Lorenz System

*Report 5 on the course "Numerical Analysis".

1st Chen Yihang
Peking University
1700010780

CONTENTS

I Problem Statement 1

II Simulation method 1

III Results 1

I. Problem Statement

Consider the famous Lorenz equation

$$\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y-x) \\ \frac{\mathrm{d}y}{\mathrm{d}t} = \rho x - y - xz \\ \frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z \end{cases} \tag{1}$$

1) For $\sigma = 10, \rho = 28, \beta = 8/3$, choose different initial values, and observe the results of simulation: whether the solution is bounded, periodic or converged.

2) Choose different σ, ρ, β and initial values, and observe the results of simulation.

II. Simulation method

For the equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x,y) \tag{2}$$

We use the Runge-Kutta formula:

$$y_{n+1} = y_n + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

where,

$$\begin{cases} K_1 = f(x_n, y_n) \\ K_2 = f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hK_1) \\ K_3 = f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hK_2) \\ K_4 = f(x_n + h, y_n + hK_3) \end{cases} \tag{3}$$

III. Results

The results are plotted in Lorenz.py. We can have the following oberservation.

a) $\sigma = 10, \rho = 28, \beta = 8/3$*:* We analyze the case in Figure 1, Figure 2.

- Figure 1 shows that small perturbations of initialization results in significant difference as the system evolves.
- Figure 2 shows that significant different initialization will eventually become periodic in the same region as the system evolves.

b) Other settings: We adjust the paramters in Lorenz system and perform simulations. We find that

- In general, σ, ρ, β should be positive to ensure the trajectory is bounded, as Figure 7 shows.
- The size of the paramters greatly influences the shape of attractor. It could have two symmetric attractors with shape "o", or one attractor with shape "8". Middle cases are more complicated.

