5.2 Characteristic Equation and Similarity

Invertible Matrix Theorem (Continued):

- Matrix A is invertible iff 0 is NOT an eigenvalue of A
- Determinant of A is NOT 0

Characteristic Equation: An equation used for finding the eigenvalues of a square matrix

- Solutions to the characteristic equation will be the eigenvalues

Characteristic Equation is given by $\det(A - \lambda I) = 0$

- After calculating eigenvalues given by the characteristic equation, you can then use them to solve for eigenvectors
- How to find eigenvectors?
  1) Plug in each λ_n into $A - \lambda I$,
  2) Reduce down matrix to REF or RREF
  3) Solve $(A - \lambda I)x = 0$ for the null space

Example: if we have $A = \begin{bmatrix} 3 & 5 \\ -1 & -3 \end{bmatrix}$, let's say we're given $\lambda = 2$ and $\lambda = -2$.

$\lambda = 2$: $A - 2I = \begin{bmatrix} 3-2 & 5 \\ -1 & -3-2 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -1 & -5 \end{bmatrix} \to \begin{bmatrix} 1 & 5 \\ 0 & 0 \end{bmatrix}$

$\to x_1 = 5x_2 \to v_1 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

$\lambda = -2$: $A + 2I = \begin{bmatrix} 3+2 & 5 \\ -1 & -3+2 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ -1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$\to x_1 = x_2 \to v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

So our eigenvectors are: $\{v_1, v_2\} = \left\{ \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

Similarity: Two $n \times n$ matrices A and B are "similar" when there exists an invertible matrix P such that $A = P^{-1}BP$

- P is an invertible matrix
- This is NOT the same as row equivalence

Significance: Similar matrices have the same eigenvalues and eigenvectors

If A and B similar, they represent the same linear transformation but under a different basis

Just b/c two matrices have same eigenvalues, does NOT mean they are similar