

# Homework - Reinforcement Learning - Part B (60/100 points)

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Computational Cognitive Modeling

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This homework is due before midnight on March 7, 2024.

## Learning and deciding in an unknown world

Part A of the homework explored various solution to a sequential decision making problem in a case where considerable information about the environment was known or was provided (e.g., the probabilities of transitioning between different states and the magnitude of rewards available in particular states). However, if reinforcement learning could only be applied to cases with full, explicit knowledge than it would be much less of a compelling model of human learning. In many cases, humans and other animals learn even when there is much more ambiguity, and as a result, a good model of experiential learning for humans would apply to cases where less is known a-priori about the world.

 No description has been provided for this image

In this part of the homework, we will shift to think about learning and deciding in an unknown environment. This is a fairly complex topic with many different solutions and types of problems. However, we will focus on one particular problem class known as the **n-armed bandit**. N-armed bandits are optimization problems that mimic many real-world problems faced by humans, organizations, and machine learning agents. The term "bandit" comes from the name of the casino games where you pull a lever to enter a lottery. The bandits have one arm (the arm you pull down) and they steal your money (see above).

 No description has been provided for this image

An N-armed bandit is a problem where a decision maker is presented with a bandit with  $n$  arms instead of just one (see Octopus cartoon). The task for the agent is, on each trial or moment in time, to choose bandits that are good while avoiding those that are less good. Since nothing may be known about the bandits a-priori, the problem is difficult and requires a balance of *exploration* (trying new things in order to learn) and *exploitation* (choosing options known to be good).

If each bandit paid out a fixed amount every time it was selected, then the problem would be solved with very simple exhaustive search process (visit each bandit once and then select the best one for the remaining time). However, the sequential search strategy just described doesn't capture the *opportunity cost* of exploration. For example, imagine that there is 100 armed bandits. Further assume that you know that 98 give zero reward, one gives a reward of 10, and one gives a reward of 20. If on the first pull you receive 10 units of reward then you are lucky and landed on a good one. However, is it worth going searching for the 20 point bandit? Given that you will have to pull a lot of zero reward bandits, it might actually be more rewarding over a finite period to continue to pull the 10 point bandit arm. Thus, exploration and exploitation act more like a tradeoff depending on the structure of the problem.

In addition, when the reward received from each bandit is probabilistic or stochastic, and furthermore the quality of the bandits might change over time, the problem becomes much more difficult. These cases require the agent to learn from the past but also be willing to adjust their beliefs based on more recent information.

Bandit tasks come up in many areas of cognitive science and machine learning. For example, there is a way to view A/B testing on websites as a **particular type of bandit problem** (your goal is to ensure conversions or purchases on your website, and your bandit arms are the different web designs you might try out). Similarly, the very real human problem of deciding where to eat lunch is a bit like a bandit problem -- should you return to your favorite restaurant or try a new one? Is the exploration worth giving up a reliably good meal?

In this part of the homework you will explore different simple algorithms for bandit problems.

### Starter code

**Warning!** Before running other cells in this notebook you must first successfully execute the following cell which includes some libraries.

```
In [2]: # The typical imports
import numpy as np
import pandas as pd
%matplotlib inline
import matplotlib.pyplot as plt
import random
import math
import seaborn as sns

/tmp/ipykernel_96/2890879120.py:3: DeprecationWarning:
Pyarrow will become a required dependency of pandas in the next major release of pandas (pandas 3.0),
(to allow more performant data types, such as the Arrow string type, and better interoperability with other libraries)
but was not found to be installed on your system.
If this would cause problems for you,
please provide us feedback at https://github.com/pandas-dev/pandas/issues/54466

    import pandas as pd
```

## A simple bandit environment

The first class provided here creates a set of simple, stationary multi-arm bandits. The bandits are stateless in that the reward from choosing each action is simply a probabilistic function of the bandit itself, and there are no other cues you can use to decide which action to take. The parameters to the constructor of the bandit environment are:

- `mus` : the mean of the distribution from which each bandit is drawn from. This should have  $k$  numbers (for the  $k$ -armed bandit)
- `sds` : the standard deviation of the distribution from which the bandit means are drawn from (also  $k$  numbers for  $k$  bandits)

```
In [2]: class KArmBanditStationary:
    def __init__(self, mus, sds):
        self.action_means = mus
        self.action_sds = sds
        self.optimal = np.argmax(self.action_means)
        self.k = len(mus)

    def step(self, action):
        return (
            np.random.normal(self.action_means[action], self.action_sds[action]),
            action == self.optimal,
        )
```

Your job in this first exercise is to write a simple RL agent which samples from these bandits and attempts to earn as much reward as possible. The following cell gives an example of how to initialize the bandit and how to draw from it

```
In [3]: bandit = KArmBanditStationary([0, 50, 10, 4], [10, 10, 10, 10])
action = 0 # pull the 0th bandit
bandit.step(
    action
) # return values are the reward, and if the action is actually optimal or not
```

Out[3]: (9.125781902999293, False)

```
In [24]: bandit = KArmBanditStationary([0, 50, 10, 4], [10, 10, 10, 10])
action = 1 # pull the 1st bandit to ensure True is returned
bandit.step(
    action
) # return values are the reward, and if the action is actually optimal or not
```

Out[24]: (66.503062606926, True)

When we initialize the `KArmBanditStationary` we in some sense know exactly which arm is optimal (the one with the higher mean), and also how hard the problem is (the standard deviation of the rewards on each arm determines the difficulty... low SD generally is an easier task due to signal-noise relationship).

However, we are going to be implementing agents that do not have access to this information. For example, this first cell implements a simple random agent. You will want to modify this class to create an agent that can learn.

```
In [5]: class RandomAgent:
    def __init__(self, k):
        self.num_actions = k
        # you could add parameters to your agent here
        pass

    def choose(self):
```

```

    return np.random.randint(self.num_actions)

def learn(self, reward, action): # this agent doesn't learn
    pass

```

This cell helps you plot the reward history including a smoothed average reward earned by the agents over the last 30 trials

```

In [6]: def plot_results(results_df, window=25):
    # set up figure
    palette = iter(sns.color_palette("Set2"))
    fig = plt.figure(constrained_layout=True, figsize=(18, 5))
    gs = fig.add_gridspec(2, 5)

    # add three axes
    rew_ax = fig.add_subplot(gs[0, :-2])
    opt_ax = fig.add_subplot(gs[1, :-2])
    runs_ax = fig.add_subplot(gs[:, -2:])

    #fig, (rew_ax, opt_ax) = plt.subplots(nrows=2, ncols=1, figsize=(18,8))
    smooth_df = results_df.groupby("run").rolling(window, on="timepoint").mean()
    sns.lineplot(
        x="timepoint",
        y="reward_history",
        data=smooth_df,
        ax=rew_ax,
        label="reward",
        color=next(palette),
    )
    rew_ax.legend(loc="upper right")

    sns.lineplot(
        x="timepoint",
        y="opt",
        data=smooth_df,
        ax=opt_ax,
        label="p(optimal)",
        color=next(palette),
    )
    opt_ax.set_ylim(0, 1)
    opt_ax.legend(loc="upper right")

    sns.distplot(
        results_df.groupby("run")["opt"].mean(), ax=runs_ax, color=next(palette)
    )
    runs_ax.set_title("proportion optimal choices across runs")
    runs_ax.set_xlim(0, 1)

```

Finally, this is an example of the random agent's performance in the environment.

```

In [7]: np.random.seed(100) # fix a seed for repeatable experiments

# parameters of simulation
n_timesteps = 300
n_runs = 500

# set up bandit options
n_bandits = 4
bandit_means = [0, 50, 10, 4]
bandit_sds = [1] * n_bandits

reward_history = []
opt_history = []
run_history = []
timestep = []

for run in range(n_runs): # repeat a given number random repetitions of the experiment
    agent = RandomAgent(n_bandits)
    np.random.shuffle(bandit_means) # randomize location of "best"
    bandit = KArmBanditStationary(bandit_means, bandit_sds) # create a new bandit
    for i in range(n_timesteps): # for a certain number of time steps
        choice = agent.choose() # let the agent choose
        reward, opt = bandit.step(choice)
        reward_history.append(reward)
        opt_history.append(opt)
        run_history.append(run)
        timestep.append(i)

# plot the results
sim_result_df = pd.DataFrame(

```

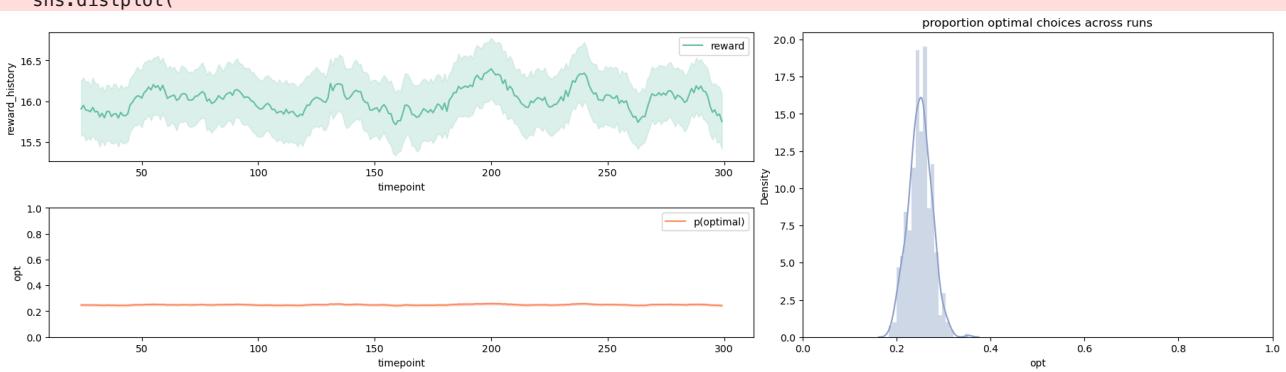
```

    }
    "run": run_history,
    "timepoint": timestep,
    "reward_history": reward_history,
    "opt": opt_history,
}
)
plot_results(sim_result_df)

/tmp/ipykernel_94/2681617309.py:35: UserWarning:
`distplot` is a deprecated function and will be removed in seaborn v0.14.0.
Please adapt your code to use either `displot` (a figure-level function with
similar flexibility) or `histplot` (an axes-level function for histograms).

For a guide to updating your code to use the new functions, please see
https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751

```



Here the plot shows the average reward earned for each trial fo the task across the repeated runs/experiments. The lower panel shows the proportion of optimal choices made by the agent across trials. The error bars on both of these plots are bootstrapped confidence intervals across runs of the agent. Finally, the right panel shows a histogram of the proportion of optimal choices made across the runs.

Everything looks as expected here. The random agent makes about the average reward of the task and chooses the optimal choice about 25% of the time.

Can you use what we have learned in lecture to do better?

