1. Is "Johann Sebastian Bach is the greatest of all the Baroque composers" a proposition? Explain.

   Answer: Yes, the statement "Johann Sebastian Bach is the greatest of all the Baroque composers" is a proposition. A **proposition** is a declarative sentence that has a definite truth value (either true or false, but not both).

2. Write the negation of "Hikaru is taller than Yutaka":

   Answer: Yutaka is as tall as or taller than Hikaru.

3. (a) Write the fully simplified negation of $3 < x \leq 4$:

   Answer: $x \leq 3 \vee x > 4$.

   (b) Negate verbally: "all people weigh at least 100 pounds."

   Answer: "There exists someone who weighs less than 100 pounds."

4. Is the conditional statement "If a human being has 7 heads, then they have 11 arms" true or false?

   Answer: The conditional "If a human being has 7 heads, then they have 11 arms" is true. A conditional is false only if its antecedent is true and its consequent is false. Since no human being has 7 heads (the antecedent is never satisfied), there is no case that makes the conditional false, so it is true.

5. Rephrase in contrapositive form: "If you are taller than 6 ft, then it is unpleasant for you to travel in economy class."

   Answer: If traveling in economy class is comfortable for you, then you are not taller than 6 ft.

6. Rephrase verbally in equivalent only if, sufficient, necessary, contrapositive and unless form: "if we had an FTL drive, then we could visit the stars".

   Answer:

- **Only if:** "We could visit the stars only if we had an FTL drive."
- **Sufficient:** "Having an FTL drive is sufficient for visiting the stars."
- **Necessary:** "Visiting the stars requires an FTL drive."
- **Contrapositive:** "If we could not visit the stars, then we did not have an FTL drive."
- **Unless:** "We cannot visit the stars unless we have an FTL drive."

7. Write the formal negation of $\forall x \, \exists y \, (x > y)$. Your negation must not contain any explicit negation symbols.

   Answer: $\exists x \, \forall y \, (x \leq y)$.

8. Use logical equivalences to simplify $(p \to q) \to (\neg p \to \neg q)$ until you have at most one occurrence of each variable p; q remaining. Identify all logical equivalences by name.

   Answer: The expression $(p \to q) \to (\neg p \to \neg q)$ simplifies to:

   $$p \vee \neg q.$$

   Steps:

   (a) **Material Implication:** Rewrite the inner implications:

   $$p \to q \equiv \neg p \vee q \quad \text{and} \quad \neg p \to \neg q \equiv p \vee \neg q.$$

   (b) Substitute into the original expression:

   $$(p \to q) \to (\neg p \to \neg q) \equiv (\neg p \vee q) \to (p \vee \neg q).$$

   (c) **Implication Equivalence for the Outer Conditional:** Rewrite the outer implication:

   $$(\neg p \vee q) \to (p \vee \neg q) \equiv \neg(\neg p \vee q) \vee (p \vee \neg q).$$

   (d) **De Morgan's Law:** Apply it to the negated part:

   $$\neg(\neg p \vee q) \equiv p \wedge \neg q.$$

   (e) The expression now becomes:

   $$(p \wedge \neg q) \vee (p \vee \neg q).$$

   (f) **Absorption Law:** Use it to simplify the expression:

   $$(p \wedge \neg q) \vee (p \vee \neg q) \equiv p \vee \neg q.$$

9. **(a) Answer:** Each bit a_k can be reconstructed by computing:

$$a_k = b_k \text{ XOR } c_k.$$

- **Case 1** : If $b_k = 0$ and $c_k = 0$, then a_k must be 0 because $0 \text{ XOR } 0 = 0$.
- **Case 2** : If $b_k = 0$ and $c_k = 1$, then a_k must be 1 because $0 \text{ XOR } 1 = 1$.
- **Case 3** : If $b_k = 1$ and $c_k = 0$, then a_k must be 1 because $1 \text{ XOR } 0 = 1$.
- **Case 4** : If $b_k = 1$ and $c_k = 1$, then a_k must be 0 because $1 \text{ XOR } 1 = 0$.

In each case, the value of a_k is uniquely determined by b_k and c_k.

(b) Answer: No, the reconstruction property does not hold if $c_k = a_k$ AND b_k. When $b_k = 0$, regardless of a_k (whether 0 or 1), the result of a_k AND b_k is 0. Therefore, c_k would always be 0 in this case, and we cannot distinguish whether a_k was 0 or 1.

(c) Answer: No, the reconstruction property does not hold if $c_k = a_k$ OR b_k. When $b_k = 1$, regardless of a_k (whether 0 or 1), the result of a_k OR b_k is 1. Therefore, c_k is always 1 in this case, and a_k cannot be uniquely determined.

10. Is the statement $\exists x\, \forall y\, (xy = 0)$ true or false? The domain of discourse is the set of real numbers.

    Answer: The statement $\exists x\, \forall y\, (xy = 0)$ is true. Choosing $x = 0$ gives $0 \cdot y = 0$ for every real number y.

11. If P and Q are predicates over some domain, and if it is true that $\forall x\, (P(x) \vee Q(x))$, must $\forall x\, P(x) \vee \forall x\, Q(x)$ also be true?

    Answer: No; $\forall x\, (P(x) \vee Q(x))$ only means that for each x, either $P(x)$ or $Q(x)$ is true, but it does not imply that either $P(x)$ is true for all x or $Q(x)$ is true for all x.

12. Translate the formal expression $\forall x\, \exists y\, \exists z\, \big(y \neq z \wedge P(x, y) \wedge P(x, z)\big)$ into English. Do not use symbols such as x, y, and z in your translation.

    Answer: Every person has at least two distinct friends.

13. Let P be defined as in the previous problem. Is $\forall x\, \exists y\, \exists z \big(y \neq z \;\; \rightarrow P(x, y) \wedge P(x, z)\big)$ true or false?

    Answer: The statement $\forall x\, \exists y\, \exists z \left(y \neq z \;\rightarrow\; (P(x, y) \wedge P(x, z))\right)$ is true. By choosing y and z to be the same individual, condition $y \neq z$ is false, making the implication true regardless of any friendship relationship.

14. **(Extra Credit)** Write a Python program that prints a complete truth table for $(a \rightarrow b) \rightarrow (c \rightarrow d)$ and $(a \rightarrow (b \rightarrow c)) \rightarrow d$