5.4 Eigenvalues and Linear Transformations

Definition : Let V be a vector space. An eigenvector of a linear transformation $T: V \to V$ is a nonzero vector x in V such that $T(x) = \lambda x$ for some scalar λ.

→ λ is called the eigenvalue of T if there are nontrivial solutions x of $T(x) = \lambda x$, such that x is called the eigenvector corresponding to λ

Express coordinate vector of transformation T, on a specific vector x of basis B, then :

$$[T(x)_C] = M[x]_B \rightarrow M = [T(b_1)]_B \; [T(b_2)]_B \cdots [T(b_n)]_B$$

- M is the matrix for T relative to basis B

Want to transform the vector which has current basis B, we can write the transformed vector's coefficient (coordinate) in the basis of C as above

$[T(x)]_C$ gives the coefficients needed to express $T(x)$ as a linear combination of basis vectors in C

For a linear transformation V to V:

$\rightarrow$ $[T(x)]_B = [T]_T [x]_B$

Diagonal matrix representation

- $A = PDP^{-1}$, where D is a diagonal $n \times n$ matrix

$B \rightarrow$ basis for $\mathbb{R}^n$ formed from columns of P, then D is the B-matrix for transformation $x \mapsto Ax$

- So "M" is just our D