# A Probabilistic Framework for Nonlinear Predictive Control With Adaptive Model Structure

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*Abstract*— **Many nonlinear systems exhibit multiple modes of behavior, frequently with only partial information on the conditions for transitioning between the different modes. This paper presents an algorithm for predictive control of nonlinear systems using multiple models for different operating regions in a probabilistic framework. The conditions for transitioning between regions are parameterized; the parameters are unknown but have known initial probability distributions. The proposed controller predicts an ensemble of state trajectories based on the probabilistic model for mode transitions. The controller learns about and adapts to the transition conditions over time using Bayesian estimation to update the probabilistic description with every new measurement. We demonstrate the algorithm on a model of a kilohertz-excited atmospheric-pressure plasma jet (APPJ) for biomedical applications. Simulations demonstrate how the proposed controller accounts for the uncertain system behavior and updates the probability distributions through the accumulation of data from the measurements. After a brief period of time where predictions from several models have a high degree of agreement with the system observations, the model-structure estimator successfully identifies the correct model.**

## I. INTRODUCTION

This paper develops a probabilistic framework for adaptive nonlinear model predictive control (NMPC) [1] for systems with uncertain model structures and multiple regions of operation. The problem is motivated by the challenge of computationally efficient high-precision control of complex systems that change behavior over operating regions with uncertainty in how to best model the behavior in a given region.

Recent years have witnessed growing interest in the notion of adaptive control based on multiple models (e.g., see [2] and the references therein). The multiple-model adaptive control methods, which are primarily developed for linear controllers, commonly entail switching between the models and controllers, meaning the control inputs are at any given time computed using one model rather than an ensemble of models that all achieve low prediction error.

In contrast to other approaches to control with different models for a set of operating conditions, like gain scheduling (see, e.g., [3] and references therein), the approach developed in this paper includes learning of uncertain conditions for switching between operating regions as well as a set of multiple model hypotheses for each of the regions.

This paper is organized as follows. Section II formulates the optimal control problem (OCP) in a probabilistic framework and outlines some of the challenges associated with solving the problem. Our proposed solution approach, which involves estimation of model probabilities and conditions for transitioning between operating regions, is presented in Section III; Section IV contains the control algorithm. The example demonstrating the algorithm is presented with simulation results in Section V. Section VI concludes the paper and contains some remarks on ongoing and future work.

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*Notation*

$\mathbb{Z}_{[a,b]}$ is the set of integers on the interval $[a, b]$. $\Pr(Z = z)$ is the probability that the unknown variable $Z$ has value $z$. $\mathrm{E}[Z \mid X]$ is the conditional expected value of $Z$ given $X$.

## II. PROBLEM FORMULATION

This paper proposes a probabilistic framework for predictive control of nonlinear systems that exhibit different types of behavior. The behavior depends on the region of operation, with several model hypotheses for each region. The different regions are characterized with parametric conditions on the states and/or control inputs and the parameters used in the characterization are not known. To simplify the notation we here consider two distinct regions, defined in terms of a function $R(\bar{x}^{[i]}(t), u(t), \theta^{[i]})$: $R(\cdot) \leq 0$ in region 1 and $R(\cdot) > 0$ in region 2. Here, $\bar{x}^{[i]}(t) \in \mathbb{R}^{n_x}$ is the predicted system state from model $i$ at time $t$, $u(t) \in \mathbb{R}^{n_x}$ is the control input, and $\theta^{[i]} \in \mathbb{R}^{n_\theta}$ is a set of model parameters, some of which are unknown. The unknown parameters that define the operating region boundaries are estimated online. To simplify the presentation we here consider the case of one model in the region where $R(\cdot) \leq 0$ and $n_\mathrm{m}$ models in the region where $R(\cdot) > 0$. The complete model is then written

$$\dot{\bar{x}}^{[0]} = f^{[0]}\big(\bar{x}^{[0]}(t), u(t), \theta^{[0]}\big), \qquad \text{if } R(\cdot) \leq 0 \quad \text{(1a)}$$
$$\dot{\bar{x}}^{[i]} = f^{[i]}\big(\bar{x}^{[i]}(t), u(t), \theta^{[i]}\big),\ i \in \mathbb{Z}_{[1,n_\mathrm{m}]}, \quad \text{if } R(\cdot) > 0 \quad \text{(1b)}$$

