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AI for theorem proving in Isabelle/HOL

This work was supported by the project AI&Reasoning (reg. no. CZ.02.1.01/0.0/0.0/15_003/0000466).

Yutaka Nagashima
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CVUT, CTU, CIIRC

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AI for theorem proving? in Isabelle?/HOL

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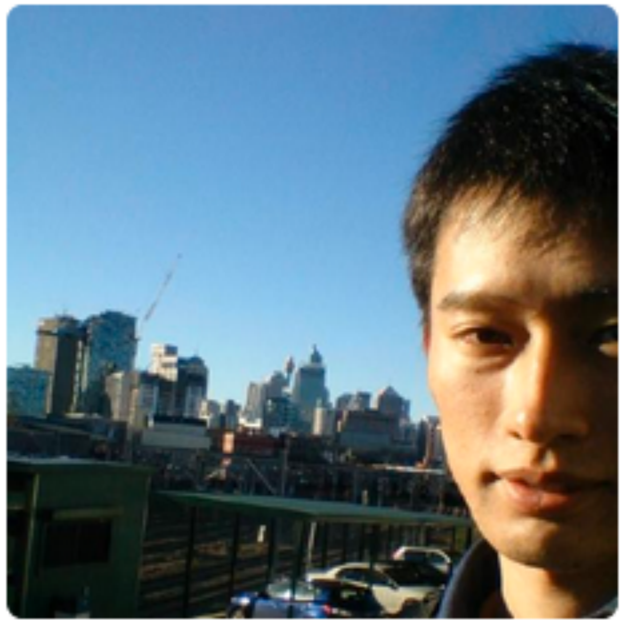

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CVUT, CTU, CIIRC

Why theorem proving?

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To build trustworthy software (Complete Formal Verification)!

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1. Specify what we want.

Why theorem proving?

To build trustworthy software (Complete Formal Verification)!

1. **Specify what we want.**
2. **Implement what we want.**

Why theorem proving?

To build trustworthy software (Complete Formal Verification)!

1. **Specify what we want.**
2. **Implement what we want.**
3. **Prove the implementation satisfies the specification.**

Why theorem proving?

To build trustworthy software (Complete Formal Verification)!

1. Specify what we want.

2. Implement what we want.

3. Prove the implementation satisfies the specification.

Example

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Why theorem proving?

To build trustworthy software (Complete Formal Verification)!

1. Specify what we want.

2. Implement what we want.

3. Prove the implementation satisfies the specification.

Example

developer | proof assistant / ITP | implementation

+

Isabelle

=

seL4

Security. Performance. Proof.

Gewin Klein et. al

Isabelle/HOL

verified micro kernel, seL4

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Why theorem proving?

To build trustworthy software (Complete Formal Verification)!

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developer

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Mathematics	Informatics	Physics
Number Theory	Language	Acoustics
Analysis	Algorithms	Astrophysics
Algebra	Data Structures	Electromagnetism
Geometry	Architecture	Molecular Physics
Probability Theory	Software Engineering	Quantum Physics
etc.	Formal Verification theorem proving	etc.

Mathematics

Number Theory

Analysis

Algebra

Geometry

Probability Theory

etc.

Informatics

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Algorithms

Data Structures

Architecture

Software Engineering

Formal Verification

theorem proving

Physics

Acoustics

Astrophysics

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Molecular Physics

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etc.

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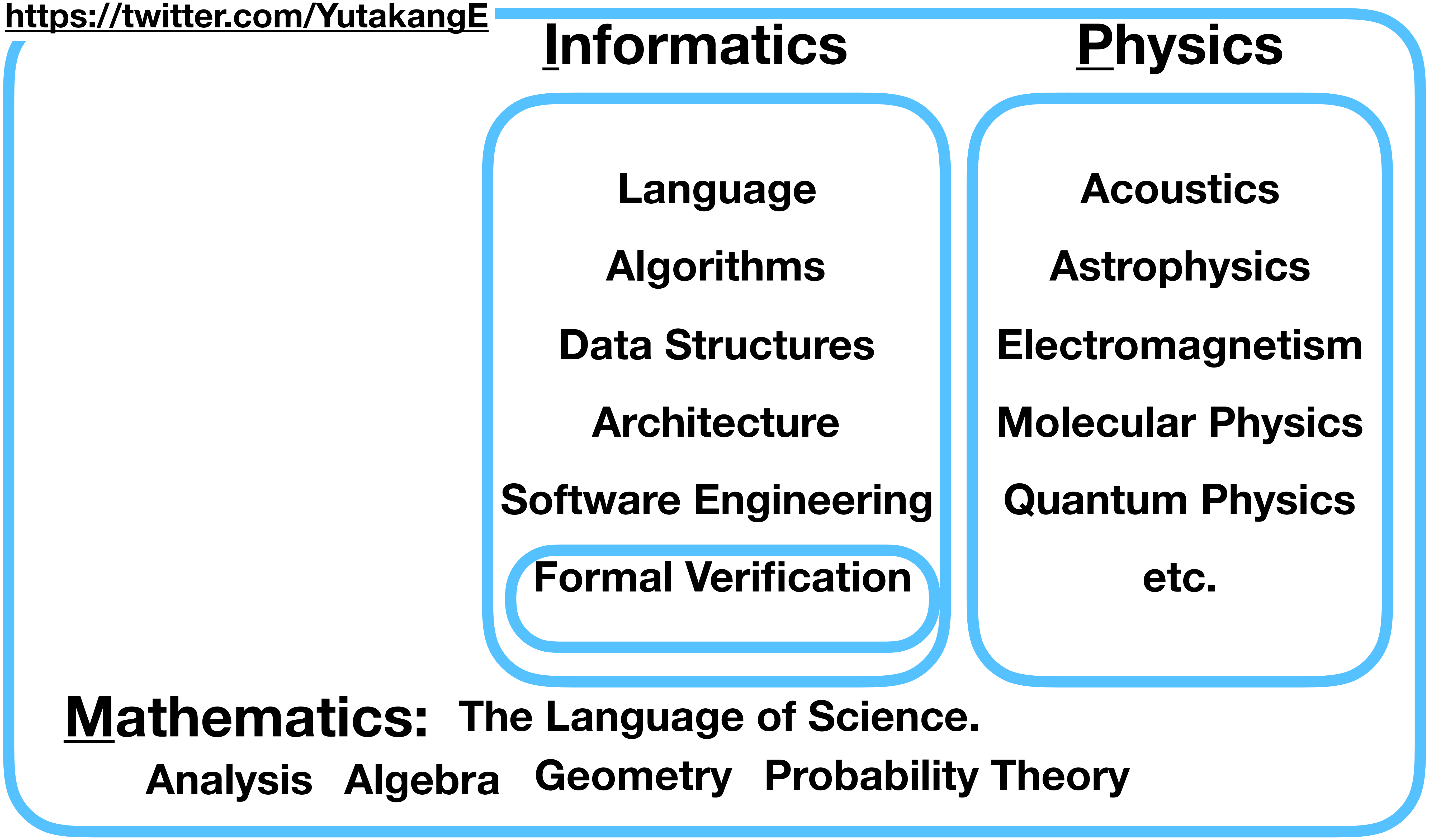

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Informatics

Language

Algorithms

Data Structures

Architecture

Software Engineering

Formal Verification

Physics

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etc.

Mathematics: The Language of Science.

Analysis Algebra Geometry Probability Theory

Logic: the Foundation of Mathematics.

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Informatics

- Language
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`Automate Logic using AI to Accelerate Science!`

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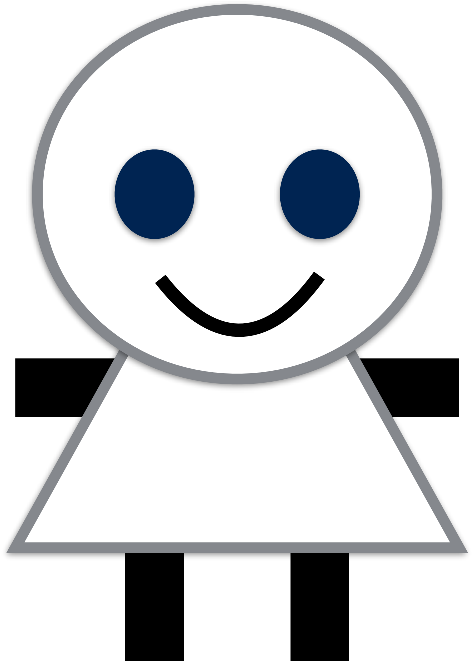
Interactive theorem proving with Isabelle/HOL



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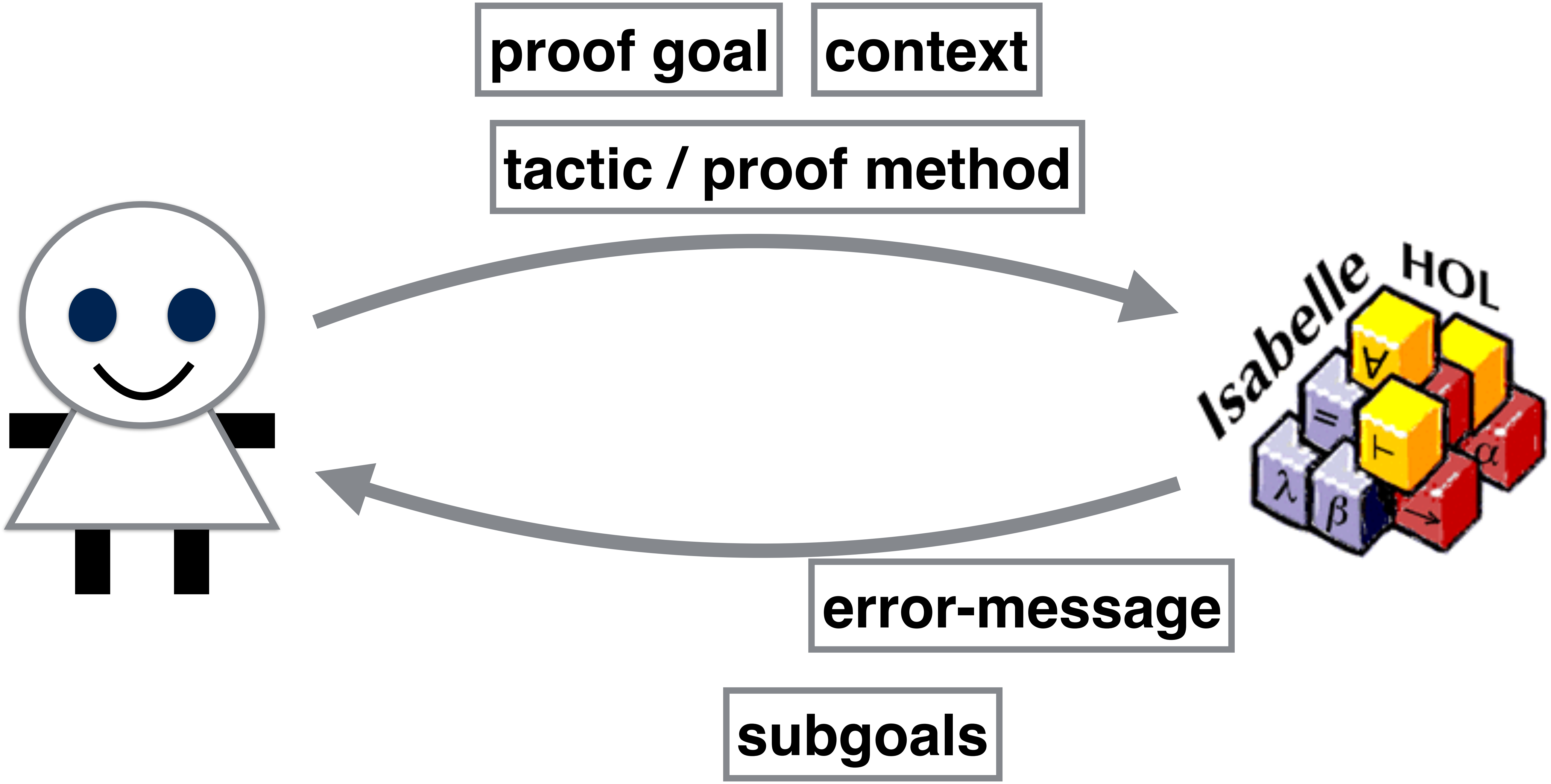
Interactive theorem proving with Isabelle/HOL

proof goal

context

tactic / proof method

Interactive theorem proving with Isabelle/HOL



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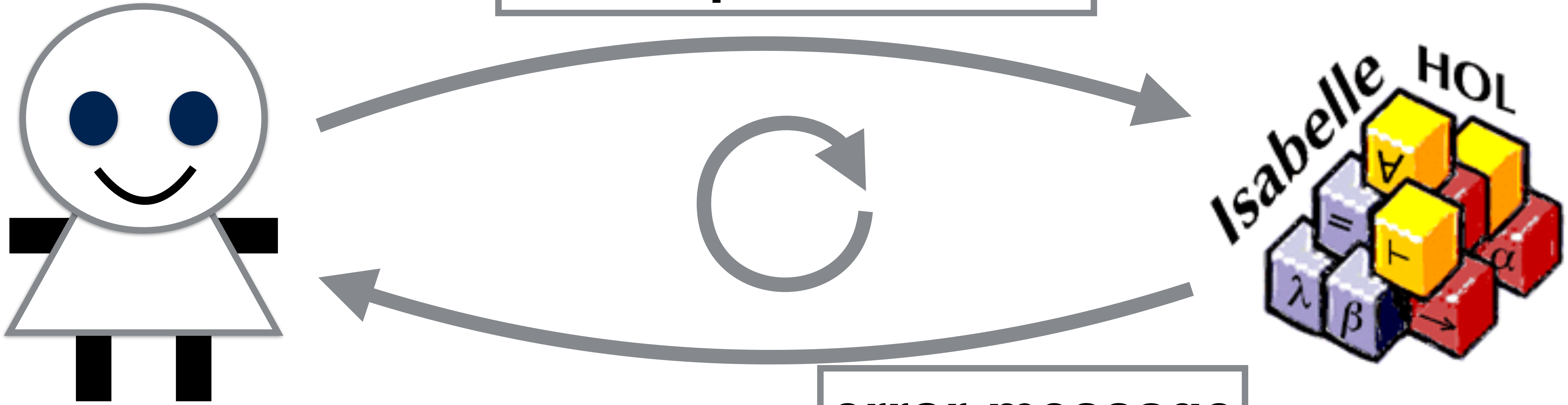
Interactive theorem proving with Isabelle/HOL

proof goal **context**

tactic / proof method



error-message

subgoals

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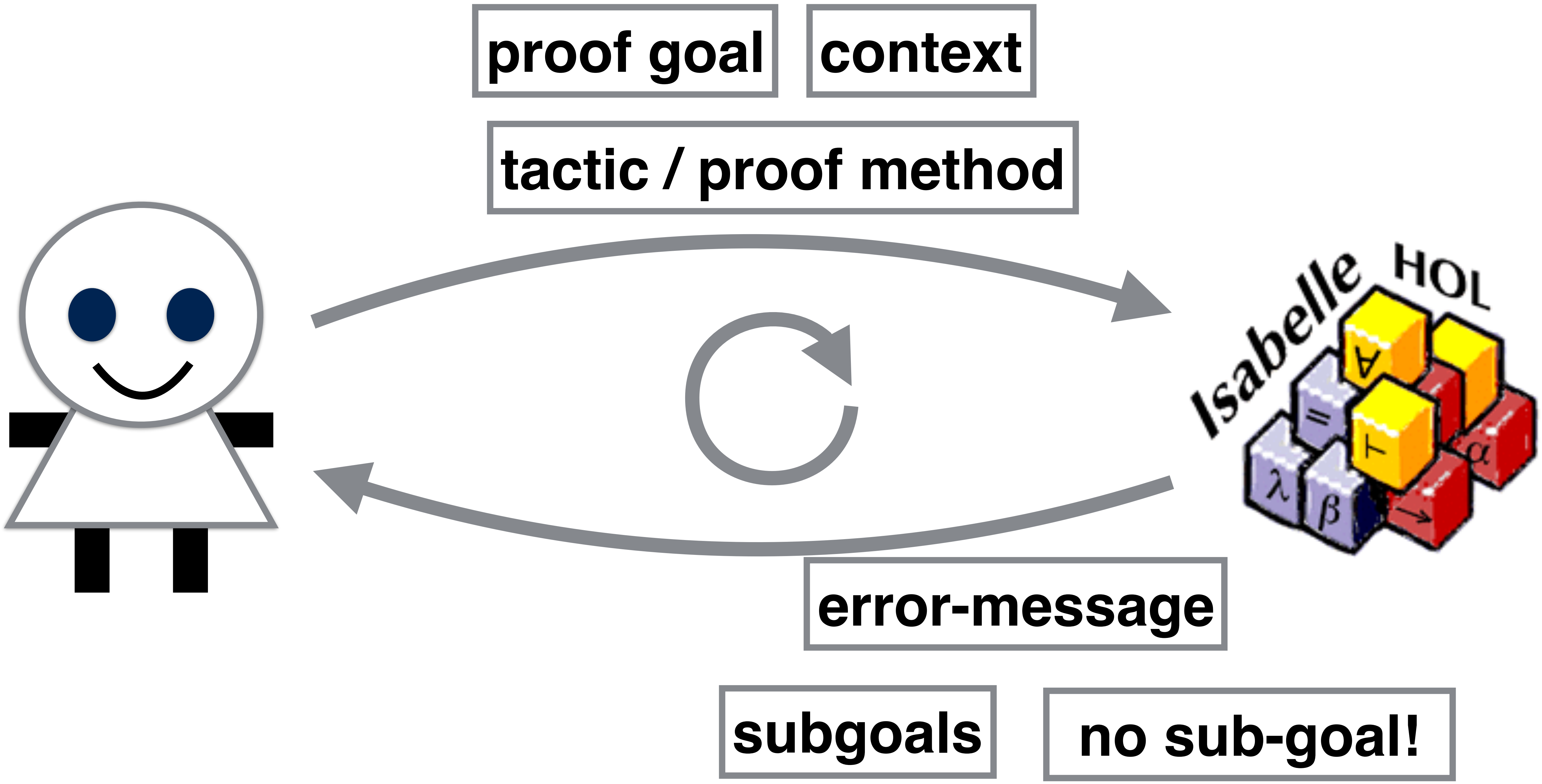
Interactive theorem proving with Isabelle/HOL



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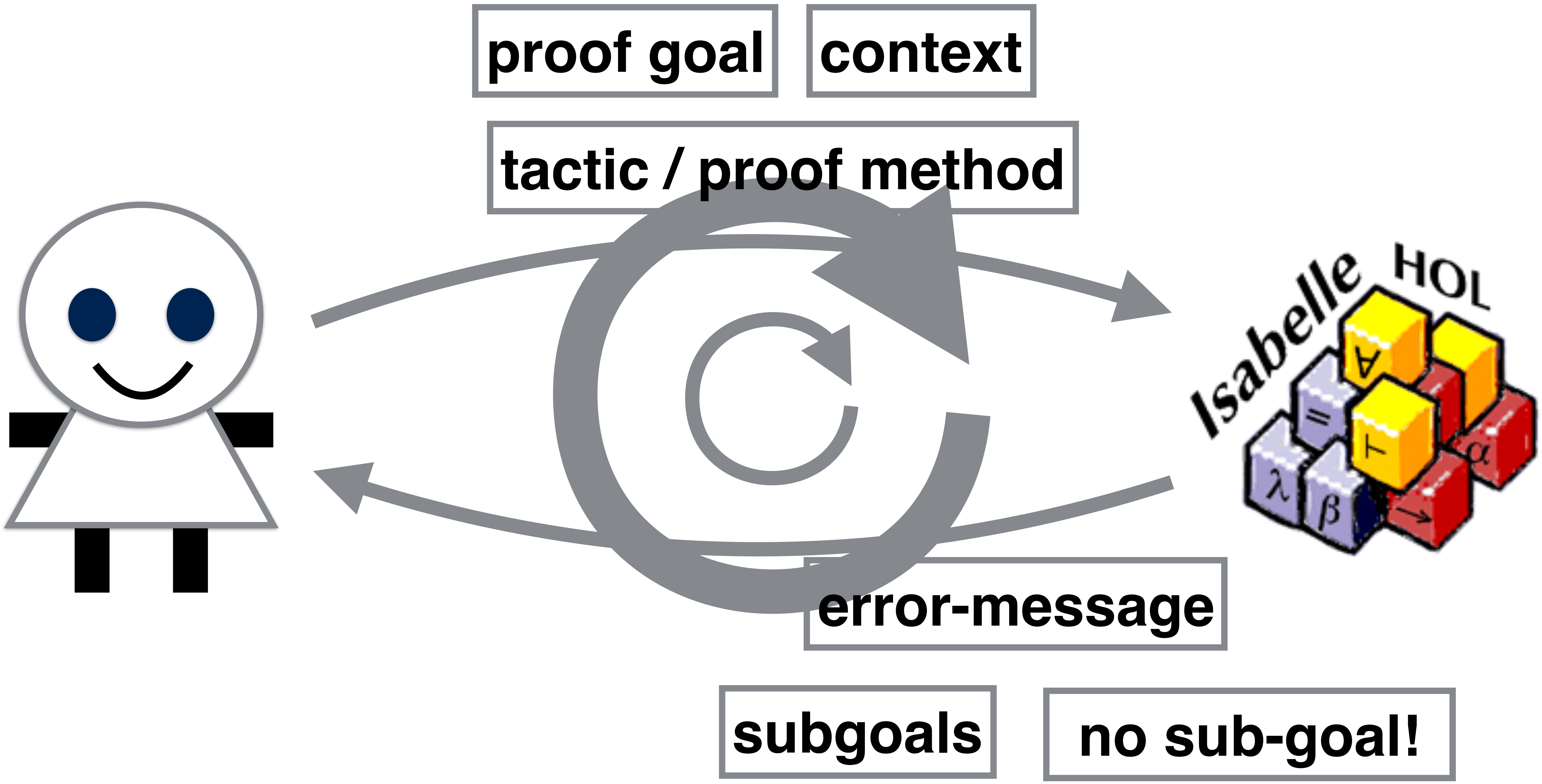

