ME 562 Inelastic Continuum Mechanics 2March2011

50 Min. Closed Book Exam

The weights (square brackets) associated with the problems indicate the relative degree of thoroughness (and time) that should govern your answers.

1. [25] What is meant by each of the following: (i) spherical part of a second-order tensor, (ii) deviatoric part of a second-order tensor, (iii) projection operator, (iii) orthogonal projection operators, and (v) spherical projector.

2. [33] (i) What is meant by an orthonormal basis and how are vectors expressed using such a basis?
(ii) What is meant by a tensor product and how is this definition used to define a tensor basis. How are tensors expressed using such a basis?
(iii) Derive the "conventional" transformation relation for components of second-order tensors.
(iv) What is the Voigt-Mandel representation of a second-order tensor? Why is the Voigt-Mandel representation introduced?
(v) Derive the transformation relation for second-order tensors using the Voigt-Mandel representation.
(vi) What are the advantages, if any, of the $\sqrt{2}$ term that appears in the Voigt-Mandel representation of second-order tensors?

3. [10] You are given a full matrix representing the Voigt-Mandel components of the elasticity tensor in some arbitrary basis. Someone states that they think the material is orthotropic. Describe fully how you would determine whether or not the material is indeed orthotropic. Include your definition of orthotropy. Why would one want to know whether or not a material is orthotropic?

4. [10] Someone gives you the components of a fourth-order tensor and claims the tensor is isotropic. How would you check the assertion of isotropy?

5. (a) [4] Define what is meant by a gradient with respect to (i) a vector and (ii) a second-order tensor.
(b) [18] For vectors $\mathbf{u}$ and $\mathbf{v}$, and a second-order tensor $\mathbf{T}$, find the following gradients:
(i) of $(\mathbf{u}\cdot\mathbf{u})$ with respect to $\mathbf{u}$,
(ii) of $(\mathbf{u}\cdot\mathbf{u})^{-1/3}\mathbf{u}$ with respect to $\mathbf{u}$,
(iii) of $\mathbf{T}\cdot\mathbf{u}$ with respect to $\mathbf{u}$ (where $\mathbf{T}$ is a constant second-order tensor) ,
(iv) of $(\mathbf{u}\cdot\mathbf{T}\cdot\mathbf{v})$ with respect to $\mathbf{T}$ ($\mathbf{u}$ and $\mathbf{v}$ are constants) ,
(v) of $\mathbf{T}$ with respect to $\mathbf{T}$, and
(vi) of $(\mathbf{T}\cdot\mathbf{T}\cdot\mathbf{T})$ with respect to $\mathbf{T}$.

ME 562 Inelastic Continuum Mechanics (8:00 – 8:50 AM) 27 April 2011

Closed Book Exam

Write a "summary" article providing an overview of the basic concepts associated with the theory of plasticity and material stability. Credit will be allocated as follows:

[25] Item 1. General flow and logical structure of your article, completeness and the introduction of terms as necessary that are not listed below.

[75] Item 2. Technical definitions and arguments. This credit is broken down as listed below and is based on the accuracy of your descriptions and definitions that are included in your article. Both word descriptions and compact mathematical equations are expected.

1. [15] What is meant by the stress paths of uniaxial stress, triaxial compression and pure shear? What would you expect to be the difference between experimental data for metals and data for concrete or rock? What features of experimental data warrant the use of a plasticity model?

2. [24] What is meant by (i) Mises stress, (ii) plastic strain, (iii) effective plastic strain, (iv) yield function, (v) yield surface, (vi) evolution equations, (vii) evolution functions, and (viii) consistency?

3. [6] Give a complete set of the elastic-plastic constitutive equations

4. [6] What is the difference between isotopic and kinematic hardening? What features of experimental data or problem type would suggest you choose one form over the other?

5. [6] What is meant by rate-dependence and rate independence? If your formulation is rate-independent, how would you add in a rate dependence?

6. [9] What is meant by tangent tensor, acoustic tensor, and material instability.

7. [9] With regard to the tangent tensor, what is meant by minor symmetries? What is meant by major symmetry? Under what condition, if any, does the tangent tensor satisfy major symmetry? What is the implication of major symmetry on material stability?

ME 562 Inelastic Continuum Mechanics (7:30 – 9:30 AM) 11 May 2011

Final Exam - Closed Book

Credit is shown in square brackets. This indicates roughly the relative weighting you should place on your effort.

1. Write a "summary" article providing an overview of the basic concepts associated with the decohesive model of material failure. Credit will be allocated as follows:

Item 1 [5]. General flow and logical structure of your article, completeness and the introduction of terms as necessary that are not listed below.

Item 2. Technical definitions and arguments. This credit is broken down as listed below and is based on the accuracy of your descriptions and definitions that are included in your article. Both word descriptions and compact mathematical equations are expected.

A [15]. In the course of your article, answer the following questions:

(i) What is a failure model expected to provide?

(ii) What are the classical models of Rankine, Tresca and Mohr-Coulomb, and what are their relative advantages and deficiencies.

B [20]. Provide definitions or descriptions for each of the following:

(i) Material failure, (ii) displacement discontinuity, (iii) decohesion function, (iv) failure surface, (v) evolution equations, (vi) evolution functions, (vii) a complete set of constitutive equations, (viii) axial splitting, (ix) mode of failure, and (x) orientation of failure surface.

2. [8] Define what is meant by the gradient of a function with respect to a second-order tensor, $\mathbf{T}$? What is the gradient with respect to $\mathbf{T}$ of (i) $\mathbf{T}$, and (ii) $(\mathbf{T} \cdot\cdot \mathbf{T})^{1/2}$?

3. [7] What is the Voigt-Mandel representation of a second-order tensor and why is it used?

4. [10] What is meant by material instability? How do you determine if material instability is a possibility?

5. [15] Isotropy is used in several contexts of material models. With regard to linear elasticity, what is the basic definition of isotropic elasticity? What is the implication of the assumption of isotropy on the form of the elasticity tensor? What is meant by an invariant of a tensor? Which invariants of strain (or stress), if any, appear in the expression for isotropic-elastic strain energy?

6. [20] What is meant by plasticity and models of plasticity? What is meant by a yield function? What are the general features of the class of Mises models for plasticity? Which invariants of stress, if any, are used in the yield functions for this class of models? In general, do invariants of stress have to be used in a yield function, and why? What is meant by the phrase "isotropic hardening"?