6.2 Orthogonal Sets

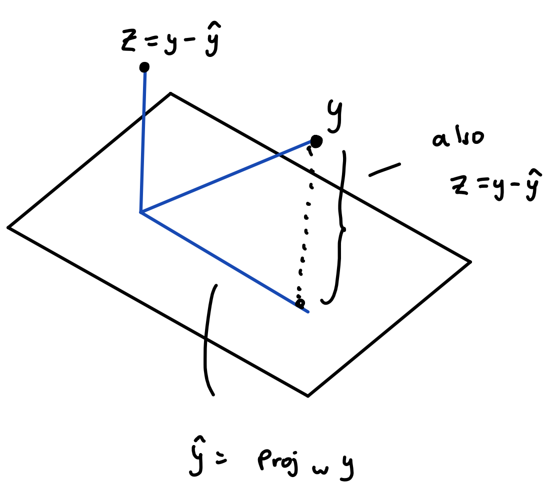
An orthogonal set is a set of vectors where each pair of distinct vectors is orthogonal

Theorem 4: If $S = \{v_1, v_2, \ldots v_n\}$ is an orthogonal set of non-zero vectors in $\mathbb{R}^n$, then S is linearly independent and thus a basis for the subspace spanned by S

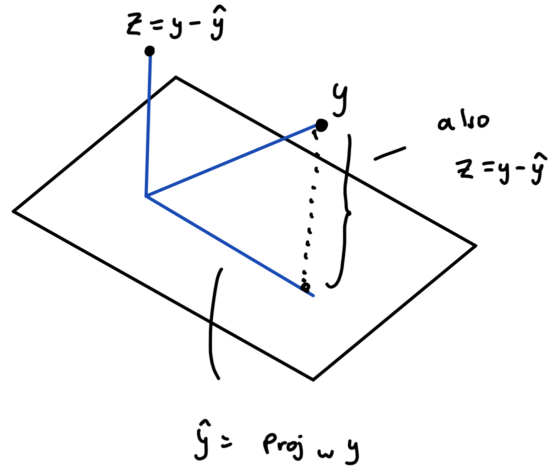
Orthogonal Basis for subspace W of $\mathbb{R}^n$ is a basis for W that is also an orthogonal set

Let $\{u_1, u_2, \ldots u_p\}$ be an orthogonal basis for a subspace W of $\mathbb{R}^n$. For each y in W, the weights in the linear combination $y = c_1 u_1 + c_2 u_2 + \ldots c_p u_p$ are given by:

$$\frac{y \cdot u_j}{u_j \cdot u_j}$$



- An orthogonal projection is a decomposition of a vector into the sum of two orthogonal vectors



So, $y = \hat{y} + z$

The formula for $\hat{y}$ is given below:

$$\hat{y} = \text{proj}_L\, y = \frac{y \cdot u}{u \cdot u} \cdot u$$

We define an orthonormal set as an orthogonal set of unit vectors

- If W is a subspace spanned by such a set, then $\{u_1, u_2, \dots u_p\}$ is an orthonormal basis for W, since the set is also linearly independent
- Simplest example of an orthonormal set is the standard basis $\{e_1, e_2, \dots e_n\}$ for $\mathbb{R}^n$

Theorem 6: An $m \times n$ matrix U has orthonormal columns if and only if

$$U^T U = I$$

Theorem 7: Let U be an $m \times n$ matrix with orthonormal columns, and let x and y be in $\mathbb{R}^n$

$$\| Ux \| = \| x \|$$

$$(Ux) \cdot (Uy) = x \cdot y$$

$$(Ux) \cdot (Uy) = 0 \quad \text{iff} \quad x \cdot y = 0$$

ORTHOGONAL Matrix: a square matrix where its rows or columns must be an orthonormal set of vectors