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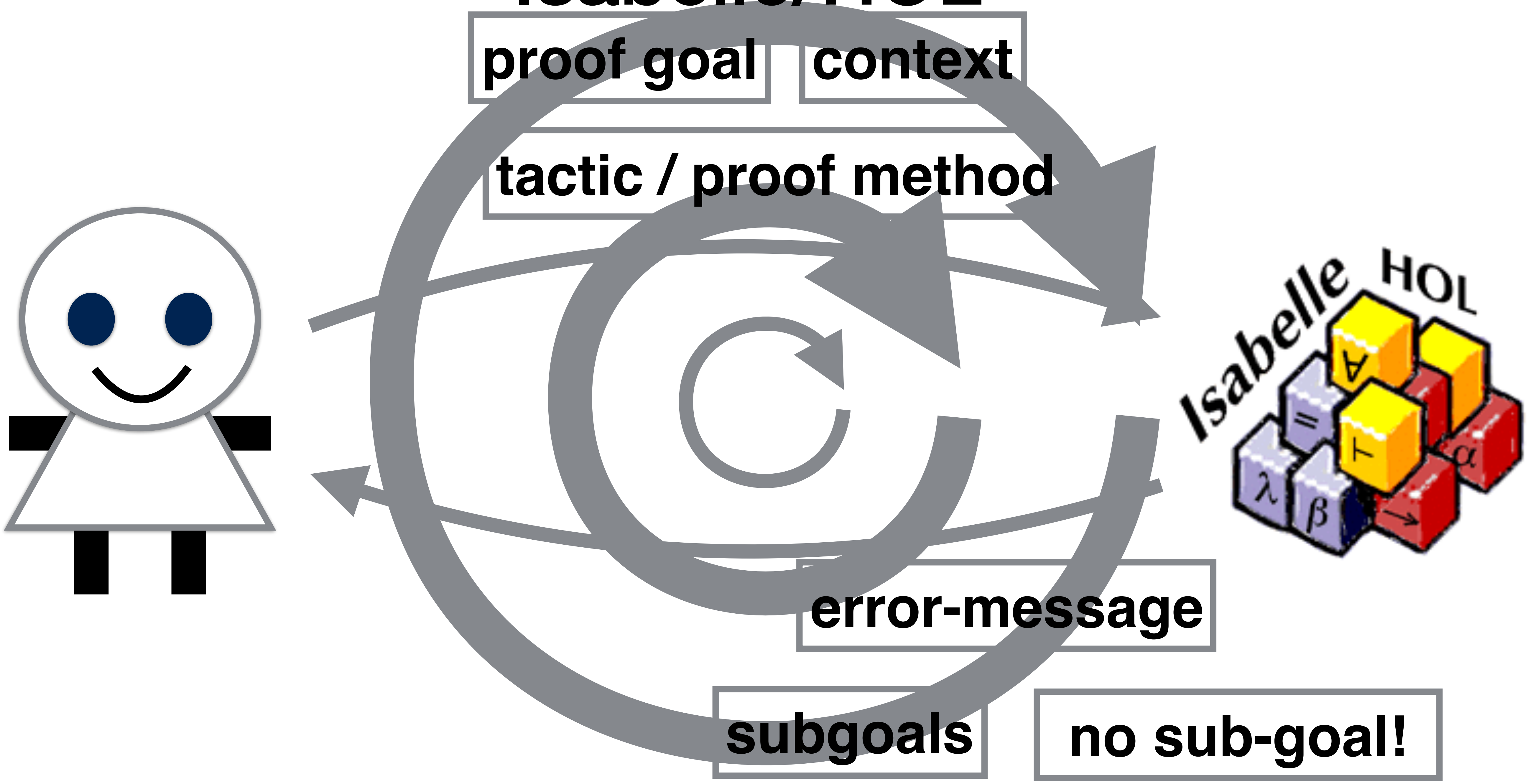

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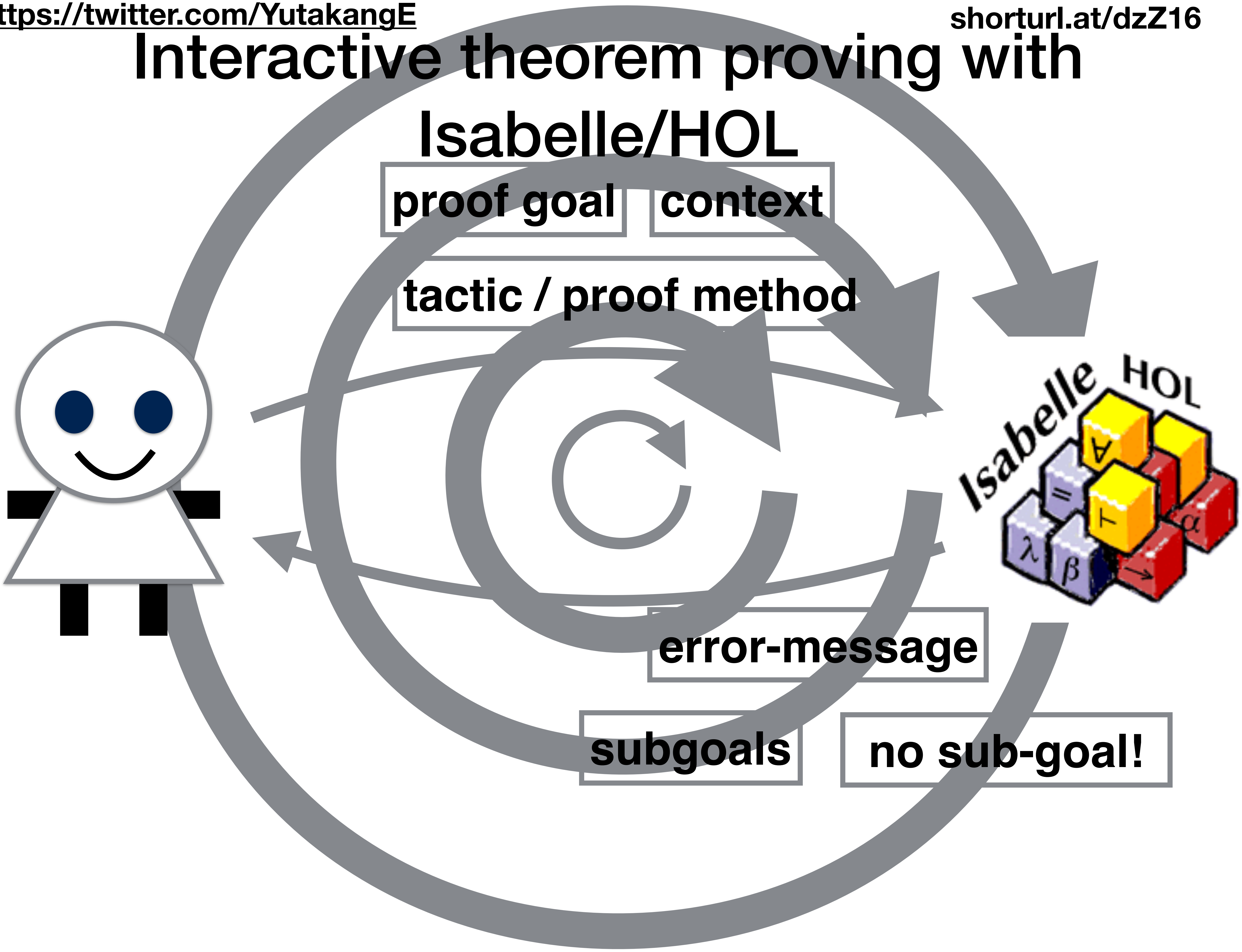

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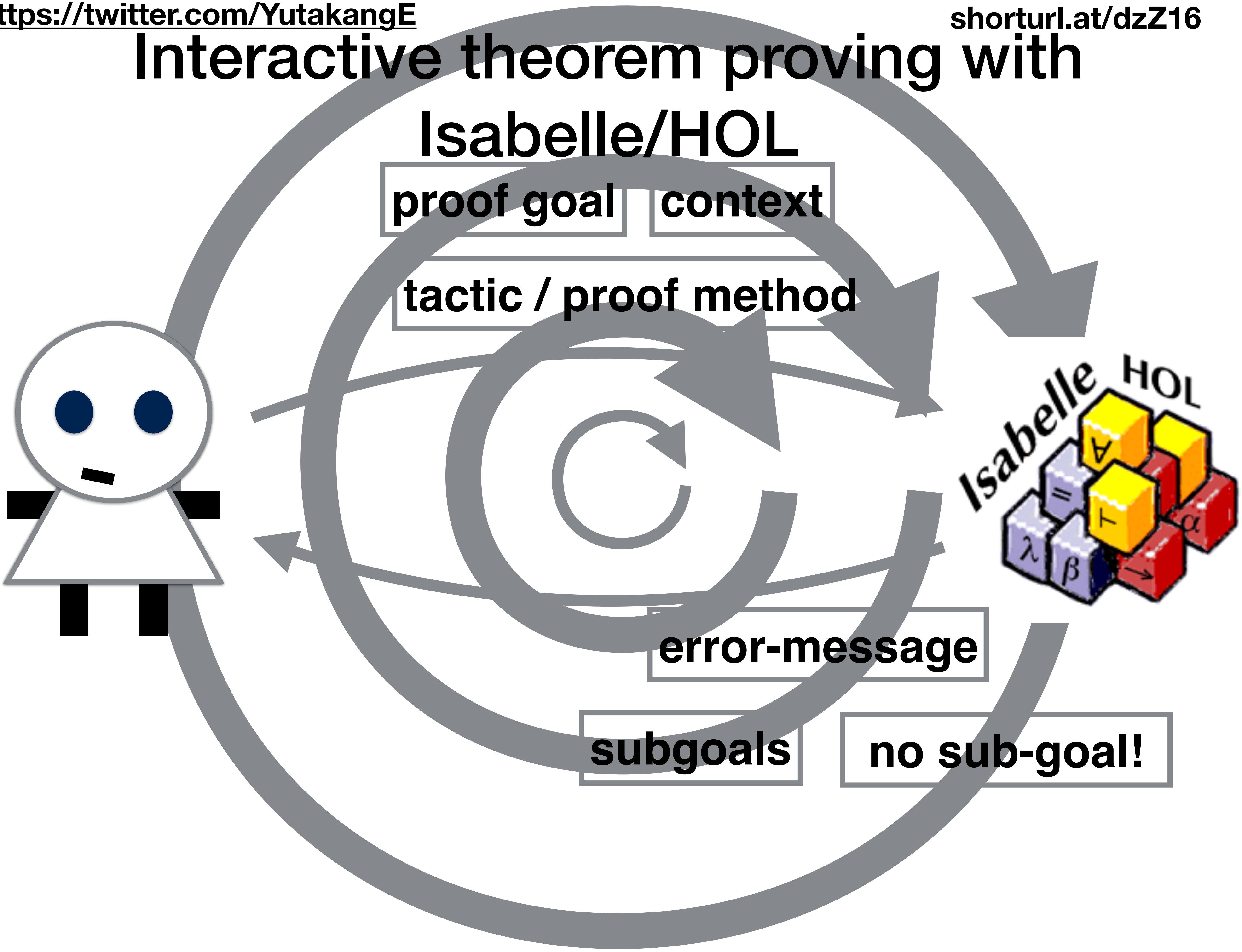
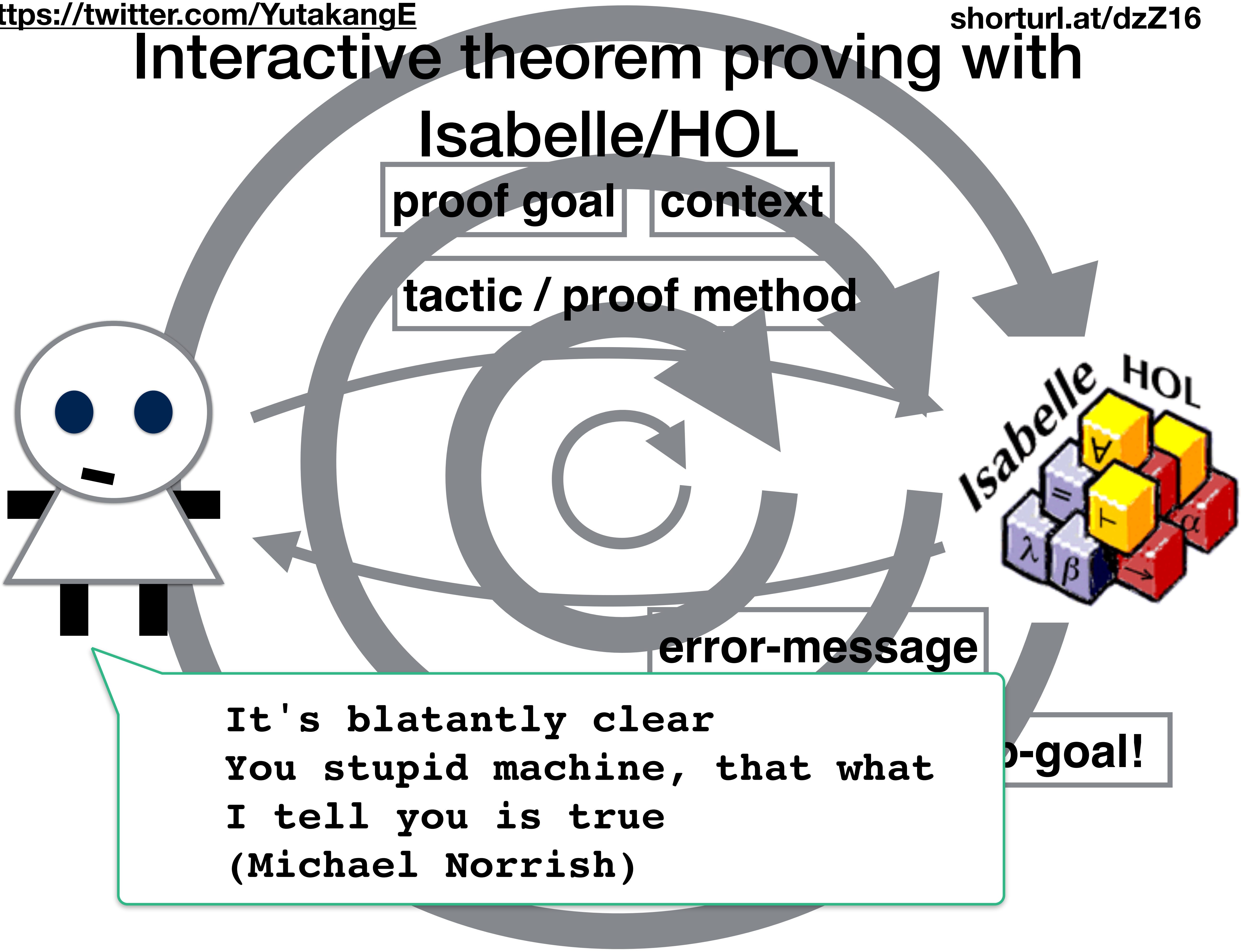


Interactive theorem proving with Isabelle/HOL



```
It's blatantly clear
You stupid machine, that what
I tell you is true
(Michael Norrish)
```

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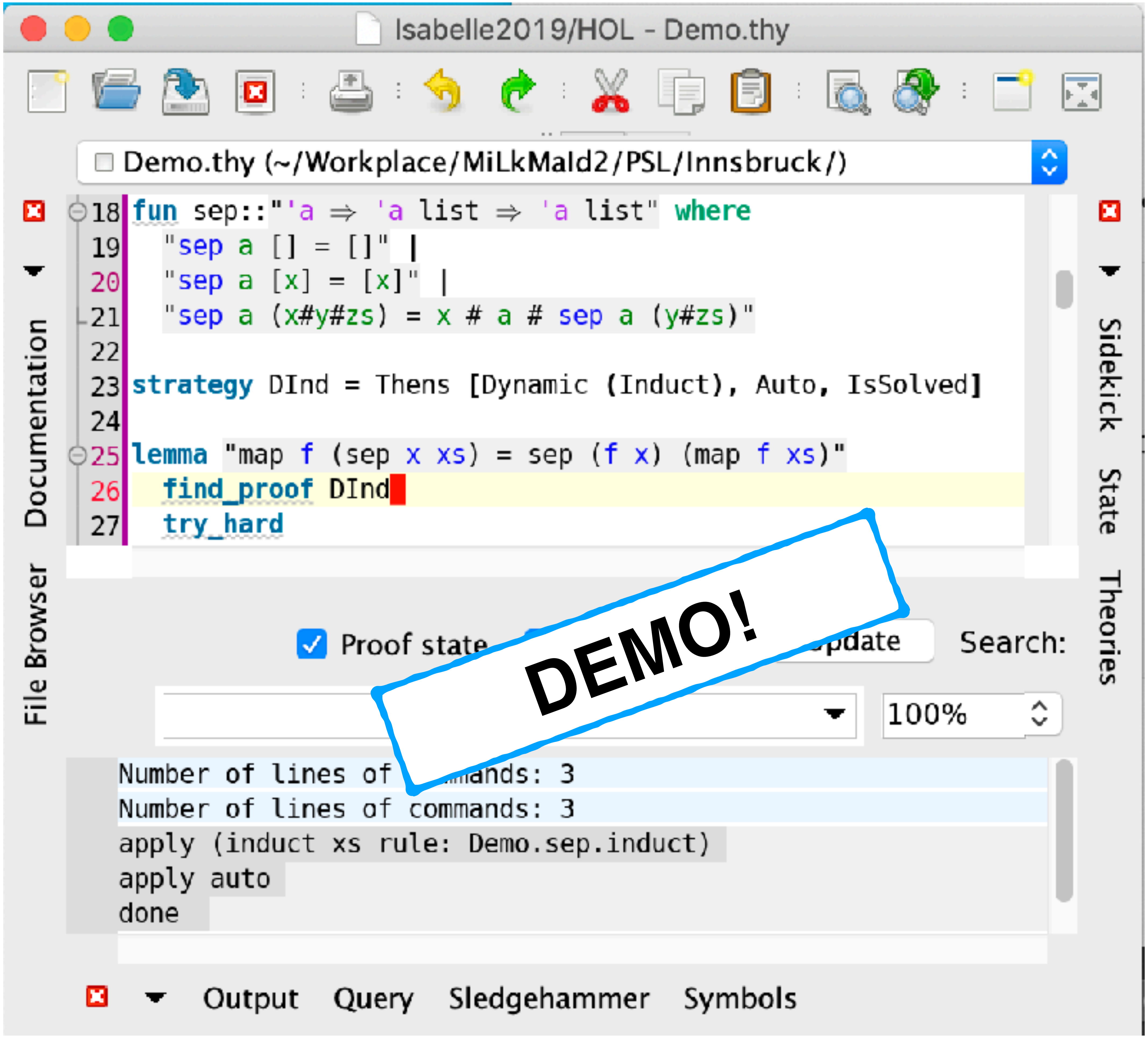

```
fun sep::"'a ⇒ 'a list ⇒ 'a list" where
  "sep a [] = []" |
  "sep a [x] = [x]" |
  "sep a (x#y#zs) = x # a # sep a (y#zs)"

strategy DInd = Thens [Dynamic (Induct), Auto, IsSolved]

lemma "map f (sep x xs) = sep (f x) (map f xs)"
  find_proof DInd
  try_hard
```

```
Number of lines of commands: 3
Number of lines of commands: 3
apply (induct xs rule: Demo.sep.induct)
apply auto
done
```

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```
lemma "map f (sep x xs) = sep (f x) (map f xs)"
find_proof DInd(*= Thens [Dynamic (Induct), Auto, IsSolved]*)
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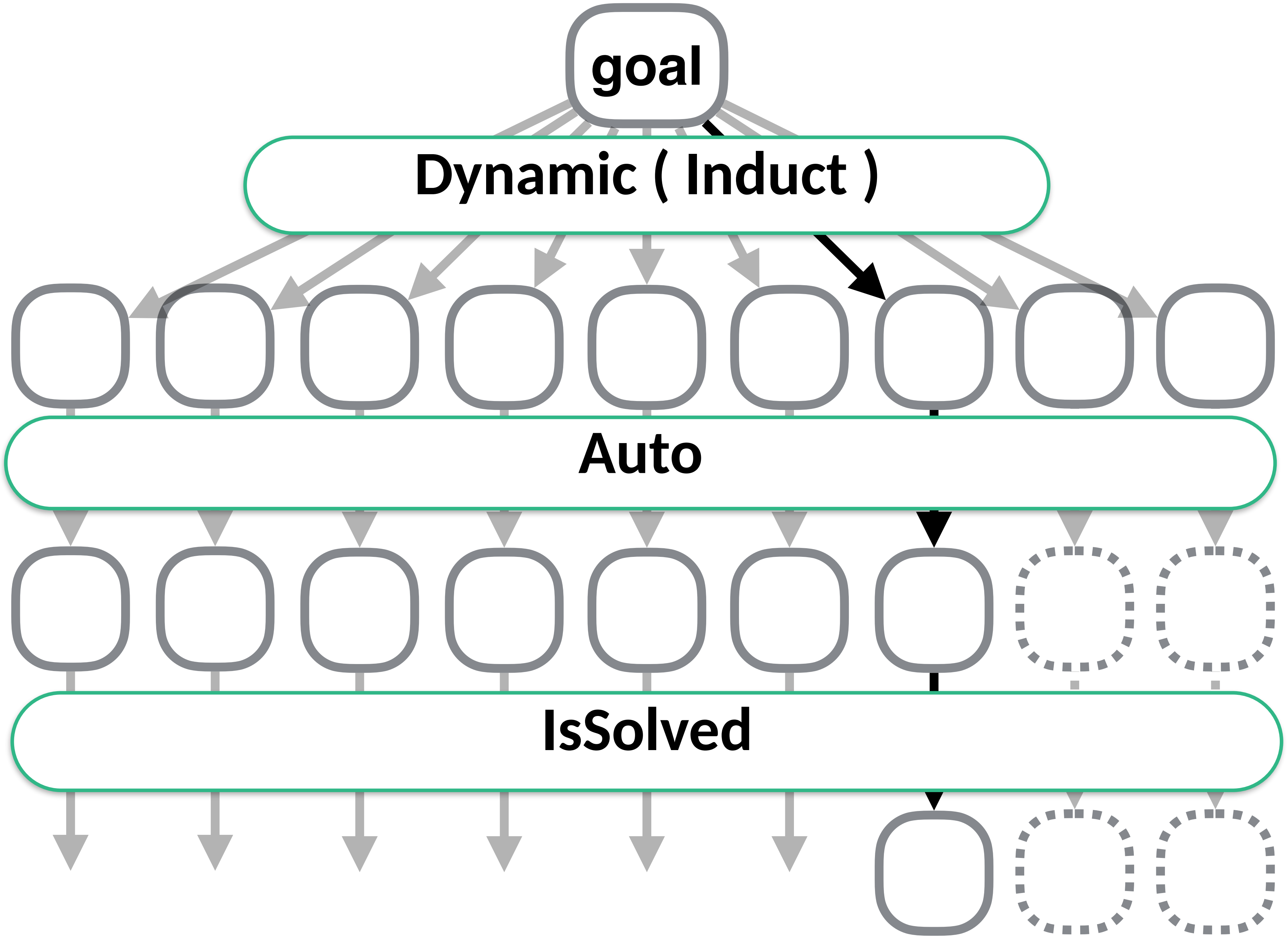