### Problem 4 (15 points)

Create a new agent class based on `RandomAgent` called `EpsilonGreedyAgentIncremental()` which keeps track of the average reward earned from each draw of the bandit. This agent will include a parameter `epsilon` which will determine the probability of choosing a random action, otherwise it should choose the best so far. To update the value of each bandit use the incremental implementation of mean-tracking which was first introduced on the slides blending monte carlo methods (this is a incremental approach to calculating the mean as a new observation arrive). Make a plot similar to the ones above showing the performance of the agent on the three measures we have considered when the environment is initialized in the same way (i.e., means are a random shuffling of [0,.50,.10,.4.] with sd=1). Next, show with a couple examples how changes in epsilon determine the shape of that plot. You should show the final code for your agent and the plots along with a single markdown cell describing your solution (1-2 paragraphs). In your answer be sure to explain if your new agent does better than the random agent and why you think that is the case. In all cases run your agent for 300 time steps and average over 500 runs.

```
In [35]: class EpsilonGreedyAgentIncremental():
    def __init__(self, k, epsilon):
        self.num_actions = k
        self.epsilon = epsilon
        self.action_values = np.zeros(k)
        self.action_counts = np.zeros(k)

    def choose(self):
        # Explore by choosing random action with probability epsilon
        if np.random.rand() < self.epsilon:
            return np.random.randint(self.num_actions)
        # Exploit with probability 1 - epsilon
        else:
            return np.argmax(self.action_values)
```

```

def learn(self, reward, action):
    self.action_counts[action] += 1
    n = self.action_counts[action]
    old_value = self.action_values[action]
    new_value = old_value + (1/n) * (reward - old_value) # Incremental implementation formula taken from the s
    self.action_values[action] = new_value

```

Utilizing an epsilon value of 0.2

```

In [36]: np.random.seed(100) # fix a seed for repeatable experiments

# parameters of simulation
n_timesteps = 300
n_runs = 500

# set up bandit options
n_bandits = 4
bandit_means = [0, 50, 10, 4]
bandit_sds = [1] * n_bandits

reward_history = []
opt_history = []
run_history = []
timestep = []

for run in range(n_runs): # repeat a given number random repetitions of the experiment
    agent = EpsilonGreedyAgentIncremental(n_bandits, 0.2) # Epsilon value of 0.2
    np.random.shuffle(bandit_means) # randomize location of "best"
    bandit = KArmBanditStationary(bandit_means, bandit_sds) # create a new bandit
    for i in range(n_timesteps): # for a certain number of time steps
        choice = agent.choose() # let the agent choose
        reward, opt = bandit.step(choice)
        agent.learn(reward, choice)

        reward_history.append(reward)
        opt_history.append(opt)
        run_history.append(run)
        timestep.append(i)

# plot the results
sim_result_df = pd.DataFrame(
    {
        "run": run_history,
        "timepoint": timestep,
        "reward_history": reward_history,
        "opt": opt_history,
    }
)
plot_results(sim_result_df)

```

```

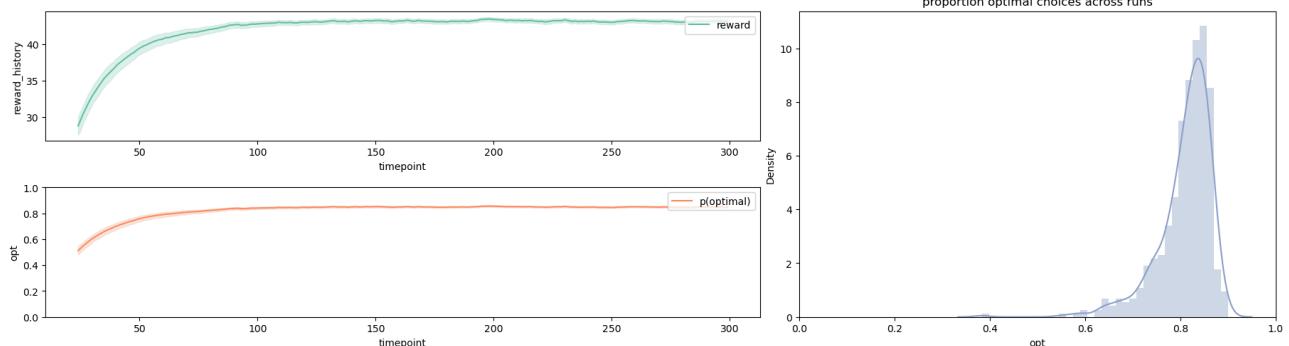
/tmp/ipykernel_94/2681617309.py:35: UserWarning:
`distplot` is a deprecated function and will be removed in seaborn v0.14.0.

```

Please adapt your code to use either `displot` (a figure-level function with similar flexibility) or `histplot` (an axes-level function for histograms).

For a guide to updating your code to use the new functions, please see  
<https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751>

```
sns.distplot(
```



Utilizing an epsilon value of 0.6

```
In [45]: np.random.seed(100) # fix a seed for repeatable experiments

# parameters of simulation
n_timesteps = 300
n_runs = 500

# set up bandit options
n_bandits = 4
bandit_means = [0, 50, 10, 4]
bandit_sds = [1] * n_bandits

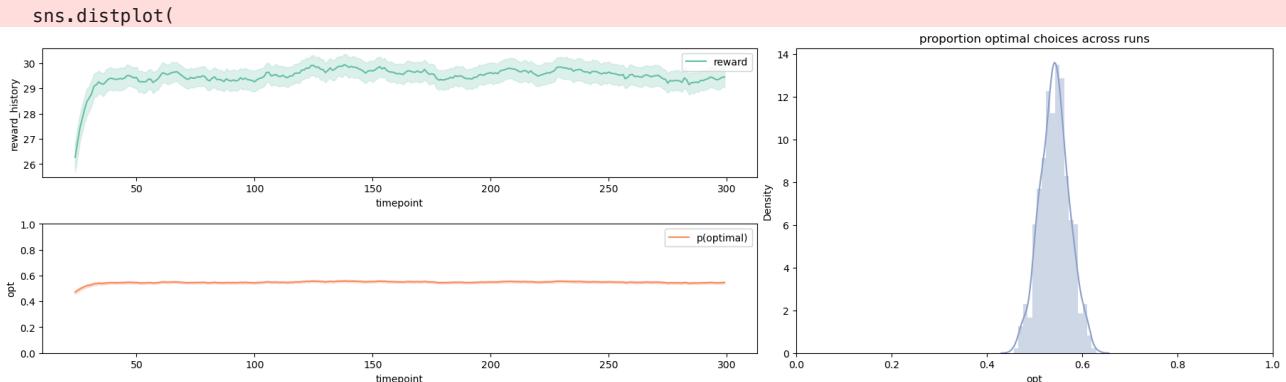
reward_history = []
opt_history = []
run_history = []
timestep = []

for run in range(n_runs): # repeat a given number random repetitions of the experiment
    agent = EpsilonGreedyAgentIncremental(n_bandits, 0.6) # Epsilon value of 0.6
    np.random.shuffle(bandit_means) # randomize location of "best"
    bandit = KArmBanditStationary(bandit_means, bandit_sds) # create a new bandit
    for i in range(n_timesteps): # for a certain number of time steps
        choice = agent.choose() # let the agent choose
        reward, opt = bandit.step(choice)
        agent.learn(reward, choice)

        reward_history.append(reward)
        opt_history.append(opt)
        run_history.append(run)
        timestep.append(i)

# plot the results
sim_result_df = pd.DataFrame(
{
    "run": run_history,
    "timepoint": timestep,
    "reward_history": reward_history,
    "opt": opt_history,
}
)
plot_results(sim_result_df)
```

/tmp/ipykernel\_94/2681617309.py:35: UserWarning:  
`distplot` is a deprecated function and will be removed in seaborn v0.14.0.  
Please adapt your code to use either `displot` (a figure-level function with similar flexibility) or `histplot` (an axes-level function for histograms).  
For a guide to updating your code to use the new functions, please see  
<https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751>



Utilizing an epsilon value of 0.01

```
In [40]: np.random.seed(100) # fix a seed for repeatable experiments

# parameters of simulation
n_timesteps = 300
n_runs = 500

# set up bandit options
n_bandits = 4
bandit_means = [0, 50, 10, 4]
```

```

bandit_sds = [1] * n_bandits

reward_history = []
opt_history = []
run_history = []
timestep = []

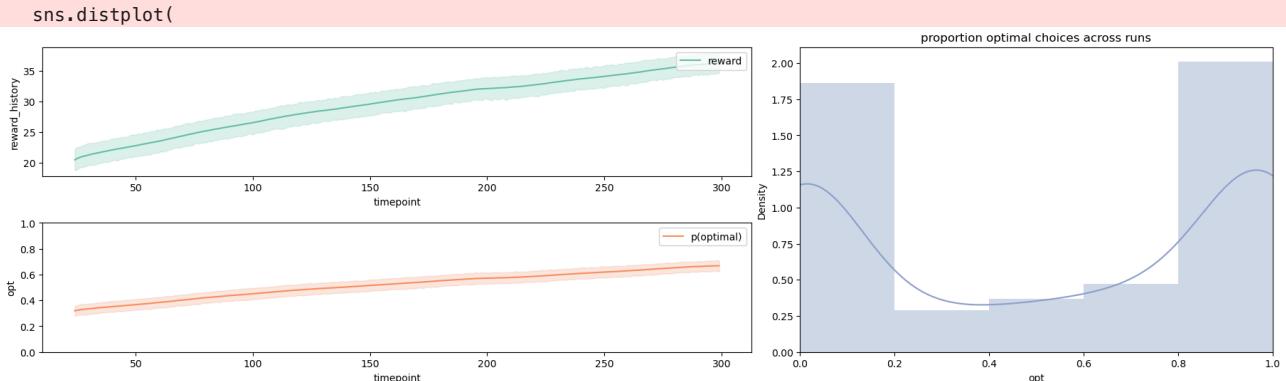
for run in range(n_runs): # repeat a given number random repetitions of the experiment
    agent = EpsilonGreedyAgentIncremental(n_bandits, 0.01) # Epsilon value of 0.01
    np.random.shuffle(bandit_means) # randomize location of "best"
    bandit = KArmBanditStationary(bandit_means, bandit_sds) # create a new bandit
    for i in range(n_timesteps): # for a certain number of time steps
        choice = agent.choose() # let the agent choose
        reward, opt = bandit.step(choice)
        agent.learn(reward, choice)

        reward_history.append(reward)
        opt_history.append(opt)
        run_history.append(run)
        timestep.append(i)

# plot the results
sim_result_df = pd.DataFrame(
{
    "run": run_history,
    "timepoint": timestep,
    "reward_history": reward_history,
    "opt": opt_history,
}
)
plot_results(sim_result_df)

```

/tmp/ipykernel\_94/2681617309.py:35: UserWarning:  
`distplot` is a deprecated function and will be removed in seaborn v0.14.0.  
Please adapt your code to use either `displot` (a figure-level function with similar flexibility) or `histplot` (an axes-level function for histograms).  
For a guide to updating your code to use the new functions, please see  
<https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751>



Utilizing an epsilon value of 0.1

```

In [44]: np.random.seed(100) # fix a seed for repeatable experiments

# parameters of simulation
n_timesteps = 300
n_runs = 500

# set up bandit options
n_bandits = 4
bandit_means = [0, 50, 10, 4]
bandit_sds = [1] * n_bandits

reward_history = []
opt_history = []
run_history = []
timestep = []

for run in range(n_runs): # repeat a given number random repetitions of the experiment
    agent = EpsilonGreedyAgentIncremental(n_bandits, 0.1) # Epsilon value of 0.1
    np.random.shuffle(bandit_means) # randomize location of "best"

