Discrete Mathematics with Applications, 4th edition
Susanna S. Epp

Supplementary Exercises: Chapter 4

1. **Section 4.1**: Write an "outline" of a proof of the following statement by indicating the "starting point" and "conclusion to be shown." (Do not prove the statement!)

$$\text{For all integers } a,\ b,\ \text{and } c,\ \text{if } \ a \mid b \text{ and } a \mid c,\ \text{then } a^2 \mid (5b^2 + 7c^3).$$

2. **Section 4.1**: Prove the following statement directly from the definitions of even and odd: For all integers a, if a is even, then $5(a+3)$ is odd.

3. **Section 4.1**: Show that the following statement is false: For all real numbers a and b, if $ab > 1$ then at least one of a or b is greater than 1.

4. **Section 4.2**: Prove the following statement directly from the definition of rational: For all real numbers r and s, if r and s are rational then $2r + 3s$ is rational.

5. **Section 4.3**: Prove the following statement directly from the definition of divisibility: For any integers a and b, if $3 \mid a$ and $a \mid b$ then $3 \mid b$.

6. **Section 4.4**: Prove that for all real numbers a and b, $||a| - |b|| \le |a - b|$.

7. **Section 4.4**: Prove: For all integers m and n, if $m \ mod\ 5 = 2$ and $n \ mod\ 6 = 3$ then $mn \ mod\ 3 = 0$.

8. **Section 4.5**: Is the following statement true or false: For all real numbers x, $\lceil x \rceil^2 = \lceil x^2 \rceil$. Prove the statement if it is true and give a counterexample if it is false.

9. **Section 4.6**: When asked to prove that the difference of any irrational number minus any rational number is irrational, a student begins as follows: "Proof: Suppose the difference of any irrational number minus any rational number is rational" What is the student's mistake?

10. **Section 4.7**: Prove by contradiction: $4+5\sqrt{2}$ is irrational. (You may use the fact that $\sqrt{2}$ is irrational.)

11. **Section 4.7**: Find the mistake: The square of any irrational number is rational. "Proof: Suppose not. That is, suppose the square of any irrational number is irrational. Consider the number $\sqrt{2}$. $(\sqrt{2})^2 = 2$, which is rational. This contradicts the supposition, and so the supposition is false and the given statement is true."

12. **Section 4.8**: Integers a and b are defined to be **relatively prime** if, and only if, their greatest common divisor is 1. Use the Euclidean algorithm to determine whether the following integers are relatively prime: 6,728 and 34,391.