Chapter 6 – Additional Problems with Solution – Helpful for the Homework, and Chapter Quiz on Chapter 6

Problem 1:

Consider the set, V = {x, y}. Find the Power Set of V

Solution:

The power set is, P(V) = {{}, {x}, {y}, {x, y}}

Problem 2:

Define a function S : Z+ → Z+ as follows: For each positive integer n,
S(n) = the sum of the positive divisors of n.
Find the following:
S(12) = ?
S(15) = ?
S(20) = ?

Solution:

S(12) = 1 + 2 + 3 + 4 + 6 + 12 = 28

S(15) = 1 + 3 + 5 + 15 = 24

S(20) = 1 + 2 + 4 + 5 + 10 + 20 = 42

Problem 3:

Define sets A and B as follows:

A = {n ∈ Z | n = 8r - 3 for some integer r} and
B = {m ∈ Z | m = 4s + 1 for some integer s}.

Is A a subset of B?

Is B a subset of A?

Solution:

A = {...5, 13, 21...} and B = {...1,5,9,13, 17, 21, ...};

9, 17∈ B but 9, 17 ∉ A.

A is a subset of B but B is not a subset of A.

Problem 4:

Disprove the following statement by finding a counterexample.
For all sets A, B, and C, $A \cup (B \cap C) \subseteq (A \cup B) \cap C$.

Solution:

One counterexample follows:

A={1,2,3,4}, B={2,3,6,7}, and C={3,4,5,6}

Therefore, $A \cup (B \cap C) = \{1,2,3,4\} \cup \{3,6\} = \{1,2,3,4,6\}$

$(A \cup B) \cap C = \{1,2,3,4,6,7\} \cap \{3,4,5,6\} = \{3,4,6\}$

{1,2,3,4,6} is not a subset of {3,4,6} ($\{1,2,3,4,6\} \not\subset \{3, 4, 6\}$)