

Solutions for Supplementary Exercises: Chapter 3

1. Section 3.1

- (a) \forall dogs d , d has a tail.
- (b) \forall fish f , f does not have live births.
- (c) \forall integers n , n is not the cube of 250,048.
Or: \forall integers n , $n \neq (250,048)^3$.
- (d) \exists a student S such that S lives on campus.

2. Section 3.2

There exist real numbers a and b such that a and b are irrational and $a + b$ is not irrational.

3. Section 3.2

\forall positive integers p , if p is not equal to 1, then p is not a divisor of 1.

4. Section 3.2

Converse: \forall real numbers x , if $x^2 < 36$ then $0 < x < 6$.

Contrapositive: \forall real numbers x , if $x^2 \geq 36$ then $0 \geq x$ or $x \geq 6$.

Inverse: \forall real numbers x , if $0 \geq x$ or $x \geq 6$ then $x^2 \geq 36$.

5. Section 3.3: In (a)–(d) below, (i) rewrite the statement without using variables and expressing your answer as simply as possible, and (ii) write a negation for the statement (either with or without variables).

- (a) (i) *Statement:* Given any positive real number, there is a positive real number that is smaller than the given number.
(ii) *Negation:* There is a smallest positive real number.
- (b) (i) *Statement:* There is a smallest positive real number.
(ii) *Negation:* Given any positive real number, there is a positive real number that is smaller than the given number.