```

```

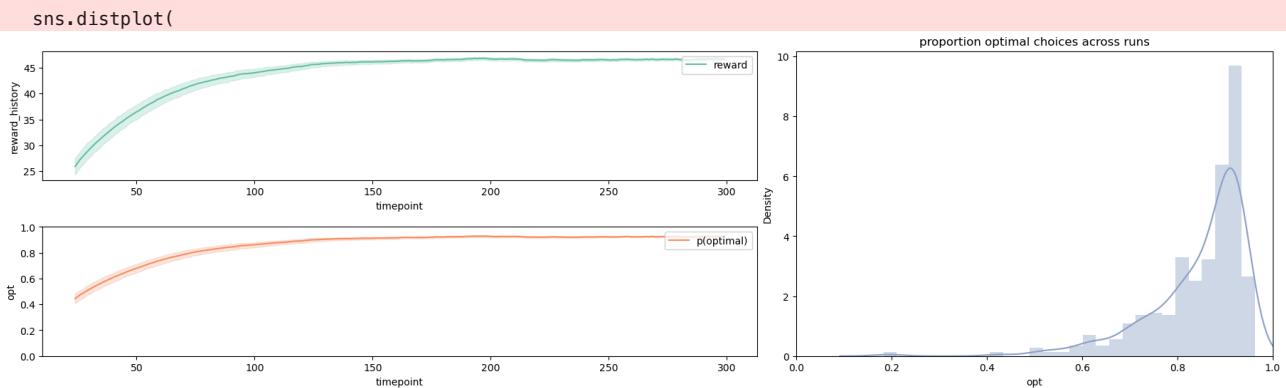
bandit = KArmBanditStationary(bandit_means, bandit_sds) # create a new bandit
for i in range(n_timesteps): # for a certain number of time steps
    choice = agent.choose() # let the agent choose
    reward, opt = bandit.step(choice)
    agent.learn(reward, choice)

    reward_history.append(reward)
    opt_history.append(opt)
    run_history.append(run)
    timestep.append(i)

# plot the results
sim_result_df = pd.DataFrame(
{
    "run": run_history,
    "timepoint": timestep,
    "reward_history": reward_history,
    "opt": opt_history,
}
)
plot_results(sim_result_df)

```

/tmp/ipykernel\_94/2681617309.py:35: UserWarning:  
`distplot` is a deprecated function and will be removed in seaborn v0.14.0.  
Please adapt your code to use either `displot` (a figure-level function with similar flexibility) or `histplot` (an axes-level function for histograms).  
For a guide to updating your code to use the new functions, please see <https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751>



Grid search to ensure epsilon of 0.1 is the best option:

```

In [54]: # Initialize random seed and simulation parameters
np.random.seed(100)
n_timesteps = 300
n_runs = 500
n_bandits = 4
bandit_means = [0, 50, 10, 4]
bandit_sds = [1] * n_bandits

# Grid search parameters
epsilons = np.arange(0.05, 1.0, 0.05)

# Dictionary to store the average reward for each epsilon
average_rewards = {}

# Start the grid search
for epsilon in epsilons:
    total_reward = 0

    for run in range(n_runs): # repeat for a given number of random repetitions of the experiment
        # Initialize the agent with the current epsilon value
        agent = EpsilonGreedyAgentIncremental(n_bandits, epsilon)
        np.random.shuffle(bandit_means) # randomize location of "best"
        bandit = KArmBanditStationary(bandit_means, bandit_sds) # create a new bandit

        for i in range(n_timesteps): # for a certain number of time steps
            choice = agent.choose() # let the agent choose
            reward, _ = bandit.step(choice)
            agent.learn(reward, choice)
            total_reward += reward
    average_rewards[epsilon] = total_reward / n_runs

```

```

# Calculate the average reward for this epsilon value
average_reward = total_reward / (n_runs * n_timesteps)
average_rewards[epsilon] = average_reward

# Find the epsilon value with the highest average reward
best_epsilon = max(average_rewards, key=average_rewards.get)
best_average_reward = average_rewards[best_epsilon]

print(f"Best epsilon: {best_epsilon}")
print(f"Best average reward: {best_average_reward}")

```

Best epsilon: 0.1  
Best average reward: 43.305251564507856

With incremental learning, we are able to recursively update / track V(s) incrementally for each action over each iteration as opposed to maintaining a list or table of rewards that we cannot access until we visit the states hundreds or thousands of times. This allows us to better understand the environment after each iteration, and is far more efficient in relation to memory.

With epsilon set to 0.2 and 0.1, we can see that there is a good balance between exploration and exploitation, as the histogram of optimal choices exhibits a peak around 0.85 when epsilon = 0.2 and 0.92 when epsilon = 0.1. Further, the reward history plots demonstrate that these agents exhibit consistent rewards after around 100 timesteps for epsilon = 0.2, and 150 timesteps when epsilon = 0.1 (though, there is greater reward when epsilon = 0.1).

When we set epsilon to 0.6 to allow for more exploration, however, the reward history seems to be lower overall than when epsilon is set to 0.2, as the agent is choosing the optimal arm less frequently and is thus gaining rewards from suboptimal arms more often. We also see this in the histogram plot, as the peak is now around 0.55 instead of 0.85. Though, this is not to say that more exploration always results in lower reward. In this example, the environment is stationary, and thus we know exactly which arm is the optimal choice. In non-stationary environments where the optimal choice changes over time or when the agent has not yet learned the values of each action, higher epsilon values may result in greater rewards.

With epsilon set to 0.01, we see a bimodal distribution in the histogram: one significant peak at the extreme left of the histogram, and the other peak at the extreme right. This is rather surprising, as there seems to be a high number of runs where the agent infrequently chose the optimal bandit, though, given the low epsilon value, it may be that the agent converged to what it thought was the optimal choice without exploring for long enough to realize that there were other choices of higher value. The next peak that is observed is very close to 1 on the x-axis, indicating that in many runs, the agent does indeed select the optimal bandit a very high proportion of the time. This was more expected with the very low epsilon value, since the agent is expected to identify the optimal choice very quickly, and then exploit that choice. The two peaks at the extreme ends exemplify that while a very low epsilon value can lead to exploitation of the best-known action, there's a chance that it may converge on a suboptimal choice since it is not engaging in much exploration.

This agent definitely outperforms the random agent, as we can gain the optimal reward up to 92% of the time when epsilon = 0.1 as opposed to a mere 25% with the random agent. Moving forward, we will utilize epsilon = 0.1, as I confirmed that the agent performs best with that epsilon value through the grid search.

### Problem 5 (15 points)

Create a new agent class based on `RandomAgent` called `EpsilonGreedyAgentConstant()` which keeps track of the average reward earned from each draw of the bandit. This agent will be nearly identical to `EpsilonGreedyIncremental()`. However, in addition to the parameter `epsilon` which will determine the probability of choosing a random action, this agent should use the "constant step size" update rule related to temporal-different learning to update the value of each action. The step size parameter ('alpha') should be added as a new input parameter to your agent (hint: small values of this parameter are often better). Make a plot similar to the ones above showing the performance of the agent on the three measures we have considered. Is the performance of this agent the same or different than the previous agent you coded for this environment? Try a few parameter combinations with your agent and in your response show 1-2 examples to help make your point. You should show the final code for your agent and the plots along with a single markdown cell describing your solution (1-2 paragraphs). Be sure that your answer includes the answer to this key question: Does the constant agent out perform the incremental agent? And does it do better than the random agent? You don't need to do statistics but just a general visual comparison of the quality of the algorithm is enough. In all cases run your agent for 300 time steps and average over 500 runs.

In [42]:

```

class EpsilonGreedyAgentConstant():
    def __init__(self, k, alpha, epsilon):
        self.num_actions = k
        self.alpha = alpha # remember that small values of this parameter are often better
        self.epsilon = epsilon
        self.action_values = np.zeros(k)

```

```

def choose(self):
    # Explore by choosing random action with probability epsilon
    if np.random.rand() < self.epsilon:
        return np.random.randint(self.num_actions)
    # Exploit with probability 1 - epsilon
    else:
        return np.argmax(self.action_values)

def learn(self, reward, action):
    old_value = self.action_values[action]
    new_value = old_value + self.alpha * (reward - old_value) # Constant Step Size implementation formula take
    self.action_values[action] = new_value

```

Utilizing an alpha value of 0.2 and an epsilon value of 0.1

```
In [64]: np.random.seed(100) # fix a seed for repeatable experiments

# parameters of simulation
n_timesteps = 300
n_runs = 500

# set up bandit options
n_bandits = 4
bandit_means = [0, 50, 10, 4]
bandit_sds = [1] * n_bandits

reward_history = []
opt_history = []
run_history = []
timestep = []

for run in range(n_runs): # repeat a given number random repetitions of the experiment
    agent = EpsilonGreedyAgentConstant(n_bandits, 0.2, 0.1) # Alpha value of 0.2; Epsilon value of 0.1
    np.random.shuffle(bandit_means) # randomize location of "best"
    bandit = KArmBanditStationary(bandit_means, bandit_sds) # create a new bandit
    for i in range(n_timesteps): # for a certain number of time steps
        choice = agent.choose() # let the agent choose
        reward, opt = bandit.step(choice)
        agent.learn(reward, choice)

        reward_history.append(reward)
        opt_history.append(opt)
        run_history.append(run)
        timestep.append(i)

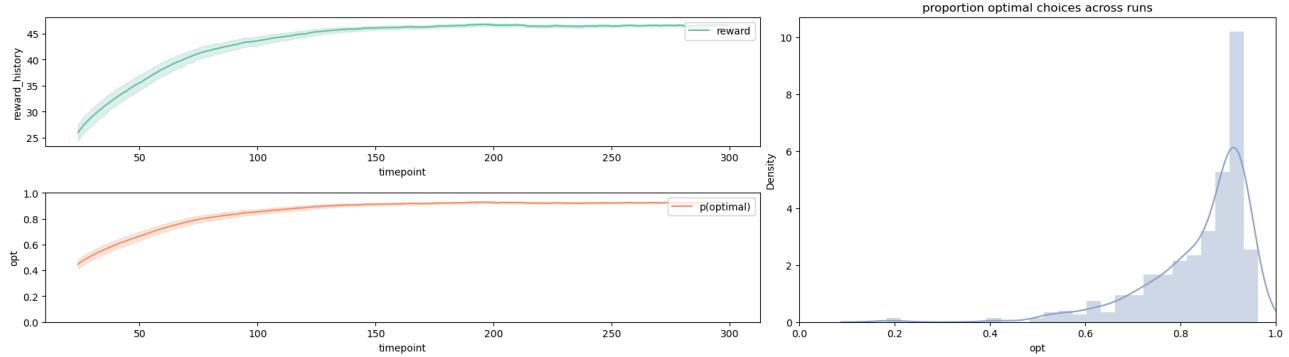
# plot the results
sim_result_df = pd.DataFrame(
{
    "run": run_history,
    "timepoint": timestep,
    "reward_history": reward_history,
    "opt": opt_history,
}
)
plot_results(sim_result_df)
```

```
/tmp/ipykernel_94/2681617309.py:35: UserWarning:
`distplot` is a deprecated function and will be removed in seaborn v0.14.0.

Please adapt your code to use either `displot` (a figure-level function with
similar flexibility) or `histplot` (an axes-level function for histograms).

For a guide to updating your code to use the new functions, please see
https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751

sns.distplot()
```



