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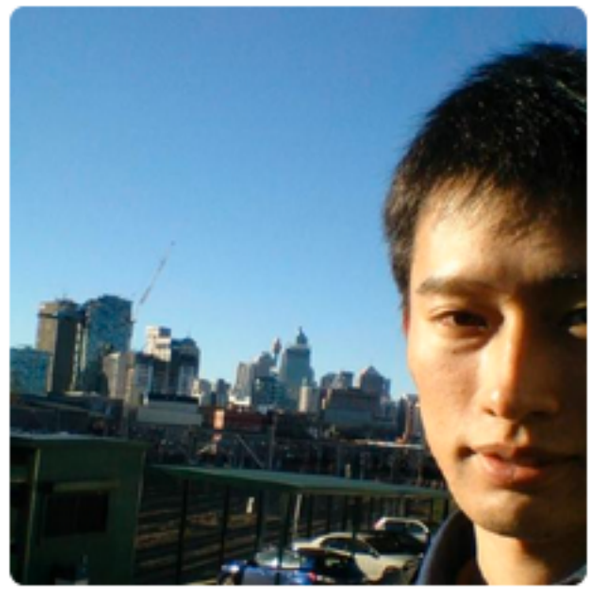
AI for theorem proving in Isabelle/HOL

This work was supported by the project AI&Reasoning (reg. no. CZ.02.1.01/0.0/0.0/15_003/0000466).

Yutaka Nagashima
University of Innsbruck
Czech Technical University



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yutakang

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CVUT, CTU, CIIRC

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in Isabelle?/HOL

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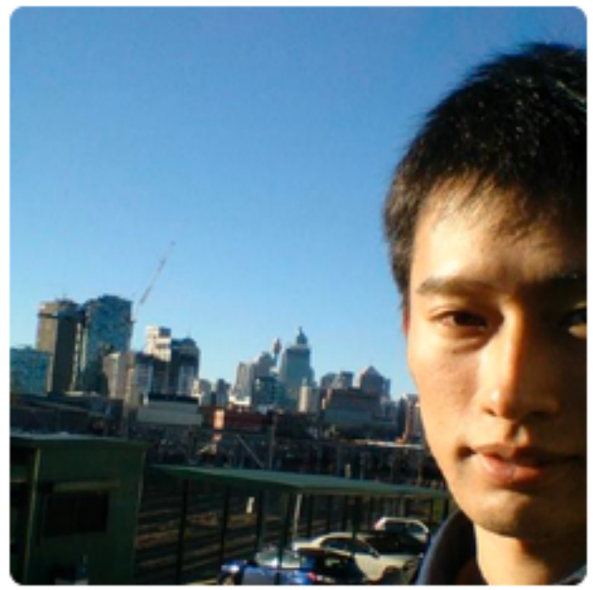

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Why theorem proving?

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To build trustworthy software (Complete Formal Verificaton)!

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1. Specify what we want.

Why theorem proving?

To build trustworthy software (Complete Formal Verificaton)!

1. **Specify what we want.**
2. **Implement what we want.**

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To build trustworthy software (Complete Formal Verificaton)!

1. **Specify what we want.**
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3. **Prove the implementation satisfies the specification.**

Why theorem proving?

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Example

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Example

developer	proof assistant / ITP		implementation
	+	=	
Gewin Klein et. al	Isabelle/HOL		verified micro kernel, seL4

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Example

developer proof assistant / ITP implementation

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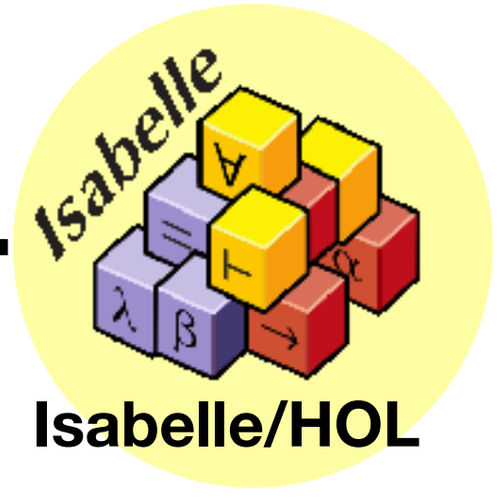

Isabelle/HOL

=

verified micro kernel, seL4

Mathematics

Number Theory

Analysis

Algebra

Geometry

Probability Theory

etc.

Informatics

Language

Algorithms

Data Structures

Architecture

Software Engineering

Formal Verification

theorem proving

Physics

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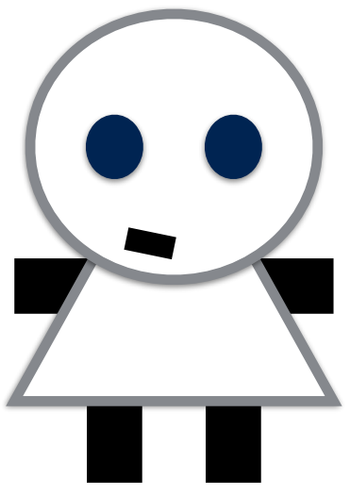

A tiny field inside Informatics. Who cares?

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`Automate Logic using AI to Accelerate Science!`

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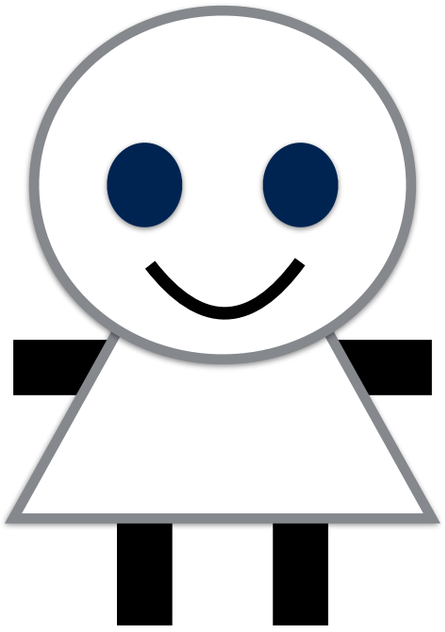
Interactive theorem proving with Isabelle/HOL



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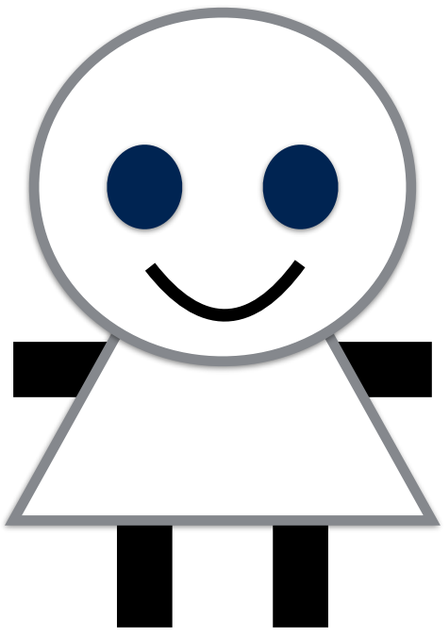
Interactive theorem proving with Isabelle/HOL

proof goal **context**

tactic / proof method



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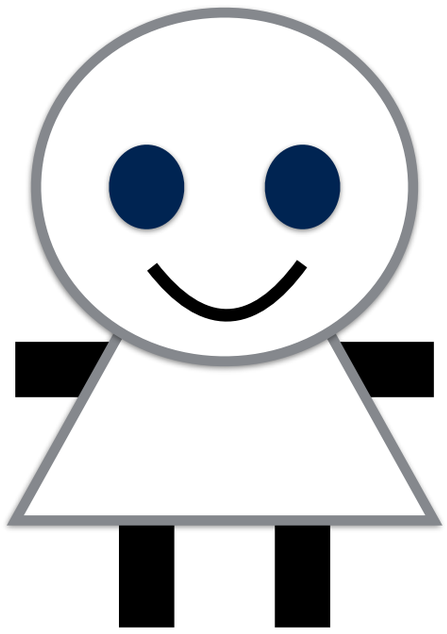

error-message

subgoals

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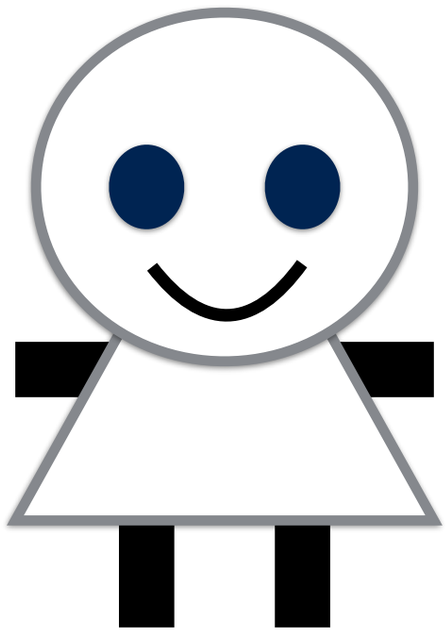

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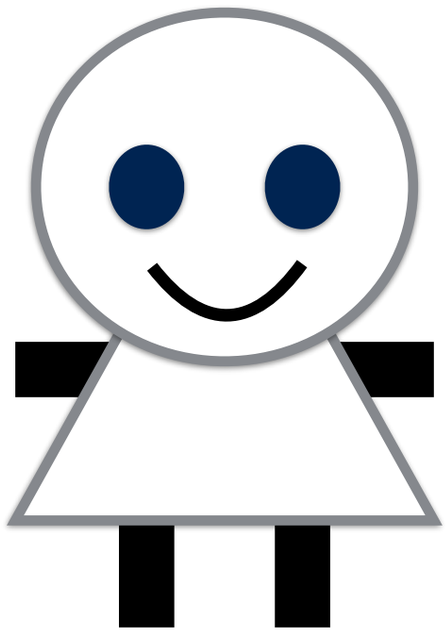

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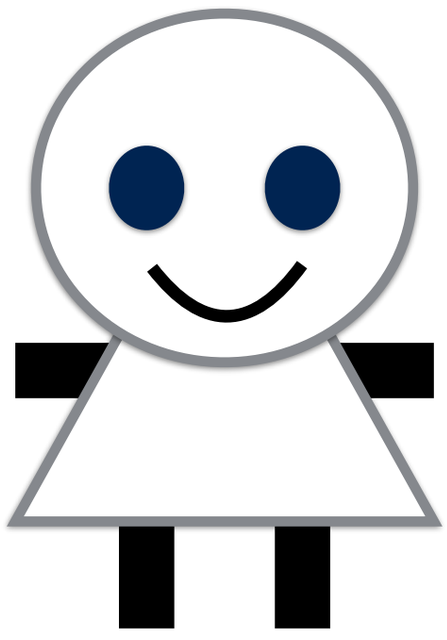

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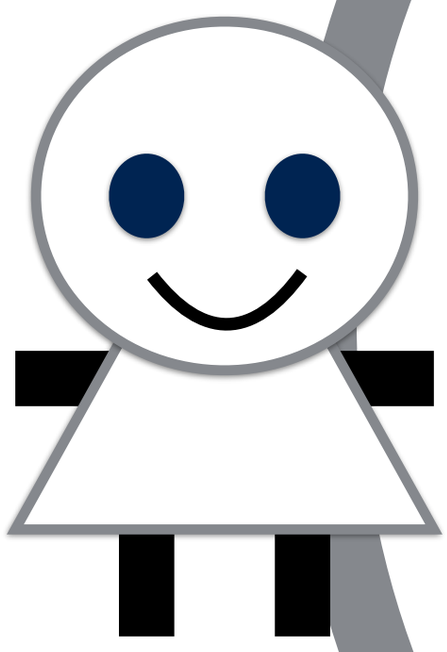

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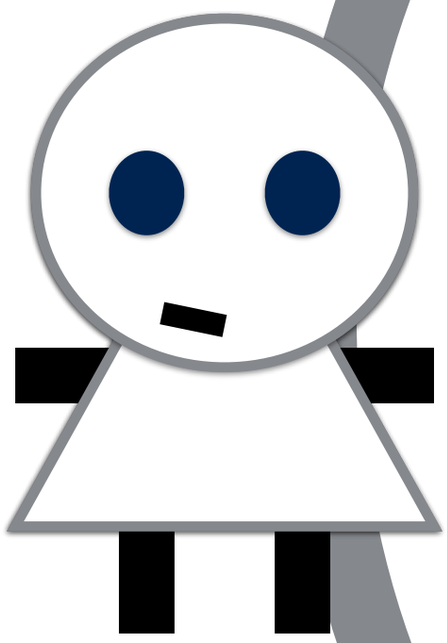

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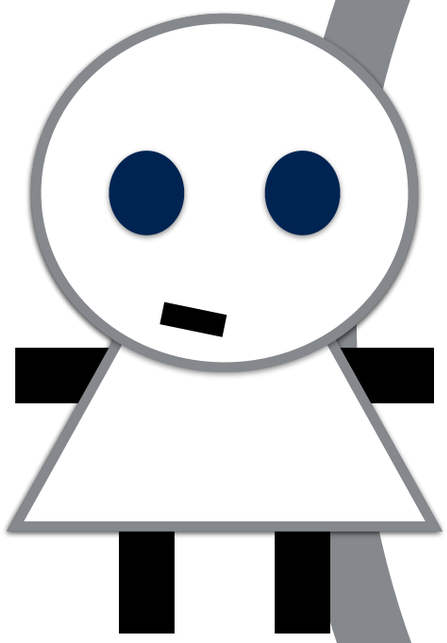

error-message

o-goal!

