5.1 Eigenvalues and Eigenvectors

- An eigenvector of an $n \times n$ matrix is a non-zero vector y such that $Ax = \lambda$ where λx is a scalar.
- A scalar λ is called an eigenvalue of A if $\exists$ a nontrivial solution x of $Ax = \lambda x$
- So when we multiply vector x by matrix A, we get back x but just scalar multiplied
- So A must be square
- This constant, λ, is what we call the eigenvalue of matrix A
- There can be no more eigenvalues then the number of rows / columns in A
  - Each λ_n has an associated v_i ← This is called the eigenvector

If we want to show if a certain value is an eigenvalue, multiply this value by I and subtract from A $(A - \lambda I)$

- If the resulting matrix is linearly independent, this means λ is NOT an eigenvalue
- If linearly dependent, then it IS an eigenvalue

So essentially, $\det(A - \lambda I) = 0$, and $A - \lambda I$ cannot be invertible

In finding eigenvectors, row reduce $A - \lambda I$ and solve for free variables:

→ $x_1 = 3x_2 \rightarrow v_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $x_1 = \frac{x_2}{2} \rightarrow v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Basis of Eigenspace : look at the basis for the null space of $A - \lambda I$

- The eigen vectors form a basis for the eigenspace

Theorem 1: The Eigenvalues of a Triangular matrix are the entries along its main diagonal

Theorem 2: If $v_1, \dots v_n$ are eigenvectors corresponding to $\lambda_1, \dots \lambda_n$ for an $n \times n$ matrix, then the set $\{v_1, v_2, v_3, \dots v_n\}$ must be linearly independent.

- So distinct eigenvalues and their corresponding eigenvalues must be linearly independent