

# MATH 128 Computational Assignment

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Tables with all data are at bottom of document.

1. i) The integral is solved as follows:

$$\begin{aligned}\int_0^1 x e^{-x^2} dx &= -\frac{1}{2} \int_0^{-1} e^u du & u &= -x^2 \\ & & du &= -2x dx \\ &= -\frac{1}{2} [e^u]_0^{-1} \\ &= -\frac{1}{2} (e^{-1} - e^0) \\ &= -\frac{1}{2} \left( \frac{1}{e} - 1 \right) \\ &= -\frac{1}{2} \left( \frac{1-e}{e} \right) \\ &= \frac{e-1}{2e} \\ &\approx 0.31606\end{aligned}$$

- ii) Summary of results:

Sum type	# iterations	Sum	Error
Left-hand	4 iterations	0.262898	0.053163
	40 iterations	0.311391	0.00467
Right-hand	4 iterations	0.354868	0.038807
	40 iterations	0.320588	0.004527
Trapezoidal	4 iterations	0.308883	0.007178
	40 iterations	0.315989	0.000071

- (a) Using the left-hand rule and  $n = 4$  rectangles, the Riemann sum comes out to be 0.262898, and the error is 0.053163.  
With  $n = 40$  rectangles, the sum is 0.311391, and the error is 0.00467.
- (b) Using the right-hand rule and  $n = 4$  rectangles, the Riemann sum comes out to be 0.354868, and the error is 0.038807.  
With  $n = 40$  rectangles, the sum is 0.320588, and the error is 0.004527.
- (c) Using the right-hand rule and  $n = 4$  rectangles, the Riemann sum comes out to be 0.308883, and the error is 0.007178.  
With  $n = 40$  rectangles, the sum is 0.315989, and the error is 0.000071.

2. i) The value of  $\int_0^1 e^{-x^2} dx$  is approximately equal to 0.746824 (according to WolframAlpha).  
 ii) With 4 iterations, the sum is 0.742984 and the error is 0.426924.  
 With 40 iterations, the sum is 0.746786 and the error is 0.000038.

Summary of results:

# iterations	Sum	Error
4 iterations	0.742984	0.426924
40 iterations	0.746786	0.000038

3. It took 30 iterations for the sum to stabilize to four decimal places. The resulting sum was 2.39475.

## Raw data

The following data was calculated with a self-made JavaScript script.

Table 1: Riemann sum approximation for  $\int_0^1 x e^{-x^2} dx$  with  $n = 4$ .

$x_i$	$x_{\text{lower}}$	$x_{\text{upper}}$	$f(x_{\text{lower}})$	$f(x_{\text{upper}})$	Left-handed		Right-handed		Trapezoidal	
					Area	Sum	Area	Sum	Area	Sum
1	0	0.25	0	0.234853	0	0	0.058713	0.058713	0.029357	0.029357
2	0.25	0.5	0.234853	0.3894	0.058713	0.058713	0.09735	0.156063	0.078032	0.107388
3	0.5	0.75	0.3894	0.427337	0.09735	0.156063	0.106834	0.262898	0.102092	0.209481
4	0.75	1	0.427337	0.367879	0.106834	0.262898	0.09197	0.354868	0.099402	0.308883

Table 2: Riemann sum approximation for  $\int_0^1 x e^{-x^2} dx$  with  $n = 40$ .

