22.7.1 Time domain viscoelasticity

Products: Abaqus/Standard Abaqus/Explicit Abaqus/CAE

References

- "Material library: overview," Section 21.1.1
- "Elastic behavior: overview," Section 22.1.1
- "Frequency domain viscoelasticity," Section 22.7.2
- *VISCOELASTIC
- *SHEAR TEST DATA
- *VOLUMETRIC TEST DATA
- *COMBINED TEST DATA
- *TRS
- "Defining time domain viscoelasticity" in "Defining elasticity," Section 12.9.1 of the Abaqus/CAE User's Guide

Overview

The time domain viscoelastic material model:

- describes isotropic rate-dependent material behavior for materials in which dissipative losses primarily caused by "viscous" (internal damping) effects must be modeled in the time domain;
- assumes that the shear (deviatoric) and volumetric behaviors are independent in multiaxial stress states (except when used for an elastomeric foam);
- can be used only in conjunction with "Linear elastic behavior," Section 22.2.1; "Hyperelastic behavior of rubberlike materials," Section 22.5.1; or "Hyperelastic behavior in elastomeric foams," Section 22.5.2, to define the continuum elastic material properties;
- can be used in Abaqus/Explicit with "Linear elastic traction-separation behavior" in "Defining the constitutive response of cohesive elements using a traction-separation description," Section 32.5.6;
- is active only during a transient static analysis ("Quasi-static analysis," Section 6.2.5), a transient implicit dynamic analysis ("Implicit dynamic analysis using direct integration," Section 6.3.2), an explicit dynamic analysis ("Explicit dynamic analysis," Section 6.3.3), a steady-state transport analysis ("Steady-state transport analysis," Section 6.4.1), a fully coupled temperature-displacement analysis ("Fully coupled thermal-stress analysis," Section 6.5.3), a fully coupled thermal-electrical-structural analysis ("Fully coupled thermal-electrical-structural analysis," Section 6.7.4), or a transient (consolidation) coupled pore fluid diffusion and stress analysis ("Coupled pore fluid diffusion and stress analysis," Section 6.8.1);
- can be used in large-strain problems;

- can be calibrated using time-dependent creep test data, time-dependent relaxation test data, or frequency-dependent cyclic test data; and
- can be used to couple viscous dissipation with the temperature field in a fully coupled temperature-displacement analysis ("Fully coupled thermal-stress analysis," Section 6.5.3) or a fully coupled thermal-electrical-structural analysis ("Fully coupled thermal-electrical-structural analysis," Section 6.7.4).

Defining the shear behavior

Time domain viscoelasticity is available in Abaqus for small-strain applications where the rate-independent elastic response can be defined with a linear elastic material model and for large-strain applications where the rate-independent elastic response must be defined with a hyperelastic or hyperfoam material model.

Small strain

Consider a shear test at small strain in which a time varying shear strain, $\gamma(t)$, is applied to the material. The response is the shear stress $\tau(t)$. The viscoelastic material model defines $\tau(t)$ as

$$\tau(t) = \int_0^t G_R(t-s)\dot{\gamma}(s)\,ds,$$

where $G_R(t)$ is the time-dependent "shear relaxation modulus" that characterizes the material's response. This constitutive behavior can be illustrated by considering a relaxation test in which a strain γ is suddenly applied to a specimen and then held constant for a long time. The beginning of the experiment, when the strain is suddenly applied, is taken as zero time, so that

$$\tau(t) = \int_0^t G_R(t-s)\dot{\gamma}(s)\,ds = G_R(t)\gamma \quad (\text{since } \dot{\gamma} = 0 \text{ for } t > 0),$$

where γ is the fixed strain. The viscoelastic material model is "long-term elastic" in the sense that, after having been subjected to a constant strain for a very long time, the response settles down to a constant stress; i.e., $G_R(t) \rightarrow G_\infty$ as $t \rightarrow \infty$.

The shear relaxation modulus can be written in dimensionless form:

$$g_R(t) = G_R(t)/G_0,$$

where $G_0 = G_R(0)$ is the instantaneous shear modulus, so that the expression for the stress takes the form

$$\tau(t) = G_0 \int_0^t g_R(t-s)\dot{\gamma}(s)\,ds.$$

The dimensionless relaxation function has the limiting values $g_R(0) = 1$ and $g_R(\infty) = G_\infty/G_0$.

Anisotropic elasticity in Abaqus/Explicit

The equation for the shear stress can be transformed by using integration by parts:

$$\tau(t) = G_0 \left(\gamma + \int_0^t \dot{g}_R(s)\gamma(t-s)\, ds \right).$$

It is convenient to write this equation in the form

$$\tau(t) = \tau_0(t) + \int_0^t \dot{g}_R(s)\tau_0(t-s)\, ds,$$

where $\tau_0(t)$ is the instantaneous shear stress at time *t*. This can be generalized to multi-dimensions as

$$\boldsymbol{\tau}(t) = \boldsymbol{\tau}_0(t) + \int_0^t \dot{g}_R(s)\boldsymbol{\tau}_0(t-s)\, ds,$$

where $\boldsymbol{\tau}(t)$ is the deviatoric part of the stress tensor and $\boldsymbol{\tau}_0(t)$ is the deviatoric part of the instantaneous stress tensor. Here the viscoelasticity is assumed to be isotropic; i.e., the relaxation function is independent of the loading direction.

