Practice

Find the general solution of the augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

So, $\begin{cases} x_1 \\ x_2 \text{ is free} \\ x_3 = 3 \end{cases}$

1.3 Vector Equations

Vectors in $\mathbb{R}^2$

Vector - An ordered list of numbers

Column Vector - A vector with only one column. We often use these for ordered pairs, triplets, etc

Vectors in $\mathbb{R}^2$ - The set of all vectors with 2 entries..

$\mathbb{R} \to$ Real numbers $\quad 2 \to$ number of entries

This is the set of all points in a plane

Operations with Vectors - Same as w/ other matrices

Scalar - multiply by a constant

Addition - Add corresponding values

Multiplication - Nope! Dimensions don't work.

Example:

If $\vec{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Find $\vec{u} + \vec{v}$ and $-2u + 4\vec{v}$

- $\vec{u} + \vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \boxed{\begin{bmatrix} 1 \\ 5 \end{bmatrix}}$
- $-2u + 4v = -2\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 4\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \boxed{\begin{bmatrix} -8 \\ 2 \end{bmatrix}}$

Vectors in $\mathbb{R}^n$

If $n \in \mathbb{R}$, then $\mathbb{R}^n$ is the collection of all Lists of ordered n-Tuples of N real numbers written as $n \times 1$ column matrices

$$\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

Zero vector: The vector whose entries are all 0, $\vec{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Algebraic properties of $\mathbb{R}^n$

These correspond to properties of real #'s, pertaining to vectors u, v and w and scalars c and D

$\vec{u} + \vec{v} = \vec{v} + \vec{u}$ $\qquad$ $(\vec{u}+\vec{v}) + \vec{w} = \vec{u} + (\vec{v}+\vec{w})$

$\vec{u} + \vec{0} = 0 + \vec{u} = \vec{u}$ $\qquad$ $\vec{u} + (-\vec{u}) = (-\vec{u}) + \vec{u} = 0$

$c(\vec{u}+\vec{v}) = c\vec{u} + c\vec{v}$ $\qquad$ $(c+d)\vec{u} = c\vec{u} + d\vec{u}$

$c(d\vec{u}) = (cd)\vec{u}$ $\qquad$ $1\vec{u} = \vec{u}$

Linear Combinations

Vector defined by $y = c_1v_1 + c_2v_2 + \dots c_nv_n$ where c_i are scalars and v_i are vectors, is called a linear combination of $\vec{v_1}, \vec{v_2}, \dots \vec{v_n}$ with weights $c_1, c_2, \dots c_n$.

Example: $\vec{v_1} = \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$, determine if $b = \begin{bmatrix} 7 \\ 4 \\ -3 \end{bmatrix}$ can be written as a linear combination of v_1 and v_2

$$\left[\begin{array}{cc|c} 1 & 2 & 7 \\ -2 & 5 & 4 \\ -5 & 6 & -3 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right], \quad \text{so} \quad c_1 = 3, \quad c_2 = 2$$

Vector Equation $x_1\vec{a_1} + x_2\vec{a_2} + \dots x_n\vec{a_n} = \vec{b}$ has the same solution set as the linear system whose augmented matrix is $[\vec{a_1}\ \vec{a_2}\ \dots\ \vec{a_n} \mid b]$. So, a vector equation only has a solution if the system is consistent

If $\vec{v_1}, \vec{v_2}, \vec{v_p}$ are in $\mathbb{R}^n$, then the set of Linear combinations is denoted by Span $\{\vec{v_1}, \vec{v_2}, \dots \vec{v_p}\}$ and is called the subset of $\mathbb{R}^n$ spanned.

Span $\{\vec{v_1}, \dots \vec{v_p}\}$ is all vectors that can be written in the form $c_1\vec{v_1} + c_2\vec{v_2} + \dots c_p\vec{v_p} = b$ with c_i scala