# PHYS 124 Final Exam Question 4

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April 23, 2020

**4. Certain structures called quantum wires behave as metals in which its gas of electrons is confined to a line (one spatial dimension). The density of states for a quantum wire of length $L$ is $g(E) = \frac{l}{\hbar\pi}\sqrt{\frac{m}{2E}}$ where $m$ is the mass of an electron.**

**(a) Find the number of electrons $N$ in a quantum wire in terms of $L$, $E$, and $m$.**

Using the Fermi-Dirac distribution and the density of states, we can calculate the number of electrons. We can define a variable $dN$ which represents the number of electrons in an infinitessimal energy band $dE$:

$$\begin{aligned} dN &= \frac{g(E)}{\exp\left(\frac{E-E_\mathrm{F}}{kT}\right)+1}\,dE \\ &= \frac{\frac{l}{\hbar\pi}\sqrt{\frac{m}{2E}}}{\exp\left(\frac{E-E_\mathrm{F}}{kT}\right)+1}\,dE \end{aligned}$$

where $E_\mathrm{F}$ is the Fermi energy. When $T$ approches zero, this becomes

$$\frac{l}{\hbar\pi}\sqrt{\frac{m}{2E}}.$$

I can integrate this from 0 to $E_\mathrm{F}$:

$$\begin{aligned} N &= \int_0^{E_\mathrm{F}} \frac{l}{\hbar\pi}\sqrt{\frac{m}{2E}}\,dE \\ &= \frac{\sqrt{m}}{\sqrt{2}\hbar\pi}\int_0^{E_\mathrm{F}} \frac{1}{\sqrt{E}}\,dE \\ &= \frac{\sqrt{m}}{\sqrt{2}\hbar\pi}\left[2E^{1/2}\right]_0^{E_\mathrm{F}} \\ &= \frac{2\sqrt{m}}{\sqrt{2}\hbar\pi}E_\mathrm{F}^{1/2} \end{aligned}$$