This form allows a straightforward generalization to anisotropic elastic deformations, where the deviatoric part of instantaneous stress tensor is computed as $\boldsymbol{\tau}_0(t) = \overline{\mathbf{D}}_0 : \mathbf{e}$. Here $\overline{\mathbf{D}}_0$ is the instantaneous deviatoric elasticity tensor, and $\mathbf{e}$ is the deviatoric part of the strain tensor.

Large strain

The above form also allows a straightforward generalization to nonlinear elastic deformations, where the deviatoric part of the instantaneous stress $\boldsymbol{\tau}_0(t)$ is computed using a hyperelastic strain enery potential. This generalization yields a linear viscoelasticity model, in the sense that the dimensionless stress relaxation function is independent of the magnitude of the deformation.

In the above equation the instantaneous stress, $\boldsymbol{\tau}_0$, applied at time $t-s$ influences the stress, $\boldsymbol{\tau}$, at time *t*. Therefore, to create a proper finite-strain formulation, it is necessary to map the stress that existed in the configuration at time $t-s$ into the configuration at time *t*. In Abaqus this is done by means of the "standard-push-forward" transformation with the relative deformation gradient $\mathbf{F}_{t-s}(t)$:

$$\mathbf{F}_{t-s}(t) = \frac{\partial \mathbf{x}(t)}{\partial \mathbf{x}(t-s)},$$

which results in the following hereditary integral:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_0 + \text{dev}\left[\int_0^t \dot{g}_R(s) \bar{\mathbf{F}}_t^{-1}(t-s) \cdot \boldsymbol{\tau}_0(t-s) \cdot \bar{\mathbf{F}}_t^{-T}(t-s)\, ds \right],$$

where $\boldsymbol{\tau}$ is the deviatoric part of the Kirchhoff stress.

The finite-strain theory is described in more detail in "Finite-strain viscoelasticity," Section 4.8.2 of the Abaqus Theory Guide.

Defining the volumetric behavior

The volumetric behavior can be written in a form that is similar to the shear behavior:

$$p(t) = -\mathrm{K}_0 \int_0^t k_R(t-s)\, \dot{\varepsilon}^{vol}(s)\, ds,$$

where p is the hydrostatic pressure, K_0 is the instantaneous elastic bulk modulus, $k_R(t)$ is the dimensionless bulk relaxation modulus, and ε^{vol} is the volume strain.

The above expansion applies to small as well as finite strain since the volume strains are generally small and there is no need to map the pressure from time $t-s$ to time t.

Defining viscoelastic behavior for traction-separation elasticity in Abaqus/Explicit

Time domain viscoelasticity can be used in Abaqus/Explicit to model rate-dependent behavior of cohesive elements with traction-separation elasticity ("Defining elasticity in terms of tractions and separations for cohesive elements" in "Linear elastic behavior," Section 22.2.1). In this case the evolution equation for the normal and two shear nominal tractions take the form:

$$t_n(t) = t_n^0(t) + \int_0^t \dot{k}_R(s) t_n^0(t-s)\, ds,$$

$$t_s(t) = t_s^0(t) + \int_0^t \dot{g}_R(s) t_s^0(t-s)\, ds,$$

$$t_t(t) = t_t^0(t) + \int_0^t \dot{g}_R(s) t_t^0(t-s)\, ds,$$

where $t_n^0(t)$, $t_s^0(t)$, and $t_t^0(t)$ are the instantaneous nominal tractions at time t in the normal and the two local shear directions, respectively. The functions $g_R(t)$ and $k_R(t)$ now represent the dimensionless shear and normal relaxation moduli, respectively. Note the close similarity between the viscoelastic formulation for the continuum elastic response discussed in the previous sections and the formulation for cohesive behavior with traction-separation elasticity after reinterpreting shear and bulk relaxation as shear and normal relaxation.

For the case of uncoupled traction elasticity, the viscoelastic normal and shear behaviors are assumed to be independent. The normal relaxation modulus is defined as

$$k_R(t) = K_{nn}(t)/K_{nn}^0,$$

where K_{nn}^0 is the instantaneous normal moduli. The shear relaxation modulus is assumed to be isotropic and, therefore, independent of the local shear directions:

$$g_R(t) = K_{ss}(t)/K_{ss}^0 = K_{tt}(t)/K_{tt}^0,$$

where K_{ss}^0 and K_{tt}^0 are the instantaneous shear moduli.

For the case of coupled traction-separation elasticity the normal and shear relaxation moduli must be the same, $g_R(t) = k_R(t)$, and you must use the same relaxation data for both behaviors.

Temperature effects

The effect of temperature, θ, on the material behavior is introduced through the dependence of the instantaneous stress, τ_0, on temperature and through a reduced time concept. The expression for the linear-elastic shear stress is rewritten as

$$\tau(t) = G_0(\theta) \int_0^t g_R\big(\xi(t) - \xi(s)\big)\dot{\gamma}(s)\, ds,$$

where the instantaneous shear modulus G_0 is temperature dependent and $\xi(t)$ is the reduced time, defined by

$$\xi(t) = \int_0^t \frac{ds}{A\big(\theta(s)\big)},$$

where $A\big(\theta(t)\big)$ is a shift function at time *t*. This reduced time concept for temperature dependence is usually referred to as thermo-rheologically simple (TRS) temperature dependence. Often the shift function is approximated by the Williams-Landel-Ferry (WLF) form. See "Thermo-rheologically simple temperature effects" below, for a description of the WLF and other forms of the shift function available in Abaqus.

