5.3 Diagonalization

- The process of taking a matrix and writing it as a product of matrices
- Specifically, it is the process of writing $A = XDX^{-1}$
  - D is a diagonal matrix, meaning that D only values along its diagonal

- If we multiply both sides to get D alone, we will find that $X^{-1}AX = D$, where X is said to "diagonalize" A

- This process is only unique for matrices A, if A has unique eigenvalues
  - Or if there are linearly independent eigenvectors

In $X^{-1}AX = D$, D is composed of the eigenvalues of A, while X is composed of the eigenvectors of A

- If we just find the corresponding eigenvalues, we can get the eigenvectors and create X

If we find eigenvectors, and the total # of them is less than the # of total eigenvalues, or don't form a basis for $\mathbb{R}^n$, then A is NOT diagonalizable

Powers of a diagonal matrix :

If $A = PDP^{-1}$, then $A^k = PD^kP^{-1}$

A square matrix is diagonalizable if A is similar to a diagonal matrix

Theorem 5: A is diagonalizable iff A has n linearly independent eigenvectors

$A = PDP^{-1}$ → n linearly independent eigenvectors of A

→ diagonal entries of D are the eigenvalues of A, and P is composed of the corresponding eigenvectors of those $\lambda_1, \dots \lambda_n$

Theorem 6: An n×n matrix is diagonalizable if it has n distinct eigenvalues

Theorem 7:

Let A be an $n \times n$ matrix whose distinct eigenvalues are $\lambda_1, \dots, \lambda_p$.

a. For $1 \leq k \leq p$, the dimension of the eigenspace for λ_k is less than or equal to the multiplicity of the eigenvalue λ_k.

b. The matrix A is diagonalizable if and only if the sum of the dimensions of the eigenspaces equals n, and this happens if and only if (*i*) the characteristic polynomial factors completely into linear factors and (*ii*) the dimension of the eigenspace for each λ_k equals the multiplicity of λ_k.

c. If A is diagonalizable and $\mathcal{B}_k$ is a basis for the eigenspace corresponding to λ_k for each k, then the total collection of vectors in the sets $\mathcal{B}_1, \dots, \mathcal{B}_p$ forms an eigenvector basis for $\mathbb{R}^n$.