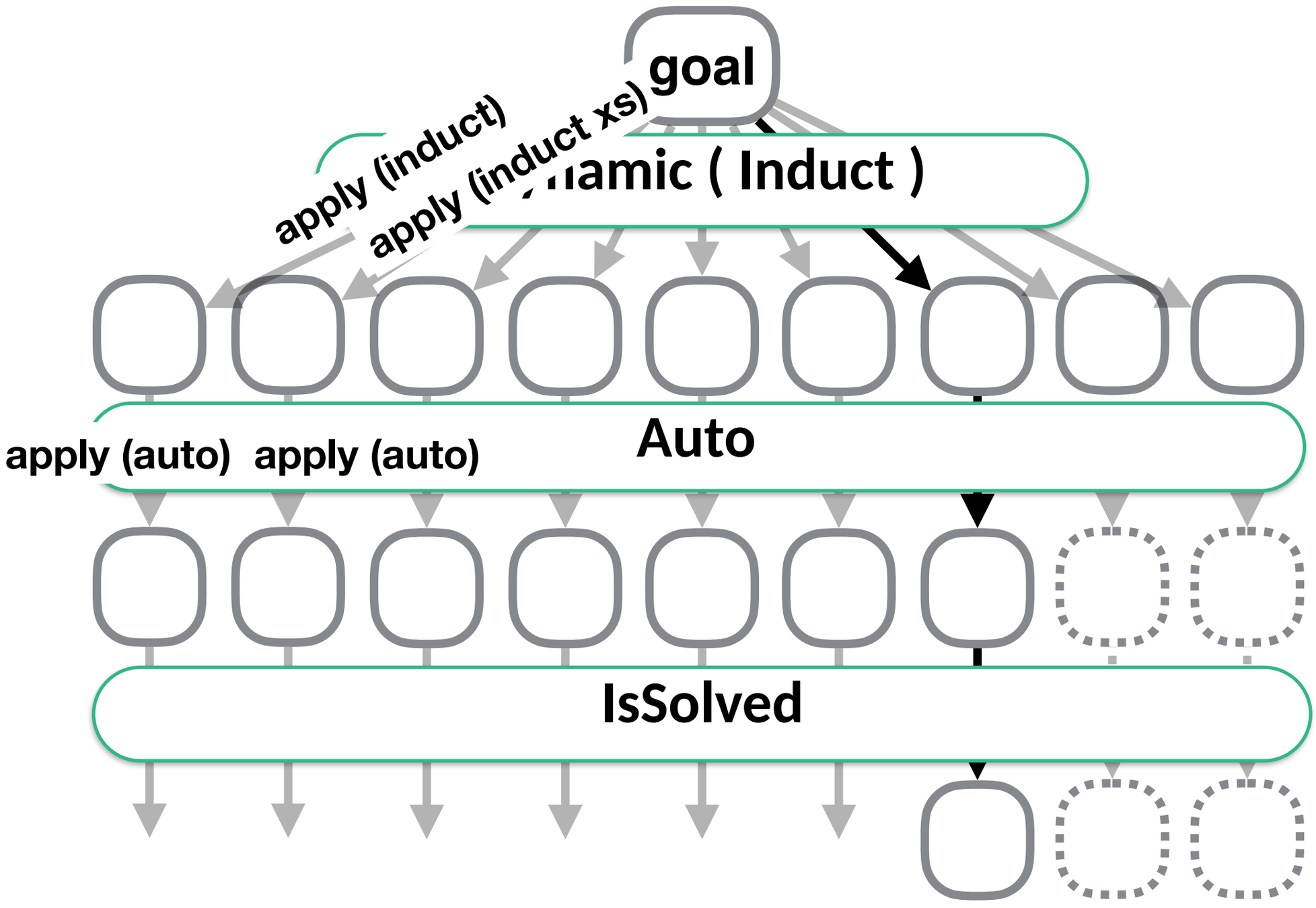

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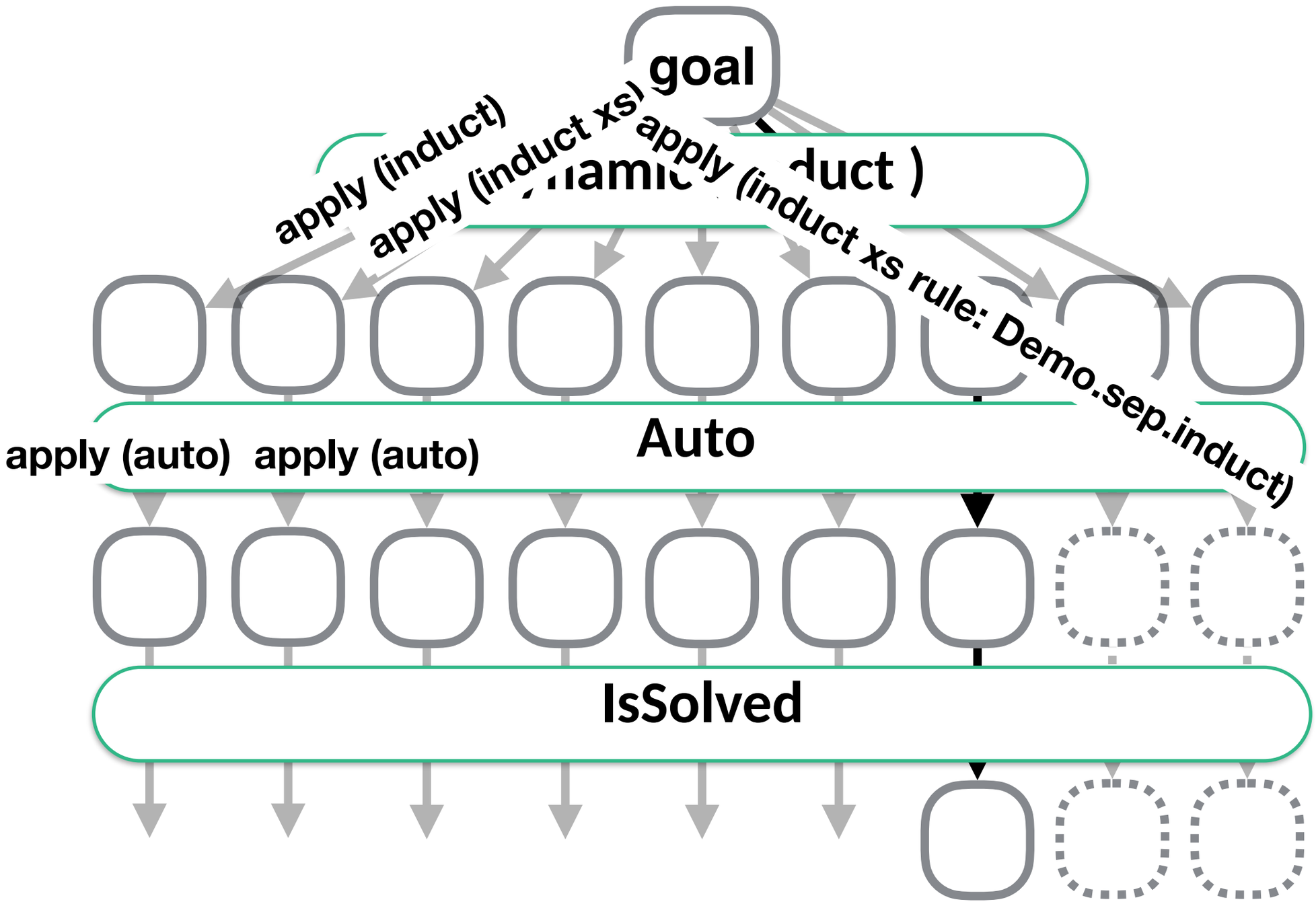

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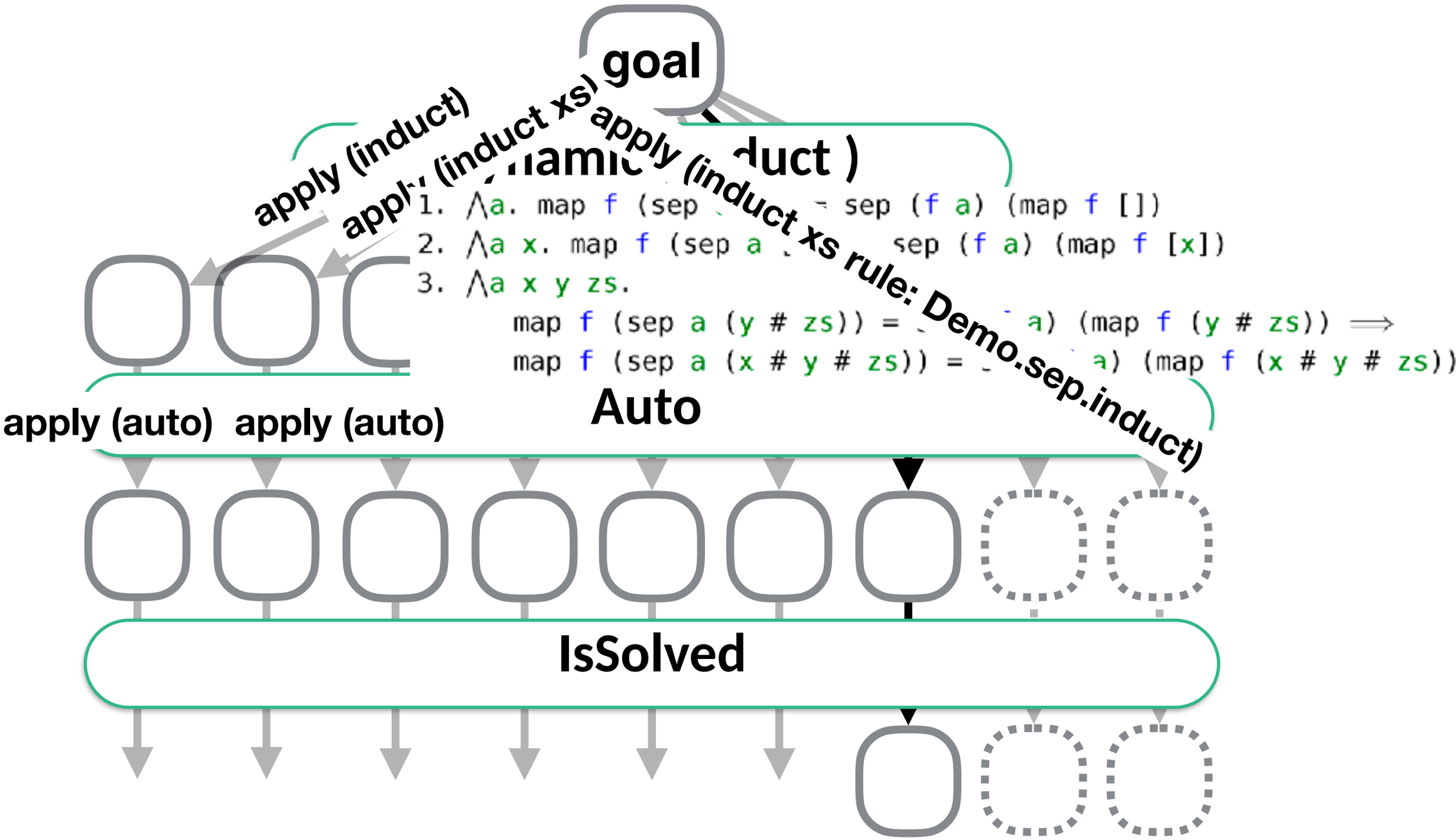
lemma "map f (sep x xs) = sep (f x) (map f xs)"

find_proof DInd(*= Thens [Dynamic (Induct), Auto, IsSolved]*)
```



https://twitter.com/YutakangE

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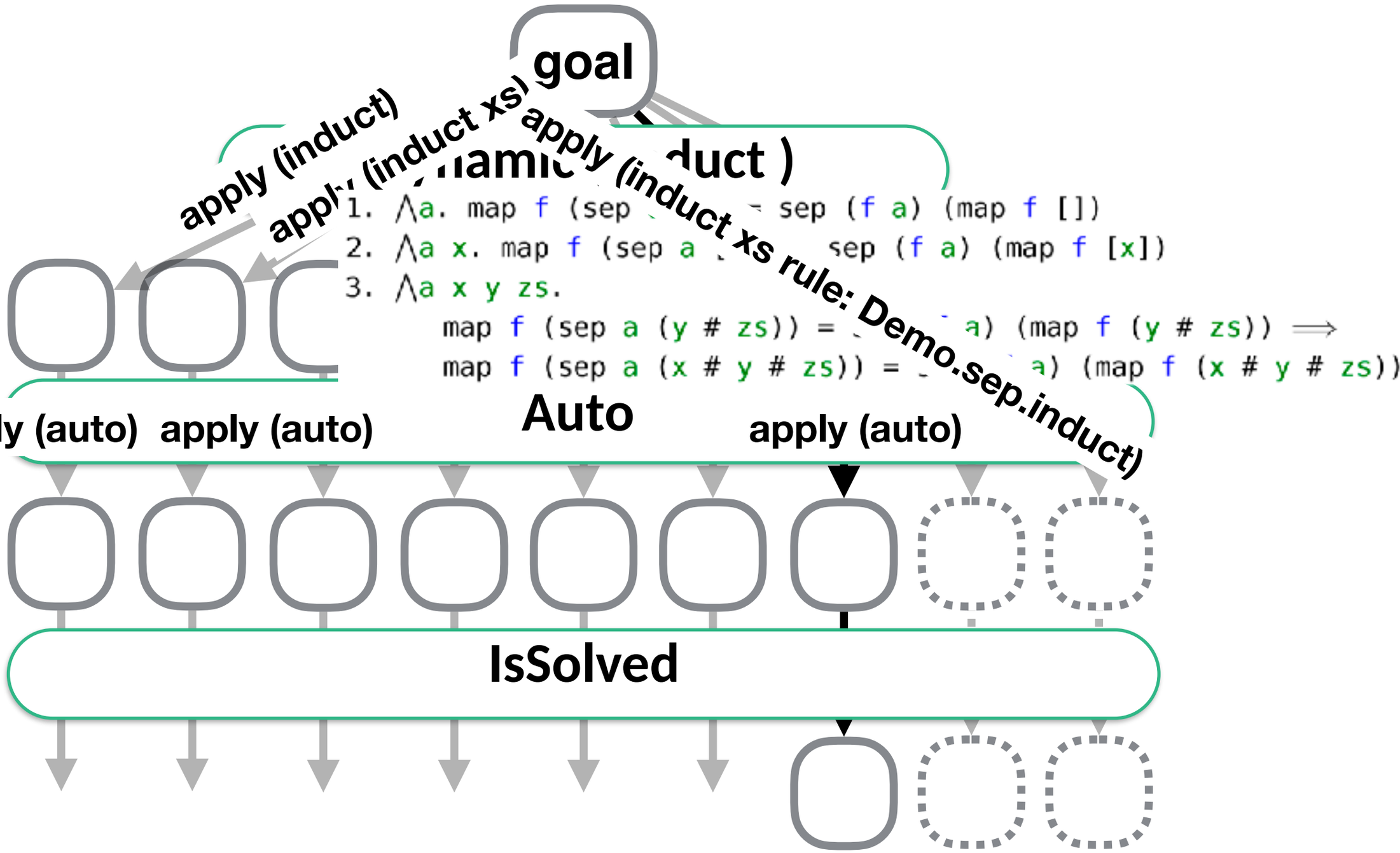

