B 9.6 (Complex Eigenvalues)

- Now suppoze that when we find the eigenvalues for $x'(t) = A\,x(t)$ we end up with $r_1 = \alpha + \beta i$ and $r_2 = \alpha - \beta i$ with associated eigenvectors $z_1 = a + ib$, $z_2 = a - ib$, then two linearly indipendent real vector solutions to $x'(t) = A\,x(t)$ are:

$$e^{\alpha t}\cos(\beta t)a - e^{\alpha t}\sin(\beta t)\,b$$

$$e^{\alpha t}\sin(\beta t)a + e^{\alpha t}\cos(\beta t)\,b$$

Recall that a is the <u>real</u> portion of the eigenvector, b is the <u>imaginary</u> portion

Example: Find the eigenvectors given that $\lambda_1 = -1 + 2i$, $\lambda_2 = -1 - 2i$.

$$A = \begin{bmatrix} -2 & -1 \\ 5 & 0 \end{bmatrix}$$

$\rightarrow$ $A - \lambda_1 I = \begin{bmatrix} -2-(-1+2i) & -1 \\ 5 & -(-1+2i) \end{bmatrix} = \begin{bmatrix} -1+2i & -1 \\ 5 & 1-2i \end{bmatrix}$, we can

choose that $v_1 = \begin{bmatrix} 1 \\ (-1+2i) \end{bmatrix}$ or $\begin{bmatrix} 1 \\ -1 \end{bmatrix} + i \begin{bmatrix} 0 \\ 2 \end{bmatrix}$