MATH 114 Assignment 3

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1. Find a set of vectors, each with a norm of 5, such that the sum of the vectors is $\begin{bmatrix}0\\1\end{bmatrix}$.

Two vectors that satisfy these conditions are $\vec{a} = \begin{bmatrix}\sqrt{\frac{99}{4}}\\ \frac{1}{2}\end{bmatrix}$ and $\vec{b} = \begin{bmatrix}-\sqrt{\frac{99}{4}}\\ \frac{1}{2}\end{bmatrix}$.

Both vectors have norm 5:

$$\begin{aligned}\|\vec{a}\| &= \sqrt{\left(\sqrt{\frac{99}{4}}\right)^2 + \left(\frac{1}{2}\right)^2}\\ &= \sqrt{\frac{99}{4} + \frac{1}{4}}\\ &= \sqrt{\frac{100}{4}}\\ &= \sqrt{25}\\ &= 5\end{aligned}$$

$$\begin{aligned}\|\vec{b}\| &= \sqrt{\left(-\sqrt{\frac{99}{4}}\right)^2 + \left(\frac{1}{2}\right)^2}\\ &= \sqrt{\frac{99}{4} + \frac{1}{4}}\\ &= \sqrt{\frac{100}{4}}\\ &= \sqrt{25}\\ &= 5\end{aligned}$$

And they sum to $\begin{bmatrix}0\\1\end{bmatrix}$:

$$\begin{aligned}\begin{bmatrix}\sqrt{\frac{99}{4}}\\ \frac{1}{2}\end{bmatrix} + \begin{bmatrix}-\sqrt{\frac{99}{4}}\\ \frac{1}{2}\end{bmatrix} &= \begin{bmatrix}\sqrt{\frac{99}{4}} - \sqrt{\frac{99}{4}}\\ \frac{1}{2} + \frac{1}{2}\end{bmatrix}\\ &= \begin{bmatrix}0\\1\end{bmatrix}\end{aligned}$$

2. (a) Consider the vectors $\vec{u} = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 5 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$.

i. Find a vector $\vec{x}$ such that $\vec{x}$ is orthogonal to both $\vec{u}$ and $\vec{v}$.