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Homework 1

Problem 1

i) Linear equations:

$$x(0) = x_1 = 0$$
$$y(0) = y_1 = 0$$
$$x(t_f) = x_1 + x_2 t_f + x_3 t_f^2 + x_4 t_f^3 = 5$$
$$y(t_f) = y_1 + y_2 t_f + y_3 t_f^2 + y_4 t_f^3 = 5$$
$$\dot{x}(0) = x_2 = V(0)\cos(\theta(0)) = 0.5\cos\left(-\frac{\pi}{2}\right)$$
$$\dot{y}(0) = y_2 = V(0)\sin(\theta(0)) = 0.5\sin\left(-\frac{\pi}{2}\right)$$
$$\dot{x}(t_f) = x_2 + 2x_3 t_f + 3x_4 t_f^2 = V(t_f)\cos\left(\theta(t_f)\right) = 0.5\cos\left(-\frac{\pi}{2}\right)$$
$$\dot{y}(t_f) = y_2 + 2y_3 t_f + 3y_4 t_f^2 = V(t_f)\sin\left(\theta(t_f)\right) = 0.5\sin\left(-\frac{\pi}{2}\right)$$

ii) If $V(t_f) = 0$, then linear matrix A will be singular, or in other words, the det(A) = 0

iii) To compute the coefficients $\boldsymbol{x} = [x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4]^T$, we need to solve $A\boldsymbol{x} = \boldsymbol{b}$

where A represents the left-hand side of the linear equations defined in i)

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & t_f & t_f^2 & t_f^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2t_f & 3t_f^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2t_f & 3t_f^2 \end{bmatrix}$$

where b represents the right-hand side of the linear equations defined in i)

$$b = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 5 \\ 0.5 \\ -0.5 \\ 0.5 \\ -0.5 \end{bmatrix}$$

In compute_traj, theta is given as:

$$\theta = \tan^{-1}\frac{\dot{y}}{\dot{x}}$$

In compute_controls, V and om are given as:

$$V = \sqrt{(\dot{x}^2 + \dot{y}^2)}$$

$$\omega = \dot{\theta} = \frac{d}{dt}\left(\tan^{-1}\frac{\dot{y}}{\dot{x}}\right) = \frac{1}{1+\frac{\dot{y}}{\dot{x}}^2}\frac{d\left(\frac{\dot{y}}{\dot{x}}\right)}{dt} = \frac{1}{1+\frac{\dot{y}}{\dot{x}}^2}\left(\frac{\ddot{y}}{\dot{x}} - \frac{\dot{y}\ddot{x}}{\dot{x}^2}\right)$$

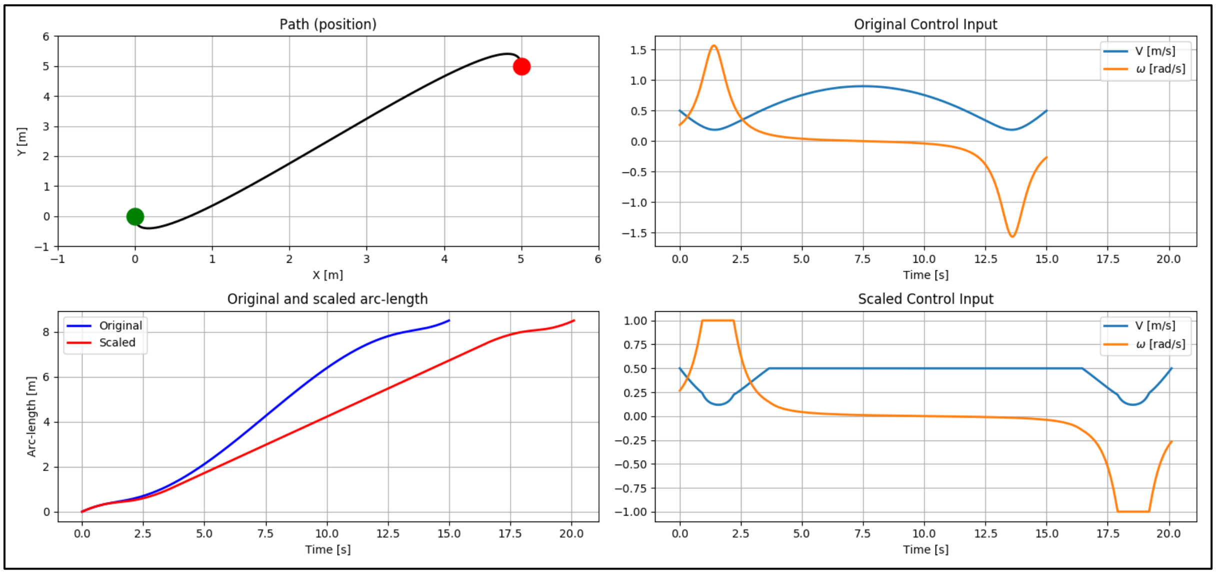
iv) In the code, the two contraints enforced while rescaling V were as follows:

