MATH 114 Final Exam Question 3

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3. **Suppose S_1 and S_2 are 3-dimensional figures. The volume of S_1 is 10, and the volume of S_2 is 4. Also suppose that A_1 and A_2 are matrices such that A_1 transforms S_1 to S_3 and A_2 transforms S_2 to the same S_3.**

   (a) **Is $\frac{\det(A_1)}{\det(A_2)}$ guaranteed to exist? Explain.**

   The only way for $\frac{\det(A_1)}{\det(A_2)}$ to not exist is to have $\det(A_2) = 0$. This would only be the case when S_3 has a volume of zero (i.e., A_2 transforms a shape of nonzero volume to one with zero volume). In all other cases, $\frac{\det(A_1)}{\det(A_2)}$ exists.

   (b) **If it does exist, what is it?**

   A_1 transforms an object from one that has a volume of 10 to one that has a volume of $V \neq 0$. So its determinant must be $\frac{V}{10}$. Likewise, A_2 transforms an object from one that has a volume of 4 to one that has a volume of that same V. So its determinant must be $\frac{V}{4}$. Putting these together, we get

$$
\begin{aligned}
\frac{\det(A_1)}{\det(A_2)} &= \frac{\frac{V}{10}}{\frac{V}{4}} \\
&= \frac{4V}{10V} \\
&= \frac{2}{5}
\end{aligned}
$$