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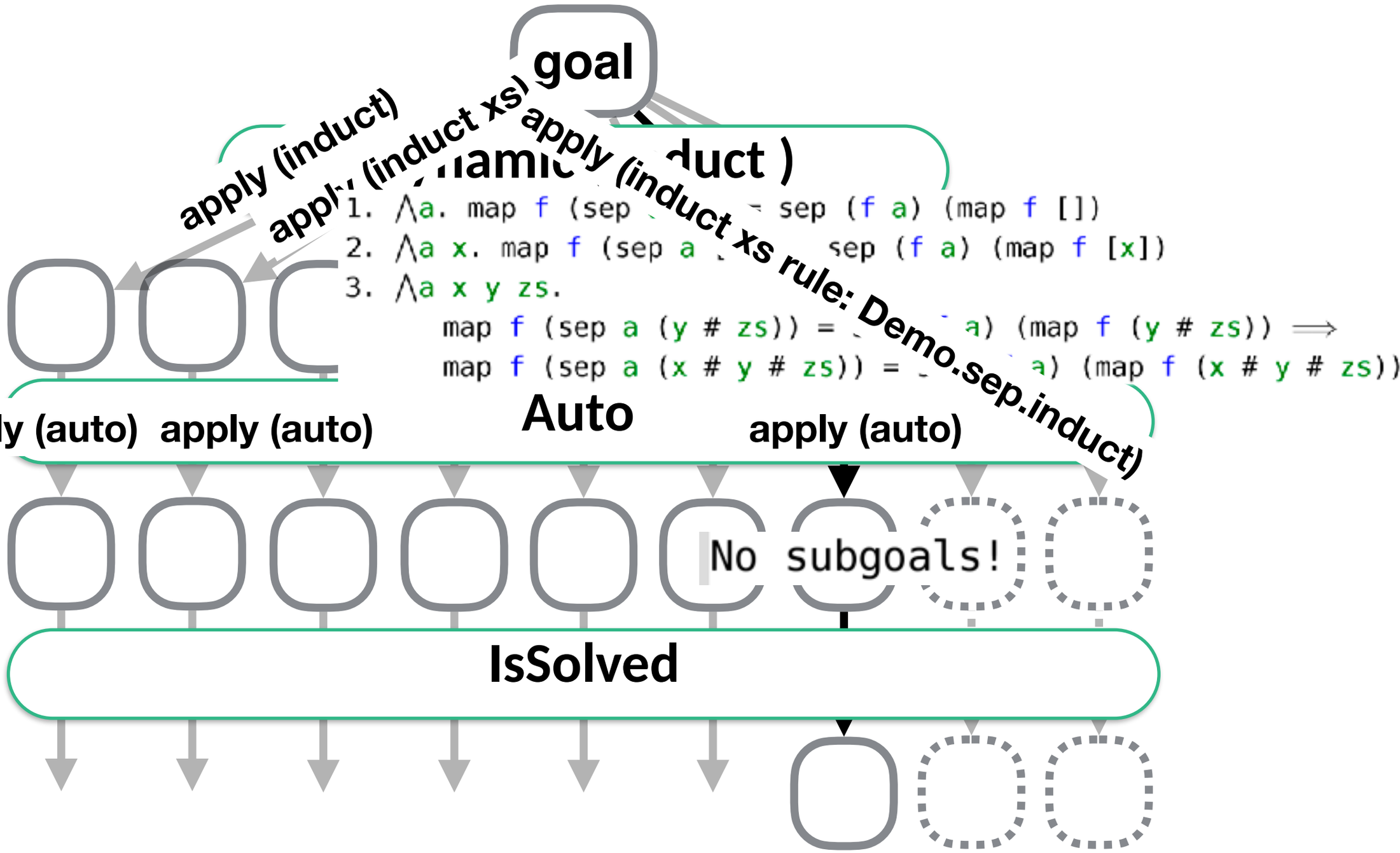

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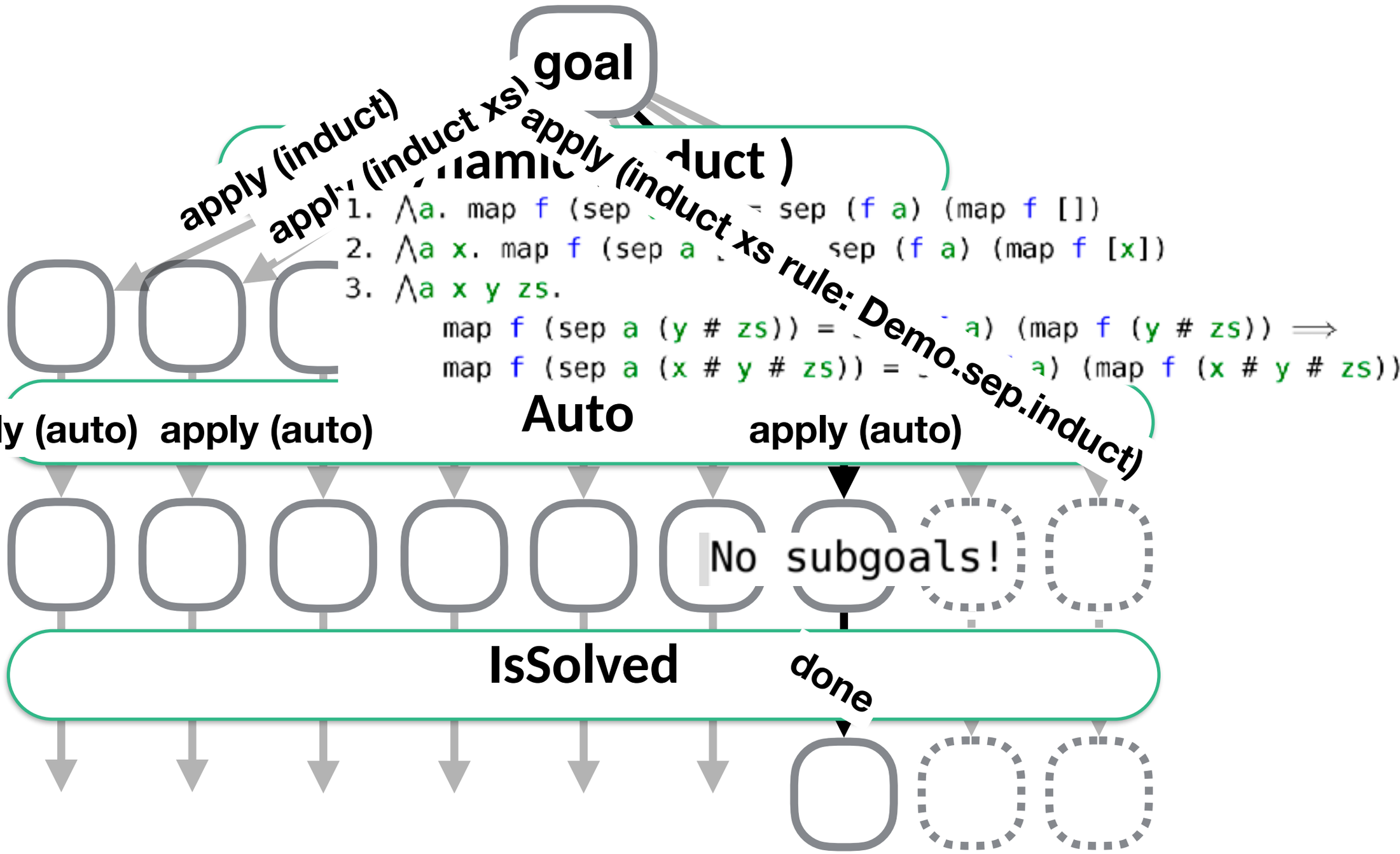

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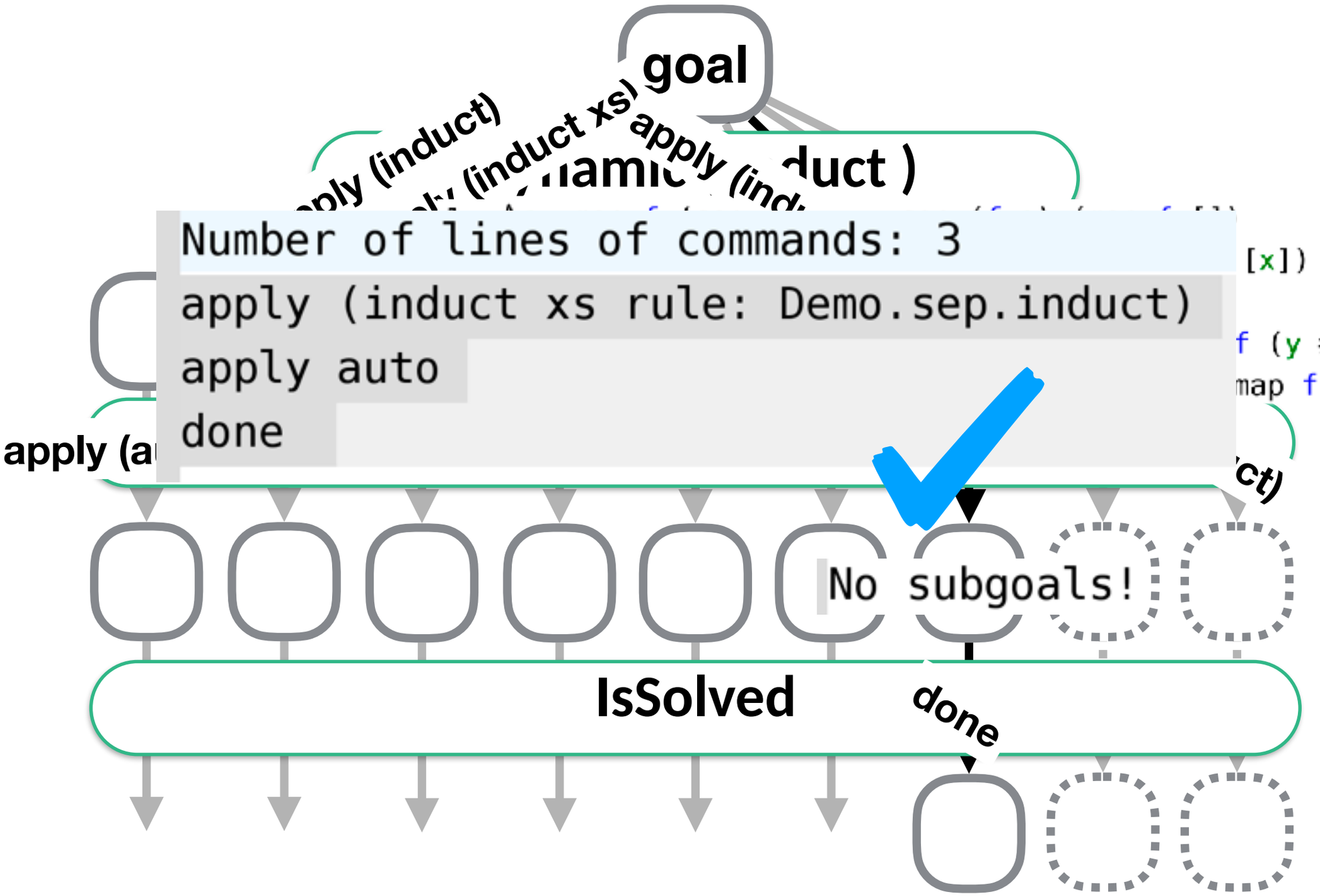


# Try_Hard: the default strategy

```
strategy Try_Hard =
Ors [Thens [Subgoal, Basic],
      Thens [DInductTac, Auto_Solve],
      Thens [DCaseTac, Auto_Solve],
