Chapter 4 – Additional Problems with Solution – Helpful for the Homework, and Chapter Quiz on Chapter 4

Problem 1:

For the following formulae, if the formula is true, prove it; if false, find a counterexample.

For all real numbers x, | x-1 | = | x | - 1

Solution: Counterexample:
x = -1.2
| x-1 | = | -1.2 -1 | = | -2.2 | = 2.2
| x | - 1 = | -1.2 | - 1 = 1.2 − 1 = 0.2
Therefore, | x-1 | ≠ | x | - 1

Problem 2:
Prove by contraposition: if the sum of two real numbers is less than 50, then at least one of the numbers is less than 25.

Solution:

Contraposition: In a sum of two real numbers, if both (not at least one (~∃) is both (∀)) the numbers are greater than or equals to 25, then the sum is greater than or equals to 50.
Suppose: a, b ∈ R, and a ≥ 25, b ≥ 25.
Let a = 25 + x, x ∈ R and x ≥ 0
b = 25 + y, y ∈ R and y ≥ 0
If both x and y are 0, a + b = 25 + 25 = 50
If either x or y or both greater than 0, then a + b = 25 + 25 + x + y = 50 + x + y > 50

Problem 3: Write the following as a ratio of two integers.

0.575757575757….

Solution:

Let n = 0.575757575757….
100n = 57.575757575757….

100n − n = 57
Or, 99n = 57
Therefore, **n = 57 / 99**

Problem 4:

-33 mod 9 =?

Solution:

-33 = -4×9+3

Therefore, -33 mod 9 = +3