The reduced time concept is also used for the volumetric behavior, the large-strain formulation, and the traction-separation formulation.

Numerical implementation

Abaqus assumes that the viscoelastic material is defined by a Prony series expansion of the dimensionless relaxation modulus:

$$g_R(t) = 1 - \sum_{i=1}^{N} \bar{g}_i^P (1 - e^{-t/\tau_i^G}),$$

where *N*, $\bar{g}_i^P$, and τ_i^G, $i = 1, 2, \ldots, N$, are material constants. For linear isotropic elasticity, substitution in the small-strain expression for the shear stress yields

$$\tau(t) = G_0 \left(\gamma - \sum_{i=1}^{N} \gamma_i \right),$$

where

$$\gamma_i = \frac{\bar{g}_i^P}{\tau_i^G} \int_0^t e^{-s/\tau_i^G} \gamma(t - s)\, ds.$$

The γ_i are interpreted as state variables that control the stress relaxation, and

$$\gamma^{cr} = \sum_{i=1}^{N} \gamma_i$$

is the “creep” strain: the difference between the total mechanical strain and the instantaneous elastic strain (the stress divided by the instantaneous elastic modulus). In Abaqus/Standard γ^{cr} is available as the creep strain output variable CE (“Abaqus/Standard output variable identifiers,” Section 4.2.1).

A similar Prony series expansion is used for the volumetric response, which is valid for both small- and finite-strain applications:

$$p = -K_0 \left(\varepsilon^{vol} - \sum_{i=1}^{N} \varepsilon_i^{vol} \right),$$

where

$$\varepsilon_i^{vol} = \frac{\bar{k}_i^P}{\tau_i^K} \int_0^t e^{-s/\tau_i^K} \varepsilon^{vol}(t-s)\, ds.$$

Abaqus assumes that $\tau_i^G = \tau_i^K = \tau_i$.

For linear anisotropic elasticity, the Prony series expansion, in combination with the generalized small-strain expression for the deviatoric stress, yields

$$\boldsymbol{\tau} = \boldsymbol{\tau}_0 - \sum_{i=1}^{N} \boldsymbol{\tau}_i,$$

where

$$\boldsymbol{\tau}_i = \frac{\bar{g}_i^P}{\tau_i^G} \int_0^t e^{-s/\tau_i^G} \boldsymbol{\tau}_0(t-s)\, ds.$$

The $\boldsymbol{\tau}_i$ are interpreted as state variables that control the stress relaxation.

For finite strains, the Prony series expansion, in combination with the finite-strain expression for the shear stress, produces the following expression for the deviatoric stress:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_0 - \sum_{i=1}^{N} \mathrm{dev}\left(\boldsymbol{\tau}_i\right),$$

where

$$\boldsymbol{\tau}_i = \frac{\bar{g}_i^P}{\tau_i^G} \int_0^t e^{-s/\tau_i^G} \bar{\mathbf{F}}_t^{-1}(t-s) \cdot \boldsymbol{\tau}_0(t-s) \cdot \bar{\mathbf{F}}_t(t-s)\, ds.$$

The $\boldsymbol{\tau}_i$ are interpreted as state variables that control the stress relaxation.

For traction-separation elasticity, the Prony series expansion yields

$$\mathbf{t} = \begin{Bmatrix} t_n \\ t_s \\ t_t \end{Bmatrix} = \begin{Bmatrix} t_n^0 \\ t_s^0 \\ t_t^0 \end{Bmatrix} - \sum_{i=1}^{N} \begin{Bmatrix} t_n^i \\ t_s^i \\ t_t^i \end{Bmatrix} = \mathbf{t}^0 - \sum_{i=1}^{N} \mathbf{t}^i,$$

where

$$t_n^i = \frac{\bar{k}_i^P}{\tau_i^K} \int_0^t e^{-s/\tau_i^K} t_n^0(t-s)\, ds,$$

$$t_s^i = \frac{\bar{g}_i^P}{\tau_i^G} \int_0^t e^{-s/\tau_i^G} t_s^0(t-s)\, ds,$$

$$t_t^i = \frac{\bar{g}_i^P}{\tau_i^G} \int_0^t e^{-s/\tau_i^G} t_t^0(t-s)\, ds.$$

The $\mathbf{t}^i$ are interpreted as state variables that control the relaxation of the traction stresses.

If the instantaneous material behavior is defined by linear elasticity or hyperelasticity, the bulk and shear behavior can be defined independently. However, if the instantaneous behavior is defined by the hyperfoam model, the deviatoric and volumetric constitutive behavior are coupled and it is mandatory to use the same relaxation data for both behaviors. For linear anisotropic elasticity, the same relaxation data should be used for both behaviors when the elasticity definition is such that the deviatoric and volumetric response is coupled. Similarly, for coupled traction-separation elasticity you must use the same relaxation data for the normal and shear behaviors.

In all of the above expressions temperature dependence is readily introduced by replacing e^{-s/τ_i^G} by $e^{-\xi(s)/\tau_i^G}$ and e^{-s/τ_i^K} by $e^{-\xi(s)/\tau_i^K}$.

Determination of viscoelastic material parameters

The above equations are used to model the time-dependent shear and volumetric behavior of a viscoelastic material. The relaxation parameters can be defined in one of four ways: direct specification of the Prony series parameters, inclusion of creep test data, inclusion of relaxation test data, or inclusion of frequency-dependent data obtained from sinusoidal oscillation experiments. Temperature effects are included in the same manner regardless of the method used to define the viscoelastic material.