We assume that there exists a solution to the equation $R(\bar{x}^{[i]}(t), u(t), \theta^{[i]}) = 0$, although unknown, and that this solution is unique. Each model hypothesis $i$ has an associated probability $p_k^{[i]}$ of being the most accurate description of the system in its operating region, updated at every sampling time $k$, with $kh = t$ where $h$ is the sampling interval. That is, $p_k^{[i]} := \Pr(\mathcal{I} = i \mid \mathcal{Y}_k)$ where $\mathcal{I}$ is the index of the best model (in the sense of lowest prediction error) and $\mathcal{Y}_k$ is the set of system observations available at time $k$. Note that we do not assume that one model best describes the system at all times; which model best describes the system may change over time. The model probabilities add up to one for

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each region and are updated with every new measurement. We assume that the entire state vector is measured at every sampling time $k$ and that the measurements $y_k \in \mathbb{R}^{n_x}$ are corrupted with additive zero-mean Gaussian noise $v_k \in \mathbb{R}^{n_x}$ with covariance $P_v$.

The control inputs $u_j$ are bounded above and below by $u_{\min} \in \mathbb{R}^{n_u}$ and $u_{\max} \in \mathbb{R}^{n_u}$, respectively, on the prediction horizon $j \in \mathbb{Z}_{[k,k+N-1]}$:

$$u_{\min} \leq u_j \leq u_{\max}, \quad \forall\, j \in \mathbb{Z}_{[k,k+N-1]} \tag{2}$$

Similarly, for every model with associated probability at or below some lower bound $p_{\min}$, the controller enforces the lower and upper bounds $x_{\min} \in \mathbb{R}^{n_x}$ and $x_{\max} \in \mathbb{R}^{n_x}$, respectively, on the predicted states at every sampling time on the prediction horizon. That is, for all $i \in \mathcal{M}_p := \{i' \mid p_k^{[i']} \geq p_{\min}\}$,

$$x_{\min} \leq \bar{x}_{j+1}^{[i]} \leq x_{\max}, \quad \forall\, j \in \mathbb{Z}_{[k+1,k+N]}, \tag{3}$$

The control objective is minimization of the probability-weighted cost function

$$J_k(x_k) = \sum_{j=k}^{k+N-1} \sum_{i \in \mathcal{M}_p} p_k^{[i']} \Big( \ell_j(\bar{x}_j^{[i]}, u_j) + \ell_{k+N}(\bar{x}_{k+N}^{[i]}) \Big) \tag{4}$$

over the receding prediction horizon from sampling time $k$ to $k+N-1$. Here, $\ell_j$ is a stage cost for $j < k+N$ and a terminal cost for $j = k+N$. Note that state trajectories from every model $i \in \{i' \mid p_k^{[i']} \geq p_{\min}\}$ are included in the cost function.

### A. Solution Challenges

The primary challenge in the control problem considered in this paper is using measurements to determine the model probabilities $p_k^{[i]}$ and thereby identify which model best fits the data at any given sampling time $k$. Furthermore, since the exact conditions for switching between operating regions are not known, each model prediction is uncertain in where the change in behavior occurs. Another challenge is that the optimal control problem that results from the problem formulation is rendered nonsmooth by the transitions from one region to another, exacerbating the challenge of minimizing the control cost.

## III. SOLUTION APPROACH

This section contains the components of the approach we develop to solve the problem formulated in Section II. We first discuss the Bayesian scheme for estimating which model best describes the system and then present how the controller predicts an ensemble of state trajectories for minimizing the probability-weighted cost function.

