1. **Answer:** By the multiplication rule, the number of customizations is

$$3 \times 2 \times 3 \times 12,$$

in unevaluated form.

2. **Answer:** Each character has 10 digits, 26 uppercase letters, 26 lowercase letters, and 10 special characters, for a total of $10 + 26 + 26 + 10 = 72$ choices. Hence the total number of passwords is

$$72^{16}.$$

If you can test 10^{12} passwords per second, then testing all 72^{16} in the worst case takes

$$\frac{72^{16}}{10^{12}} \text{ seconds}$$

and in years

$$\frac{72^{16}}{10^{12} \times 60 \times 60 \times 24 \times 365} \approx 10^{10} \text{ years}$$

3. A password has length k where $12 \le k \le 16$. Each character can be a digit (10 choices), an uppercase letter (26), a lowercase letter (26), or a special character (10), giving $10 + 26 + 26 + 10 = 72$ total symbols. Without restriction, there are

$$72^k$$

possible strings of length k. We exclude strings consisting solely of letters (uppercase or lowercase), of which there are

$$52^k$$

all-letter strings. Thus, for each k, the number of valid passwords is

$$72^k - 52^k,$$

and summing over $k = 12$ to 16 yields

$$\boxed{\sum_{k=12}^{16} \left(72^k - 52^k\right)}$$

This enforces at least one digit or special character.

4. The dignitaries are received in an ordered sequence. The number of ways is the number of permutations of 4 items:

$$\boxed{P(4,4) = 4! = 4 \times 3 \times 2 \times 1}$$

We use permutations since the ordering of all 4 distinct dignitaries matters.

5. The lottery selects an unordered council of 99 from 100,000, so I used combinations. The number of ways is

$$\boxed{\binom{100000}{99}}$$

Order does not matter, hence combinations.

6. The total number of functions $f : \{1, 2, 3, 4, 5\} \to \{1, 2, \ldots, 7\}$ is

$$7^5.$$

Those that are one-to-one are the injections counted by permutations:

$$P(7, 5) = 7 \times 6 \times 5 \times 4 \times 3.$$

Therefore, the number of functions that are *not* one-to-one is

$$\boxed{7^5 - 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}$$

Explanation: subtract the number of injective functions from the total.

7. There are 8 age $\times 7$ income $\times 5$ gender $\times 6$ ethnicity $= 8 \cdot 7 \cdot 5 \cdot 6 = 1680$ possible demographic profiles. By the pigeonhole principle, to guarantee at least 3 respondents share the same profile, one must sample

$$\boxed{2 \times 1680 + 1 = 3361}$$

people. If at most 2 per profile, you can cover $2 \cdot 1680$ people without a triple; one more forces a third in some category.

8. (a) The maximum overlap is the smaller of the two sets: $\min\{20, 35\} = 20$. This occurs if every dog owner also owns a cat.

   (b) Suppose $|D| = 20$, $|C| = 35$, and total attendees $|D \cup C| = 40$. By inclusion–exclusion,

$$|D \cup C| = |D| + |C| - |D \cap C| \le 40,$$

so

$$20 + 35 - |D \cap C| \le 40 \quad \Longrightarrow \quad |D \cap C| \ge 15.$$

Therefore, at least 15 people own both cats and dogs, which follows from rearranging the inclusion–exclusion inequality.

9. **Extra Credit**

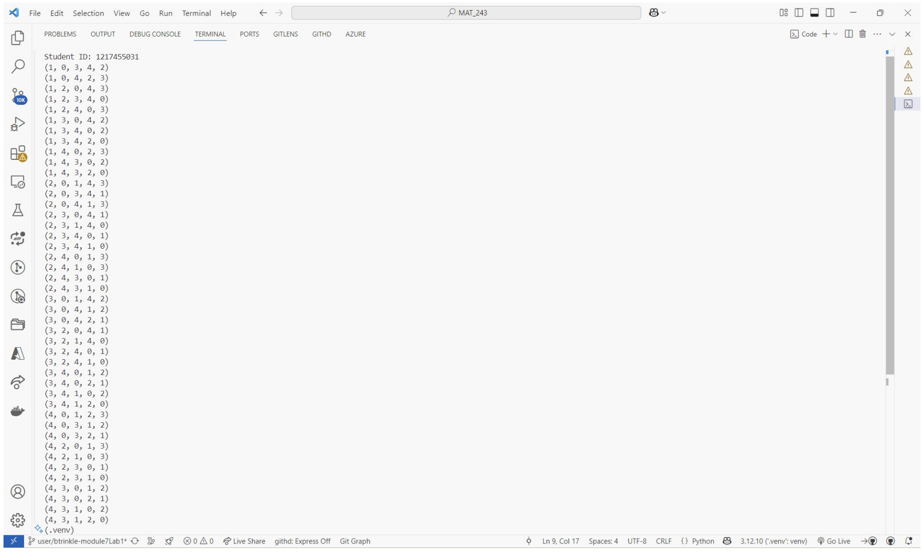

Figure 1: Output

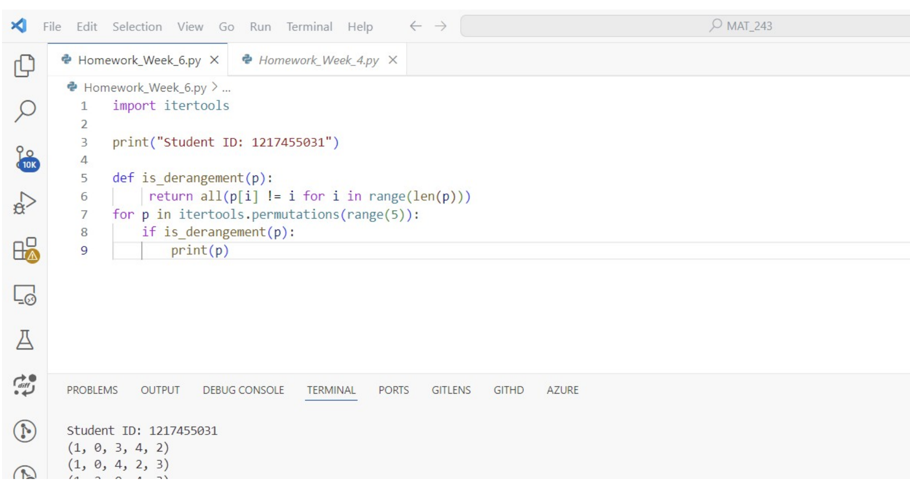

Figure 2: Code