2.3 Characterizations of Invertible Matrices

The Invertible Matrix Theorem

Let A be an $n \times n$ square matrix. The following are equivalent

- A is an invertible matrix
- $A \sim I_n$
- A has n pivot positions
- The columns of A span $\mathbb{R}^n$
- A^T is an Invertible Matrix
- The columns of A form a linearly independent set
- The Linear transformation $x \mapsto Ax$ is one-to-one
- $Ax = b$ has at least one solution for each $b \in \mathbb{R}^n$
- The linear transformation $x \mapsto Ax$ maps $\mathbb{R}^n$ to $\mathbb{R}^n$
- There is an $n \times n$ matrix C such that $CA = I_n$
- There is an $n \times n$ matrix D such that $AD = I_n$