### A. Bayesian Estimation

With the assumption of additive Gaussian measurement noise $v_k$, the likelihood $L_k^{[i]} := \Pr(y_k \mid \theta_{\text{th}} = \theta_{\text{th}}^{[i]})$ of the observation or measurement $y_k$ given $\theta_{\text{th}} = \theta_{\text{th}}^{[i]}$ is

$$L_k^{[i]} = \exp\left( -\frac{1}{2} \big(e_k^{[i]}\big)^{\top} P_v^{-1} e_k^{[i]} \right) \tag{5}$$

where

$$e_k^{[i]} = y_k - \mathrm{E}\big[y_k \mid \theta_{\text{th}} = \theta_{\text{th}}^{[i]}\big] \tag{6}$$

Note that large covariances $P_v$ in the measurement noise Using Bayes' rule, each probability is updated with the new information in $y_k$ as follows:

$$p_k^{[i]} = \frac{p_{k-1}^{[i]} L_k^{[i]}}{\sum_{j=1}^{n_{\text{m}}} p_{k-1}^{[j]} L_k^{[j]}} \tag{7}$$

Similar to [4], we introduce a lower bound $p_{\min}$ on the probabilities $p_k^{[i]}$ to ensure that no probability assumes a value so close to zero that it cannot assume nonzero values later. The probabilities are normalized after this correction so that $\sum_{i=1}^{n_{\text{m}}} p_{k-1}^{[i]} L_k^{[i]} = 1$.

### B. Model prediction

Models $i$ with a threshold-location probability $p_k^{[i]} < p_{\min}$ are not used to predict the probabilistic ensemble of system states.

With the given assumption on existence and uniqueness of a solution to $R(\bar{x}^{[i]}(t), u(t), \theta^{[i]}) = 0$, each model candidate $i \in \mathbb{Z}_{[i,n_{\text{m}}]}$ intersects model 0 at a unique threshold value $\theta_{\text{th}}^{[i]}$. Hence, the region of validity for each model candidate $i$ is well defined and resolving the model-structure uncertainty also resolves the uncertainty in where the different modes of behavior occur.

## IV. CONTROL ALGORITHM

The control input at sampling time $k$, $u_k$, is determined by solving the following optimal control problem:

$$\min_{\{u_j\}_{j=k}^{k+N-1}} \quad J_k(x_k) \tag{8a}$$

s. t. :

$$\text{the multiple models in (1)}, \qquad \forall\, t \in [k, (k+N-1)h] \tag{8b}$$

$$u_{\min} \leq u_j \leq u_{\max}, \qquad \forall\, j \in \mathbb{Z}_{[k,k+N-1]} \tag{8c}$$

$$x_{\min} \leq \bar{x}_{j+1}^{[i]} \leq x_{\max}, \qquad \forall\, j \in \mathbb{Z}_{[k+1,k+N]}, \tag{8d}$$

$$x_k = y_k \tag{8e}$$

The solution to this OCP is the open-loop optimal control-input sequence $\{u_j^{\text{o}}\}_{j=k}^{k+N-1}$.

The full algorithm for MPC with structural uncertainty and multiple models over different operating regions is as follows. At time $k$:

1) Record measurement $y_k$.
2) Use $y_k$ and the predicted states $\bar{x}_k^{[i]}$ (given $y_{k-1}$) from all models $i$ to determine the likelihoods $L_k^{[i]}$ using (5) and model/threshold probabilities $p_k^{[i]}$ using (7).
3) Minimize the probability-weighted cost (4) over the horizon $Z_{[k,k+N-1]}$ based on the predicted evolution of the states from every model $i$ that has a probability $p_k^{[i]} \geq p_{\min}$.
4) Store the one-step-ahead predicted states $\bar{x}_{k+1}^{[i]}$ for determining the likelihoods $L_k^{[i]}$ at the next sampling interval.

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5) Apply the first element of the solution $\{u_j^{\mathrm{o}}\}_{j=k}^{k+N-1}$ to the plant: $u_k = u_k^{\mathrm{o}}$.
6) Wait for the sampling interval $h$, set $k \leftarrow k+1$, and go to step 1).

