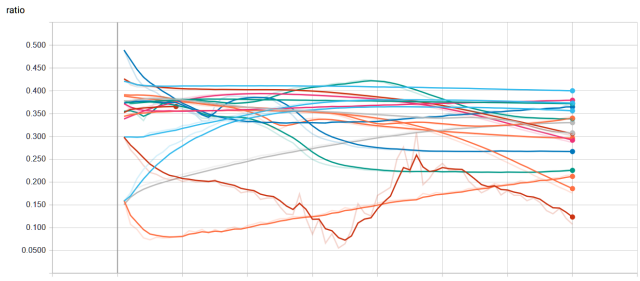

Figure 1: The proportion of error not explained by a linear model of known invariants. Model demonstrated by the lowest red line achieved a ratio of 0.98 demonstrating nearly all of the variance was explained by known invariants.

In this period we discovered optimized learning procedures through a sweep over model parameters. We then applied this result to pendulum models and visualized the learned parameters. Additionally, the performance of pre-existing pendulum simulation was suspect and work was completed on an improved pendulum solver to produce quicker and more varied trajectories.

Following the work on a framework for hyper-parameter search on learning invariants in planetary trajectories, multiple sweeps were conducted to determine optimization parameters that provided a learned constant that was consistent across held out points along the orbital trajectories.

These learned parameters served as a strong initial parameters for learning an invariant over trajectories of pendulum. This parameter-optimized model was trained to minimize the objective:

$$\min_{\phi} \sum_{t=1}^{T} \left(\left| \frac{\dot{\boldsymbol{r}}_t \cdot \bigtriangledown \phi \left(\boldsymbol{r}_t \right)}{\| f_t \|_2^2 * \| \bigtriangledown \phi(\boldsymbol{r}_t) \|_2^2} \right| - \left(1 - \| \bigtriangledown \phi \left(\boldsymbol{r}_t \right) \|_2^2 \right)^2 \right) \tag{1}$$

and the learned invariant displayed near zero variance along simulated trajectories while varying over the set of training trajectories. Figure 2 demonstrates the performance of the optimized model learning a non trivial function ϕ quickly and with near zero gradient along pendulum trajectories. During evaluation of this model, the sparse sampling of pendulum trajectories was identified to bias models, thus a faster RK4 differential equation solver was implemented to provide denser sampling pendulum trajectories.

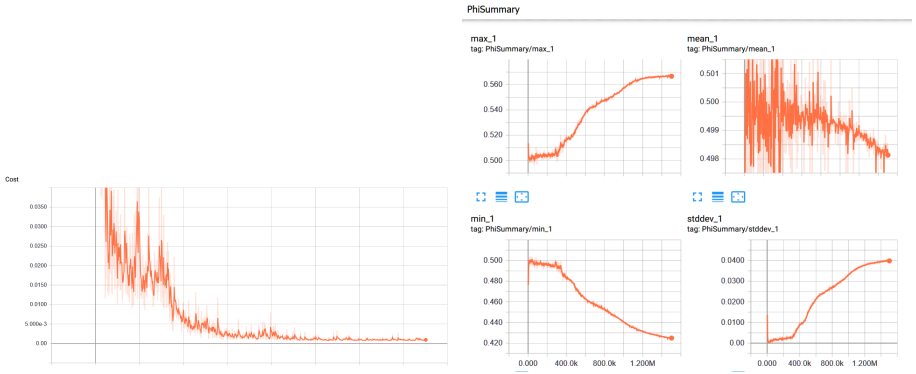

Figure 2: (Left) $\dot{\boldsymbol{r}}_t \cdot \bigtriangledown \phi\left(\boldsymbol{r}_t\right)$ along test pendulum trajectories (Right) Summary statistics of learned invariant $\phi\left(\boldsymbol{r}_t\right)$ over test set while training. Note the learned function is both non-trivial and constant along sampled trajectories

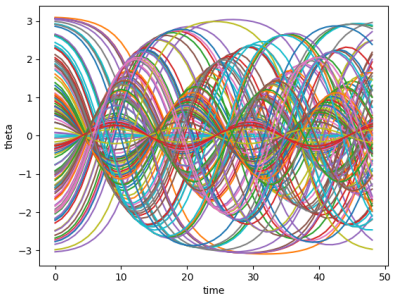

Figure 3: Sampled pendulum trajectories simulated by RK4 solver