

1. (6 points) Let S be the set of all bitstrings of length 3.

Let R be the relation defined on S by aRb iff $a \text{ XOR } b = 000$ for $a, b \in S$. Here, XOR is the bitwise exclusive OR operator.

- (a) Is R reflexive?
- (b) Is R symmetric?
- (c) Is R antisymmetric?
- (d) Is R transitive?
- (e) Is R an equivalence relation?

Explain your answers.

2. (6 points) Let S be the set of all continuous functions $f : [0, \infty) \rightarrow \mathbb{R}$. Let R be the relation defined on S by $(f, g) \in R$ iff $f(x)$ is $O(g(x))$.

- (a) Is R reflexive?
- (b) Is R antisymmetric?
- (c) is R symmetric?
- (d) is R transitive?

Explain your answer in details Use the definition of big-O to justify your answer if you think R has a certain property or give a counter example if you think R does not have a certain property.

3. (extra credit, 8 points) Write a Python function that determines whether a given relation R on $S = \{1, 2, 3, 4\}$ is transitive but not reflexive. Then apply this functions to all relations R on S to count how many of them are transitive but not reflexive. Show your program and the number of transitive but not reflexive relations on S it found.

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