CMSC 207- Lecture 1
CHAPTER 1: SPEAKING MATHEMATICALLY

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Variables-1

There are two uses of a variable.

First Use: Checking whether a certain number theoretic property is satisfied; such as:

Is there a number with the following property: doubling it, and adding 3 gives the same result as squaring it?

Here, one can introduce a variable to replace the potentially ambiguous word "it":

Speaking Mathematically (Speaking Formally):

Is there a number x with the property that $2x + 3 = x^2$?

Variables-2

A variable allows you to give a temporary name or symbol assigned to it so that you can perform concrete computations with that name or symbol to help discover its possible values.

Second use of variables: Used to maintain the generality of the statement

Speaking Mathematically (Speaking Formally):

For any number, if it is greater than 2, then its square is greater than 4.

Variables-3

- Introducing a variable to give a temporary name to the number enables you to maintain the generality of the statement, and replacing all instances of the word "it" by the name ensures that possible ambiguity is avoided. Therefore, the above generality may be stated as using **Speaking Mathematically (Speaking Formally):**
- For any number n, if n is greater than 2, then n^2 is greater than 4.

Variables-4

Use variables to rewrite the following sentences formally.

a. Are there numbers such that the sum of their squares equals the square of their sum?

b. Given any real number, its square is nonnegative.

Solution:

a. Speaking Mathematically,

Are there numbers a and b with the property that

$$a^2 + b^2 = (a + b)^2?$$

Or**:** Are there numbers a and b such that

$$a^2 + b^2 = (a + b)^2?$$

Example 1 – *Solution*

b. Speaking Mathematically,

Given any real number $\boldsymbol{r}$, $\boldsymbol{r}^{2}$ is nonnegative.

***Or*: For any real number r, $r^2 \geq 0$.**

***Or*: For all real numbers r, $r^2 \geq 0$.**

Or: For all real numbers r, r^2 ***NOT*** **<** 0.

Important Mathematical Statements

3 of the most important kinds of sentences in Discrete Math are

- **Universal Statements** – Statements that are universally true. States that a certain property is true for all elements in a set. **Ex.** For any real number n, its square n^2 is non-negative.
- **Conditional Statements** – If one thing is true then some other thing is also true. **Ex.** If n is even, then its square, n^2 is also even.
- **Existential Statements** – States that for certain property, there is at least one element for which the property is true. **Ex.** There is a prime number, which is even.

2 is a prime number and it's even

Speaking Mathematically with Mathematical Statements

Universal statements contain some variation of the words **“for all”** and conditional statements contain versions of the words **“if-then.”** Existential statements contain variation of the words “**there exists**” or “**there is some**”

Hybrid Statements

- Hybrid statements combine two of the three basic forms of statements to create more complex statement forms
- Examples Include:
  - Universal Conditional Statements
  - Universal Existential Statements
  - Existential Universal Statements, etc.
- Some hybrid statement forms are discussed in following couple of slides

Universal Conditional Statements

A ***universal conditional statement*** is a statement that is **both universal** and **conditional.** Here is an example:

For all animals x, if x is a human being, then x is mortal.

Universal conditional statements can be rewritten in ways that make them appear to be **purely universal** or **purely conditional**.

In-class Assignment-1 (take 5 minutes to complete)

Fill in the blanks to rewrite the following statement: **For all integer *x*, if *x* is nonzero then x^2 is positive.**

a. If an integer is nonzero, then its square _____.

b. For all nonzero integer *x*, ____.

c. If *x* ____, then ____.

d. The square of any nonzero integer is ____.

e. All nonzero integers have ____.

Check Your Solution: In-class Assignment 1

a. is positive

b. x^2 is positive

c. is a nonzero integer; x^2 is positive

d. positive

e. positive squares (*or*: squares that are positive)

Universal Existential Statements-1

A ***universal existential statement*** is a statement that is universal because its first part says that a certain property is true for all objects of a given kind, and it is existential because its second part asserts the existence of something.

Speaking Mathematically, there exists something that is true for all elements of a kind

Example:

Every real number has an additive inverse.

In this statement, the property "has an additive inverse" applies universally to all real numbers.

Universal Existential Statements-2

“Has an additive inverse” asserts the **existence of** something—**an additive inverse**—for each real number.

However, the nature of the additive inverse depends on the real number; different real numbers have different additive inverses.

In-class Assignment-2 (take 5 minutes to complete)

Fill in the blanks to rewrite the following statement:

Every animal has a life span.

a. All animals _____.

b. For all animals A, there is ____.

c. For all animals A, there is a life span L such that _____.

