

CptS 223 Homework #3 - Heaps, Hashing, Sorting

Please complete the homework problems on the following page using a separate piece of paper. Note that this is an individual assignment and all work must be your own. Be sure to show your work when appropriate. Please scan the assignment and upload the PDF to Git, but also bring a printed out copy for the grading TAs. We've found that many of the scans are difficult to read and notate.

1. [6] Starting with an empty hash table with a fixed size of 11, insert the following keys in order into four distinct hash tables (one for each collision mechanism): {12, 9, 1, 0, 42, 98, 70, 3}. You are only required to show the final result of each hash table. In the very likely event that a collision resolution mechanism is unable to successfully resolve, simply record the state of the last successful insert and note that collision resolution failed. For each hashtable type, compute the hash as follows:

$$\text{hashkey}(\text{key}) = (\text{key} * \text{key} + 3) \% 11$$

Separate Chaining (buckets)

	3		0	12 1 98			9 42	70		
0	1	2	3	4	5	6	7	8	9	10

To probe, start at $i = \text{hashkey}$ and do $i++$ if collisions continue

Linear Probing: $\text{probe}(i') = (i + 1) \% \text{TableSize}$

	3		0	12	1	98	9	42	70	
0	1	2	3	4	5	6	7	8	9	10

Quadratic Probing: $\text{probe}(i') = (i * i + 5) \% \text{TableSize}$

	3		0	12		42	9	98	70	1
0	1	2	3	4	5	6	7	8	9	10

2. [3] For implementing a hash table. Which of these would probably be the best initial table size to pick?

Table Sizes:

1 100 101 15 500

Why?

101 would need to be chosen because it is the only prime number, so would avoid collisions.

3. [4] For our running hash table, you'll need to decide if you need to rehash. You just inserted a new item into the table, bringing your data count up to 53491 entries. The table's vector is currently sized at 106963 buckets.

- Calculate the load factor (λ):

$$53491/106963 = 0.50009$$

- Given a linear probing collision function should we rehash? Why?

You would need to rehash because the load factor is greater than 0.5

- Given a separate chaining collision function should we rehash? Why?

You would not need to rehash because the load factor is not greater than 1.

4. [4] What is the Big-O of these actions for a well designed and properly loaded hash table with N elements?

Function	Big-O complexity
Insert(x)	$O(1)$
Rehash()	$O(N)$
Remove(x)	$O(1)$
Contains(x)	$O(1)$

5. [3] If your hash table is made in C++11 with a vector for the table, has integers for the keys, uses linear probing for collision resolution and only holds strings... would we need to implement the Big Five for our class? Why or why not?

Yes because you will still need to use the assignment operators for move and copy.

6. [6] Enter a reasonable hash function to calculate a hash key for these prototypes:

```
int hashit( int key, int TS )  
{
```

```
    return key % TS;
```

```
}
```

```
int hashit( string key, int TS )  
{
```

```
}
```

7. [3] I grabbed some code from the Internet for my linear probing based hash table because the Internet's always right. The hash table works, but once I put more than a few thousand entries, the whole thing starts to slow down. Searches, inserts, and contains calls start taking *way* longer than $O(1)$ time and my boss is pissed because it's slowing down the whole application services backend I'm in charge of. I think the bug is in my rehash code, but I'm not sure where. Any ideas why my hash table starts to suck as it grows bigger?

```
/**
 *      Rehashing      for      linear      probing      hash      table.
 */
void      rehash(      )
{
    vector<HashEntry> oldArray = array;

    //      Create      new      double-sized,      empty      table
    array.resize(      2      *      oldArray.size(      )      );
    for(      auto      &      entry      :      array      )
        entry.info = EMPTY;

    //      Copy      table      over
    currentSize = 0;
    for(      auto      &      entry      :      oldArray      )
        if(      entry.info == ACTIVE      )
            insert(      std::move(      entry.element      )      );
}
```

The reason it is slowing down is because when rehashing a larger number is chosen that is not prime.

8. [4] Time for some heaping fun! What's the time complexity for these functions in a binary heap of size N ?

Function	Big-O complexity
insert(x)	$O(\log N)$
findMin()	$O(1)$
deleteMin()	$O(\log N)$
buildHeap(vector<int>{1...N})	$O(1)$

9. [4] What would a good application be for a priority queue (a binary heap)? Describe it in at least a paragraph of why it's a good choice for your example situation.

A priority queue would be a good choice for a medical application. In a hospital there are many patients that need to get helped in a line but if there is someone that comes in with life threatening injuries then they will move to the front of the line because they would have priority over the others. Everyone will have different priorities which will determine when they get helped.

10. [4] For an entry in our heap (root @ index 1) located at position i , where are it's parent and children?

Parent: $i/2$

Children: right child: $2i+1$

Left child: $2i$

What if it's a d-heap?

