Braided Sixth-Chord Geometry: From the Circle of Keys to a 3D Sphere

Abstract

We present a geometric framework for representing sixth chords and their functional dynamics using a progressive embedding: a numbered circle (1–12) representing keys and pitch classes, thickened into a sphere only when planar representations saturate. Sixth chords are modeled not as static points or linear vectors but as braided trajectories that preserve ambiguity and cooperative motion without forced resolution. Using two exemplars—iii^6 and IV^6—we show how this geometry scales naturally from music theory to social coordination, identifying where sixth-chord motion enables alignment without dominance and where resolution should remain local and reversible.

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1. The Circle as a Numbered Musical Primitive

We begin with a circle labeled 1 through 12, preserving the utility of both a number line and the circle of keys. This structure simultaneously encodes cyclic recurrence, transpositional equivalence, and adjacency relations central to tonal theory. At this stage, each key is a point; motion between keys is legible and finite.

The circle is sufficient for triads and dominant motion. It becomes insufficient once relational chords—particularly sixth chords—are drawn as internal connections. These chords introduce crossings that obscure voice-leading and functional independence. The representational failure is geometric, not musical.

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2. Sixth Chords as Relational Objects

A sixth chord is defined by a root and its sixth degree, creating a diagonal relation across the circle (e.g., 1–6). Unlike triads, which collapse toward a tonal center, or dominants, which exert directed pull, sixth chords sustain openness. They imply cooperation between stability and color rather than demand resolution.

When multiple sixth chords are drawn on the plane, their lines intersect and overwrite one another. The plane cannot encode their independence.

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3. Necessity of the Sphere: Thickening the Circle

To preserve the original circle while gaining capacity, we lift it into three dimensions. The numbered circle becomes the equator of a sphere. No musical relationships are discarded; depth is added only to separate overlapping relations.

This move is minimal and motivated: the sphere exists because the plane runs out of room.

4. Braided Vectors on the Sphere

On the sphere, each sixth chord is represented as a curved arc that departs from the equator, wraps around the surface, and returns. Crucially, the chord is not a single arc but a braid of two coupled trajectories: - a strand representing tonal anchoring (root tendency) - a strand representing the sixth as openness or delay

These strands twist without collapsing, maintaining identity through motion. Braiding encodes cooperation without fusion.

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5. Two Exemplars: iii^6 and IV^6

iii^6 (Mediant Sixth)

The iii^6 chord originates near scale degree 3 and rises gently above the equator. Its braid is loose, its orbit long, and its curvature shallow. Functionally, it modulates identity without urgency. It neither prepares nor resolves; it explores.

IV^6 (Subdominant Sixth)

The IV^6 chord originates near scale degree 4 and remains closer to the equator. Its braid is tighter and curves toward known resolution regions without entering them. Functionally, it aligns collective motion and prepares shared action without forcing closure.

These two chords are sufficient to demonstrate distinct modes of non-resolution.

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6. Resolution as Local Basins

Resolution is not represented at the center of the sphere. The center corresponds to total collapse—forced consensus or authoritarian dominance. Instead, resolutions appear as shallow basins near the equator at culturally stable degrees (e.g., 1, 5).

Sixth-chord trajectories may approach, touch, or orbit these basins, but they are not absorbed. Healthy systems allow repeated entry and exit.

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7. Societal Mapping

In social terms: - Keys correspond to stable positions or norms - Sixth chords correspond to cooperative states that preserve plurality - Braiding corresponds to parallel interests that do not merge - Resolution basins correspond to decisions, rituals, or agreements

Sixth-chord motion is beneficial during coalition-building, pre-legislative phases, diplomacy, education, and creative collaboration. Failure occurs when systems bypass this phase and collapse prematurely into dominant resolution.

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8. Conclusion

We begin with a numbered circle because meaning is cyclic and countable. We move to a sphere only when relationships overlap. Sixth chords demand depth because cooperation cannot be drawn flat. This framework preserves musical rigor while offering a scalable model for understanding coordination without dominance.