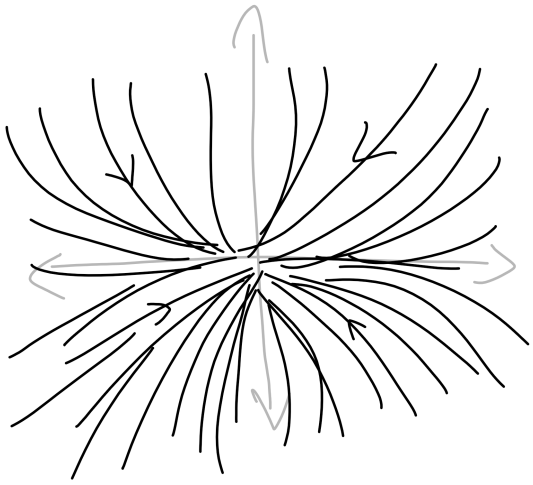
MATH 114 Assignment 9, Q1

Brandon Tsang

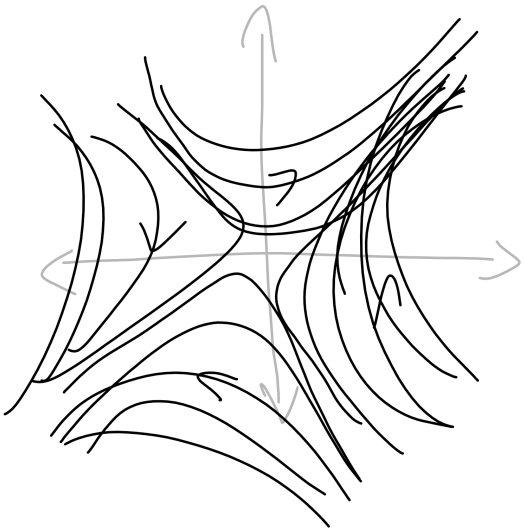
April 3, 2020

1. (a) Draw a sketch of how particles on the plane would move when repeatedly transformed by a matrix with:

- **Eigenvector $\begin{bmatrix}1\\0\end{bmatrix}$, eigenvalue $0 < \lambda_1 < 1$, and**



- **Eigenvector $\begin{bmatrix}1\\1\end{bmatrix}$, eigenvalue $\lambda_2 > 1$.**



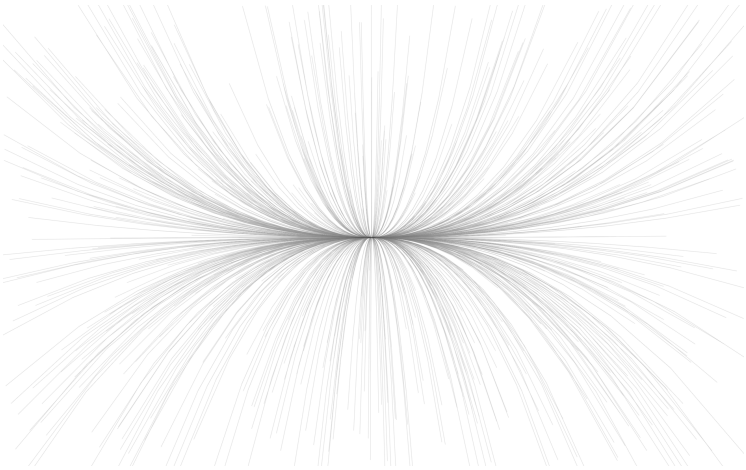
(b) Find such a matrix, and verify it looks nice in the visualizer. You can include a picture if you like.

- **Eigenvector $\begin{bmatrix}1\\0\end{bmatrix}$, eigenvalue $0 < \lambda_1 < 1$, and**

  I chose $\lambda_1 = 0.8$. To find the matrix that would satisfy this, I used the following relation:

  $$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}u\\0\end{bmatrix} = \begin{bmatrix}0.8u\\0\end{bmatrix}$$

  For this relation to hold true, a must be 0.1, and c must be 0. b and d can really be anything. The matrix that created the picture below was $\begin{bmatrix}0.8 & 0\\ 0 & 0.6\end{bmatrix}$.



- **Eigenvector $\begin{bmatrix}1\\1\end{bmatrix}$, eigenvalue $\lambda_2 > 1$.**

  I chose $\lambda_2 = 1.4$. Using a similar approach as before:

  $$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}u\\u\end{bmatrix} = \begin{bmatrix}1.4u\\1.4u\end{bmatrix}$$

  For this relation to hold true, $a + b$ and $c + d$ must be 1.4. I chose the matrix $\begin{bmatrix}1.08 & 0.32\\ 0.32 & 1.08\end{bmatrix}$ (those numbers made the most symmetrical picture).

(The pictures above were made with a tool I coded myself.)