PHYS 234 Assignment 2

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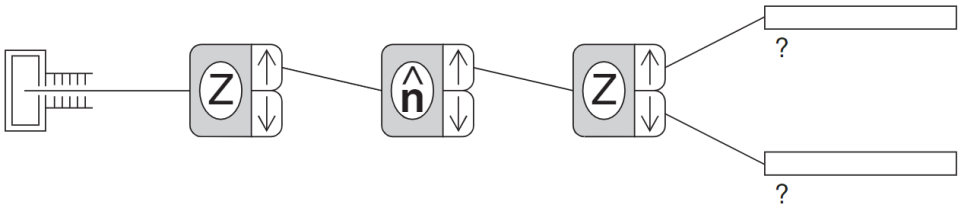
3. Three Stern-Gerlach Analyzers with Arbitrary Direction

A beam of spin-$\frac{1}{2}$ particles is sent through a series of three S-G analyzers, as shown in the figure. The second S-G analyzer is aligned along the $\hat{n}$-direction.



(a) Find the probability that particles transmitted through the first S-G analyzer are measured to have spin down at the third S-G analyzer.

This would be the probability of the particles going through the first two analyzers multiplied by the probability of those particles going through the third analyzer.

$$\begin{aligned}
\left|\langle +|+\rangle_n|^2|_n\langle +|-\rangle\right|^2 &= \left|\langle +|\left(\cos\left(\frac{\theta}{2}\right)|+\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\phi}|-\rangle\right)\right|^2\left|\left(\cos\left(\frac{\theta}{2}\right)\langle +| + \sin\left(\frac{\theta}{2}\right)e^{-i\phi}\langle -|\right)|-\rangle\right|^2 \\
&= \left|\cos\left(\frac{\theta}{2}\right)\right|^2\left|\sin\left(\frac{\theta}{2}\right)e^{-i\phi}\right|^2 \\
&= \cos^2\left(\frac{\theta}{2}\right)\sin^2\left(\frac{\theta}{2}\right) \\
&= \frac{\sin^2\theta}{4}
\end{aligned}$$

(b) How must the angle θ of the second S-G analyzer be oriented so as to maximize the probability that particles are meaured to have spin down at the third S-G analyzer? What is this maximum fraction?

Since the maximum of $\sin\theta$ is 1, we mush find a θ which will set the probability to $\frac{1}{4}$:

$$\begin{aligned}
\frac{\sin^2\theta}{4} &= \frac{1}{4} \\
\sin^2\theta &= 1 \\
\theta &= \frac{\pi}{2}
\end{aligned}$$

In other words, the second analyzer must be oriented along the xy-plane, and the maximum probability is $\frac{1}{4}$.