Throughout this section, we shall make the following assumptions, denoted (H 1), (H 2) and (H 3), whose statements will not be repeated.

(H 1) We consider a *regular family of triangulations* $\mathcal{T}_h$ in the following sense:

(i) There exists a constant σ such that

$$\forall K \in \bigcup_h \mathcal{T}_h, \quad \frac{h_K}{\rho_K} \leqslant \sigma. \tag{3.2.1}$$

(ii) The quantity

$$h = \max_{K \in \mathcal{T}_h} h_k \tag{3.2.2}$$

approaches zero.

In other words, *the family formed by the finite elements* (K, P_K, Σ_K), $K \in \bigcup_h \mathcal{T}_h$, *is a regular family of finite elements*, in the sense of Section 3.1.

Remark 3.2.1. There is of course an ambiguity in the meaning of h, which was first considered as a defining parameter of both families $(\mathcal{T}_h)$ and (X_h), and which was next specifically defined in (3.2.2). We have nevertheless conformed to this often followed usage. □

(H 2) All the finite elements (K, P_K, Σ_K), $K \in \bigcup_h \mathcal{T}_h$, are affine-equivalent to a single reference finite element $(\hat{K}, \hat{P}, \hat{\Sigma})$. In other words, *the family* (K, P_K, Σ_K), $K \in \mathcal{T}_h$ *for all* h, *is an affine family of finite elements*, in the sense of Section 2.3.

(H 3) *All the finite elements* (K, P_K, Σ_K), $K \in \bigcup_h \mathcal{T}_h$, *are of class* $\mathscr{C}^0$.

We shall say that a family of triangulations satisfies an *inverse assumption*, in view of the *inverse inequalities* to be established in the next theorem, if there exists a constant ν such that

$$\forall K \in \bigcup_h \mathcal{T}_h, \quad \frac{h}{h_K} \leq \nu. \tag{3.2.28}$$

Notice that this is by no means a restrictive condition in practice.

For such families, we are able to estimate the equivalence constants between familiar semi-norms (we remind the reader that σ is the constant which appears in the regularity assumption; cf. (3.2.1)).

Theorem 3.2.6. *Let there be given a family of triangulations which satisfies hypotheses* (H 1), (H 2) *and an inverse assumption, and let there be given two pairs* (l, r) *and* (m, q) *with* $l, m \geq 0$ *and* $(r, q) \in [1, \infty]$ *such that*

$$l \leq m \quad \text{and} \quad \hat{P} \subset W^{l,r}(\hat{K}) \cap W^{m,q}(\hat{K}). \tag{3.2.29}$$

Then there exists a constant $C = C(\sigma, \nu, l, r, m, q)$ *such that*

$$\forall v_h \in X_h, \quad \Big(\sum_{K \in \mathcal{T}_h} |v_h|^q_{m,q,K}\Big)^{1/q} \leq \frac{C}{(h^n)^{\max\{0,(1/r)-(1/q)\}} h^{m-l}} \Big(\sum_{K \in \mathcal{T}_h} |v_h|^r_{l,r,K}\Big)^{1/r} \tag{3.2.30}$$

if $p, q < \infty$, *with*

$$\max_{K \in \mathcal{T}_h} |v_h|_{m,\infty,K} \quad \textit{in lieu of} \quad \Big(\sum_{K \in \mathcal{T}_h} |v_h|^q_{m,q,K}\Big)^{1/q} \quad \textit{if } q = \infty,$$

$$\max_{K \in \mathcal{T}_h} |v_h|_{l,\infty,K} \quad \textit{in lieu of} \quad \Big(\sum_{K \in \mathcal{T}_h} |v_h|^r_{l,r,K}\Big)^{1/r} \quad \textit{if } r = \infty.$$