

## 1.2: Row Reduction and Echelon Forms

### Echelon Form vs. Reduced Row Echelon Form (RREF)

- 1) All non-zero rows are above all zero rows
- 2) Each leading entry of a row is in a column to the right of the leading entry of the row above it
- 3) All entries in a column below a leading entry are zeroes

RREF: All conditions AND:

- 4) The leading entry in each non-zero row is 1
- 5) Each leading 1 is the only non-zero entry in the column

REF:

$$\left[ \begin{array}{cccc|c} 2 & 7 & 6 & 1 & 3 \\ 0 & 1 & 2 & 1 & 4 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

RREF

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 1 & 2 \end{array} \right]$$

## Pivot!

$$\left[ \begin{array}{cccc|c} \boxed{1} & 4 & 2 & 3 & 9 \\ 0 & 0 & \boxed{2} & 7 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

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□ - pivot values

• pivot columns

Pivot position : corresponds to 1 in RREF

Pivot column : The column that contains the pivot

Pivot : Nonzero number in pivot position used to create zeroes in row operations

There's a specific "algorithm" to convert a matrix to REF or RREF. It would take too long for me to write that, so consult your textbook or watch videos. You'll also build that intuition as you solve more problems

## Consistent System with Infinitely many solutions

$$\left[ \begin{array}{cccc|c} 1 & 0 & -5 & 1 & 1 \\ 0 & 1 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \rightarrow \begin{cases} x_1 = 5x_3 + 1 \\ x_2 = 4 - x_3 \\ x_3 \text{ is free} \end{cases}$$

- Free variables can take on any value. Once you choose a value for your free variable, it will determine the values of the other (basic) variables

## Practice

Find the general solution of the augmented matrix

$$\left[ \begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{array} \right] \sim \left[ \begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$\text{So, } \begin{cases} x_1 \\ x_2 \text{ is free} \\ x_3 = 3 \end{cases}$$

## 1.3 Vector Equations

### Vectors in $\mathbb{R}^2$

Vector - An ordered list of numbers

Column Vector - A vector with only one column. We often use these for ordered pairs, triplets, etc

Vectors in  $\mathbb{R}^2$  - The set of all vectors with 2 entries..

$\mathbb{R} \rightarrow$  Real numbers       $2 \rightarrow$  number of entries

This is the set of all points in a plane

Operations with Vectors - Same as w/ other matrices

Scalar - multiply by a constant

Addition - Add corresponding values

Multiplication - Nope! Dimensions don't work.