

1. (2 points) You buy a laptop computer with customization options. There are 3 choices for the OS, 2 for the processor, 3 for the amount of RAM and 12 choices for the SSD. In how many different ways can this computer be customized? Give your answer in un-evaluated form. You don't need to explain it.
2. (4 points) A password must consist of 16 characters. Each character can be a digit (0-9), an uppercase or lowercase letter (A-Z, a-z) or one out of 10 special characters. How many valid passwords are there? Give your answer in unevaluated form. You don't need to explain it.

If you have forgotten your password, but can test 1 trillion passwords per second, how much time would you require to find the password in the worst-case scenario that your forgotten password is the last one tested? Give the answer in years, rounded to the nearest power of 10.
3. (6 points) A password is required to be 12 to 16 characters in length. Characters can be digits (0-9), upper or lower-case letters (A-Z, a-z) or special characters. There are 10 permitted special characters. There is an additional rule that not all characters can be letters (i.e. there has to be at least one digit or one special character.) How many permitted passwords are there? Give your answer in un-evaluated/un-simplified form and explain it fully.
4. (2 points) The British Queen is to receive 4 foreign dignitaries. The dignitaries are sensitive to the order in which they are received. In how many ways can the Queen receive them? Give your answer in terms of permutations or combinations and explain your choice. You do not have to evaluate.
5. (2 points) The Martian Colonies elect their government through a lottery. There are 100,000 people living on Mars, and every year, a council of 99 co-equal leaders is randomly selected from the population. In how many ways can the leadership be elected? Give your answer in terms of permutations or combinations and explain your choice. You do not have to evaluate.
6. (4 points) How many functions are there from a set of 5 elements to a set of 7 elements that are *not* 1-1? Explain your reasoning fully.
7. (4 points) A survey company asks all respondents for demographic information. There are 8 age and 7 income groups, 5 gender choices, and 6 choices for ethnicity. How many people would this company have to poll so it's inevitable that at least three people in the sample have identical demographic information? Give the number in un-evaluated form and explain.
8. At the pet owner's meeting, there are 20 people who own dogs, and 35 people who own cats, and some people own both.

Justify your answers to the following questions precisely, by using inequalities and the inclusion-exclusion principle. Use C for the set of cat owners and D for the set of dog owners.

- (a) (2 points) What is the largest number of people who could own a dog and a cat, based on the given information? Explain your reasoning.
 - (b) (4 points) Suppose there are a total of 40 people at the meeting. What number of people who own both dog(s) and cat(s) is guaranteed? Explain your reasoning.
9. (extra credit, 6 points) Write a Python program that prints all *derangements* of 0, 1, 2, 3, 4. Derangements are permutations in which no element is in its original position. For example, 1, 2, 3, 4, 0 is a derangement of 0, 1, 2, 3, 4, while 0, 2, 1, 4, 3 is not.

Show your program and the output of the program.