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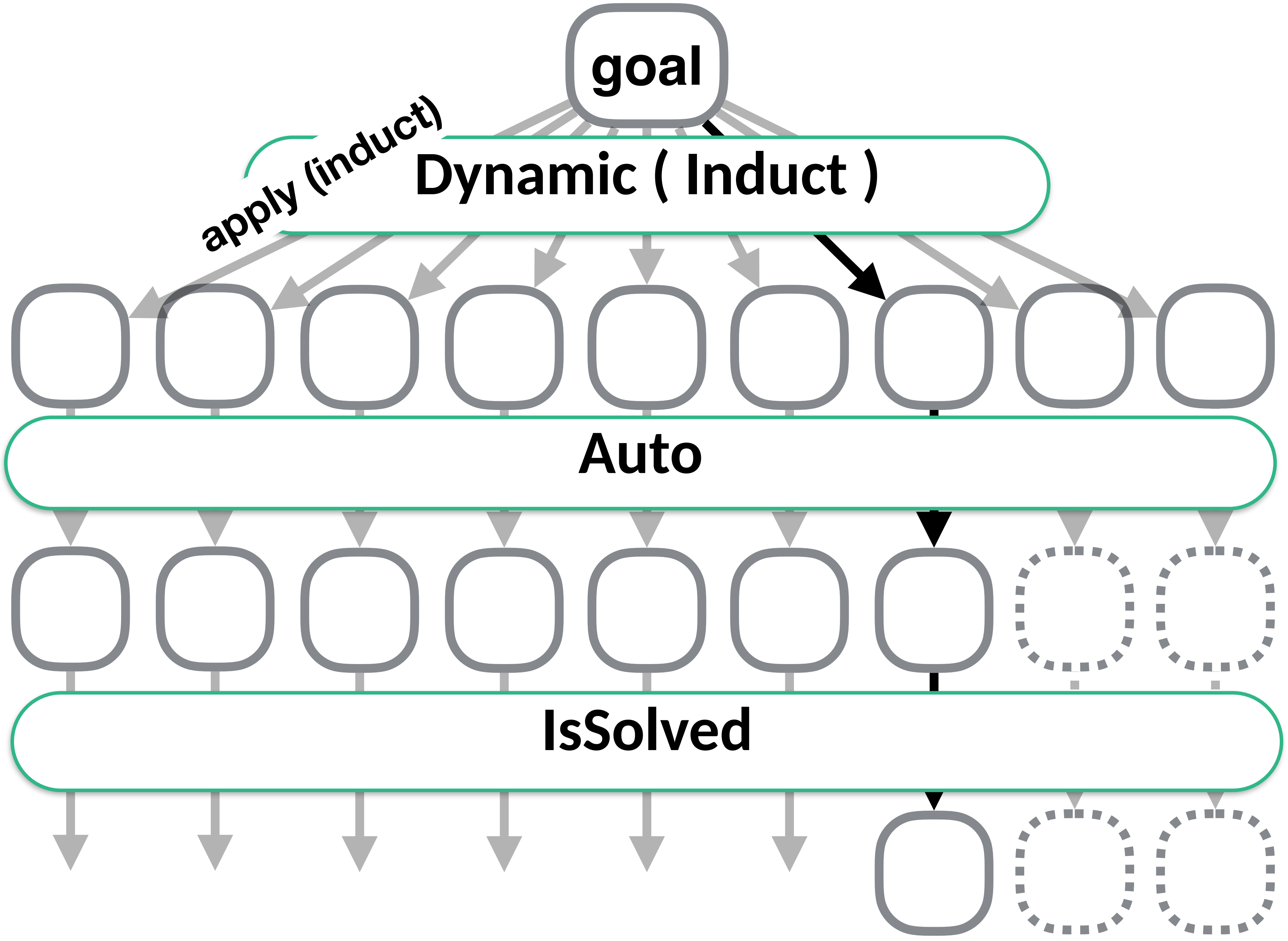

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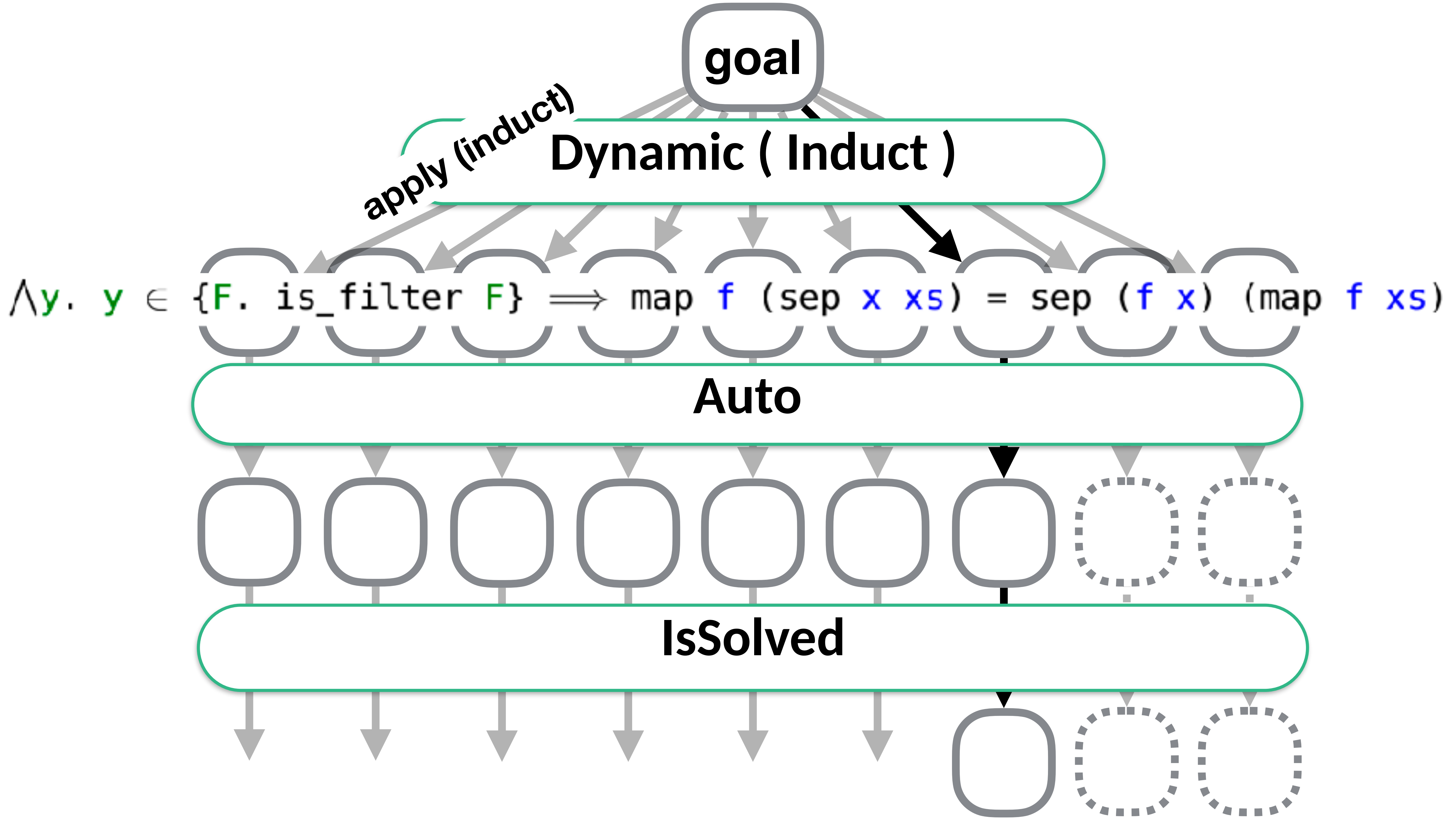
goal
apply (induct)
Dynamic (Induct)
⋀y. y ∈ {F. is_filter F} ⟹ map f (sep x xs) = sep (f x) (map f xs)
Auto
IsSolved

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shorturl.at/dzZ16

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find_proof DInd(*= Thens [Dynamic (Induct), Auto, IsSolved]*)
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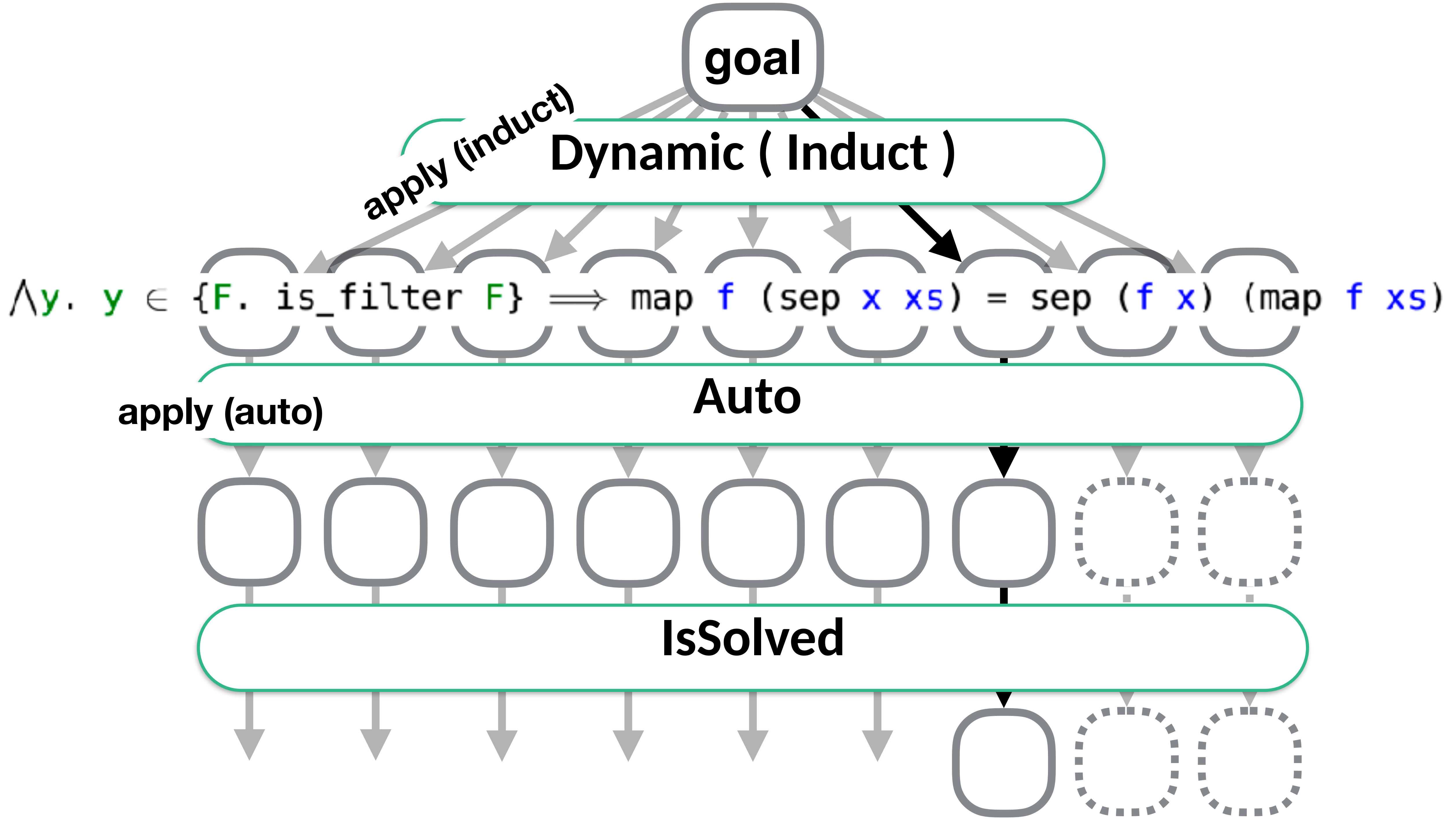

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⋀y. y ∈ {F. is_filter F} ⟹ map f (sep x xs) = sep (f x) (map f xs)
Auto
apply (auto)
IsSolved

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shorturl.at/dzZ16

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find_proof DInd(*= Thens [Dynamic (Induct), Auto, IsSolved]*)
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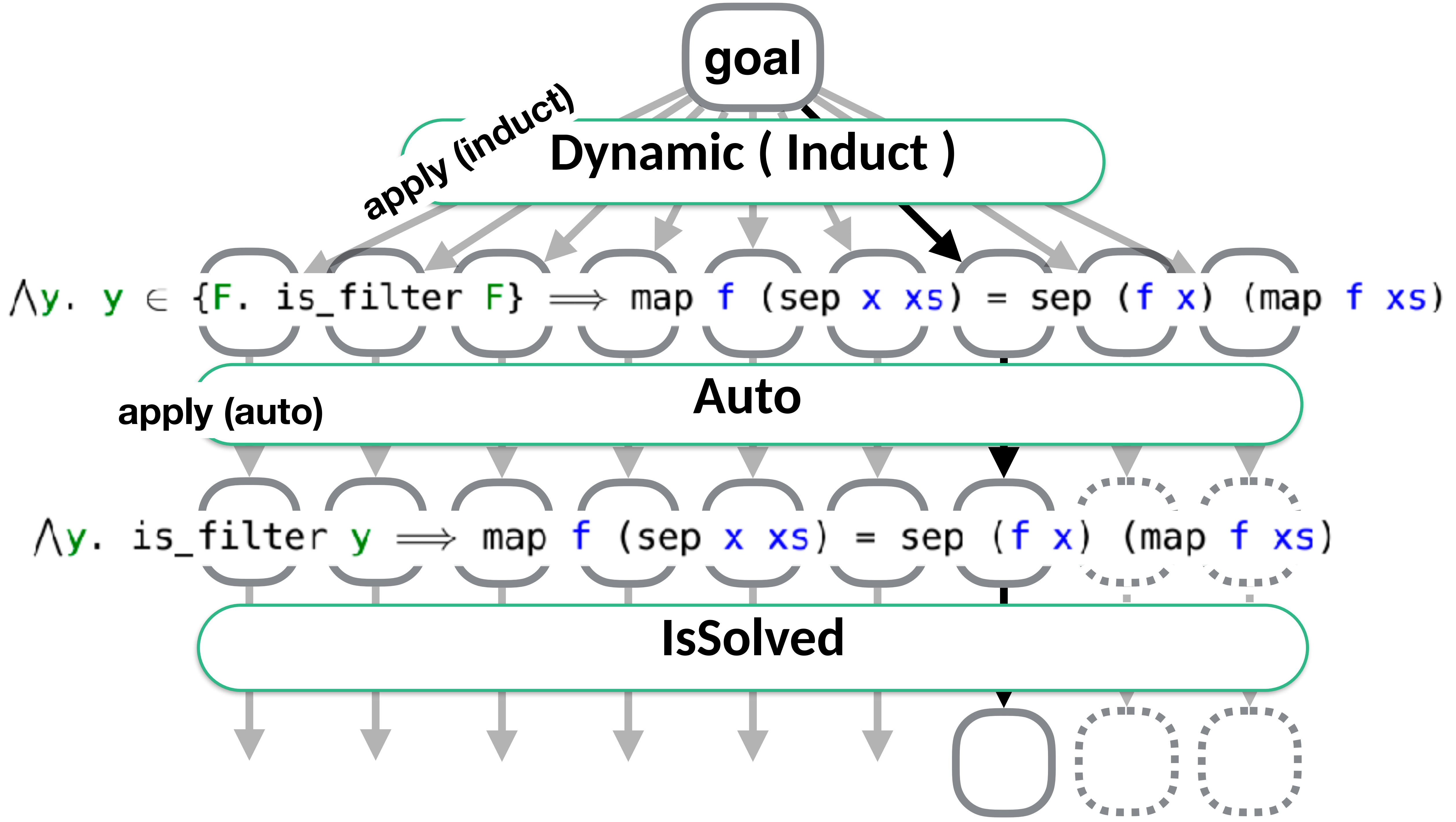
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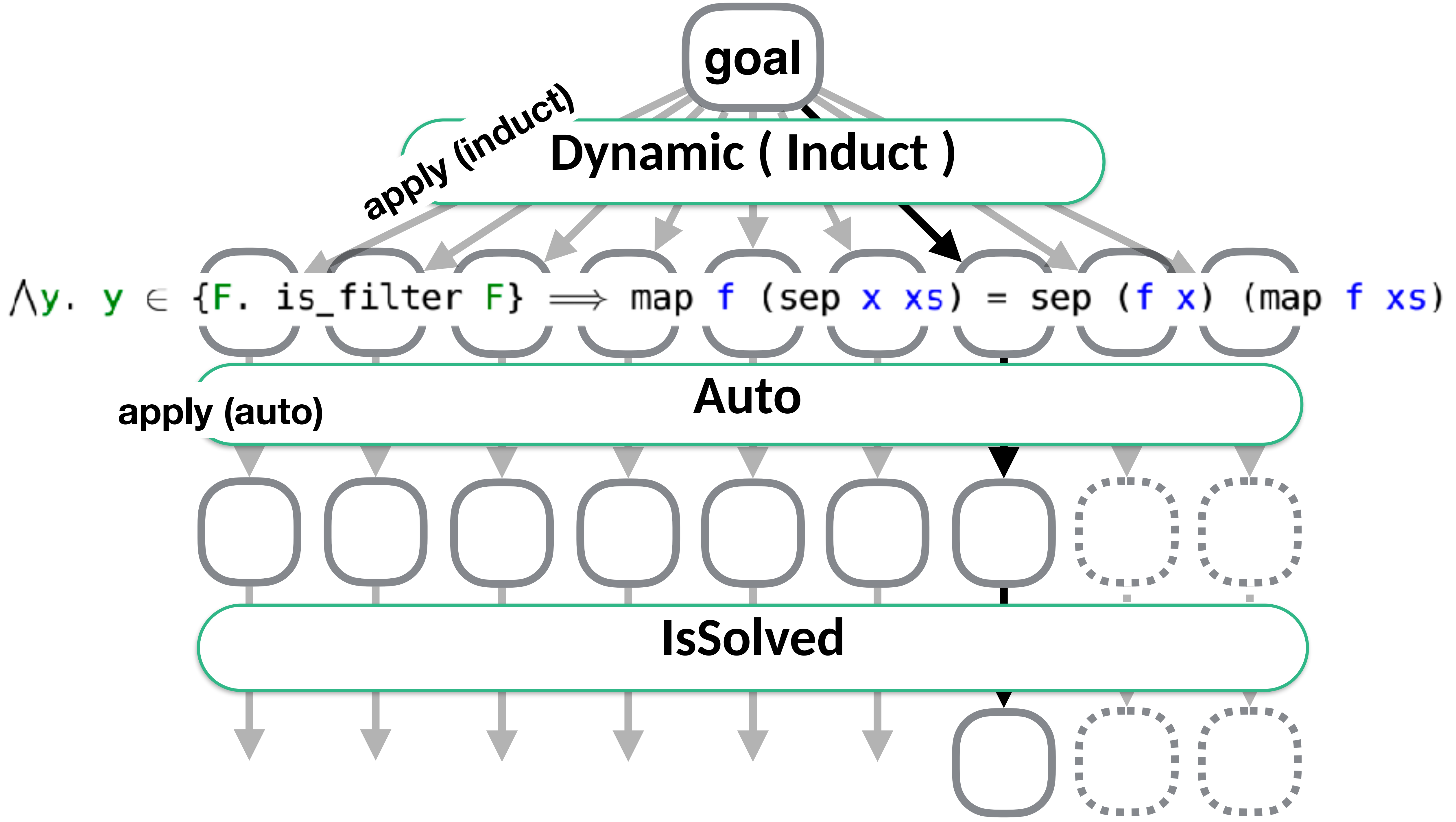
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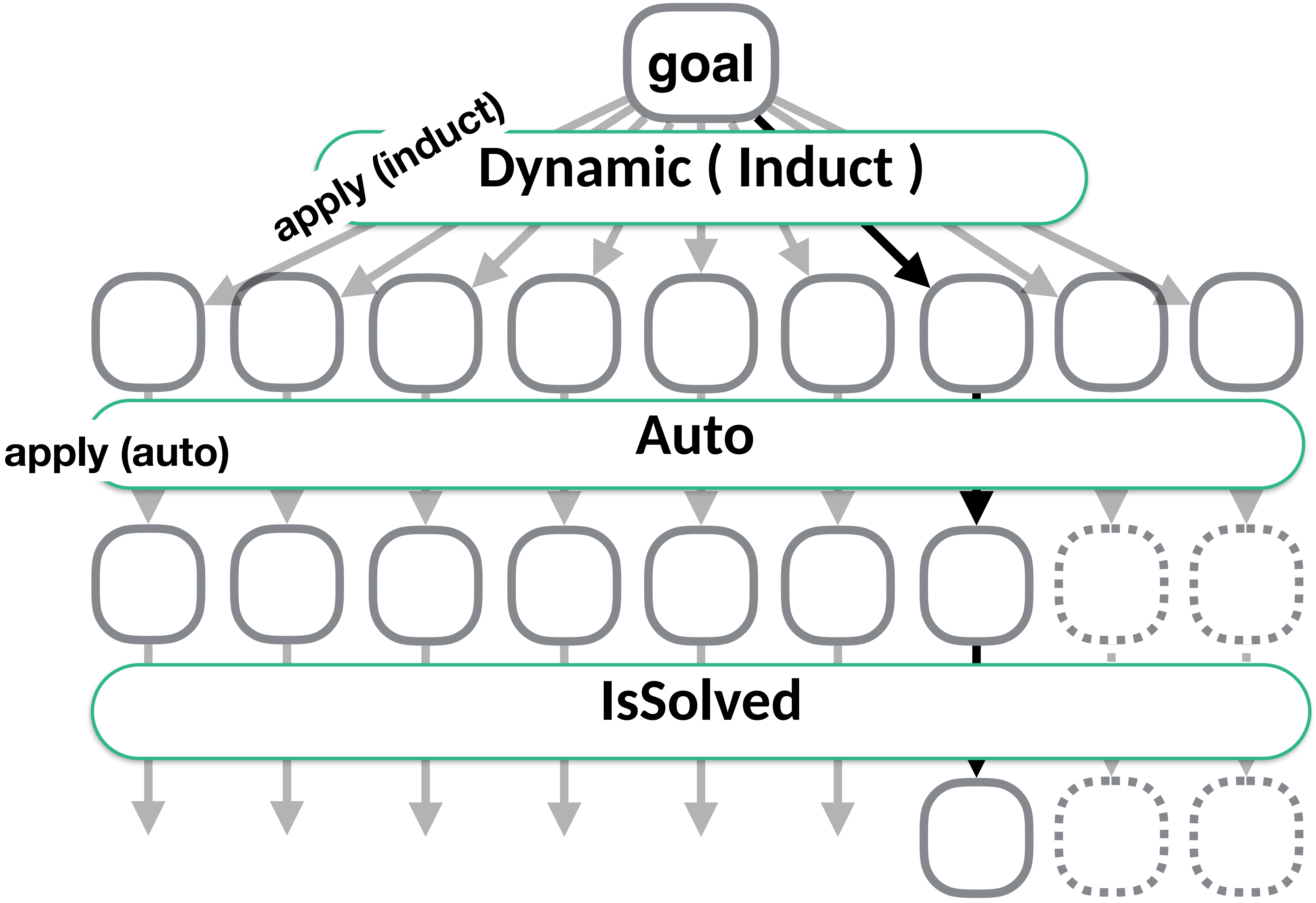