Abaqus/CAE allows you to evaluate the behavior of viscoelastic materials by automatically creating response curves based on experimental test data or coefficients. A viscoelastic material can be evaluated only if it is defined in the time domain and includes hyperelastic and/or elastic material data. See "Evaluating hyperelastic and viscoelastic material behavior," Section 12.4.7 of the Abaqus/CAE User's Guide.

Direct specification

The Prony series parameters $\bar{g}_i^P$, $\bar{k}_i^P$, and τ_i can be defined directly for each term in the Prony series. There is no restriction on the number of terms that can be used. If a relaxation time is associated with only one of the two moduli, leave the other one blank or enter a zero. The data should be given in ascending order of the relaxation time. The number of lines of data given defines the number of terms in the Prony series, *N*. If this model is used in conjunction with the hyperfoam material model, the two modulus ratios have to be the same (

$\bar{g}_i^P = \bar{k}_i^P$).

Input File Usage: *VISCOELASTIC, TIME=PRONY

The data line is repeated as often as needed to define the first, second, third, etc. terms in the Prony series.

Abaqus/CAE Usage: Property module: material editor: **Mechanical⟶Elasticity⟶Viscoelastic**: **Domain: Time** and **Time: Prony**

Enter as many rows of data in the table as needed to define the first, second, third, etc. terms in the Prony series.

Creep test data

If creep test data are specified, Abaqus will calculate the terms in the Prony series automatically. The normalized shear and bulk compliances are defined as

$$j_S(t) = G_0\, J_S(t) \qquad \text{and} \qquad j_K(t) = K_0\, J_K(t),$$

where $J_S(t) = \gamma(t)/\tau_0$ is the shear compliance, $\gamma(t)$ is the total shear strain, and τ_0 is the constant shear stress in a shear creep test; $J_K(t) = \varepsilon^{vol}(t)/p_0$ is the volumetric compliance, $\varepsilon^{vol}(t)$ is the total volumetric strain, and p_0 is the constant pressure in a volumetric creep test. At time $t = 0$, $j_S(0) = j_K(0) = 1$.

The creep data are converted to relaxation data through the convolution integrals

$$\int_0^t g_R(s) j_S(t-s) ds = t \qquad \text{and} \qquad \int_0^t k_R(s) j_K(t-s) ds = t.$$

Abaqus then uses the normalized shear modulus $g_R(t)$ and normalized bulk modulus $k_R(t)$ in a nonlinear least-squares fit to determine the Prony series parameters.

Using the shear and volumetric test data consecutively

The shear test data and volumetric test data can be used consecutively to define the normalized shear and bulk compliances as functions of time. A separate least-squares fit is performed on each data set; and the two derived sets of Prony series parameters, $(\bar{g}_i^P, \tau_i^G)$ and $(\bar{k}_i^P, \tau_i^K)$, are merged into one set of parameters, $(\bar{g}_i^P, \bar{k}_i^P, \tau_i)$.

Input File Usage: Use the following three options. The first option is required. Only one of the second and third options is required.
*VISCOELASTIC, TIME=CREEP TEST DATA
*SHEAR TEST DATA
*VOLUMETRIC TEST DATA

Abaqus/CAE Usage: Property module: material editor: **Mechanical⟶Elasticity⟶Viscoelastic**: **Domain: Time** and **Time: Creep test data**

In addition, select one or both of the following:

Test Data⟶Shear Test Data
Test Data⟶Volumetric Test Data

Using the combined test data

Alternatively, the combined test data can be used to specify the normalized shear and bulk compliances simultaneously as functions of time. A single least-squares fit is performed on the combined set of test data to determine one set of Prony series parameters, $(\bar{g}_i^P, \bar{k}_i^P, \tau_i)$.

Input File Usage: Use both of the following options:
*VISCOELASTIC, TIME=CREEP TEST DATA
*COMBINED TEST DATA

Abaqus/CAE Usage: Property module: material editor: **Mechanical⟶Elasticity⟶Viscoelastic**: **Domain: Time**, **Time: Creep test data**, and **Test Data⟶Combined Test Data**

Relaxation test data

As with creep test data, Abaqus will calculate the Prony series parameters automatically from relaxation test data.

Using the shear and volumetric test data consecutively

Again, the shear test data and volumetric test data can be used consecutively to define the relaxation moduli as functions of time. A separate nonlinear least-squares fit is performed on each data set; and the two derived sets of Prony series parameters, $(\bar{g}_i^P, \tau_i^G)$ and $(\bar{k}_i^P, \tau_i^K)$, are merged into one set of parameters, $(\bar{g}_i^P, \bar{k}_i^P, \tau_i)$.

Input File Usage: Use the following three options. The first option is required. Only one of the second and third options is required.
*VISCOELASTIC, TIME=RELAXATION TEST DATA
*SHEAR TEST DATA
*VOLUMETRIC TEST DATA

Abaqus/CAE Usage: Property module: material editor: **Mechanical⟶Elasticity⟶Viscoelastic**: **Domain: Time** and **Time: Relaxation test data**

In addition, select one or both of the following:

Test Data⟶Shear Test Data
Test Data⟶Volumetric Test Data

Using the combined test data

Alternatively, the combined test data can be used to specify the relaxation moduli simultaneously as functions of time. A single least-squares fit is performed on the combined set of test data to determine one set of Prony series parameters, $(\bar{g}_i^P, \bar{k}_i^P, \tau_i)$.

