Chapter 4 HW

① $\forall$ integers m and n, if $2m+n$ is odd then m and n are both odd

Disproof by counterexample:

$\exists$ integers m and n such that $2m+n$ is odd, and m and n are not both odd

let $m=2, n=1$

$2(2)+1=5$ by substitution

$2m+n$ is odd but m is even, this disproves the original statement. Q.E.D

② $\forall$ irrational numbers a and b, a times b is irrational

Disproof by counterexample:

$\exists$ irrational numbers a and b such that a times b is not irrational

let $a=\sqrt{2}$ and $b=\sqrt{2}$

$a \cdot b = \sqrt{2} \cdot \sqrt{2}$ by substitution

$\sqrt{2} \cdot \sqrt{2} = 2$

both a and b are irrational numbers, but their product is a rational number.
Q.E.D