Due:

(1) Let $A = \{4, 5, 6\}$ and $B = \{15, 16, 17, 18, 19\}$. Define a relation R from A to B as follows:

For all $(x, y) \in A \times B$, $(x, y) \in R$ if and only if $\dfrac{y}{x-1}$ is an integer.

(a) Write R as a set of ordered pairs.
(b) Draw an arrow diagram for R.
(c) Is R a function? Explain.

(2) Define functions F and G from R to R by the formulas $F(x) = (x + 1)(x + 3)$ and $G(x) = (x + 2)^2 - 1$. Does $F = G$? If so, explain why, if not, give a value of x that shows that they are not equal.

(3) Consider the sentence "Every student has studied calculus." Rewrite the sentence by filling in these blanks:
(a) All students____.
(b) For every student x,____.

(4) Consider the sentence "Some student in this class has visited Mexico." Rewrite the sentence by filling in these blanks:
(a) There is a student x in this class such that_________________________.
(b) There exists____.

(5) Let A be the set of all integers n such that n is a factor of 12.
(a) Use set roster notation to indicate the elements in the set.
(b) Use set builder notation to describe the sets.

(6) Simplify the expression $(q \vee \sim p) \to (r \vee p)$ to include no implications and as few symbols and logical operators as possible. State each law that you are using.

(7) Create a truth table for the statement form $(q \vee \sim p) \to (r \vee p)$.

(8) True or false:
(a) $\{3\} \subseteq \{2, 3, 5\}$
(b) $\{3\} \subseteq \{1, \{2\}, \{3\}, 5\}$
(c) $3 \in \{2, 3, \{5\}\}$.

(9) Suppose x is a particular (although unknown) number. Write the negation of $-2 \le x < 0$. Expand the result – do not leave a $\sim$.

(10) Write the negation, converse, inverse, and contrapositive of the sentence "If I am an adult, then I am big or I am brave." Use words, not variables.

(11) Consider the following argument:

If the entry is small, then the output is predictable.
The output is not predictable.
If the entry is giant, then the output is negative.
Either the entry is small or the entry is giant.
$\therefore$, The output is negative.

Use the variables

s = the entry is small,
g = the entry is giant,
p = the output is predictable, and
n = the output is negative.

Prove that this argument is valid in two ways:

(a) Deduce the conclusion from the premises using known rules of inference (see page 61).

(b) Use a truth table to prove the argument is valid.

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Example: Design a circuit for the following input/output table.

Input			Output
P	Q	R	S
1	1	1	1
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	0

(a) First, create a circuit in the most naive way, by tying together six recognizers.
(b) Now look for a simpler circuit.

(13) Compute $-23 + (-71)$ using signed 8-bit arithmetic.

(14) Write the negation of this statement without using any $\sim$ symbols:

For any building x in the city, there is a fire station y such that the distance between x and y is at most 2 miles.

(15) Rewrite this with and/or statements: Being odd is a necessary condition for the square of a number to be odd.

What if the word "necessary" is replaced with "sufficient"?