Input File Usage: Use both of the following options:
*VISCOELASTIC, TIME=RELAXATION TEST DATA
*COMBINED TEST DATA

Abaqus/CAE Usage: Property module: material editor: **Mechanical⟶Elasticity⟶Viscoelastic**: **Domain: Time**, **Time: Relaxation test data**, and **Test Data⟶Combined Test Data**

Frequency-dependent test data

The Prony series terms can also be calibrated using frequency-dependent test data. In this case Abaqus uses analytical expressions that relate the Prony series relaxation functions to the storage and loss moduli. The expressions for the shear moduli, obtained by converting the Prony series terms from the time domain to the frequency domain by making use of Fourier transforms, can be written as follows:

$$G_s(\omega) = G_0[1 - \sum_{i=1}^{N} \bar{g}_i^P] + G_0 \sum_{i=1}^{N} \frac{\bar{g}_i^P \tau_i^2 \omega^2}{1 + \tau_i^2 \omega^2},$$

$$G_\ell(\omega) = G_0 \sum_{i=1}^{N} \frac{\bar{g}_i^P \tau_i \omega}{1 + \tau_i^2 \omega^2},$$

where $G_s(\omega)$ is the storage modulus, $G_\ell(\omega)$ is the loss modulus, ω is the angular frequency, and *N* is the number of terms in the Prony series. These expressions are used in a nonlinear least-squares fit to determine the Prony series parameters from the storage and loss moduli cyclic test data obtained at *M* frequencies by minimizing the error function χ^2:

$$\chi^2 = \sum_{i=1}^{M} \frac{1}{G_\infty^2} [(G_s - \bar{G}_s)_i^2 + (G_\ell - \bar{G}_\ell)_i^2],$$

where $\bar{G}_s$ and $\bar{G}_\ell$ are the test data and G_0 and G_∞, respectively, are the instantaneous and long-term shear moduli. The expressions for the bulk moduli, $\mathrm{K}_s(\omega)$ and $\mathrm{K}_\ell(\omega)$, are written analogously.

The frequency domain data are defined in tabular form by giving the real and imaginary parts of ωg^* and ωk^* —where ω is the circular frequency—as functions of frequency in cycles per time. $g^*(\omega)$ is the Fourier transform of the nondimensional shear relaxation function $g(t) = \frac{G_R(t)}{G_\infty} - 1$. Given the frequency-dependent storage and loss moduli $G_s(\omega)$, $G_\ell(\omega)$, $\mathrm{K}_s(\omega)$, and $\mathrm{K}_\ell(\omega)$, the real and imaginary parts of ωg^* and ωk^* are then given as

$$\omega \Re(g^*) = G_\ell / G_\infty, \;\; \omega \Im(g*) = 1 - G_s / G_\infty, \quad \omega \Re(k^*) = \mathrm{K}_\ell / \mathrm{K}_\infty, \;\; \omega \Im(k^*) = 1 - \mathrm{K}_s / \mathrm{K}_\infty,$$

where G_∞ and K_∞ are the long-term shear and bulk moduli determined from the elastic or hyperelastic properties.

Input File Usage: *VISCOELASTIC, TIME=FREQUENCY DATA

Abaqus/CAE Usage: Property module: material editor: **Mechanical⟶Elasticity⟶Viscoelastic**: **Domain: Time** and **Time: Frequency data**

Calibrating the Prony series parameters

You can specify two optional parameters related to the calibration of Prony series parameters for viscoelastic materials: the error tolerance and N_{max}. The error tolerance is the allowable average root-mean-square error of data points in the least-squares fit, and its default value is 0.01. N_{max} is the maximum number of terms *N* in the Prony series, and its default (and maximum) value is 13. Abaqus will perform the least-squares fit from $N = 1$ to $N = N_{max}$ until convergence is achieved for the lowest *N* with respect to the error tolerance.

The following are some guidelines for determining the number of terms in the Prony series from test data. Based on these guidelines, you can choose N_{max}.

- There should be enough data pairs for determining all the parameters in the Prony series terms. Thus, assuming that *N* is the number of Prony series terms, there should be a total of at least $2N$ data points in shear test data, $2N$ data points in volumetric test data, $3N$ data points in combined test data, and $2N$ data points in the frequency domain.
- The number of terms in the Prony series should be typically not more than the number of logarithmic "decades" spanned by the test data. The number of logarithmic "decades" is defined as $\log_{10}(t_{max}/t_{min})$, where t_{max} and t_{min} are the maximum and minimum time in the test data, respectively.

Input File Usage: *VISCOELASTIC, ERRTOL=*error_tolerance*, NMAX=N_{max}

Abaqus/CAE Usage: Property module: material editor: **Mechanical**⟶**Elasticity**⟶**Viscoelastic**: **Domain: Time**; **Time: Creep test data**, **Relaxation test data**, or **Frequency data**; **Maximum number of terms in the Prony series:** N_{max}; and **Allowable average root-mean-square error:** *error_tolerance*

Thermo-rheologically simple temperature effects

Regardless of the method used to define the viscoelastic behavior, thermo-rheologically simple temperature effects can be included by specifying the method used to define the shift function. Abaqus supports the following forms of the shift function: the Williams-Landel-Ferry (WLF) form, the Arrhenius form, and user-defined forms.

Thermo-rheologically simple temperature effects can also be included in the definition of equation of state models with viscous shear behavior (see "Viscous shear behavior" in "Equation of state," Section 25.2.1).

