1. Let $S = \{\text{all bitstrings of length } 3\}$, and define aRb iff $a \oplus b = 000$.

   (a) **Reflexive?** Yes. For every $a \in S$, $a \oplus a = 000$, so aRa.

   (b) **Symmetric?** Yes. If aRb, then $a \oplus b = 000$; since XOR is symmetric, $b \oplus a = 000$, hence bRa.

   (c) **Antisymmetric?** Yes. Antisymmetry requires: if aRb and bRa, then $a = b$. Here aRb already implies $a = b$, so the condition is satisfied.

   (d) **Transitive?** Yes. If aRb and bRc, then $a = b$ and $b = c$, so $a = c$ and aRc.

   (e) **Equivalence relation?** Yes. R is reflexive, symmetric, and transitive, hence an equivalence relation.

2. Let $S = \{ f : [0, \infty) \to \mathbb{R} \mid f \text{ continuous}\}$, and $(f, g) \in R$ iff $f(x) = O(g(x))$.

   (a) **Reflexive?** Yes, since $f(x) = O(f(x))$ via the bound $|f(x)| \leq 1 \cdot |f(x)|$.

   (b) **Antisymmetric?** No. Since $f(x) = x$ and $g(x) = 2x$ satisfy $f = O(g)$ and $g = O(f)$ but $f \neq g$.

   (c) **Symmetric?** No. Since $f(x) = x$ and $g(x) = x^2$ give $f = O(g)$ (for $x \geq 1$) but not $g = O(f)$.

   (d) **Transitive?** Yes. If $f = O(g)$ and $g = O(h)$, then there exist constants C_1, C_2 and x_0, x_1 so that

   $$|f(x)| \leq C_1|g(x)|, \quad |g(x)| \leq C_2|h(x)| \quad \text{for large } x.$$

   Hence $|f(x)| \leq C_1C_2\,|h(x)|$ for sufficiently large x, so $f = O(h)$.

3. **Extra Credit**

Homework_Week_6.py

Homework_Week_6.py > ...

```python
S = [1,2,3,4]
pairs = [(a,b) for a in S for b in S]
def is_transitive(R):
    return all((a,d) in R for (a,b) in R for (c,d) in R if b==c)
def is_reflexive(R):
    return all((a,a) in R for a in S)
count = 0
for mask in range(1 << len(pairs)):
    R = {pairs[i] for i in range(len(pairs)) if (mask >> i) & 1}
    if is_transitive(R) and not is_reflexive(R):
        count += 1
print("Student ID: 1217455031")
print(count)
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS GITLENS GITHD AZURE

```
$ python -u "c:\Users\Btrin\OneDrive\Desktop\ASU_IFT\Projects\MAT_243\Homework_Week_6.py"
Student ID: 1217455031
3639
(.venv)
Btrin@DESKTOP-J8TQDQE MINGW64 /c/Users/Btrin/OneDrive/Desktop/ASU_IFT/Projects/MAT_243 (user/btrinkle-module7Lab1)
$
```