$x_i$	$x_{\text{lower}}$	$x_{\text{upper}}$	$f(x_{\text{lower}})$	$f(x_{\text{upper}})$	Left-handed Area Sum		Right-handed Area Sum		Trapezoidal Area Sum	
1	0	0.025	0	0.024984	0	0	0.000625	0.000625	0.000312	0.000312
2	0.025	0.05	0.024984	0.049875	0.000625	0.000625	0.001247	0.001871	0.000936	0.001248
3	0.05	0.075	0.049875	0.074579	0.001247	0.001871	0.001864	0.003736	0.001556	0.002804
4	0.075	0.1	0.074579	0.099005	0.001864	0.003736	0.002475	0.006211	0.00217	0.004974
5	0.1	0.125	0.099005	0.123062	0.002475	0.006211	0.003077	0.009288	0.002776	0.007749
6	0.125	0.15	0.123062	0.146663	0.003077	0.009288	0.003667	0.012954	0.003372	0.011121
7	0.15	0.175	0.146663	0.169722	0.003667	0.012954	0.004243	0.017197	0.003955	0.015076
8	0.175	0.2	0.169722	0.192158	0.004243	0.017197	0.004804	0.022001	0.004523	0.019599
9	0.2	0.225	0.192158	0.213893	0.004804	0.022001	0.005347	0.027349	0.005076	0.024675
10	0.225	0.25	0.213893	0.234853	0.005347	0.027349	0.005871	0.03322	0.005609	0.030284
11	0.25	0.275	0.234853	0.25497	0.005871	0.03322	0.006374	0.039594	0.006123	0.036407
12	0.275	0.3	0.25497	0.274179	0.006374	0.039594	0.006854	0.046449	0.006614	0.043021
13	0.3	0.325	0.274179	0.292423	0.006854	0.046449	0.007311	0.053759	0.007083	0.050104
14	0.325	0.35	0.292423	0.309647	0.007311	0.053759	0.007741	0.0615	0.007526	0.05763
15	0.35	0.375	0.309647	0.325806	0.007741	0.0615	0.008145	0.069645	0.007943	0.065573
16	0.375	0.4	0.325806	0.340858	0.008145	0.069645	0.008521	0.078167	0.008333	0.073906
17	0.4	0.425	0.340858	0.354768	0.008521	0.078167	0.008869	0.087036	0.008695	0.082602
18	0.425	0.45	0.354768	0.367509	0.008869	0.087036	0.009188	0.096224	0.009028	0.09163
19	0.45	0.475	0.367509	0.379058	0.009188	0.096224	0.009476	0.1057	0.009332	0.100962
20	0.475	0.5	0.379058	0.3894	0.009476	0.1057	0.009735	0.115435	0.009606	0.110568
21	0.5	0.525	0.3894	0.398526	0.009735	0.115435	0.009963	0.125398	0.009849	0.120417
22	0.525	0.55	0.398526	0.406433	0.009963	0.125398	0.010161	0.135559	0.010062	0.130479
23	0.55	0.575	0.406433	0.413123	0.010161	0.135559	0.010328	0.145887	0.010244	0.140723
24	0.575	0.6	0.413123	0.418606	0.010328	0.145887	0.010465	0.156352	0.010397	0.15112
25	0.6	0.625	0.418606	0.422896	0.010465	0.156352	0.010572	0.166925	0.010519	0.161639
26	0.625	0.65	0.422896	0.426014	0.010572	0.166925	0.01065	0.177575	0.010611	0.17225
27	0.65	0.675	0.426014	0.427985	0.01065	0.177575	0.0107	0.188275	0.010675	0.182925
28	0.675	0.7	0.427985	0.428838	0.0107	0.188275	0.010721	0.198996	0.01071	0.193635
29	0.7	0.725	0.428838	0.42861	0.010721	0.198996	0.010715	0.209711	0.010718	0.204353
30	0.725	0.75	0.42861	0.427337	0.010715	0.209711	0.010683	0.220395	0.010699	0.215053
31	0.75	0.775	0.427337	0.425063	0.010683	0.220395	0.010627	0.231021	0.010655	0.225708
32	0.775	0.8	0.425063	0.421834	0.010627	0.231021	0.010546	0.241567	0.010586	0.236294
33	0.8	0.825	0.421834	0.417698	0.010546	0.241567	0.010442	0.252009	0.010494	0.246788
34	0.825	0.85	0.417698	0.412706	0.010442	0.252009	0.010318	0.262327	0.01038	0.257168
35	0.85	0.875	0.412706	0.406913	0.010318	0.262327	0.010173	0.2725	0.010245	0.267413
36	0.875	0.9	0.406913	0.400372	0.010173	0.2725	0.010009	0.282509	0.010091	0.277505
37	0.9	0.925	0.400372	0.393141	0.010009	0.282509	0.009829	0.292338	0.009919	0.287423
38	0.925	0.95	0.393141	0.385277	0.009829	0.292338	0.009632	0.30197	0.00973	0.297154
39	0.95	0.975	0.385277	0.376837	0.009632	0.30197	0.009421	0.311391	0.009526	0.30668
40	0.975	1	0.376837	0.367879	0.009421	0.311391	0.009197	0.320588	0.009309	0.315989

 Table 3: Riemann sum approximation for  $\int_0^1 e^{-x^2} dx$  with  $n = 4$ .

$x_i$	$x_{\text{lower}}$	$x_{\text{upper}}$	$f(x_{\text{lower}})$	$f(x_{\text{upper}})$	Left-handed Area Sum		Right-handed Area Sum		Trapezoidal Area Sum	
1	0	0.25	1	0.939413	0.25	0.25	0.234853	0.234853	0.242427	0.242427
2	0.25	0.5	0.939413	0.778801	0.234853	0.484853	0.1947	0.429553	0.214777	0.457203
3	0.5	0.75	0.778801	0.569783	0.1947	0.679553	0.142446	0.571999	0.168573	0.625776
4	0.75	1	0.569783	0.367879	0.142446	0.821999	0.09197	0.663969	0.117208	0.742984

Table 4: Riemann sum approximation for  $\int_0^1 e^{-x^2} dx$  with  $n = 40$ .

