1. (1 point) Is "Johann Sebastian Bach is the greatest of all the Baroque composers" a proposition? Explain.

2. (1 point) Write the negation of "Hikaru is taller than Yutaka". Your (verbally given) negation must not contain any words or phrases that explicitly express negation, such as "not", "it is untrue that", "is false", "is incorrect", etc.

3. (a) (1 point) Write the fully simplified negation of $3 < x \leq 4$.
   (b) (1 point) Negate verbally: "all people weigh at least 100 pounds."

4. (1 point) Is the conditional statement "If a human being has 7 heads, then they have 11 arms" true or false? Explain.

5. (1 point) Rephrase in contrapositive form: "If you are taller than 6 ft, then it is unpleasant for you to travel in economy class." Your contrapositive must not contain explicit references to negation. Assume that the negation of "unpleasant" is "pleasant".

6. (5 points) Rephrase verbally in equivalent *only if*, *sufficient*, *necessary*, *contrapositive* and *unless* form: "if we had an FTL drive, then we could visit the stars".

7. (1 point) Write the formal negation of $\forall x \exists y(x > y)$. Your negation must not contain any explicit negation symbols.

8. (4 points) Use logical equivalences to simplify $(p \to q) \to (\neg p \to \neg q)$ until you have at most one occurrence of each variable p, q remaining. Identify all logical equivalences by name. You will not receive credit for a truth table solution.

9. We have two mass storage devices, A and B with the same specs. Each can store N bits, but read/write operations are slow. We mitigate this problem by turning the two devices into a storage array: data is alternatingly written to A and B. The array can now store $2N$ bits, and read and write at almost twice the speed of the individual devices, assuming that the time it takes to send data to A and B is negligible, compared to the time it takes each device to write the data.

   There is however a flaw in this plan. Since data is distributed over two devices, the total chance of failure has now doubled. The failure of just one device causes the failure of the array.

   We could eliminate this problem by copying all of A's data redundantly to an additional devices C, and all of B's data to an additional device D. This insures against single and double drive failure, but at the cost of doubling the required amount of storage devices.

   We decide that simultaneous drive failure is sufficiently unlikely to ignore that possibility, and we will guard against data loss from single drive failure only. This requires only one extra device C, as follows:

We abstract each device as a list of bits, $A = a_1, a_2, ..., a_N$, $B = b_1, b_2, ..., b_N$ and $C = c_1, c_2, ..., c_N$.

At the time the data is written to drive A and B, a "checksum" is also written to drive C, according to:

$$c_k = a_k \text{ XOR } b_k$$

for each $k = 1, 2, ..., N$. Here, XOR is the bitwise exclusive or operation.

For example, all this means that if the original file is 01100110, then A stores 0101 and B stores 1010, and C stores 1111. Make sure you understand this example. If not, you are not understanding this question, and cannot answer it correctly.

(a) (2 points) Assume that after the data has been written to drives A, B and C as explained above, drive A is destroyed. Only drives B and C are left. Explain how each bit a_k can be reconstructed from the knowledge of b_k and c_k. Your explanation must be specific and detailed. It is insufficient to declare that "A can be reconstructed from B and C" or such.

Hint: consider the four cases for $(b_k, c_k) : (1, 0), (1, 1), (0, 0)$ and $(0, 1)$. Explain in each case how a_k can be reconstructed.

(b) (2 points) Would the same reconstruction property still hold if the data on drive C had been written according to: $c_k = a_k$ AND b_k for each $k = 1, ..., N$? You must fully explain your answer.

(c) (2 points) Would the same reconstruction property still hold if the data on drive C had been written according to: $c_k = a_k$ OR b_k for each $k = 1, ..., N$? You must fully explain your answer.

10. (2 points) Is the statement $\exists x \forall y (xy = 0)$ true or false? The domain of discourse is the set of real numbers. Explain.

11. (2 points) If P and Q are predicates over some domain, and if it is true that $\forall x (P(x) \vee Q(x))$, must $\forall x P(x) \vee \forall x Q(x)$ also be true? Explain.

12. (2 points) Suppose P is the predicate defined by $P(x, y) = x$ is friends with y, where x and y are people. (No one is considered to be friends with themselves.) *Translate* the formal expression $\forall x \exists y \exists z (y \neq z \wedge P(x, y) \wedge P(x, z))$ into English. Don't use symbols such as x, y and z in the English translation.

13. (2 points) Let P be defined as in the previous problem. Is $\forall x \exists y \exists z (y \neq z \rightarrow P(x, y) \wedge P(x, z))$ true or false? Explain.

14. (4 points extra credit) The goal of this problem is to disprove a logical equivalence programmatically. Write a Python program that prints a complete truth table for $(a \rightarrow b) \rightarrow (c \rightarrow d)$ and $(a \rightarrow (b \rightarrow c)) \rightarrow d$, and

marks the rows where the two statements have different truth values. You have to build the truth table from scratch. Don't use a build-in truth table generator. Show your program and the output.