Utilizing an alpha value of 0.5 and an epsilon value of 0.1

```
In [67]: np.random.seed(100) # fix a seed for repeatable experiments

# parameters of simulation
n_timesteps = 300
n_runs = 500

# set up bandit options
n_bandits = 4
bandit_means = [0, 50, 10, 4]
bandit_sds = [1] * n_bandits

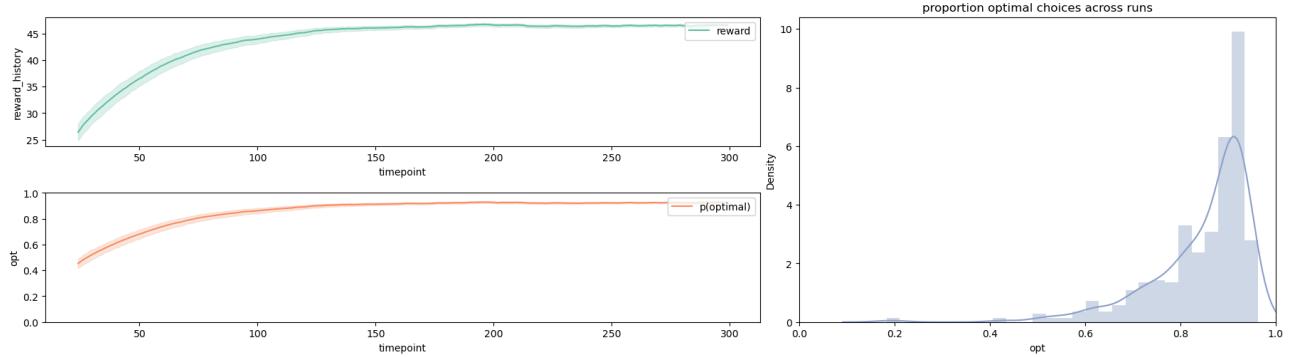
reward_history = []
opt_history = []
run_history = []
timestep = []

for run in range(n_runs): # repeat a given number random repetitions of the experiment
    agent = EpsilonGreedyAgentConstant(n_bandits, 0.5, 0.1) # Alpha value of 0.2; Epsilon value of 0.1
    np.random.shuffle(bandit_means) # randomize location of "best"
    bandit = KArmBanditStationary(bandit_means, bandit_sds) # create a new bandit
    for i in range(n_timesteps): # for a certain number of time steps
        choice = agent.choose() # let the agent choose
        reward, opt = bandit.step(choice)
        agent.learn(reward, choice)

        reward_history.append(reward)
        opt_history.append(opt)
        run_history.append(run)
        timestep.append(i)

# plot the results
sim_result_df = pd.DataFrame(
    {
        "run": run_history,
        "timepoint": timestep,
        "reward_history": reward_history,
        "opt": opt_history,
    }
)
plot_results(sim_result_df)
```

```
/tmp/ipykernel_94/2681617309.py:35: UserWarning:
`distplot` is a deprecated function and will be removed in seaborn v0.14.0.
Please adapt your code to use either `displot` (a figure-level function with
similar flexibility) or `histplot` (an axes-level function for histograms).
For a guide to updating your code to use the new functions, please see
https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751
sns.distplot()
```



Utilizing an alpha value of 0.1 and an epsilon value of 0.1

```
In [60]: np.random.seed(100) # fix a seed for repeatable experiments

# parameters of simulation
n_timesteps = 300
n_runs = 500

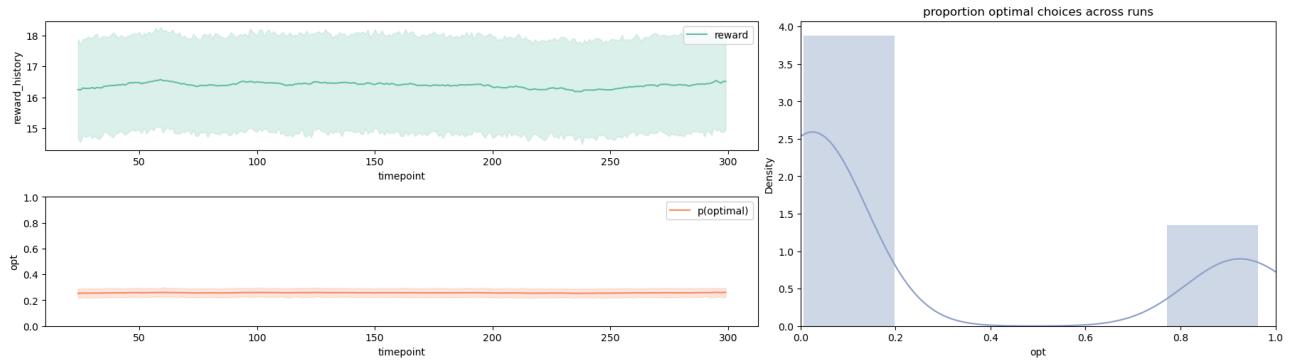
# set up bandit options
n_bandits = 4
bandit_means = [0, 50, 10, 4]
bandit_sds = [1] * n_bandits

reward_history = []
opt_history = []
run_history = []
timestep = []

for run in range(n_runs): # repeat a given number random repetitions of the experiment
    agent = EpsilonGreedyAgentConstant(n_bandits, 0.1, 0.1) # Alpha value of 0.1; Epsilon value of 0.1
    np.random.shuffle(bandit_means) # randomize location of "best"
    bandit = KArmBanditStationary(bandit_means, bandit_sds) # create a new bandit
    for i in range(n_timesteps): # for a certain number of time steps
        choice = agent.choose() # let the agent choose
        reward, opt = bandit.step(choice)
        reward_history.append(reward)
        opt_history.append(opt)
        run_history.append(run)
        timestep.append(i)

# plot the results
sim_result_df = pd.DataFrame(
    {
        "run": run_history,
        "timepoint": timestep,
        "reward_history": reward_history,
        "opt": opt_history,
    }
)
plot_results(sim_result_df)
```

```
/tmp/ipykernel_94/2681617309.py:35: UserWarning:
'distplot' is a deprecated function and will be removed in seaborn v0.14.0.
Please adapt your code to use either `displot` (a figure-level function with
similar flexibility) or `histplot` (an axes-level function for histograms).
For a guide to updating your code to use the new functions, please see
https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751
sns.distplot()
```



Utilizing an alpha value of 0.01 and an epsilon value of 0.1

```
In [61]: np.random.seed(100) # fix a seed for repeatable experiments

# parameters of simulation
n_timesteps = 300
n_runs = 500

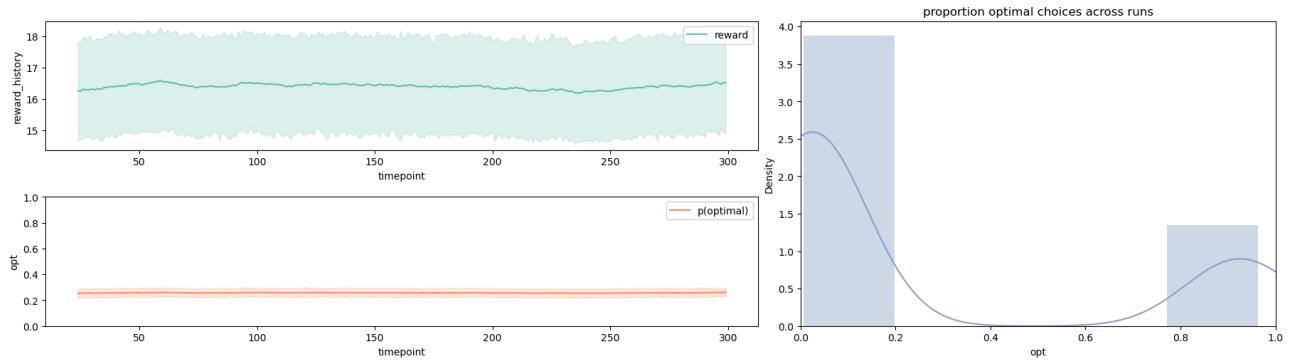
# set up bandit options
n_bandits = 4
bandit_means = [0, 50, 10, 4]
bandit_sds = [1] * n_bandits

reward_history = []
opt_history = []
run_history = []
timestep = []

for run in range(n_runs): # repeat a given number random repetitions of the experiment
    agent = EpsilonGreedyAgentConstant(n_bandits, 0.01, 0.1) # Alpha value of 0.01; Epsilon value of 0.1
    np.random.shuffle(bandit_means) # randomize location of "best"
    bandit = KArmBanditStationary(bandit_means, bandit_sds) # create a new bandit
    for i in range(n_timesteps): # for a certain number of time steps
        choice = agent.choose() # let the agent choose
        reward, opt = bandit.step(choice)
        reward_history.append(reward)
        opt_history.append(opt)
        run_history.append(run)
        timestep.append(i)

# plot the results
sim_result_df = pd.DataFrame(
    {
        "run": run_history,
        "timepoint": timestep,
        "reward_history": reward_history,
        "opt": opt_history,
    }
)
plot_results(sim_result_df)
```

```
/tmp/ipykernel_94/2681617309.py:35: UserWarning:
'distplot' is a deprecated function and will be removed in seaborn v0.14.0.
Please adapt your code to use either `displot` (a figure-level function with
similar flexibility) or `histplot` (an axes-level function for histograms).
For a guide to updating your code to use the new functions, please see
https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751
sns.distplot()
```