## V. CASE STUDY

We now demonstrate the algorithm on a model of a kilohertz-excited atmospheric-pressure plasma jet (APPJ) for biomedical applications, similar to the model developed in [5]. Absent a first-principles model for the relationship between the applied voltage and the plasma current, this part of the model is empirical and based on experimental data. Based on the data, we identify two distinct modes of behavior, similar to those in [6], that are estimated through two different linear parameterizations. The results do not provide precise information on the condition for the change in behavior, but the condition is formulated in terms of a threshold location that by the given assumptions is unique to each model hypothesis, and is thereby estimated together with the model probabilities.

The control objective is to track a time-varying surface-temperature reference that increases linearly before assuming a constant value.

### A. Model

The surface temperature $T_{\mathrm{s}}$ varies with time according to the differential equation

$$\rho_{\mathrm{t}} c_{\mathrm{pt}} \dot{T}_{\mathrm{s}} = k_0 \cdot (T_0 - T_{\mathrm{s}}) - k_{\mathrm{inf}} \cdot (T_{\mathrm{s}} - T_{\mathrm{inf}}) \tag{9}$$

where $\rho_{\mathrm{t}}$ is the tube density, $c_{\mathrm{pt}}$ is the specific heat capacity of the tube, $k_0$ and $k_{\mathrm{inf}}$ are lumped heat-transfer coefficients, and $T_{\mathrm{inf}}$ is ambient temperature, all of which are constant. $T_0$ is the gas temperature and depends on the applied power $P_{\mathrm{in}}$ and the gas inlet flow velocity $v_{\mathrm{in}}$ through

$$-\rho_{\mathrm{in}} c_{\mathrm{p}} A_{\mathrm{c}} v_{\mathrm{in}} \cdot (T_{\mathrm{in}} - T_0) = \eta P_{\mathrm{in}} \tag{10}$$

where $\rho_{\mathrm{in}}$ is the density of the gas, $c_{\mathrm{p}}$ is the specific heat capacity of the gas $A_{\mathrm{c}}$ is the tube cross-sectional area, $T_{\mathrm{in}}$ is the gas inlet temperature, and $\eta$ is an efficiency factor, all of which are constant. $v_{\mathrm{in}}$ is a control input, whereas $P_{\mathrm{in}}$ depends on the applied power $V_{\mathrm{app}}$ and the plasma current $i_{\mathrm{p}}$ through

$$P_{\mathrm{in}} = \frac{1}{2} i_{\mathrm{p}}^{[i]} V_{\mathrm{app}} \cos\phi^{[i]}, \qquad i \in \mathbb{Z}_{[0,n_{\mathrm{m}}]} \tag{11}$$

where $\phi^{[i]}$ is the phase angle. The plasma current and phase angle both depend on $V_{\mathrm{app}}$. From analyzing experimental data, the current takes on two distinct behaviors: one when $V_{\mathrm{app}}$ takes on low values and another when $V_{\mathrm{app}}$ is high. Similarly, experimental data on how $i_{\mathrm{p}}$ and $V_{\mathrm{app}}$ are related show two distinct modes of dependence, one for low values of $V_{\mathrm{app}}$ and another for high values of $V_{\mathrm{app}}$. We fit one linear model to the low-voltage region but because of the higher variability in the high-voltage region, we consider five datasets and fit one linear model to each. The linear models are of the form

$$i_{\mathrm{p}}^{[i]} = c^{[i]} V_{\mathrm{app}} + d^{[i]}, \qquad i \in \mathbb{Z}_{[0,n_{\mathrm{m}}]} \tag{12}$$

where $i = 0$ is the index of the low-voltage model while the five high-voltage models have indices 1 through 5, and $c^{[i]}$ and $d^{[i]}$ are determined from experimental data. Each of the five high-voltage lines intersects the low-voltage line at different threshold voltages $V_{\mathrm{th}}$. It is not clear *a priori* which linear model and corresponding threshold voltage best matches the system when operated under feedback control; the controller therefore estimates the probability of each model hypothesis being the best description of the system in terms of prediction error. The model hypotheses can be summarized as

$$i_{\mathrm{p}}^{[i]} = \begin{cases} c^{[0]} V_{\mathrm{app}} + d^{[0]}, & \text{if } V_{\mathrm{app}} \le V_{\mathrm{th}}^{[i]} \\ c^{[i]} V_{\mathrm{app}} + d^{[i]},\ i \in \mathbb{Z}_{[1,n_{\mathrm{m}}]}, & \text{if } V_{\mathrm{app}} > V_{\mathrm{th}}^{[i]} \end{cases} \tag{13}$$

