MATH 114 Assignment 8

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1. **(a) A matrix M transformed the figure A to the figure B. Calculate $\det(M)$.**

The area of A (found by counting grid squares) is 5, and the area of B is 2. Therefore, the transformation from M decreased the area by a factor of $\frac{2}{5}$, and $\det(M) = \frac{2}{5}$.

(b) Given $N = \begin{bmatrix} 7 & 3 \\ 8 & 4 \end{bmatrix}$, calculate $\det(N)$.

$\det(N)$ is calculated as follows:

$$\begin{aligned} \det(N) = \begin{vmatrix} 7 & 3 \\ 8 & 4 \end{vmatrix} &= (7)(4) - (3)(8) \\ &= 28 - 24 \\ &= 4 \end{aligned}$$

(c) In the previous assignment, we created the following matrix P:

$$P = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} & 0 & 0 \\ -\frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{10}} & 0 & 0 \\ \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Calculate $\det(P)$.

First, we find the determinants of the matrices which make up P, which we will call P_1, P_2, P_3, P_4, and P_5. We will utilize the shortcut method discussed in class.

$$\begin{aligned} \det(P_1) = \begin{vmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} &= (1)(1)(1)(1) \\ &= 1 \end{aligned}$$

$$
\begin{aligned}
\det(P_2) = \begin{vmatrix} \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} & 0 & 0 \\ -\frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} &\sim \begin{vmatrix} \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} & 0 & 0 \\ 0 & \frac{3}{\sqrt{10}} + \frac{1}{3\sqrt{10}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \\
&\sim \begin{vmatrix} \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} & 0 & 0 \\ 0 & \frac{10}{3\sqrt{10}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \\
&= \left(\frac{3}{\sqrt{10}}\right)\left(\frac{10}{3\sqrt{10}}\right) \\
&= \frac{30}{30} \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
\det(P_3) = \begin{vmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} &= \cos\theta \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} - 0 \begin{vmatrix} 0 & 0 & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \\
&\quad + \sin\theta \begin{vmatrix} 0 & 1 & 0 \\ -\sin\theta & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} - 0 \begin{vmatrix} 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \\ 0 & 0 & 0 \end{vmatrix} \\
&= \cos\theta(1)(\cos\theta)(1) - 0 - \sin\theta \begin{vmatrix} -\sin\theta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} - 0 \\
&= \cos^2\theta - \sin\theta(-\sin\theta)(1)(1) \\
&= \cos^2\theta + \sin^2\theta \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
\det(P_1) = \begin{vmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} &= (1)(1)(1)(1) \\
&= 1
\end{aligned}
$$

Finally,

$$
\begin{aligned}
\det(P) &= \det(P_1)\det(P_2)\det(P_3)\det(P_4)\det(P_5) \\
&= 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \\
&= 1
\end{aligned}
$$

2. (a) **Compute the determinant of $Q = \begin{bmatrix} 2 & 0 & 0 & 1 \\ -4 & 1 & 0 & -2 \\ -3 & 0 & 1 & 0 \\ -3 & 1 & 0 & -2 \end{bmatrix}$. What can you say about $\det(Q^{-1})$?**

The determinant of Q is calculated as follows:

$$\begin{aligned}
\det(Q) = \begin{vmatrix} 2 & 0 & 0 & 1 \\ -4 & 1 & 0 & -2 \\ -3 & 0 & 1 & 0 \\ -3 & 1 & 0 & -2 \end{vmatrix} &\sim \begin{vmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 \\ -3 & 1 & 0 & -2 \end{vmatrix} \\
&\sim \begin{vmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 \\ 0 & 1 & 0 & -\frac{1}{2} \end{vmatrix} \\
&\sim \begin{vmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -\frac{1}{2} \end{vmatrix} \\
&= (2)(1)(1)\left(-\frac{1}{2}\right) \\
&= -1
\end{aligned}$$

(b) **Compute the determinant of $R = \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 2 & 0 & 6 & 6 \end{bmatrix}$. What can you say about $\det(R^{-1})$?**

The determinant of R is calculated as follows:

$$\det(R) = \begin{vmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 2 & 0 & 6 & 6 \end{vmatrix} \sim \begin{vmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

Since the bottom row is all zeroes, this matrix has no determinant.