It's blatantly clear
You stupid machine, that what
I tell you is true
(Michael Norrish)

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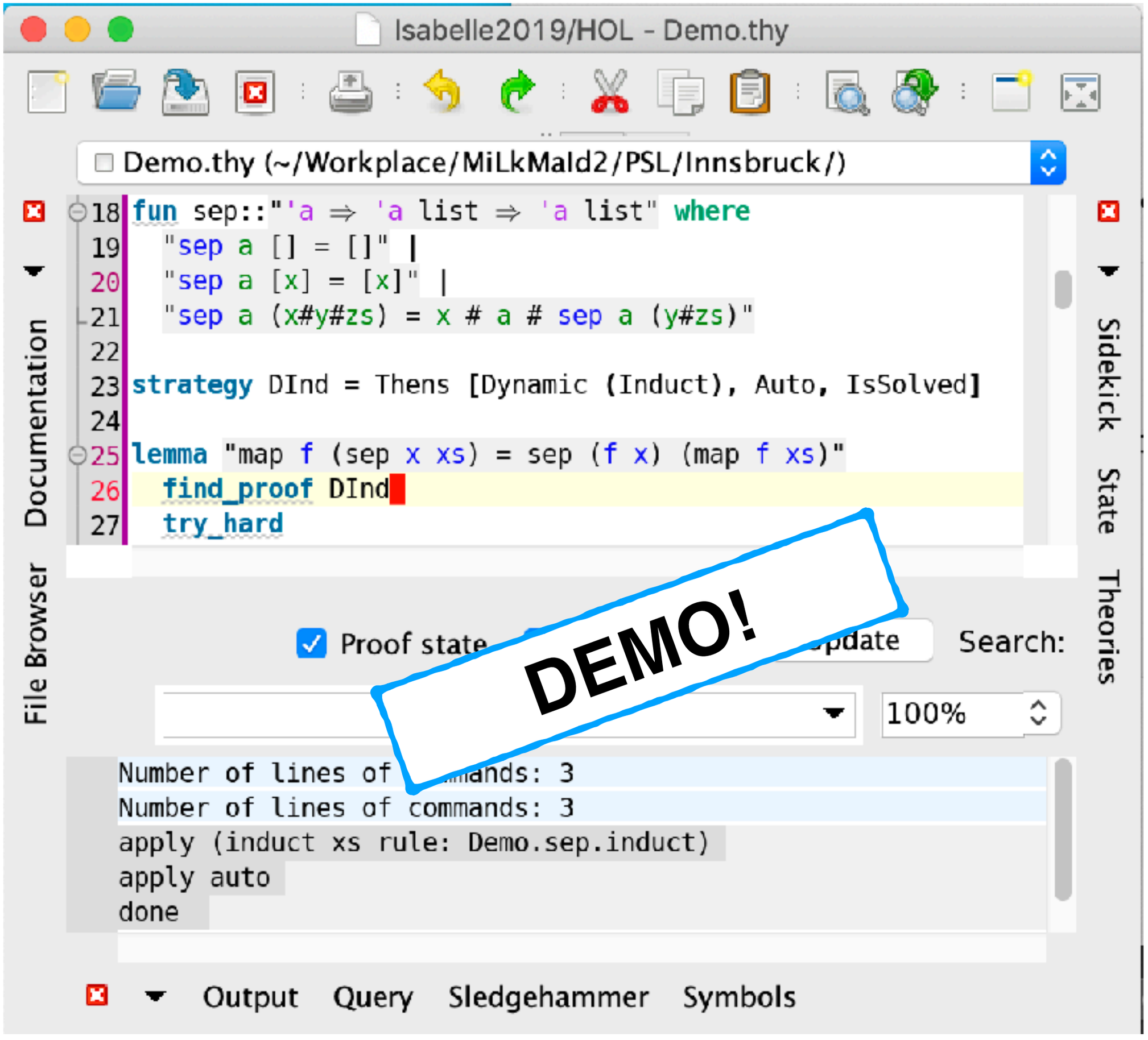

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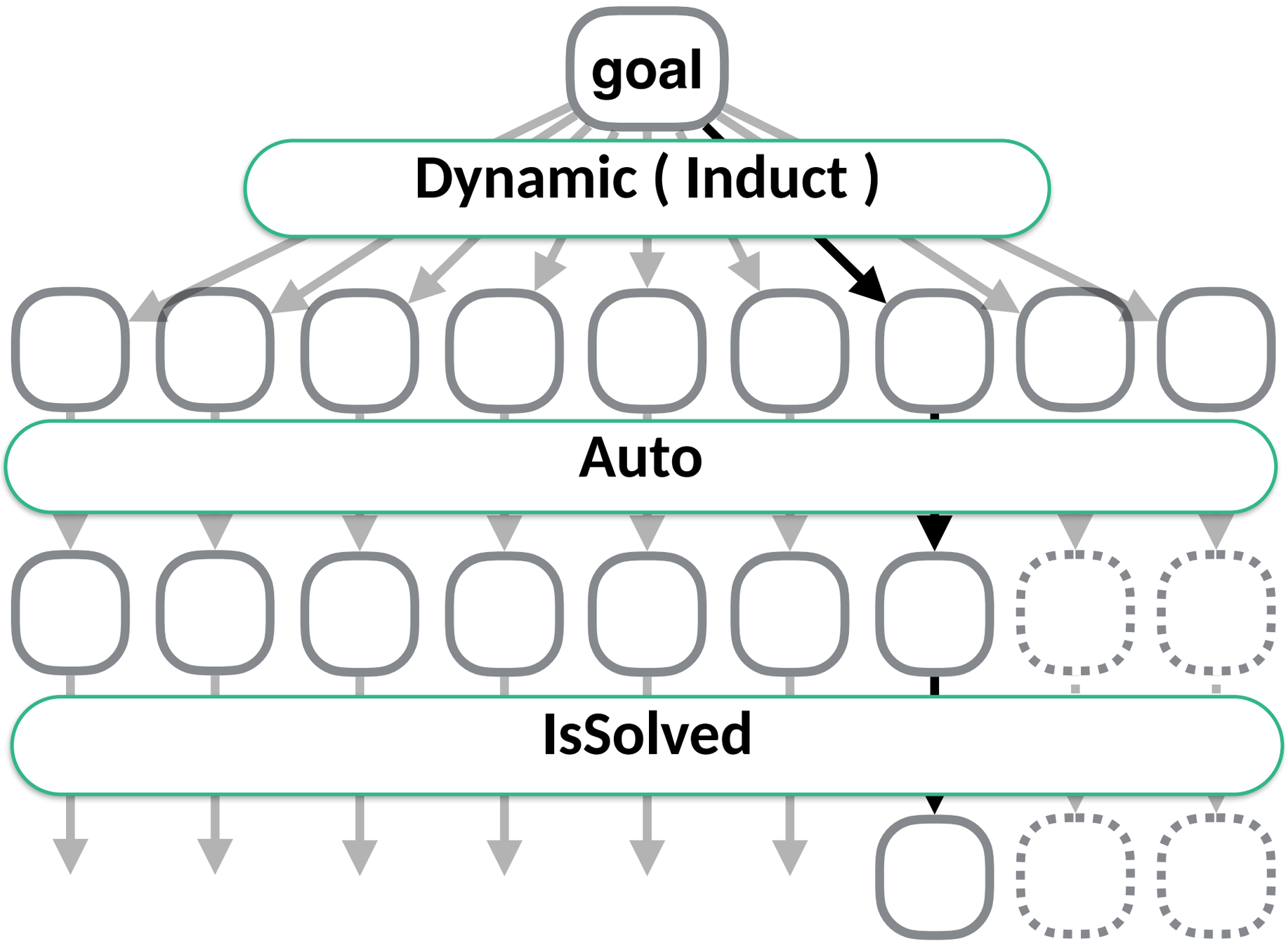

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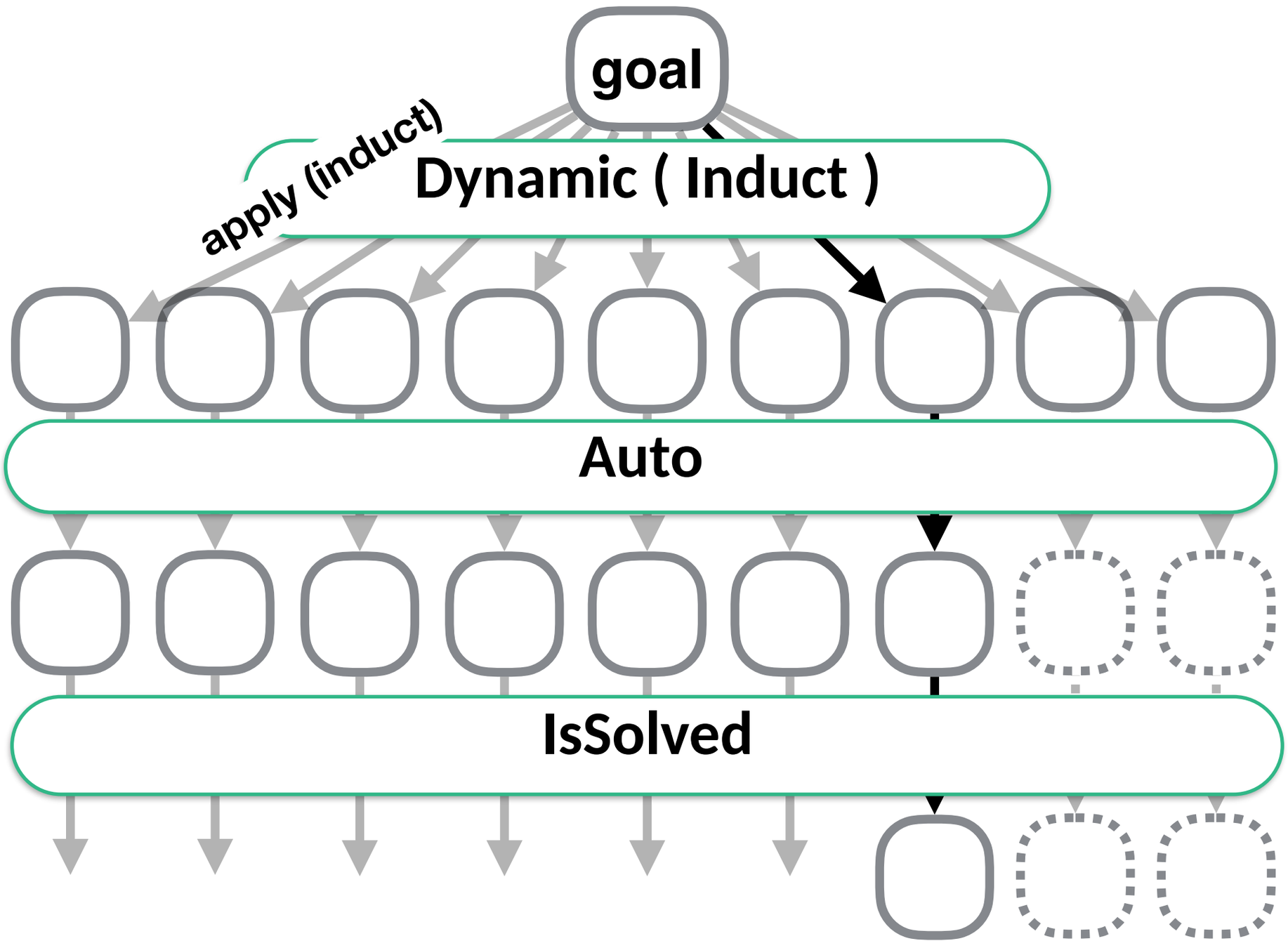

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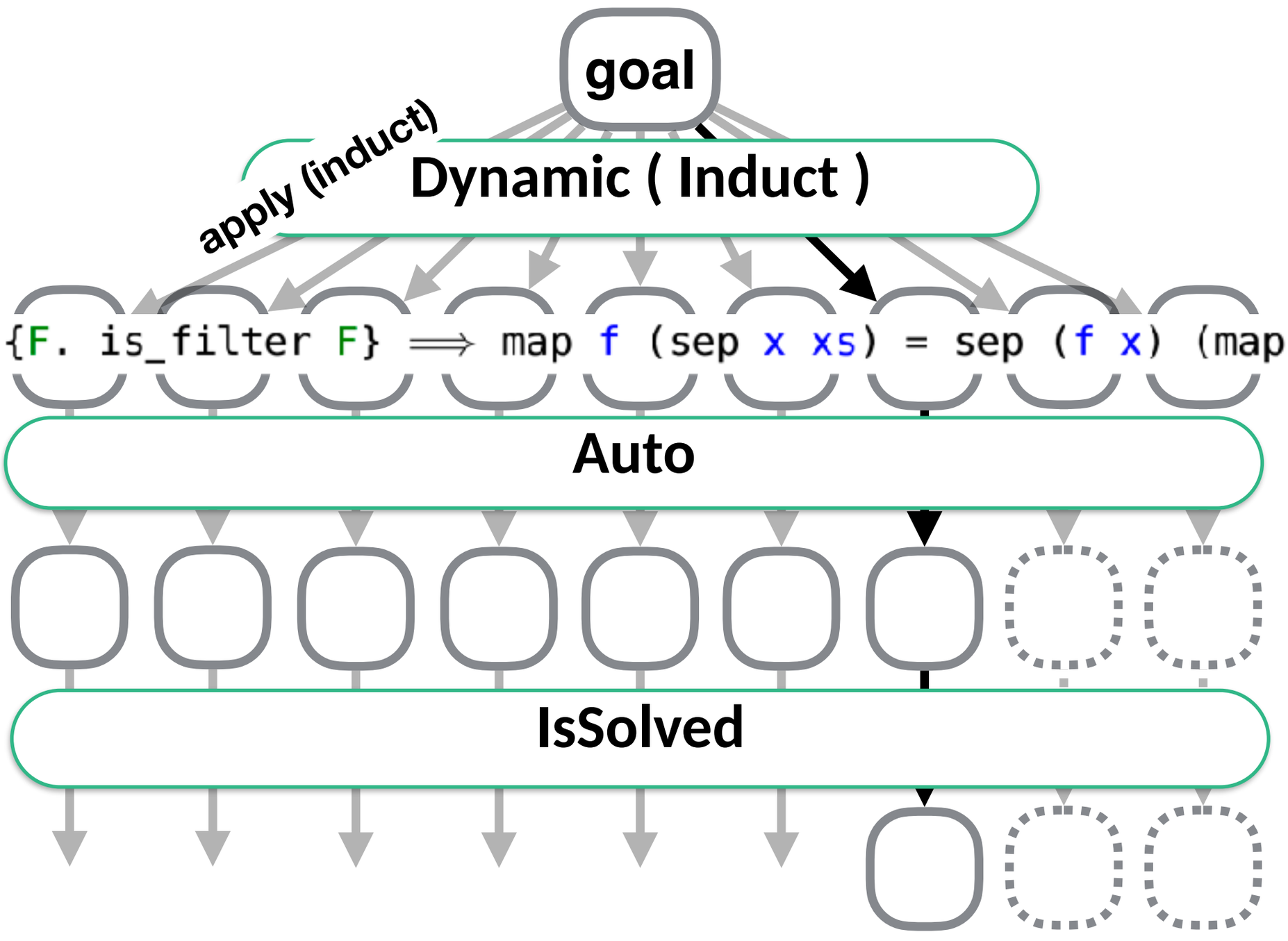

