1. Is the following a valid argument or fallacy? "If it is Sunday, then the store is closed. The store is closed. Therefore, it is Sunday."

   Answer: This is a fallacy, specifically affirming the consequent fallacy. The argument states:

   If it is Sunday, then the store is closed.
   The store is closed.
   $\therefore$ It is Sunday.

   This argument is invalid because the store might be closed for reasons other than it being Sunday. Affirming the consequent incorrectly assumes that if the consequent is true, then the antecedent must also be true.

2. Name the argument form of the following argument: Dogs eat meat. Fluffy does not eat meat. Therefore, Fluffy is not a dog.

   Answer: Argument: Modus Tollens

   $$p \to q, \quad \neg q \quad \therefore \quad \neg p$$

3. Use the rules of inference to prove the conclusion r given the four premises listed below. Write your solution as a numbered sequence of statements, clearly identifying each as a premise or a derived statement. Specify the rule of inference and refer by number to the statements used.

   Proof:

   (a) $p \to \neg q$ (premise)
   (b) $p \vee u$ (premise)
   (c) q (premise)
   (d) $((r \wedge t) \vee p) \vee \neg u$ (premise)
   (e) $\neg p$ (Modus Tollens from statements 1 and 3)
   (f) u (Disjunctive Syllogism from statements 2 and 5)
   (g) $(r \wedge t) \vee p$ (Disjunctive Syllogism from statements 4 and 6)
   (h) $r \wedge t$ (Disjunctive Syllogism from statements 7 and 5)
   (i) r (Simplification from statement 8)

   Answer: r is logically proven.

4. Lions hunt antelopes. Ramses is a lion. Ramses does not hunt Sylvester. Therefore, Sylvester is not an antelope.

   Predicates: $H(x, y)$: "x hunts y"; $L(x)$: "x is a lion"; $A(x)$: "x is an antelope" The domain of discourse is all animals.

   Premise:

   (a) $\forall x \forall y (L(x) \land A(y) \rightarrow H(x, y))$ (Lions hunt antelopes)
   (b) $L(Ramses)$
   (c) $\neg H(Ramses, Sylvester)$

   Answer:

   (a) Assume $A(Sylvester)$ (Assumption for Contradiction)
   (b) $L(Ramses) \land A(Sylvester)$ (Conjunction from 2, 4)
   (c) $H(Ramses, Sylvester)$ (Universal Instantiation from 1, 5)
   (d) Contradiction from 3, 6
   (e) Therefore, $\neg A(Sylvester)$ (Proof by contradiction from assumption 4)

   Thus, Sylvester is not an antelope.

5. Prove directly that the product of an even and an odd number is even.

   Proof: Let the even number be $2a$ and the odd number be $2b + 1$, where $a, b \in \mathbb{Z}$. The product is:

   $$(2a)(2b + 1) = 4ab + 2a = 2(2ab + a)$$

   Since $(2ab + a)$ is an integer, the product is divisible by 2, thus it is even.

6. If x is irrational, then $\frac{1}{x}$ is irrational, for arbitrary $x \neq 0$.

   Step 1:
   The contrapositive of the given statement is:
   If $\frac{1}{x}$ is rational, then x is rational.

   Step 2:
   Assume $\frac{1}{x}$ is rational. By definition of rational numbers, there exist integers m, n (with $n \neq 0$) such that:

   $$\frac{1}{x} = \frac{m}{n}$$

Step 3:
Since $x \neq 0$, we can solve for x:

$$x = \frac{n}{m}$$

Because $m, n \in \mathbb{Z}$ and $m \neq 0$, $x = \frac{n}{m}$ is clearly a rational number.

Answer:
By contraposition, we have proven the original statement. Specifically, if x is irrational, then $\frac{1}{x}$ must also be irrational for arbitrary $x \neq 0$.

7. Prove that there is a positive integer n that satisfies:

$$2n + 1 \geq 33$$

Proof (Existence): $n = 16$.

$$2(16) + 1 = 32 + 1 = 33$$

Since $33 \geq 33$, the inequality is satisfied.

Answer: Therefore, the positive integer $n = 16$ satisfies the condition, proving the existence.

8. Prove that for any positive integer n, there exists an even positive integer k satisfying:

$$\frac{1}{n+2} \leq \frac{1}{k-1} < \frac{1}{n}.$$

Step 1: Rewrite the inequality

$$n < k - 1 \leq n + 2.$$

Step 2: Choose a suitable integer

$$k = n + 2.$$

Step 3: Confirm that k is even

- If n is even, adding 2 yields another even integer.
- If n is odd, adding 2 (odd + 2 = odd + even) yields an odd number plus 1, which is even.
- $k = n + 2$ **is always even.**

Step 4: Substitute $k = n + 2$ into the inequality

$$n < (n + 2) - 1 = n + 1 \leq n + 2$$

- $n < n + 1$ is true for all positive integers n.
- $n + 1 \leq n + 2$ is also true for all positive integers n.

Step 5: Reconfirm original inequality

$$\frac{1}{n+2} \leq \frac{1}{(n+2)-1} = \frac{1}{n+1} < \frac{1}{n}.$$

This is true, since:

$$n + 2 > n + 1 > n.$$

Answer: Therefore, the integer $k = n+2$ satisfies the required conditions for all positive integers n.

9. Write a Python program that iterates through all q values.

Answer: