Example Case Study

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1 Instruction

Planterbox Inc., a plant care software business, just acquired a regional plant care operator named SaaSafras based in the Southeastern US.

SaaSafras' business has been stable with some organic growth and some natural churn. Planterbox has identified you as a rising star within the organization, and you've been hand-picked to run the newly acquired business after the previous CEO of SaaSafras retired and sold their business. You're excited about the opportunity, and your first project is to figure out how you can grow SaaSafras' revenue (Planterbox decided they loved the name too much, even though they acquired the company they'll keep the SaaSafras name).

As part of the acquisition, you've inherited 20 great people who can do it all between sales, account management, and support. Unfortunately, doing it all has come at a cost, and productivity has been lower than if they had specialized in any of the individual roles (and they are begging for specialization as well). Going forward, each person will only perform one core role at a time. While each person will only perform one role at a time, they can switch to another role at the start of any month - one person could cycle through every job without a loss in productivity.

Your goal is to maximize SaaSafras' cumulative revenue over the next 24 months (the sum of all revenue generated over 24 months). Your responsibility is to determine where these 20 people will work each month for the next 24 months to maximize the cumulative revenue generated over the entire period.

The three roles are:

- **New Business Acquisition:** These people are responsible for selling and getting new customers in the door.
- **Account Management:** These people help existing customers; they drive revenue growth from the customers they work with and improve retention.
- **Support:** These people solve customer problems; they improve retention for any active customer.

SaaSafras has some key metrics that won't change with the acquisition. The business currently has 1,000 customers. SaaSafras acquires 25 customers a month organically through great branding and customer referrals. Some customers turn to other, more specialized solutions, and monthly churn is 10%. There has been a standalone support organization in addition to your swiss army knife team, and support's CSAT (Customer Satisfaction) has been steady at 70% for several years. SaaSafras doesn't offer any discounts, and every active customer pays a baseline fee of $100 a month for the core product.

Team specifics are below:

- Each New Business Acquisition team member can acquire 5 new customers a month.

- Account Managers increase revenue by 20% month-over-month for accounts they manage up to a cap of 6 months. To be clear, this revenue increase compounds by 20% each month up to the 6th month, at which point it maxes out and remains flat for the remaining duration of time that the customer has an account manager (for example, if a customer has an account manager in month 1 they'll pay $120, in month 2 they'll pay $144, etc). If a customer has an account manager and then loses the account manager, their metrics (revenue per month) return to the baseline, but there isn't a negative consequence. Each Account Manager can carry 25 customers.
- Each support agent increases CSAT by 1 percentage point. Each point of CSAT leads to a 15% relative decrease in churn.

SaaSafras needs you to make 3 key decisions:

1. How many people will work on acquiring new business, account management, and support each month from month 1 to month 24?
2. Why are they working there? We care about your understanding of the why behind your decisions just as much as we care about the actual decision, so please explain your reasoning, not just the conclusion (i.e., don't rely too much on solver / other tools).
3. **Bonus:** If you had a magic wand, what is the one variable you would try to improve going into year 3? How would you approach moving that variable? By "variable" here, we mean one of your team members' core metrics (customer acquisition, CSAT, revenue increase, relative churn decrease, etc).

Output for the Case Study

In line with our actual work, the ideal output from this exercise is a written document outlining your thinking backed by analysis.

2 Introduction

As the newly appointed CEO of SaaSafras, a SaaS company recently acquired by Planterbox Inc, I am tasked with guiding the company through its next growth phase. Our primary goal is to maximize revenue growth with a strategic emphasis on long-term value, reflective of Planterbox's acquisition strategy to integrate SaaSafras for sustainable long-term success.

In order to maximize revenue I will need to strategically deploy resources. The challenge lies in optimally allocating our 20 staff members among three roles—New Business Acquisition, Account Management, and Support—to maximize revenue growth.

