B9.5 (Homogenous Linear Systems with Linear Coefficients)

- So, how do we actually go about finding a general solution to

  $x'(t) = Ax(t)$

- Let $A = [a_{ij}]$ be an $n \times n$ constant matrix. The eigenvalues of A are those numbers (real or complex) numbers r for which $(A - rI)u = 0$ has at least one nontrivial (real or complex) solution to u

- The corresponding nontrivial solutions u are called the eigenvectors of A associated with r

- **Theorem 5:** Suppose the $n \times n$ constant matrix A has n linearly independent eigenvectors $u_1, u_2, u_3, \dots u_n$, and let r_i be the eigenvalue corresponding to u_i. Then, $\{e^{r_1 t}u_1, e^{r_2 t}u_2, e^{r_3 t}u_3, \dots e^{r_n t}u_n\}$ is a fundamental solution set and $X(t) = [e^{r_1 t}u_1 \;\; e^{r_2 t}u_2 \; \dots \; e^{r_n t}u_n]$ is a fundamental matrix on $(-\infty, \infty)$ for homogenous system $x' = Ax$.

Thus, our general solution of $x' = Ax$ is: $x(t) = c_1 e^{r_1 t}u_1 + c_2 e^{r_2 t}u_2 + \dots c_n e^{r_n t}u_n$

Theorem 6: If $r_1, \dots r_m$ are distinct eigenvalues of the matrix A and u_i is an eigenvector associated with r_i, Then $u_1, \dots u_m$ are linearly independent

- When solving for eigenvalues, always have left side "alone"