where the value of $V_{\mathrm{th}}$ that best describes the system is unknown. The function $R(\cdot)$ defining the operating regions in this problem is then a function of the control input $V_{\mathrm{app}}$ and the threshold voltage $V_{\mathrm{th}}$:

$$R(\bar{x}^{[i]}(t), u(t), \theta^{[i]}) = R(V_{\mathrm{app}}, V_{\mathrm{th}}^{[i]}) = V_{\mathrm{app}} - V_{\mathrm{th}}^{[i]} \tag{14}$$

The surface-temperature measurements $y_k$ at each sampling interval are corrupted with additive zero-mean Gaussian noise $v_k$: $y_k = T_{\mathrm{s},k} + v_k$.

The predicted surface temperature $\bar{T}_{\mathrm{s}}$ is constrained below and above by $T_{\mathrm{s,min}}$ and $T_{\mathrm{s,max}}$ respectively, while the control inputs $V_{\mathrm{app}}$ and $v_{\mathrm{in}}$ are bounded below and above by $V_{\mathrm{app,min}}$, $V_{\mathrm{app,max}}$, $v_{\mathrm{in,min}}$, and $v_{\mathrm{in,max}}$, respectively.

The stage cost in the control objective is the squared setpoint-tracking error $\ell_j = (\bar{T}_{\mathrm{s},j}^{[i]} - T_{\mathrm{s,ref},j})^2$; there is no terminal cost. The reference temperature is linearly increasing from 37 °C to 42 °C over the course of 120 s, and then remains at 42 °C for the remainder of the simulation.

The model parameters are given in Table I; the parameters and threshold locations for the linear $i_{\mathrm{p}}$-$v_{\mathrm{app}}$ model hypotheses are given in Table II; the control parameters are given in Table III. The simulation is performed with model 1 being the an exact representation of the plant. Note that the algorithm is not developed with the unrealistic assumption that the system is contained in the model set (see [7]); we here make this assumption for the purpose of clearer simulation results. Fig. 1 shows a hypothetical ensemble of model predictions as part of solving the OCP (8). Note how the predicted state trajectories diverge one by one as $V_{\mathrm{app}}$ crosses the different threshold candidates $V_{\mathrm{th}}^{[i]}$, $i \in \mathbb{Z}_{[i,n_{\mathrm{m}}]}$.

### B. Simulation Results

The simulation results are shown in Fig. 2. The first plot shows that the controller is able to keep the temperature-tracking error low, even in the first 50 seconds, a period in which the model probabilities do not reflect which model is the best representation of the system. As noted above, model 1 is an exact representation of the plant; however, the initial probability for this model is the somewhat low value $p_0^{[1]} = 0.1$. After the first 50 seconds, the model-structure and threshold estimator (5)–(7) has correctly identified that model 1 is the best representation of the system. After this

