

CMSC 207 Additional Problems with Solutions – To Support: Homework 2 & Quiz 2 Chapter 2

Following are some additional CMSC 207 Chapter 2 Materials related problems with solutions. These problems will help you with your Homework Assignment 2 on Chapter 2 as well as the Quiz 2 on Chapter 2 that you will take in class.

**In the following three problems, you are traveling in a certain country where every inhabitant is either a truthteller who always tells the truth, or a liar who always lies.

IMPORTANT: In exercises 1, 2 and 3, you are travelling in a certain country where every citizen or inhabitant of the country is either a TRUTHTELLER WHO ALWAYS TELLS THE TRUTH or a LIAR WHO ALWAYS LIES.

#1 You meet two inhabitants of this country. Their names are Percival and Llewellyn.

Percival says: "At least one of us is a liar."

Is Percival a liar or a truthteller?

What about Llewellyn? Explain your answer.

***** Here is what one possible answer could be for #1: *****

Case 1: Suppose Percival is a LIAR. Then his stating that
"At least one of us is a liar"
is TRUE.

This is a contradiction or impossibility.

Guess what! A liar could NEVER make the above statement!!

So, Percival cannot be a LIAR. LIARs cannot make ANY TRUE
statements! ---

Case 2: Suppose Percival is a TRUTHTELLER.

Then the statement "At least one of us is a liar"
has to be TRUE, which means that the other character,
Llewellyn, has to be a LIAR.

Conclusion: PERCIVAL is a TRUTHTELLER, and
LLEWELLYN is a LIAR

Another way of looking at it would be to draw a table like
a truth table showing the four row-combination of possibilities:

Percival	Llewellyn	Percival said: "At least one of us is a liar."	
-----	-----		
-----	-----	Statement is:	
Liar	Liar	True	He can't make it!
Liar	TruthTeller	True	He can't state it!
TruthTeller	Liar	True	He can make this statement.
TruthTeller	TruthTeller	False	He could NOT tell a lie!

#2: You meet Merlin and Meredith.

CMSC 207 Additional Problems with Solutions – To Support: Homework 2 & Quiz 2 Chapter 2

Merlin states the implication,

IF I Merlin am a TRUTHTELLER THEN Meredith is a TRUTHTELLER.

--

Write down the TRUTH TABLE for the implication.

P	Q	IF P THEN Q	P	Q	P ----> Q
T	T	T	1	1	1
T	F	F	1	0	0
F	T	T	0	1	1
F	F	T	0	0	1

Case #1: Merlin is a TRUTH TELLER.

Can we conclude anything from that, or does it lead to a contradiction, an impossibility???

Case #2: Merlin is a LIAR

(Merlin is a Truth Teller)' where ' means not

Analyze the situation and statement

where its FALSE that "Merlin is a Truth Teller"

i.e. "I am a truth teller" with the pronoun I referring to Merlin.

"I am a Truth Teller" -----> "Meredith is a Truth Teller"

True	TRUE	True
True	FALSE	False
False	TRUE	True
False	TRUE	False

P	----->	Q
P	implies	Q
if P	then	Q

That is enough hints for problem #2. If you lay it out like this on paper, and review what you know about the IF P THEN Q implication statements and logic, you will see and be able to explain your answer, eventually.

***** Exercise #3 *****

Finally, you meet Rothwold and Grymlin. Rothwold says,

"Either I am a liar or Grymlin is a truth teller"

Is Rothwold a liar or a truth teller? What about Grymlin?

Explain your answer.

Case #1: Rothwold is a Truth Teller:

a. Then the statement has to be TRUE, cause Rothwold is

CMSC 207 Additional Problems with Solutions – To Support: Homework 2 & Quiz 2 Chapter 2

a TRUTH TELLER.

b. Is it possible for the statement to be true?