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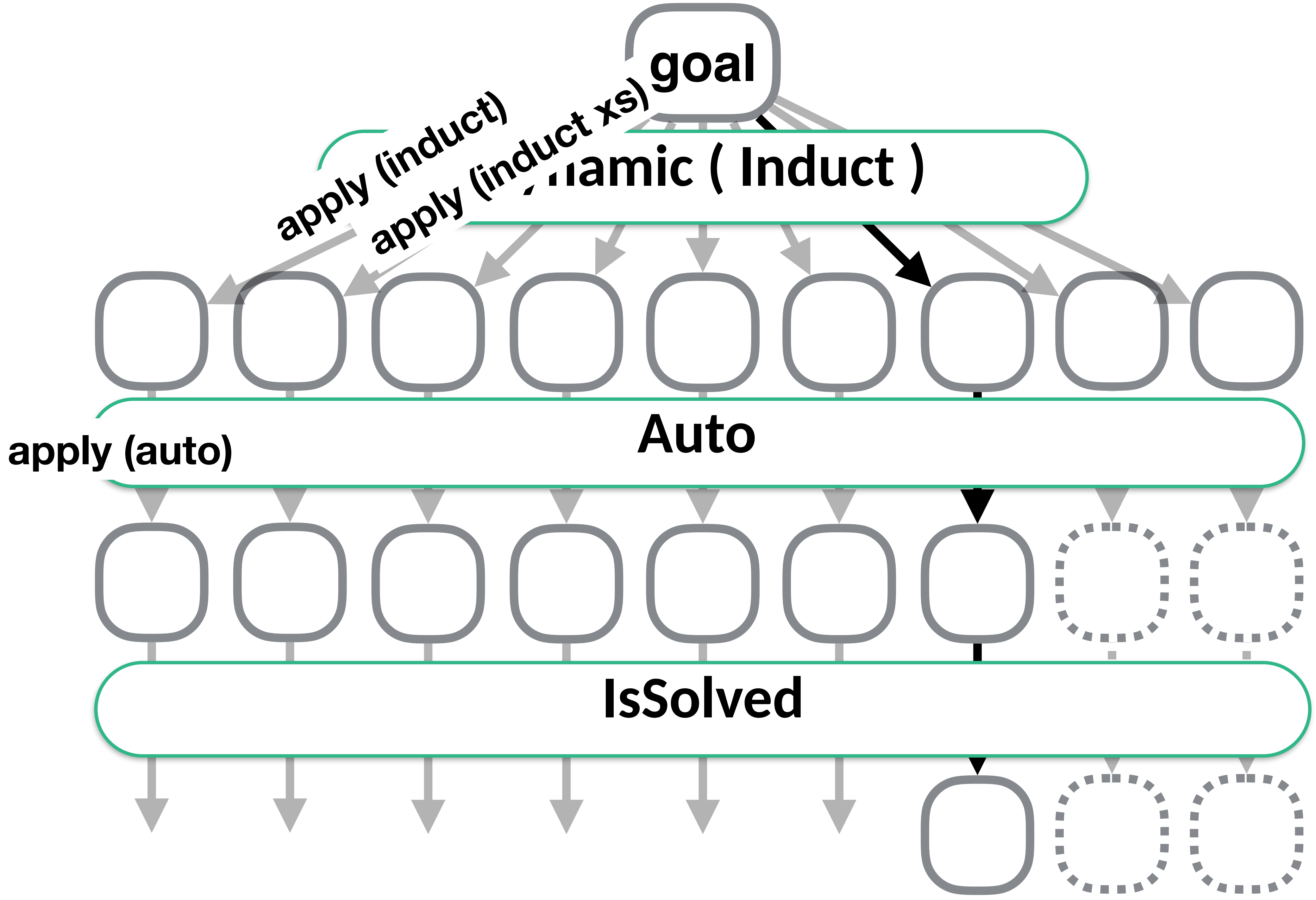

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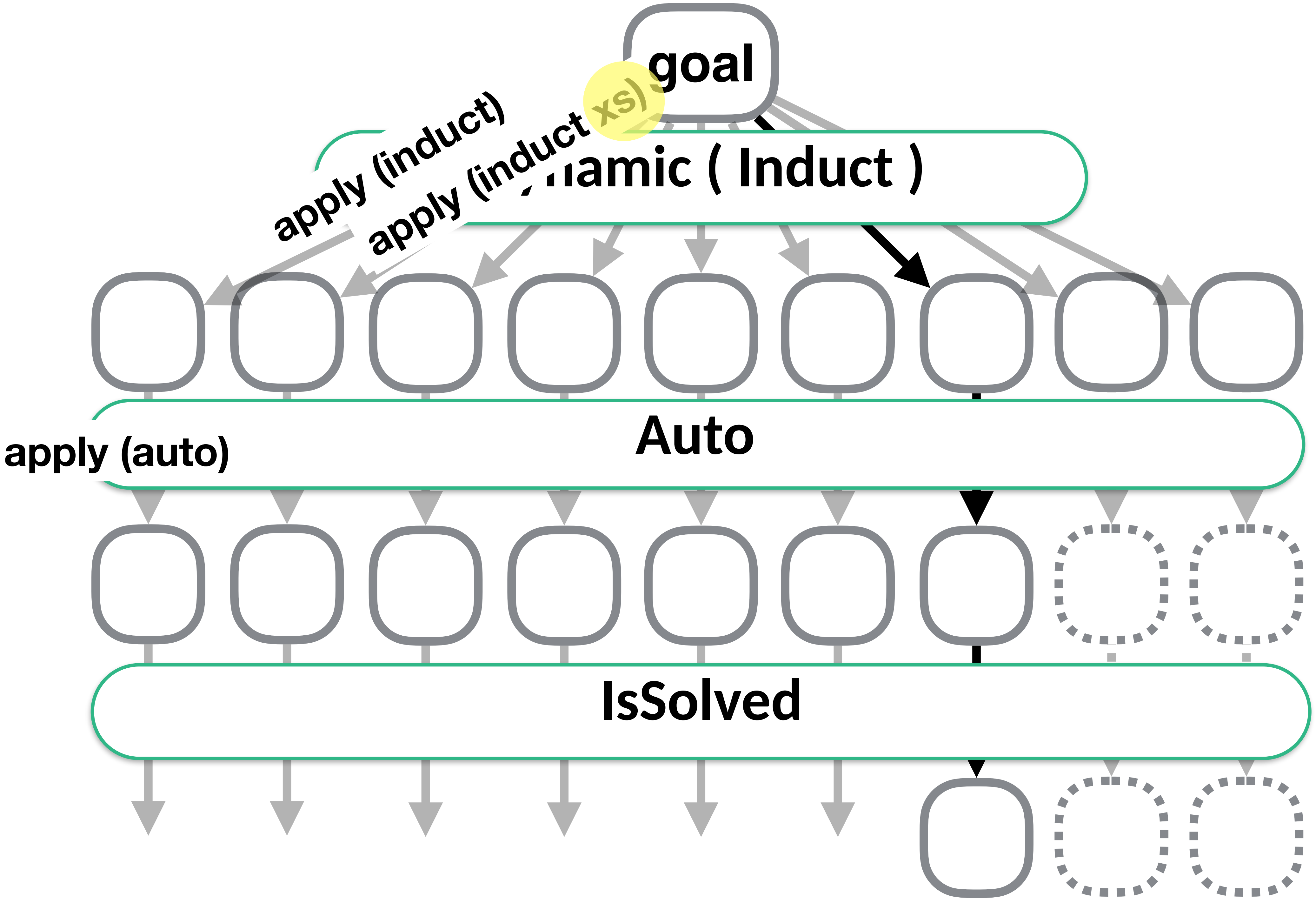

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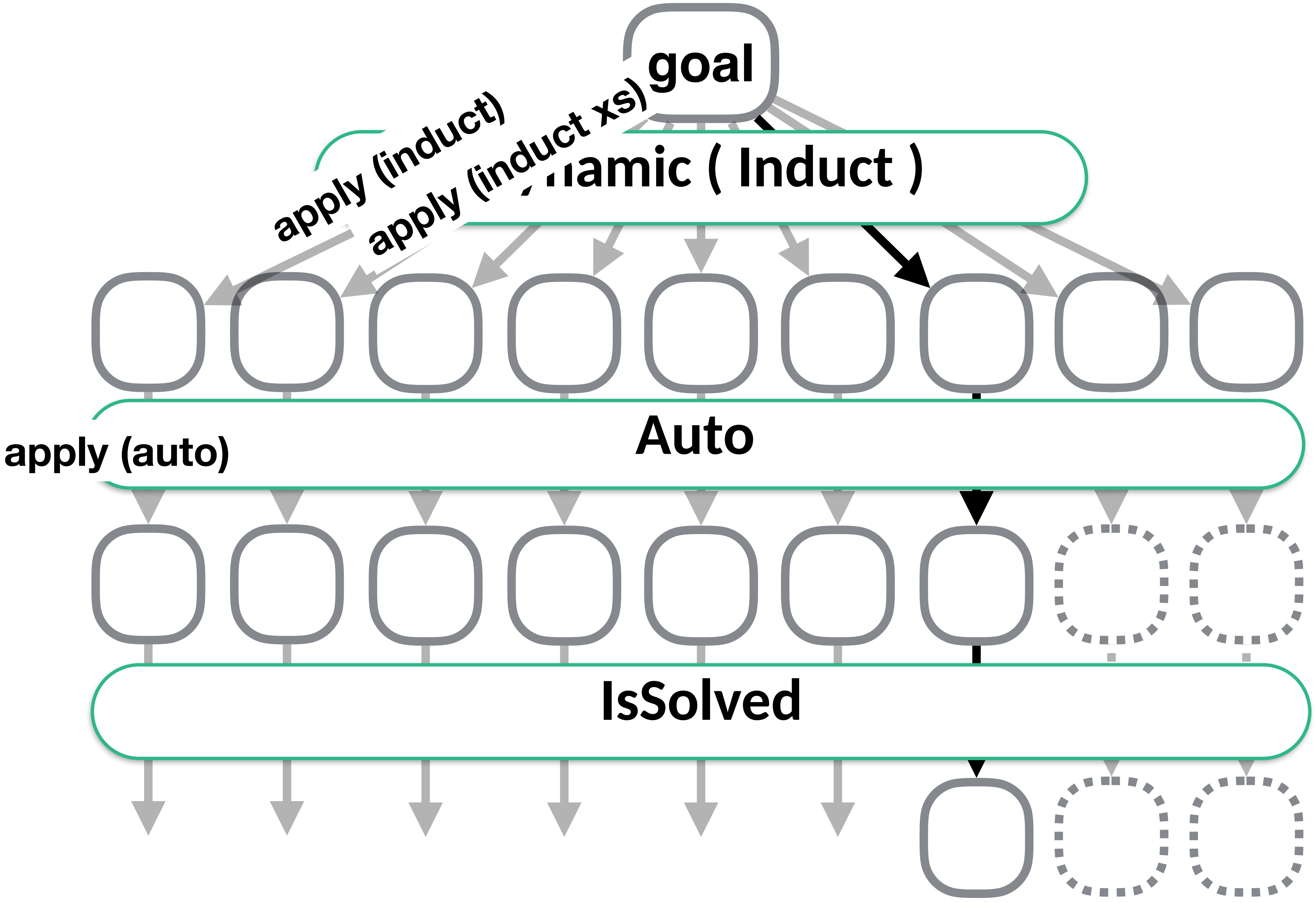

