4.5) Dimensions of a Vector Space

9) If vector space V has a basis $\mathcal{B} = \{b_1, b_2, \dots b_n\}$, then any set in V containing more than N vectors must be linearly dependent

10) If vector space V has a basis of n vectors, then every basis of V must consist of exactly n vectors

If V is spanned by a finite set, then V is said to be finite-dimensional, and the dimension of V, written as $\dim V$, is the number of vectors in a basis for V. The dimension of $\{0\}$ vector space is defined to be zero. If V is not spanned by a finite set, then V is said to be infinite-dimensional

The subspaces of $\mathbb{R}^3$ can be classified by dimension

0 dimensional subspace

1 dimensional subspace

2 dimensional subspace

3 dimensional subspace

Theorem 11: Let H be the subspace of a finite dimensional vector space V. Any linearly independent set in H can be expanded, if necessary, to a basis for H. Also, H is finite dimensional and $\dim H \leq \dim V$

Theorem 12: Basis Theorem

V is a p-dimensional vector space, $p \geq 1$, any linearly independent set of exactly p vectors in V is automatically a basis for V. Any set of exactly p elements that spans V is automatically a basis for V

Rank Theorem: $\dim \text{Col } A + \dim \text{Nul } A = n$ for an $m \times n$ matrix

- Dimension of a column space and the dimension of a null space will always be equal to the total # of columns in our matrix