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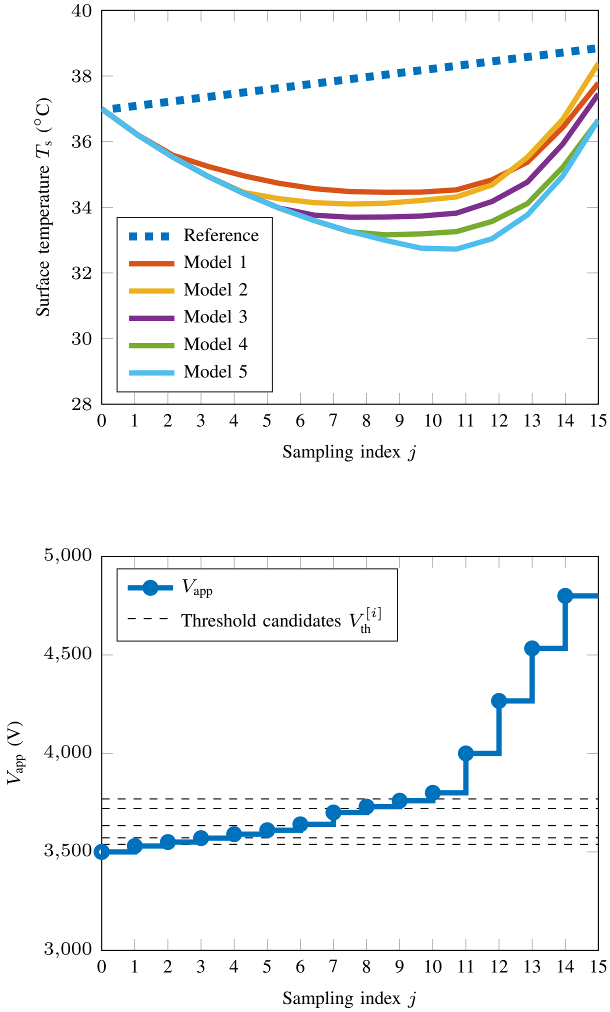


Fig. 1. Illustration of a hypothetical ensemble of model predictions (top plot) as part of solving the OCP (8) as a function of the input $V_{\text{app}}$ that gradually crosses the threshold candidates (bottom plot); $v_{\text{in}}$ held constant at $10\,\mathrm{m\,s^{-1}}$.

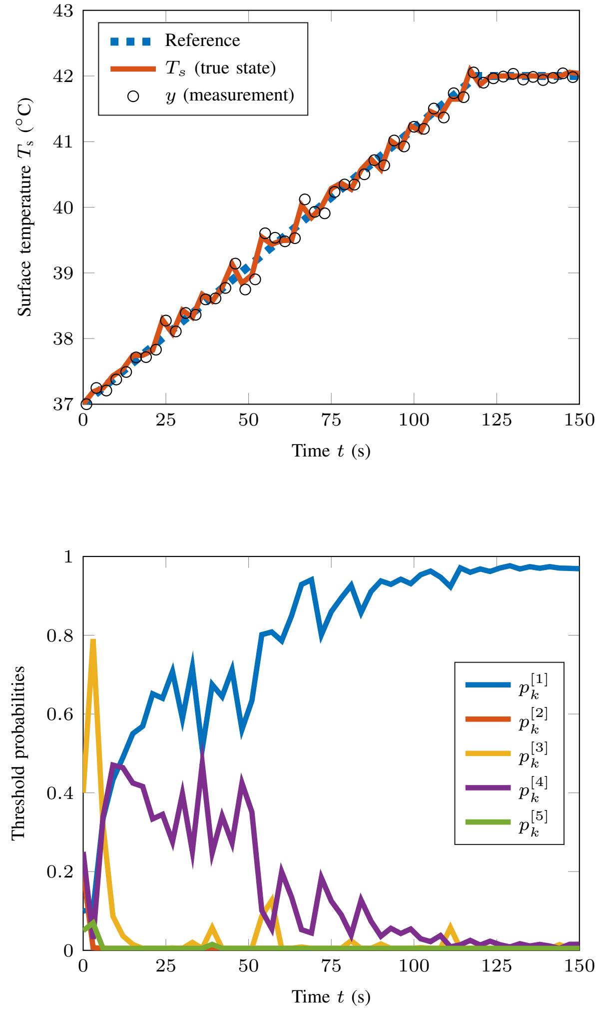


Fig. 2. Simulation results. The first plot shows the temperature reference, the temperature, and the measurements; the second plot shows the probabilities of each threshold location.

time, almost all additional system observations are consistent with the estimate, and $p_k^{[1]}$ approaches its maximum value.