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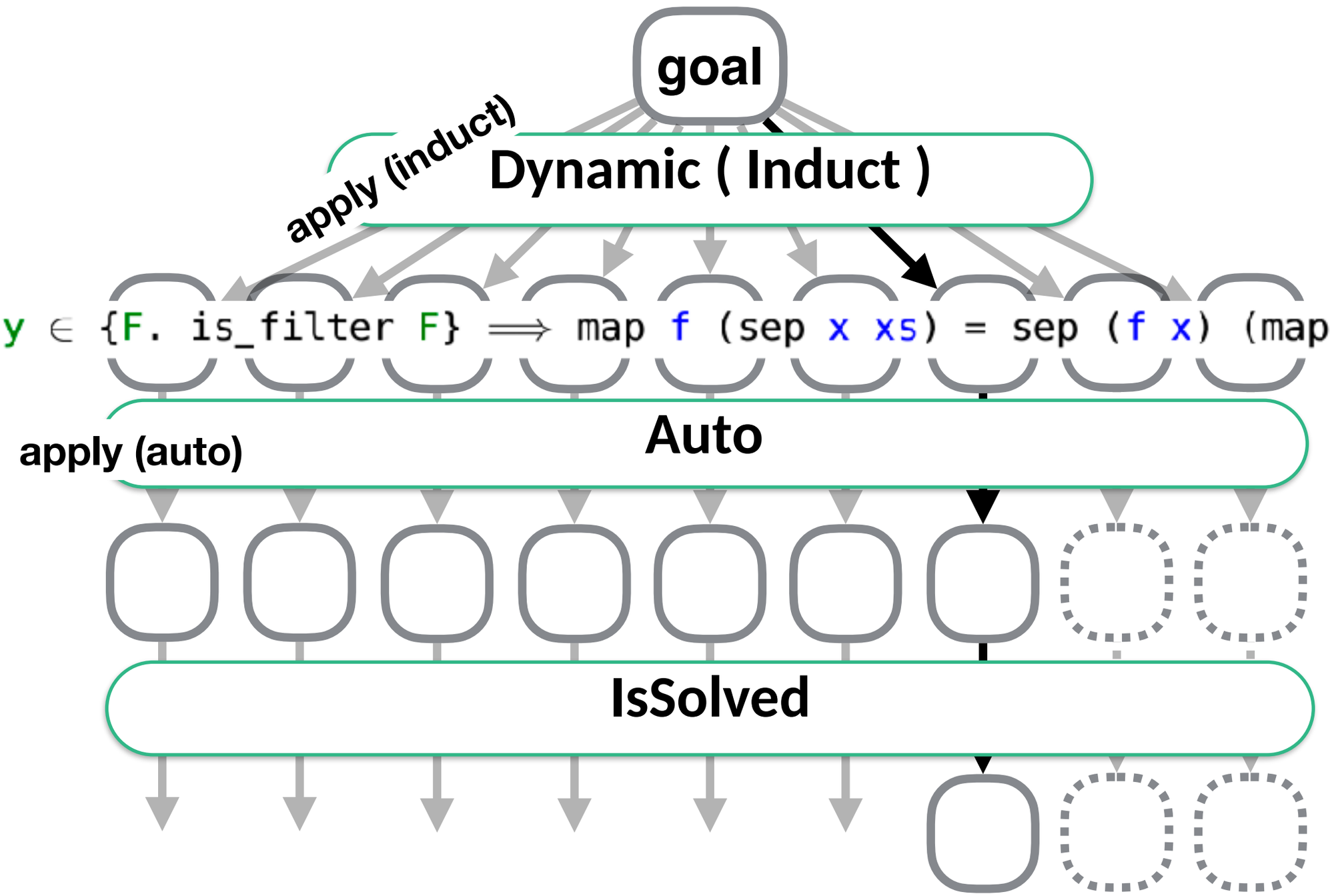

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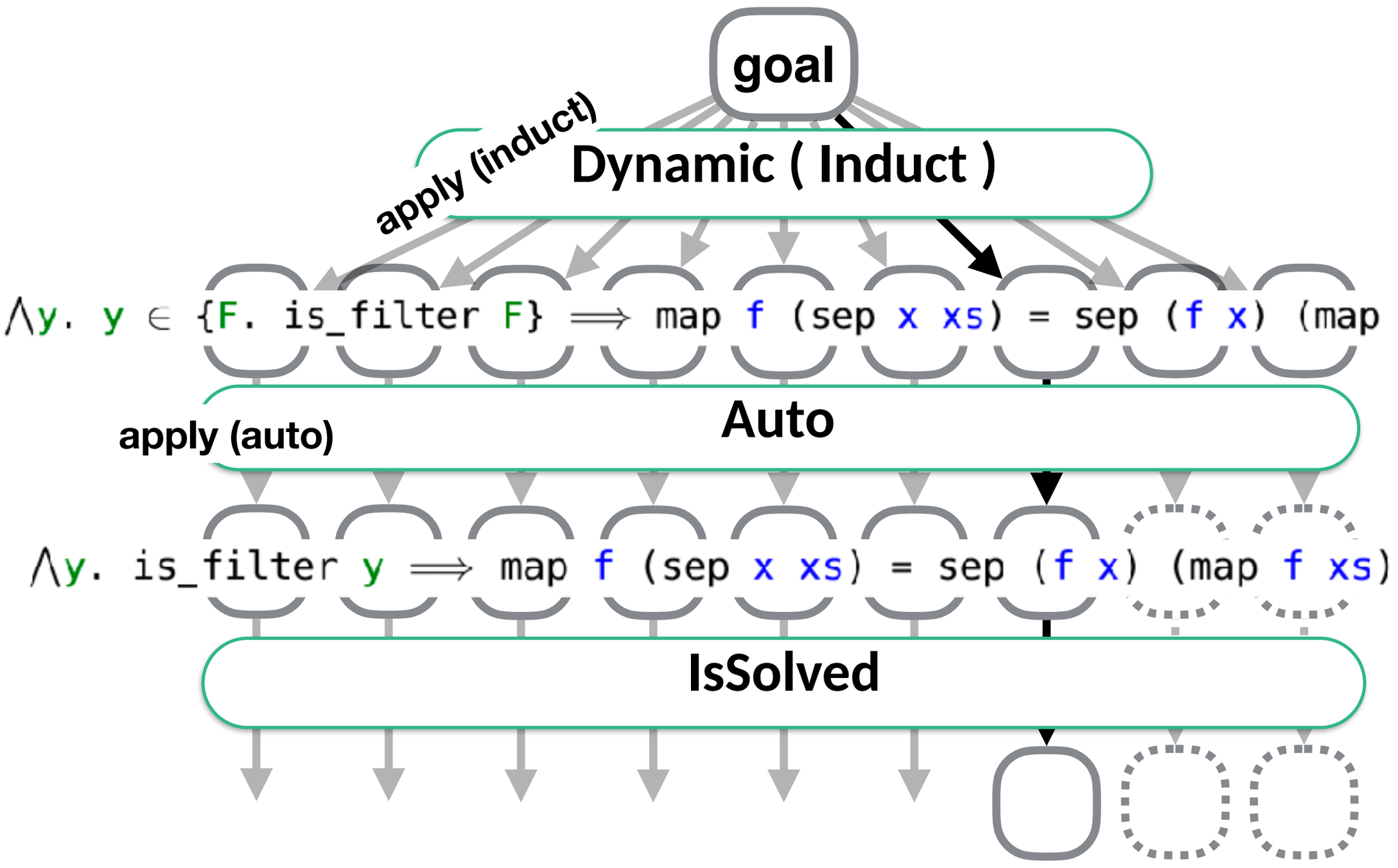

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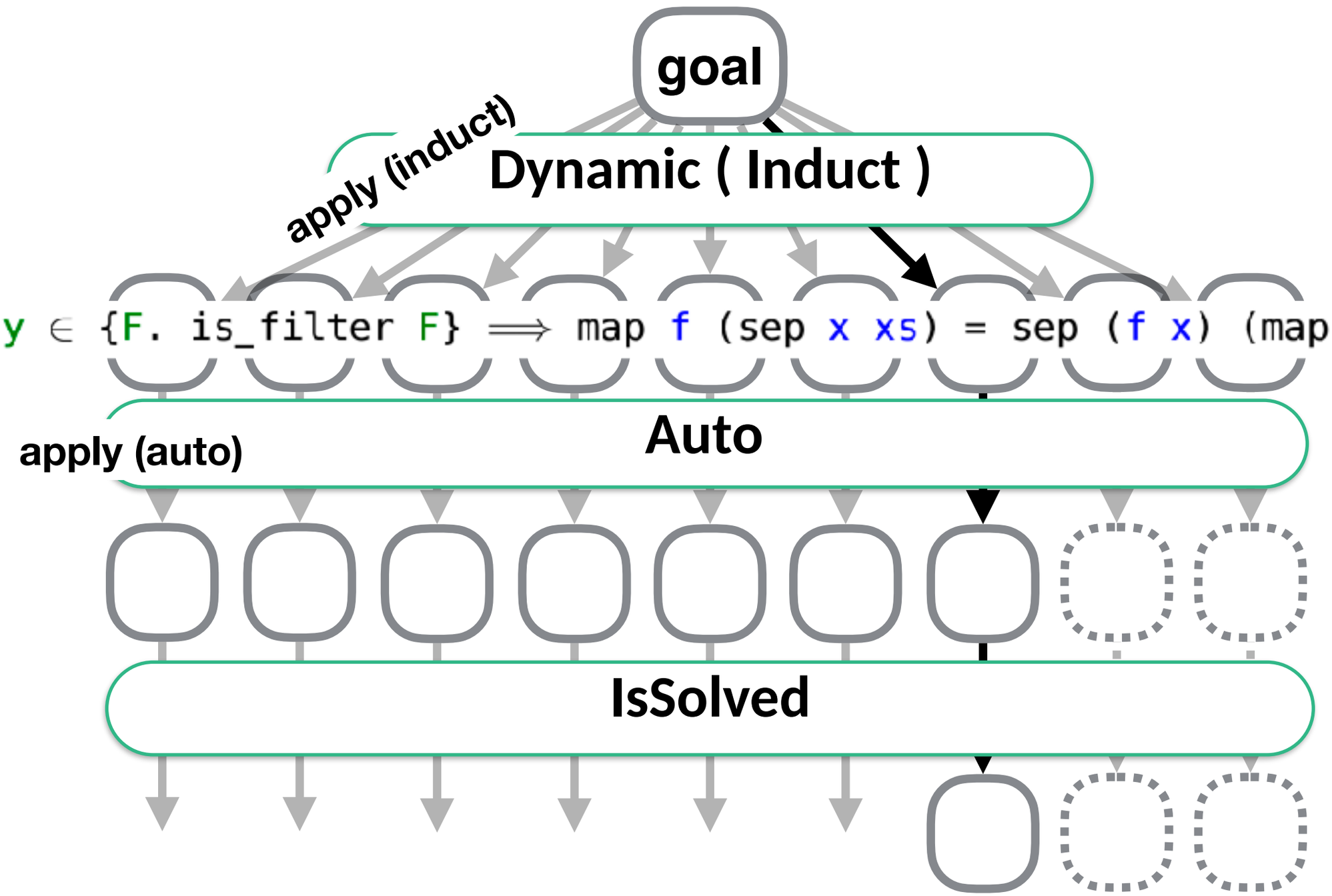