In software businesses like ours, we can assume that the marginal cost per customer is virtually zero, which typically incentives customer base expansion. I will apply marginal analysis to the three different positions to precisely optimize staff allocation and drive maximum business growth.

3 Marginal Analysis

In this section, we apply marginal analysis to determine the optimal allocation of team members across different roles. Marginal analysis is a staple of economic theory, commonly used to maximize outputs such as revenue or operational efficiency in a labor context. In our scenario, marginal value refers to the additional revenue generated by assigning an extra team member to a specific role.

To facilitate our analysis, we define the following variables to represent the distribution of team members:

- i: Number of team members allocated to New Business Acquisition.
- j: Number of team members allocated to Account Management.
- k: Number of team members allocated to Support.

For quantifying marginal rates, we use $\Delta_x y$ to denote the marginal change in y with respect to a marginal change in x. This notation, while varied, aligns with other common differential expressions:

$$\delta y/\delta x \approx MY/MX \approx y'(x) \approx \Delta_x y \tag{1}$$

We calculate the marginal value V_x for roles $x \in \{i, j, k\}$ as follows:

$$V_x = \text{customer} \cdot \text{price} \cdot \text{LTV factor} \tag{2}$$

Given our total of 20 team members, we establish a constraint that ensures full utilization of our workforce for maximum productivity:

$$i + j + k = 20 \tag{3}$$

This constraint effectively outlines our operational limits and serves as the basis for our resource allocation strategy.

3.1 New Business Acquisition

To accurately assess the impact of each team member, we begin by considering the net revenue they can potentially generate each month in a position. Each member of new business acquisition brings in 5 new customers monthly, with each customer contributing $p = 100$ dollars per month. The long-term value (LTV) of these customers is crucial, as it incorporates all future revenues, not just what is generated this month. We include the infinite sum for LTV to capture the total potential revenue from each customer over the lifetime as a customer, adjusted for customer attrition due to churn. This approach ensures we account for the full economic value of customer acquisition:

$$V_i = i \cdot p \cdot 5 \cdot \sum_{n=0}^{\infty} (1 - r_c)^n \tag{4}$$

To manage the complexity of calculating the infinite series, we apply the formula for the sum of an infinite geometric series:

$$S = \sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \text{ for } |r| < 1$$

With $r = 1 - r_c$, this simplifies to:

$$\sum_{n=0}^{\infty} (1 - r_c)^n = \frac{1}{r_c} \tag{5}$$

We then calculate the marginal value of adding an additional team member to new business acquisition:

$$\Delta_i V_i = p \cdot 5 \cdot \frac{1}{r_c} \tag{6}$$

This marginal value, $\Delta_i V_i$, quantifies the additional revenue generated by each new team member and will be compared against the marginal values of other roles to determine the most effective allocation of resources.

3.2 Account Management

Account managers play a critical role in capitalizing on our existing customer base to drive incremental revenue. Each account manager can manage up to 25 customers, with revenue increasing by a factor of 1.2^t during the first 6 months, after which it remains constant. Initially, we consider $t = 6$ to estimate the peak value provided by an account manager, treating earlier deficits as a fixed and can be negligible cost over long-term horizons.

$$V_j = j \cdot 25 \cdot p \cdot 1.2^6 \tag{7}$$

To determine the marginal value of adding an account manager, we calculate:

$$\Delta_j V_j = V_j(j+1) - V_j(j) = j \cdot 25 \cdot p \cdot 1.2^6 - (6 - j) \cdot 25 \cdot p \cdot 1.2^6 - 25 \cdot p \tag{8}$$

$$\Delta_j V_j = 25 \cdot 100 \cdot (2.99 - 1) = 25 \cdot 100 \cdot 1.99 = 4975 \tag{9}$$

3.2.1 New Business Acquisition Vs Account Management

Now to compare the between two roles we can compare use their marginal value. Since the marginal values for both New Business Acquisition and Account Management are constants, their Marginal Rate of Substitution (MRS) is also constant, indicating they are perfect substitutes. In practical terms, this implies that one role is either equally valuable as or more valuable than the other, allowing us to potentially eliminate the less effective role reduce the complexity.

$$\Delta_i V_i = 5000, \tag{10}$$

$$\Delta_j V_j = 4975, \tag{11}$$

results in the inequality:

$$\Delta_i V_i > \Delta_j V_j. \tag{12}$$

Therefore, allocating resources to New Business Acquisition is mathematically more valuable than Account Management under current conditions. For example, if the goal is the maximize revenue over a small period there are situations where account managers would be more valuable. This conclusion allows us to simplify our allocation strategy:

$$j = 0, \quad i + k = 20 \tag{13}$$

By reducing the constraint to 2-dimensions the optimization becomes much simpler and exact.

Eliminating Account Management from consideration also allows for a further simplification to the problem. Since every customer pays the same price and both New Business Acquisition (NBA) and Support directly influence monthly changes in customer numbers, we can focus exclusively on maximizing customer count.

3.3 Support

The support team plays a crucial role in enhancing customer satisfaction and retention by effectively reducing churn. Their efforts are directly linked to maintaining a stable and growing customer base, underscoring the importance of optimizing their allocation.

Let x_t denote the number of customers in period t, where:

$$x_t = x_{t-1} \cdot (1 - r_c) \tag{14}$$

Here, r_c is the churn rate, representing the proportion of customers lost each period without effective support intervention.

To account for the impact of support staff on reducing churn, we modify the customer growth equation as follows:

$$x_t = x_{t-1} \cdot \left(1 - r_c \cdot (1 - r_d)^k\right) + 5i + 25 \tag{15}$$

In this model, r_d represents the rate of churn reduction per support staff member, set at 15%, and k denotes the number of support staff. The terms $5i$ and 25 account for new customers gained through New Business Acquisition efforts and organic growth, respectively.

In order to compare Support against New Business Acquisition, we calculate the marginal gain in customers resulting from the addition of one more support staff member:

$$\Delta_k x_t = x_{t-1} \cdot (1 - r_c \cdot (1 - r_d)^{k+1}) - x_{t-1} \cdot (1 - r_c \cdot (1 - r_d)^k) \tag{16}$$

Simplifying, the marginal gain becomes:

$$\Delta_k x_t = x_{t-1} \cdot r_c \cdot r_d \cdot (1 - r_d)^k \tag{17}$$

This equation provides a basis for comparing the effectiveness of allocating additional resources to support versus New Business Acquisition.

4 Optimizing Allocation

4.1 Inflection Points

This subsection evaluates the marginal effects on customer numbers from allocating additional staff to New Business Acquisition (NBA) versus Support roles. We aim to identify the inflection points where the value of adding support staff surpasses that of NBA. Given that $\Delta_i x_t$ is constant while $\Delta_k x_t$ varies, yielding a one-dimensional set of optimal solutions. Understanding these dynamics enables us to strategically distribute our staff to maximize customer growth.

$$\Delta_i x_t < \Delta_k x_t \tag{18}$$

Here, $x_{\min}$ is the minimum customer base required for the additional impact of one support staff member to surpass that of one NBA staff member in terms of customer acquisition. We calculate $x_{\min}$ as follows:

$$x_{\min} = \frac{5}{r_c \cdot r_d \cdot (1 - r_d)^k} \tag{19}$$

This value represents the threshold at which adding another support staff member becomes more beneficial than adding a new business acquisition staff in terms of reducing churn and increasing the customer base.

Table 1: Required Minimum Number of Customers ($x_{\min}$) for Each Number of Support Staff

k	$x_{\min}$	k	$x_{\min}$
0	334	11	1992
1	393	12	2344
2	462	13	2757
3	543	14	3244
4	639	15	3816
5	752	16	4490
6	884	17	5282
7	1040	18	6214
8	1224	19	7311
9	1440	20	8601
10	1694		

This table helps set clear thresholds for when it is beneficial to allocate resources to support over NBA, facilitating transparent communication and between the leadership and staff. Additionally, a system involving volunteer shifts or random assignment could be implemented to allow team members to transition roles based on operational needs and personal preferences.

4.2 Equilibrium

When analyzing recursive functions, a crucial aspect is determining the equilibrium point where the function stabilizes—referred to as the customer equilibrium (x_e). In our scenario, calculating x_e is complex because each staff allocation level (k) yields a distinct x_e, which may vary above or below the x_{min} for the subsequent k. To establish the equilibrium where the customer count remains constant from one period to the next:

Assuming a stable customer equilibrium, where $x_t = x_{t-1} = x_e$, the equation simplifies to:

$$x_e = x_e \cdot \left(1 - r_c \cdot (1 - r_d)^k\right) + 5(20 - k) + 25 \tag{20}$$

This equation balances the loss and gain of customers over a period. Solving for x_e involves rearranging the terms:

$$x_e(1 - r_c \cdot (1 - r_d)^k) = 100 - 5k + 25 \tag{21}$$

$$x_e \cdot r_c \cdot (1 - r_d)^k = 125 - 5k \tag{22}$$

$$x_e = \frac{125 - 5k}{r_c \cdot (1 - r_d)^k} \tag{23}$$

This final equation determines x_e based on the number of support staff k, facilitating decisions on optimal staffing to maximize customer retention.

Table 2: Equilibrium Number of Customers (x_e) for Various Numbers of Support Team Members (k)

k	x_e	k	x_e
0	1250.00	11	4183.02
1	1411.76	12	4569.68
2	1591.70	13	4962.55
3	1791.17	14	5351.77
4	2011.47	15	5723.82
5	2253.75	16	6060.52
6	2518.89	17	6337.80
7	2807.44	18	6524.20
8	3119.37	19	6579.03
9	3453.98	20	6450.03
10	3809.54		

The table provides a clear overview of how customer equilibrium x_e changes with different numbers of support staff, aiding in strategic planning and resource allocation.

4.3 Max Possible Customer

Using the inflection points and equilibrium conditions, we determine the largest value of k until which the customer base can be maximized, governed by the condition $x_{\min} < x_e$.

Given the equations:

$$x_{\min} = \frac{5}{r_c \cdot r_d \cdot (1 - r_d)^k} \tag{24}$$

$$x_e = \frac{125 - 5k}{r_c \cdot (1 - r_d)^k} \tag{25}$$

We seek the values of k for which $x_{\min} > x_e$. By simplifying the inequality, we eliminate common terms:

$$\frac{5}{r_c \cdot r_d \cdot (1 - r_d)^k} > \frac{125 - 5k}{r_c \cdot (1 - r_d)^k} \tag{26}$$

Canceling out r_c and $(1 - r_d)^k$ (assuming they are non-zero), and simplifying further, we derive:

$$5 > 125 - 5k \tag{27}$$

$$k < 25 - \frac{5}{r_d} \tag{28}$$

$$k < \frac{2.75}{0.15} \approx 18.33 \tag{29}$$

This calculation identifies the upper limit for k, suggesting that increasing the number of support staff beyond 18 does not yield additional benefits in terms of customer retention, as the equilibrium number of customers (x_e) will not support further increases effectively.

Table 3: Comparison of Required Minimum Number of Customers ($x_{\min}$) and Equilibrium Number of Customers (x_e) for Various Numbers of Support Staff (k)

k	$x_{\min}$	x_e	k	$x_{\min}$	x_e
0	334	1250	11	1992	4183
1	393	1411	12	2344	4569
2	462	1591	13	2757	4962
3	543	1791	14	3244	5351
4	639	2011	15	3816	5723
5	752	2253	16	4490	6060
6	884	2518	17	5282	6337
7	1040	2807	18	6214	6524
8	1224	3119	19	7311	6579
9	1440	3453	20	8601	6450
10	1694	3809			

Since k must be an integer, we round up because the inequality $k < 18.33$ implies that k will reach 18 but not 19. Therefore, $k < 19$. Therefore we can determine the largest possible customer pool to be 6524 customers.

5 Calculations

5.1 Projections

To estimate projected revenue, we conducted a Python simulation using the recursive function x_t with dynamic adjustment of support staff k based on customer count thresholds. This deterministic approach ensures consistent results across simulations, eliminating the need for multiple runs.

Table 4: Projected Revenue and Customer Growth

t	k	x_{t-1}	Δx_t	t	k	x_{t-1}	Δx_t
0	7	1000	0	13	10	1643	43
1	7	1057	57	14	10	1685	42
2	8	1113	56	15	10	1726	41
3	8	1167	54	16	11	1767	41
4	8	1220	53	17	11	1807	40
5	8	1271	51	18	11	1846	39
6	9	1321	50	19	11	1885	39
7	9	1370	49	20	11	1923	38
8	9	1418	48	21	11	1960	37
9	9	1465	47	22	11	1997	37
10	10	1511	46	23	12	2033	36
11	10	1556	45	24	12	2069	36
12	10	1600	44				

This simulation project more than a doubling of our customer base over two years. For a visual analysis of growth trends, refer to the following plots, which illustrate the projected customer growth over different time spans.

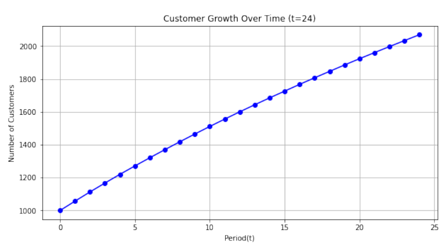

Figure 1: Graph of Customer Growth Over 24 Months

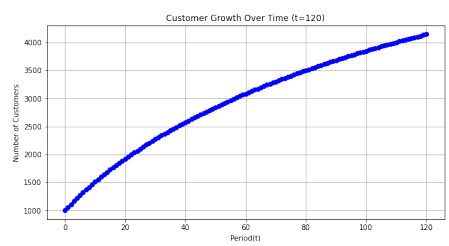

Figure 2: Graph of Customer Growth Over 120 Months

The growth trajectory over 24 months appears linear, suggesting steady growth without significant accelerations or declines. Over a longer period, as shown in the graph of customer growth over 120 months, the growth pattern becomes asymptotic, highlighting a slowdown as the market saturates. Ideally, strategic adjustments are necessary to shift from linear to exponential growth to sustain long-term business viability.

5.2 Alternative Comparison

In order to demonstrate the success of our strategy, we compare the outcomes of simulations with different allocations: $k = 0$, $k = 20$, and an optimal k. This comparison allows us to assess the efficacy of dynamic staff reallocation based on customer growth needs.

Table 5: Comparison of Customer Growth Across Different Support Levels

t	$k=0$	$k=20$	$k=$ **opt**	t	$k=0$	$k=20$	$k=$ **opt**
0	1000	1000	1000	13	1184	1262	1643
1	1025	1021	1057	14	1190	1282	1685
2	1047	1042	1113	15	1196	1302	1726
3	1067	1062	1167	16	1201	1321	1767
4	1085	1082	1220	17	1205	1340	1807
5	1101	1102	1271	18	1209	1359	1846
6	1115	1122	1321	19	1213	1378	1885
7	1128	1142	1370	20	1216	1397	1923
8	1140	1162	1418	21	1219	1416	1960
9	1151	1182	1465	22	1222	1435	1997
10	1160	1202	1511	23	1224	1454	2033
11	1169	1222	1556	24	1226	1473	2069
12	1177	1242	1600				

Figure 3: Graph of Customer Growth Over 24 Months

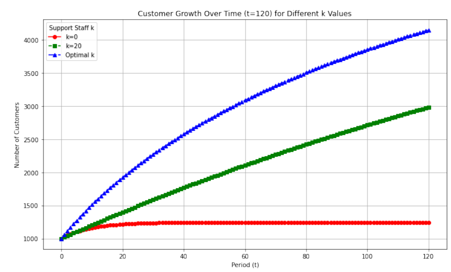

Figure 4: Graph of Customer Growth Over 120 Months

The table and graphs show a clear trend where the optimal allocation strategy outperforms both static scenarios (k=0 and k=20) by wide margins, confirming the effectiveness of dynamically adjusting staff based on calculated growth trajectories.

