Input: $oldsymbol{A} \in \mathbb{R}_+^{M imes N}, \quad K, au \in \mathbb{N}, lpha \in \mathbb{R}_+.$ Output: $oldsymbol{B} \in \mathbb{R}_+^{M imes K}, oldsymbol{C} \in \mathbb{R}_+^{K imes N}$

1. Initialize $\boldsymbol{B}^0, \boldsymbol{C}^0, t = 0$.

repeat
2. $B \leftarrow B^t$, $C^{\text{old}} \leftarrow C^t$.

Algorithm 2 FNMAI

for i = 1 to τ do

3.1. Compute the gradient matrix $\nabla_{\mathbf{C}} \mathcal{F}(\mathbf{B}; \mathbf{C}^{\text{old}})$.

3.2. Compute fixed set I_+ for C^{old} . 3.3. Update C^{old} as:

 $egin{aligned} oldsymbol{U} \leftarrow \mathcal{Z}_+ig[
abla_{oldsymbol{C}}\mathcal{F}(oldsymbol{B};oldsymbol{C}^{ ext{old}})ig]; & oldsymbol{U} \leftarrow \mathcal{Z}_+ig[oldsymbol{C}^{ ext{old}}-lphaoldsymbol{U}ig]; \\ oldsymbol{C}^{ ext{new}} \leftarrow \mathcal{P}_+ig[oldsymbol{C}^{ ext{old}}-lphaoldsymbol{U}ig]. \end{aligned}$

3.4. $C^{\text{old}} \leftarrow C^{\text{new}}$.

end for c^{t+1}

4. $C^{t+1} \leftarrow C^{\text{old}}$. 5. $C \leftarrow C^{t+1}$, $B^{\text{old}} \leftarrow B^k$.

for i = 1 to τ do

6.1. Compute the gradient matrix $\nabla_{\boldsymbol{B}} \mathcal{F}(\boldsymbol{B}^{\text{old}}; \boldsymbol{C})$.

6.2. Compute fixed set I_+ for $\boldsymbol{B}^{\text{old}}$. 6.3. Update $\boldsymbol{B}^{\text{old}}$ as:

6.3. Update B^{**} as: $U \leftarrow \mathcal{Z}_+[
abla_B \mathcal{F}(B^{old}; C)]; \quad U \leftarrow \mathcal{Z}_+[U(CC^T)^{-1}];$

 $\boldsymbol{B}^{\text{new}} \leftarrow \mathcal{P}_{+} \big[\boldsymbol{B}^{\text{old}} - \alpha \boldsymbol{U} \big].$

end for

6.4. $\mathbf{B}^{old} \leftarrow \mathbf{B}^{new}$.

7. $\boldsymbol{B}^{t+1} \leftarrow \boldsymbol{B}^{old}$. 8. $t \leftarrow t+1$.

until Stopping criteria are met