PHYS 122L Experiment 9
Electrostatic Forces and the Permittivity of Free Space

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Purpose

Using a parallel-plate capcitor setup along with an ammeter and voltmeter, today's experiment will attempt to measure the permittivity of free space ϵ_0, one of the most fundamental constants in physics.

Results

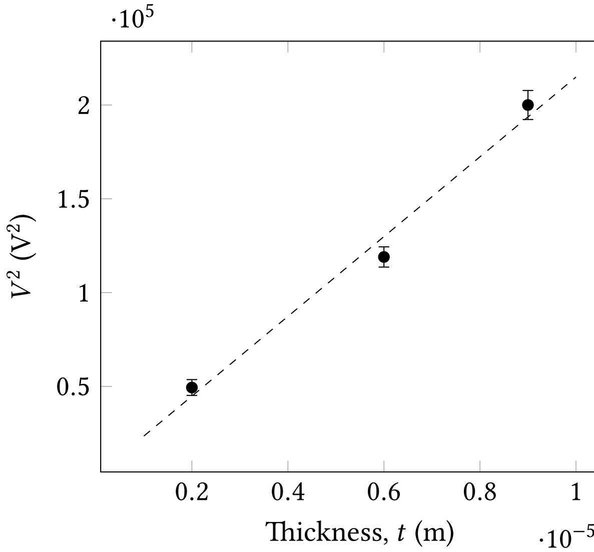
First, we determined the distance d between the two plates. To do this, we measured the thickness of the foam separating them. Since the foam is compressible, we determined that our uncertainty would be 0.1 mm.

Zero error:	-0.005 mm
Measured thickness of foam:	1.997 mm
Uncertainty:	0.1 mm
Final measurement:	(1.997 ± 0.100) mm

Here are the results for the parallel plate capacitor:

Thickness (m)	Plate voltage, V (V)			Average plate voltage, V (V)	ΔV (V)	V^2 (V^2)	ΔV^2 (V^2)
	1	2	3				
$2 \cdot 10^{-6}$	213	222	232	222	9.50	49400	4230
$6 \cdot 10^{-6}$	354	341	340	345	7.81	119000	5390
$9 \cdot 10^{-6}$	442	442	457	447	8.66	200000	7740

Making a plot of V^2 vs. t:



Slope, m:	$2.1262 \times 10^{10}\ \mathrm{V^2\,m^{-1}}$
Uncertainty in slope, Δm:	$2.7 \times 10^{9}\ \mathrm{V^2\,m^{-1}}$

Now, we use the data to calculate ϵ_0:

$$\begin{aligned}
m &= \frac{2d^2\rho g}{\epsilon_0} \\
2.1262 \times 10^{10}\ \mathrm{V^2\,m^{-1}} &= \frac{2(0.001\,99\ \mathrm{m})^2(2710\ \mathrm{kg\,m^{-3}})(9.8067\ \mathrm{m\,s^{-2}})}{\epsilon_0} \\
\epsilon_0 &= \frac{2(0.001\,99\ \mathrm{m})^2(2710\ \mathrm{kg\,m^{-3}})(9.8067\ \mathrm{m\,s^{-2}})}{2.1262 \times 10^{10}\ \mathrm{V^2\,m^{-1}}} \\
&= 9.8997 \times 10^{-12}\ \mathrm{s^4\,A^2\,m^{-3}\,kg^{-1}}
\end{aligned}$$

To find the relative uncertainty:

$$\begin{aligned}
\frac{\Delta\epsilon_0}{\epsilon_0} &= \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{2\Delta d}{d}\right)^2} \\
&= \sqrt{\left(\frac{2.7 \times 10^{9}\ \mathrm{V^2\,m^{-1}}}{2.1262 \times 10^{10}\ \mathrm{V^2\,m^{-1}}}\right)^2 + \left(\frac{2(0.0001\ \mathrm{m})}{0.001\,997\ \mathrm{m}}\right)^2} \\
&= 0.16 \\
&= 16\%
\end{aligned}$$

This means that the abolute uncertainty is $9.8997 \times 10^{-12}\ \mathrm{s^4\,A^2\,m^{-3}\,kg^{-1}} \cdot 0.16 = 1.6 \times 10^{-12}\ \mathrm{s^4\,A^2\,m^{-3}\,kg^{-1}}$. Finally, we get that $\epsilon_0 = (9.9 \pm 1.6) \times 10^{-12}\ \mathrm{s^4\,A^2\,m^{-3}\,kg^{-1}}$.

Discussion

1. **How does your value for the permittivity of free space ϵ_0 compare to the accepted value? Do they agree within experimental error? What does this tell you about the random or systematic nature of the uncertainty in your measurement?**

   The accepted value for ϵ_0 is $8.854 \times 10^{-12}\,\mathrm{s^4\,A^2\,m^{-3}\,kg^{-1}}$, which fits in our experimental uncertainty.

2. **Qualitatively, equation (9) indicates that your graph is a straight line with zero intercept. Comment on how well your results agree with this theory.** The intercept of our graph was $V^2 = 2273$. This value is relatively small, considering our V^2 values were in the 10^5 range. In fact, the uncertainties in our V^2 values are all larger than 2273.

3. **Assess any strengths or weaknesses of this experiment.** The scale of the experiment makes the values imprecise. For example, Any bends on the foil make for a significant change in d over different parts of the foil. Also, relying on forces on this scale can lead to inaccurate results since there are many other forces which also act at this scale (including friction, air resistance, etc.).