Instructions: Please note that handwritten assignments **will not be graded**. Use the provided LaTeX template to complete your homework. Please do not alter the order or spacing of questions (keep each question on its own page). When you submit to Gradescope, you must mark which page(s) correspond to each question. **You may not receive credit for unmarked questions**.
When including graphical figures, we encourage the use of tools such as graphviz or packages like tikz for simple and complex figures. However, these may be handwritten only if they are neat and legible (as defined by the grader).

List any collaborators (besides TAs or professors) here:

1. (5 points) [W12, ⋆] NP-Completeness. For the following questions, select whether the statement is true or false, and write a *brief* explanation of your reasoning.

   (a) Consider two decision problems A and B where A is known to be NP-Complete, and every instance of B can be reduced to an instance of A in polynomial size and time. B is NP-Complete.

   ■ True □ False

   If A is NP-Complete, and B can be reduced to A in polynomial time, then B is also NP-Complete.

   (b) If a problem, X, is NP-Complete, it is unsolvable

   □ True ■ False

   NP-Complete problems are solvable, but they are not solvable in polynomial time.

   (c) Let $\propto$ denote a polynomial-time reduction. Then, for problems X, Y and Z,

   $$(X \propto Y) \wedge (Y \propto Z) \rightarrow (X \propto Z).$$

   In other words, the polynomial-time reduction property is transitive

   ■ True □ False

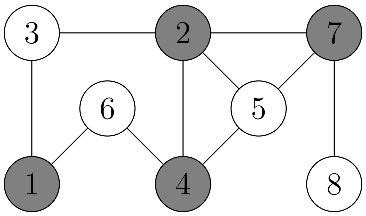
   If X can be reduced to Y in polynomial time, and Y can be reduced to Z in polynomial time, then X can be reduced to Z in polynomial time.

2. (10 points) [W12, ★★★] Satisfiability. Provide a set of boolean assignments which will satisfy the following expression:

$$(x_1 \vee x_2 \vee x_3) \wedge (x_2 \vee \neg x_3 \vee x_4 \vee \neg x_5) \wedge (\neg x_1 \vee \neg x_5) \wedge (x_3 \vee \neg x_5) \wedge (x_3 \vee x_5) \wedge (\neg x_3 \vee \neg x_4) \wedge (\neg x_2 \vee x_4 \vee x_5)$$

   (a) x_1: □ True ■ False
   (b) x_2: □ True ■ False
   (c) x_3: ■ True □ False
   (d) x_4: □ True ■ False
   (e) x_5: □ True ■ False

3. (10 points) [W12, ⋆⋆⋆] Vertex Cover. Indicate which vertices are needed in the optimal vertex cover for the following graph.



   (a) 1: ■ Included
   (b) 2: ■ Included
   (c) 3: □ Included
   (d) 4: ■ Included
   (e) 5: □ Included
   (f) 6: □ Included
   (g) 7: ■ Included

4. (10 points) [W12, ⋆] Satisfiability. Bill has an n-SAT problem with 10 clauses that he wishes to reduce to 3-SAT. His instance has:

   - 1 one-literal clause
   - 2 two-literal clauses
   - 3 four-literal clauses
   - 4 five-literal clauses

   (a) (5 points) Using the reduction discussed in class, how many clauses will Bill end up with?

   $\boxed{12}$

   (b) (5 points) Assuming Bill doesn't reuse dummy variables, how many dummy variables will Bill add?

   $\boxed{6}$

# of literals, k	# of clauses	new dummy var	# 3-literal clause
1	1	2	4
2	2	$1 \times 2 = 2$	$2 \times 2 = 4$
4	3	0	1
5	4	$(5 - 3) = 2$	$(5 - 2) = 3$
total	10	6	12