4.6 Change of Basis

- The basis for a vector space can be describe as the set of linearly independent vectors can be used to represent any vector within the vector space through linear combinations

Change of basis: Can be simply described by finding a transformation that allow us to express vectors from our old basis is our new basis

SO, we want to express our original vector own old basis, and find its equivalent in our other basis

You can think of it as: the same "kind" of combination is being applied (same weights) but in terms of vectors from the other basis

Consider the scenario:

$B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$ for vector space V such that

$b_1 = 4c_1 + c_2$ and $b_2 = -6c_1 + c_2$

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- find the exact vector with weight $[3, 1]$ in C

$\hookrightarrow$ $[x]_C = \begin{bmatrix} 4 & -6 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$

Theorem 15 : Suppose $B = \{ b_1, \ldots b_n \}$ and $C = \{ c_1, c_2, \ldots c_n \}$ be bases for vector space V. Then, $\exists$ a unique $n \times n$ matrix $P_{C \leftarrow B}$

such that $[x]_C = P_{C \leftarrow B} [x_B]$

The columns of $P_{C \leftarrow B}$ are the C - coordinate matrix from B to C , and each of the $[b_i]$ describe the relationship from b_i and C for c_1 to c_n.

written in terms of b

This allows you to express basis B in term of C

$P_{C \leftarrow B}$ columns are linearly independent , as they are the coordinate vectors of linearly independent set B

$P_{C \leftarrow B}$ is square, must be invertible as well

$P_{C \leftarrow B} = [c_1 \; c_2 \mid b_1 \; b_2]$

$[x]_C = P_{C \leftarrow B} [x]_b$

$\rightarrow$ $[x]_b = P_{C \leftarrow B}^{-1} [x]_C$

$\longrightarrow$ $(P_{C \leftarrow B})^{-1} = P_{B \leftarrow C}$