1. **Simplify and find a function g such that $f(x) = O(g(x))$.**

   Given:

   $$f(x) = \left(x^{2.3} + x\,\ln(x^5)\right)\left(1.1\,x + 1 + 1.2\,x\right) \;+\; \left(x^2 + 1.2\,x\right)\left(x^3 + 0.92\,x\right).$$

   Step 1:

   $$x^{2.3} + x\ln(x^5) = O\left(x^{2.3}\right), \quad 1.1\,x + 1 + 1.2\,x = O(x).$$

   Their product is $O\left(x^{3.3}\right)$.

   Step 2:

   $$x^2 + 1.2\,x = O\left(x^2\right), \quad x^3 + 0.92\,x = O\left(x^3\right).$$

   Their product is $O\left(x^5\right)$.

   Step 3: Add the two results:

   $$f(x) = O\left(x^{3.3}\right) + O\left(x^5\right).$$

   Step 4: Since x^5 dominates $x^{3.3}$ for large x,

   $$f(x) = O\left(x^5\right).$$

   Answer:$g(x) = x^5$.

2. **Is there a smallest real number a such that $x^2\,6^x$ is $O(a^x)$?**

   (a) If $x^2\,6^x \le C\,a^x$ for large x, then $\left(\frac{6}{a}\right)^x$ must eventually shrink faster than x^{-2}.

   (b) This requires $\frac{6}{a} < 1$, or $a > 6$.

   (c) No real number $a \le 6$ can work, and there is no minimum real > 6.

   Answer:

   No. We must have $a > 6$, but there is no smallest real number.

3. **Hex addition $\texttt{CA10} + \texttt{4F57}$.**

   $$\begin{array}{r} \texttt{CA10} \\ + \;\texttt{4F57} \\ \hline \end{array}$$

   (a) Add the rightmost digits:

   $$0 + 7 = 7.$$

(b) Next digits:
$$1 + 5 = 6.$$

(c) Next, add:
$$A_{16} + F_{16} = 10_{10} + 15_{10} = 25_{10}.$$

In hexadecimal, $25_{10} = 19_{16}$; write down 9 and carry 1.

(d) Finally, add the leftmost digits with the carry:
$$C_{16} + 4_{16} + 1(\text{carry}) = 12_{10} + 4_{10} + 1_{10} = 17_{10}.$$

Since $17_{10} = 11_{16}$, write down 1 and carry 1 (which becomes a new digit at the front).

Answer: 11967_{16}

In hexadecimal addition, when the sum of digits (and any carry) exceeds F_{16}, convert the total sum to hexadecimal. For example, 1+1+1 in binary yields 1 with a carry of 1 because the result exceeds the single-digit value in that base.

4. **Binary multiplication** 11011×1001**.**

$$\begin{array}{r} 11011 \\ \times\ 1001 \\ \hline 11011 \\ 000000 \\ 0000000 \\ +11011000 \\ \hline 11110011 \end{array}$$

In binary addition, when adding three 1's together, $1 + 1 + 1$, the result is 1 with a carry of 1 since binary digits are only 0 and 1.

Answer: 11110011_2

5. **Use the Euclidean Algorithm to show** $\gcd(621, 82) = 1$**.**
Steps:

(a) $621 = 82 \cdot 7 + 47$; $\gcd(621, 82) = \gcd(82, 47)$.

(b) $82 = 47 \cdot 1 + 35$; $\gcd(82, 47) = \gcd(47, 35)$.

(c) $47 = 35 \cdot 1 + 12$; $\gcd(47, 35) = \gcd(35, 12)$.

(d) $35 = 12 \cdot 2 + 11$; $\gcd(35, 12) = \gcd(12, 11)$.

(e) $12 = 11 \cdot 1 + 1$; $\gcd(12, 11) = \gcd(11, 1)$.

(f) $11 = 1 \cdot 11 + 0$; $\gcd(11, 1) = 1$.

Answer: $\gcd(621, 82) = 1$.

6. **Show that you can multiply n by 35 using only five multiplications by 2 and two additions**
Steps:
   (a) $35_{10} = 32 + 2 + 1 = 2^5 + 2^1 + 2^0$.
   (b) Double n five times to get $32n$.
   (c) Then add $(32n) + (2n) + n = 35n$.

Answer: $35n = (2^5 n) + (2n) + n$.

7. **Multiply an octal number n by $(10)_8$. Illustrate with $(741)_8$.**

Multiplying any number by its base shifts the digits one place to the left. In octal:

$$(10)_8 = 8 \quad \text{(in decimal)}.$$

Thus, multiplying $(741)_8$ by $(10)_8$ shifts every digit one place to the left:

$$(741)_8 \times (10)_8 = (7410)_8.$$

Answer: 7410_8.

8. **Divide an octal number n by $(10)_8$. Illustrate with $(741)_8$.**
Steps:
   (a) In octal, dividing by $(10)_8$ is the same as removing the last digit as remainder.
   (b) Example: $(741)_8 \div (10)_8$ equals $(74)_8$ and remainder $(1)_8$.

Answer: $(74)_8$, remainder $(1)_8$.

9. **Fast exponentiation for $32^n \bmod 13$.**

Step 1: Reduce the Base

$$32 \div 13 = 2 \quad \text{with remainder } 32 - 2 \cdot 13 = 32 - 26 = 6,$$

$$32 \equiv 6 \pmod{13}.$$

Thus, for any exponent n,

$$32^n \equiv 6^n \pmod{13}.$$

Step 2: Verify for Small Exponents

- For $n = 0$:
$$6^0 = 1.$$
- For $n = 1$:
$$6^1 = 6.$$
- For $n = 2$:
$$6^2 = 36.$$
  Dividing 36 by 13:
$$36 \div 13 = 2 \quad (\text{since } 13 \times 2 = 26), \quad 36 - 26 = 10,$$
  so
$$6^2 \equiv 10 \pmod{13}.$$

Step 3: Apply Euler's Theorem

Because 13 is prime, its Euler totient is

$$\varphi(13) = 13 - 1 = 12.$$

Since 6 and 13 share no common factors other than 1 (coprime), Euler's theorem tells us:

$$6^{12} \equiv 1 \pmod{13}.$$

This means the powers of 6 modulo 13 are periodic with period 12.

Step 4: Reduce the Exponent

For any exponent n, we can reduce it modulo 12:

$$6^n \equiv 6^{\,n \bmod 12} \pmod{13}.$$

$$\textbf{Answer: } 6^{\,n \bmod 12} \pmod{13}.$$

10. **Closed-form for the hex number of n copies of 'A'.**

Step 1: Express the Number as a Sum

An n-digit number where each digit is A can be written as

$$\underbrace{AA\cdots A}_{n \text{ digits}} = \sum_{k=0}^{n-1} A \cdot 16^k.$$

Since $A = 10$, this becomes

$$\sum_{k=0}^{n-1} 10 \cdot 16^k = 10 \sum_{k=0}^{n-1} 16^k.$$

Step 2: Sum the Geometric Series

The sum of the geometric series with n terms and common ratio 16 is given by:

$$\sum_{k=0}^{n-1} 16^k = \frac{16^n - 1}{16 - 1} = \frac{16^n - 1}{15}.$$

This becomes:

$$10 \cdot \frac{16^n - 1}{15}.$$

Step 3: Simplify the Expression

$$10 \cdot \frac{16^n - 1}{15} = \frac{10(16^n - 1)}{15} = \frac{2(16^n - 1)}{3}.$$

$$\textbf{Answer: } \frac{2(16^n - 1)}{3}.$$

11. **Extra Credit**