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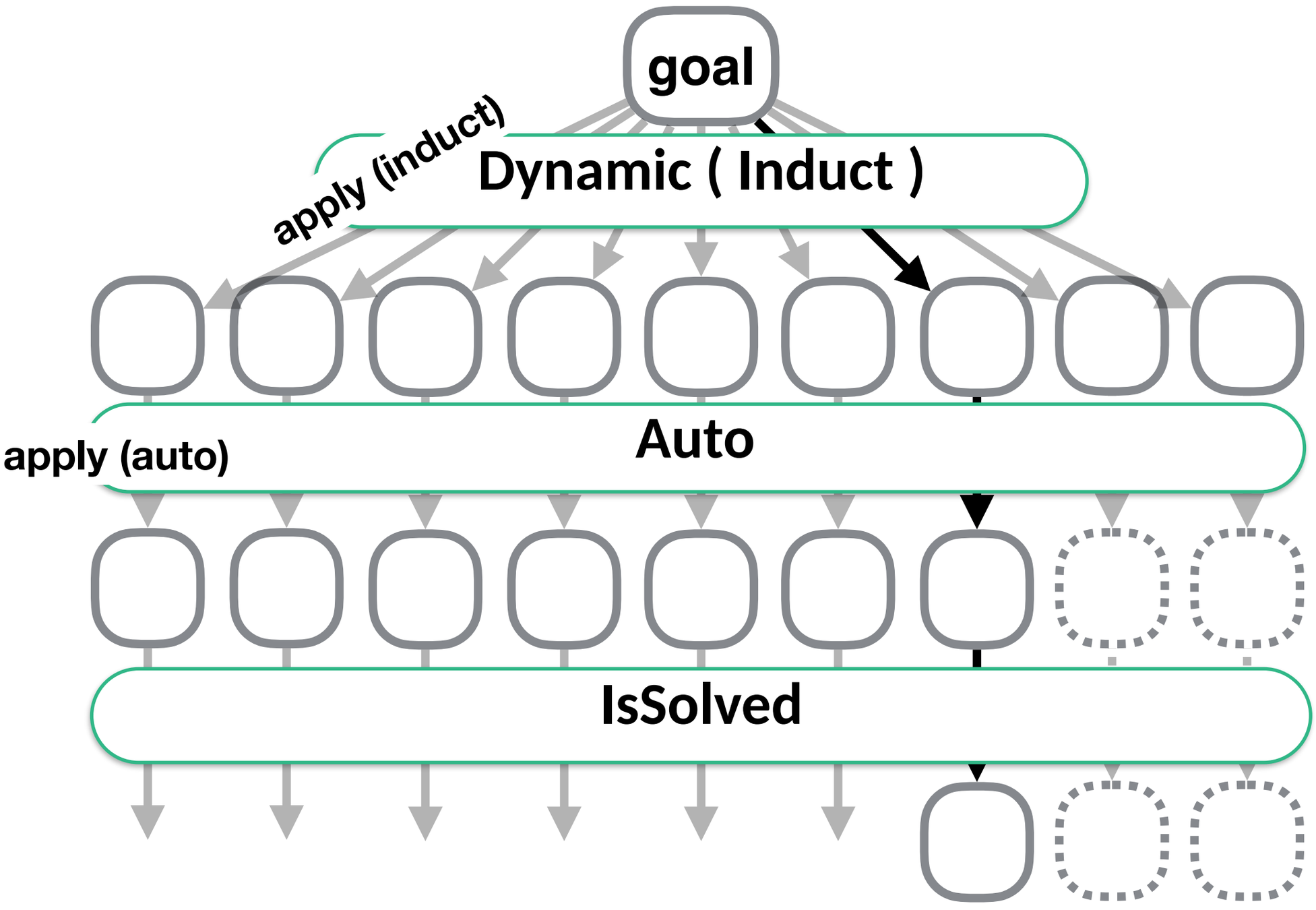

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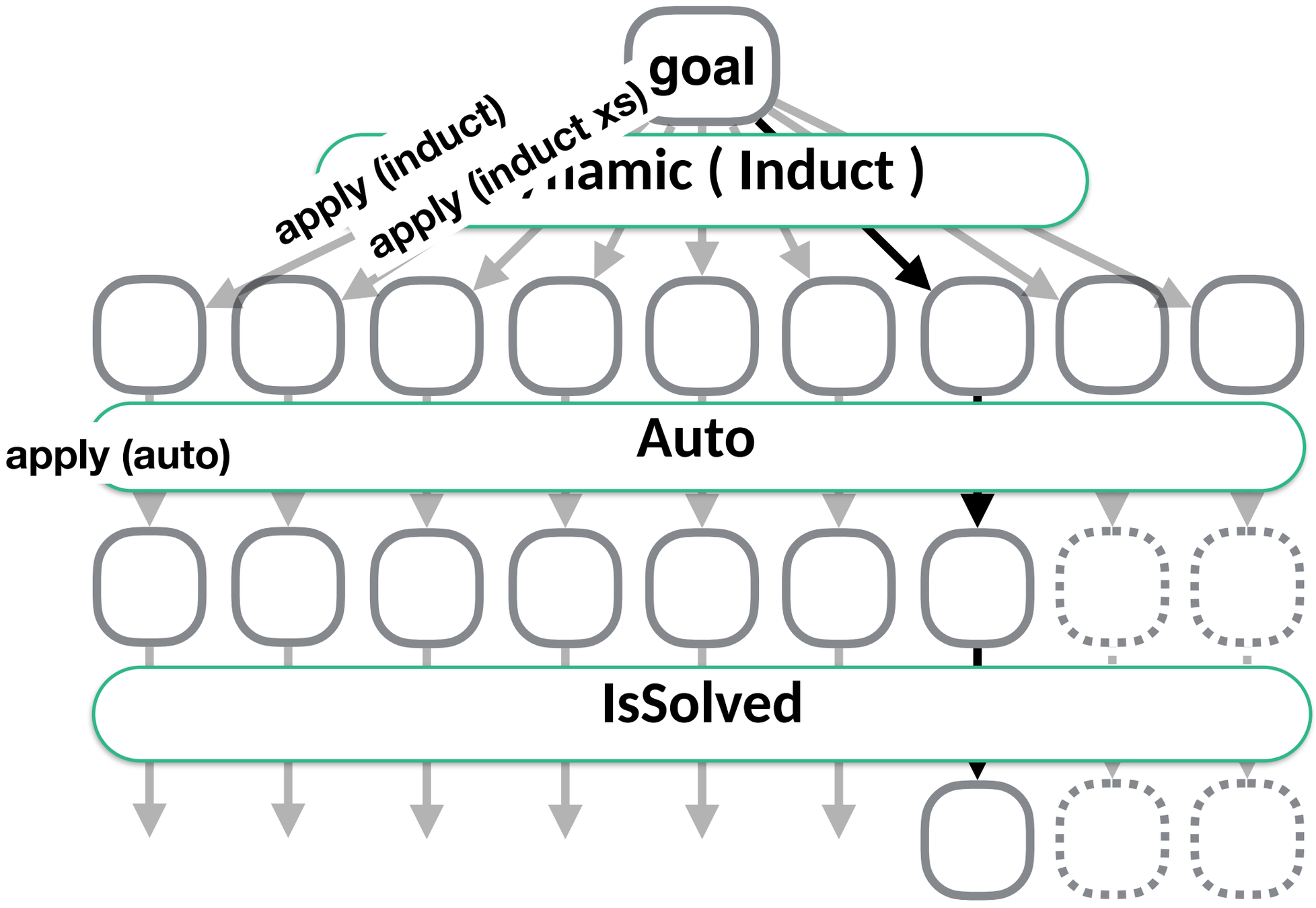

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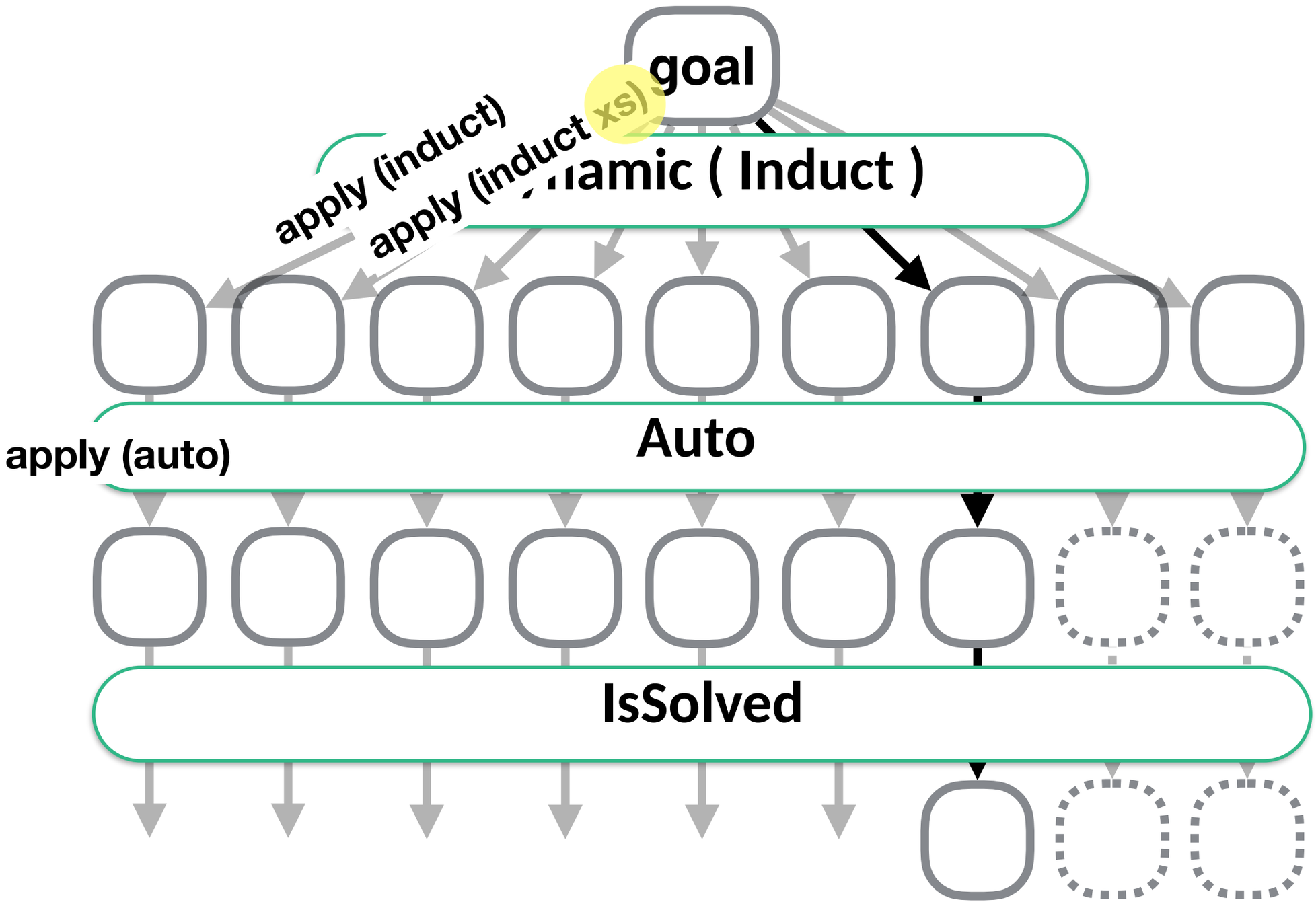

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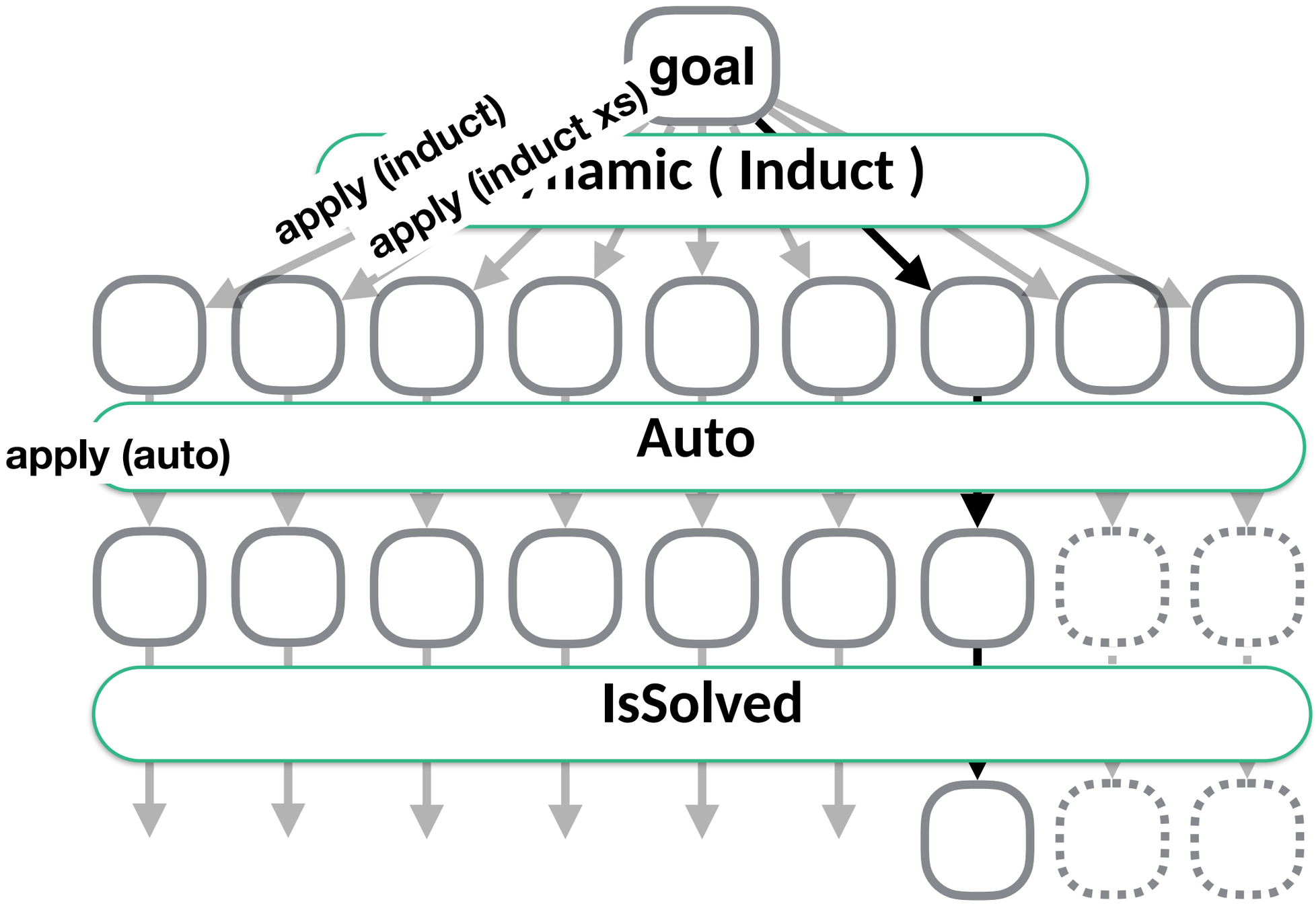

