Northwestern University

Fourier Transforms, U and V Matrices

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The fourier transform $\hat{h}(\omega)$ of a real or complex function of time $h(t)$ is defined as

$$\hat{h}(\omega) \equiv \int_{-\infty}^{\infty} h(t)e^{-i\omega t}dt \tag{0.1}$$

For purely real $h(t)$ the following relation holds. Note that the complex conjugation and reversal operations commute:

$$[\hat{h}(\omega)]^* = \int_{-\infty}^{\infty} h(t)e^{i\omega t}dt \tag{0.2}$$

$$\rightarrow [\hat{h}(-\omega)]^* = \int_{-\infty}^{\infty} h(t)e^{-i\omega t}dt \tag{0.3}$$

$$\rightarrow [\hat{h}(-\omega)]^* = \hat{h}(\omega) \tag{0.4}$$

In the case of a complex function of time $h(t) \equiv h_+(t) - ih_\times(t)$ the same sequence of operations yields

$$\hat{h}(\omega) = \int_{-\infty}^{\infty} [h_+(t) - ih_\times(t)]e^{-i\omega t}dt \tag{0.5}$$

$$\rightarrow [\hat{h}(\omega)]^* = \int_{-\infty}^{\infty} [h_+(t) + ih_\times(t)]e^{i\omega t}dt \tag{0.6}$$

$$\rightarrow [\hat{h}(-\omega)]^* = \int_{-\infty}^{\infty} [h_+(t) + ih_\times(t)]e^{-i\omega t}dt \tag{0.7}$$

$$\rightarrow [\hat{h}(-\omega)]^* = \widehat{h^*}(\omega) \tag{0.8}$$

The right hand side of the last formula is the *fourier transform of the complex conugate of* $h(t)$. The "Complex conjugate in time" operation is defined as

$$\mathcal{I}\hat{h}(\omega) = \int_{-\infty}^{\infty} h^*(t)e^{-i\omega t}dt \tag{0.9}$$

This is also the fourier transform of the couplex conjugate of the time domain function $h(t)$, and therefore equivalent, for any complex function, to:

$$\mathcal{I}\hat{h}(\omega) = [\hat{h}(-\omega)]^* \tag{0.10}$$

This is true for any arbitrary function that can be written $h(t) = h_+(t) + ih_\times(t)$.