14.4 Radiation with Conduction and Convection

Physical situations that involve radiation with conduction and convection are fairly common. Examples include automobile radiators and heat transfer in the furnace of boilers and incinerators. The energy equations become more complex as they comprise both temperature differences coming from convection and temperature derivatives coming from conduction. Hence, there are no classical methods of solution, but numerical methods and specific methods for particular problems.

This section provides the solution of several examples where radiation is combined with the other modes of heat transfer.

Example 14.4

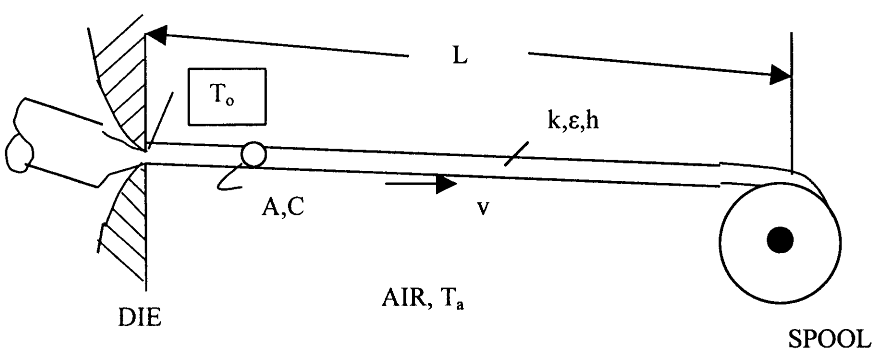

Figure 14.7 Extrusion of a thin wire.

Problem: A thin wire is extruded at a fixed velocity, v, through a die at a temperature of T_o. The wire then passes through air at T_a until its temperature is reduced to T_L. The heat transfer coefficient to the air is h, and the wire emissivity is ε. Find T as a function of wire velocity v and distance L. Derive the differential equation for the wire temperature as a function of the distance from the die.

Solution

Consider the heat balance for flow in and out of a control volume fixed in space, as shown below.

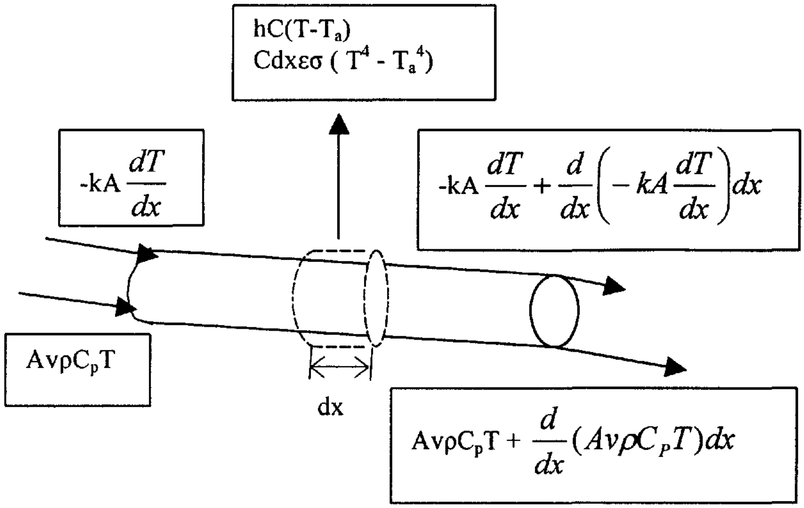

Figure 14.8 Sketch for Example 14.4.

For steady-state conditions, energy in = energy out. Hence,

$$-kA\frac{dT}{dx} + Av\rho C_pT = -kA\frac{dT}{dx} + \frac{d}{dx}\left(-kA\frac{dT}{dx}\right)dx + Av\rho C_pT$$

$$+\frac{d}{dx}(Av\rho C_pT)dx + hCdx(T-T_a) + Cdx\varepsilon\sigma\left(T^4 - T_a^4\right) \qquad \text{(i)}$$

For constant k, the equation reduces to

$$-kA\frac{d^2T}{dx^2} + Av\rho C_p\frac{dT}{dx} + hC(T-T_a) + C\varepsilon\sigma\left(T^4 - T_a^4\right) = 0$$

or

$$\frac{d^2T}{dx^2} - \frac{v\rho C_p}{k}\frac{dT}{dx} - \frac{hC}{kA}(T - T_a) - \frac{C\varepsilon\sigma}{kA}(T^4 - T_a^4) = 0.$$
(ii)

The boundary conditions are as follows:

At $x = 0$, $T = T_o$

At $x = L$, we assume that there is no heat lost after the wire is wound, that is, $\left.\frac{dT}{dx}\right|_L = 0$.

Example 14.5

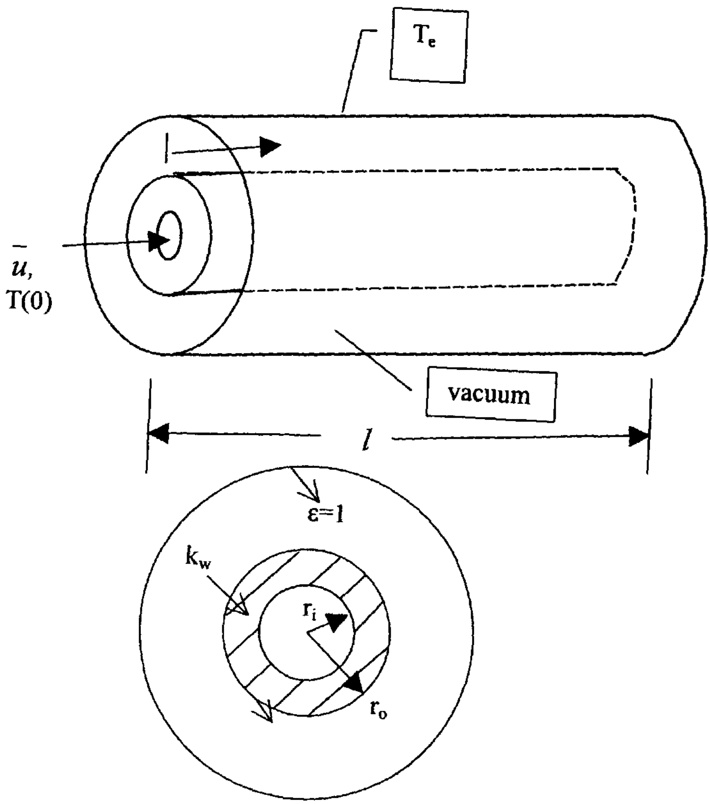

Figure 14.9 Sketch for Example 14.5.