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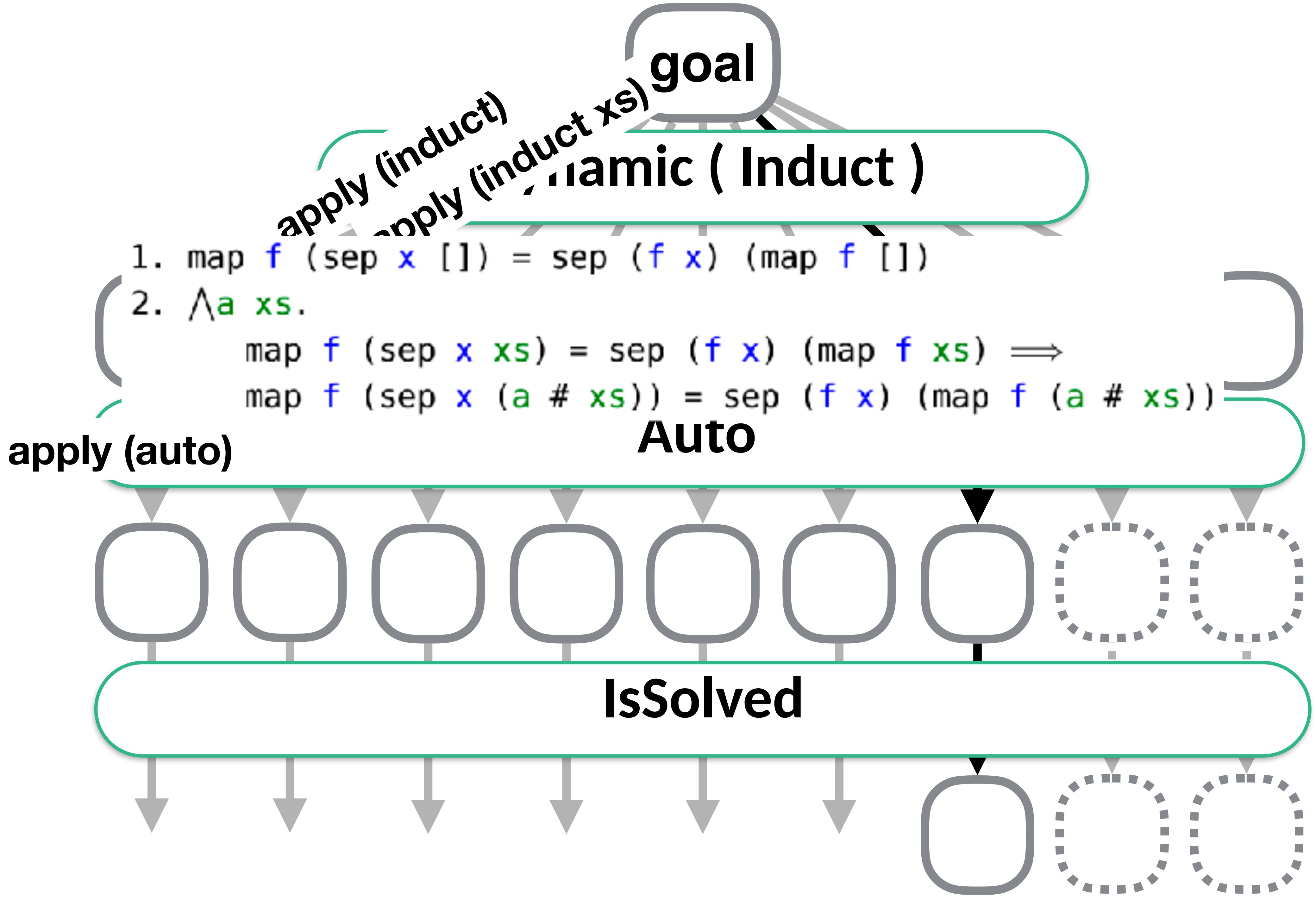

```
1. map f (sep x []) = sep (f x) (map f [])
2. ⋀a xs.
      map f (sep x xs) = sep (f x) (map f xs) ⟹
      map f (sep x (a # xs)) = sep (f x) (map f (a # xs))
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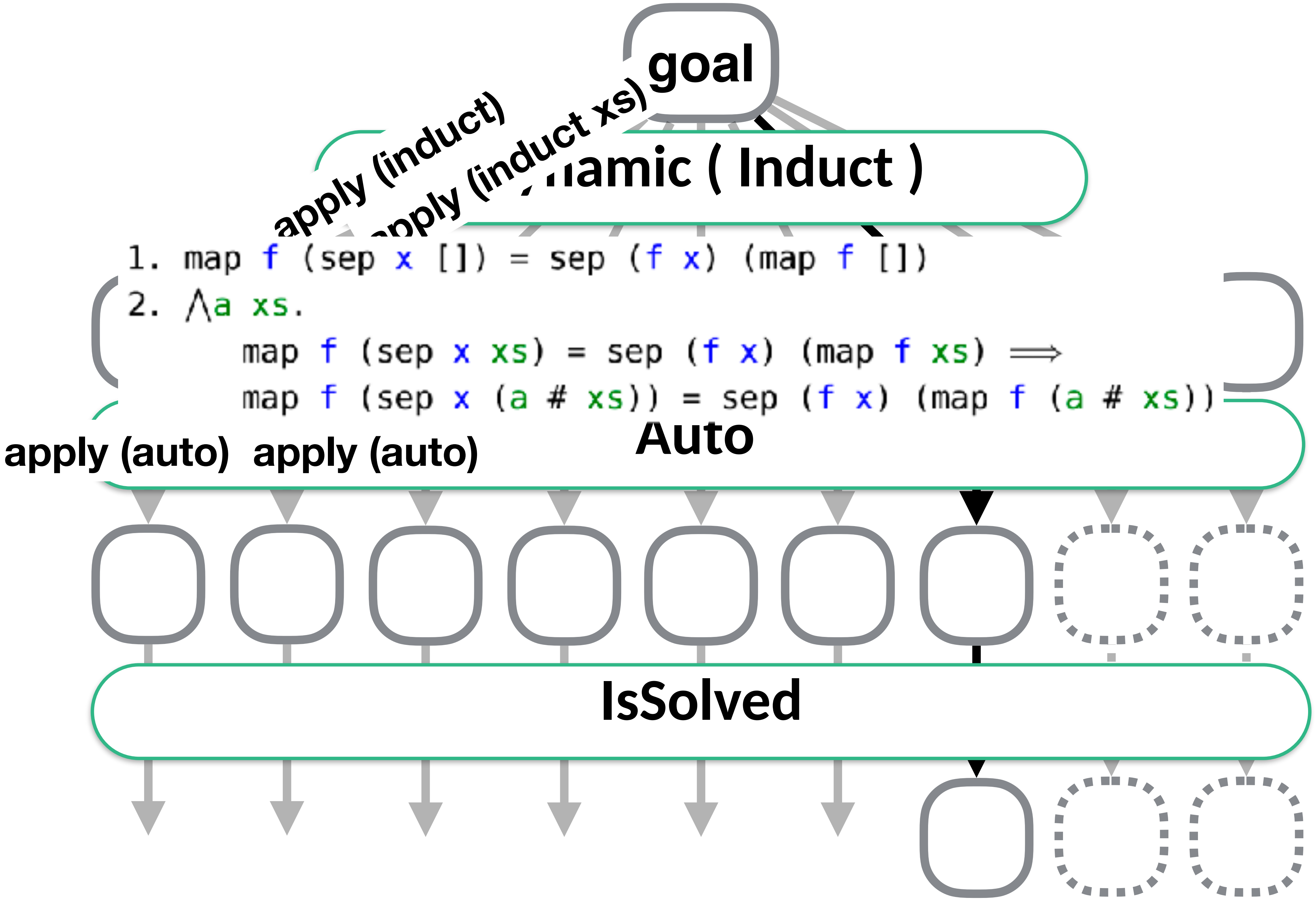

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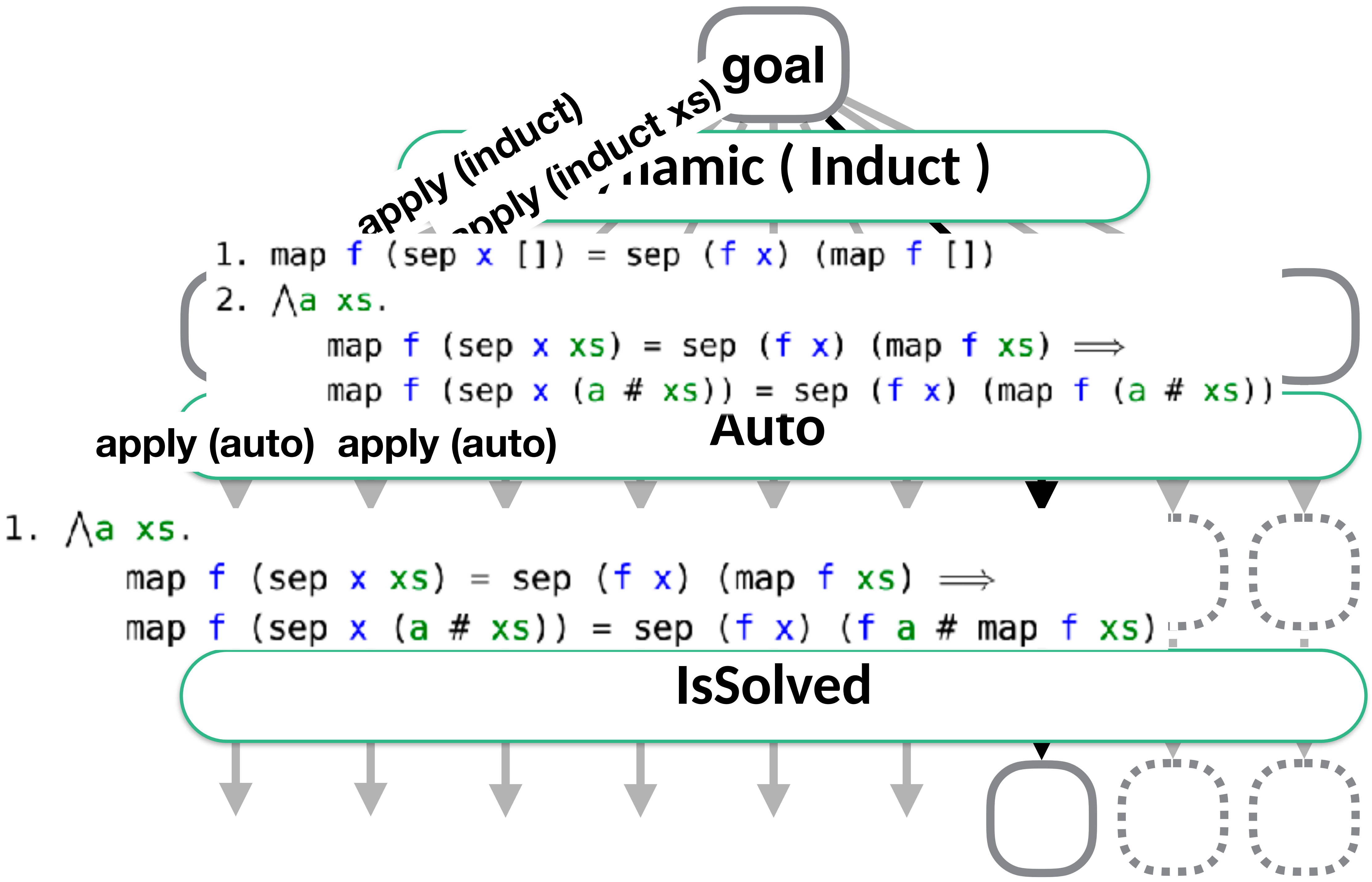

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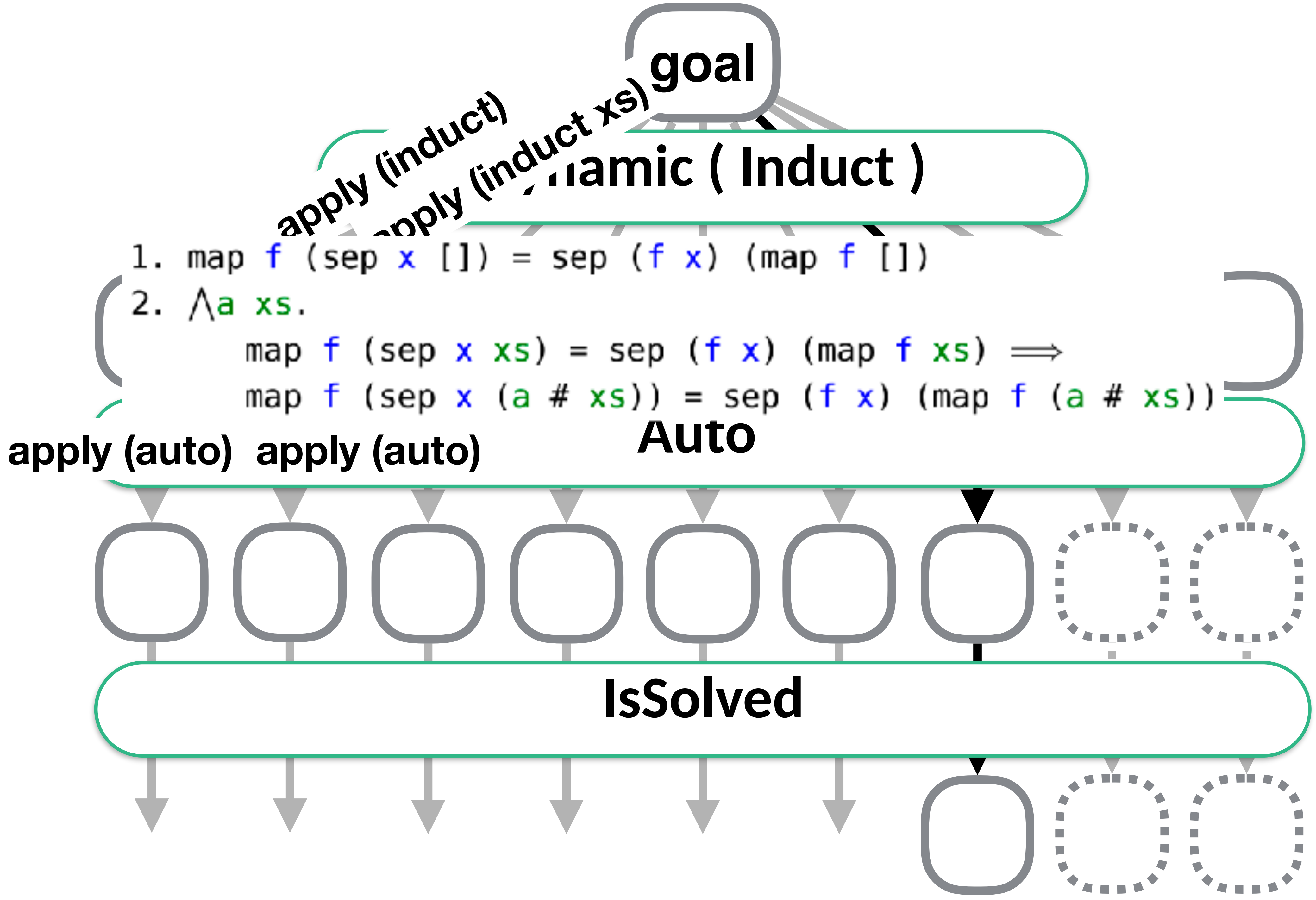

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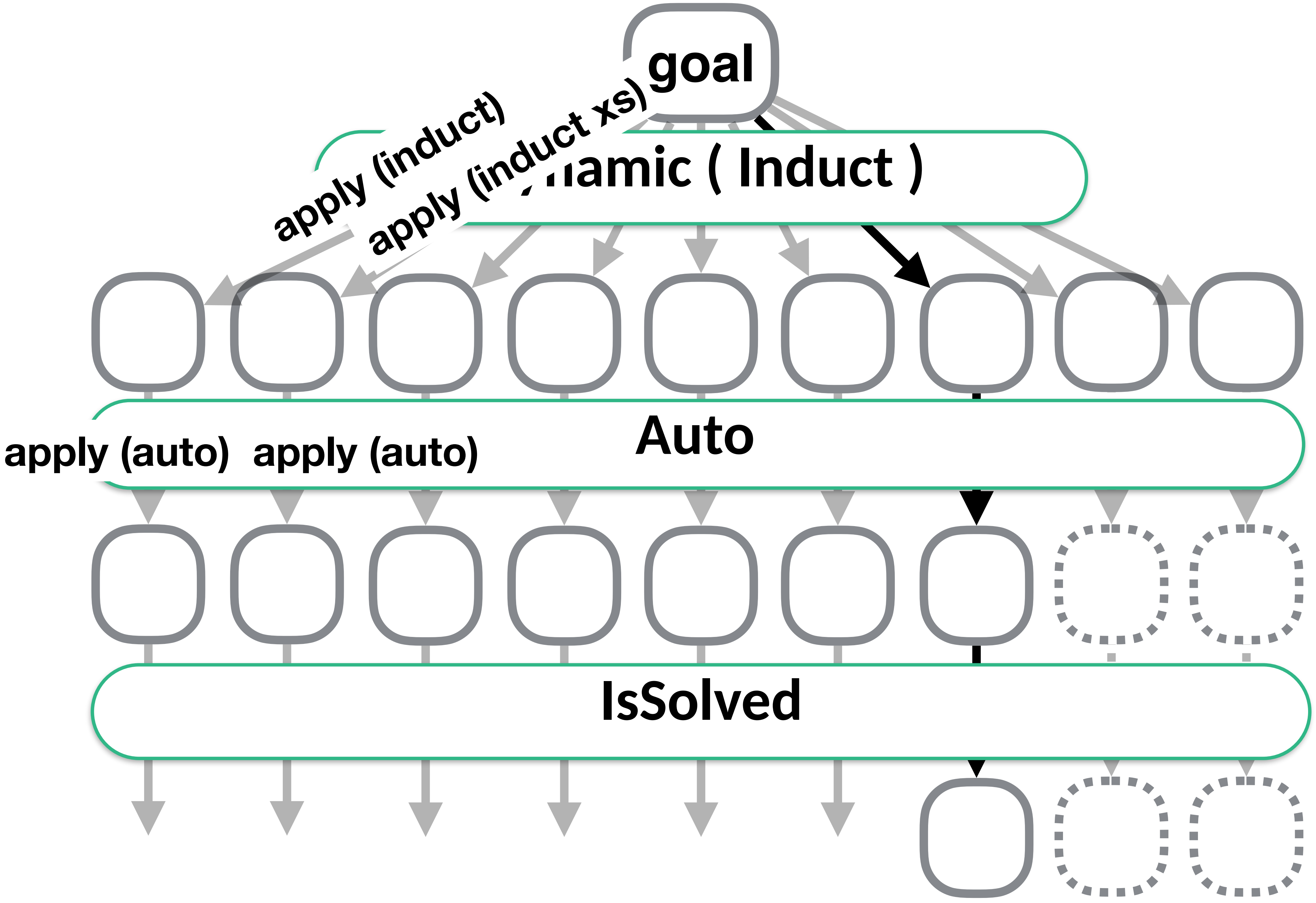

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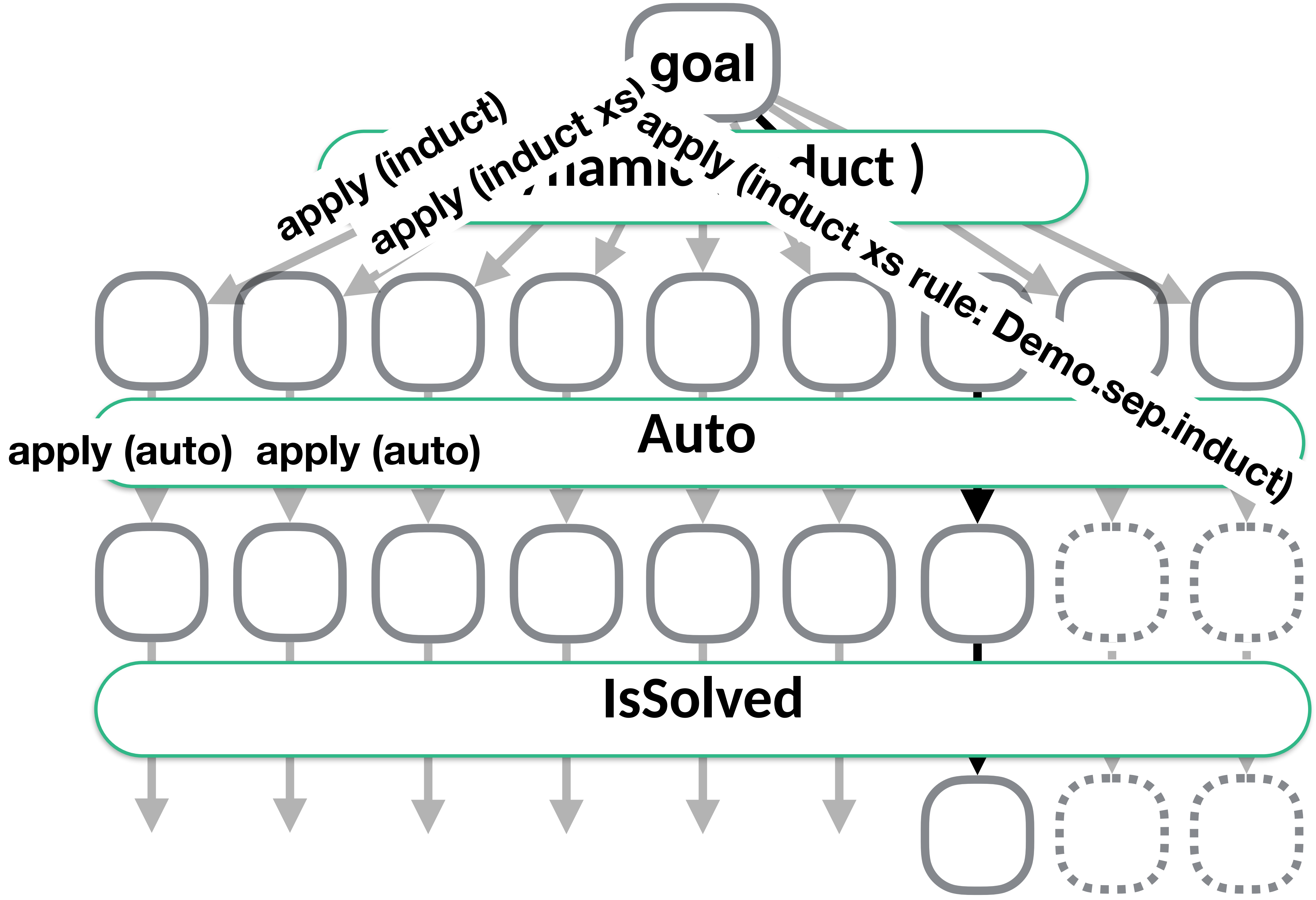

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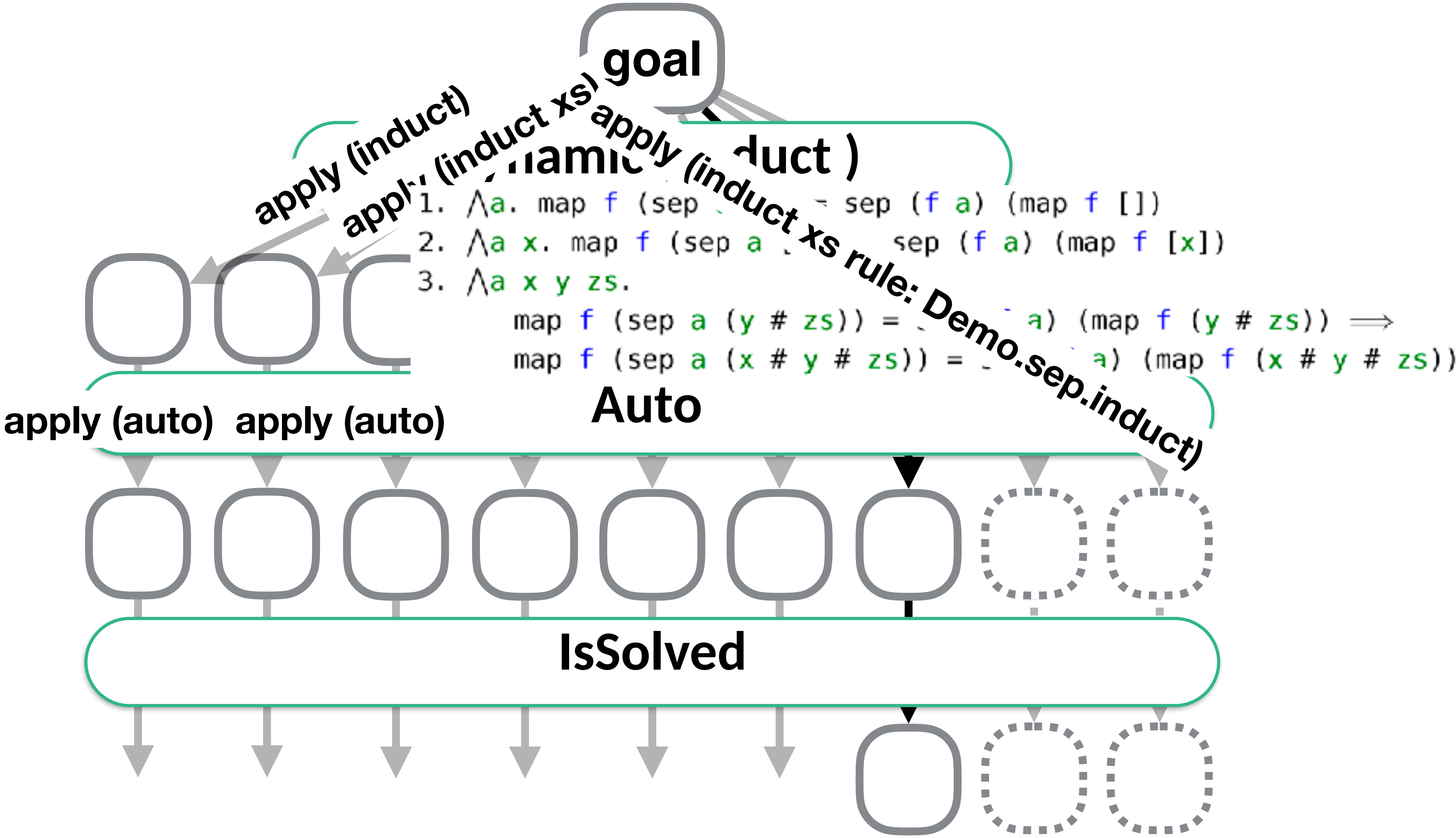

```
1. ⋀a. map f (sep
2. ⋀a x. map f (sep a
3. ⋀a x y zs.
     map f (sep a (y # zs)) =
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```

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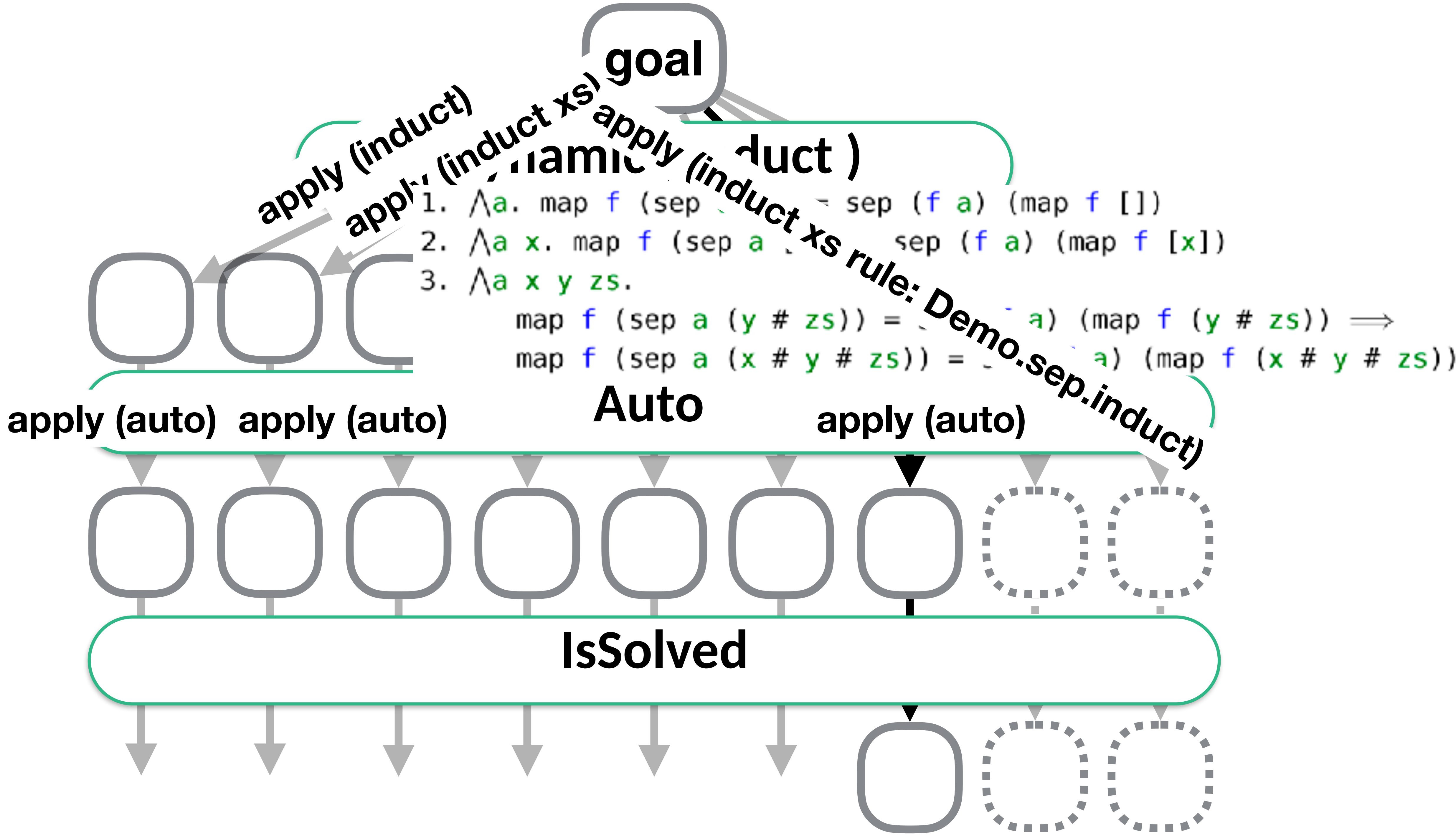

