Discrete Mathematics with Applications, 4th edition
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Solutions for Supplementary Exercises: Chapter 2

1. **Section 2.1**
   (a) $p \vee q$
   (b) $p \wedge r$
   (c) $(p \vee q) \wedge r$

2. **Section 2.2**
   Truth table:

p	q	$\sim p$	$\sim q$	$p \to \sim q$	$\sim p \vee \sim q$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

   Explanation: The truth table shows that $p \to \sim q$ and $\sim p \vee \sim q$ have the same truth values in all cases (i.e., regardless of the truth values of p and q). Therefore the statement forms are logically equivalent.

3. **Section 2.2**: The given statements are not logically equivalent. Let p be "Tom likes celebrating," and q be "Tom enjoys the annual gala." Then the statements have the following symbolic forms: $p \to q$ and $q \vee p$
   Truth table:

p	q	$p \to q$	$q \vee p$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F

   Explanation: The truth table shows that $p \to q$ and $q \vee p$ do not always have the same truth values. Row 2 shows that it is possible for $p \to q$ to be false and $q \vee p$ to be true, and row 4 shows that it is possible for $p \to q$ to be true and $q \vee p$ to be false.. Therefore the forms of the statements are not logically equivalent, and so the statements themselves are not logically equivalent.

4. **Section 2.2**
   (a) $-3 \geq x$ or $x > 2$
   (b) The moon was not shining and the light was not on.
   (c) The DNA matches and Mia was not at the scene of the crime.

5. **Section 2.2**
   Converse: If Lou's score was at least 88, then Lou got an A.
   Contrapositive: If Lou's score was less than 88, then Lou did not get an A.
   Inverse: If Lou did not get an A, then Lou's score was less than 88.

6. **Section 2.2**
   If Cinderella does not do all her chores, then she will not be allowed to go to the ball.
   Or: If Cinderella goes to the ball, then she did all her chores.

7. **Section 2.3**: Let p be "Line 10 is correct," and let q be "The variable is declared."

   Logical form:

$$\begin{array}{ll} & p \to q \\ & q \vee \sim p \\ \text{Therefore,} & p \vee q \end{array}$$

   Truth table:

			premises		conclusion
p	q	$\sim p$	$p \to q$	$q \vee \sim p$	$p \vee q$
T	T	F	T	T	T
T	F	F	F	F	
F	T	T	T	T	T
F	F	T	T	T	F

   Explanation: The fourth row shows that it is possible for an argument of this form to have true premises and a false conclusion. Therefore, the argument is invalid.

8. **Section 2.3**: Let p be "Tom is guilty," q be "Sue is guilty," and r be "Ana is guilty."

   Tom is guilty or Sue is guilty.
   If Ana is guilty or Sue is guilty, then Tom is not guilty.
   ∴ Tom is not guilty or Ana is not guilty.

   Logical form:

$$\begin{array}{ll} & p \vee q \\ & r \vee q \to \sim p \\ \therefore & \sim p \vee \sim r \end{array}$$

   Truth table:

						premises		conclusion
p	q	r	$\sim p$	$\sim r$	$p \vee q$	$r \vee q$	$r \vee q \to \sim p$	$\sim p \vee \sim r$
T	T	T	F	F	T	T	F	
T	T	F	F	T	T	T	F	
T	F	T	F	F	T	T	F	
T	F	F	F	T	T	F	T	
F	T	T	T	F	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	F	F	T	T	T
F	F	F	T	T	F	F	T	

   Explanation: The only situations in which both premises are true are rows 5, 6, and 7. In each of these rows the conclusion is also true. Therefore, the argument is valid.

9. **Section 2.4**

   The output is 1.

10. **Section 2.4**

    $(P \wedge \sim Q \wedge R) \vee (\sim P \wedge \sim Q \wedge R)$

11. **Section 2.5**

    (a) $101011_2 = 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 32 + 8 + 2 + 1 = 43_{10}$

    (b) $86 = 64 + 16 + 4 + 2 = 2^6 + 2^4 + 2^2 + 2^1 = 1010110_2$

(c)

$$
\begin{array}{rr}
 & 110011_2 \\
+ & 11011_2 \\
\hline
 & 1001110_2
\end{array}
$$