      Thens [Subgoal, Advanced],
      Thens [DCaseTac, Solve_Many],
      Thens [DInductTac, Solve_Many] ]
```

```
strategy Basic =
  Ors [
      Auto_Solve,
      Blast_Solve,
      FF_Solve,
      Thens [IntroClasses, Auto_Solve],
      Thens [Transfer, Auto_Solve],
      Thens [Normalization, IsSolved],
      Thens [DInduct, Auto_Solve],
      Thens [Hammer, IsSolved],
      Thens [DCases, Auto_Solve],
      Thens [DCoinduction, Auto_Solve],
      Thens [Auto, RepeatN(Hammer), IsSolved],
      Thens [DAuto, IsSolved]]
```

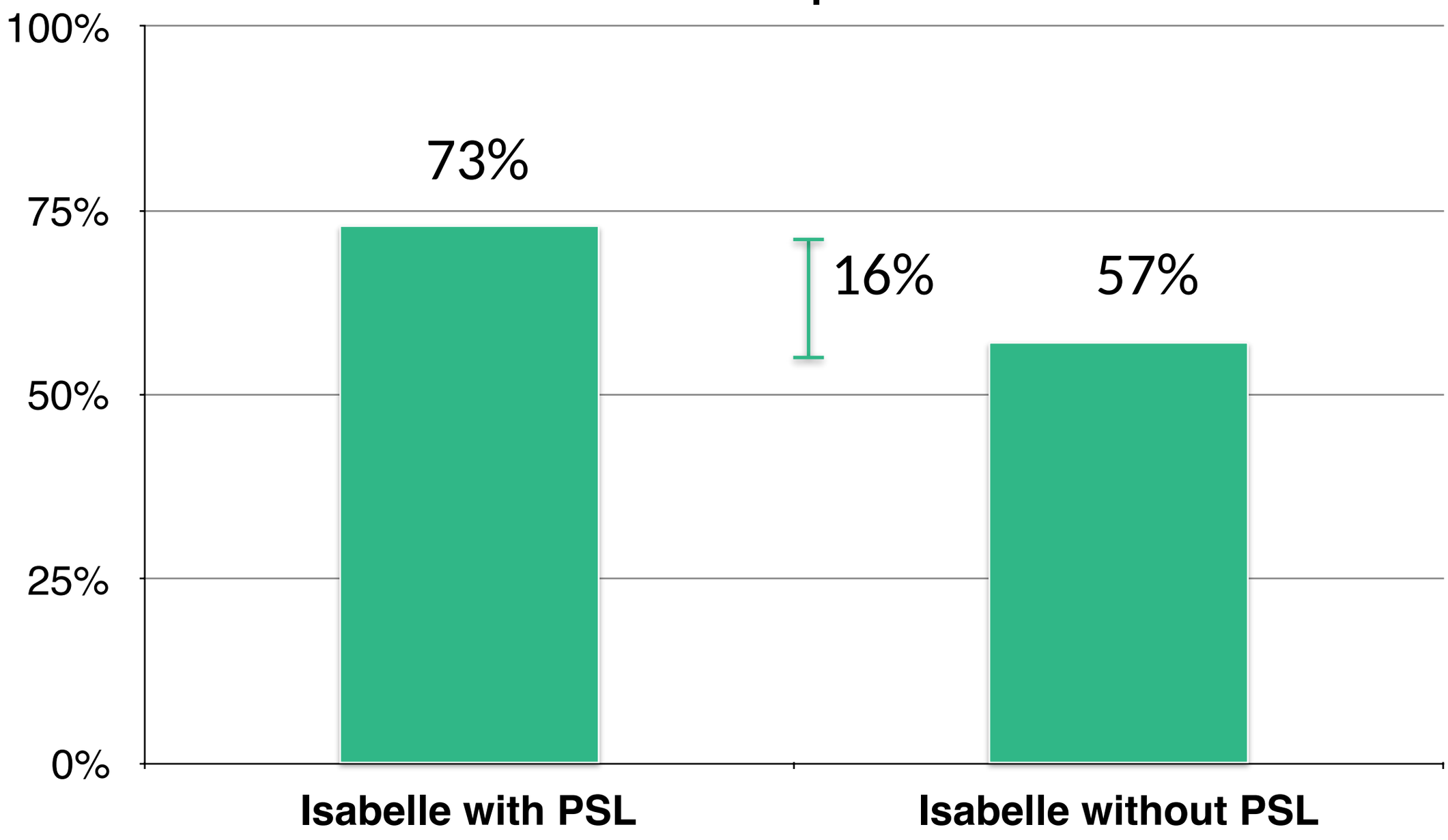
https://twitter.com/YutakangE

git clone https://github.com/data61/PSL

# Evaluation

but the search space explodes

PaMpeR: Proof Method Recommendation



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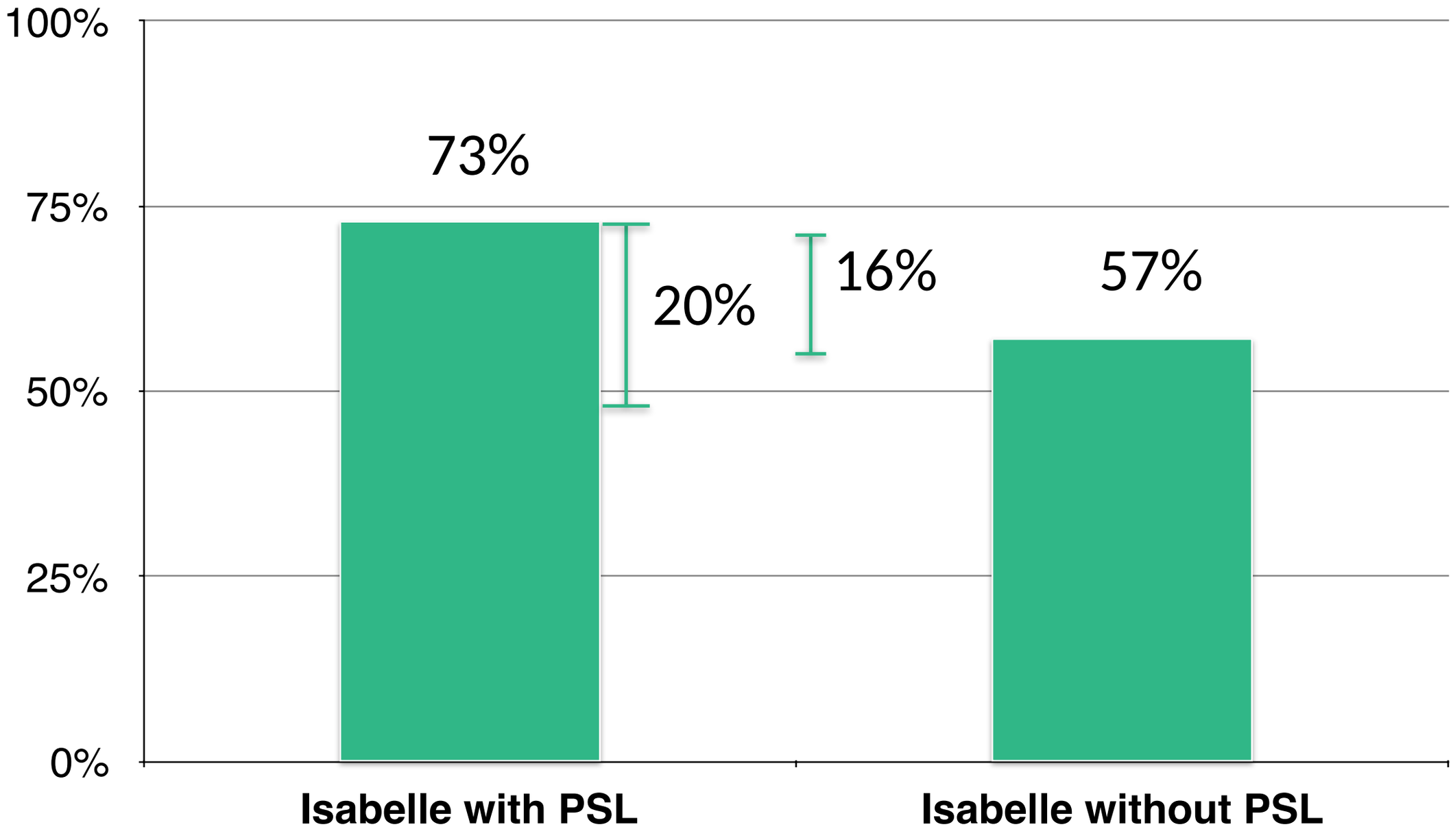
# Evaluation

but the search space explodes

PaMpeR: Proof Method Recommendation



preparation phase

