

Chapter 10 – Additional Problems with Solution – Helpful for the Homework, and Chapter Quiz on Chapter 10**Problem 1:**

If a graph has vertices of degrees 2, 3, 4, 3, and 6, how many edges does it have? Why?

Solution:

Total degree of the graph = $2 + 3 + 4 + 3 + 6 = 18$

So, according to the Handshake Theorem, **the graph has $18 / 2 = 9$ edges.**

Problem 2:

For a graph with six vertices of degrees 1, 1, 3, 2, 2, and 4, explain why such a graph exists or does not exist.

Solution:

Such a graph does not exist, because the total degree of the graph = $1 + 1 + 3 + 2 + 2 + 4 = 13$, which is an odd degree, and is not even. There are 3 vertices having degrees of 1, 1, and 3. This number of vertices is odd (which is 3), which violates the following Proposition.

By Proposition 10.1.3, in any graph there are an even number of vertices of odd degree.

Problem 3:

What is the total degree of a tree with 5 vertices?

Solution:

A tree with 5 vertices has $(5 - 1) = 4$ edges. **So, the total degree of the tree is $= 2 \times 4 = 8$.**

Problem 4:

Can a graph with eight vertices and six edges be connected?

Solution:

A graph with 8 vertices requires at least $(8 - 1)$ edges = 7 edges to be connected, which in this case, will be a tree. So a graph with 8 vertices and 6 edges is not connected.