Williams-Landel-Ferry (WLF) form

The shift function can be defined by the Williams-Landel-Ferry (WLF) approximation, which takes the form:

$$\log_{10}(A) = -\frac{C_1(\theta - \theta_0)}{C_2 + (\theta - \theta_0)},$$

where θ_0 is the reference temperature at which the relaxation data are given; θ is the temperature of interest; and C_1, C_2 are calibration constants obtained at this temperature. If $\theta \leq \theta_0 - C_2$, deformation changes will be elastic, based on the instantaneous moduli.

For additional information on the WLF equation, see "Viscoelasticity," Section 4.8.1 of the Abaqus Theory Guide.

Input File Usage: *TRS, DEFINITION=WLF

Abaqus/CAE Usage: Property module: material editor: **Mechanical**⟶**Elasticity**⟶**Viscoelastic**: **Domain:**

Time, **Time: *any method***, and **Suboptions**⟶**Trs**: **Shift function: WLF**

Arrhenius form

The Arrhenius shift function is commonly used for semi-crystalline polymers. It takes the form

$$\ln(A) = \frac{E_0}{R}\left(\frac{1}{\theta - \theta^Z} - \frac{1}{\theta_0 - \theta^Z}\right),$$

where E_0 is the activation energy, R is the universal gas constant, θ^Z is the absolute zero in the temperature scale being used, θ_0 is the reference temperature at which the relaxation data are given, and θ is the temperature of interest.

Input File Usage: Use the following option to define the Arrhenius shift function:
*TRS, DEFINITION=ARRHENIUS

In addition, use the *PHYSICAL CONSTANTS option to specify the universal gas constant and absolute zero.

Abaqus/CAE Usage: The Arrhenius shift function is not supported in Abaqus/CAE.

User-defined form

The shift function can be specified alternatively in user subroutines `UTRS` in Abaqus/Standard and `VUTRS` in Abaqus/Explicit.

Input File Usage: *TRS, DEFINITION=USER

Abaqus/CAE Usage: Property module: material editor: **Mechanical**⟶**Elasticity**⟶**Viscoelastic**: **Domain: Time**, **Time: *any method***, and **Suboptions**⟶**Trs**: **Shift function: User subroutine UTRS**

Defining the rate-independent part of the material response

In all cases elastic moduli must be specified to define the rate-independent part of the material behavior. Small-strain linear elastic behavior is defined by an elastic material model (“Linear elastic behavior,” Section 22.2.1), and large-deformation behavior is defined by a hyperelastic (“Hyperelastic behavior of rubberlike materials,” Section 22.5.1) or hyperfoam (“Hyperelastic behavior in elastomeric foams,” Section 22.5.2) material model. The rate-independent elasticity for any of these models can be defined in terms of either instantaneous elastic moduli or long-term elastic moduli. The choice of defining the elasticity in terms of instantaneous or long-term moduli is a matter of convenience only; it does not have an effect on the solution.

The effective relaxation moduli are obtained by multiplying the instantaneous elastic moduli with the dimensionless relaxation functions as described below.

Linear elastic isotropic materials

For linear elastic isotropic material behavior

$$G_R(t) = G_0\left(1 - \sum_{k=1}^{N} \bar{g}_k^P (1 - e^{-t/\tau_k})\right)$$

and

$$\mathrm{K}_R(t) = \mathrm{K}_0\left(1 - \sum_{k=1}^{N} \bar{k}_k^P (1 - e^{-t/\tau_k})\right),$$

where G_0 and K_0 are the instantaneous shear and bulk moduli determined from the values of the user-defined instantaneous elastic moduli E_0 and ν_0: $G_0 = E_0/2(1+\nu_0)$ and $\mathrm{K}_0 = E_0/3(1-2\nu_0)$.

If long-term elastic moduli are defined, the instantaneous moduli are determined from

$$G_\infty = G_0\left(1 - \sum_{k=1}^{N} \bar{g}_k^P\right) \qquad \text{and} \qquad K_\infty = K_0\left(1 - \sum_{k=1}^{N} \bar{k}_k^P\right).$$

Linear elastic anisotropic materials

For linear elastic anisotropic material behavior the relaxation coefficients are applied to the elastic moduli as

$$\overline{\mathbf{D}}_R(t) = \overline{\mathbf{D}}_0\left(1 - \sum_{k=1}^{N} \bar{g}_k^P (1 - e^{-t/\tau_k})\right)$$

and

$$\mathrm{K}_R(t) = \mathrm{K}_0\left(1 - \sum_{k=1}^{N} \bar{k}_k^P (1 - e^{-t/\tau_k})\right),$$

where $\overline{\mathbf{D}}_0$ and K_0 are the instantaneous deviatoric elasticity tensor and bulk moduli determined from the values of the user-defined instantaneous elastic moduli $\mathbf{D}_0$. If both shear and bulk relaxation coefficients are specified and they are unequal, Abaqus issues an error message if the elastic moduli $\mathbf{D}_0$ is such that the deviatoric and volumetric response is coupled.

