Corner-Event Band Structure in Discrete Rectangular Billiards with Phase-Coupled Reflections

Abstract

We study a deterministic discrete billiard system on rectangular integer grids in which corner impacts trigger a multiplicative phase update. Using a canonical 5×7 geometry with multiplier 7 and modulus 1,000,003, we demonstrate the emergence of stable, quantized bands in corner-event spacing over time. These bands persist across long runs, separate cleanly by corner type, and vary systematically under multiplier sweeps. All results are fully reproducible using an interactive simulator.

1. Introduction

Discrete billiards provide a minimal setting in which geometry, arithmetic, and dynamics intersect. While classical billiards emphasize continuous trajectories, discrete models expose structural phenomena that remain hidden in continuous limits. This work focuses on corner events, which act as arithmetic probes of the underlying geometry when coupled to a phase update rule.

2. Model Definition

The system evolves on an integer lattice bounded by a rectangle of size (X,Y). The particle moves with integer velocity and reflects elastically at walls. Corner events occur when both coordinates simultaneously reach a boundary.

At each corner event, the system records the event time, corner type, and phase. Diagonal corners increment the phase additively, while off-diagonal corners apply a multiplicative update modulo a large prime. Between corner events, the phase remains constant.

3. Phenomenology (Canonical 5×7, M=7)

Using the canonical configuration (5×7 grid, initial state (2,4), velocity (1,2)), we observe 460 corner events over 5,000 steps. The spacing between successive corner events falls into a small number of discrete bands that remain stable across time.

Diagonal and off-diagonal events occupy overlapping but statistically distinct subsets of these bands. This separation is visible in spacing-versus-time plots and persists under extended runs.

4. Trajectory and Phase-Space Interpretation

The discrete trajectory may be lifted to a toroidal representation in which reflections correspond to linear motion with folding. The phase variable introduces an additional winding dimension, turning corner events into punctures on a higher-dimensional manifold.

Visualizing phase logarithmically reveals spiral structures that encode long-term arithmetic growth. Configurations with no corner events correspond to rational slope lock-ins that never intersect a corner.

5. Discussion and Reproducibility

The observed band structure is deterministic and geometric, not stochastic. Its robustness across time and parameter sweeps indicates a structural origin rooted in lattice resonance. All figures and tables in this paper can be regenerated using the CornerReflector.sim application.

Canonical configuration: Size=(5,7), Multiplier=7, Modulus=1,000,003, Initial=(2,4), Velocity=(1,2), Steps=5000.