

## 1 Executive summary

We discovered a new method of approximating partial differential equations using deep neural networks. These methods are able to predict turbulent flow more effectively than previous methods without relying on complex non-linearities by nature of approximating arbitrary PDEs fit a powerful class of equations capable of modeling many physically based systems.

### 1.1 Formulating Approximation of Ordinary Differential Equations]

**1.1.1 What was to be tested? What was the expected outcome prior to testing?** Previous results used sequence-to-sequence models to learn an embedding of the current state, as well as a model of the dynamics using recurrent networks such as Long-Short Term Memory units (LSTMs). This worked well in practice, however represents a much more complex function approximation scheme. While this has been shown to approximate dynamical systems [Sonoda & Murata (2017)], complex and non-linear mechanics such as forget-gates do not map well to the discovery of physical laws.

**1.1.2 High-level summary of main results.** Explicitly modeling the dynamics as a PDE allows for interpolation by adjusting  $\Delta t$  in the discretized time derivative. Additionally, because many physically based systems are easily modeled using PDEs, the class of problems

High level take away is that this formulation underpins many successful neural network architectures. This result ensures that learned models can be transferred

What are PDEs used for in the real world? These are systems that can be reasoned about to learn fundamental

#### 1.1.3 Discuss relevance to project and DARPA concerns.

### 1.2 Evaluating Predictions of Approximated Ordinary Differential Equations

**1.2.1 What was to be tested? What was the expected outcome prior to testing?** We evaluate new PDE based Previous models capably predicted mid-length turbulent flows

#### 1.2.2 High-level summary of main results.

#### 1.2.3 Discuss relevance to project and DARPA concerns. etc.

## 2 Achievements

### 2.1 Scientific Breakthroughs

e.g. planned publications

### 2.2 Technology developments

e.g. software packages, significant performance increases, reduced need for data, etc.

## **2.3 Application results**

impact to external users

## **2.4 Transitions achieved**

accomplished or committed

## **3 Lessons Learned**

Surprises

Problems encountered/risks that occurred, and corresponding solutions/mitigations

Open Issues

## **4 Next Steps**

Investigate methods to reduce edge effects, a common failure point for PDEs

Extend PDE to multiple scales

Test interpolation of model dynamics (potential for new application of learned model)

Compare resistance to noise between new fully convolutional formulation and previous recurrent lstm model.

## **5 Technical details**

Technical details on goals and tests.

### **5.1 Goal 1 / Hypothesis 1**

### **5.2 Goal 1 / Hypothesis 1**

## **References**

Sho Sonoda and Noboru Murata. Double continuum limit of deep neural networks. ICML Workshop Principled Approaches to Deep Learning, 2017