$$|\tilde{V}| \leq V_{\max}$$

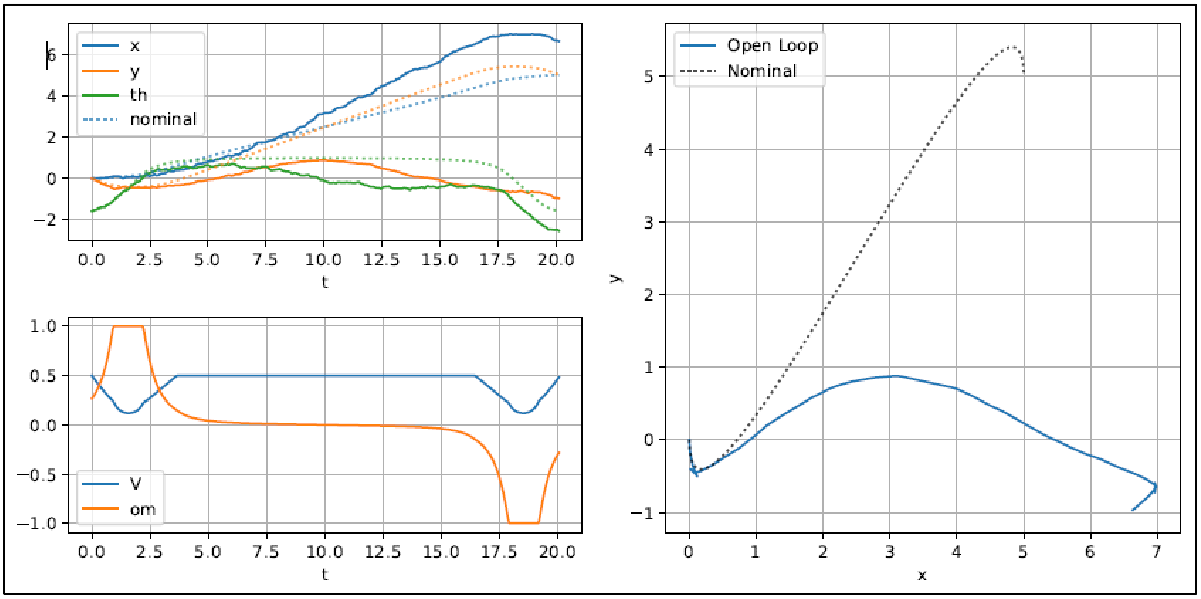
$$|\tilde{\omega}| = \frac{\omega}{V}\tilde{V} \leq \omega_{max}, or$$

