2.1 Matrix Operations

Matrix Sums : Just as we did with vectors, this is done component wise. Therefore, matrices can only be added if they have the same dimensions

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \quad B = \begin{bmatrix} -4 & -3 & -2 \\ 2 & 4 & 6 \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} -3 & 0 & 3 \\ 4 & 8 & 12 \end{bmatrix} = B + A$$

$A + C \rightarrow$ NOT VALID, as the dimensions of A and C are not the same

$B + C \rightarrow$ Not valid by same reason

$$A - B = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} - \begin{bmatrix} -4 & -3 & -2 \\ 2 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 7 \\ 0 & 0 & 0 \end{bmatrix}$$

Scalar Multiplication :

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}, \rightarrow \quad 2A = \begin{bmatrix} 2 & 6 & 10 \\ 4 & 8 & 12 \end{bmatrix}$$

$$3A - 2B = 3\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} - 2\begin{bmatrix} -4 & -3 & -2 \\ 2 & 4 & 6 \end{bmatrix} \approx \begin{bmatrix} 11 & 15 & 19 \\ 2 & 4 & 6 \end{bmatrix}$$

General Properties

$A + B = B + A$, $(A+B) + C = A + (B + C)$, $A + 0 = A$

$r(A+B) = rA + rB$, $(r+s)A = rA + sA$, $r(sA) = (rs)A$

Matrix Multiplication

If A is an $n \times n$ matrix and if B is an $n \times p$ matrix with columns $b_1, b_2, \dots b_p$, then the product AB is the $m \times p$ matrix whose columns are $Ab_1, Ab_2, \dots Ab_p$. That is $AB = A[b_1 \; b_2 \; \dots \; b_p] =$

$[Ab_1 \; Ab_2 \; \dots \; Ab_p]$

$$\underset{m \times n}{A} \cdot \underset{n \times p}{B} = \underset{m \times p}{C}$$

Example: $A = \begin{bmatrix} 4 & 1 \\ 5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 3 \\ -2 & 2 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} -2 & 6 & 12 \\ -4 & 9 & 15 \end{bmatrix}$

Row Column Rule

If the product AB is defined, then the entry in Row i and column j of A is the sum of products in the corresponding entries from row i of A and column j of B

$$(AB)_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + \dots a_{in} b_{nj}$$

More Properties:

$A(BC) = (AB)C$. $A(B+C) = AB + AC$, $(B+C)A = BA + CA$

$r(AB) = (rA)B = A(rB)$

$I_m A = A I_n = A$

In general, $AB \neq BA$

If $AB = AC$, it's not required that $B = C$

If $AB = 0$, you cannot conclude $A = 0$ or $B = 0$

Transpose of a Matrix

Given an $m \times n$ matrix A, the transpose, denoted A^T, is an $n \times n$ matrix whose columns are formed by the rows of A

Properties of Transpose

$(A^T)^T = A$, $(A+B)^T = A^T + B^T$ $(rA^T) = r(A^T)$

$(AB)^T = B^T A^T$