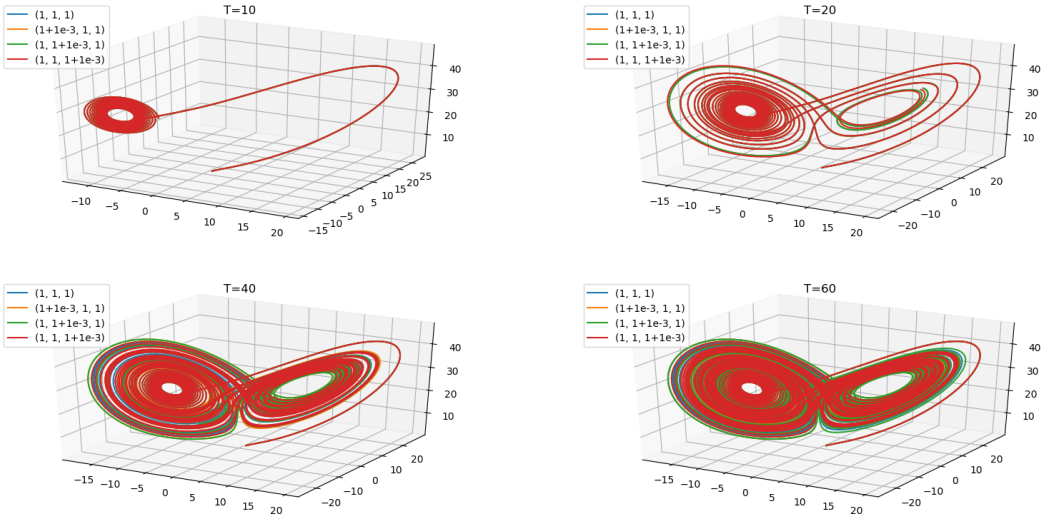

Figure 1. Case 1

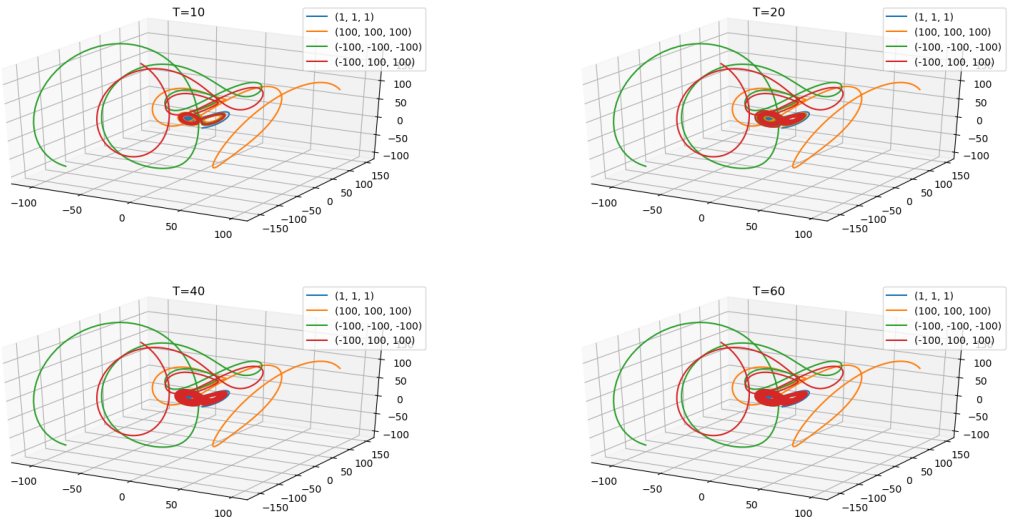

Figure 2. Case 2

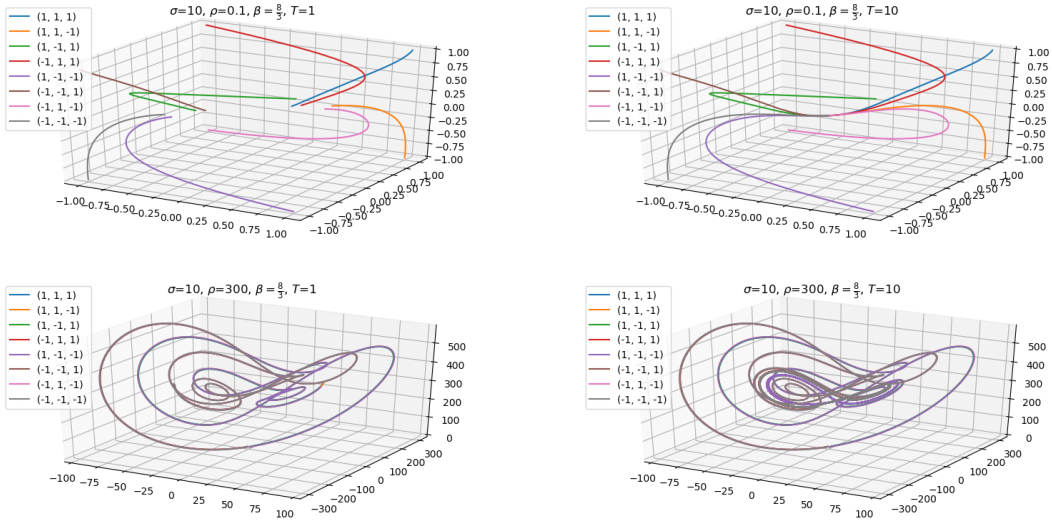

Figure 3. Case 3

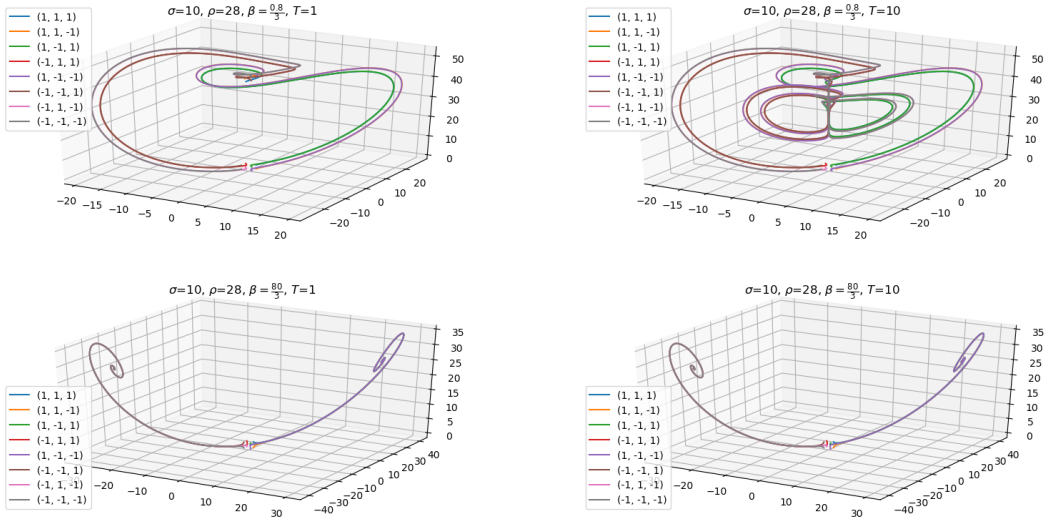