6 Extra Fun

6.1 Bonus Question

Reducing churn is crucial for sustained business growth. One effective approach is enhancing customer satisfaction (CSAT) or improving churn reduction per support staff member (r_d), which directly impacts customer retention rates.

Exit interviews are a strategic tool to identify and address potential issues that lead to customer churn. They provide an opportunity not only to save a departing customer but also to gather insights that can inform broader improvements in service and product offerings.

Regularly reaching out to customers to check if they need assistance or have concerns can significantly reduce churn. Customers are more likely to stay with a service where they feel their needs are actively addressed.

Implementing a referral program can also boost growth. Offering a month's discount for referring new customers helps cover some of the customer acquisition costs (CAC) and can slightly reduce the lifetime value (LTV) of the referring customer. However, this is often offset by the value of acquiring new customers at a lower cost. Referrals can lead to exponential growth if the rate of referrals (r_r), defined as the number of referrals per customer over a period, exceeds the net churn rate.

The following equation models customer growth under these strategies:

$$x_t = x_{t-1} \cdot \left(1 - r_c \cdot (1 - r_d)^k\right) + 5 \cdot (20 - k) + 25 + x_{t-1} \cdot r_r \tag{30}$$

This model shows that if r_r is greater than the adjusted churn rate $(1 - r_c \cdot (1 - r_d)^k)$, the customer base will experience exponential growth. This especially becomes more reasonable for large k or if churn is decreasing by other factors.

By initially focusing on improving the customer experience through exit interviews and regular outreach, this strategy enhances customer satisfaction, thereby driving more referrals.

A Python Code

A.1 Optimal Support Staff Calculation

```python
def find_opt_k(rc, rd, init_cust):
    max_cust = 0
    opt_k = 0
    for k in range(21):  # Max k could be 20
        x = init_cust * (1 - rc * (1 - rd) ** k) + 5 *
            (20 - k) + 25
        if x > max_cust:
            max_cust = x
            opt_k = k
    return opt_k
```

The function ‘find_opt_k‘ calculates the optimal number of support staff (k) that maximizes the customer base. It iterates through possible values of k and finds the one that yields the highest number of customers.

A.2 Simulating Customer Growth with Optimal k

```python
def sim_customers(rc, rd, periods, init_cust):
    opt_k = find_opt_k(rc, rd, init_cust)
    prev_x = init_cust
    results = [(0, prev_x, opt_k)]  # Store initial
        conditions

    for t in range(1, periods + 1):
        # Calculate new customer count, round down by
            converting to int
        opt_k = find_opt_k(rc, rd, prev_x)
        prev_x = int(prev_x * (1 - rc * (1 - rd) **
            opt_k) + 5 * (20 - opt_k) + 25)
        results.append((t, prev_x, opt_k))

    return results
```

The function ‘sim_customers‘ simulates the customer growth over a specified number of periods, using the optimal number of support staff (k) calculated for each period. It stores the results for each period, including the number of customers and the optimal k.

A.3 Simulation Constants and Running the Simulation

```python
# Constants for the simulation
rc = 0.10  # Churn rate
rd = 0.15  # Decreasing churn rate per support staff
periods = 120  # Simulation period length
init_cust = 1000  # Initial customer count
```

```
# Run the simulation with an optimal starting k
simulation_results = sim_customers(rc, rd, periods,
    init_cust)

# Format the results for presentation
simulation_table = [("Period", "Customers", "Support
    Staff k")]
simulation_table.extend(simulation_results)
print(simulation_table)
```

This block sets up the constants for the simulation, including the churn rate, decreasing churn rate per support staff, the number of periods, and the initial customer count. It then runs the customer growth simulation using these constants and formats the results for presentation.