Utilizing an alpha value of 0.9 and an epsilon value of 0.1

```
In [62]: np.random.seed(100) # fix a seed for repeatable experiments

# parameters of simulation
n_timesteps = 300
n_runs = 500

# set up bandit options
n_bandits = 4
bandit_means = [0, 50, 10, 4]
bandit_sds = [1] * n_bandits

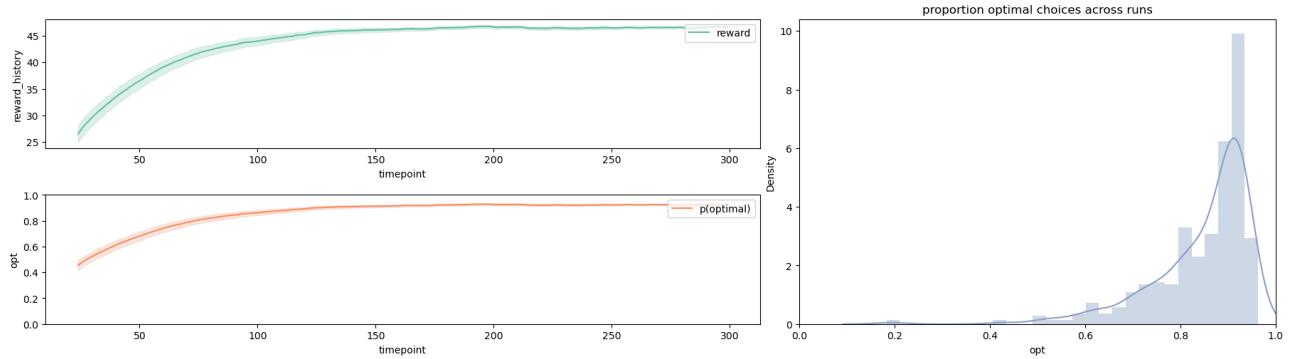
reward_history = []
opt_history = []
run_history = []
timestep = []

for run in range(n_runs): # repeat a given number random repetitions of the experiment
    agent = EpsilonGreedyAgentConstant(n_bandits, 0.9, 0.1) # Alpha value of 0.9; Epsilon value of 0.1
    np.random.shuffle(bandit_means) # randomize location of "best"
    bandit = KArmBanditStationary(bandit_means, bandit_sds) # create a new bandit
    for i in range(n_timesteps): # for a certain number of time steps
        choice = agent.choose() # let the agent choose
        reward, opt = bandit.step(choice)
        agent.learn(reward, choice)

        reward_history.append(reward)
        opt_history.append(opt)
        run_history.append(run)
        timestep.append(i)

# plot the results
sim_result_df = pd.DataFrame(
    {
        "run": run_history,
        "timepoint": timestep,
        "reward_history": reward_history,
        "opt": opt_history,
    }
)
plot_results(sim_result_df)
```

```
/tmp/ipykernel_94/2681617309.py:35: UserWarning:
`distplot` is a deprecated function and will be removed in seaborn v0.14.0.
Please adapt your code to use either `displot` (a figure-level function with
similar flexibility) or `histplot` (an axes-level function for histograms).
For a guide to updating your code to use the new functions, please see
https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751
sns.distplot()
```



```
In [55]: alphas = [0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95]
epsilons = [0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95]

best_average_reward = -np.inf
best_alpha = None
best_epsilon = None

for alpha in alphas:
    for epsilon in epsilons:
        np.random.seed(100) # fix a seed for repeatable experiments

        # parameters of simulation
        n_timesteps = 300
        n_runs = 500

        # Initialize variables to store simulation results
        average_reward_per_run = []

        for run in range(n_runs):
            # Initialize the bandit and the agent for this run
            agent = EpsilonGreedyAgentConstant(n_bandits, alpha, epsilon)
            bandit = KArmBanditStationary(bandit_means, bandit_sds)

            total_reward = 0

            for t in range(n_timesteps):
                choice = agent.choose()
                reward, _ = bandit.step(choice)
                agent.learn(reward, choice)
                total_reward += reward

            average_reward_per_run.append(total_reward / n_timesteps)

        # Compute the overall average reward for this (alpha, epsilon) combination
        average_reward = np.mean(average_reward_per_run)

        # Update best parameters if this combination outperforms the others
        if average_reward > best_average_reward:
            best_average_reward = average_reward
            best_alpha = alpha
            best_epsilon = epsilon

# After the grid search
print(f"Best average reward: {best_average_reward}")
print(f"Best alpha: {best_alpha}")
print(f"Best epsilon: {best_epsilon}")
```

Best average reward: 41.73060029732312  
 Best alpha: 0.75  
 Best epsilon: 0.1

```
In [63]: np.random.seed(100) # fix a seed for repeatable experiments

# parameters of simulation
n_timesteps = 300
n_runs = 500

# set up bandit options
n_bandits = 4
bandit_means = [0, 50, 10, 4]
bandit_sds = [1] * n_bandits

reward_history = []
opt_history = []
```

```

run_history = []
timestep = []

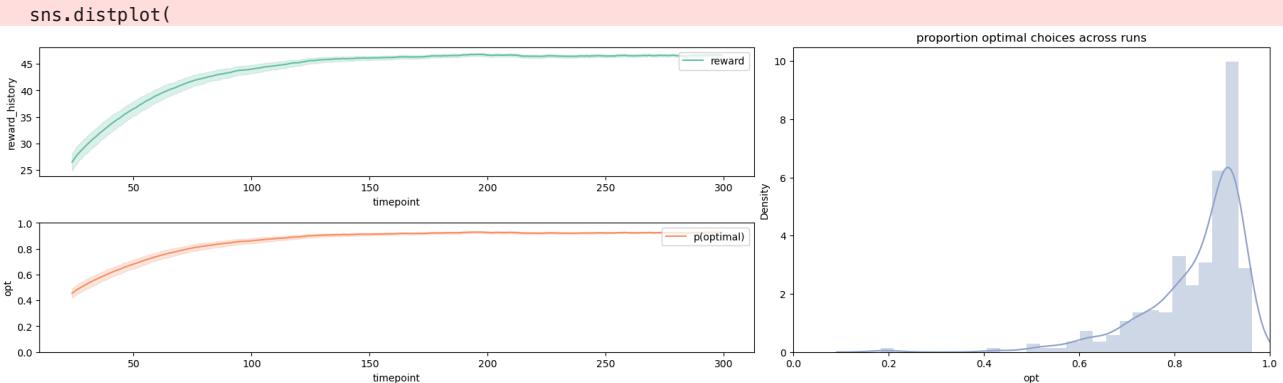
for run in range(n_runs): # repeat a given number random repetitions of the experiment
    agent = EpsilonGreedyAgentConstant(n_bandits, 0.75, 0.1) # Alpha value of 0.9; Epsilon value of 0.1
    np.random.shuffle(bandit_means) # randomize location of "best"
    bandit = KArmBanditStationary(bandit_means, bandit_sds) # create a new bandit
    for i in range(n_timesteps): # for a certain number of time steps
        choice = agent.choose() # let the agent choose
        reward, opt = bandit.step(choice)
        agent.learn(reward, choice)

        reward_history.append(reward)
        opt_history.append(opt)
        run_history.append(run)
        timestep.append(i)

# plot the results
sim_result_df = pd.DataFrame(
    {
        "run": run_history,
        "timepoint": timestep,
        "reward_history": reward_history,
        "opt": opt_history,
    }
)
plot_results(sim_result_df)

```

/tmp/ipykernel\_94/2681617309.py:35: UserWarning:  
`distplot` is a deprecated function and will be removed in seaborn v0.14.0.  
Please adapt your code to use either `displot` (a figure-level function with similar flexibility) or `histplot` (an axes-level function for histograms).  
For a guide to updating your code to use the new functions, please see  
<https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751>



Is the performance of this agent the same or different than the previous agent you coded for this environment? Try a few parameter combinations with your agent and in your response show 1-2 examples to help make your point. You should show the final code for your agent and the plots along with a single markdown cell describing your solution (1-2 paragraphs). Be sure that your answer includes the answer to this key question: Does the constant agent out perform the incremental agent? And does it do better than the random agent?

The performance of this constant step-size agent seems to parallel that of the incremental agent (when maintaining the same epsilon value), so it does not outperform the incremental agent. Based on the various outputs, it seems as though the EpsilonGreedyAgentConstant's performance is fairly insensitive to any changes in alpha aside from very low alpha values ( $\alpha \leq 0.2$ ), which produce a bimodal distribution in the histogram that is similar to when we utilize very low epsilon values with the incremental agent. Thus, given the right conditions, once the optimal bandit is identified in a stationary environment, the agent continues to exploit it regardless of whether we are using an incremental or constant agent. For example, in my simulations, I tested a wide range of alpha values (0.2, 0.5, 0.1, 0.01, 0.9, and 0.75) all while keeping epsilon constant at 0.1, and all three of the plots looked nearly identical across all three measures in question when alpha is greater than or equal to 0.2. Like the incremental agent, we see that the constant agent demonstrates superior performance when compared with the random agent, as the random agent is unable to converge and exploit the most rewarding actions.

### Problem 6 (15 points)

Below is a new type of bandit environment based on `KArmBanditStationary` class where the reward probabilities of each bandit change over time. This is sometimes known as a "restless bandit" (see the Daw et al. 2013 paper on explore exploit mentioned in Lecture). The idea is that on each time step the mean reward of each action should be modified up or down by a sample from a Gaussian distribution (e.g.,  $\mu_{t+1} = \mu_t + \mathcal{N}(0, 20)$ ). I have called the new class `KArmBanditRestless`. The mean of the arms is itself drawn initially from a random normal distribution as well. Using this environment (with the number of arms set to 4), test the `RandomAgent()`, `EpsilonGreedyAgentConstant()` and `EpsilonGreedyAgentIncremental()` agents. You may want to play with the alpha parameter of the incremental agent to see if you can find a particularly good setting. Show the final code for your agent, plots showing the average reward the agent earns over time, along with a markdown cell describing your solution in 1-2 paragraphs. Which agent performs better in this environment? Is this different than the conclusion you made from the previous environment? Be sure to answer these two questions in your response. In all cases run your agent for 300 time steps and average over 500 runs.

```
In [73]: class KArmBanditRestless:
    def __init__(self, k, mu=0, sigma=2, sd=2, walk_sd=30):
        self.k = k
        self.action_means = np.random.normal(mu, sigma, k)
        self.action_sds = sd
        self.walk_sd = walk_sd
        self.optimal = np.argmax(self.action_means)

    def step(self, action):
        out = (
            np.random.normal(self.action_means[action], self.action_sds),
            action == self.optimal,
        )
        self.action_means = self.action_means + np.random.normal(
            0.0, self.walk_sd, self.k
        )
        self.optimal = np.argmax(self.action_means)
        return out
```

```
In [74]: # initialize this bandit like this
n_bandits = 4
bandit = KArmBanditRestless(n_bandits)
```

Random Agent

```
In [75]: np.random.seed(100) # fix a seed for repeatable experiments

# parameters of simulation
n_timesteps = 300
n_runs = 500

# set up bandit options
n_bandits = 4

reward_history = []
opt_history = []
run_history = []
timestep = []

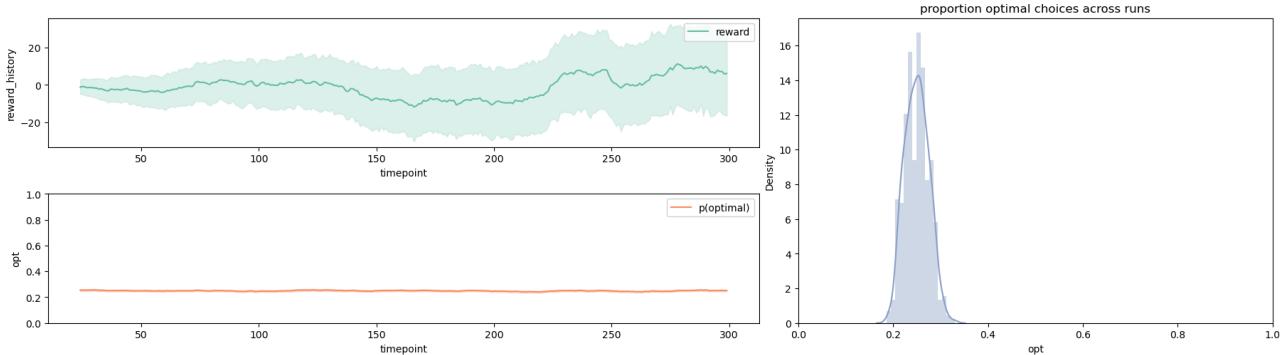
for run in range(n_runs): # repeat a given number random repetitions of the experiment
    agent = RandomAgent(n_bandits)
    bandit = KArmBanditRestless(n_bandits) # create a new Restless bandit
    for i in range(n_timesteps): # for a certain number of time steps
        choice = agent.choose() # let the agent choose
        reward, opt = bandit.step(choice)
        reward_history.append(reward)
        opt_history.append(opt)
        run_history.append(run)
        timestep.append(i)

# plot the results
sim_result_df = pd.DataFrame(
{
    "run": run_history,
    "timepoint": timestep,
    "reward_history": reward_history,
    "opt": opt_history,
}
)
plot_results(sim_result_df)
```

```
/tmp/ipykernel_94/2681617309.py:35: UserWarning:
`distplot` is a deprecated function and will be removed in seaborn v0.14.0.
Please adapt your code to use either `displot` (a figure-level function with
similar flexibility) or `histplot` (an axes-level function for histograms).

For a guide to updating your code to use the new functions, please see
https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751
```

```
sns.distplot(
```



