6.1 Inner Product, Length, Orthogonality

The inner product / Dot product of two $n \times 1$ matrices (vectors) u and v (denoted $u \cdot v$) is computed as $u^T \cdot v$, and results in a scalar

Properties of Inner Product

1) Commutative: $u \cdot v = v \cdot u$
2) $(u+v) \cdot w = u \cdot w + v \cdot w$
3) $(cu)v = c(u \cdot v) = u \cdot cv$ ← c is a constant
4) $u \cdot u$ must always be ≥ 0. if $u \cdot u = 0$, the u MUST $= 0$

Length of a Vector

- The length of a vector, or its norm, is the non-negative scalar $\|v\|$ defined by:

$\|v\| = \sqrt{v_1^2 + v_2^2 + \dots v_n^2}$ where each v_i corresponds to a component of the original vector

Distance of two vectors can be found by: $\|v - u\|$

Unit Vector: any vector with length 1

- If we want to find the unit vector for a particular vector, then we have to multiply v by $1/\|v\|$

ORTHOGONAL vectors: Two vectors are said to be orthogonal if their dot product is 0

- means they are perpendicular, $u \cdot w = 0$
- Can also determine orthogonality if $\|u+v\|^2 = \|u\|^2 + \|v\|^2$

If W is a Subspace of $\mathbb{R}^n$, and say we have vector z, which happens to be orthogonal to each vector in W, we would call the set of all vectors orthogonal to those in W, as $W^\perp$, or the Orthogonal complement

$x \in W^\perp$ if x is orthogonal to every vector in W

$W^\perp$ is also a vector space of $\mathbb{R}^n$

Theorem 3: Let A be an $n \times n$ matrix. The orthogonal complement of the row space of A is the null space of A, and the orthogonal complements of the column space of A is the null space of A^T

So, $(\text{Row } A)^\perp = \text{Nul } A$ and $(\text{Col } A)^\perp = \text{Null } A^T$