Algorithm 2 FNMA$^{\mathrm{I}}$

Input: $\boldsymbol{A} \in \mathbb{R}_+^{M\times N}, \quad K, \tau \in \mathbb{N}, \alpha \in \mathbb{R}_+$.
Output: $\boldsymbol{B} \in \mathbb{R}_+^{M\times K}, \boldsymbol{C} \in \mathbb{R}_+^{K\times N}$
1. Initialize $\boldsymbol{B}^0, \boldsymbol{C}^0, t = 0$.
repeat
  2. $\boldsymbol{B} \leftarrow \boldsymbol{B}^t, \quad \boldsymbol{C}^{\text{old}} \leftarrow \boldsymbol{C}^t$.
  for $i = 1$ to τ **do**
    3.1. Compute the gradient matrix $\nabla_{\boldsymbol{C}}\mathcal{F}(\boldsymbol{B}; \boldsymbol{C}^{\text{old}})$.
    3.2. Compute fixed set I_+ for $\boldsymbol{C}^{\text{old}}$.
    3.3. Update $\boldsymbol{C}^{\text{old}}$ as:

$$\boldsymbol{U} \leftarrow \mathcal{Z}_+\big[\nabla_{\boldsymbol{C}}\mathcal{F}(\boldsymbol{B}; \boldsymbol{C}^{\text{old}})\big]; \quad \boldsymbol{U} \leftarrow \mathcal{Z}_+\big[\big(\boldsymbol{B}^T\boldsymbol{B}\big)^{-1}\boldsymbol{U}\big];$$
$$\boldsymbol{C}^{\text{new}} \leftarrow \mathcal{P}_+\big[\boldsymbol{C}^{\text{old}} - \alpha\boldsymbol{U}\big].$$

    3.4. $\boldsymbol{C}^{\text{old}} \leftarrow \boldsymbol{C}^{\text{new}}$.
  end for
  4. $\boldsymbol{C}^{t+1} \leftarrow \boldsymbol{C}^{\text{old}}$.
  5. $\boldsymbol{C} \leftarrow \boldsymbol{C}^{t+1}, \quad \boldsymbol{B}^{\text{old}} \leftarrow \boldsymbol{B}^k$.
  for $i = 1$ to τ **do**
    6.1. Compute the gradient matrix $\nabla_{\boldsymbol{B}}\mathcal{F}(\boldsymbol{B}^{\text{old}}; \boldsymbol{C})$.
    6.2. Compute fixed set I_+ for $\boldsymbol{B}^{\text{old}}$.
    6.3. Update $\boldsymbol{B}^{\text{old}}$ as:

$$\boldsymbol{U} \leftarrow \mathcal{Z}_+\big[\nabla_{\boldsymbol{B}}\mathcal{F}(\boldsymbol{B}^{old}; \boldsymbol{C})\big]; \quad \boldsymbol{U} \leftarrow \mathcal{Z}_+\big[\boldsymbol{U}\big(\boldsymbol{C}\boldsymbol{C}^T\big)^{-1}\big];$$
$$\boldsymbol{B}^{\text{new}} \leftarrow \mathcal{P}_+\big[\boldsymbol{B}^{\text{old}} - \alpha\boldsymbol{U}\big].$$

    6.4. $\boldsymbol{B}^{old} \leftarrow \boldsymbol{B}^{new}$.
  end for
  7. $\boldsymbol{B}^{t+1} \leftarrow \boldsymbol{B}^{old}$.
  8. $t \leftarrow t + 1$.
until Stopping criteria are met