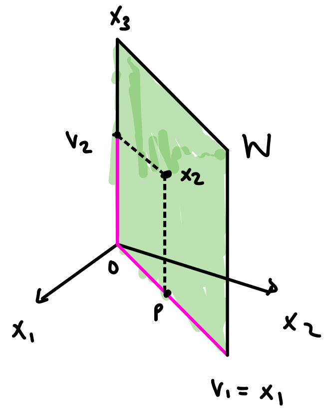
6.4 The Gram Schmidt Process

- The process for producing an orthogonal / orthonormal basis for any non zero subspace of $\mathbb{R}^n$, given vectors that already span a certain subspace



← Construction of an orthogonal basis $\{v_1, v_2\}$

Main premise: Finding vectors that are orthogonal to each other, by following steps where you find the orthogonal vector to the span of another, then 2 others, and so on

Theorem 11: Gram Schmidt Process

Given basis $\{x_1, \dots x_p\}$ for a non zero subspace W of $\mathbb{R}^n$, define

$$V_1 = X_1$$

$$V_2 = X_2 - \frac{X_2 \cdot V_1}{V_1 \cdot V_1} V_1$$

$$V_3 = X_3 - \frac{X_3 \cdot V_1}{V_1 \cdot V_1} V_1 - \frac{X_3 \cdot V_2}{V_2 \cdot V_2} V_2$$

$$\vdots$$

$$V_p = X_p - \frac{X_p \cdot V_1}{V_1 \cdot V_1} V_1 - \dots - \frac{X_p \cdot V_{p-1}}{V_{p-1} \cdot V_{p-1}} \cdot V_{p-1}$$

$\{V_1, \dots V_p\}$ is an orthogonal basis for W. In addition

$$\text{Span}\{V_1, \dots V_k\} = \text{Span}\{X_1, \dots X_k\} \quad \text{for } 1 \le k \le p$$

If you normalize the scale of all V_K, you have an orthonormal basis

Theorem 12: QR Factorization

If A is an $m \times n$ matrix with linearly independent columns, then A can be factored as $A = QR$, where Q is an $m \times n$ matrix whose columns form an orthonormal basis for Col A and R is an $n \times n$ upper triangular invertible matrix with positive entries along its diagonal

Remember, $Q^T Q = I$, so we can solve for R

by: $Q^T A$