Chapter 7 – Additional Problems with Solution – Helpful for the Homework, and Chapter Quiz on Chapter 7

Problem 1:

Find the inverse function G for the function **F(x) = 3x+9** in the real numbers.

Solution:

F(x) = 3x + 9
So, 3x = F(x) – 9
Therefore, x = (F(x) – 9) / 3
Or, **G(y) = (y – 9)/3**

Problem 2:

Is there an inverse function for **F(x) = 3x-4** in the integers?

Solution:
F(x) = 3x - 4
So, 3x = F(x) + 4
Hence, x = (F(x) + 4) / 3
Or, G(y) = (y + 4) / 3
If y = 1, then y ∈ Z
Therefore**, G(y) = (1 + 4) / 3 = 5/ 3 ∉ Z**
So, the answer is **No.**

Problem 3:

Consider an ***onto*** function F with domain X and range Y. Compare the cardinality of X with the cardinality of Y

Solution:

The Cardinality of a set is the number of elements in the set. |X| denotes the cardinality of set, X

Since the function, F is onto from domain, X to range, Y, each element of set Y is being mapped onto by at least one element (may be more than one element) from the set X. Therefore, **|X| ≥ |Y|.**

Problem 4:

Consider an ***one-to-one*** function F with domain X and range Y. Compare the cardinality of X with the cardinality of Y.

Solution:

Since the function is one-to-one, each element of the domain X maps exactly onto one (1) element in range Y. To satisfy the condition for being a function, all elements of domain, X will need to map onto the elements in range, Y. So, the range, Y contains at least as many elements as domain, X, and may be more. Some additional elements in range, Y is possible which are not being mapped onto.

Therefore, $|\mathbf{Y}| \geq |\mathbf{X}|$ or, $|\mathbf{X}| \leq |\mathbf{Y}|$