$$|\tilde{V}| \leq \frac{\omega_{max}}{\omega} V$$

v) Differential Flatness Figure

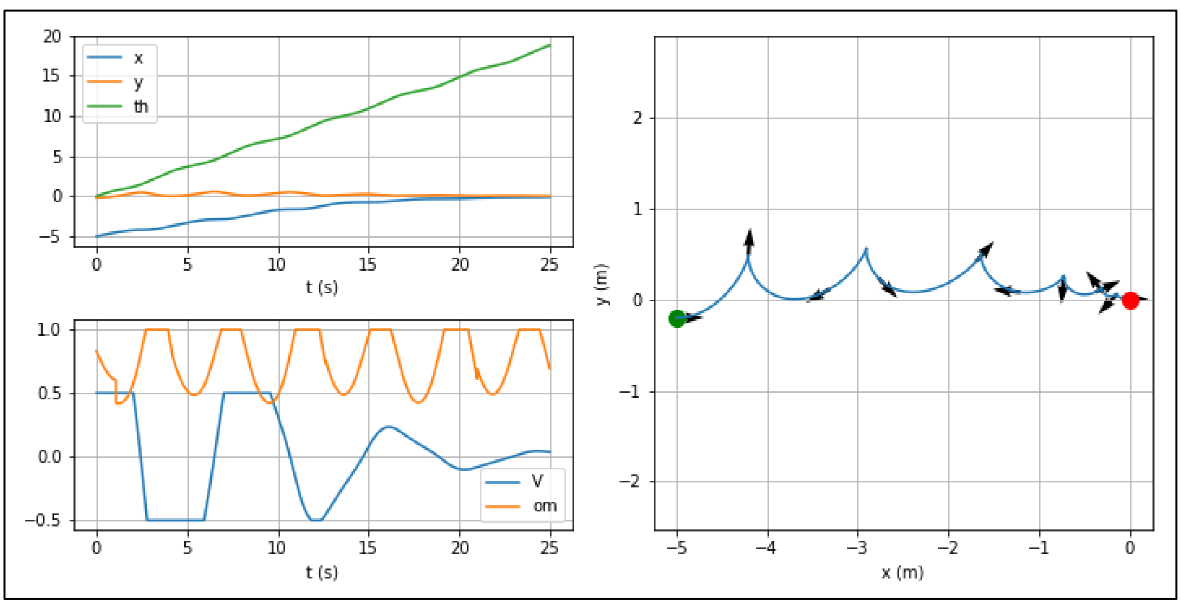


vi) Open Loop

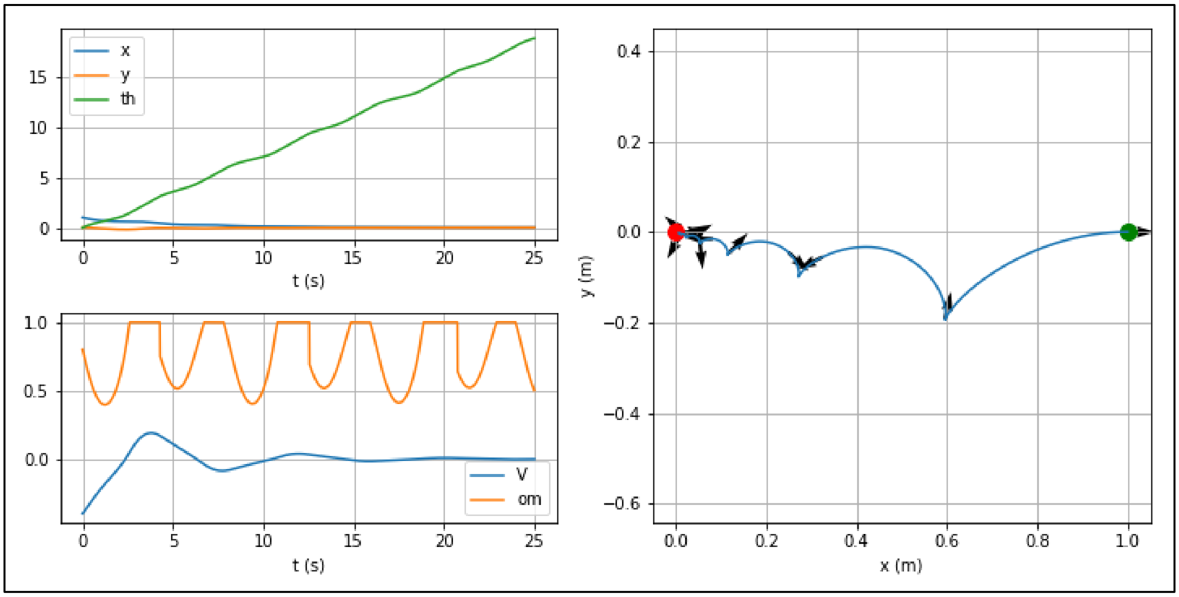


Problem 2

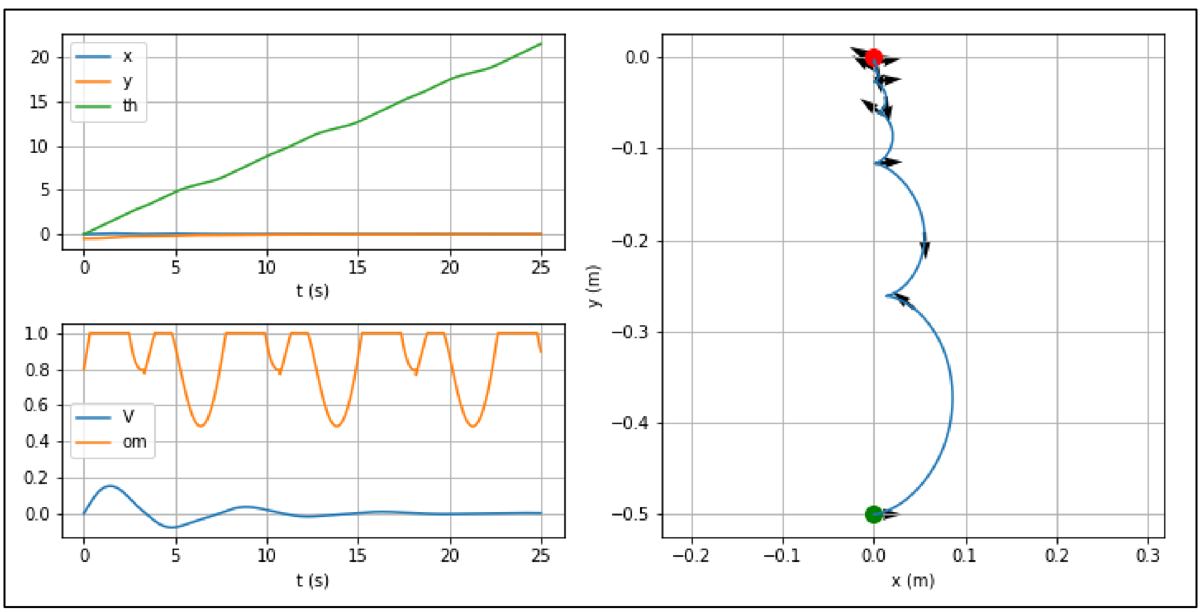
Forward



Reverse



Parallel



Problem 3

i) From the notes:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -V\sin(\theta) \\ \sin(\theta) & V\cos(\theta) \end{bmatrix} \begin{bmatrix} a \\ \omega \end{bmatrix} := \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

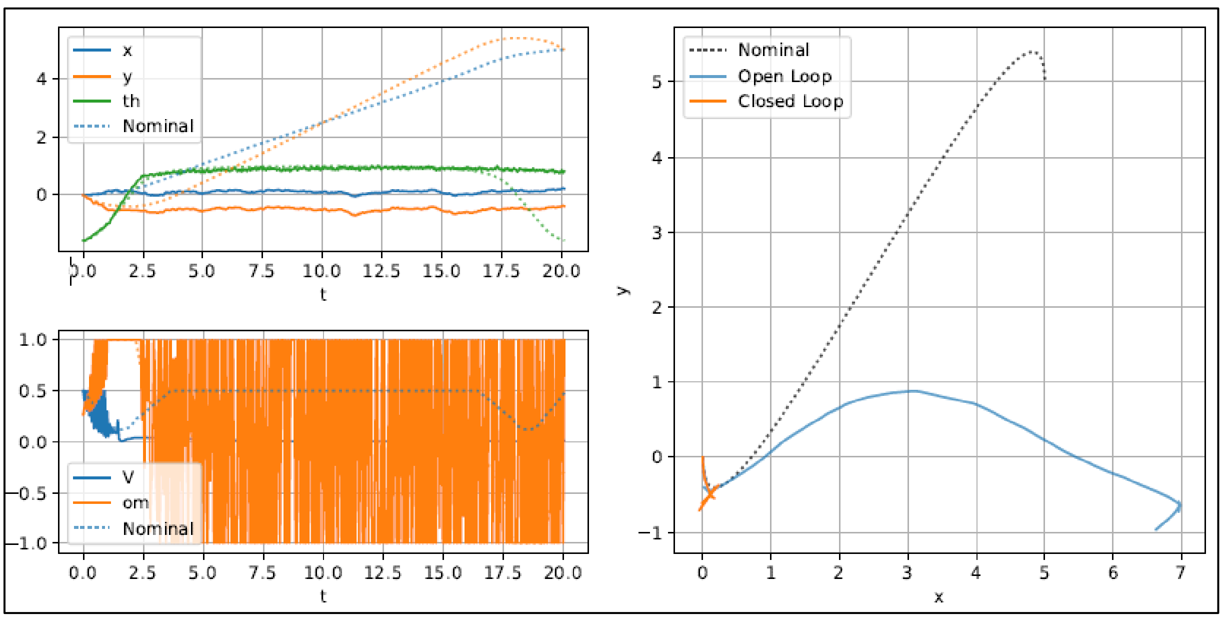
$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \ddot{x}_d + k_{px}(x_d - x) + k_{dx}(\dot{x}_d - \dot{x}) \\ \ddot{y}_d + k_{py}(y_d - y) + k_{dy}(\dot{y}_d - \dot{y}) \end{bmatrix}$$

