Brandon Horner
Homework 7
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1. evaluate expression $((a\times d)+(c\times b))\div(b\times d)$ in environment $\sigma = [a \to 1, b \to 2, c \to 2, d \to 3]$

$A[[((a\times d)+(c\times b))\div(b\times d)]]\sigma = (A[[((a\times d)+(c\times b))]]\sigma) \div A[[(b\times d)]]\sigma$ **last case in (6)**
$= ((A[[(a\times d)]]\sigma) + (A[[(c\times b)]]\sigma)) \div A[[(b\times d)]]\sigma$ **last case in (3)**
$= (((A[[a]]\sigma \times A[[d]]\sigma)) + (A[[c]]\sigma \times A[[b]]\sigma)) \div (A[[b]]\sigma \times A[[d]]\sigma)$
last case in (5)*3
$= ((1 \times 3) + (2 \times 2)) \div (2 \times 3)$ **by case (1) & σ(a)=1, σ(b)=2, σ(c)=2, σ(d)=3**
= **7/6**

2. evaluate expression $((a \times d) + (c \times b)) \div (b \times d)$ in environment $\sigma = [a \to 1, c \to 2, d \to 3]$

$A[[((a\times d)+(c\times b))\div(b\times d)]]\sigma = (A[[((a\times d)+(c\times b))]]\sigma) \div A[[(b\times d)]]\sigma$ **last case in (6)**
$= ((A[[(a\times d)]]) + (A[[(c\times b)]]\sigma)) \div A[[(b\times d)]]\sigma$ **last case in (3)**
$= (((A[[a]]\sigma \times A[[d]]\sigma)) + (A[[(c \times b)]]\sigma)) \div (A[[b \times d]]\sigma)$ **last case in (5)**
$= (((A[[a]]\sigma \times A[[d]]\sigma)) + (A[[c]]\sigma \times \bot\text{undef}) \div (A[[b \times d]]\sigma)$ **third case in (5)**
= **⊥undef** **⊥undef if A[[e1]]σ ∈ Z and A[[e2]]σ = ⊥undef**

3. evaluate expression $((a\times d)+(c\times b))\div(b\times d)$ in environment $\sigma = [a \to 1, b \to 0, c \to 2, d \to 3]$

$A[[((a\times d)+(c\times b))\div(b\times d)]]\sigma = (A[[((a\times d)+(c\times b))]]\sigma) \div A[[(b\times d)]]\sigma$ **last case in (6)**
$= ((A[[(a\times d)]]\sigma) + (A[[(c\times b)]]\sigma)) \div A[[(b\times d)]]\sigma$ **last case in (3)**
$= (((A[[a]]\sigma \times A[[d]]\sigma)) + (A[[c]]\sigma \times A[[b]]\sigma)) \div (A[[b]]\sigma \times A[[d]]\sigma)$
last case in (5)*3
$= ((1 \times 3) + (2 \times 0)) \div (0 \times 3)$ **by case (1) & σ(a)=1, σ(b)=0, σ(c)=2, σ(d)=3**
$= (3) \div (0)$ **4th case in (6)**
= **⊥div0** **⊥div0 if A[[e1]]σ ∈ Z and A[[e2]]σ = ⊥div0**

4. evaluate expression $!(x = (x \div 0))$ in environment $\sigma = [z \to 1]$

$B[[\ !(x = A[[(x \div 0))]]\sigma\ ]]\sigma =$ **⊥undef** **1st case in (6) ⊥undef if A[[e1]]σ = ⊥undef**
Alternative:
$B[[\ !(x = A[[(x \div 0))]]\sigma\ ]]\sigma =$ **⊥undef** **1st case in (11) ⊥ if A[[a1]]σ = ⊥**

5. evaluate expression $0 < (((a \times d) + (c \times b)) \div (b \times d))$ in environment $\sigma = [a \to 1, b \to 2, c \to 2, d \to 3]$

$B[[\ 0 < (A[[((a\times d)+(c\times b))\div(b\times d)]]\sigma)\ ]]\sigma$
$= B[[\ 0 < (((A[[((a\times d)+(c\times b))]]\sigma) \div A[[(b\times d)]]\sigma))\ ]]\sigma$ **last case in (6)**
$= B[[\ 0 < (((A[[(a\times d)]]\sigma) + (A[[(c\times b)]]\sigma)) \div A[[(b\times d)]]\sigma)\ ]]\sigma$ **last case in (3)**
$= B[[\ 0 < (((A[[a]]\sigma \times A[[d]]\sigma)) + (A[[c]]\sigma \times A[[b]]\sigma)) \div (A[[b]]\sigma \times A[[d]]\sigma))\ ]]\sigma$
last case in (5)*3
$= B[[\ 0 < ((1 \times 3) + (2 \times 2)) \div (2 \times 3)\ ]]\sigma$
by case (1) & σ(a)=1, σ(b)=2, σ(c)=2, σ(d)=3
$= B[[\ 0 < 7/6\ ]]\sigma$
= True **third case in (12)**