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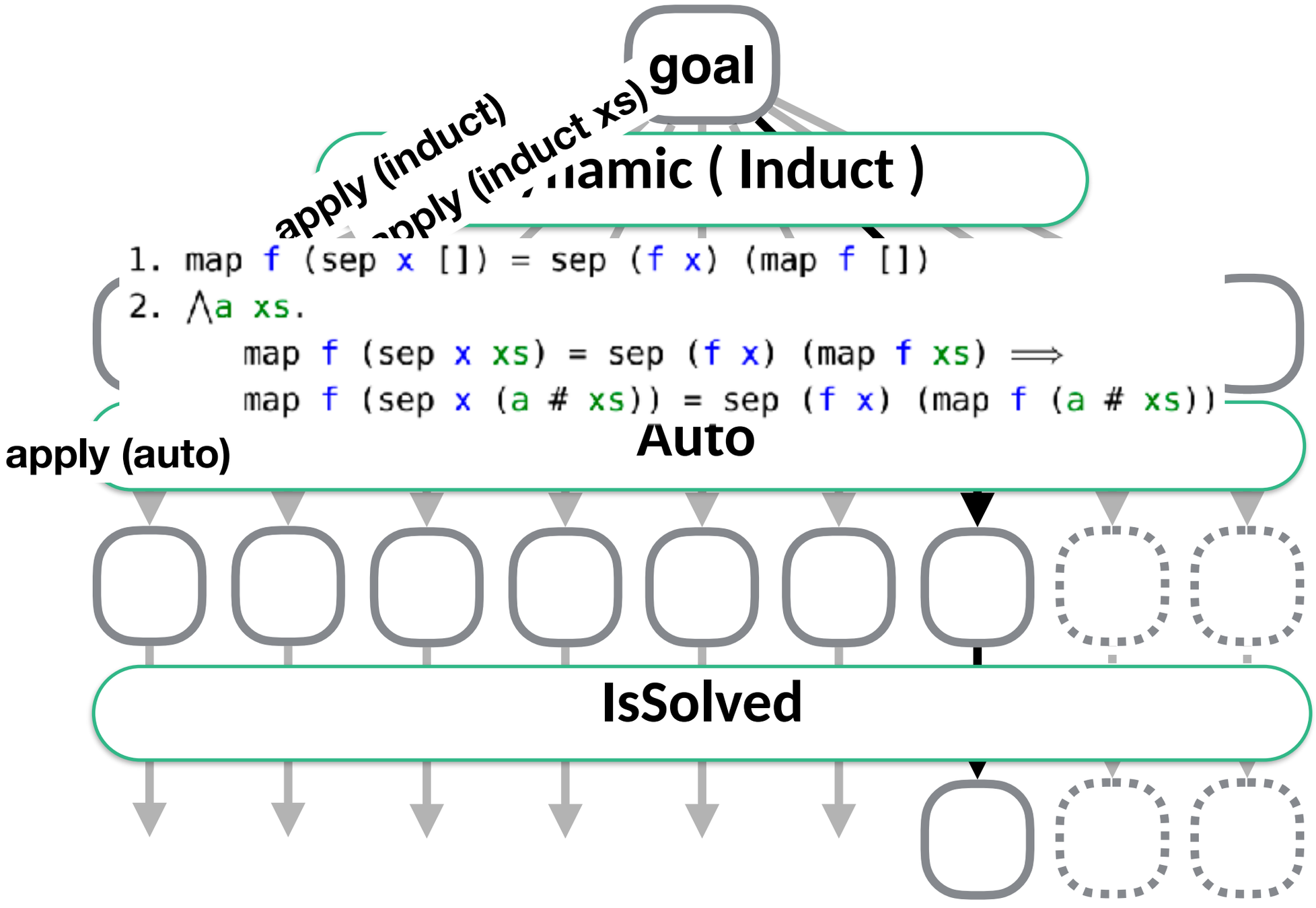

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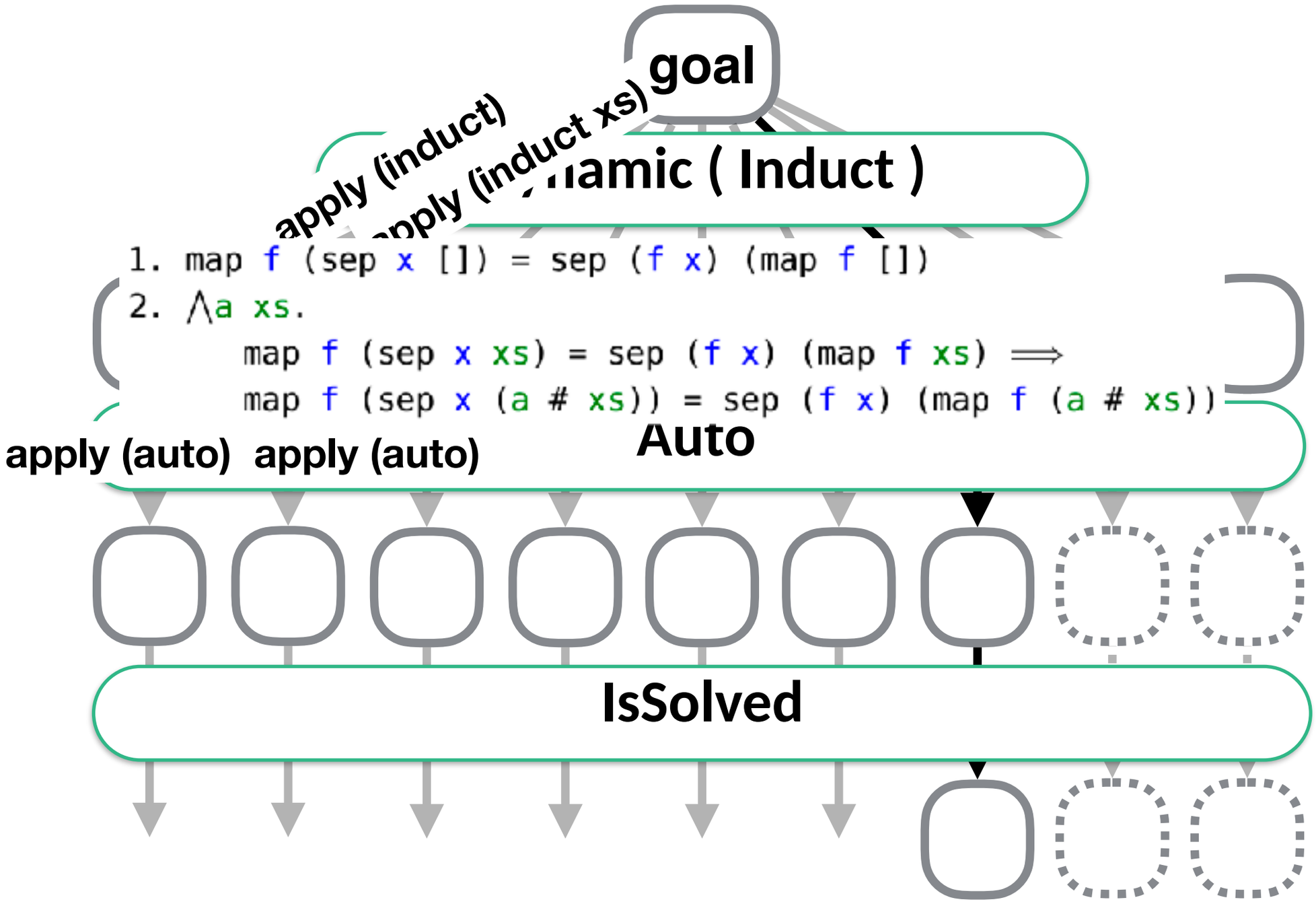

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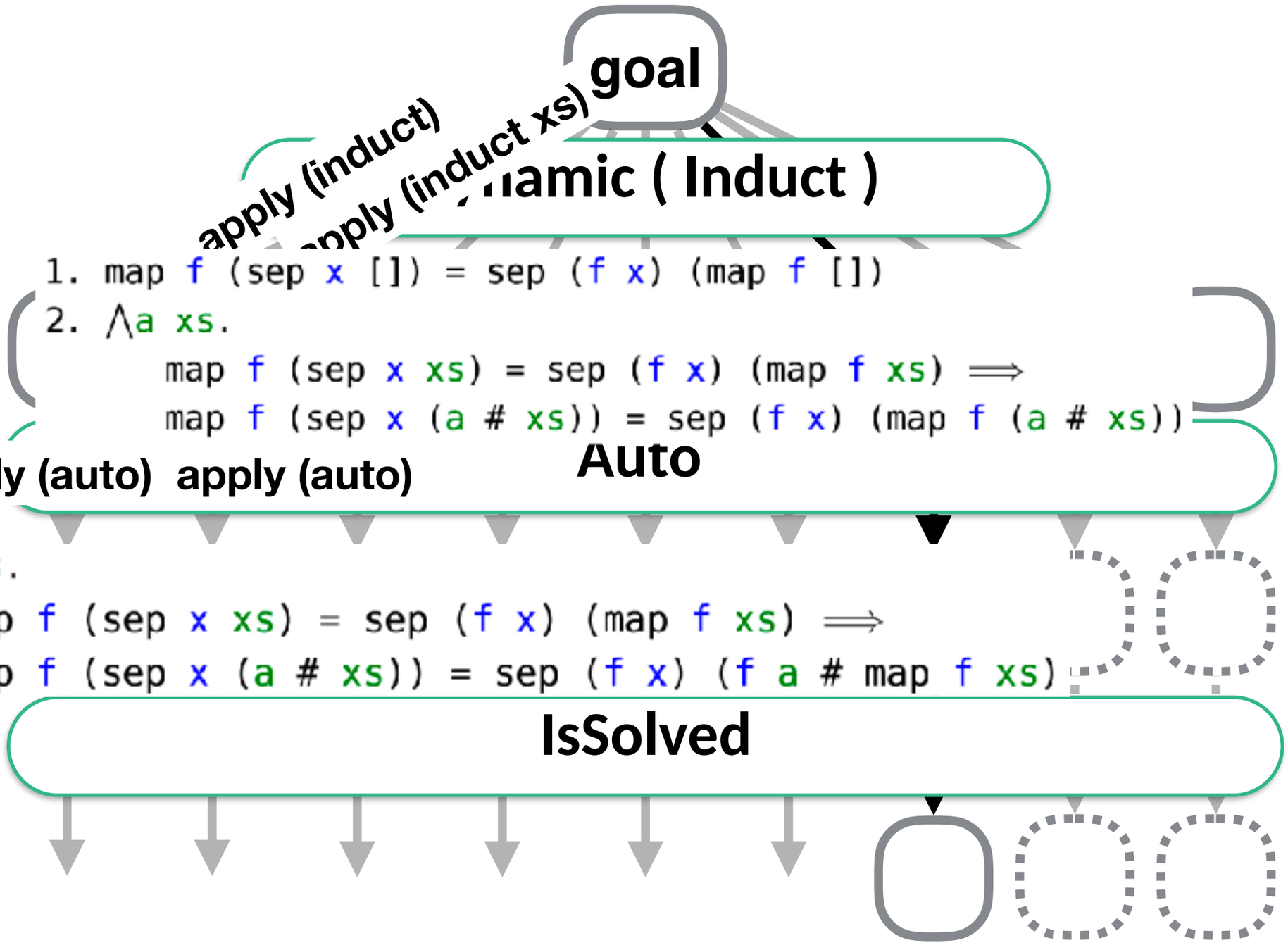

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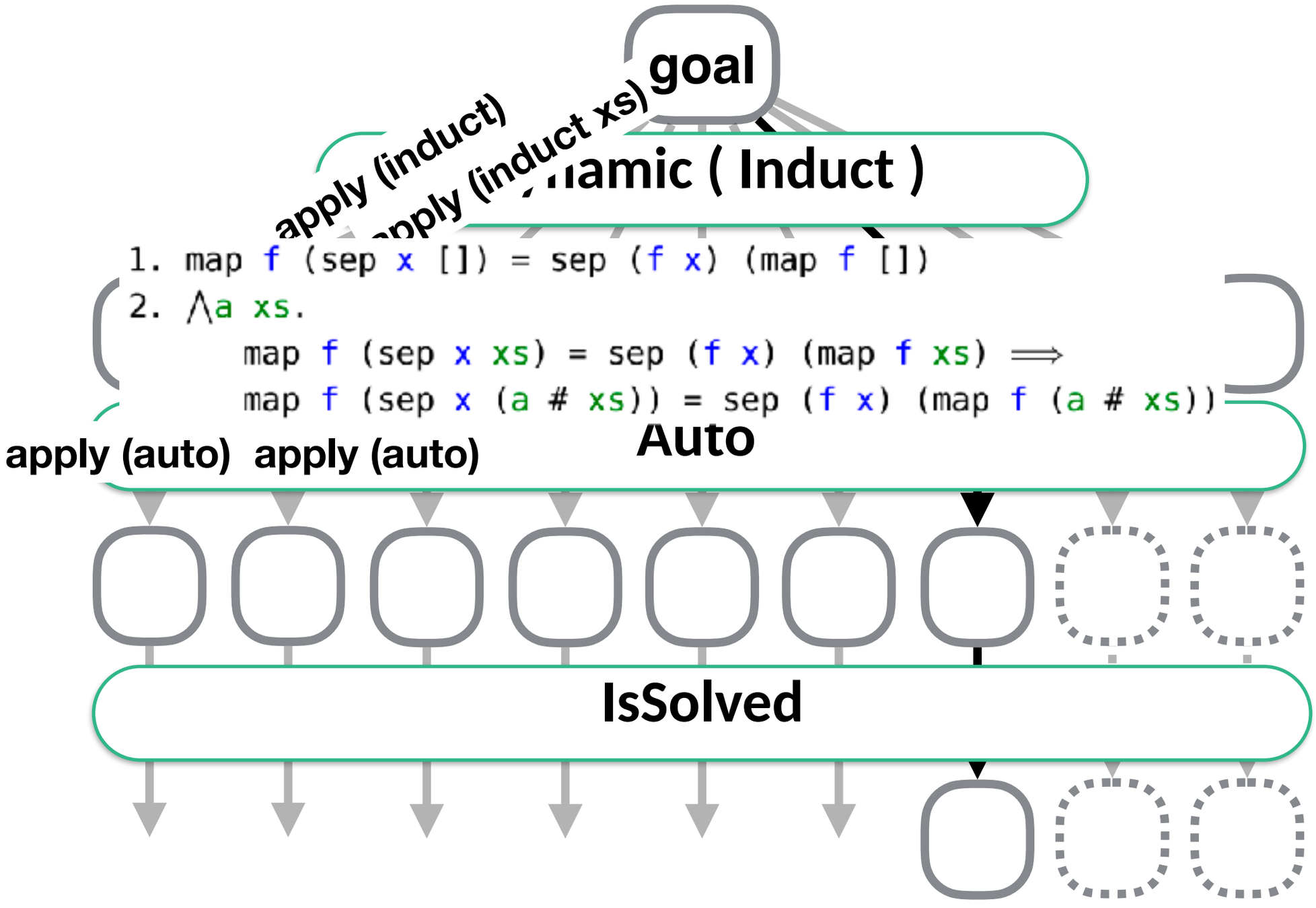