Incremental

```
In [76]: np.random.seed(100) # fix a seed for repeatable experiments
```

```
# parameters of simulation
n_timesteps = 300
n_runs = 500

# set up bandit options
n_bandits = 4

reward_history = []
opt_history = []
run_history = []
timestep = []

for run in range(n_runs): # repeat a given number random repetitions of the experiment
    agent = EpsilonGreedyAgentIncremental(n_bandits, 0.1) # Epsilon value of 0.1
    bandit = KArmBanditRestless(n_bandits) # create a new Restless bandit
    for i in range(n_timesteps): # for a certain number of time steps
        choice = agent.choose() # let the agent choose
        reward, opt = bandit.step(choice)
        agent.learn(reward, choice)

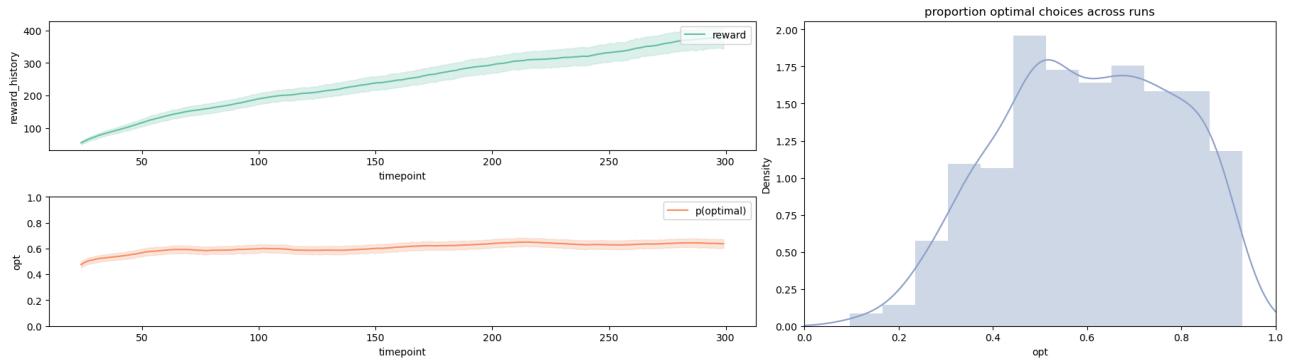
        reward_history.append(reward)
        opt_history.append(opt)
        run_history.append(run)
        timestep.append(i)

# plot the results
sim_result_df = pd.DataFrame(
    {
        "run": run_history,
        "timepoint": timestep,
        "reward_history": reward_history,
        "opt": opt_history,
    }
)
plot_results(sim_result_df)
```

```
/tmp/ipykernel_94/2681617309.py:35: UserWarning:
`distplot` is a deprecated function and will be removed in seaborn v0.14.0.
Please adapt your code to use either `displot` (a figure-level function with
similar flexibility) or `histplot` (an axes-level function for histograms).

For a guide to updating your code to use the new functions, please see
https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751
```

```
sns.distplot(
```



Grid search to identify best epsilon value for the restless bandit:

```
In [77]: # Initialize random seed and simulation parameters
np.random.seed(100)
n_timesteps = 300
n_runs = 500
n_bandits = 4
bandit_means = [0, 50, 10, 4]
bandit_sds = [1] * n_bandits

# Grid search parameters
epsilons = np.arange(0.05, 1.0, 0.05)

# Dictionary to store the average reward for each epsilon
average_rewards = {}

# Start the grid search
for epsilon in epsilons:
    total_reward = 0

    for run in range(n_runs): # repeat for a given number of random repetitions of the experiment
        # Initialize the agent with the current epsilon value
        agent = EpsilonGreedyAgentIncremental(n_bandits, epsilon)
        bandit = KArmBanditRestless(n_bandits) # create a new bandit

        for i in range(n_timesteps): # for a certain number of time steps
            choice = agent.choose() # let the agent choose
            reward, _ = bandit.step(choice)
            agent.learn(reward, choice)
            total_reward += reward

    # Calculate the average reward for this epsilon value
    average_reward = total_reward / (n_runs * n_timesteps)
    average_rewards[epsilon] = average_reward

# Find the epsilon value with the highest average reward
best_epsilon = max(average_rewards, key=average_rewards.get)
best_average_reward = average_rewards[best_epsilon]

print(f"Best epsilon: {best_epsilon}")
print(f"Best average reward: {best_average_reward}")
```

Best epsilon: 0.1

Best average reward: 257.1304204289822

Constant

```
In [78]: np.random.seed(100) # fix a seed for repeatable experiments

# parameters of simulation
n_timesteps = 300
n_runs = 500

# set up bandit options
n_bandits = 4

reward_history = []
opt_history = []
run_history = []
timestep = []

for run in range(n_runs): # repeat a given number random repetitions of the experiment
    agent = EpsilonGreedyAgentConstant(n_bandits, 0.75, 0.1) # Alpha value of 0.75; Epsilon value of 0.1
    bandit = KArmBanditRestless(n_bandits) # create a new Restless bandit
```

```

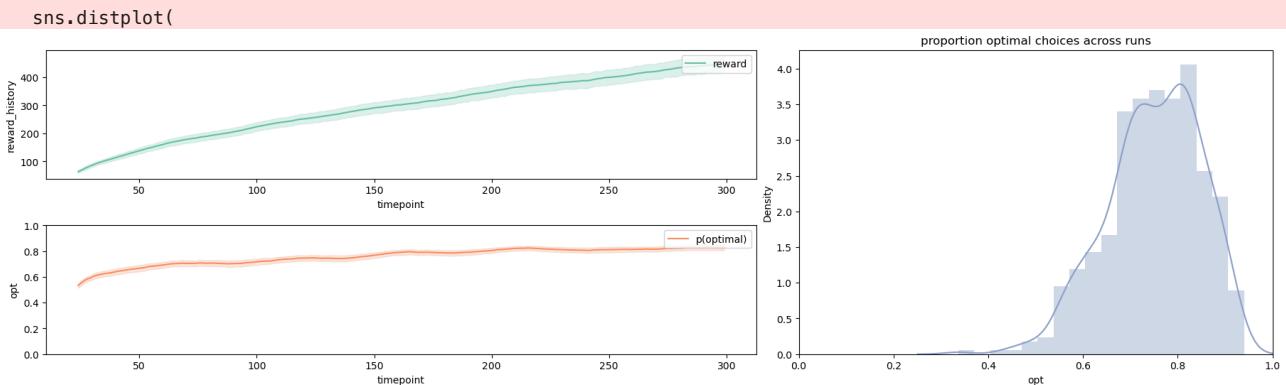
for i in range(n_timesteps): # for a certain number of time steps
    choice = agent.choose() # let the agent choose
    reward, opt = bandit.step(choice)
    agent.learn(reward, choice)

    reward_history.append(reward)
    opt_history.append(opt)
    run_history.append(run)
    timestep.append(i)

# plot the results
sim_result_df = pd.DataFrame(
{
    "run": run_history,
    "timepoint": timestep,
    "reward_history": reward_history,
    "opt": opt_history,
}
)
plot_results(sim_result_df)

```

/tmp/ipykernel\_94/2681617309.py:35: UserWarning:  
`distplot` is a deprecated function and will be removed in seaborn v0.14.0.  
Please adapt your code to use either `displot` (a figure-level function with similar flexibility) or `histplot` (an axes-level function for histograms).  
For a guide to updating your code to use the new functions, please see  
<https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751>



Grid search to identify best alpha and epsilon values for the restless bandit:

```

In [79]: alphas = [0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95]
epsilons = [0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95]

best_average_reward = -np.inf
best_alpha = None
best_epsilon = None

for alpha in alphas:
    for epsilon in epsilons:
        np.random.seed(100) # fix a seed for repeatable experiments

        # parameters of simulation
        n_timesteps = 300
        n_runs = 500

        # Initialize variables to store simulation results
        average_reward_per_run = []

        for run in range(n_runs):
            # Initialize the bandit and the agent for this run
            agent = EpsilonGreedyAgentConstant(n_bandits, alpha, epsilon)
            bandit = KArmBanditRestless(n_bandits)

            total_reward = 0

            for t in range(n_timesteps):
                choice = agent.choose()
                reward, _ = bandit.step(choice)
                agent.learn(reward, choice)
                total_reward += reward

            average_reward_per_run.append(total_reward/n_timesteps)

        if np.mean(average_reward_per_run) > best_average_reward:
            best_alpha = alpha
            best_epsilon = epsilon
            best_average_reward = np.mean(average_reward_per_run)

```

```

        average_reward_per_run.append(total_reward / n_timesteps)

    # Compute the overall average reward for this (alpha, epsilon) combination
    average_reward = np.mean(average_reward_per_run)

    # Update best parameters if this combination outperforms the others
    if average_reward > best_average_reward:
        best_average_reward = average_reward
        best_alpha = alpha
        best_epsilon = epsilon

    # After the grid search
print(f"Best average reward: {best_average_reward}")
print(f"Best alpha: {best_alpha}")
print(f"Best epsilon: {best_epsilon}")

```

Best average reward: 297.7114533111353  
 Best alpha: 0.9  
 Best epsilon: 0.05

In [80]:

```

np.random.seed(100) # fix a seed for repeatable experiments

# parameters of simulation
n_timesteps = 300
n_runs = 500

# set up bandit options
n_bandits = 4

reward_history = []
opt_history = []
run_history = []
timestep = []

for run in range(n_runs): # repeat a given number random repetitions of the experiment
    agent = EpsilonGreedyAgentConstant(n_bandits, 0.9, 0.05) # Alpha value of 0.75; Epsilon value of 0.1
    bandit = KArmBanditRestless(n_bandits) # create a new Restless bandit
    for i in range(n_timesteps): # for a certain number of time steps
        choice = agent.choose() # let the agent choose
        reward, opt = bandit.step(choice)
        agent.learn(reward, choice)

        reward_history.append(reward)
        opt_history.append(opt)
        run_history.append(run)
        timestep.append(i)

# plot the results
sim_result_df = pd.DataFrame(
    {
        "run": run_history,
        "timepoint": timestep,
        "reward_history": reward_history,
        "opt": opt_history,
    }
)
plot_results(sim_result_df)

```

```

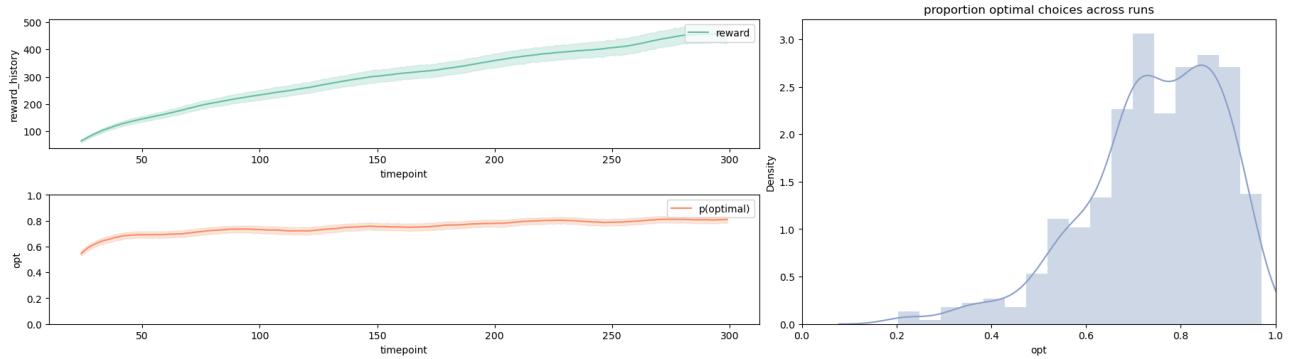
/tmp/ipykernel_94/2681617309.py:35: UserWarning:
`distplot` is a deprecated function and will be removed in seaborn v0.14.0.

Please adapt your code to use either `displot` (a figure-level function with
similar flexibility) or `histplot` (an axes-level function for histograms).

For a guide to updating your code to use the new functions, please see
https://gist.github.com/mwaskom/de44147ed2974457ad6372750bbe5751

sns.distplot()

```



Which agent performs better in this environment? Is this different than the conclusion you made from the previous environment? Be sure to answer these two questions in your response. In all cases run your agent for 300 time steps and average over 500 runs.

