3.2 Properties of Determinants

1) If we interchange rows / swap roles, we switch the sign of our determinant

2) If we scale a row, we also scale our determinant by that same factor

3) If we replace one row with a scalar combination of another, our determinant actually stays the same

An Example

Compute $A = \begin{bmatrix} 1 & -4 & 2 \\ -2 & 8 & -9 \\ -1 & 7 & 0 \end{bmatrix}$

Swapping rows

$$\det A = \begin{vmatrix} 1 & -4 & 2 \\ -2 & 8 & -9 \\ -1 & 7 & 0 \end{vmatrix} \to \begin{vmatrix} 1 & -4 & 2 \\ 0 & 0 & -5 \\ 0 & 3 & 2 \end{vmatrix} = - \begin{vmatrix} 1 & -4 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & -5 \end{vmatrix}$$

$\det A = -(1)(3)(-5) = 15$ ✓

Another Example : compute $\begin{vmatrix} 2 & 2 & 0 \\ 4 & -4 & 3 \\ -3 & 1 & 4 \end{vmatrix}$

$$2\begin{vmatrix} 1 & 1 & 0 \\ 4 & -4 & 3 \\ -3 & 1 & 4 \end{vmatrix} \sim 2(4)\begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 11 \\ 0 & 1 & 1 \end{vmatrix} \to -2(4)\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 11 \end{vmatrix}$$

$= -8(11) = -88$ ✓

Two more properties:

1) If A is an $n \times n$ matrix, then $\det A^T = \det A$

↳ $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$

$\det(A) = 4 - (2)(-3) = 10$ $\qquad$ $\det(A^T) = 4 - (-3)(2) = 10$ ✓

2) If A and B are $n \times n$ matrices, then $\det AB = \det A \cdot \det B$

↳ $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$

↳ $AB = \begin{bmatrix} 2 & 8 \\ 4 & 6 \end{bmatrix}$

$\det A = 10$, $\det B = -2$ → $\det A \cdot \det B = -20$

$\det AB = 12 - 32 = -20$ ✓