Chapter 8 – Additional Problems with Solution – Helpful for the Homework, and Chapter Quiz on Chapter 8

Problem 1:

Let S = { (0,0), (0,3), (1,0), (1,2), (2,0), (3,2) }

Find S', the transitive closure of S.

Solution:

S' has all the elements of S union these new elements: { (0,2), (1,3), (2,2), (2,3), (3,0), (3,3) } to make the relation transitive.

Problem 2:

Sets R and S are transitive. Give a counter example to disprove: **R ∩ S is Transitive.**

Solution:
Counterexample:
Let Relation, R = {(1, 2), (2, 3), (1, 3)}

Let Relation, S = {(2, 3), (3, 4), (2, 4)}

Therefore, **R ∩ S = {(2, 3)},** which is not transitive.

Problem 3:

For the relation given, do the following:

1. state whether the relation is reflexive
2. state whether the relation is symmetric
3. state whether the relation is transitive

If the relation is NOT (reflexive, symmetric, transitive), you must give a counterexample that proves it.

R1 = { (0,0), (0,1), (0,3), (1,1), (1,0), (2,3), (3,3) }

Solution:

The relation is not reflexive. Counterexample: (2, 2) $\notin$ R1

The relation is not symmetric. Counterexample: (3, 2) $\notin$ R1

The relation is not transitive. Counterexample: (1, 0), (0, 3) $\in$ R1. However, (1, 3) $\notin$ R1

Problem 4:

Let A = {2, 3, 4, 5}

Let B = {3, 4, 5, 6}

Let S be the *less than* relation, so for all (x, y) ordered pairs, $xSy \equiv x < y$

($x < y$ means x is less than y)

part 1: state explicitly which ordered pairs are in **S**

part 2: state explicitly which ordered pairs are in $\mathbf{S^{-1}}$

Solution:

S = {(2, 3), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 5), (4, 6), (5, 6)}

$\mathbf{S^{-1}}$ **= {(3, 2), (4, 2), (5, 2), (6, 2), (4, 3), (5, 3), (6, 3), (5, 4), (6, 4), (6, 5)}**

$\mathbf{S^{-1}}$ **is the greater than relation. This means,** $\mathbf{x\, S^{-1} y \equiv x > y}$