PHYS 234 Assignment 1

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4. <u>Matrix Operations</u>
Given the following matrices:

$$\boldsymbol{A} = \begin{bmatrix} -1 & 0 & i \\ 3 & 0 & 2 \\ i & -2i & 2 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 2 & 0 & i \\ 0 & i & 0 \\ 1 & 2 & 2 \end{bmatrix}$$

compute the following:

(a) $\boldsymbol{A + B}$

$$\begin{aligned} A + B &= \begin{bmatrix} -1 & 0 & i \\ 3 & 0 & 2 \\ i & -2i & 2 \end{bmatrix} + \begin{bmatrix} 2 & 0 & i \\ 0 & i & 0 \\ 1 & 2 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 2i \\ 3 & i & 2 \\ 1+i & 2-2i & 4 \end{bmatrix} \end{aligned}$$

(b) $\boldsymbol{AB}$

$$\begin{aligned} AB &= \begin{bmatrix} -1 & 0 & i \\ 3 & 0 & 2 \\ i & -2i & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 & i \\ 0 & i & 0 \\ 1 & 2 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -2+i & 2i & i \\ 8 & 4 & 4+3i \\ 2+2i & 6 & 3 \end{bmatrix} \end{aligned}$$

(c) $\boldsymbol{[A, B]}$ **(commutator of** $\boldsymbol{A}$ **and** $\boldsymbol{B}$**)**

The commutator of two matrices is defined as

$$[A, B] = AB - BA.$$

We already know AB, so we must find BA.

$$\begin{aligned} BA &= \begin{bmatrix} 2 & 0 & i \\ 0 & i & 0 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & i \\ 3 & 0 & 2 \\ i & -2i & 2 \end{bmatrix} \\ &= \begin{bmatrix} -3 & 2 & 4i \\ 3i & 0 & 2i \\ 5+2i & -4i & 8+i \end{bmatrix} \end{aligned}$$

Then,

$$[A, B] = AB - BA = \begin{bmatrix} -2+i & 2i & i \\ 8 & 4 & 4+3i \\ 2+2i & 6 & 3 \end{bmatrix} - \begin{bmatrix} -3 & 2 & 4i \\ 3i & 0 & 2i \\ 5+2i & -4i & 8+i \end{bmatrix}$$

$$= \begin{bmatrix} -5+i & -2+2i & -3i \\ 8-3i & 4 & 4+i \\ -3 & 6+4i & -5-i \end{bmatrix}$$

(d) A^{T} (transpose)

$$A^{\mathrm{T}} = \begin{bmatrix} -1 & 3 & i \\ 0 & 0 & -2i \\ i & 2 & 2 \end{bmatrix}$$

(e) $A^{\dagger}$ (complex transpose)

$$A^{\mathrm{T}} = \begin{bmatrix} -1 & 3 & -i \\ 0 & 0 & 2i \\ -i & 2 & 2 \end{bmatrix}$$

(f) Verify by direct calculation that $(AB)^{\mathrm{T}} = B^{\mathrm{T}} A^{\mathrm{T}}$.

First, we calculate $(AB)^{\mathrm{T}}$:

$$(AB)^{\mathrm{T}} = \begin{bmatrix} -2+i & 8 & 2+2i \\ 2i & 4 & 6 \\ i & 4+3i & 3 \end{bmatrix}$$

Next, we calculate $B^{\mathrm{T}} A^{\mathrm{T}}$:

$$B^{\mathrm{T}} A^{\mathrm{T}} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & i & 2 \\ i & 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 3 & i \\ 0 & 0 & -2i \\ i & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2+i & 8 & 2+2i \\ 2i & 4 & 6 \\ i & 4+3i & 3 \end{bmatrix}$$

We can see that they are the same, so it is true that $(AB)^{\mathrm{T}} = B^{\mathrm{T}} A^{\mathrm{T}}$.