Solving the linear system yields a and ω. To get V, we need another ODE:

$$a = \dot{V} = \frac{d}{dt}\left(\sqrt{\dot{x}_d{}^2 + \dot{y}_d{}^2}\right) = \frac{\dot{x}_d\ddot{x}_d + \dot{y}_d\ddot{y}_d}{\sqrt{\dot{x}_d{}^2 + \dot{y}_d{}^2}} = \frac{\dot{x}_d\ddot{x}_d + \dot{y}_d\ddot{y}_d}{V}$$

$$V = \left(\frac{\dot{x}_d\ddot{x}_d + \dot{y}_d\ddot{y}_d}{a}\right)$$

ii) Closed Loop (Can't seem to find the instability here)



Problem 4

i) Hamiltonian:

$$H = g + pa = \lambda + V^2 + \omega^2 + p_1 V\cos(\theta) + p_2 V\sin(\theta) + p_3\omega$$

NOCs:

$$\dot{x}_1 = \frac{\partial H}{\partial p_1} = V\cos(\theta), \qquad \dot{x}_2 = \frac{\partial H}{\partial p_2} = V\sin(\theta), \qquad \dot{x}_3 = \frac{\partial H}{\partial p_3} = \omega$$

$$\dot{p}_1 = -\frac{\partial H}{\partial x_1} = 0, \qquad \dot{p}_2 = -\frac{\partial H}{\partial x_2} = 0, \qquad \dot{p}_3 = -\frac{\partial H}{\partial x_3} = p_1 V\sin(\theta) - p_2 V\cos(\theta)$$

$$0 = 2V + p_1\cos(\theta) + p_2\sin(\theta)\,, \qquad 0 = 2\omega + p_3$$

Initial BC:

$$x(0) = 0, \qquad y(0) = 0, \qquad \theta(0) = -\frac{\pi}{2}$$

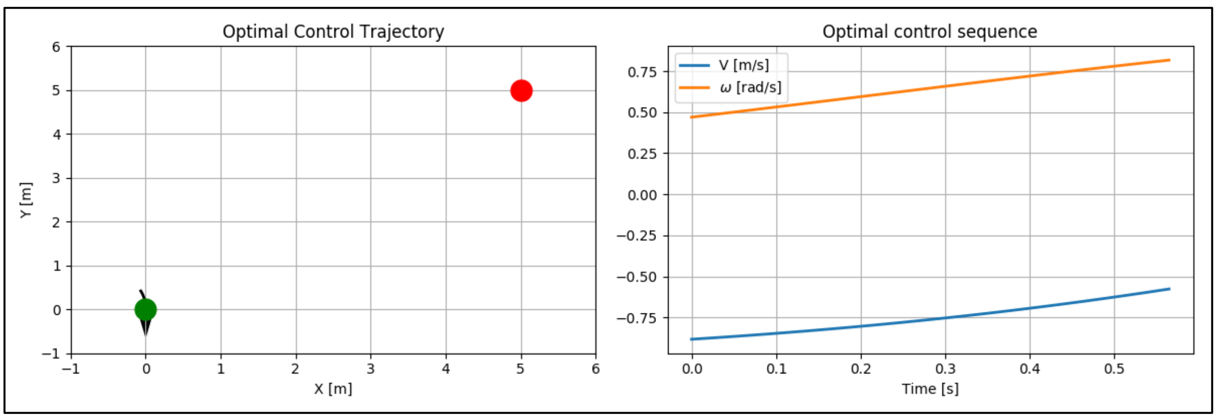
Final BC:

$$x(t_f) = 5, \qquad y(t_f) = 5, \qquad \theta(t_f) = -\frac{\pi}{2}$$

$$H(t_f) = \lambda + V(t_f)^2 + \omega(t_f)^2 + p_1(t_f)V(t_f)\cos\left(\theta(t_f)\right) + p_2(t_f)V(t_f)\sin\left(\theta(t_f)\right) + p_3(t_f)\omega(t_f)$$

where $V(t_f) = -\frac{p_1(t_f)\cos(\theta(t_f)) + p_2(t_f)\sin(\theta(t_f))}{2}$ and $\omega(t_f) = -\frac{p_3(t_f)}{2}$

Optimal Control Plot



iv-v) Could not complete because I could not get my code to work. Normally I would take the extra days to complete, but I have a quiz in another class I have to study for.