Now, in this non-stationary environment where the probabilities of the bandits change over time, it is important to utilize an agent that is adaptive, or rather, an agent that can forget older information that is no longer relevant and weigh the recent rewards more heavily. This instantly made me think of the EpsilonGreedyAgentConstant agent, as I recalled Todd discussing how it can be advantageous to keep a constant step size in order to forget older information over time in non-stationary environments. As expected, both the incremental and constant agents outperform the random agent, given its inability to learn anything about the environment. After examining the constant and incremental agents, though, it is clear that the constant agent indeed outperforms the incremental agent, as it is able to adapt to the environment's fluctuations more quickly and shows a much higher distribution of optimal choices in the histogram. We also see this in the reward history plot, as the reward gained surpasses all other agents. In this non-stationary environment, the best performance of the constant agent was found when epsilon = 0.05 and when alpha = 0.9. While the incremental agent still outperformed the random agent, its inability to adapt in the same way as the constant agent resulted in a lower distribution of optimal choices. These differences show the importance of tailoring your learning strategy to the dynamics of the environment in question.

### Problem 7 (15 points)

For this problem, we return to the grid world task we considered in Part A. Specifically, you should combine the ideas about explore-exploit and incremental learning of q-values to implement a temporal-difference solution to the grid world problem you explored in Part A of the homework. You can reuse the code from that notebook in building your solution. This solution should also obtain similar results to the policy-iteration and monte-carlo versions you explored, but is learned more incrementally and online.

The basic setup of the GridWorld environment is provided again for you below. Your solution to this problem should involve modifications to the solution to the Monte-Carlo problems in Part A. In particular, instead of waiting until a particular episode ends to update the values of the Q-values, use the Q-learning equation to incrementally updates these values as an episode unfolds. To balance exploration and exploitation try any of the methods you developed in the earlier parts of this assignment.

As a reminder the question for updating the Q values in Q-learning is as follows:

$$Q(s, a) = Q(s, a) + \alpha[r + \gamma \max_{a'} Q(s', a') - Q(s, a)]$$

The pseudo code for this algorithm is:

```

Initialize, for all $s$ in $S$, $a$ in $A(s)$:
    $Q(s,a)$ <- arbitrary

Repeat many times (for each episode):
    a) Initialize $ss$ at the start state
    b) Repeat
        1. Choose action $a$ from $ss$ using policy derived from Q values in that state (e.g., SoftMax)
        2. Take action $a$, observe $r$, $s'$
        3. Update $Q(s,a)$
            Find max $Q(s',a')$ over all action $a'$ in state $s'$
            $Q(s,a) = Q(s,a) + \alpha [r + \gamma \max Q(s',a') - Q(s,a)]$
            $ss <- s'$
    Until $ss$ is the goal state
  
```

```
In [3]: # import the gridworld library
import numpy as np
import random
import math
import statistics
from copy import deepcopy
from IPython.display import display, Markdown, Latex, HTML
from gridworld import GridWorld, random_policy

In [4]: gridworld = [
    ["o", "o", "o", "o", "o", "o", "x", "g"],
    ["o", "x", "x", "o", "x", "x", "o", "o"],
    ["s", "o", "o", "o", "o", "o", "o", "x"],
] # the problem described above, 'x' is a wall, 's' is start, 'g' is goal, and 'o' is a normal room

mygrid = GridWorld(gridworld)
mygrid.raw_print() # print out the grid world
mygrid.index_print() # print out the indicies of each state
mygrid.coord_print() # print out the coordinates of each state (helpful in your code)

# define the rewards as a hash table
rewards = {}

# mygrid.transitions contains all the pairwise state-state transitions allowed in the grid
# for each state transition initialize the reward to zero
for start_state in mygrid.transitions:
    for action in mygrid.transitions[start_state].keys():
        next_state = mygrid.transitions[start_state][action]
        rewards[str([start_state, action, next_state])] = 0.0

# now set the reward for moving up into state 8 (the goal state) to +10
rewards[str([17, "up", 8])] = 10

# now set the penalty for walking off the edge of the grid and returning to state 45 (the start state)
for i in [0, 1, 2, 3, 4, 5, 6, 7]:
    rewards[str([i, "up", 45])] = -1
for i in [0, 9, 18, 27, 36, 45]:
    rewards[str([i, "left", 45])] = -1
for i in [45, 46, 47, 48, 49, 50, 51, 52, 53]:
    rewards[str([i, "down", 45])] = -1
for i in [8, 17, 26, 35, 44, 53]:
    rewards[str([i, "right", 45])] = -1
```

## Welcome to your new Grid World!

### Raw World Layout

```

o o o o o o o x g
o x x o x x o x o
o x x o x x o x o
o x x o x x o o o
o x x o x x x o o
s o o o o o o o x

```

### Indexes of each grid location as an id number

0	1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16	17
18	19	20	21	22	23	24	25	26
27	28	29	30	31	32	33	34	35
36	37	38	39	40	41	42	43	44
45	46	47	48	49	50	51	52	53

### Indexes of each grid location as a tuple

(0,0)	(0,1)	(0,2)	(0,3)	(0,4)	(0,5)	(0,6)	(0,7)	(0,8)
(1,0)	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)	(1,8)
(2,0)	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)	(2,8)
(3,0)	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)	(3,7)	(3,8)
(4,0)	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)	(4,7)	(4,8)
(5,0)	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)	(5,7)	(5,8)

The following code sets up the major things you need to track. Note that unlike in the Monte Carlo solution you do not need a separate accounting of the returns as you are updating those to the Q-values directly.

Also you shouldn't need to update the policy table until you have run many episodes through the maze. The final update to the `policy_table` should just be to print out your final greedy solution and compare it to the solutions you obtained in Part A.

Write new functions and incorporate the ones we need from prior problems:

```
In [5]: # this is a helper function that will take a set of q-values and convert them into a greedy decision strategy
def be_greedy(q_values):
    if len(q_values) == 0:
        return {}

    keys = list(q_values.keys())
    vals = [q_values[i] for i in keys]
    maxqs = [i for i, x in enumerate(vals) if x == max(vals)]
    if len(maxqs) > 1:
        pos = random.choice(maxqs)
    else:
        pos = maxqs[0]
    policy = deepcopy(q_values)
    for i in policy.keys():
        policy[i] = 0.0
    policy[keys[pos]] = 1.0
    return policy

# Select action using epsilon-greedy strategy
def choose(state, q_values, epsilon):
    # Explore by choosing random action with probability epsilon
    if np.random.rand() < epsilon:
        return np.random.choice(list(q_values[state].keys()))
    # Exploit with probability 1 - epsilon
    else:
        return max(q_values[state], key=q_values[state].get)
```

Try with alpha = 0.9

```
In [6]: starting_state = 45
goal_state = 8 # terminate the MC roll out when you get to this state
GAMMA = 0.9
EPSILON = 0.2
ITERATIONS = 50000 # this may need to be 100,000 or more!
PRINT_EVERY = 1000 # how often to print out our progress
random.seed(5000) # try multiple random seed to verify your code works
alpha = 0.9

# set up initial data structures that might be useful for you
# q(s,a) - the q-values for each action in each state
def zero_q_values():
    qvals = {"up": 0.0, "right": 0.0, "down": 0.0, "left": 0.0}
    return qvals

q_value_table = [
    [zero_q_values() for i in range(mygrid.ncols)] for j in range(mygrid.nrows)
]

# pi - the policy table
policy_table = [
    [random_policy() for i in range(mygrid.ncols)] for j in range(mygrid.nrows)
]

display(Markdown("Initial (randomized) policy"))
mygrid.pretty_print_policy_table(policy_table)
```

```

# Initialize Q-values
q_values = {state: {action: 0.0 for action in mygrid.actions} for state in mygrid.valid_states.keys()}

# Q-learning algorithm with choose function
for episode in range(ITERATIONS):
    state = random.choice(list(mygrid.valid_states.keys()))
    while state != goal_state:
        action = choose(state, q_values, EPSILON)

        next_state = mygrid.transitions[state][action]

        reward_key = str([state, action, next_state])
        reward = rewards.get(reward_key, 0.0) # Default to 0.0 if not explicitly defined

        # Q-learning update
        best_future_action = max(q_values[next_state], key=q_values[next_state].get)
        td_target = reward + GAMMA * q_values[next_state][best_future_action]
        q_values[state][action] += alpha * (td_target - q_values[state][action])

        # Update state
        state = next_state

    # Update policy based on new Q-values
    for state in mygrid.valid_states.keys():
        state_x, state_y = mygrid.index_to_coord(state) # Assuming this method translates state index to grid coordinates
        policy_table[state_x][state_y] = be_greedy(q_values[state])

    # Displaying the policy periodically (assuming a function exists to visualize your policy table)
    if episode % PRINT_EVERY == 0:
        display(Markdown(f"**Policy after {episode} episodes**"))
        mygrid.pretty_print_policy_table(policy_table)

display(Markdown("**Improved policy**"))
mygrid.pretty_print_policy_table(policy_table)

```

**Initial (randomized) policy**

→	↓	←	←	↑	←	↑	█	←
↓	█	█	←	█	█	→	█	↓
←	█	█	↑	█	█	→	█	→
↑	█	█	↑	█	█	↓	←	←
←	█	█	→	█	█	█	↑	↓
↑	↓	↓	↑	←	←	↓	↓	█

**Policy after 0 episodes**

→	←	→	↓	↓	→	↓	█	↑
→	█	█	↓	█	█	→	█	↑
↑	█	█	←	█	█	←	█	↑
↓	█	█	↓	█	█	↑	↑	↓
↓	█	█	↑	█	█	█	→	→
↑	↑	→	↑	↓	→	→	→	█

**Policy after 1000 episodes**

→	→	→	→	→	→	↓	█	↑
↑	█	█	↑	█	█	↓	█	↑
↑	█	█	↑	█	█	↓	█	↑
↑	█	█	↓	█	█	→	→	↑
↑	█	█	↓	█	█	█	↑	↑
↑	←	←	→	→	→	→	↑	█

**Policy after 2000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 3000 episodes**

→	→	→	→	→	→	→	↓	█	→
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	→	↑	█

**Policy after 4000 episodes**

→	→	→	→	→	→	→	↓	█	→
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	→	↑	█

**Policy after 5000 episodes**

→	→	→	→	→	→	→	↓	█	↓
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	→	↑	█

**Policy after 6000 episodes**

→	→	→	→	→	→	→	↓	█	←
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	→	↑	█

**Policy after 7000 episodes**

→	→	→	→	→	→	→	↓	█	→
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	→	↑	█

**Policy after 8000 episodes**

→	→	→	→	→	→	↓	█	→	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 9000 episodes**