Case #2: Rothwold is not a Truth Teller, i.e. Rothwold is a LIAR.

a. Then the statement

"Either I (Rothwold) am a liar
OR
Grymlin is a truthteller"

HAS TO BE FALSE!!!!

b. Review the TRUTH TABLE for P or Q ($P \vee Q$)

"I am a liar"	OR	"Grymlin is a truthteller"
-----		-----
T	TRUE	T
T	TRUE	F
F	TRUE	T
F	FALSE	F
---	----	---
P	OR	Q

Suppose Rothwold is a liar. Then his statement is True. So, he is NOT A LIAR. Therefore, it is a Contradiction. Thus, Rothwold is a truthteller. If Rothwold is a truthteller. If Rothwold is a truthteller, then his statement can only be true if Grymlin is a truthteller. **So, both Rothwold and Grymlin are truthteller.**

Two types of people live on the island, Knights and Knaves.

Knights always tell the truth. Knaves always lie.

You run into a group of six natives from the island.

U, V, W, X, Y and Z (They have short names on this island).

U says: None of us is a knight.

V says: At least three of us are knights.

W says: At most three of us are knights.

X says: Exactly five of us are knights.

Y says: Exactly two of us are knights.

Z says: Exactly one of us is a knight.

Which are the knights, and which are the knaves?

U is a knave, a liar.

None of us is a knight would be true, if none of U,V,W,X,Y or Z were knights, but that is a contradiction, because if it were TRUE, U would have to be a knight. But if he were a knight, it would be FALSE.

Fact #1: U is a knave. (It's NOT the case there are 0 knights)

CMSC 207 Additional Problems with Solutions – To Support: Homework 2 & Quiz 2 Chapter 2

Consider the last statement, by Z.

Z says: Exactly one of us is a knight.

Suppose that this statement is TRUE and Z is a knight.

Oops! That would make W's statement true too.

W says: At most three of us are knights.

But if W's statement is true, W is a knight too, along with Z, so Z's statement cannot possibly be true, because it leads to a CONTRADICTION, or IMPOSSIBILITY or ABSURDITY.

Fact #2: Z is a knave. (It's NOT the case there is ONE knight).

Since we have that U is a knave and Z is a knave, we now have a total of 2 knaves.

Since we only have V, W, X and Y left, we now know that X is a knave!

X says that exactly five of us are knights, and at this point we know that there are FOUR OR FEWER knights.

Therefore, X is a liar, the knave, indeed.

Fact #3: X is a knave. (It's NOT the case there are FIVE knights).

What do we have left now?

U, X and Z are knaves, for a total of THREE knaves.

V says: At least three of us are knights.

W says: At most three of us are knights.

Y says: Exactly two of us are knights.

Consider Y's statement, that there are EXACTLY TWO knights.

**** "There are EXACTLY TWO knights", has EXACTLY TWO cases. ****

Case #1: Suppose it's TRUE.

Y is a knight.

W's statement that AT MOST THREE OF US ARE KNIGHTS, is true.

V's statement that AT LEAST THREE OF US ARE KNIGHTS, if false.

Y is knight. W is knight. V is knave. This is possible....

Case #2: Suppose it's FALSE.

Y is a knave. There are NOT exactly TWO knights.

W's statement that AT MOST THREE OF US ARE KNIGHTS, is true.

CMSC 207 Additional Problems with Solutions – To Support: Homework 2 & Quiz 2 Chapter 2

V's statement that AT LEAST THREE OF US ARE KNIGHTS, is false.

W is a knight, but Y and V are both knaves.

But from above, we already have that U, X, and Z are knaves.
So, U, V, W, X and Z are knaves and W is the one knight.

But that is impossible, because ONE KNIGHT would make Z's
statement TRUE, and that is impossible. It was ruled out above.

Conclusion: W and Y are knights. U, V, X, and Z are knaves.

In the Knight and Knave problem, two natives C and D approach you but only C speaks. C says: Both of us are knaves.
What are C and D?

Solution: If C is a knight, then his statement must be true, and hence he is a knave.
This is a contradiction. Therefore, C is a knave. Since C is a knave, his statement must be false. Therefore, D is a knight.

You then encounter natives E and F. E says: F is a knave. F says: E is a knave. How many knaves are there?

Solution:

If E and F are both knaves, then E's statement is true, which is a contradiction since he is a knave. If E and F are both knights, then E's statement is false, which is a contradiction since he is a knight. Thus, one of E and F is a knight and one is a knave. Therefore, there is one knave.