Figure 4. Case 4

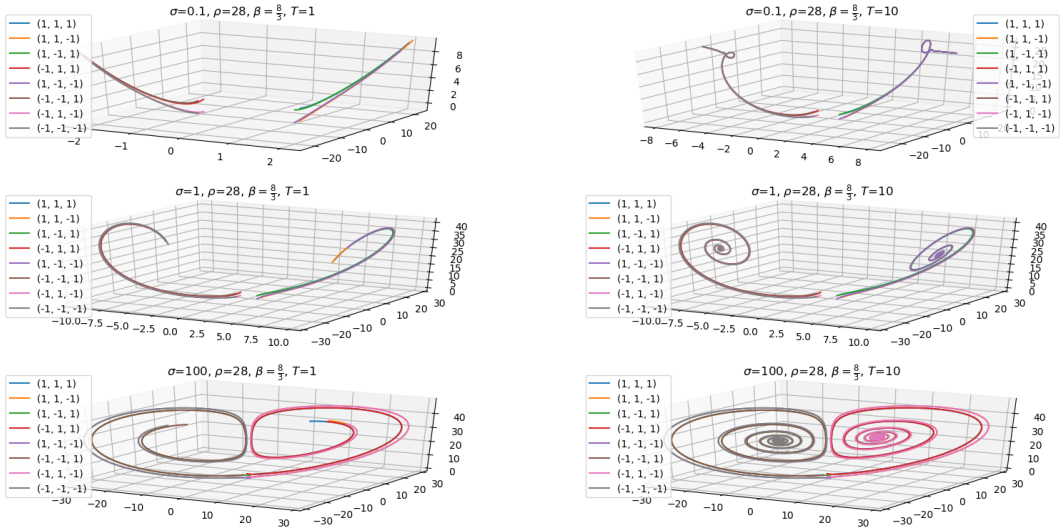

Figure 5. Case 5

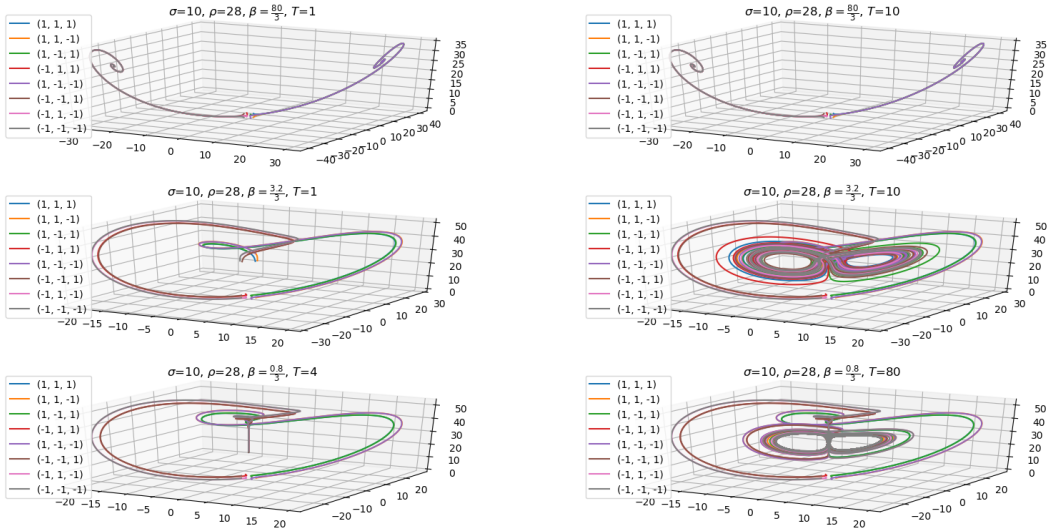

Figure 6. Case 6

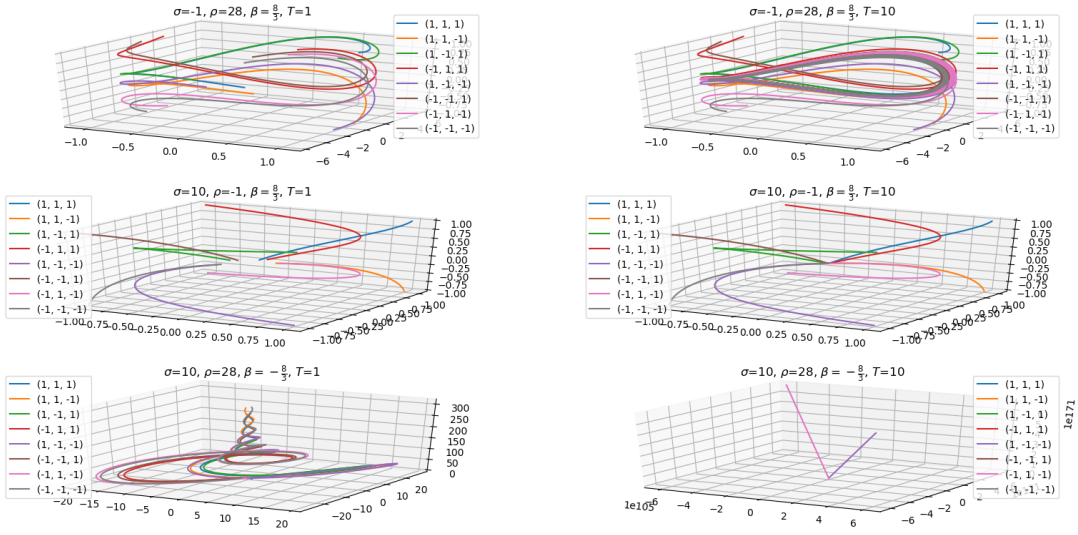

Figure 7. Case 7