4.3) Linearly Independent sets and Bases

$\{V_1, \dots V_p\}$ in $\mathbb{R}^n$ is linearly independent if $x_1V_1 + x_2V_2 + \dots x_pV_p = 0$

has only the trivial solutions

Linear Dependence

- Non-trivial solution exists
- Vectors are multiple of one another
- A vector is a linear combination of other vectors in the set
- There are more vectors than entries in each vector

Basis :

Let H be a subspace of vector space V. $B = \{b_1, b_2, \dots b_p\}$

is a basis for H if

1) B is linearly independent

2) $H = \text{span}\{b_1, b_2, \dots b_p\}$

Ex: $\beta: \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^2$

An efficient spanning set contains no unnecessary vectors.

Spanning Set Theorem:

Let $S = \{v_1, v_2, \dots v_p\}$ be a set in V and let

$H = \text{span}\{v_1, \dots v_p\}$

1) If $v_k \in S$ and v_k is a linear combination of the remaining vectors in S, then $S \setminus \{v_k\}$ still spans H.

b) If $H \neq \vec{0}$, some subset of S is a basis for H