→	→	→	→	→	→	→	↓	█	→
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 10000 episodes**

→	→	→	→	→	→	→	↓	█	←
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 11000 episodes**

→	→	→	→	→	→	→	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 12000 episodes**

→	→	→	→	→	→	→	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 13000 episodes**

→	→	→	→	→	→	→	↓	█	←
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 14000 episodes**

→	→	→	→	→	→	↓	█	←	
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 15000 episodes**

→	→	→	→	→	→	→	↓	█	←
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 16000 episodes**

→	→	→	→	→	→	→	↓	█	←
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 17000 episodes**

→	→	→	→	→	→	→	↓	█	→
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 18000 episodes**

→	→	→	→	→	→	→	↓	█	↓
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 19000 episodes**

→	→	→	→	→	→	→	↓	█	←
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 20000 episodes**

→	→	→	→	→	→	↓	█	←	
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 21000 episodes**

→	→	→	→	→	→	→	↓	█	→
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 22000 episodes**

→	→	→	→	→	→	→	↓	█	↓
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 23000 episodes**

→	→	→	→	→	→	→	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 24000 episodes**

→	→	→	→	→	→	→	↓	█	→
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 25000 episodes**

→	→	→	→	→	→	→	↓	█	←
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 26000 episodes**

→	→	→	→	→	→	↓	█	←	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 27000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 28000 episodes**

→	→	→	→	→	→	↓	█	←	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 29000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 30000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 31000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 32000 episodes**

→	→	→	→	→	→	↓	█	↓	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 33000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 34000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 35000 episodes**

→	→	→	→	→	→	↓	█	←	
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 36000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 37000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 38000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 39000 episodes**

→	→	→	→	→	→	↓	█	↓	
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 40000 episodes**

→	→	→	→	→	→	↓	█	←	
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 41000 episodes**

→	→	→	→	→	→	↓	█	↓	
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 42000 episodes**

→	→	→	→	→	→	↓	█	←	
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 43000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 44000 episodes**

→	→	→	→	→	→	↓	█	→	
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 45000 episodes**

→	→	→	→	→	→	→	↓	█	←
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 46000 episodes**

→	→	→	→	→	→	→	↓	█	↓
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 47000 episodes**

→	→	→	→	→	→	→	↓	█	←
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

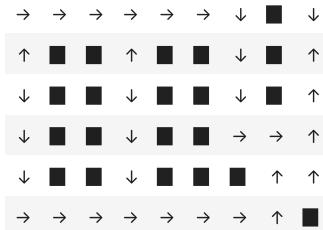
**Policy after 48000 episodes**

→	→	→	→	→	→	→	↓	█	↓
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 49000 episodes**

→	→	→	→	→	→	→	↓	█	→
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Improved policy**



```
In [94]: starting_state = 45
goal_state = 8 # terminate the MC roll out when you get to this state
GAMMA = 0.9
EPSILON = 0.2
ITERATIONS = 50000 # this may need to be 100,000 or more!
PRINT_EVERY = 1000 # how often to print out our progress
random.seed(5000) # try multiple random seed to verify your code works
alpha = 0.1

# set up initial data structures that might be useful for you
# q(s,a) - the q-values for each action in each state
def zero_q_values():
    qvals = {"up": 0.0, "right": 0.0, "down": 0.0, "left": 0.0}
    return qvals

q_value_table = [
    [zero_q_values() for i in range(mygrid.ncols)] for j in range(mygrid.nrows)
]

# pi - the policy table
policy_table = [
    [random_policy() for i in range(mygrid.ncols)] for j in range(mygrid.nrows)
]

display(Markdown(""))
mygrid.pretty_print_policy_table(policy_table)

# Initialize Q-values
q_values = {state: {action: 0.0 for action in mygrid.actions} for state in mygrid.valid_states.keys()}

# Q-learning algorithm with choose function
for episode in range(ITERATIONS):
    state = random.choice(list(mygrid.valid_states.keys()))
    while state != goal_state:
        action = choose(state, q_values, EPSILON)

        next_state = mygrid.transitions[state][action]

        reward_key = str([state, action, next_state])
        reward = rewards.get(reward_key, 0.0) # Default to 0.0 if not explicitly defined

        # Q-learning update
        best_future_action = max(q_values[next_state], key=q_values[next_state].get)
        td_target = reward + GAMMA * q_values[next_state][best_future_action]
        q_values[state][action] += alpha * (td_target - q_values[state][action])

        # Update state
        state = next_state

    # Update policy based on new Q-values
    for state in mygrid.valid_states.keys():
        state_x, state_y = mygrid.index_to_coord(state) # Assuming this method translates state index to grid coord
        policy_table[state_x][state_y] = be_greedy(q_values[state])

    # Displaying the policy periodically (assuming a function exists to visualize your policy table)
    if episode % PRINT_EVERY == 0:
        display(Markdown(""))
        mygrid.pretty_print_policy_table(policy_table)

display(Markdown(""))
mygrid.pretty_print_policy_table(policy_table)
```

**Initial (randomized) policy**

→	↓	←	←	↑	←	↑	█	←	
↓	█	█	█	←	█	█	→	█	↓
←	█	█	█	↑	█	█	→	█	→
↑	█	█	█	↑	█	█	↓	█	←
←	█	█	█	→	█	█	█	↑	↓
↑	↓	↓	↑	←	←	↓	↓	█	

**Policy after 0 episodes**

→	←	→	↓	↓	→	↓	█	↑	
→	█	█	█	↓	█	█	→	█	↑
↑	█	█	█	←	█	█	←	█	↑
↓	█	█	█	↓	█	█	↑	↑	↓
↓	█	█	█	↑	█	█	█	→	→
↑	↑	→	↑	→	→	←	→	█	

**Policy after 1000 episodes**

→	→	→	→	→	→	↓	█	←	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	→	→	↑
↑	█	█	█	↑	█	█	█	↑	↑
↑	↖	→	↑	↖	↖	↖	↖	↑	█

**Policy after 2000 episodes**

→	→	→	→	→	→	↓	█	↓	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	→	→	↑
↑	█	█	█	↑	█	█	█	↑	↑
↑	↖	→	↑	↖	↖	↖	↖	↑	█

**Policy after 3000 episodes**

→	→	→	→	→	→	↓	█	→	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	→	→	↑
↑	█	█	█	↑	█	█	█	↑	↑
↑	↖	→	↑	↖	↖	↖	↖	↑	█

**Policy after 4000 episodes**

→	→	→	→	→	→	↓	█	↓	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	→	→	↑
↑	█	█	█	↑	█	█	█	↑	↑
↑	→	→	↑	↖	↖	↖	↖	↑	█

**Policy after 5000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	→	→	↑
↑	█	█	█	↑	█	█	█	↑	↑
↑	→	→	↑	←	←	→	↑	█	

**Policy after 6000 episodes**

→	→	→	→	→	→	→	↓	█	↓
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	→	→	↑
↑	█	█	█	↑	█	█	█	↑	↑
↑	→	→	↑	←	←	→	↑	█	

**Policy after 7000 episodes**

→	→	→	→	→	→	→	↓	█	↓
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	→	→	↑
↑	█	█	█	↑	█	█	█	↑	↑
↑	→	→	↑	←	←	→	↑	█	

**Policy after 8000 episodes**

→	→	→	→	→	→	→	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	→	→	↑
↑	█	█	█	↑	█	█	█	↑	↑
↑	→	→	↑	←	←	→	↑	█	

**Policy after 9000 episodes**

→	→	→	→	→	→	→	↓	█	→
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	→	→	↑
↑	█	█	█	↑	█	█	█	↑	↑
↑	→	→	→	→	→	→	→	↑	█

**Policy after 10000 episodes**

→	→	→	→	→	→	→	↓	█	←
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	→	↑	█

**Policy after 11000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 12000 episodes**

→	→	→	→	→	→	↓	█	↓	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 13000 episodes**

→	→	→	→	→	→	↓	█	←	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↑	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 14000 episodes**

→	→	→	→	→	→	↓	█	→	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↑	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 15000 episodes**

→	→	→	→	→	→	↓	█	→	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 16000 episodes**

→	→	→	→	→	→	↓	█	→	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 17000 episodes**

→	→	→	→	→	→	↓	█	→	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 18000 episodes**

→	→	→	→	→	→	→	↓	█	→
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 19000 episodes**

→	→	→	→	→	→	→	↓	█	→
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 20000 episodes**

→	→	→	→	→	→	→	↓	█	→
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 21000 episodes**

→	→	→	→	→	→	→	↓	█	→
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 22000 episodes**

→	→	→	→	→	→	→	↓	█	→
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 23000 episodes**

→	→	→	→	→	→	↓	█	↓	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 24000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 25000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 26000 episodes**

→	→	→	→	→	→	↓	█	→	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 27000 episodes**

→	→	→	→	→	→	↓	█	→	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 28000 episodes**

→	→	→	→	→	→	↓	█	←	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 29000 episodes**

→	→	→	→	→	→	↓	█	→	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 30000 episodes**

→	→	→	→	→	→	→	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 31000 episodes**

→	→	→	→	→	→	→	↓	█	←
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 32000 episodes**

→	→	→	→	→	→	→	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 33000 episodes**

→	→	→	→	→	→	→	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 34000 episodes**

→	→	→	→	→	→	→	↓	█	←
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 35000 episodes**

→	→	→	→	→	→	↓	█	→	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 36000 episodes**

→	→	→	→	→	→	→	↓	█	→
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 37000 episodes**

→	→	→	→	→	→	→	↓	█	←
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 38000 episodes**

→	→	→	→	→	→	→	↓	█	↓
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 39000 episodes**

→	→	→	→	→	→	→	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 40000 episodes**

→	→	→	→	→	→	→	↓	█	←
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 41000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 42000 episodes**

→	→	→	→	→	→	→	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 43000 episodes**

→	→	→	→	→	→	→	↓	█	↓
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 44000 episodes**

→	→	→	→	→	→	→	↓	█	←
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 45000 episodes**

→	→	→	→	→	→	→	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 46000 episodes**

→	→	→	→	→	→	→	↓	█	↓
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 47000 episodes**

→	→	→	→	→	→	↓	█	↑	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 48000 episodes**

→	→	→	→	→	→	↓	█	↓	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	→	↑
→	→	→	→	→	→	→	↑	█	

**Policy after 49000 episodes**

→	→	→	→	→	→	↓	█	↓	
↑	█	█	█	↑	█	█	↓	█	↑
↑	█	█	█	↑	█	█	↓	█	↑
↓	█	█	█	↓	█	█	→	→	↑
↓	█	█	█	↓	█	█	█	↑	↑
→	→	→	→	→	→	→	↑	█	

Here we see that when alpha = 0.1, it takes ~15,000 episodes to converge, whereas when alpha = 0.9, it only takes ~2000 episodes.

## Turning in homework

When you are finished with this notebook. Save your work in order to turn it in. To do this select *File->Download As...->HTML*.

No description has been provided for this image

You can turn in your assignments using Gradescope for the course (available on <https://home.nyu.edu>). **Make sure you complete all parts (A and B) of this homework.**