If long-term elastic moduli are defined, the instantaneous moduli are determined from

$$\overline{\mathbf{D}}_\infty = \overline{\mathbf{D}}_0\left(1 - \sum_{k=1}^{N} \bar{g}_k^P\right) \qquad \text{and} \qquad K_\infty = K_0\left(1 - \sum_{k=1}^{N} \bar{k}_k^P\right).$$

Hyperelastic materials

For hyperelastic material behavior the relaxation coefficients are applied either to the constants that define the energy function or directly to the energy function. For the polynomial function and its particular cases (reduced polynomial, Mooney-Rivlin, neo-Hookean, and Yeoh)

$$C_{ij}^R(t) = C_{ij}^0 \left(1 - \sum_{k=1}^{N} \bar{g}_k^P (1 - e^{-t/\tau_k})\right),$$

for the Ogden function

$$\mu_i^R(t) = \mu_i^0 \left(1 - \sum_{k=1}^{N} \bar{g}_k^P (1 - e^{-t/\tau_k})\right),$$

for the Arruda-Boyce and Van der Waals functions

$$\mu^R(t) = \mu^0 \left(1 - \sum_{k=1}^{N} \bar{g}_k^P (1 - e^{-t/\tau_k})\right),$$

and for the Marlow function

$$U_{dev}^R(t) = U_{dev}^0 \left(1 - \sum_{k=1}^{N} \bar{g}_k^P (1 - e^{-t/\tau_k})\right).$$

For the coefficients governing the compressible behavior of the polynomial models and the Ogden model

$$D_i^R(t) = D_i^0 / \left(1 - \sum_{k=1}^{N} \bar{k}_k^P (1 - e^{-t/\tau_k})\right),$$

for the Arruda-Boyce and Van der Waals functions

$$D^R(t) = D^0 / \left(1 - \sum_{k=1}^{N} \bar{k}_k^P (1 - e^{-t/\tau_k})\right),$$

and for the Marlow function

$$U_{vol}^R(t) = U_{vol}^0 \left(1 - \sum_{k=1}^{N} \bar{k}_k^P (1 - e^{-t/\tau_k})\right).$$

If long-term elastic moduli are defined, the instantaneous moduli are determined from

$$C_{ij}^\infty = C_{ij}^0 \left(1 - \sum_{k=1}^{N} \bar{g}_k^P\right), \quad \text{or} \quad \mu_i^\infty = \mu_i^0 \left(1 - \sum_{k=1}^{N} \bar{g}_k^P\right), \quad \text{or} \quad \mu^\infty = \mu^0 \left(1 - \sum_{k=1}^{N} \bar{g}_k^P\right),$$

while the instantaneous bulk compliance moduli are obtained from

$$D_i^\infty = D_i^0 / \left(1 - \sum_{k=1}^{N} \bar{k}_k^P\right), \quad \text{or} \quad D^\infty = D^0 / \left(1 - \sum_{k=1}^{N} \bar{k}_k^P\right);$$

for the Marlow functions we have

$$U_{dev}^{\infty} = U_{dev}^{0}\left(1 - \sum_{k=1}^{N} \bar{g}_k^P\right) \quad \text{and} \quad U_{vol}^{\infty} = U_{vol}^{0}\left(1 - \sum_{k=1}^{N} \bar{k}_k^P\right).$$

Mullins effect

If long-term moduli are defined for the underlying hyperelastic behavior, the instantaneous value of the parameter m in Mullins effect is determined from

$$m^{\infty} = m^{0}\left(1 - \sum_{k=1}^{N} \bar{g}_k^P\right).$$

Elastomeric foams

For elastomeric foam material behavior the instantaneous shear and bulk relaxation coefficients are assumed to be equal and are applied to the material constants μ_i in the energy function:

$$\mu_i^R(t) = \mu_i^0\left(1 - \sum_{k=1}^{N} \bar{g}_k^P (1 - e^{-t/\tau_k})\right) = \mu_i^0\left(1 - \sum_{k=1}^{N} \bar{k}_k^P (1 - e^{-t/\tau_k})\right).$$

If only the shear relaxation coefficients are specified, the bulk relaxation coefficients are set equal to the shear relaxation coefficients and vice versa. If both shear and bulk relaxation coefficients are specified and they are unequal, Abaqus issues an error message.

If long-term elastic moduli are defined, the instantaneous moduli are determined from

$$\mu_i^{\infty} = \mu_i^0\left(1 - \sum_{k=1}^{N} \bar{g}_k^P\right) = \mu_i^0\left(1 - \sum_{k=1}^{N} \bar{k}_k^P\right).$$

Traction-separation elasticity

For cohesive elements with uncoupled traction-separation elastic behavior:

$$K_{nn}(t) = K_{nn}^0\left(1 - \sum_{k=1}^{N} \bar{k}_k^P (1 - e^{-t/\tau_k})\right),$$

$$K_{ss}(t) = K_{ss}^0\left(1 - \sum_{k=1}^{N} \bar{g}_k^P (1 - e^{-t/\tau_k})\right),$$

and

$$K_{tt}(t) = K_{tt}^0\left(1 - \sum_{k=1}^{N} \bar{g}_k^P (1 - e^{-t/\tau_k})\right),$$

where K_{nn}^0 is the instantaneous normal modulus and K_{ss}^0 and K_{tt}^0 are the instantaneous shear moduli. If long-term elastic moduli are defined, the instantaneous moduli are determined from

$$K_{nn}^{\infty}/K_{nn}^{0} = \left(1 - \sum_{k=1}^{N} \bar{k}_k^P\right), \qquad \text{and} \qquad K_{ss}^{\infty}/K_{ss}^{0} = K_{tt}^{\infty}/K_{tt}^{0} = \left(1 - \sum_{k=1}^{N} \bar{g}_k^P\right).$$