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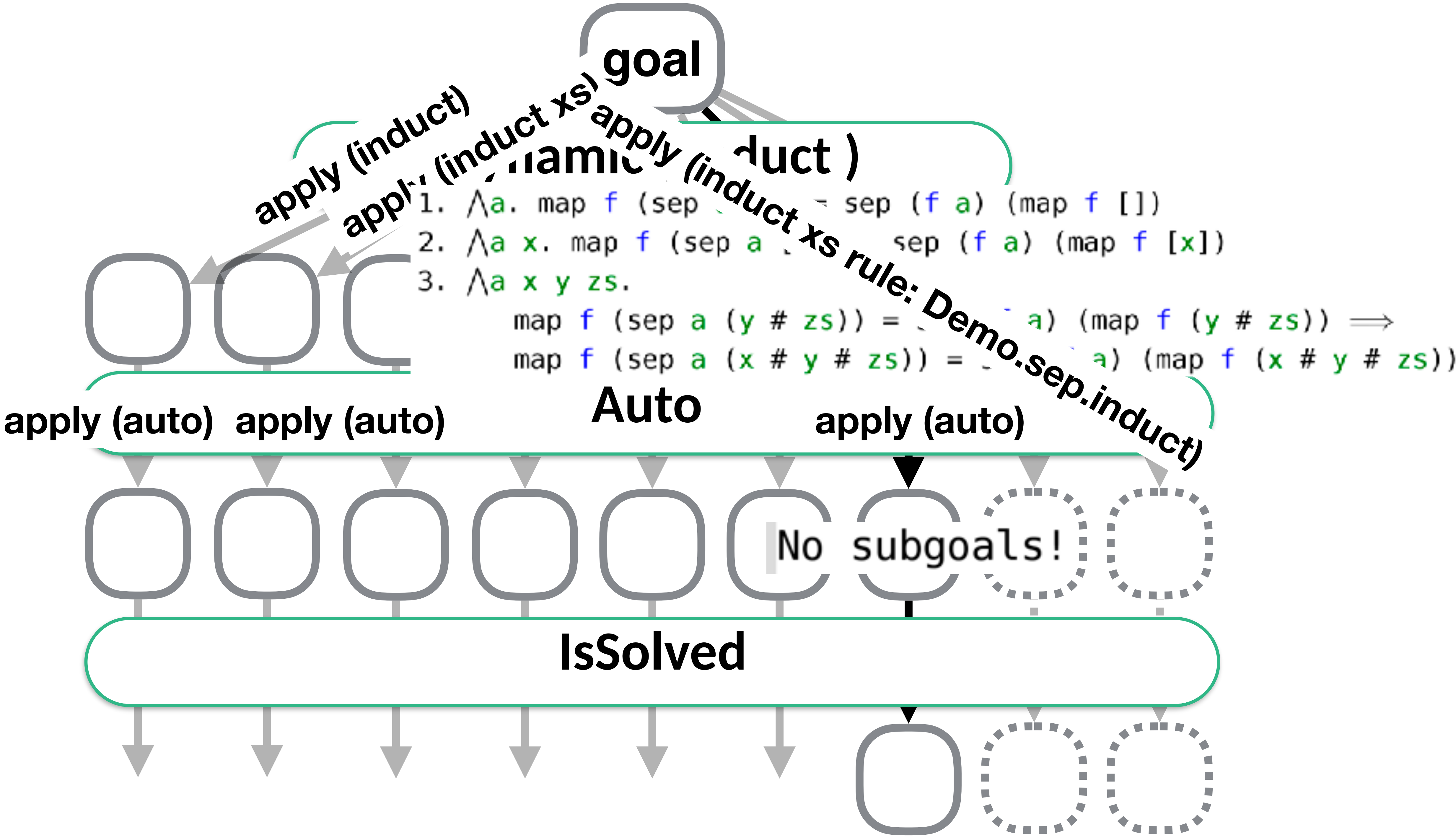

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3. ⋀a x y zs.
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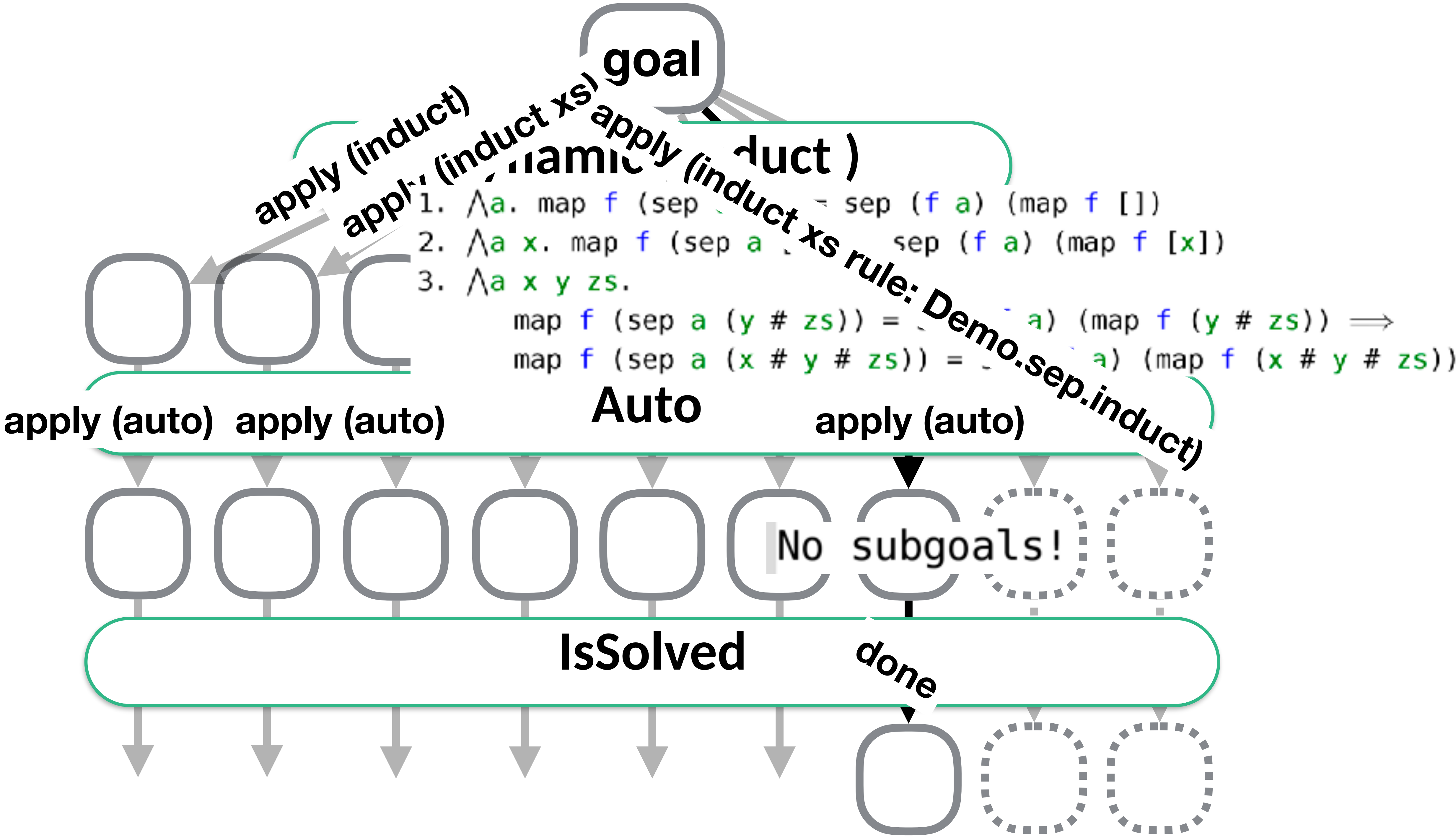

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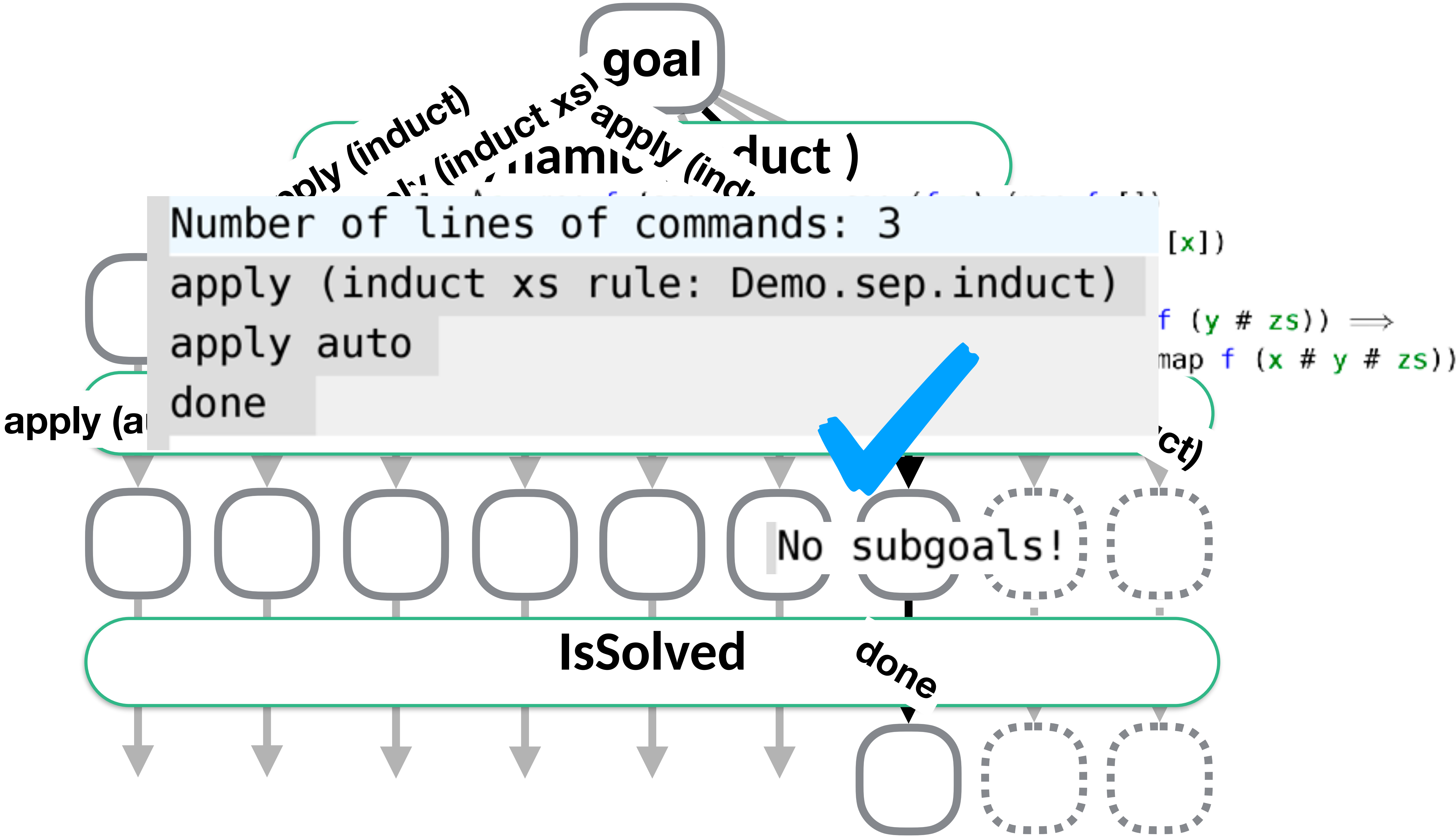


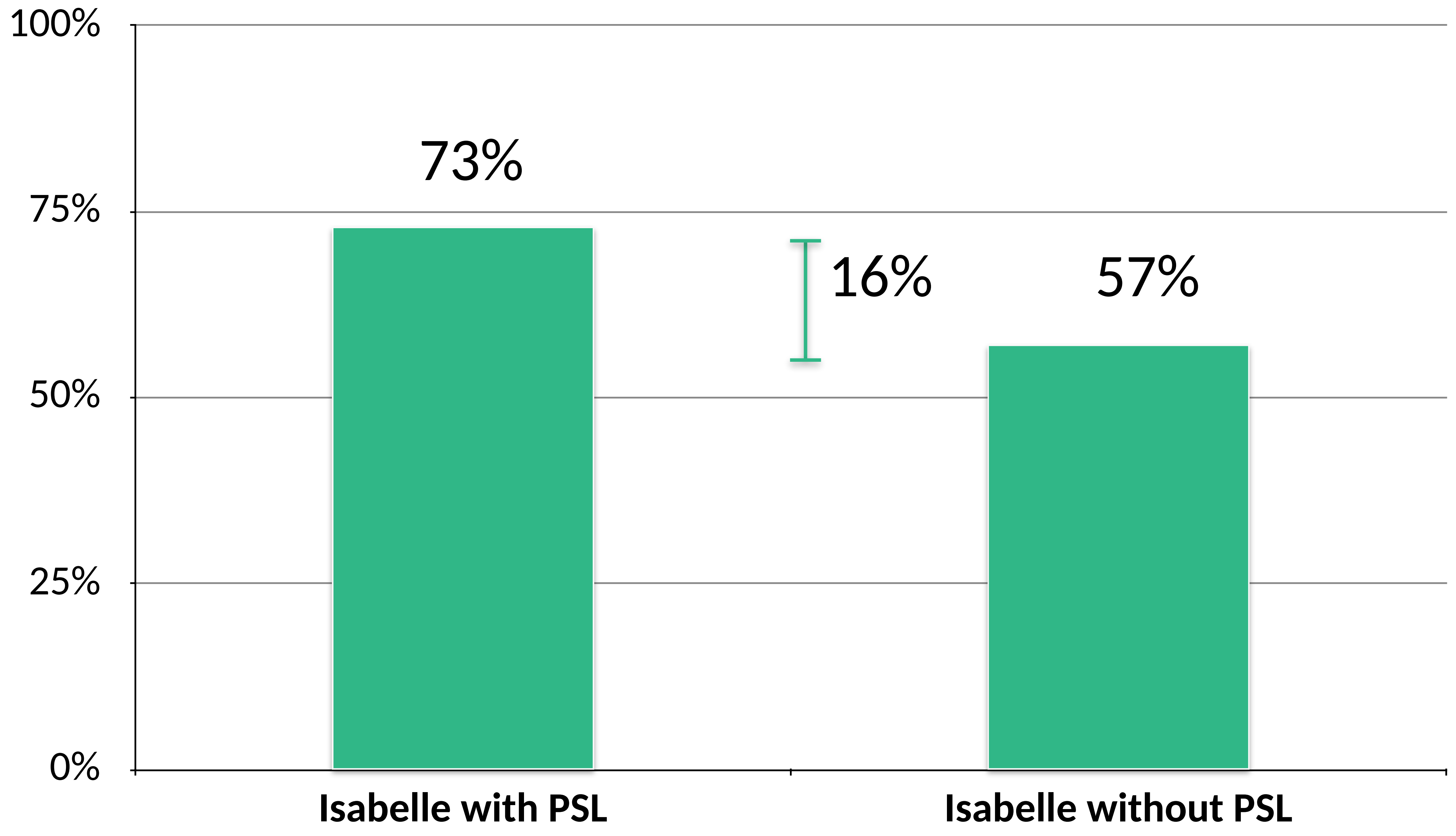
```
Number of lines of commands: 3
apply (induct xs rule: Demo.sep.induct)
apply auto
done
```

Try_Hard: the default strategy

```
strategy Try_Hard =
Ors [Thens [Subgoal, Basic],
     Thens [DInductTac, Auto_Solve],
     Thens [DCaseTac, Auto_Solve],
     Thens [Subgoal, Advanced],
     Thens [DCaseTac, Solve_Many],
     Thens [DInductTac, Solve_Many] ]
```

```
strategy Basic =
  Ors [
      Auto_Solve,
      Blast_Solve,
      FF_Solve,
      Thens [IntroClasses, Auto_Solve],
      Thens [Transfer, Auto_Solve],
      Thens [Normalization, IsSolved],
      Thens [DInduct, Auto_Solve],
      Thens [Hammer, IsSolved],
      Thens [DCases, Auto_Solve],
      Thens [DCoinduction, Auto_Solve],
      Thens [Auto, RepeatN(Hammer), IsSolved],
      Thens [DAuto, IsSolved]]
```