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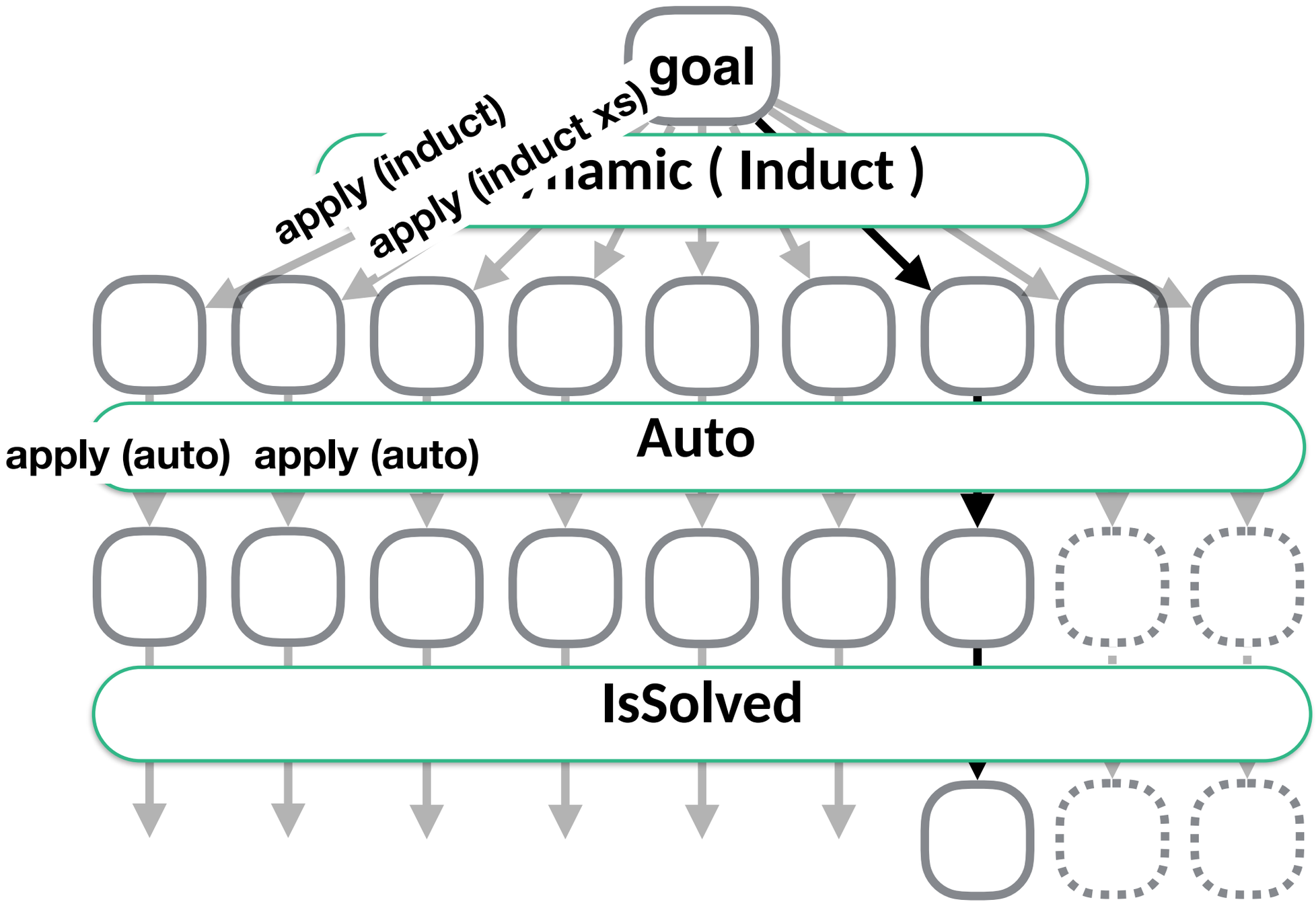

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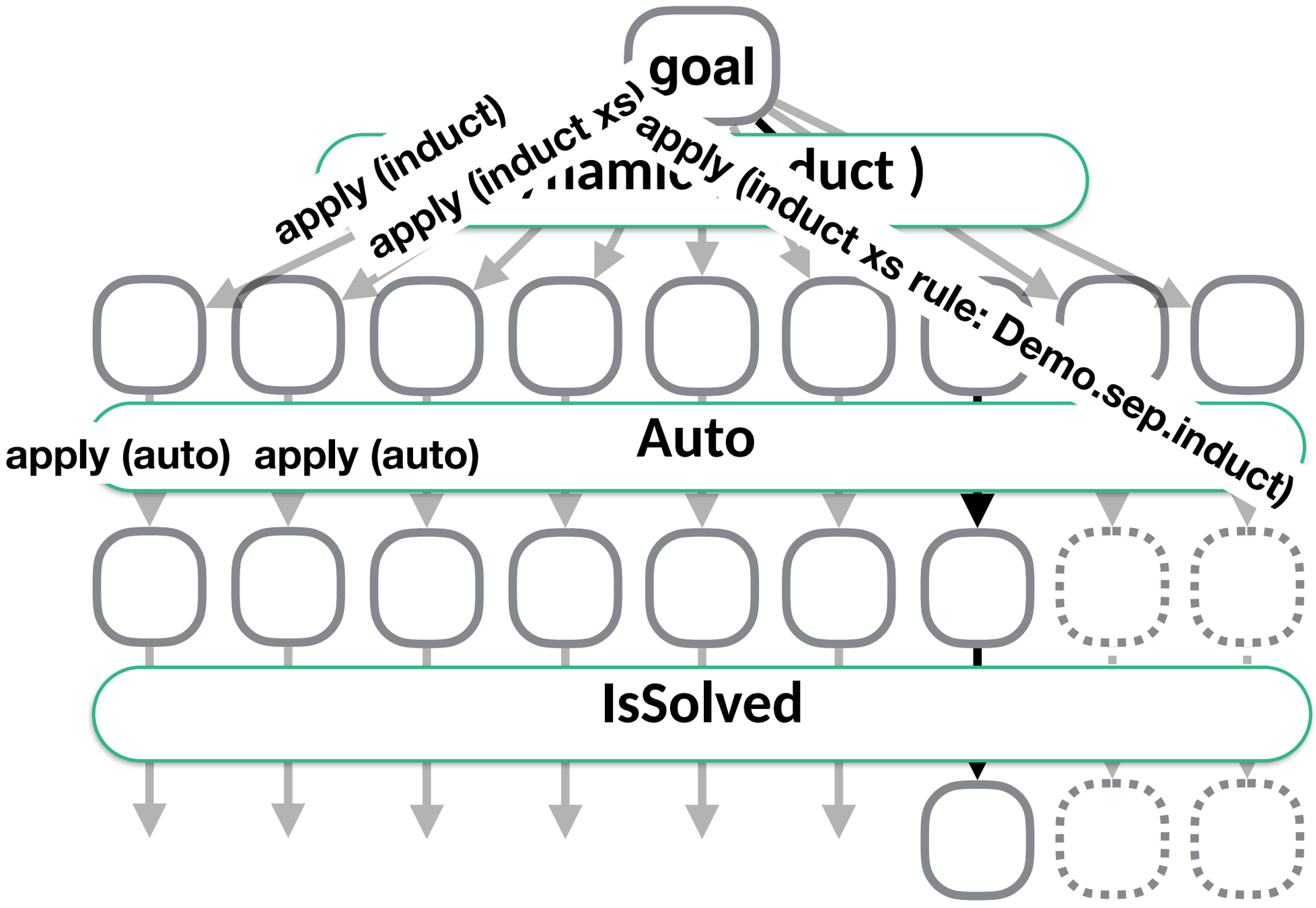

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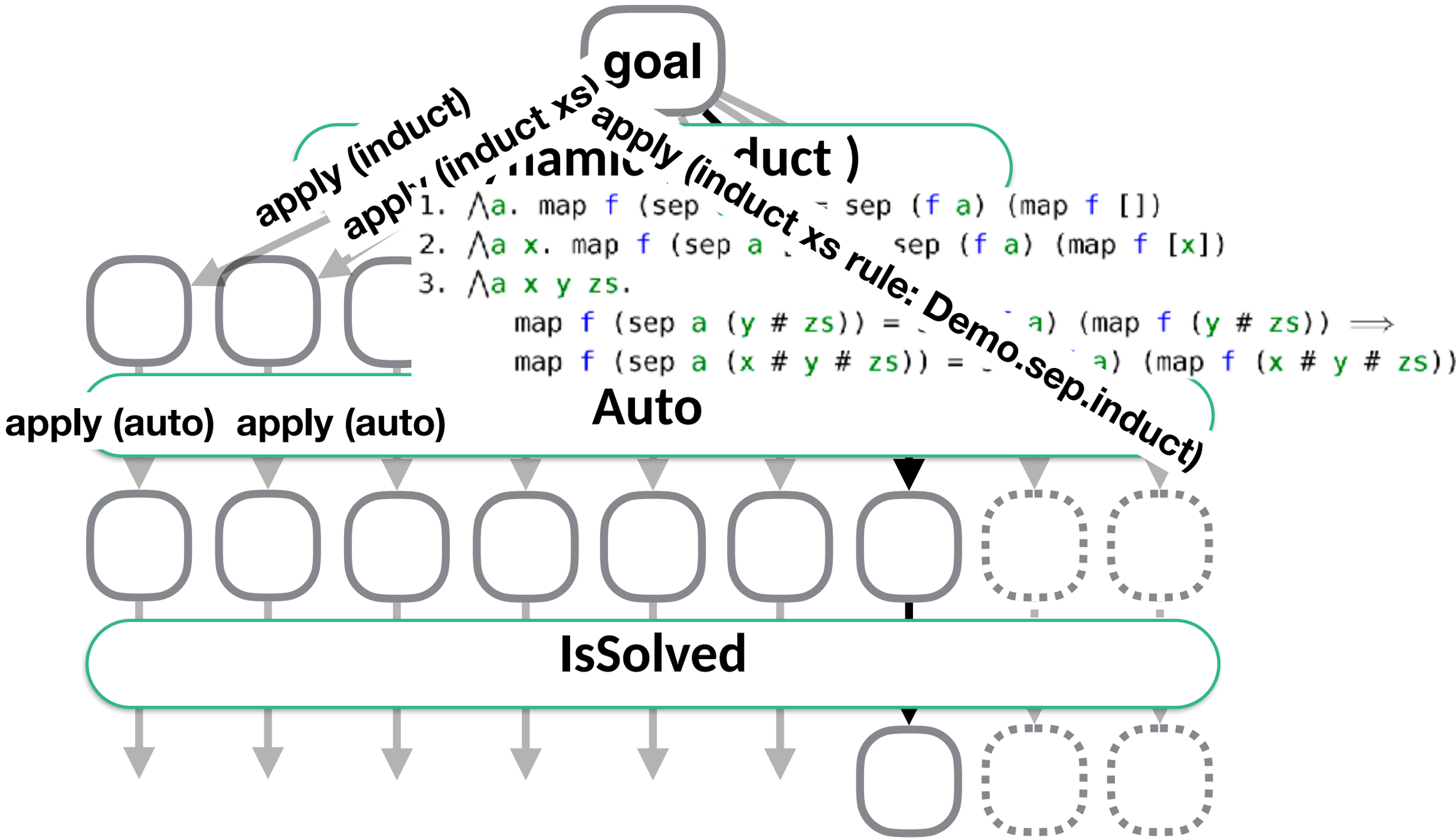

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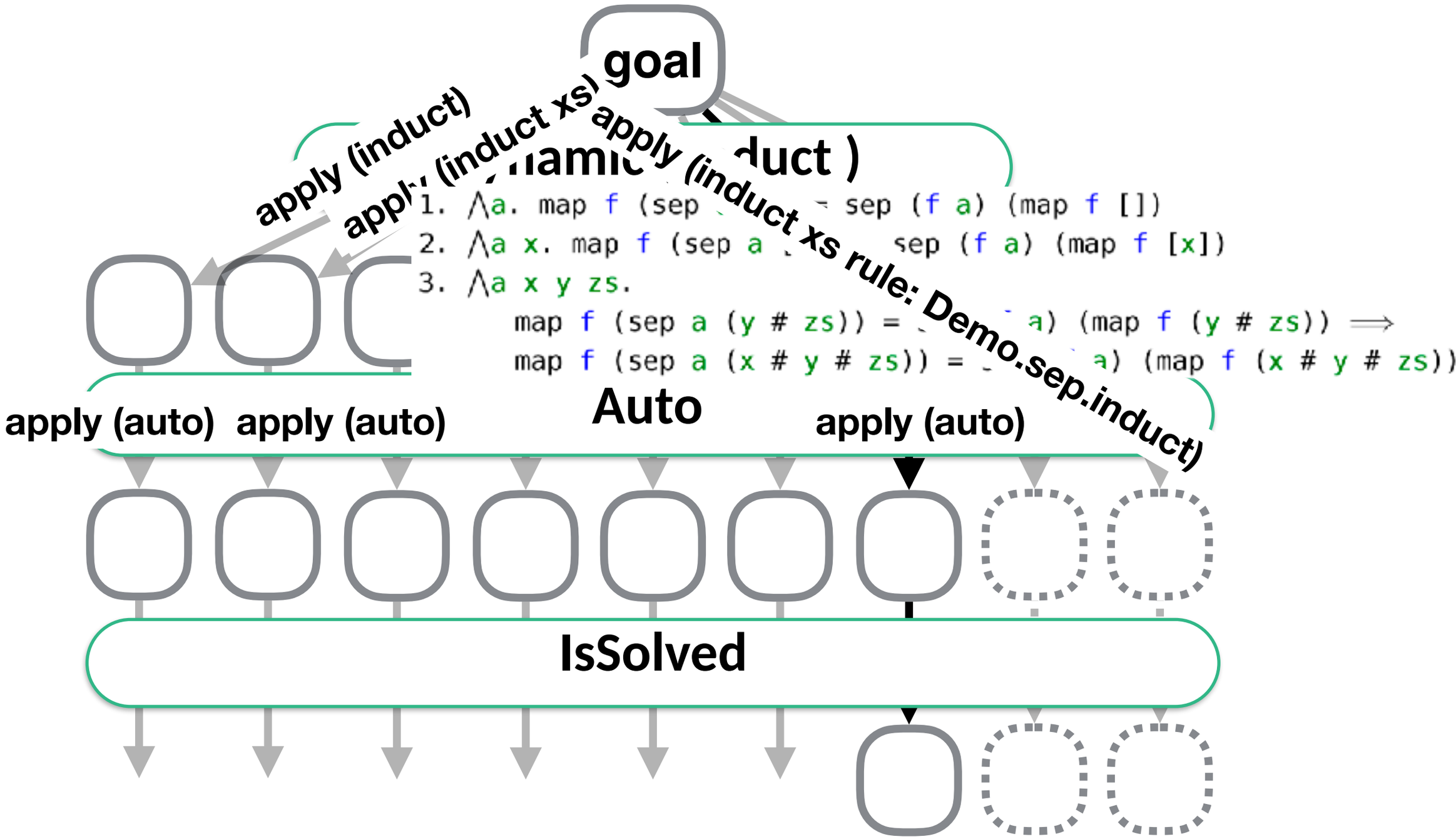

