PHYS 124 Final Exam Question 5

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5. **A particle of mass m is placed in a one-dimensional infinite square well potential of width L.**

   (a) **What is the zero-point energy of a particle placed in this well?**
   That is given by the equation
   $$E = \frac{\pi^2\hbar^2}{2mL^2}.$$

   (b) **Determine the probability P_n $(0 < x < L/a)$ that a particle in the nth energy state of $\psi_n(x)$ is observed to be in a region $1/a$ of the width of the well.**
   The wave function for the particle in the well is
   $$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}.$$
   The probability for the particle to be at any point in the well is $|\psi|^2$:
   $$|\psi_n(x)|^2 = \frac{2}{L} \sin^2\left(\frac{n\pi x}{L}\right)$$
   Integrating from 0 to L/a:
   $$\begin{aligned}
   P_n &= \int_0^{L/a} \frac{2}{L} \sin^2\left(\frac{n\pi x}{L}\right) dx \\
   &= \frac{2}{L} \int_0^{L/a} \frac{1 - \cos\left(\frac{2n\pi x}{L}\right)}{2} dx \\
   &= \frac{2}{L} \int_0^{L/a} \left(\frac{1}{2} - \frac{\cos\left(\frac{2n\pi x}{L}\right)}{2}\right) dx \\
   &= \frac{2}{L}\frac{L}{2a} - \frac{1}{L} \int_0^{L/a} \cos\left(\frac{2n\pi x}{L}\right) dx \\
   &= \frac{1}{a} - \frac{1}{L}\frac{L}{2n\pi} \left[\sin u\right]_0^{2n\pi/a} \\
   &= \frac{1}{a} - \frac{1}{2n\pi}\left(\sin\left(\frac{2n\pi}{a}\right) - 0\right) \\
   &= \frac{1}{a} - \frac{1}{2n\pi} \sin\left(\frac{2n\pi}{a}\right)
   \end{aligned}$$

   (c) **For what value of n is this probability the largest? What does your answer become if the region is chosen to be 1/3 the size of the box?**

$n = 1$ will maximize the function, since $\frac{1}{2n\pi}$ is larger for small ns, and $n > 0$.

If $a = 3$, the function becomes

$$\frac{1}{3} - \frac{1}{2n\pi} \sin\left(\frac{2n\pi}{3}\right).$$

If $a = 3$ and $n = 1$, the probability of finding the particle in the region of the box becomes $\frac{1}{3} - \frac{\sqrt{3}}{4n\pi}$.

(d) What does the probability in part (b) become as n gets large?

The probability becomes $\frac{1}{a}$.

(e) If the particle in part (b) were classical (no quantum physics), what would be the probability that it would be confined to a region of $1/a$ of the width of the well? How does this compare to your answer in part (d)?

The probability for the classical particle would be $\frac{1}{a}$. This is the same as my answer for part **(d)**. This makes sense, since high energy particles will have a higher momentum (by $E = pc$), and therefore less uncertainty in position.