11.1 Martensitic transformation involves the Bain transformation, shown schematically in Figure Ex11.1. The FCC structure is transformed into the BCC structure. Assuming that there is a 5% expansion in volume during the FCC to BCC transformation, (a) calculate the lattice parameter of the BCC structure in terms of a_0, and (b) determine the strains in the three orthogonal directions.

a) FCC volume = $a_0^{\,3}$

BCC volume = a^3

$$(1.05)a_0^{\,3} = a^3$$

$$\boxed{1.01a_0 = a}$$

b)

$$E = \frac{a - a_0}{a_0}$$

$$E_{11} = \frac{a - a_0}{a} = \frac{1.01a_0 - a_0}{a_0} = .01$$

$$E_{22} = \frac{a - \dfrac{a_0}{\sqrt{2}}}{\dfrac{a_0}{\sqrt{2}}} = \frac{\left(1.01 - \dfrac{1}{\sqrt{2}}\right)a_0}{\dfrac{1}{\sqrt{2}} - a_0} = .42$$

$$E_{33} = \frac{a - \dfrac{a_0}{\sqrt{2}}}{\dfrac{a_0}{\sqrt{2}}} = \frac{\left(1.01 - \dfrac{1}{\sqrt{2}}\right)a_0}{\dfrac{1}{\sqrt{2}} - a_0} = .42$$

11.2 Plot hydrostatic strain versus carbon content for the martensitic transformation in steel from the plot shown in Ex11.2.

$$V_{0F} = a_0 \frac{a_0}{\sqrt{2}} \frac{a_0}{\sqrt{2}}$$

$$V_{0B} = a^2 c$$

$$E_H = \frac{V_{0B} - V_{0F}}{V_{0F}}$$

From Fig. Ex 11.2

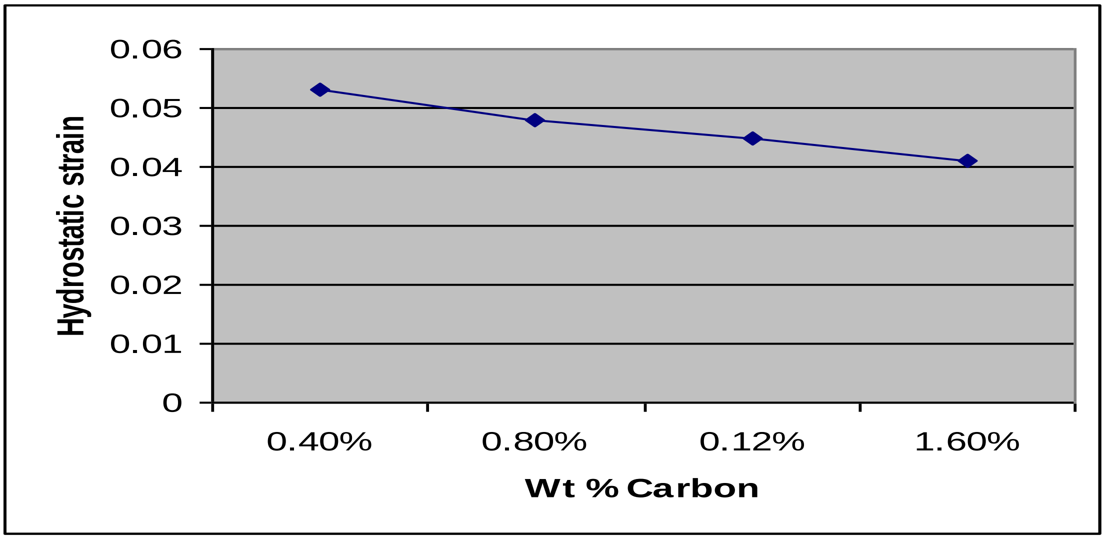
	.4%	.8%	1.2%	1.6%
~ a	.286	.286	.285	.285
~ c	.290	.294	.300	.304
~ a_0	.356	.358	.360	.362

$$V_F = \frac{a_0{}^3}{2} \qquad V_B = a^2 c$$

% carbon	V_F	V_B	E_H
.4%	.0225	.0237	.053
.8%	.0229	.0240	.048
.12%	.0233	.0243	.045
1.6%	.0237	.0248	.041