A.4 Plotting Customer Growth Over Time

```
import matplotlib.pyplot as plt

# Prepare data for plotting
periods = [result[0] for result in simulation_results]
customers = [result[1] for result in simulation_results]

# Create the plot
plt.figure(figsize=(10, 5))
plt.plot(periods, customers, marker='o', linestyle='-',
    color='b')
plt.title('Customer Growth Over Time (t=120)')
plt.xlabel('Period(t)')
plt.ylabel('Number of Customers')
plt.grid(True)
plt.show()
```

This block uses `matplotlib` to plot the customer growth over time based on the simulation results. The plot visualizes the number of customers for each period, providing a clear graphical representation of the growth trajectory.

A.5 Simulating Customer Growth with Fixed k

```
def sim_customers_fixed_k(rc, rd, periods, init_cust,
    fixed_k=None):
    if fixed_k is None:  # Use the optimal k if none
        specified
        fixed_k = find_opt_k(rc, rd, init_cust)

    prev_x = init_cust
    results = [(0, prev_x, fixed_k)]  # Store initial
        conditions with specified k
```

```
    for t in range(1, periods + 1):
        prev_x = int(prev_x * (1 - rc * (1 - rd) **
            fixed_k) + 5 * (20 - fixed_k) + 25)
        results.append((t, prev_x, fixed_k))

    return results
```

The function 'sim_customers_fixed_k' simulates customer growth over a specified number of periods using a fixed number of support staff (k). This allows comparison of customer growth under different fixed k values.

A.6 Running Simulations with Fixed k Values

```
# Constants for the simulation
rc = 0.10  # Churn rate
rd = 0.15  # Decreasing churn rate per support staff
periods = 120  # Simulation period length
init_cust = 1000  # Initial customer count

# Run the simulations with fixed k values
results_k0 = sim_customers_fixed_k(rc, rd, periods,
    init_cust, fixed_k=0)
results_k20 = sim_customers_fixed_k(rc, rd, periods,
    init_cust, fixed_k=20)

# Format the results for presentation
simulation_table_k0 = [("Period", "Customers", "Support
    Staff k=0")]
simulation_table_k20 = [("Period", "Customers", "Support
     Staff k=20")]

simulation_table_k0.extend(results_k0)
simulation_table_k20.extend(results_k20)

# Print results for verification (optional)
for table in [simulation_table_k0, simulation_table_k20
    ]:
    for entry in table:
        print(entry)
```

This block runs simulations using fixed k values of 0 and 20. It formats the results for presentation and optionally prints them for verification.

A.7 Plotting Customer Growth for Different k Values

```
# Prepare data for plotting
periods = [result[0] for result in results_k0]  #
    Periods should be the same for all
```

```
customers_k0 = [result[1] for result in results_k0]
customers_k20 = [result[1] for result in results_k20]
customers_k_opt = [result[1] for result in
    simulation_results]

# Create the plot
plt.figure(figsize=(12, 7))
plt.plot(periods, customers_k0, marker='o', linestyle='-
    ', color='red', label='k=0')
plt.plot(periods, customers_k20, marker='s', linestyle='
    --', color='green', label='k=20')
plt.plot(periods, customers_k_opt, marker='^', linestyle
    ='-.', color='blue', label='Optimal k')

plt.title('Customer Growth Over Time (t=120) for
    Different k Values')
plt.xlabel('Period (t)')
plt.ylabel('Number of Customers')
plt.legend(title='Support Staff k')
plt.grid(True)
plt.show()
```

This final block prepares the data for plotting and creates a comparative plot of customer growth over time for different fixed k values (0 and 20) as well as the optimal k. This allows visualization of how different support staff allocations impact customer growth.