1.8 Matrix + Linear Transformations

A matrix transformation is a function. It acts upon vector x by multiplication by A, and maps it to b. $A\vec{x} = b$

So $T(x)$ from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a rule that assigns each vector x in $\mathbb{R}^n$ to a vector $T(x)$ in $\mathbb{R}^m$

$\mathbb{R}^n$ is the domain

$\mathbb{R}^m$ is the codomain / Image $\quad T: \mathbb{R}^n \to \mathbb{R}^m$ or $x \mapsto Ax$

Ex: If $A = \begin{bmatrix} 1 & 4 \\ -2 & 3 \\ 0 & 1 \end{bmatrix}$ and $x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $b = \begin{bmatrix} 2+4 \\ -4+3 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ 1 \end{bmatrix}$

Linear Transformations:

Transformations that preserve the operations of vector addition and scalar multiplication. T is linear if:

i) $T(u+v) = T(u) + T(v)$

ii) $T(cu) = cT(u)$

Every matrix transformation is a linear transformation but not every linear transformation is a matrix transformation

From i and ii we can also say that:

$T(0) = 0$ and $T(cu + dv) = cT(u) + dT(v)$