$$E_H = \frac{V_B - V_F}{V_F}$$

Plot of hydrostatic strain versus carbon content



11.3 From the data of Figure 11.12, estimate the M_s temperature of the alloy at zero stress

Temperature in K	Load in N
233K	~ 1090N
243K	~ 1420N
253K	~ 1730N

243 K – 233 K = $\Delta T = 10$ 1420N – 1090 N = $\Delta N = 330$ N

253 K – 243 K = $\Delta T = 10$ 1730N – 1420 N = $\Delta N = 310$ N

$\Delta \overline{N} = \boxed{320 \text{ N}}$

$\Delta \overline{T} = 10$

$$\text{Slope} = \frac{\Delta \overline{N}}{\Delta \overline{T}} = \frac{320}{10} = 32$$

Change in load per kelvin: $\dfrac{32N}{1K}$

$$\frac{1090N}{32\frac{N}{K}} \approx 34.06$$ K change to drop the load to zero stress.

Initial minus the calculated change is needed

$233 - 34.06 \approx$ $\boxed{\text{198.93K is estimated Ms for zero stress}}$.

11.4 The steel shown in Figure 11.15(b) has a plane strain fracture toughness of 110 MPa $m^{1/2}$ and a yield stress of 320 MPa. Will the cracks shown in the figure have a catastrophic effect if a specimen is stressed to 180 MPa?

$$K_{ic} = 110 MPa \cdot m^{\frac{1}{2}}$$

$$K_{ic} = Y\sigma\sqrt{\pi a}$$

Assume: Y = 1.12 (single edge notch)

Crack Length $\approx$ $2a = 10\mu m$ from figure 11.15 b

$$\sigma = \frac{K_{ic}}{Y\sqrt{\pi a}} = \frac{110 \times 10^6}{1.12\sqrt{\pi\left(5 \times 10^{-6}\right)}}$$

$$\sigma = 2.5 \times 10^{10} > \sigma_y$$

Therefore the cracks will have catastrophic effects.

11.5 Write down all the possible martensite variants for the Kurdjumov—Sachs orientation.

The 12 variants are:

$(2\ 2\ 5)$	$(\bar{2}\ 2\ 5)$
$(2\ 5\ 2)$	$(\bar{2}\ 5\ 2)$
$(5\ 2\ 2)$	$(\bar{5}\ 2\ 2)$
$(2\ \bar{2}\ 5)$	$(2\ 2\ 5)$
$(2\ \bar{5}\ 2)$	$(2\ 5\ \bar{2})$
$(5\ \bar{2}\ 2)$	$(5\ 2\ \bar{2})$

11.8 Calculate the total strain energy associated with a martensite lens having a volume of 10 μm^3, assuming that all the energy is elastically stored. Specify the assumptions made; include both shear and longitudinal strain components from Equation 11.2.

$$V = 10 \times 10^{-6} m^3$$

We assume a linearly elastic solid under uniaxial stress

$$\begin{aligned} U &= \frac{1}{2} E \varepsilon_{ij}{}^2 \\ &= \frac{1}{2} E\left(\varepsilon_{23}{}^2 + \varepsilon_{23}{}^2 + \varepsilon_{32}{}^2\right) \\ &= \frac{1}{2}\left(210 \times 10^9\right)\left(.10^2 + .10^2 + .05^2\right) \end{aligned}$$

