Constructing effective field theory potentials to reproduce np $^1S_0$ scattering phase shifts

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In this work, a pionless effective field theory and an effective field theory including pions are developed to describe np scattering for the spin-singlet, $l = 0$ partial wave. The low-energy constants (LECs) are determined by fitting calculated scattering phase shifts to the phenomenological three-Yukawa model from Project 1. The phase shifts are compared to the Yukawa model for different orders of the effective field theories as functions of energy are compared to determine the breakdown scale. The dependence of the phase shift on the value of the cutoff used in the effective field theory is also explored.

I. INTRODUCTION

The underlying theory of the nuclear force is quantum chromodynamics (QCD). However, low-energy probes cannot resolve the short-distance details of a system. According to renormalization theory, this short-distance structure can be replaced with something simpler without altering low-energy observables. Consequently, *effective* field theories (EFTs) can be constructed to reproduce low-energy experimental data without performing a full QCD calculation.

In Project 1, the potential in MeV for the $l = 0$ partial wave of the neutron-proton interaction was given by the sum of three Yukawa terms

$$V(r) = -10.463\frac{e^{-\mu r}}{\mu r} - 1650.6\frac{e^{-4\mu r}}{\mu r} + 6484.3\frac{e^{-7\mu r}}{\mu r} \quad (1)$$

where $\mu = 0.7$ fm^{-1}, the inverse of the pion mass. Using this potential, the np $^1S_0$ scattering phase shifts for different lab-frame energies were calculated by solving the discretized Lippmann-Schwinger equation numerically. Here, an EFT is constructed to approximate the np $^1S_0$ phase shifts from Project 1.

To construct a field theory to model nucleon-nucleon (NN) interactions, one can begin by treating nucleons and mesons as fundamental degrees of freedom. An example Lagrangian including nucleons as a Dirac field and a complex pseudoscalar meson field is shown in Eq. 2.

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - M)\psi + \frac{1}{2}(\partial_\mu\phi^\dagger\partial^\mu\phi - m^2\phi^\dagger\phi) - ig\bar{\psi}\gamma^5\phi\psi \quad (2)$$

The pseudoscalar propagator is defined by Eq. 3,

$$G(x-y) \equiv \langle 0|T\{\phi(x)\phi^\dagger(y)\}|0\rangle \quad (3)$$

where $|0\rangle$ is the vacuum state in the non-interacting theory. It can be shown that this operator is a Green's function for the free pseudoscalar equation of motion (the Klein-Gordon equation).

$$(\partial^2 + m^2)G(x) = -i\delta^4(x) \quad (4)$$

In momentum-space, the propagator then takes the simple form given in Eq. 5.

$$\tilde{G}(k) = \frac{i}{k^2 - m^2 + i\epsilon} \quad (5)$$

It now becomes clear that for low-energy processes where the four-momentum transfers are small compared to the meson mass, the propagator is approximately constant ($\approx -i/m^2$). A constant momentum-space propagator corresponds to a contact interaction in coordinate-space, where the asymptotic particles couple at a single vertex. This suggests that NN scattering at energies significantly below the lightest meson mass should be possible to describe with a simpler theory in which mesons are not included as explicit degrees of freedom, and that NN interactions in this theory would take the form of contact interactions in coordinate-space.

Now we consider a pionless EFT, where there are no mesons explicitly included in the Lagrangian. The goal is to describe np $^1S_0$ scattering at low energies where even the longest-range Yukawa term in the potential of Eq. 1 is not resolved. Neglecting spin-dependent and isospin-dependent terms, the leading order term for the EFT is given by the constant

$$V^{LO}_{^1S_0}(p', p) = C_0 \quad (6)$$

and the NLO and NNLO corrections are

$$V^{NLO}_{^1S_0}(p', p) = C_2(p^2 + p'^2) \quad (7)$$

$$V^{NNLO}_{^1S_0}(p', p) = C_4(p^4 + p'^4) + C_4' p^2 p'^2 \quad (8)$$

Note that on-shell ($p' = p$), the two terms in the expression for $V^{NNLO}_{^1S_0}$ are equivalent so only one of the constants c_4 and c_4' is needed.

To construct an EFT that works at higher energies, the one-pion exchange contribution can be added to the potentials in Eqs. 6-8 above. To do this, the longest-range Yukawa term in the potential of Eq. 1 can be included as an explicit degree of freedom. This potential is

$$V(k', k) = \frac{-10.463 \text{ MeV}}{4\mu k k'} \ln\left[\frac{\mu^2 + (k + k')^2}{\mu^2 + (k - k')^2}\right] \quad (9)$$

where the first term from Eq. 1 has been converted to momentum space and $\hbar = c = 1$. While pion exchange is explicitly included in this theory, heavier mesons are not. So in addition to the Yukawa potential describing one-pion exchange, there will still be contact terms representing all of the heavier mesons, which have been excluded as explicit degrees of freedom by construction.

II. METHODS

In order to extract values for the parameters of the theory, it is necessary to calculate observable quantities (e.g. scattering phase shifts) which can be fit to experimental data. Using standard statistical techniques estimates for the physical values of the parameters can be determined.

To calculate the phase shifts, the Lippmann-Schwinger equation was solved numerically, using the code developed in Project 1, for various values of the coupling constants.

The parameter space for the theory was searched using an iterative grid search, where each parameter was first varied over many orders of magnitude to determine the scales for each. Once that was determined, the grid was sub-ranged for each parameter within its respective order of magnitude. This is somewhat of a brutish approach, and more elegant search methods certainly exists, but because the number of parameters in our theory is relatively low, we can afford to simply search this way.

Selection of the parameter was based on a χ^2-esque method, where the squared difference between the EFT phase shift and the phenomenological phase shift was minimized.

