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Detector AM Response Vectorization

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1 ComputeDetAMResponse

The output of this function are the components of what will become a complex number, F_+ and $F_\times$. These are long sums of different terms that, based on the current code, look like this:

$$
\begin{aligned}
F_+ = &[R_{0,0}X_0 + R_{0,1}X_1 + R_{0,2}X_2]X_0 - [R_{0,0}Y_0 + R_{0,1}Y_1 + R_{0,2}Y_2]Y_0 && (1.1)\\
&+ [R_{1,0}X_0 + R_{1,1}X_1 + R_{1,2}X_2]X_1 - [R_{1,0}Y_0 + R_{1,1}Y_1 + R_{1,2}Y_2]Y_1 && (1.2)\\
&+ [R_{2,0}X_0 + R_{2,1}X_1 + R_{2,2}X_2]X_2 - [R_{2,0}Y_0 + R_{2,1}Y_1 + R_{2,2}Y_2]Y_2 && (1.3)
\end{aligned}
$$

Collecting all the positive terms of this expression gives

$$
\begin{aligned}
F_+ = &R_{0,0}X_0X_0 + R_{0,1}X_0X_1 + R_{0,2}X_0X_2 && (1.4)\\
&+ R_{0,0}X_0X_0 + R_{0,1}X_0X_1 + R_{0,2}X_0X_2 && (1.5)\\
&+ R_{0,0}X_0X_0 + R_{0,1}X_0X_1 + R_{0,2}X_0X_2 && (1.6)
\end{aligned}
$$

At which point we see that the same result is computable as an outer product of the vector $\vec{X}$ using

$$F_+ = \vec{X}\mathbf{R}\vec{X} - \vec{Y}\mathbf{R}\vec{Y} \tag{1.7}$$

And the complex part of the gravitational wave is thus

$$F_\times = \vec{X}\mathbf{R}\vec{Y} + \vec{Y}\mathbf{R}\vec{X} \tag{1.8}$$

One a sample-to-sample basis, the numbers that vary are the components of the vectors $\vec{X}$ and $\vec{Y}$. Thus we need a function that takes as input a *list* vectors (or *vector*) of vectors and produces a vector with the right components as output. To that end we define the tensor X^i_j where

$$X^i = \begin{bmatrix} X^i_0 \\ X^i_1 \\ X^i_2 \end{bmatrix} \tag{1.9}$$

As well as the tensor R^i_{jk} where

$$R^i = \begin{bmatrix} R^i_{00} & R^i_{01} & R^i_{02} \\ R^i_{10} & R^i_{11} & R^i_{12} \\ R^i_{20} & R^i_{21} & R^i_{22} \end{bmatrix} \tag{1.10}$$

The tensor X is like a stack of all the different possible $\vec{X}$ coming out of the page. The Tensor R is like n copies of the matrix $\mathbf{R}$ stacked on top of each other. In this way the desired vector is obtainable with the tensor contraction

$$F^i_+ = X^{lm} R^i_{lj} X^j_m - Y^{lm} R^i_{lj} Y^j_m \tag{1.11}$$

$$F^i_\times = X^{lm} R^i_{lj} Y^j_m + Y^{lm} R^i_{lj} X^j_m \tag{1.12}$$

$$\tag{1.13}$$