$x_i$	$x_{\text{lower}}$	$x_{\text{upper}}$	$f(x_{\text{lower}})$	$f(x_{\text{upper}})$	Left-handed		Right-handed		Trapezoidal	
					Area	Sum	Area	Sum	Area	Sum
1	0	0.025	1	0.999375	0.025	0.025	0.024984	0.024984	0.024992	0.024992
2	0.025	0.05	0.999375	0.997503	0.024984	0.049984	0.024938	0.049922	0.024961	0.049953
3	0.05	0.075	0.997503	0.994391	0.024938	0.074922	0.02486	0.074782	0.024899	0.074852
4	0.075	0.1	0.994391	0.99005	0.02486	0.099782	0.024751	0.099533	0.024806	0.099657
5	0.1	0.125	0.99005	0.984496	0.024751	0.124533	0.024612	0.124145	0.024682	0.124339
6	0.125	0.15	0.984496	0.977751	0.024612	0.149145	0.024444	0.148589	0.024528	0.148867
7	0.15	0.175	0.977751	0.969839	0.024444	0.173589	0.024246	0.172835	0.024345	0.173212
8	0.175	0.2	0.969839	0.960789	0.024246	0.197835	0.02402	0.196855	0.024133	0.197345
9	0.2	0.225	0.960789	0.950635	0.02402	0.221855	0.023766	0.220621	0.023893	0.221238
10	0.225	0.25	0.950635	0.939413	0.023766	0.245621	0.023485	0.244106	0.023626	0.244863
11	0.25	0.275	0.939413	0.927164	0.023485	0.269106	0.023179	0.267285	0.023332	0.268196
12	0.275	0.3	0.927164	0.913931	0.023179	0.292285	0.022848	0.290133	0.023014	0.291209
13	0.3	0.325	0.913931	0.899762	0.022848	0.315133	0.022494	0.312628	0.022671	0.31388
14	0.325	0.35	0.899762	0.884706	0.022494	0.337628	0.022118	0.334745	0.022306	0.336186
15	0.35	0.375	0.884706	0.868815	0.022118	0.359745	0.02172	0.356466	0.021919	0.358105
16	0.375	0.4	0.868815	0.852144	0.02172	0.381466	0.021304	0.377769	0.021512	0.379617
17	0.4	0.425	0.852144	0.834748	0.021304	0.402769	0.020869	0.398638	0.021086	0.400703
18	0.425	0.45	0.834748	0.816686	0.020869	0.423638	0.020417	0.419055	0.020643	0.421346
19	0.45	0.475	0.816686	0.798017	0.020417	0.444055	0.01995	0.439005	0.020184	0.44153
20	0.475	0.5	0.798017	0.778801	0.01995	0.464005	0.01947	0.458475	0.01971	0.46124
21	0.5	0.525	0.778801	0.759098	0.01947	0.483475	0.018977	0.477453	0.019224	0.480464
22	0.525	0.55	0.759098	0.738968	0.018977	0.502453	0.018474	0.495927	0.018726	0.49919
23	0.55	0.575	0.738968	0.718475	0.018474	0.520927	0.017962	0.513889	0.018218	0.517408
24	0.575	0.6	0.718475	0.697676	0.017962	0.538889	0.017442	0.531331	0.017702	0.53511
25	0.6	0.625	0.697676	0.676634	0.017442	0.556331	0.016916	0.548247	0.017179	0.552289
26	0.625	0.65	0.676634	0.655406	0.016916	0.573247	0.016385	0.564632	0.016651	0.568939
27	0.65	0.675	0.655406	0.634052	0.016385	0.589632	0.015851	0.580483	0.016118	0.585058
28	0.675	0.7	0.634052	0.612626	0.015851	0.605483	0.015316	0.595799	0.015583	0.600641
29	0.7	0.725	0.612626	0.591186	0.015316	0.620799	0.01478	0.610578	0.015048	0.615689
30	0.725	0.75	0.591186	0.569783	0.01478	0.635578	0.014245	0.624823	0.014512	0.630201
31	0.75	0.775	0.569783	0.548469	0.014245	0.649823	0.013712	0.638535	0.013978	0.644179
32	0.775	0.8	0.548469	0.527292	0.013712	0.663535	0.013182	0.651717	0.013447	0.657626
33	0.8	0.825	0.527292	0.5063	0.013182	0.676717	0.012658	0.664375	0.01292	0.670546
34	0.825	0.85	0.5063	0.485537	0.012658	0.689375	0.012138	0.676513	0.012398	0.682944
35	0.85	0.875	0.485537	0.465043	0.012138	0.701513	0.011626	0.688139	0.011882	0.694826
36	0.875	0.9	0.465043	0.444858	0.011626	0.713139	0.011121	0.699261	0.011374	0.7062
37	0.9	0.925	0.444858	0.425017	0.011121	0.724261	0.010625	0.709886	0.010873	0.717073
38	0.925	0.95	0.425017	0.405555	0.010625	0.734886	0.010139	0.720025	0.010382	0.727455
39	0.95	0.975	0.405555	0.386499	0.010139	0.745025	0.009662	0.729687	0.009901	0.737356
40	0.975	1	0.386499	0.367879	0.009662	0.754687	0.009197	0.738884	0.00943	0.746786

Table 5: Riemann sum approximation for  $\int_0^2 \sqrt{2 - \frac{1}{1+x^2}} dx$  with  $n = 30$ .

