Chapter 3 – Additional Problems with Solution – Helpful for the Homework, and Chapter Quiz on Chapter 3

Problem 1:

Translate the statement "The sum of two positive integers is always positive" into a logical expression.

Solution: To translate this statement into a logical expression, we first rewrite it so that the implied quantifiers and a domain are shown: "For every two integers, if these integers are both positive, then the sum of these integers is positive." Next, we introduce the variables *x* and *y* to obtain "For all positive integers *x* and *y*, *x* + *y* is positive." Consequently, we can express this statement as:

$$\forall x \forall y((x > 0) \wedge (y > 0) \rightarrow (x + y > 0)),$$

where the domain for both variables consists of all integers. Note that we could also translate this using the positive integers as the domain. Then the statement "The sum of two positive integers is always positive" becomes "For every two positive integers, the sum of these integers is positive. We can express this as

$$\forall x \forall y(x + y > 0),$$

where the domain for both variables consists of all positive integers.

Problem 2:
Translate the statement "Every real number except zero has a multiplicative inverse." (A **multiplicative inverse** of a real number *x* is a real number *y* such that ***xy* = 1**.)

Solution:

We first rewrite this as "For every real number *x* except zero, *x* has a multiplicative inverse." We can rewrite this as "For every real number *x*, if $x \neq 0$, then there exists a real number *y* such that ***xy* = 1**." This can be rewritten as:

$$\forall x((x \neq 0) \rightarrow \exists y(xy = 1)).$$

Problem 3:

Translate the statement

$$\forall x(C(x) \vee \exists y(C(y) \wedge F(x, y)))$$

into English, where $C(x)$ is "x has a computer," $F(x, y)$ is "x and y are friends," and the domain for both x and y consists of all students in your school.

Solution:

The statement says that for every student x in your school, x has a computer or there is a student y such that y has a computer and x and y are friends. In other words, every student in your school has a computer or has a friend who has a computer.

Problem 4:

Translate the statement

$$\exists x \forall y \forall z((F(x, y) \wedge F(x, z) \wedge (y \neq z)) \rightarrow \neg F(y,z))$$

into English, where $F(a,b)$ means a and b are friends and the domain for x, y, and z consists of all students in your school.

Solution:

Solution: We first examine the expression $(F(x, y) \wedge F(x, z) \wedge (y \neq z)) \rightarrow \neg F(y, z)$. This expression says that if students x and y are friends, and students x and z are friends, and furthermore, if y and z are not the same student, then y and z are not friends. It follows that the original statement, which is triply quantified, says that there is a student x such that for all students y and all students z other than y, if x and y are friends and x and z are friends, then y and z are not friends. In other words, there is a student none of whose friends are also friends with each other. ◀