Evaluation



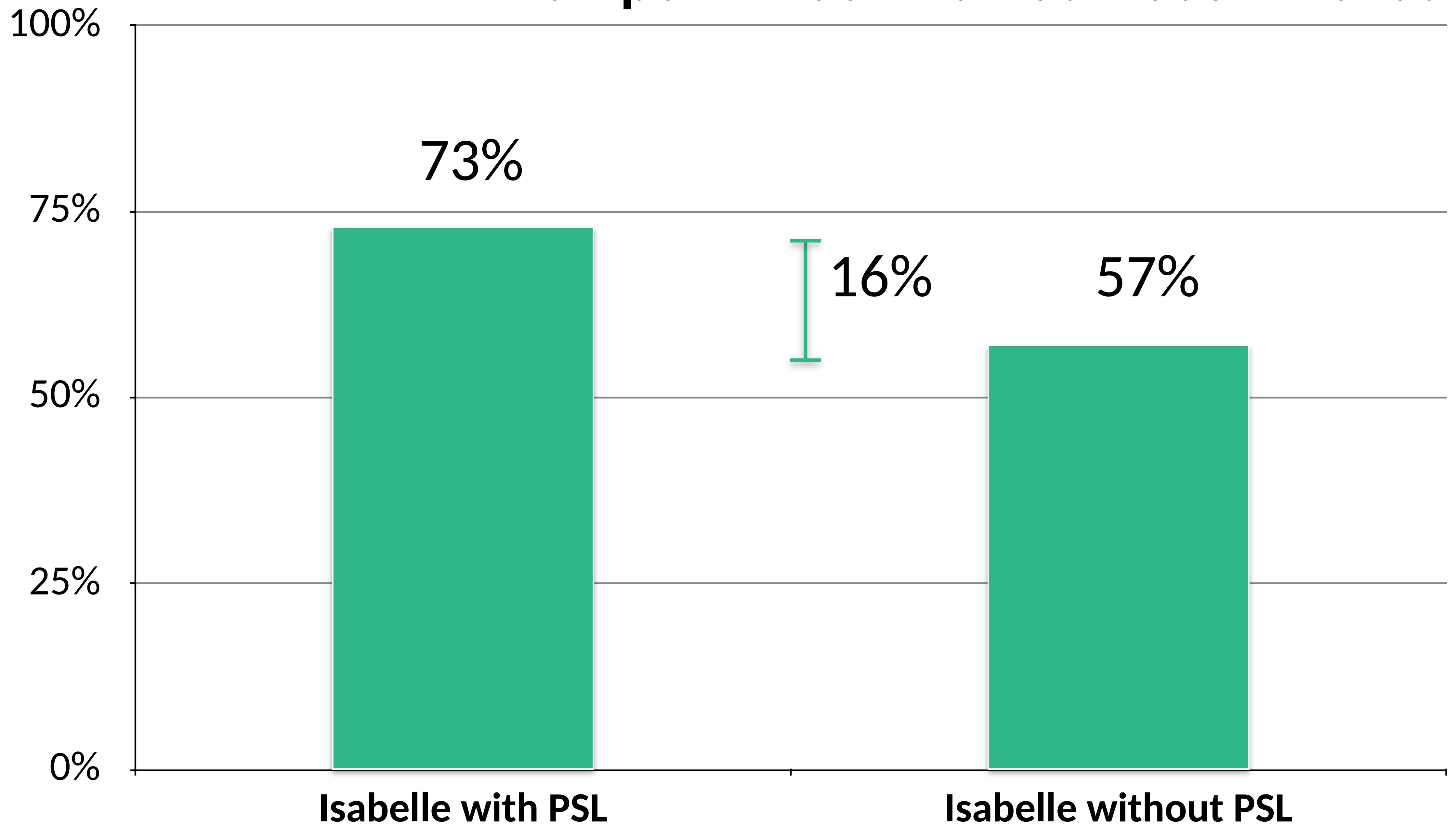
https://twitter.com/YutakangE

shorturl.at/dzZ16

Evaluation

but the search space explode

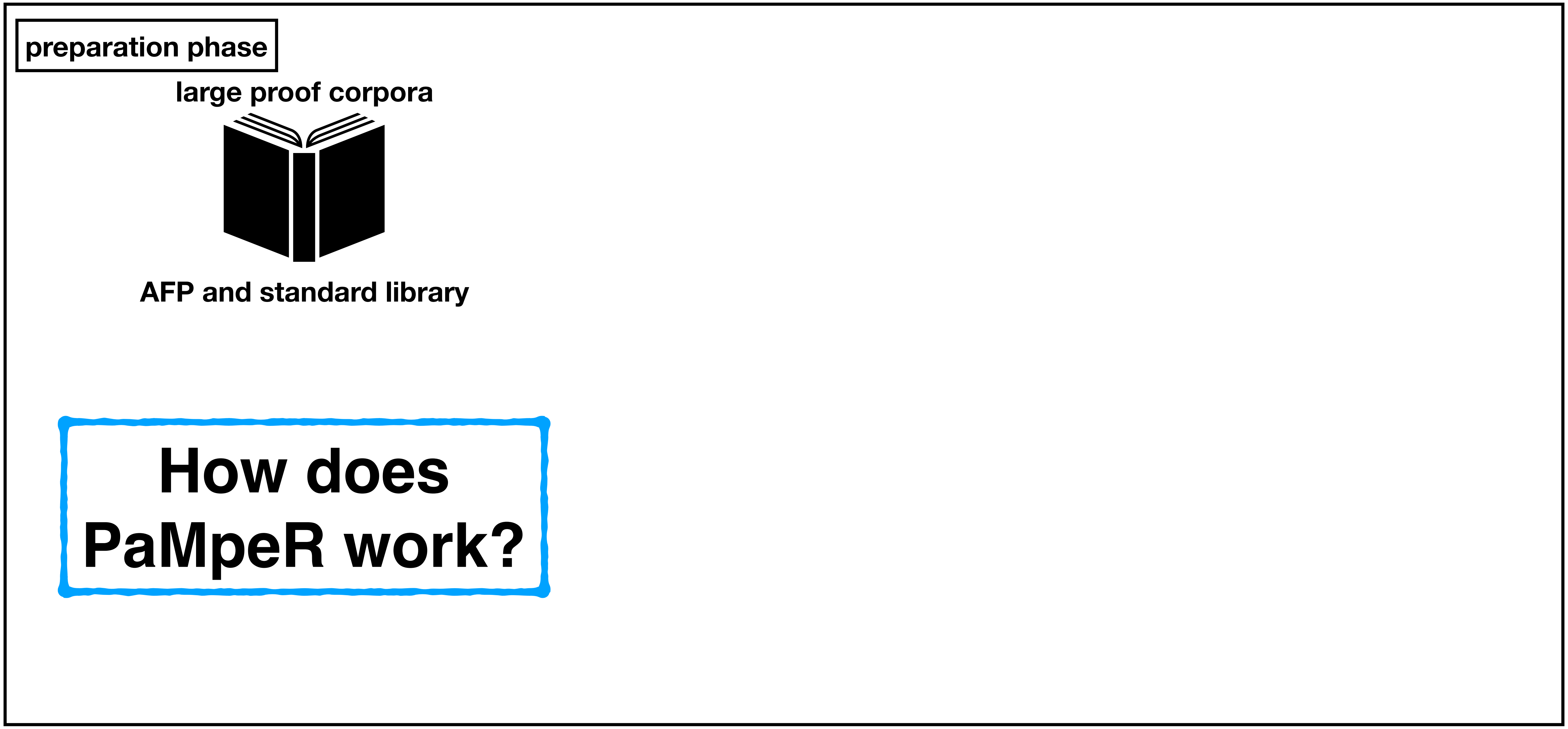
PaMpeR: Proof Method Recommendation



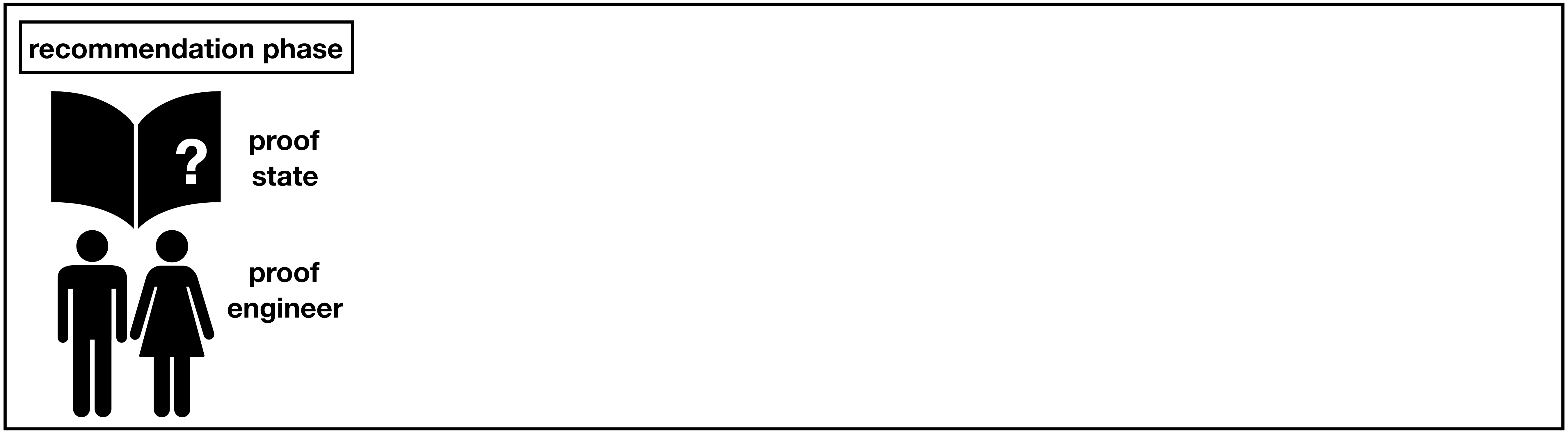
preparation phase

How does PaMpeR work?

recommendation phase



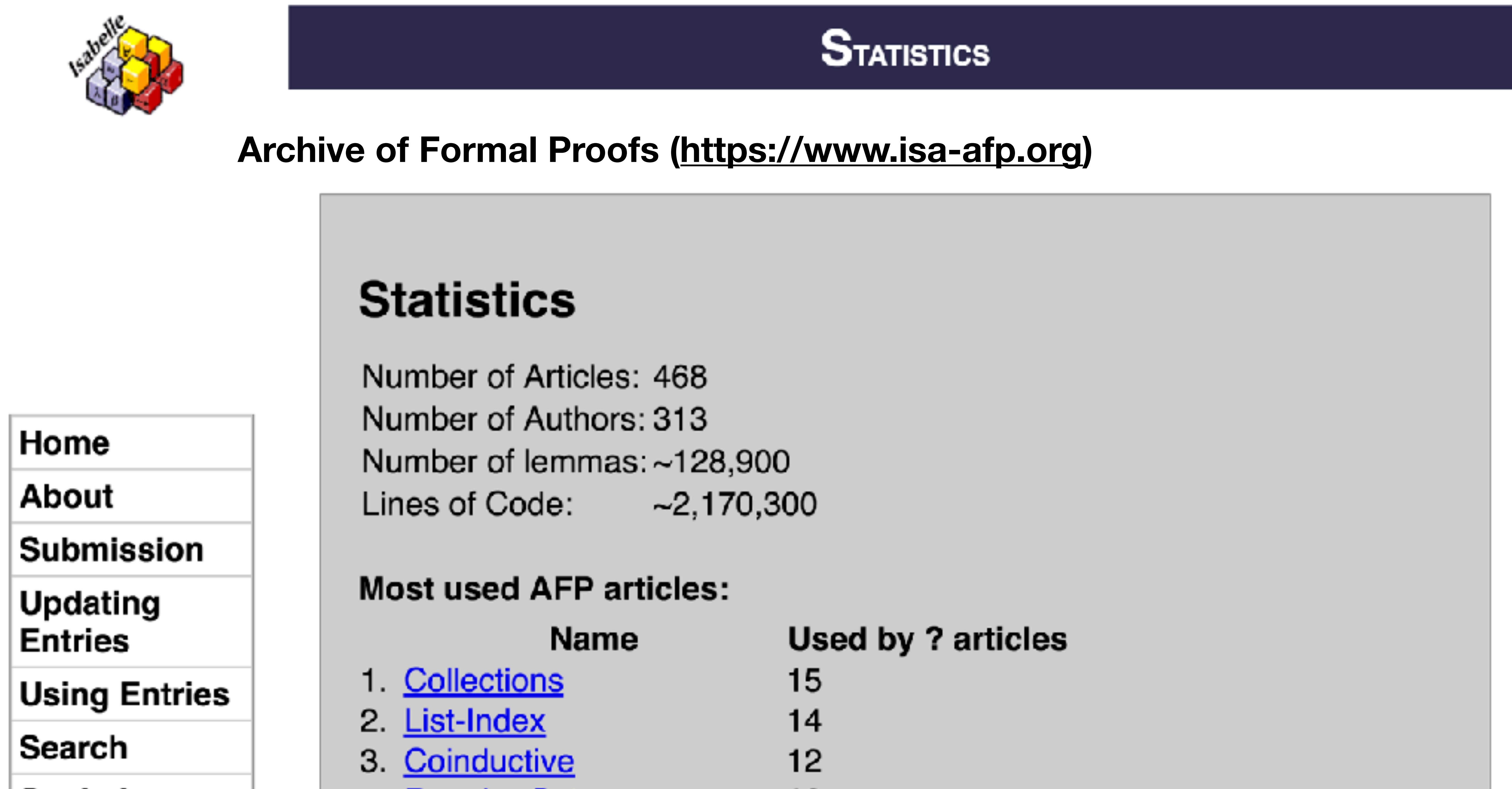
How does PaMpeR work?



preparation phase

large proof corpora

AFP and standard library



Archive of Formal Proofs (https://www.isa-afp.org)

Statistics

Number of Articles:	468
Number of Authors:	313
Number of lemmas:	~128,900
Lines of Code:	~2,170,300

Most used AFP articles:

	Name	Used by ? articles
1.	Collections	15
2.	List-Index	14
3.	Coinductive	12

preparation phase

large proof corpora

AFP and standard library

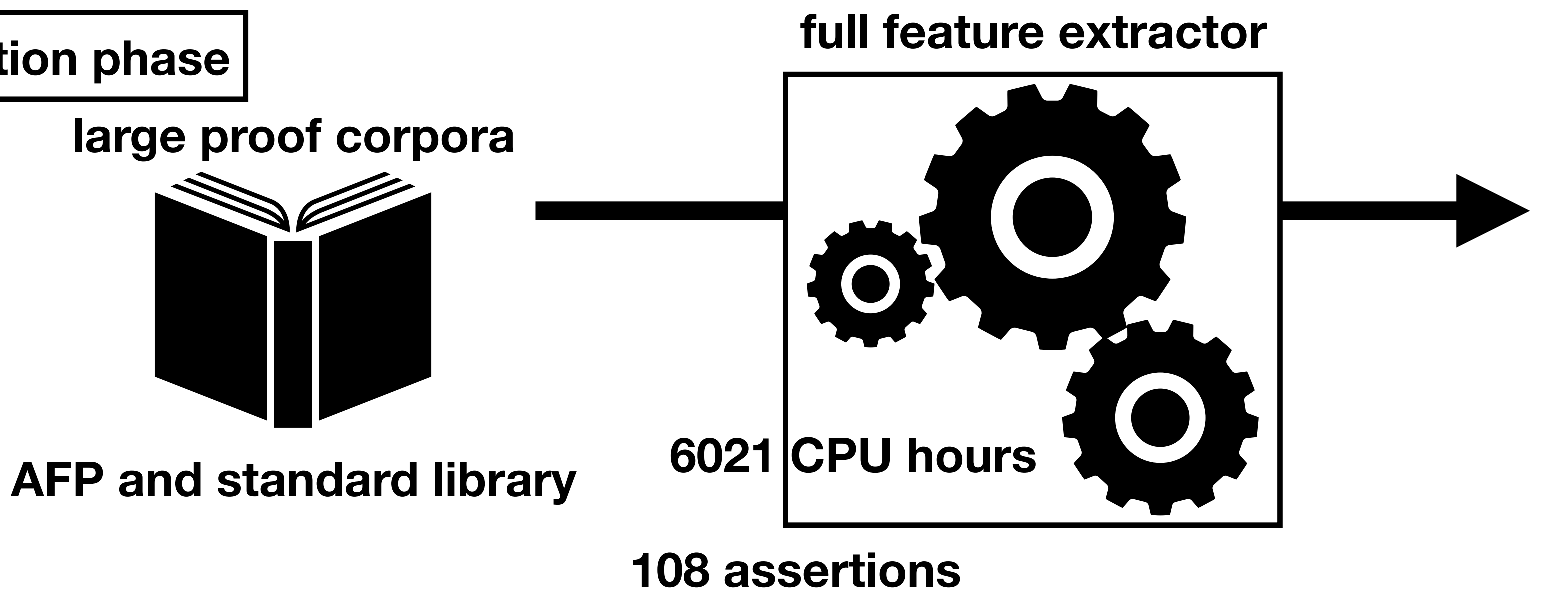
How does PaMpeR work?

recommendation phase

proof state

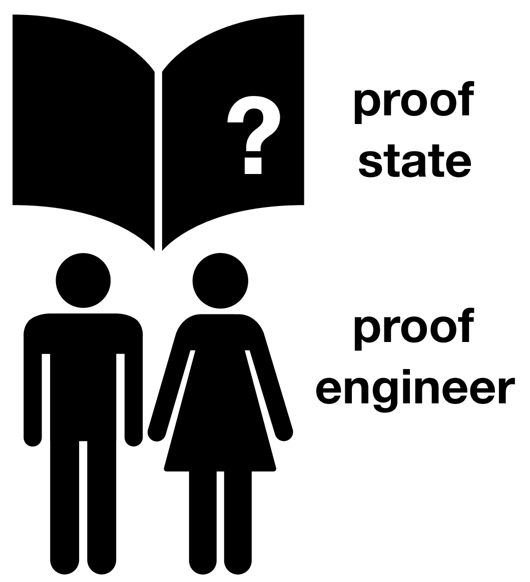
proof engineer

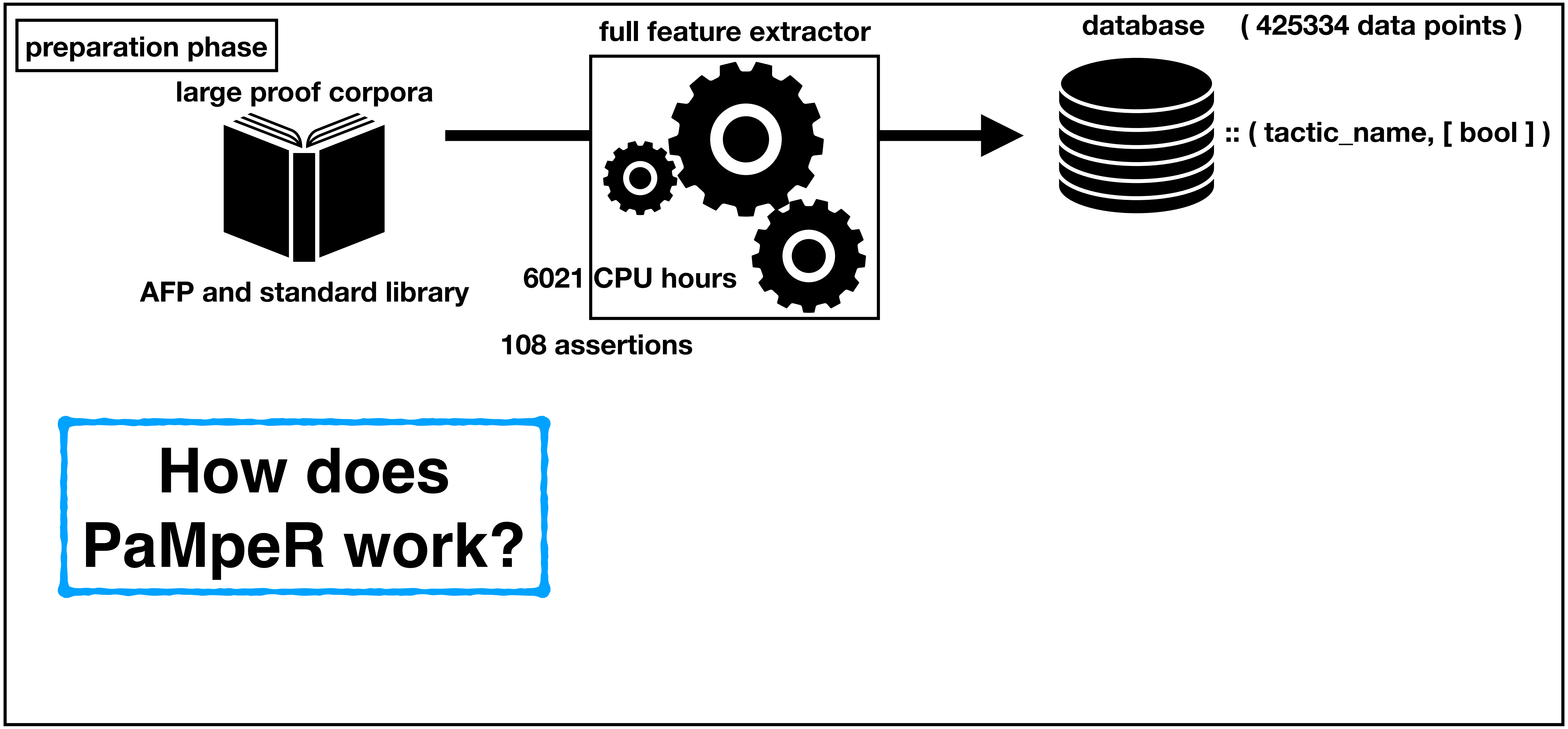
preparation phase



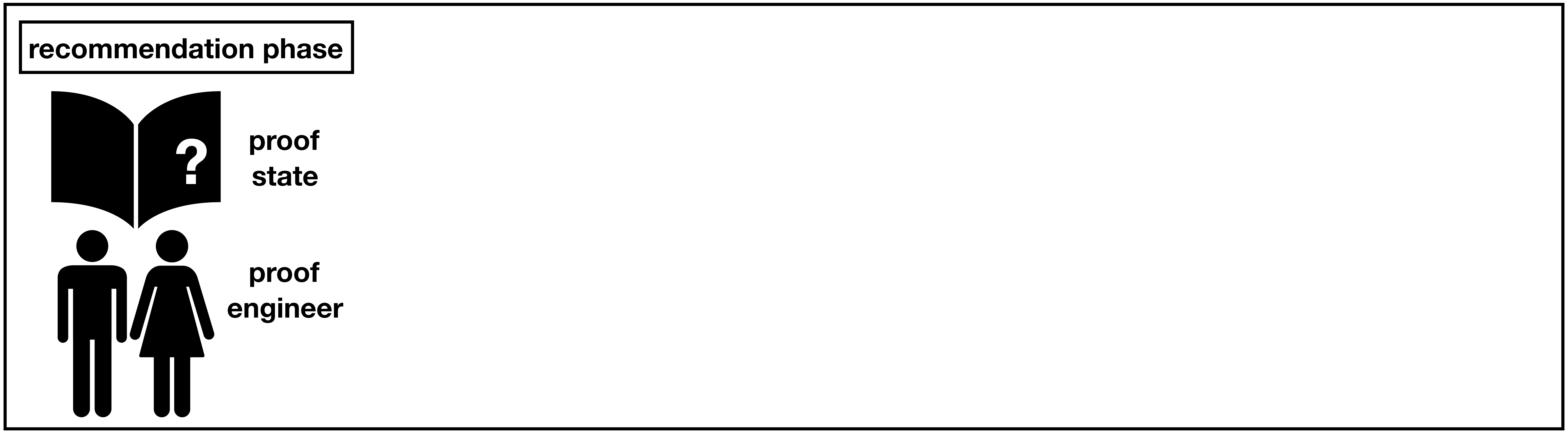
How does PaMpeR work?

