# 4.4 Coordinate Systems

The Unique Representation Theorem : Let $B = \{b_1, b_2, \dots b_n\}$ be a basis for vector space V. Then for each x in V, ∃ a unique set of scalars $c_1, c_2, \dots c_n$ such that

$$x = c_1 b_1 + c_2 b_2 + \dots + c_n b_n$$

Suppose that $B = \{b_1, b_2, \dots b_n\}$ is a basis for V and x is in V. The coordinate of x relative to the basis B (or the B coordinate of x) are the weights $c_1 b_1 + c_2 b_2 + \dots + c_n b_n$

The notation for this would be: $[x]_B$ ← weights for each basis vector in vector x

Remember, Coordinate corresponds to coefficients

$[x]_B$ is referred to as the coordinate vector of x (relative to B)

The transformation $x \longrightarrow [x]_B$ is referred to as the coordinate mapping (determined by B)

- Transforms x from one basis vector to another
- one-to-one, so each vector x in our vector space, there is only one unique coordinate mapping

Standard Basis : denoted $\{e_1, e_2, e_3, \ldots e_n\}$ is a special set of vectors which form the basis

$$e_1 \to \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0_n \end{bmatrix}, \quad e_n \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

If you have $B = \{b_1, b_2, .. b_n\}$ for $\mathbb{R}^n$ and want to express coordinate of vector v in terms $\{e_1, .. e_n\}$

You would have to express each basis in b as a linear combination of the standard basis vectors

It's possible for 2 basis be bases for the same space, or subspace of a vector space, but do not span each other.
(basis vectors from one cannot be linear combinat from another

Thm 9: $B = \{b_1, .. b_n\}$ basis for v-space V. Then $x \longrightarrow [x]_b$ is a one to transformation from V onto $\mathbb{R}^n$