# How does PaMpeR work?

recommendation phase

preparation phase

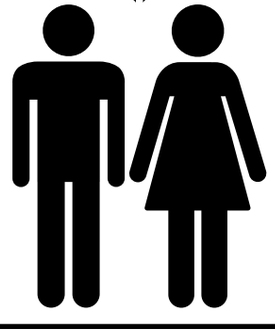
large proof corpora

AFP and standard library

# How does PaMpeR work?

recommendation phase

proof state



proof engineer

preparation phase

large proof corpora

AFP and standard library


# Statistics

## Archive of Formal Proofs (https://www.isa-afp.org)



## Statistics

Number of Articles: 468
Number of Authors: 313
Number of lemmas: ~128,900
Lines of Code: ~2,170,300

**Most used AFP articles:**

| | Name | Used by ? articles |
|---|---|---|
| 1. | Collections | 15 |
| 2. | List-Index | 14 |
| 3. | Coinductive | 12 |

preparation phase

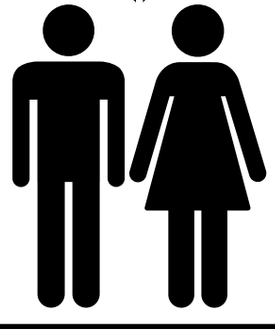
large proof corpora

AFP and standard library

# How does PaMpeR work?

recommendation phase

proof state



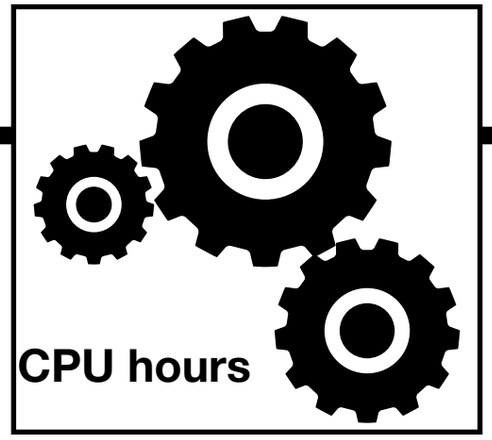
proof engineer

preparation phase

large proof corpora

AFP and standard library

full feature extractor



