(**1**) (a) $R = \{(4, 15), (4, 18), (5, 16), (6, 15)\}$.

(c) R is not a function because $(4, 15) \in R$ and $(4, 18) \in R$, meaning there are two "outputs" for the input 4.

(2) These functions are equal, because $(x + 1)(x + 3) = x^2 + 4x + 3 = x^2 + 4x + 4 - 1 = (x + 2)^2 - 1$, so the outputs are the same for every input. Furthermore, their domains are the same.

(3) (a) All students have studied calculus.

(b) For every student x, x has studied calculus.

(4) (a) There is a student x in this class such that x has visited Mexico.

(b) There exists a student in this class that has visited Mexico.

(**5**) (a) $A = \{1, 2, 3, 4, 6, 12, -1, -2, -3, -4, -6, -12\}$. Ah ha! Did you forget the negative integers?

(b) $A = \{n \in \mathbb{Z} : n \mid 12\}$.

(**6**) This will simplify to $p \vee r$. Use the Associative Law and the Absorption Law. (See page 35 for the list of logical equivalences)

(**7**)

(8) (a) True (b) False (c) True

(9) $-2 \leq x < 0$ means $(-2 \leq x) \wedge (x < 0)$, so by DeMorgan's Law, its negation is $(-2 > x) \vee (x \geq 0)$.

(10) The converse is "If I am big or I am brave, then I am an adult."

The inverse is "If I am not an adult, then I am not big and I am not brave."

The contrapositive is "If I am not big and I am not brave, then I am not an adult."

Remember that the implication $p \to q$ can be rewritten as $\sim p \vee q$, and so $\sim(p \to q) \equiv p \wedge (\sim q)$. Therefore, the negation of the given statement is "I am an adult and I am not big and I am not brave."

(11) (a) Since $s \to p$ and $\sim p$ are given, then by modus tollens, we may deduce $\sim s$. Now we know $s \vee g$ and $\sim s$ are given, so by elimination, we may deduce g. We know $g \to n$ are given, so we may deduce n by modus ponens. (See page 61 for a list of valid argument forms.)

(12) (b) Outline: To make a simpler circuit, notice that if we write the negation of S:

Input			Output
P	Q	R	S
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1

It's easy to make a circuit for this table, and then negate the final result.

(13) Answer only: 10100010.

(14)

There is a building x in the city such that for any fire station y, the distance between x and y is more than 2 miles.

(15) If x^2 is odd, then x is odd. If we replace "necessary" with "sufficient," then the statement would read: "If x is odd, then x^2 is odd."