For cohesive elements with coupled traction-separation elastic behavior the shear and bulk relaxation coefficients must be equal:

$$\mathbf{K}(t) = \mathbf{K}^0 \left(1 - \sum_{k=1}^{N} \bar{k}_k^P (1 - e^{-t/\tau_k})\right) = \mathbf{K}^0 \left(1 - \sum_{k=1}^{N} \bar{g}_k^P (1 - e^{-t/\tau_k})\right),$$

where $\mathbf{K}^0$ is the user-defined instantaneous elasticity matrix. If long-term elastic moduli are defined, the instantaneous moduli are determined from

$$\mathbf{K}^{\infty} = \mathbf{K}^0 \left(1 - \sum_{k=1}^{N} \bar{k}_k^P\right) = \mathbf{K}^0 \left(1 - \sum_{k=1}^{N} \bar{g}_k^P\right).$$

Material response in different analysis procedures

The time-domain viscoelastic material model is active during the following procedures:

- transient static analysis ("Quasi-static analysis," Section 6.2.5),
- transient implicit dynamic analysis ("Implicit dynamic analysis using direct integration," Section 6.3.2),
- explicit dynamic analysis ("Explicit dynamic analysis," Section 6.3.3),
- steady-state transport analysis ("Steady-state transport analysis," Section 6.4.1),
- fully coupled temperature-displacement analysis ("Fully coupled thermal-stress analysis," Section 6.5.3),
- fully coupled thermal-electrical-structural analysis ("Fully coupled thermal-electrical-structural analysis," Section 6.7.4), and
- transient (consolidation) coupled pore fluid diffusion and stress analysis ("Coupled pore fluid diffusion and stress analysis," Section 6.8.1).

Viscoelastic material response is always ignored in a static analysis. It can also be ignored in a coupled temperature-displacement analysis, a coupled thermal-electrical-structural analysis, or a soils consolidation analysis by specifying that no creep or viscoelastic response is occurring during the step even if creep or viscoelastic material properties are defined (see "Fully coupled thermal-stress analysis," Section 6.5.3, or "Coupled pore fluid diffusion and stress analysis," Section 6.8.1). In these cases it is assumed that the loading is applied instantaneously, so that the resulting response corresponds to an elastic solution based on instantaneous elastic moduli.

Abaqus/Standard also provides the option to obtain the fully relaxed long-term elastic solution directly in a static or steady-state transport analysis without having to perform a transient analysis. The long-term value is used for this purpose. The viscous damping stresses (the internal stresses associated with each of the Prony-series terms) are increased gradually from their values at the beginning of the step to their long-term values at the end of the step if the long-term value is specified.

Material options

The viscoelastic material model must be combined with an elastic material model. It is used with the isotropic linear elasticity model ("Linear elastic behavior," Section 22.2.1) to define classical, linear, small-strain, viscoelastic behavior or with the hyperelastic ("Hyperelastic behavior of rubberlike materials," Section 22.5.1) or hyperfoam ("Hyperelastic behavior in elastomeric foams," Section 22.5.2) models to define large-deformation, nonlinear, viscoelastic behavior. It can also be used with anisotropic linear elasticity and with traction-separation elastic behavior in Abaqus/Explicit. The elastic properties defined for these models can be temperature dependent.

Viscoelasticity cannot be combined with any of the plasticity models. See "Combining material behaviors," Section 21.1.3, for more details.

Elements

The time domain viscoelastic material model can be used with any stress/displacement, coupled temperature-displacement, or thermal-electrical-structural element in Abaqus.

Output

In addition to the standard output identifiers available in Abaqus ("Abaqus/Standard output variable identifiers," Section 4.2.1, and "Abaqus/Explicit output variable identifiers," Section 4.2.2), the following variables have special meaning in Abaqus/Standard if viscoelasticity is defined:

EE	Elastic strain corresponding to the stress state at time *t* and the instantaneous elastic material properties.
CE	Equivalent creep strain defined as the difference between the total strain and the elastic strain.

Considerations for steady-state transport analysis

When a steady-state transport analysis ("Steady-state transport analysis," Section 6.4.1) is combined with large-strain viscoelasticity, the viscous dissipation, CENER, is computed as the energy dissipated per revolution as a material point is transported around its streamline; that is,

$$W_{cr} = \oint \boldsymbol{\sigma} : d\boldsymbol{\varepsilon}.$$

Consequently, all the material points in a given streamline report the same value for CENER, and other derived quantities such as ELCD and ALLCD also have the meaning of dissipation per revolution. The recoverable elastic strain energy density, SENER, is approximated as

$$W_{el} = W_{el}^t + W_{cr}^t + \Delta W - W_{cr},$$

where ΔW is the incremental energy input and t is the time at the beginning of the current increment. Since two different units are used in the quantities appearing in the above equation, a proper meaning cannot be assigned to quantities such as SENER, ELSE, ALLSE, and ALLIE.

Considerations for large-strain viscoelasticity

In Abaqus/Standard the viscous energy dissipated is computed only approximately for large-strain viscoelasticity.

Abaqus/Explicit does not compute the viscous dissipation for performance reasons for the case of large-strain viscoelasticity. Instead, the contribution of viscous dissipation is included in the strain energy output, SENER; and CENER is output as zero. Consequently, special care must be exercised when interpreting strain energy results of large-strain viscoelastic materials in Abaqus/Explicit since they include viscous dissipation effects.