Check Your Solution: In-class Assignment 2

a. have life spans

b. a life span for *A*

c. *L* is a life span for *A*

Existential Universal Statements-1

An ***existential universal statement*** is a statement that is existential because its first part asserts that a certain object exists and is universal because its second part says that the object satisfies a certain property for all things of a certain kind.

Existential Universal Statements-2

Example:

There is a positive integer that is less than or equal to every positive integer:

This statement is true because the number ***1*** is a positive integer, and it satisfies the property of being less than or equal to every positive integer.

In-class Assignment-3 (take 6 minutes to complete)

Fill in the blanks to rewrite the following statement in three different ways:

There is a person in my class who is at least as old as every other person in my class.

a. Some _____ is at least as old as _____.

b. There is a person *p* in my class such that *p* is _____.

c. There is a person *p* in my class with the property that for every person *q* in my class, *p* is _____.

Check Your Solution: In-class Assignment 3

a. person in my class; every person in my class

b. at least as old as every person in my class

c. at least as old as *q*

The Language of Sets-1

A set is a collection of elements or objects.

Example: if *A* is the set of all fruits, then mango is an element of *A*, and if *I* is the set of all integers from 1 to 200, then the number 159 is an element of *I*.

Speaking Mathematically - Sets

S is a set of elements, and an element x is in S, then $\mathbf{x \in S}$

If x is not an element of S, $\mathbf{x \notin S}$

S = {1, 2, 3} denotes a set whose elements are 1, 2 and 3

S = {1, 2, 3, …, 1000} denotes a very large set whose elements are all integers from 1 through 1000

S = {1, 2, 3, …} denotes an infinite set whose elements are all positive integers

… is called an ellipse while **speaking mathematically** and implies **"and so forth"**

The **axiom of extension** says that a set is completely determined by its elements and not the order in which they might be listed or the fact that some elements might be listed more than once.

In-class Assignment-4 (take 8 minutes to complete)

a. Let A = {a, b, c}, B = {c, a, b}, and C = {a, a, b, c, c, c}.

What are the elements of A, B, and C? How are A, B, and C related?

b. Is $\{0\} = 0$?

c. How many elements are in the set $\{1, \{1\}\}$?

d. For each nonnegative integer n, let $U_n = \{n, -n\}$. Find U_1, U_2, and U_0.

Check Your Solution: In-class Assignment 4

a. A, B, and C have exactly the same three elements: a, b, and c. Therefore, A, B, and C are simply different ways of representing the same set.

b. $\{0\} \neq 0$ because $\{0\}$ is a set with one element, namely 0, whereas 0 is just the symbol that represents the number zero.

c. The set $\{1, \{1\}\}$ has two elements: 1, and the set whose only element is 1.

d. $U_1 = \{1, -1\}$, $U_2 = \{2, -2\}$, $U_0 = \{0, -0\} = \{0, 0\} = \{0\}$.

The Language of Sets-1

Certain sets of numbers are so frequently referred to that they are given special symbolic names. These are summarized as follows:

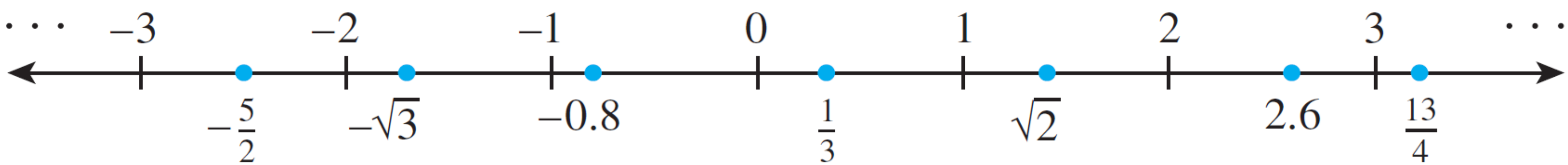
R denotes the **set of all real numbers; R^+:** Set of positive real numbers; **R^-:** Set of negative real numbers

Z denotes the **set of all integers; Z^+:** Set of positive integers; **Z^-:** Set of negative integers

Q denotes the **set of all rational numbers, or quotients of integers; Q^+:** Set of positive rational numbers; **Q^-:** Set of negative rational numbers

The Language of Sets-2

The set of real numbers is usually represented as the set of all points on a line, known as the **Number Line**, as depicted below:



The number 0 lies as the ***middle point***, called the ***origin***.

A unit of distance is marked off, and each point to the right of the origin represents a positive real number found by computing its distance from the origin. Similarly, each point to the left of the origin represents a negative real number.