$x_i$	$x_{\text{lower}}$	$x_{\text{upper}}$	$f(x_{\text{lower}})$	$f(x_{\text{upper}})$	Left-handed		Right-handed		Trapezoidal	
					Area	Sum	Area	Sum	Area	Sum
1	0	0.066667	1	1.00221	0.066667	0.066667	0.066814	0.066814	0.06674	0.06674
2	0.066667	0.133333	1.00221	1.008696	0.066814	0.133481	0.067246	0.13406	0.06703	0.133771
3	0.133333	0.2	1.008696	1.019049	0.067246	0.200727	0.067937	0.201997	0.067592	0.201362
4	0.2	0.266667	1.019049	1.032662	0.067937	0.268664	0.068844	0.270841	0.06839	0.269752
5	0.266667	0.333333	1.032662	1.048809	0.068844	0.337508	0.069921	0.340762	0.069382	0.339135
6	0.333333	0.4	1.048809	1.066739	0.069921	0.407428	0.071116	0.411878	0.070518	0.409653
7	0.4	0.466667	1.066739	1.08574	0.071116	0.478544	0.072383	0.48426	0.071749	0.481402
8	0.466667	0.533333	1.08574	1.105194	0.072383	0.550927	0.07368	0.55794	0.073031	0.554433
9	0.533333	0.6	1.105194	1.124591	0.07368	0.624607	0.074973	0.632913	0.074326	0.62876
10	0.6	0.666667	1.124591	1.143544	0.074973	0.699579	0.076236	0.709149	0.075605	0.704364
11	0.666667	0.733333	1.143544	1.161771	0.076236	0.775816	0.077451	0.7866	0.076844	0.781208
12	0.733333	0.8	1.161771	1.179086	0.077451	0.853267	0.078606	0.865206	0.078029	0.859236
13	0.8	0.866667	1.179086	1.19538	0.078606	0.931873	0.079692	0.944898	0.079149	0.938385
14	0.866667	0.933333	1.19538	1.210602	0.079692	1.011565	0.080707	1.025605	0.080199	1.018585
15	0.933333	1	1.210602	1.224745	0.080707	1.092272	0.08165	1.107255	0.081178	1.099763
16	1	1.066667	1.224745	1.237831	0.08165	1.173921	0.082522	1.189777	0.082086	1.181849
17	1.066667	1.133333	1.237831	1.249903	0.082522	1.256443	0.083327	1.273103	0.082924	1.264773
18	1.133333	1.2	1.249903	1.261017	0.083327	1.33977	0.084068	1.357171	0.083697	1.348471
19	1.2	1.266667	1.261017	1.271236	0.084068	1.423838	0.084749	1.44192	0.084408	1.432879
20	1.266667	1.333333	1.271236	1.280625	0.084749	1.508587	0.085375	1.527295	0.085062	1.517941
21	1.333333	1.4	1.280625	1.289249	0.085375	1.593962	0.08595	1.613245	0.085662	1.603604
22	1.4	1.466667	1.289249	1.297171	0.08595	1.679912	0.086478	1.699723	0.086214	1.689818
23	1.466667	1.533333	1.297171	1.304451	0.086478	1.76639	0.086963	1.786687	0.086721	1.776538
24	1.533333	1.6	1.304451	1.311145	0.086963	1.853353	0.08741	1.874096	0.087187	1.863725
25	1.6	1.666667	1.311145	1.317306	0.08741	1.940763	0.08782	1.961917	0.087615	1.95134
26	1.666667	1.733333	1.317306	1.322981	0.08782	2.028583	0.088199	2.050115	0.08801	2.039349
27	1.733333	1.8	1.322981	1.328213	0.088199	2.116782	0.088548	2.138663	0.088373	2.127722
28	1.8	1.866667	1.328213	1.333044	0.088548	2.20533	0.08887	2.227533	0.088709	2.216431
29	1.866667	1.933333	1.333044	1.337509	0.08887	2.294199	0.089167	2.3167	0.089018	2.305449
30	1.933333	2	1.337509	1.341641	0.089167	2.383366	0.089443	2.406142	0.089305	2.394754