## VI. CONCLUSIONS AND FUTURE WORK

### A. Conclusion

This paper presents a probabilistic framework for nonlinear predictive control with adaptation to model-structure uncertainty. The algorithm is developed for problems with multiple model hypotheses in several distinct regions of operation. We define the operating regions as conditions on states, inputs, and unknown parameters that are estimated online. To simplify the presentation we state the problem formulation for two distinct regions with an unknown parameter describing their boundary, and with multiple model hypotheses for only one of the regions. The algorithm recursively updates the probabilities of each of the models being the one that best describes the data generated by and observed from the system, with the implicit assumption that the most most suitable model may change over time. A case study applying the algorithm to an atmospheric-pressure plasma jet illustrates how the algorithm predicts an ensemble of state trajectories and minimizes a probability-weighted control cost. The simulation results show that the algorithm successfully identifies the most suitable model (in this case the "true" model) from the data and successfully keeps the output close to the reference trajectory, even when there are multiple model candidates that appear equally good.

### B. Future Work

We are currently extending the algorithm to also identify and adapt to unknown model parameters, enabling high-precision control of complex systems with a high level of uncertainty. Future work includes investigating the performance of algorithm on problems in which the operational regions have a more complex boundary, as well as improving the efficiency of the algorithm to enable its application to real-time control.

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TABLE I

PARAMETERS FOR THE APPJ MODEL IN SECTION V-A.

| Parameter | Value | Unit |
|---|---|---|
| $\rho_\mathrm{t}$ | 2200 | $\mathrm{kg\,m^{-3}}$ |
| $c_\mathrm{pt}$ | 670 | $\mathrm{J\,kg^{-1}\,K^{-1}}$ |
| $k_0$ | 9500 | $\mathrm{J\,m^{-3}\,K^{-1}}$ |
| $k_\mathrm{inf}$ | $50,000$ | $\mathrm{J\,m^{-3}\,K^{-1}}$ |
| $T_\mathrm{inf}$ | 298 | K |
| $\rho_\mathrm{in}$ | 0.1561 | $\mathrm{kg\,m^{-3}}$ |
| $c_\mathrm{p}$ | 5193 | $\mathrm{J\,kg^{-1}\,K^{-1}}$ |
| $A_\mathrm{c}$ | $7.06\times10^{-6}$ | $\mathrm{m^2}$ |
| $T_\mathrm{in}$ | 298 | K |
| $\eta$ | 0.9 | – |

TABLE II

COEFFICIENTS, THRESHOLD VALUES, AND INITIAL PROBABILITIES FOR THE MULTIPLE LINEAR MODELS (5).

| $i$ | $c^{[i]}$ (A) | $d^{[i]}$ (A/V) | $\phi^{[i]}$ | $V_\mathrm{th}^{[i]}$ (V) | $p_0^{[i]}$ |
|---|---|---|---|---|---|
| 0 | $3.764\times10^{-7}$ | 0.0037 | $75°$ | | |
| 1 | $4.691\times10^{-6}$ | $-0.0115$ | $70°$ | $3.5383\times10^{3}$ | 0.10 |
| 2 | $6.131\times10^{-6}$ | $-0.0168$ | $70°$ | $3.5716\times10^{3}$ | 0.20 |
| 3 | $5.429\times10^{-6}$ | $-0.0146$ | $70°$ | $3.6334\times10^{3}$ | 0.40 |
| 4 | $5.183\times10^{-6}$ | $-0.0142$ | $70°$ | $3.7203\times10^{3}$ | 0.25 |
| 5 | $6.812\times10^{-6}$ | $-0.0205$ | $70°$ | $3.7690\times10^{3}$ | 0.05 |

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TABLE III

PARAMETERS FOR THE OCP SOLVED IN THE CASE STUDY IN SECTION V-B.

| Parameter | Value | Unit |
|---|---|---|
| $N$ | 15 | – |
| $h$ | 3 | s |
| $T_\mathrm{s,min}$ | 310 | K |
| $T_\mathrm{s,max}$ | 318 | K |
| $V_\mathrm{app,min}$ | $2\times10^3$ | V |
| $V_\mathrm{app,max}$ | $2\times10^3$ | V |
| $v_\mathrm{in,min}$ | 10 | $\mathrm{m\,s^{-1}}$ |
| $v_\mathrm{in,max}$ | 35 | $\mathrm{m\,s^{-1}}$ |
| $P_v$ | $(0.05)^2$ | $\mathrm{K^2}$ |
| $T_\mathrm{s,0}$ | 310 | K |

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