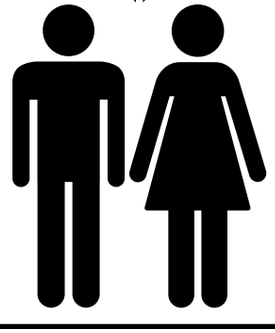
6021 CPU hours

108 assertions

# How does PaMpeR work?

recommendation phase

proof state



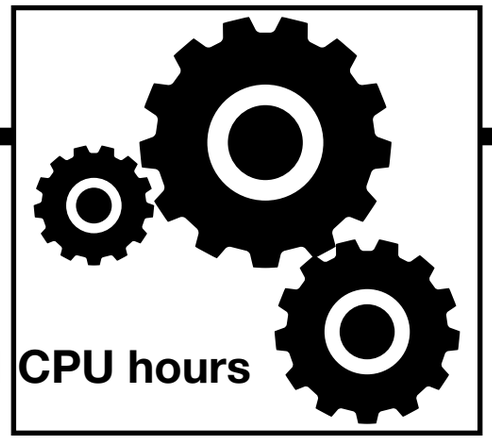
proof engineer

preparation phase

large proof corpora

AFP and standard library

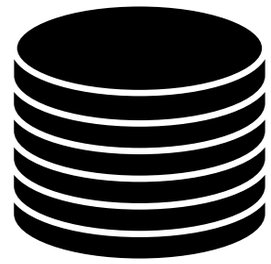
full feature extractor



6021 CPU hours

108 assertions

database ( 425334 data points )

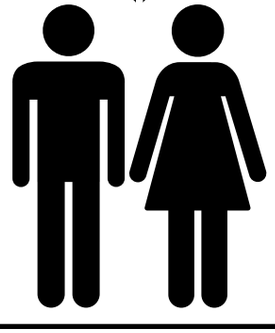


:: ( tactic_name, [ bool ] )