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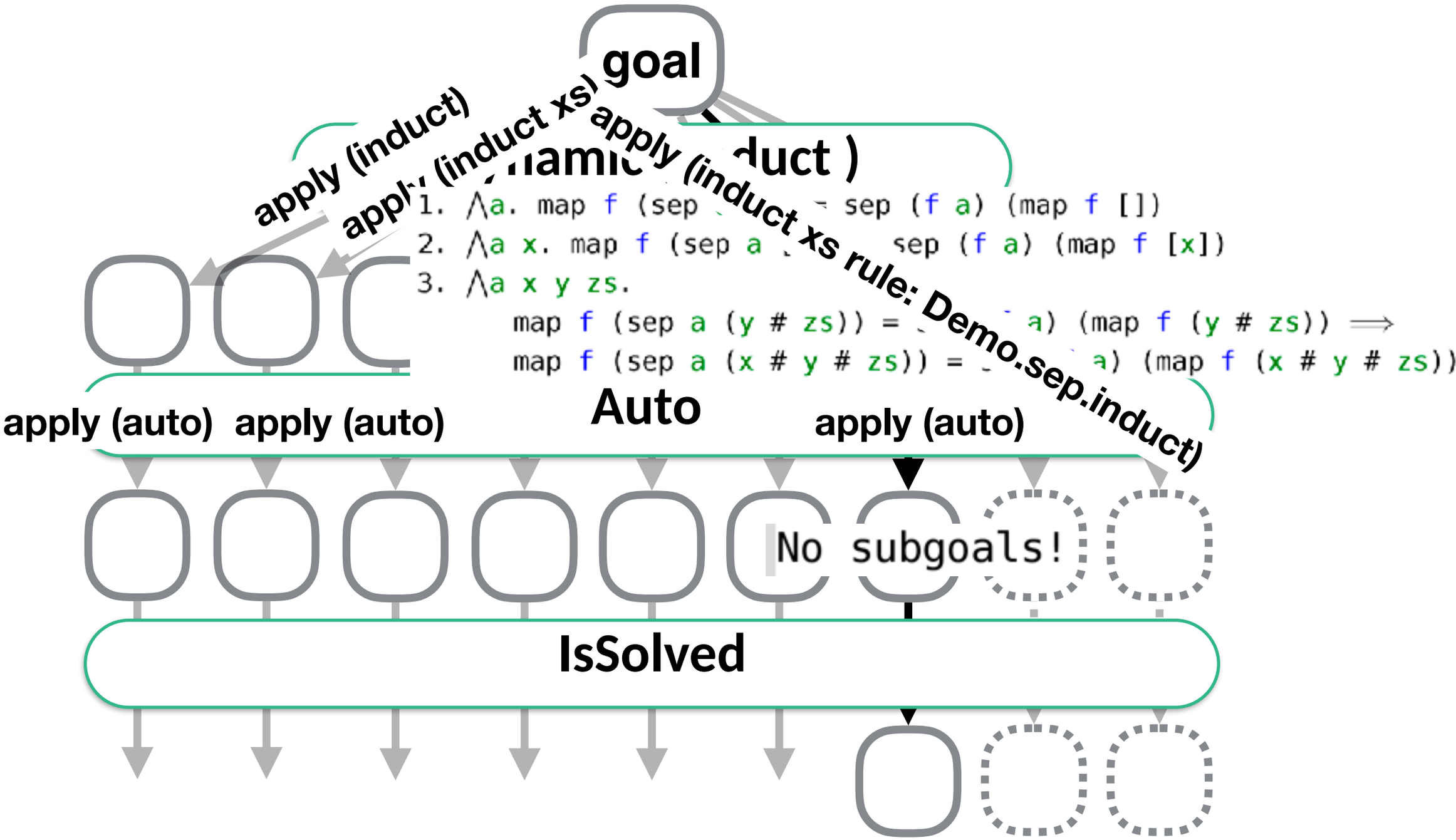

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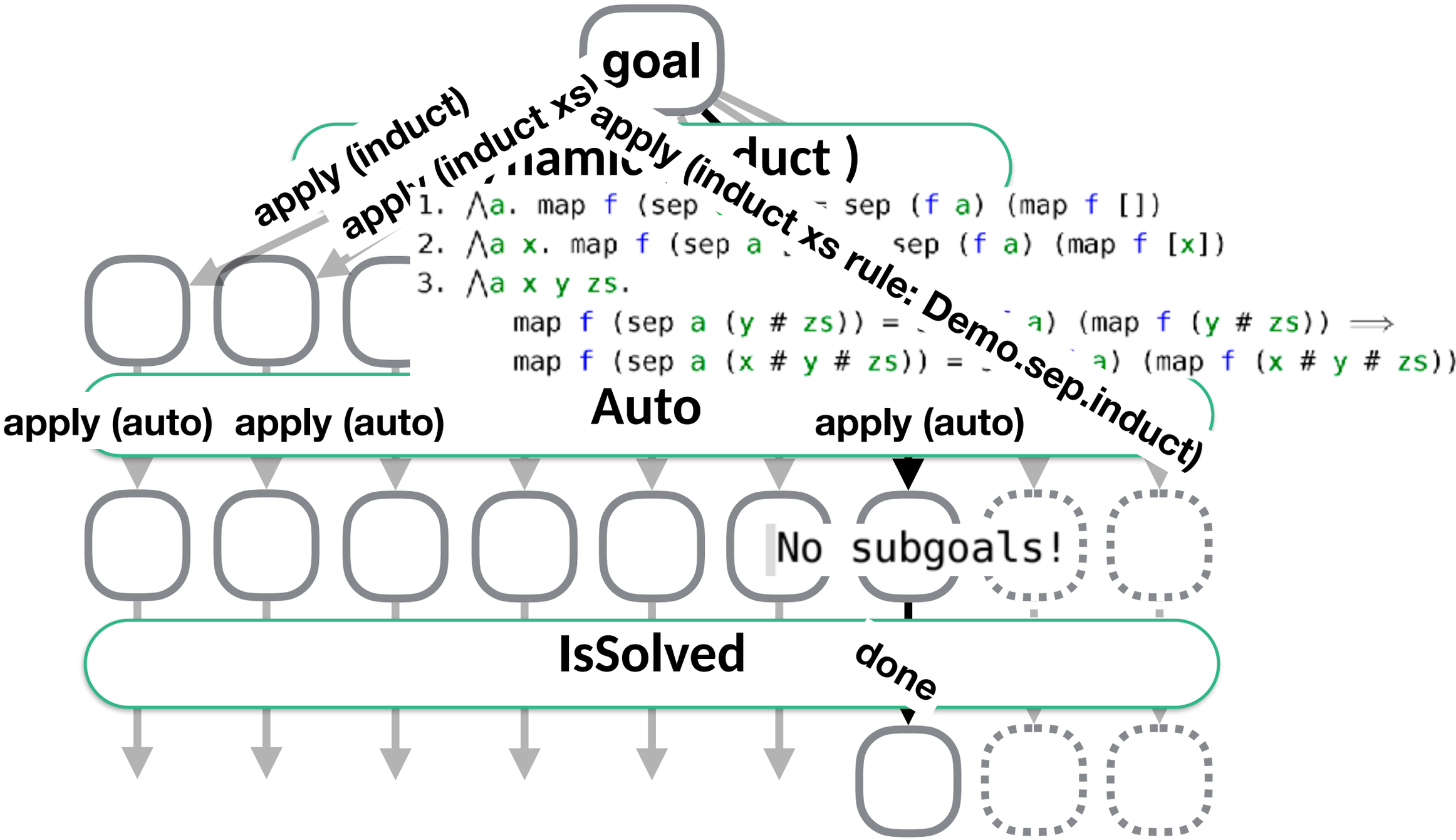
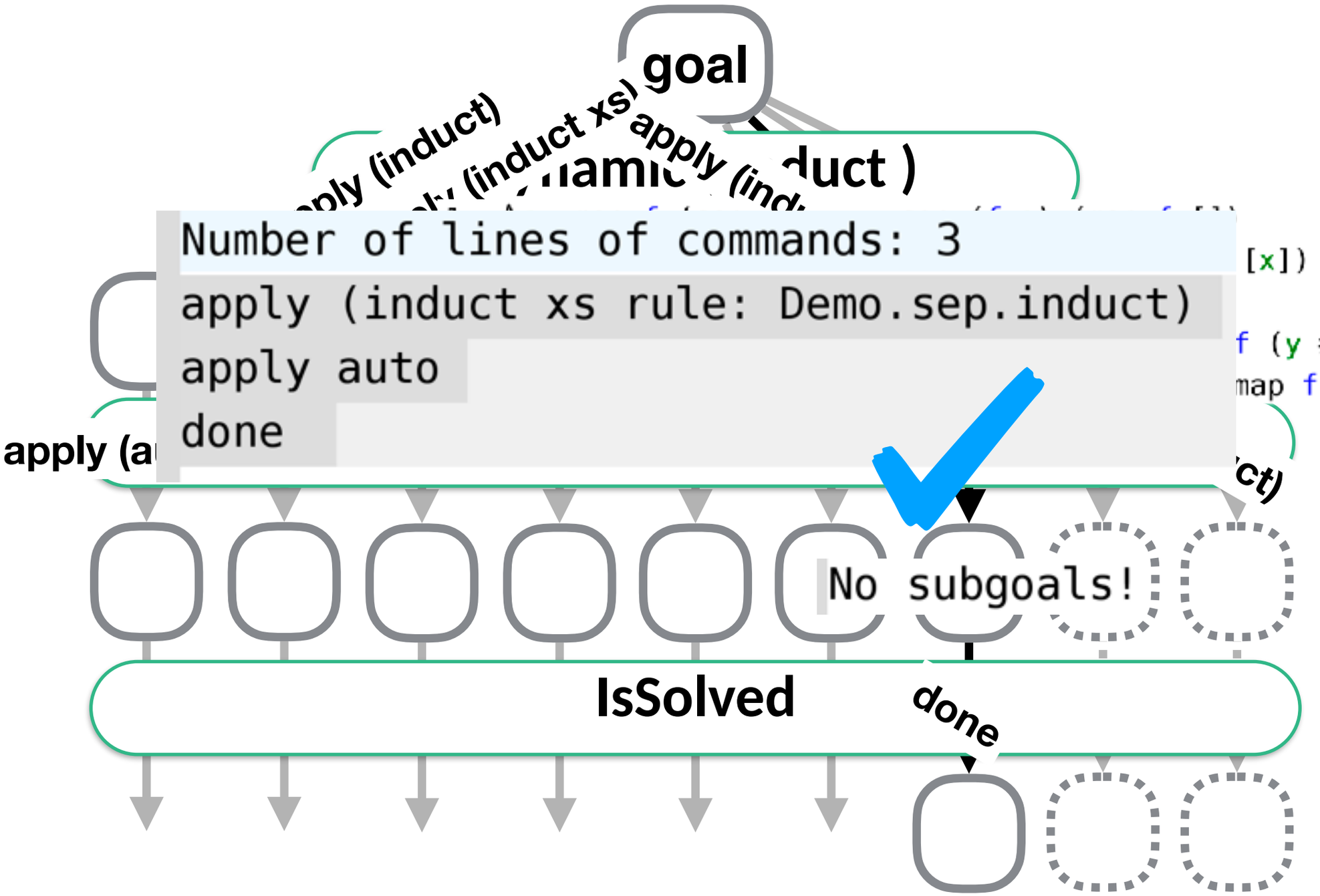
```



https://twitter.com/YutakangE

shorturl.at/dzZ16

```
lemma "map f (sep x xs) = sep (f x) (map f xs)"
find_proof DInd(*= Thens [Dynamic (Induct), Auto, IsSolved]*)
```



Try_Hard: the default strategy

```
strategy Try_Hard =
Ors [Thens [Subgoal, Basic],
      Thens [DInductTac, Auto_Solve],
      Thens [DCaseTac, Auto_Solve],
      Thens [Subgoal, Advanced],
      Thens [DCaseTac, Solve_Many],
      Thens [DInductTac, Solve_Many] ]
```