recommendation phase

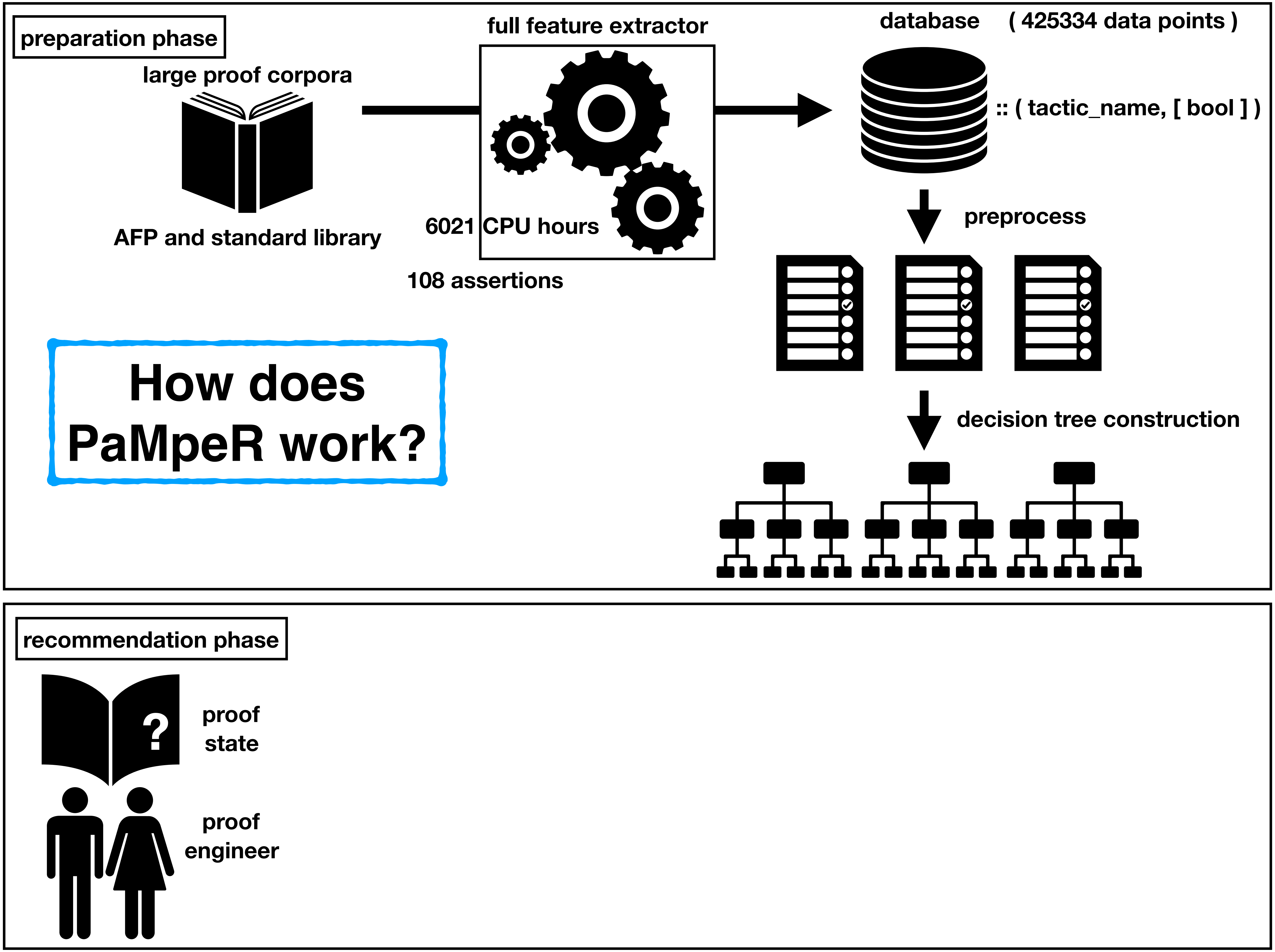




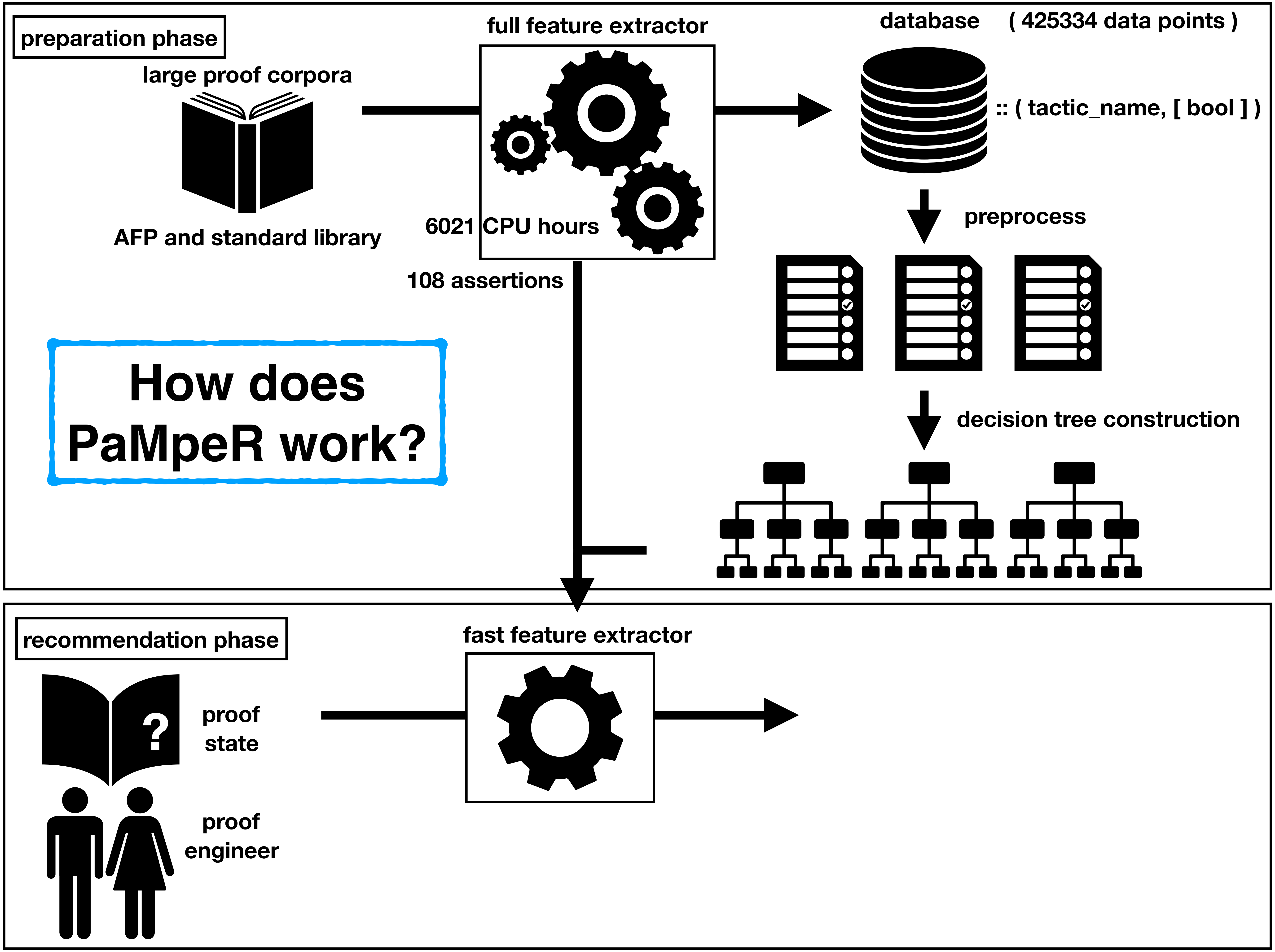
How does PaMpeR work?

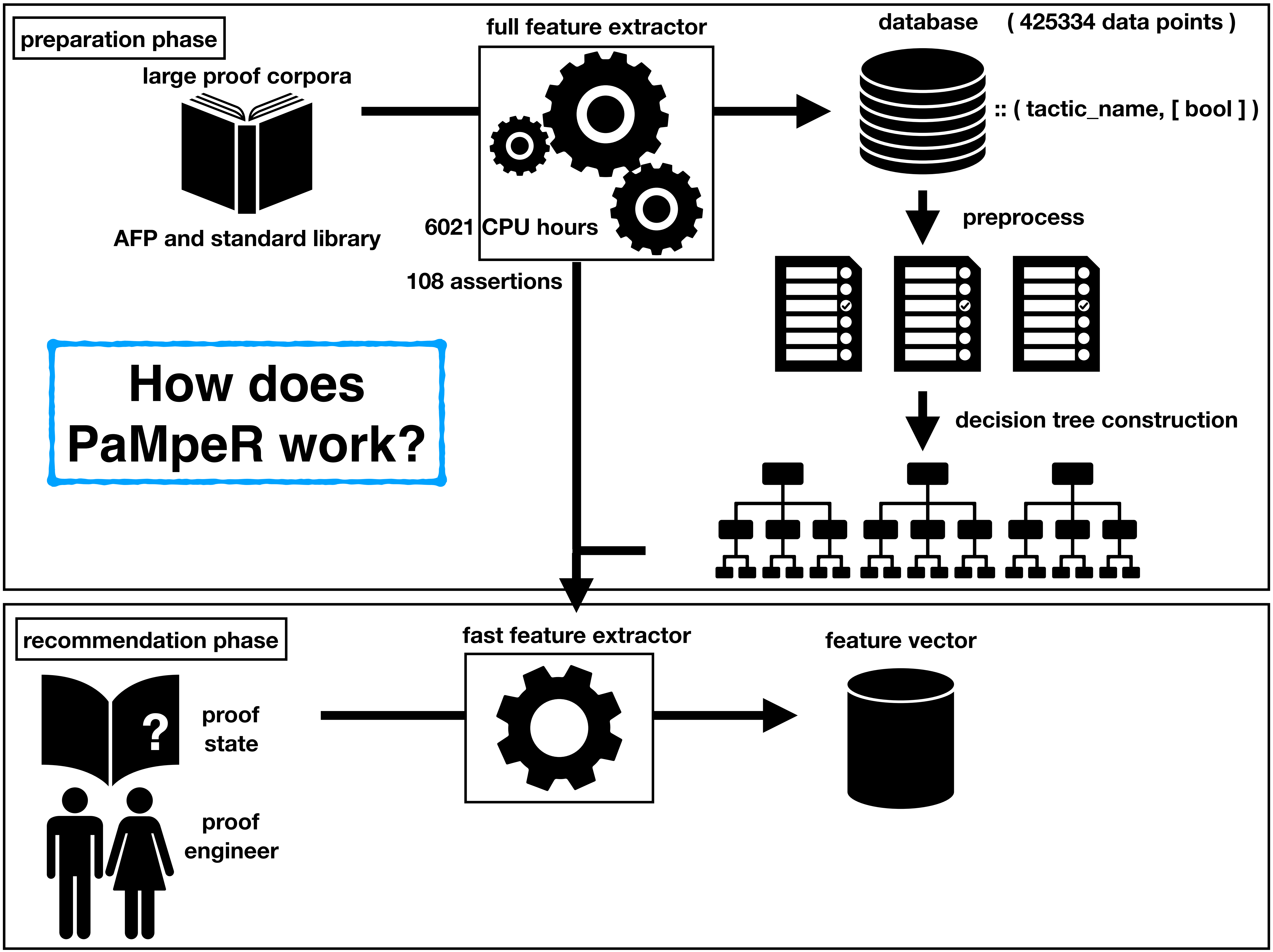


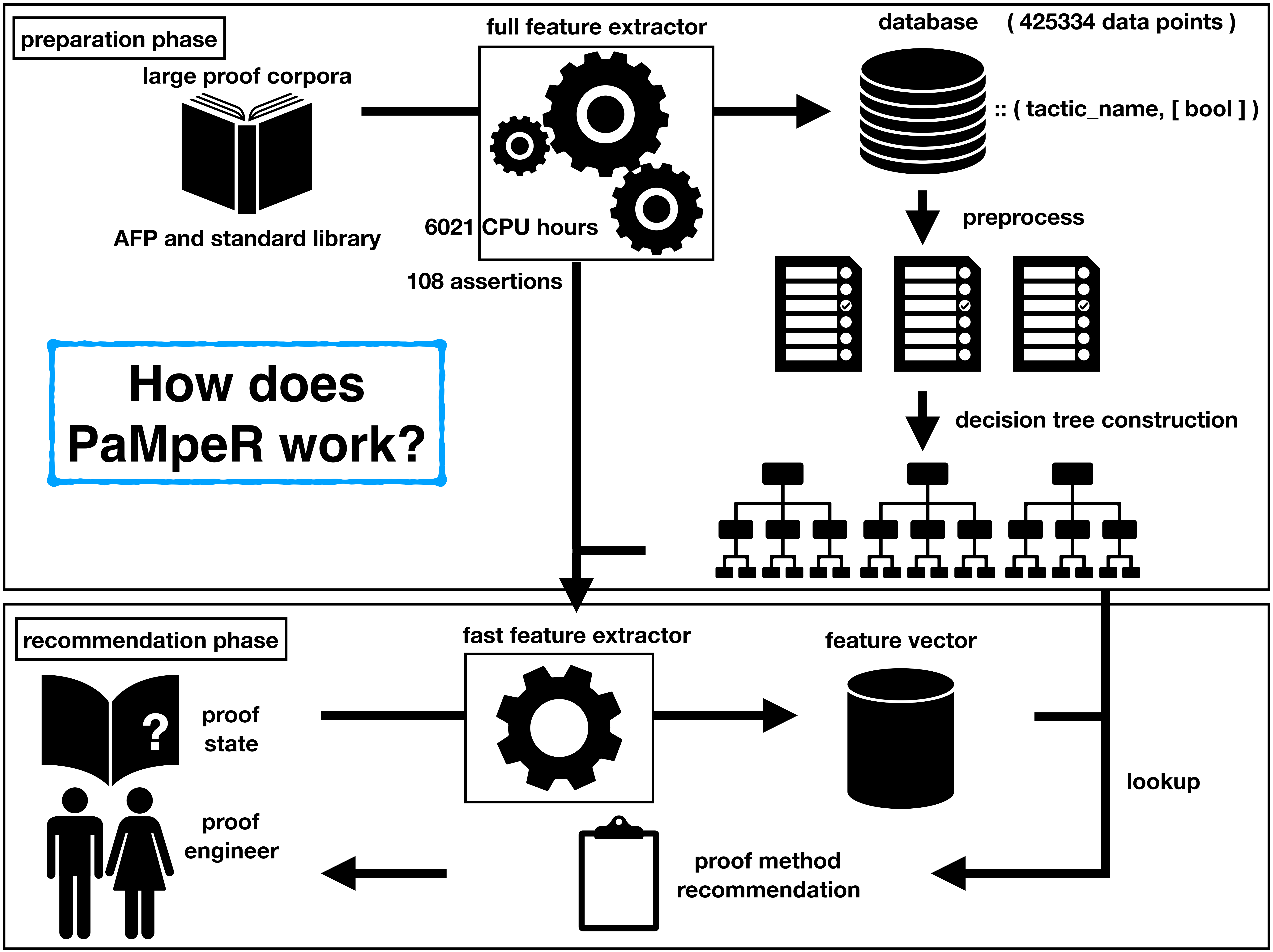
How does PaMpeR work?



How does PaMpeR work?




preparation phase
large proof corpora
AFP and standard library
full feature extractor
6021 CPU hours
108 assertions
database (425334 data points)
:: (tactic_name, [bool])
preprocess
decision tree construction
How does PaMpeR work?
recommendation phase
proof state
proof engineer
fast feature extractor
feature vector


preparation phase
large proof corpora
AFP and standard library
full feature extractor
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preprocess
decision tree construction
How does PaMpeR work?
recommendation phase
proof state
proof engineer
fast feature extractor
feature vector
lookup
proof method recommendation

Summary

PSL can find how to apply induction for easy problems.

CADE2017 (https://link.springer.com/10.1007/978-3-319-63046-5_32)

PaMpeR recommends which proof methods to use.

ASE2018 (https://dx.doi.org/10.1145/3238147.3238210)

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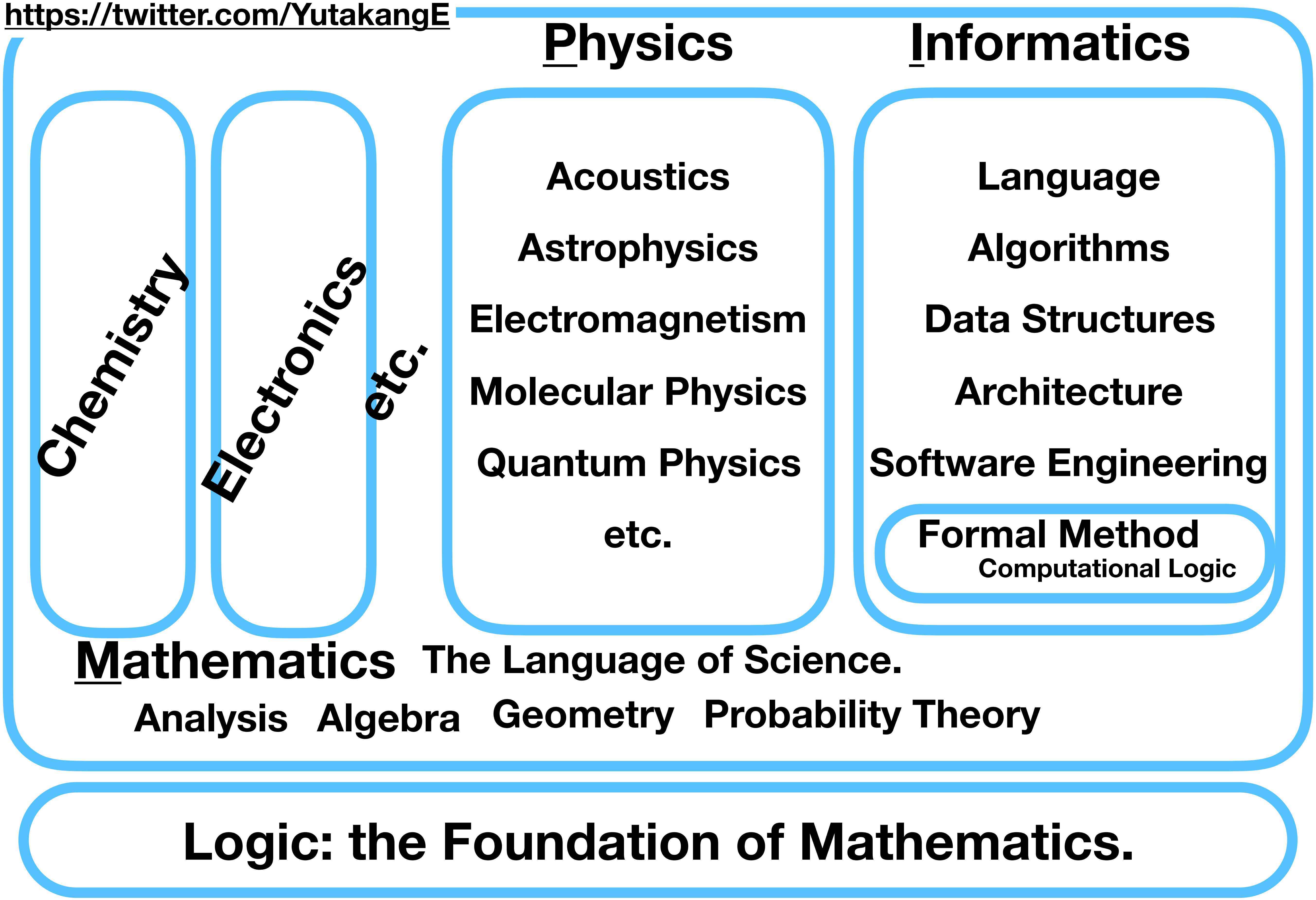
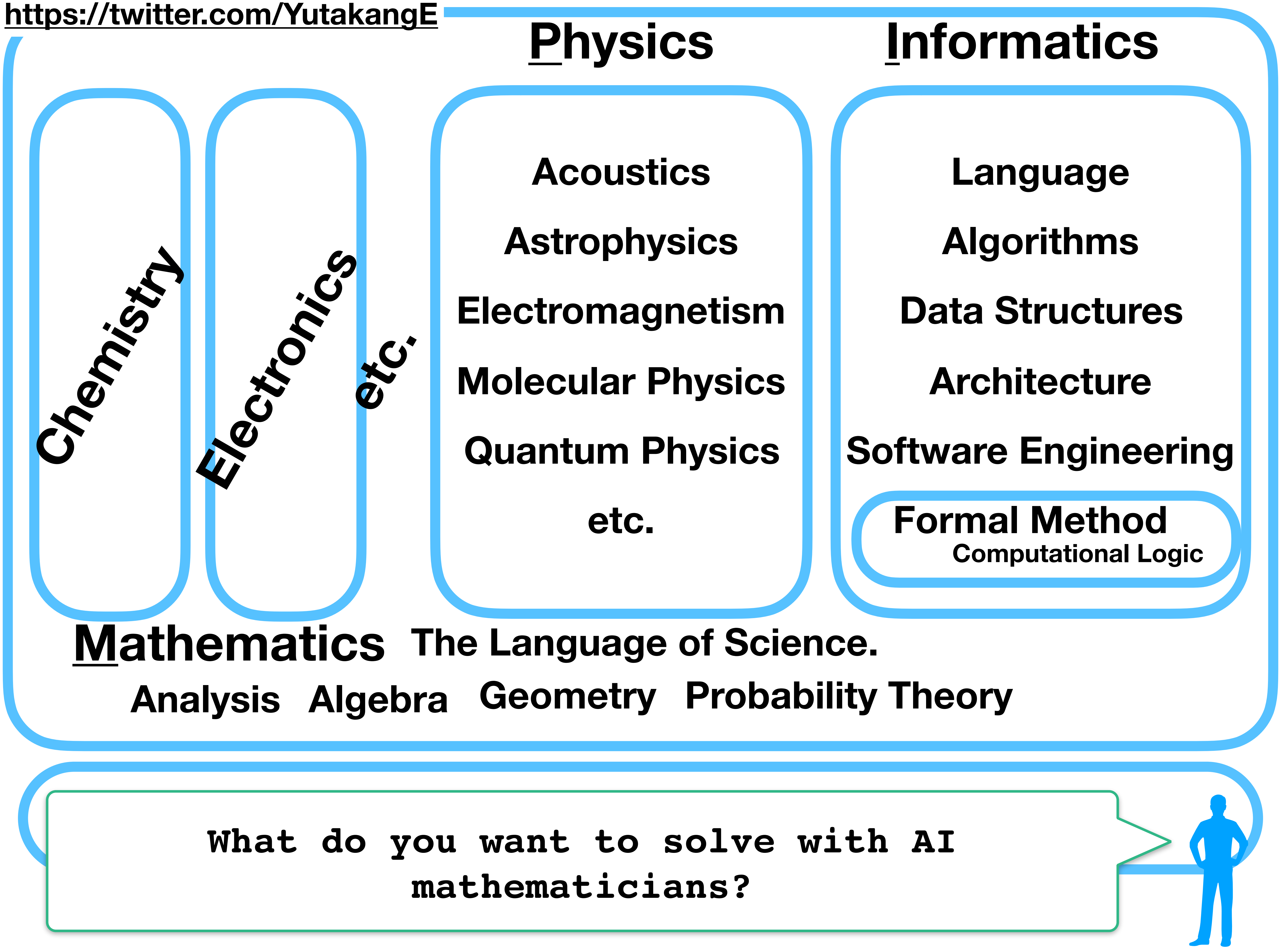


https://twitter.com/YutakangE



What do you want to solve with AI mathematicians?

https://twitter.com/YutakangE

Physics

Acoustics

Astrophysics

Electromagneti

Mole

Informatics

Language

Algorithms

Data Structures

Architecture

Software Engineering

Formal Method
Computational Logic

Chemistry

Electronics

etc.

Q & A

Mathematics The Language of Science.

Analysis Algebra Geometry Probability Theory

```
What do you want to solve with AI
mathematicians?
```