5.5 Complex Eigenvalues and Eigenvectors

- when characteristic equation : $\det(A - \lambda I) = 0$ does not return Real valued eigen values

- If $\lambda_1 = a + bi$, a complex number, and specific eigen vector $\begin{bmatrix} c+di \\ e \end{bmatrix}$
- Then, for the conjugate of $\lambda_1 \to \lambda_2$, $\lambda_2 = a - bi$ with associated eigenvector $\begin{bmatrix} c-di \\ e \end{bmatrix}$

  $\to$ These two vectors are linearly independent

- For specific $v_1 = \begin{bmatrix} c-di \\ e \end{bmatrix}$, the real parts are $\begin{bmatrix} c \\ e \end{bmatrix}$ and imaginary $\begin{bmatrix} -d \\ 0 \end{bmatrix}$

Theorem 9 : Let A be a 2×2 matrix with $\lambda_1 = a - bi$ $(b \neq 0)$ and associated eigen vector $v_1 \in \mathbb{C}^2$. Then,

$$A = PCP^{-1} \text{ where } P = [\text{Real } v \quad \text{Imaginary } v] \text{ and } C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

Example of finding complex eigenvectors

Example: Find the eigenvectors given that $\lambda_1 = -1 + 2i$, $\lambda_2 = -1 - 2i$.

$$A = \begin{bmatrix} -2 & -1 \\ 5 & 0 \end{bmatrix}$$

must equal 0 when added up (multiplied by factor)

$$\rightarrow \quad A - \lambda_1 I = \begin{bmatrix} -2-(-1+2i) & -1 \\ 5 & -(-1+2i) \end{bmatrix} = \begin{bmatrix} -1+2i & -1 \\ 5 & 1-2i \end{bmatrix}$$

, we can choose that $v_1 = \begin{bmatrix} 1 \\ (-1+2i) \end{bmatrix}$ or $\begin{bmatrix} 1 \\ -1 \end{bmatrix} + i \begin{bmatrix} 0 \\ 2 \end{bmatrix}$