```
strategy Basic =
  Ors [
      Auto_Solve,
      Blast_Solve,
      FF_Solve,
      Thens [IntroClasses, Auto_Solve],
      Thens [Transfer, Auto_Solve],
      Thens [Normalization, IsSolved],
      Thens [DInduct, Auto_Solve],
      Thens [Hammer, IsSolved],
      Thens [DCases, Auto_Solve],
      Thens [DCoinduction, Auto_Solve],
      Thens [Auto, RepeatN(Hammer), IsSolved],
      Thens [DAuto, IsSolved]]
```

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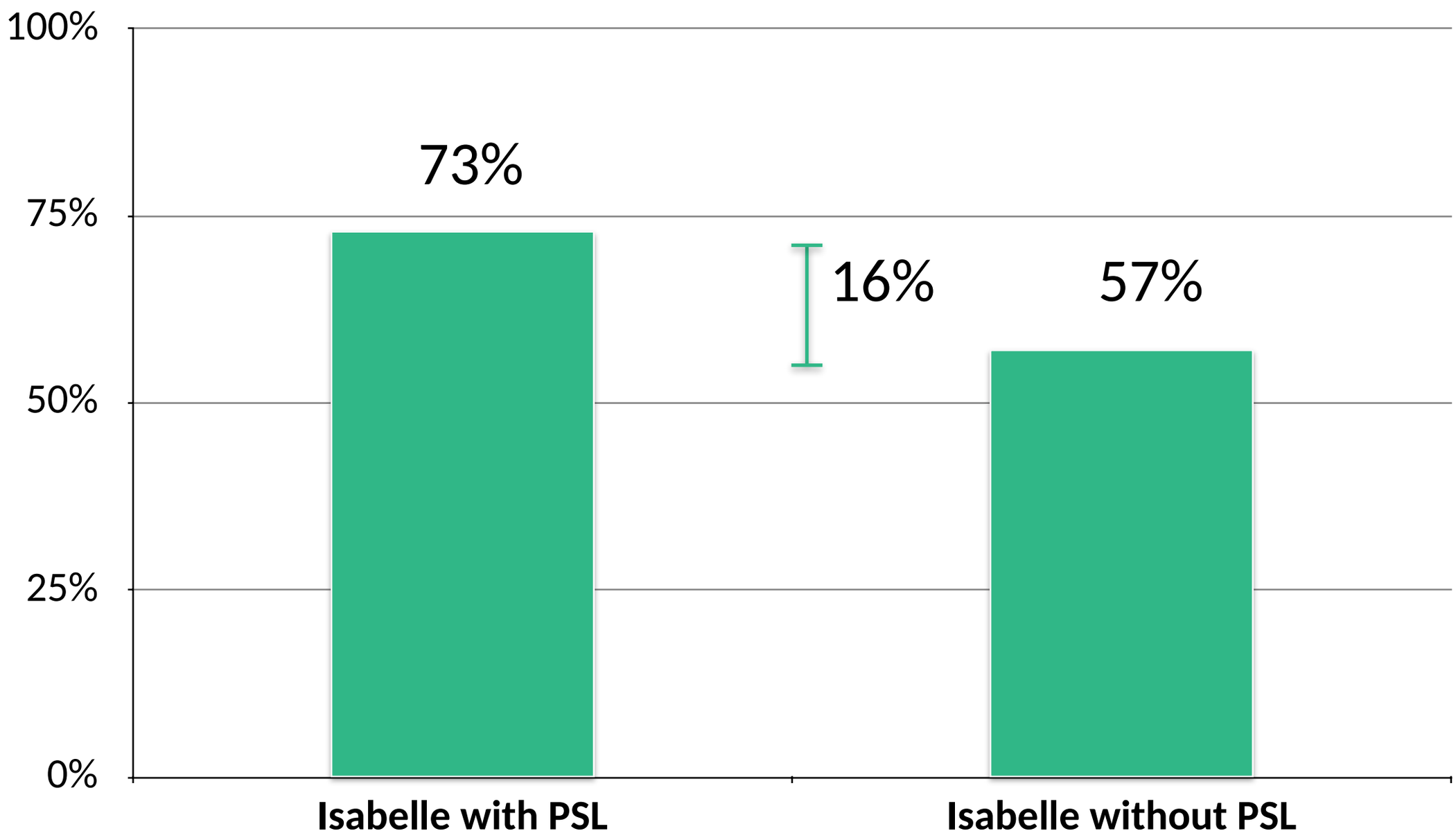
shorturl.at/dzZ16

Evaluation



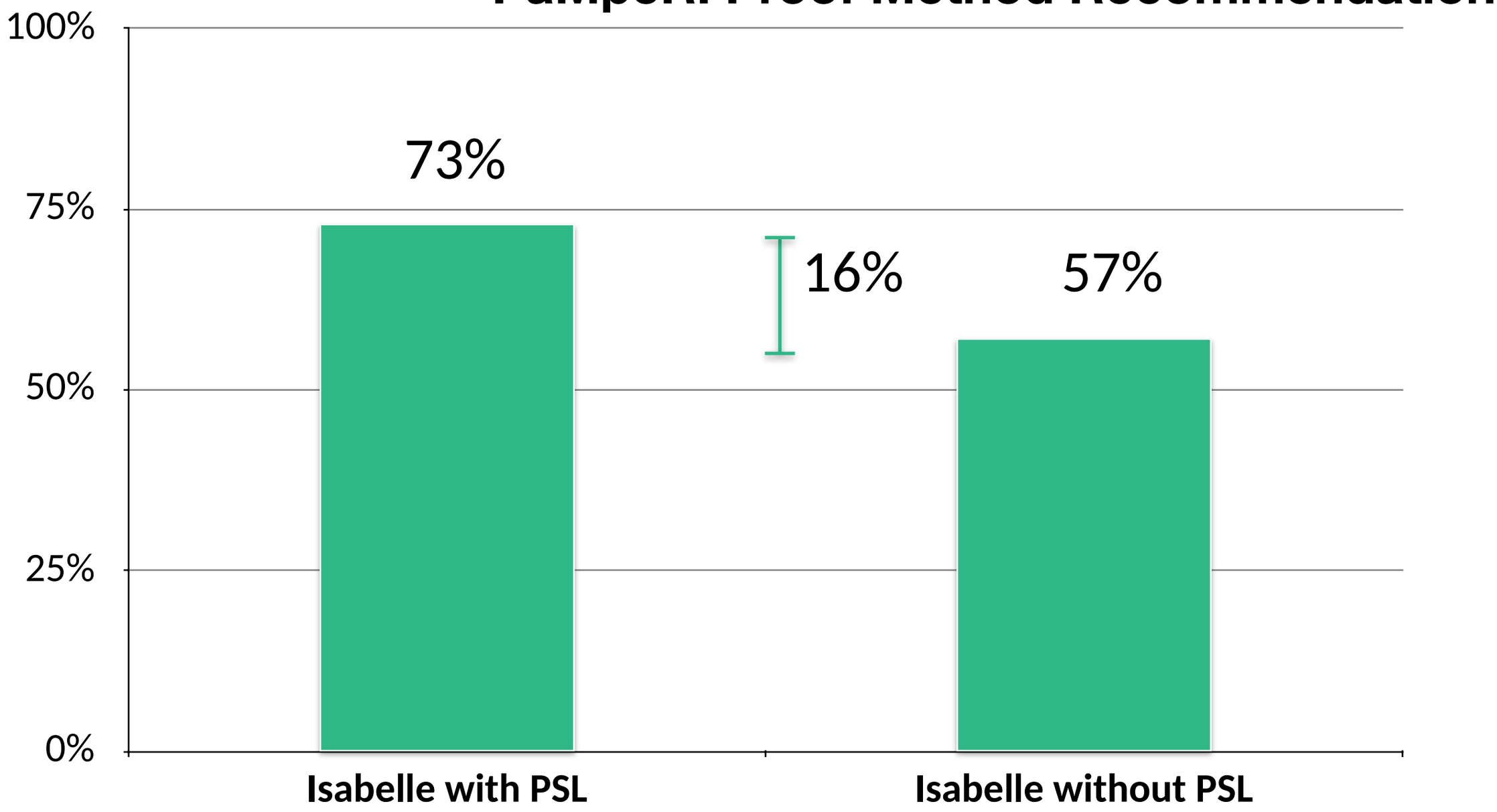
https://twitter.com/YutakangE

shorturl.at/dzZ16

Evaluation

but the search space explode

PaMpeR: Proof Method Recommendation



preparation phase

How does PaMpeR work?

recommendation phase

preparation phase

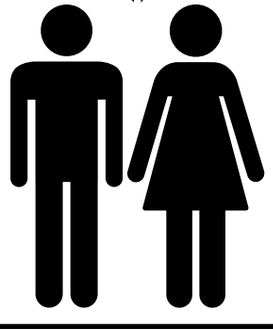
large proof corpora

AFP and standard library

How does PaMpeR work?

recommendation phase

proof state



proof engineer

preparation phase

large proof corpora

AFP and standard library

Statistics

Archive of Formal Proofs (https://www.isa-afp.org)

Statistics

Number of Articles: 468
Number of Authors: 313
Number of lemmas: ~128,900
Lines of Code: ~2,170,300

Most used AFP articles:

	Name	Used by ? articles
1.	Collections	15
2.	List-Index	14
3.	Coinductive	12

preparation phase

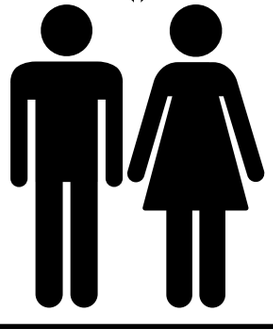
large proof corpora

AFP and standard library

How does PaMpeR work?

recommendation phase

proof state



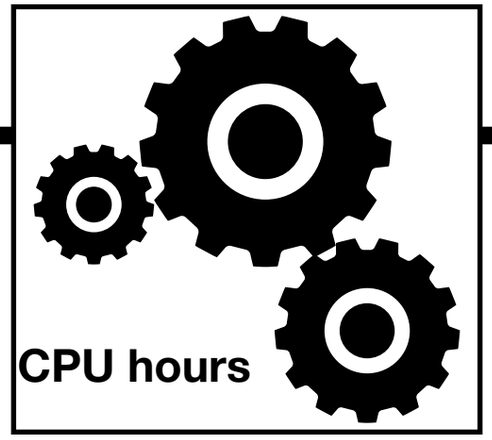
proof engineer

preparation phase

large proof corpora

AFP and standard library

full feature extractor



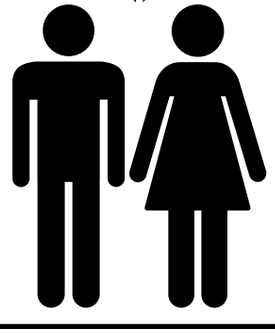
6021 CPU hours

108 assertions

How does PaMpeR work?

recommendation phase

proof state



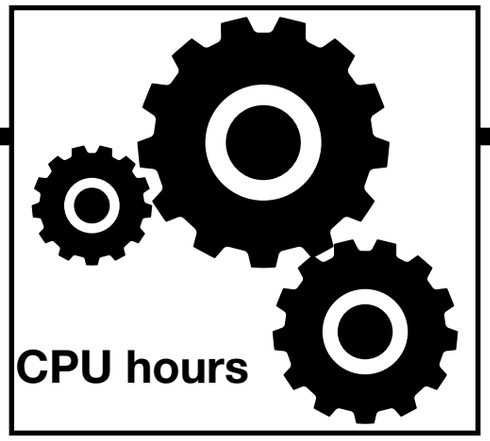
proof engineer

preparation phase

large proof corpora

AFP and standard library

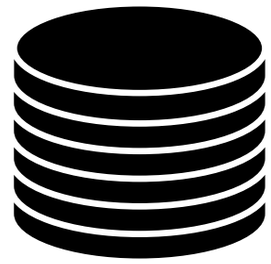
full feature extractor



6021 CPU hours

108 assertions

database (425334 data points)

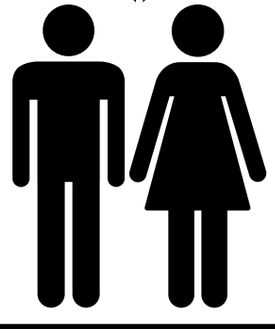


:: (tactic_name, [bool])

How does PaMpeR work?

recommendation phase

proof state



proof engineer

preparation phase

large proof corpora

AFP and standard library

full feature extractor

6021 CPU hours

108 assertions

database (425334 data points)

:: (tactic_name, [bool])

preprocess

decision tree construction

How does PaMpeR work?

recommendation phase

proof state

proof engineer

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full feature extractor

6021 CPU hours

108 assertions

database (425334 data points)

:: (tactic_name, [bool])

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How does PaMpeR work?

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fast feature extractor

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full feature extractor

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108 assertions

database (425334 data points)

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How does PaMpeR work?

recommendation phase

proof state

proof engineer

fast feature extractor

feature vector

How does PaMpeR work?

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full feature extractor

6021 CPU hours

108 assertions

database (425334 data points)

:: (tactic_name, [bool])

preprocess

decision tree construction

recommendation phase

proof state

proof engineer

fast feature extractor

feature vector

lookup

proof method recommendation

Summary

PSL can find how to apply induction for easy problems.
CADE2017 (https://link.springer.com/10.1007/978-3-319-63046-5_32)

PaMpeR recommends which proof methods to use.
ASE2018 (https://dx.doi.org/10.1145/3238147.3238210)

https://twitter.com/YutakangE

Chemistry

Electronics

etc.

Physics

Acoustics

Astrophysics

Electromagnetism

Molecular Physics

Quantum Physics

etc.

Informatics

Language

Algorithms

Data Structures

Architecture

Software Engineering

Formal Method
Computational Logic

Mathematics The Language of Science.

Analysis Algebra Geometry Probability Theory

Logic: the Foundation of Mathematics.

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Chemistry

Electronics

etc.

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Mathematics The Language of Science.

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What do you want to solve with AI mathematicians?

https://twitter.com/YutakangE

Chemistry

Electronics

etc.

Physics

Acoustics

Astrophysics

Electromagnetic

Mole

Q & A

Informatics

Language

Algorithms

Data Structures

Architecture

Software Engineering

Formal Method
Computational Logic

Mathematics The Language of Science.

Analysis Algebra Geometry Probability Theory

What do you want to solve with AI mathematicians?