We chose to match the EFT phase shift to the phenomenological theory developed in Project 1 rather than to experimental data because it allows for more flexibility in terms of the momentum grid points at which we do the matching. It is advised in this kind of theory to fit the LECs to the most infrared possible “data”, and our phenomenological model can be evaluated at energies lower than where data are available.

Contributions to the momentum-space potential at greater-than-leading order are UV-divergent, and must be regulated in order for the numerical solver to converge. A naïve sharp UV cutoff would lead to Gibbs overshoot, so a super-Gaussian function was chosen instead to regulate the divergences in a smooth way. Using the cutoff, the momentum-space potential becomes

$$V(p', p) \to e^{-p'^4/\Lambda^4} V(p', p) e^{-p^4/\Lambda^4} \qquad (10)$$

As a first pass, the value of the cutoff was chosen to be approximately equal to the expected breakdown scale for the theory (the pion mass).

III. RESULTS

Here we examine the results of the effective field theory calculations by constructing the log-log error plots of Lepage. Note that the comparisons are all done relative to the phase shift results from the phenomenological Yukawa potential from Project 1 shown in Eq. 1. The vertical axes for all figures is the absolute value of the difference of this phase shift and the phase shift calculated using effective field theory at multiple orders. All parameters were adjusted to reproduce the phase shifts for energies between 0.001 MeV to 1 MeV. It is important to note that due to our approach to fitting our parameters, the values of the parameters used could correspond to a local minimum of the squared difference between the phase shifts for the effective field theory and the phase shifts for the potential from Project 1. This is especially apparent in the NNLO case.

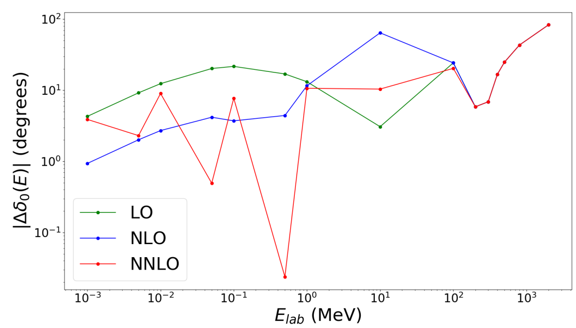

FIG. 1. Comparison for pionless EFT calculations at multiple orders. The breakdown scale is clearly in the region of the pion mass, as expected.

In Figure 1, we show the results for the comparison of different orders using a cutoff of $\Lambda = 138$ MeV. We can clearly pick off the breakdown scale as the point where the multiple orders converge. This is at the pion mass. The relatively poor results for NNLO could be due to the parameter set corresponding to a local minimum in squared phase shift difference.

We are able to reproduce the expected behavior for the dependence on the cutoff in Figure 2. The theoretical errors for an EFT calculation of a low-energy process of energy Q scale roughly as $\mathrm{Max}((Q/\Lambda)^n, (Q/\Lambda_b)^n)$ where Λ_b is the breakdown scale of the EFT. Since we must reconstrain the constants for every value of the cutoff, it is possible that the constants are not optimal for the observed $\Lambda_b = 138$ MeV, but we do clearly see the diminishing returns as Λ is increased beyond the breakdown scale and we see the power law dependence of the error.

Next, the one-pion exchange was included in the EFT by explicitly adding the first Yukawa term of Eq. 9 to the contact terms in Eq. 6-8. We were unable to find the optimal parameter set at NNLO after including a Yukawa term in addition to our EFT parameters. We

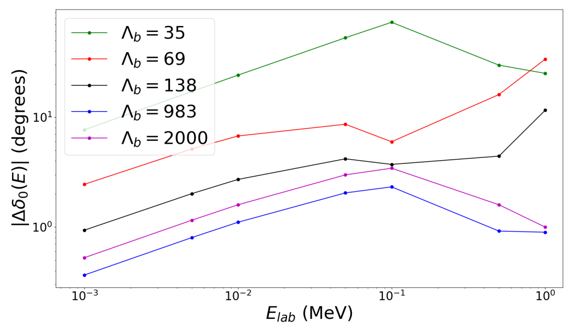

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FIG. 2. Comparison of pionless effective field theory calculation at NLO for multiple cutoff values (in MeV).

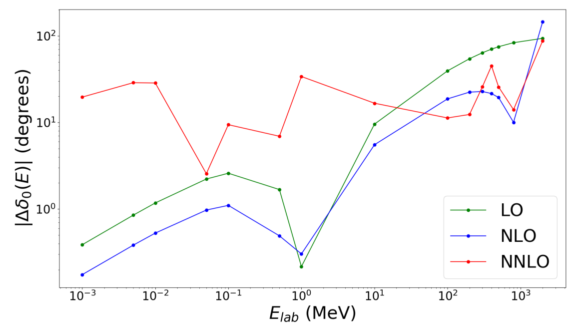

FIG. 3. Comparison for EFT calculations at multiple orders including the Yukawa term. The breakdown scale is roughly in the region of the four times the pion mass.

do however see the expected improvement from LO to NLO. The breakdown is expected to be near the four times the pion mass since that is the longest-range term of Eq. 1 not included explicitly in the EFT. The observed breakdown scale is roughly in this region.

IV. CONCLUSIONS

In conclusion, we were able to fit the constants for a pionless effective field theory and an effective field theory with one-pion exchange explicitly included to reproduce the $^1S_0$ scattering phase shift for the np system. The log-log error behavior for the EFTs was similar to the expected behavior given by Lepage. At the breakdown scale, the errors for all orders of the EFT converge and the EFT improves as the chosen cutoff approaches the breakdown scale but has minimal improvement for cutoffs beyond the breakdown scale. Our results would be considerably improved if we could determine a more sophisticated method for fitting the constants, particularly for the NNLO case.