PHYS 124 Final Exam Question 2

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2. (a) The Friedmann equations describing our universe at large scales are

$$\frac{1}{2}\left(\frac{da(t)}{dt}\right)^2 - \frac{4\pi G\rho(t)}{3}a^2(t) = -\frac{kc^2}{2l^2} \tag{1}$$

and

$$\frac{d}{dt}(\rho(t)a^3(t)) = -p(t)\frac{d}{dt}a^3(t) \tag{2}$$

for the scale factor $a(t)$. Explain the meaning of the quantities k, l, ρ, and p in these equations.

k can be either +1, 0, or −1, and it defines the shape of the universe.

ρ is the density of energy in the universe.

p is the pressure of energy in the universe.

(b) Suppose that the scale factor $a(t) = a_0 t^{1/2}$. If $k = 0$, find the dependence of ρ and p on t. What cosmic era does this correspond to?

Using equation 1:

$$\begin{aligned}
\frac{1}{2}\left(\frac{da(t)}{dt}\right)^2 - \frac{4\pi G\rho(t)}{3}a^2(t) &= -\frac{kc^2}{2l^2} \\
\frac{1}{2}\left(\frac{1}{2}a_0 t^{-1/2}\right)^2 - \frac{4\pi G\rho(t)}{3}(a_0^2 t^{1/2}) &= 0 \\
\frac{1}{8t^{3/2}}a_0^2 - \frac{4\pi G\rho(t)}{3}(a_0^2 t^{1/2}) &= 0 \\
\frac{1}{8t^{3/2}} &= \frac{4\pi G\rho(t)}{3}t^{1/2} \\
\frac{3}{8} &= 4\pi G\rho(t)t^2 \\
\rho(t) &= \frac{3}{32\pi G t^2}
\end{aligned}$$

Then, with equation 2:

$$\begin{aligned}
\frac{d}{dt}(\rho(t)a^3(t)) &= -p(t)\frac{d}{dt}a^3(t) \\
\frac{d}{dt}\left(\frac{3}{32\pi G t^2}(a_0^3 t^{3/2})\right) &= -p(t)\frac{d}{dt}(a_0^3 t^{3/2}) \\
\frac{3a_0^3}{32\pi G}\frac{d}{dt}\left(t^{-1/2}\right) &= -p(t)\left(\frac{3}{2}a_0^3 t^{1/2}\right) \\
\frac{1}{32\pi G}t^{-3/2} &= p(t)t^{1/2} \\
p(t) &= \frac{1}{32\pi G t^2}
\end{aligned}$$

[The rest of this question was skipped.]