Parent: $((i-2)/d)+1$

Children: $d(i-1)+1+(\text{amount of children})$

11. [6] Show the result of inserting 10, 12, 1, 14, 6, 5, 15, 3, and 11, one at a time, into an initially empty binary heap. Use a 1-based array like the book does. After insert(10):

10										
----	--	--	--	--	--	--	--	--	--	--

After insert (12):

10	12									
----	----	--	--	--	--	--	--	--	--	--

etc:

1	10	12								
---	----	----	--	--	--	--	--	--	--	--

1	10	12	14							
---	----	----	----	--	--	--	--	--	--	--

1	6	10	12	14						
---	---	----	----	----	--	--	--	--	--	--

1	6	5	14	12	10					
---	---	---	----	----	----	--	--	--	--	--

1	6	5	14	12	10	15				
---	---	---	----	----	----	----	--	--	--	--

1	3	5	6	12	10	15	14			
---	---	---	---	----	----	----	----	--	--	--

1	3	5	6	12	10	15	14	11		
---	---	---	---	----	----	----	----	----	--	--

12. [4] Show the same result (only the final result) of calling buildHeap() on the same vector of values: {10, 12, 1, 14, 6, 5, 15, 3, 11}

1	3	5	6	12	10	15	14	11		
---	---	---	---	----	----	----	----	----	--	--

13. [4] Now show the result of three successive deleteMin operations from the

prior heap:

3	6	5	11	12	10	15	14			
---	---	---	----	----	----	----	----	--	--	--

5	6	10	11	12	15	14				
---	---	----	----	----	----	----	--	--	--	--

6	11	10	14	12	15					
---	----	----	----	----	----	--	--	--	--	--

14. [4] What are the average complexities and the stability of these sorting algorithms:

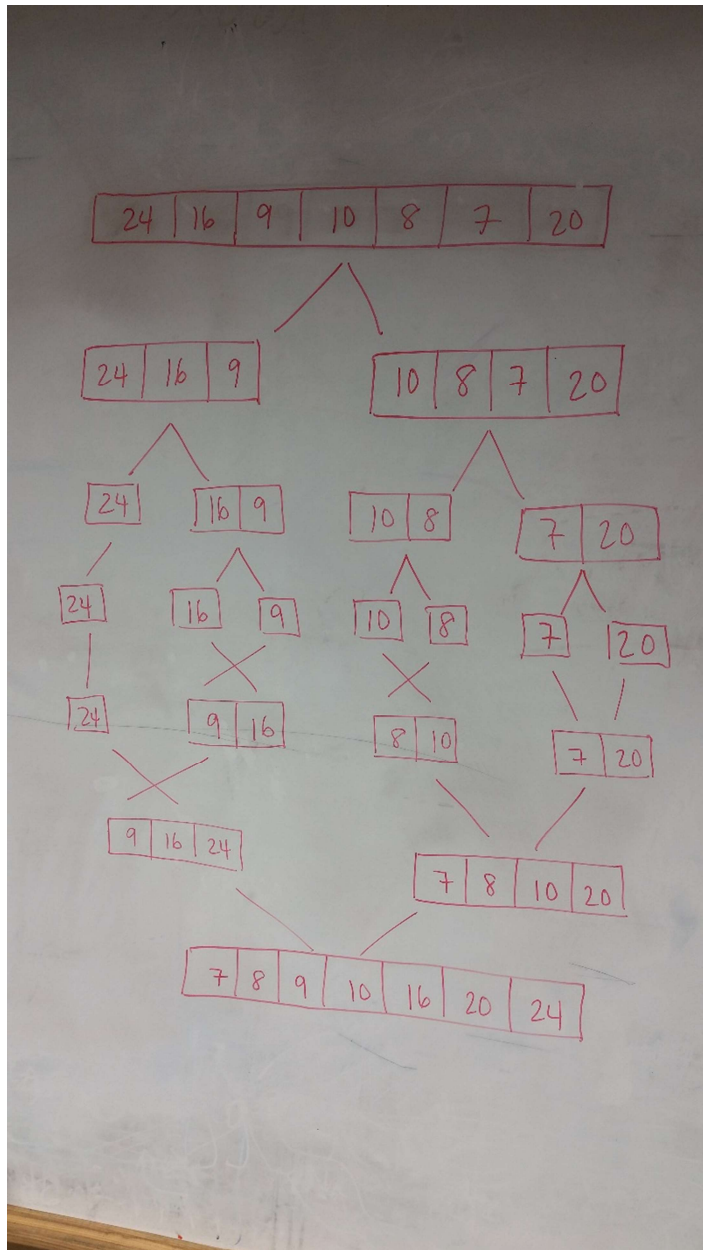
Algorithm	Average complexity	Stable (yes/no)?
Bubble Sort	$O(n^2)$	yes
Insertion Sort	$O(n^2)$	Yes
Heap sort	$O(n \log n)$	no
Merge Sort	$O(n \log n)$	yes
Radix sort	$O(n(\text{size of word}))$	yes
Quicksort	$O(n \log n)$	no

15. [2] What are the key differences between Mergesort and Quicksort? How does this influence why languages choose one over the other?

In quicksort a pivot needs to be chosen out of the data set at which point the set is divided between left and right of the pivot and sorted by calling quicksort again choosing more pivots. Mergesort splits the data set into pieces until there is only one element in each piece and then remerges the pieces to get a sorted array. Quicksort sorts as it pivots so does not need to remerge the data set.

16. [4] Draw out how Mergesort would sort this list:

24	16	9	10	8	7	20
----	----	---	----	---	---	----



17. [4] Draw how Quicksort would sort this list:

24	16	9	10	8	7	20
----	----	---	----	---	---	----