$$\text{U} = 10.52 \times 10^6$$

U = energy per unit volume

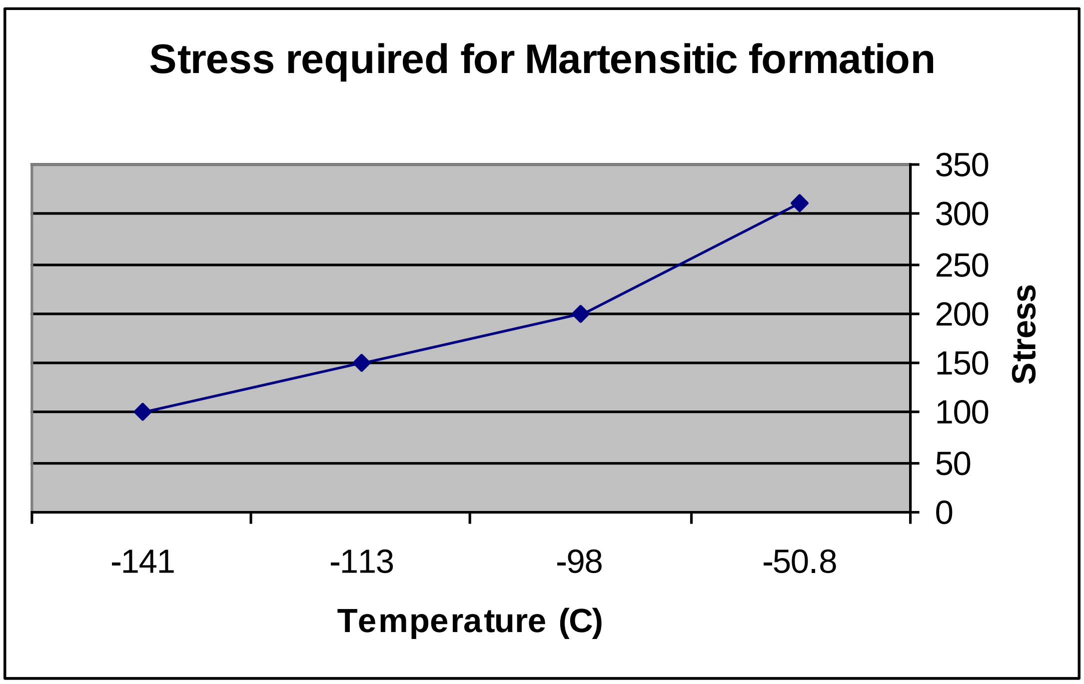
Total energy = $U \cdot V$

Total energy = $\left(10.52 \times 10^{6}\right)\left(10 \times 10^{-6}\right)$

$\boxed{= 10.52\ \text{J}}$

11.9 Plot the stress required to form martensite as a function of temperature in Figure 11.16b.

Temperature in °C	Martensite formation σ MPa
-50.5°	~ 310
-98°	~ 200
-113°	~ 150
-141°	~ 100



- Stress at which martensite forms increases when temperature increases

11.10 (a) To what radius can a wire with diameter of 1 mm be curved using the superelastic effect if the maximum strain is approximately 0.05? (b) If the wire were made of a high-strength piano wire steel ($\sigma y \sim$ 2GPa), what would be the minimum radius to which it could be curved? Take E = 210 GPa. (c) Discuss the differences obtained in (a) and (b).

$$\sigma = \frac{Mc}{I} \qquad \frac{\sigma}{\varepsilon} = E$$

$$\varepsilon = \frac{\Delta L}{L}$$

$$\varepsilon = \frac{(R + L)d\theta - Rd\theta}{Rd\theta}$$

$$\varepsilon = \frac{L}{R} \Rightarrow R = \frac{L}{\varepsilon}$$

a) $R = \dfrac{L}{\varepsilon} = \dfrac{\frac{1}{2}(1 \times 10^{-3})}{.05}$

$\boxed{\text{R = .01m or 10mm}}$

b) $R = \dfrac{L}{\varepsilon} \qquad \varepsilon = \dfrac{\sigma}{E}$

$$R = \frac{EL}{\sigma} = \frac{(210 \times 10^{9})(1 \times 10^{-3}) \cdot \frac{1}{2}}{2 \times 10^{9}}$$

$\boxed{\text{R = .0525m or 52.5mm}}$

11.11 What is the volume change associated with the tetragonal-to-monoclinic transformation in zirconia?

Given:

Monoclinic zirconia	Tetragonal zirconia
a = 0.5156 nm	a = 0.5094 nm
b = 0.5191 nm	b = 0.5304 nm
c = 0.5304 nm	
β = 98.9◦	

Volume for Monoclinic

$$V_m = abc\sin\beta$$

$$V_m = (.5156)(.5191)(.5304)Sin98.9$$

$$V_m = .14025nm^3$$

Volume for Tetragonal

$$V_T = a^2 b$$

$$V_T = (.5094)^2(.5304)$$

$$V_T = .13763nm^3$$

$$\frac{V_m \times V_T}{V_m} \times 100 = \frac{.14025 - .13763}{.14025} \times 100 = 1.86\%$$

1.86% decrease in volume can be expected in transformation from monoclinic to tetragonal zirconia.