

Two-body problem: in the previous formulation,

$$\min_{\phi} \sum_{t=1}^T \|\dot{\mathbf{r}}_t \cdot \nabla \phi(\mathbf{r}_t)\|_2^2 \quad (1)$$

This converged in less than 3 epochs and did not produce any meaningful  $\phi()$ . As found last week the solution found would simply ignore any inputs and return 0.

As discussed we experimented with adding a term to maximize the gradients of  $\phi()$  yielding the minimization:

$$\min_{\phi} \sum_{t=1}^T \left( \|\dot{\mathbf{r}}_t \cdot \nabla \phi(\mathbf{r}_t)\|_2^2 + \frac{1}{\|\nabla \phi(\mathbf{r}_t)\|_2^2} \right) \quad (2)$$

However as  $\nabla \phi()$  was highly variant solving this encountered issues with numeric stability and would underflow/overflow for various training hyper-parameters.

To combat the numerical stability we experimented with log of the norm of the gradient of phi:

$$\min_{\phi} \sum_{t=1}^T (\|\dot{\mathbf{r}}_t \cdot \nabla \phi(\mathbf{r}_t)\|_2^2 - \log(\|\nabla \phi(\mathbf{r}_t)\|_2^2)) \quad (3)$$

This minimization again suffered from numerical stability.

Given our goal of minimizing the dot product  $\dot{\mathbf{r}}_t \cdot \nabla \phi(\mathbf{r}_t)$  we minimized the cosine distance between  $\dot{\mathbf{r}}_t$  and  $\nabla \phi(\mathbf{r}_t)$  and instead of maximizing the gradient we simply enforce that  $\|\nabla \phi(\mathbf{r}_t)\| = 1$  along the trajectory:

$$\min_{\phi} \sum_{t=1}^T \left( \left| \frac{\dot{\mathbf{r}}_t \cdot \nabla \phi(\mathbf{r}_t)}{\|\dot{\mathbf{r}}_t\|_2^2 * \|\nabla \phi(\mathbf{r}_t)\|_2^2} \right| - \left(1 - \|\nabla \phi(\mathbf{r}_t)\|_2^2\right)^2 \right) \quad (4)$$

This objective results in a well defined gradient for minimization. Producing a phi that is constant across  $\mathbf{r} \in T$ . However this phi after convergence decreases consistently for each epoch across multiple planets. This may be a result of over-fitting however as it is only seen after 100k epochs for a small 50 unit fully connected neural network. I will post a histogram of phi as a function of training to validate these claims.

One of the potential concerns with the approach in (5) was unit-gradient magnitude was only enforced along the trajectory. We are currently testing the effect on regularizing the entire field to be unit length:

$$\min_{\phi} \sum_{t=1}^T \left| \frac{\dot{\mathbf{r}}_t \cdot \nabla \phi(\mathbf{r}_t)}{\|\dot{\mathbf{r}}_t\|_2^2 * \|\nabla \phi(\mathbf{r}_t)\|_2^2} \right| - \sum_{x \in R^4} \left(1 - \|\nabla \phi(x)\|_2^2\right)^2 \quad (5)$$