# How does PaMpeR work?

recommendation phase

proof state



proof engineer

preparation phase

large proof corpora

AFP and standard library

full feature extractor

6021 CPU hours

108 assertions

database ( 425334 data points )

:: ( tactic_name, [ bool ] )

preprocess

decision tree construction

# How does PaMpeR work?

recommendation phase

proof state

proof engineer

preparation phase

large proof corpora

AFP and standard library

full feature extractor

6021 CPU hours

108 assertions

database ( 425334 data points )

:: ( tactic_name, [ bool ] )

preprocess

decision tree construction

# How does PaMpeR work?

recommendation phase

proof state

proof engineer

fast feature extractor

preparation phase

large proof corpora

AFP and standard library

full feature extractor

6021 CPU hours

108 assertions

database ( 425334 data points )

:: ( tactic_name, [ bool ] )

preprocess

decision tree construction

# How does PaMpeR work?

recommendation phase

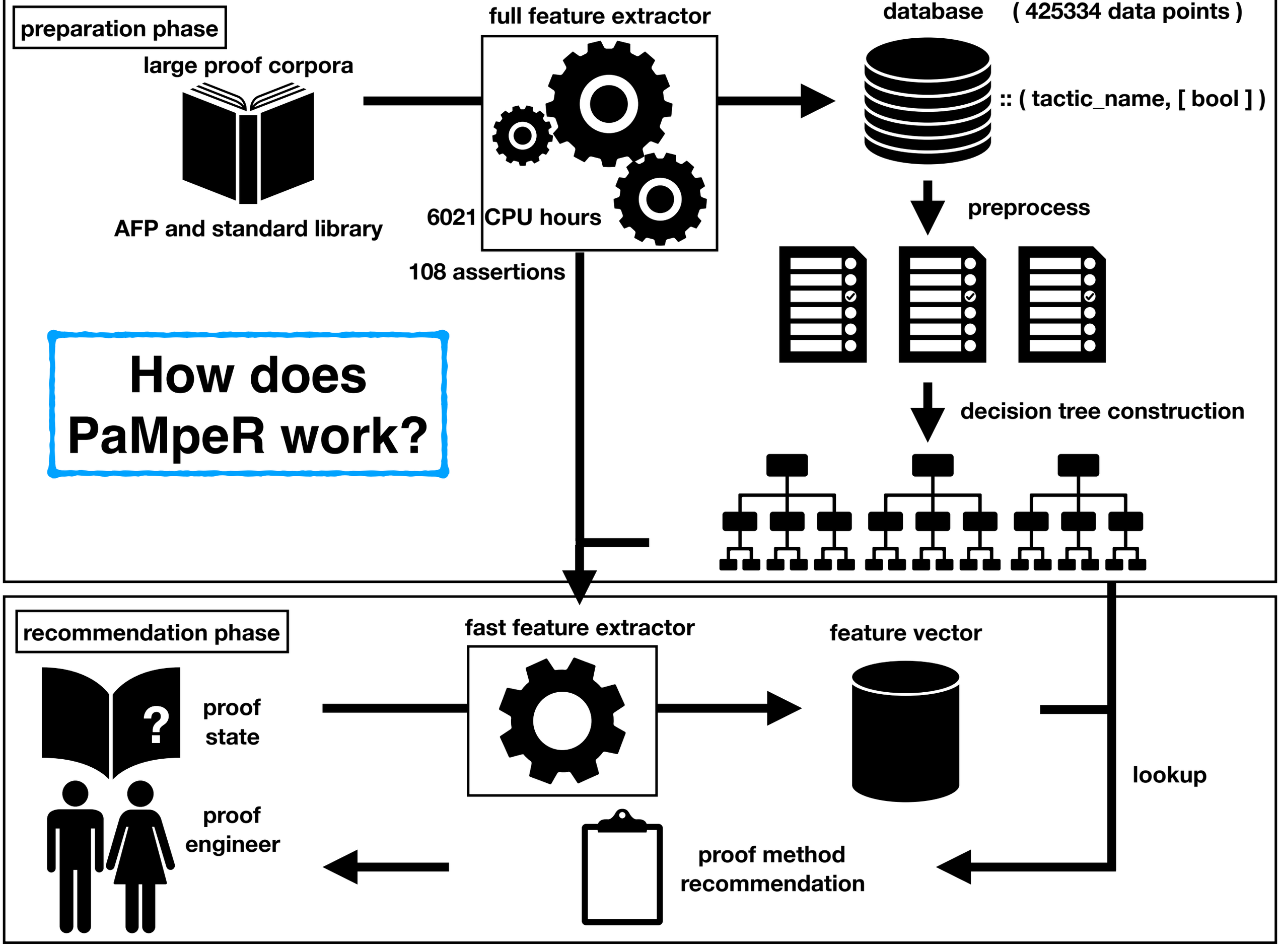
proof state

proof engineer

fast feature extractor

feature vector

# How does PaMpeR work?



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# Summary

PSL can find how to apply induction for easy problems.
CADE2017 (https://link.springer.com/10.1007/978-3-319-63046-5_32)

PaMpeR recommends which proof methods to use.
ASE2018 (https://dx.doi.org/10.1145/3238147.3238210)