Assignment 4 ME 562 Constitutive Equations Spring/2015

(25 points) Due April 16

The purpose of this part of the assignment is to add the capability to your driver program of handling the backstrain version of viscoelasticity for elementary paths to your driver program. Attempt to draw conclusions concerning the effects of increases in values of particular parameters as you provide your plots.

Recall that the back strain evolution equations are

$$e^{b} = \frac{1}{\tau}e - \frac{1}{\tau^{b}}e^{b} \qquad \text{Linear}$$

$$\left.\begin{aligned} \dot{e}^{b} = (1+\bar{\sigma}^{ve})\left[\frac{1}{\tau}e - \frac{1}{\tau^{b}}e^{b}\right] \qquad \sigma_{res} = E\frac{\tau^{b}}{\tau^{*}}e \\ \bar{\sigma}^{ve} = \frac{[(\sigma-\sigma_{res})\cdot\cdot(\sigma-\sigma_{res})]^{1/2}}{\sigma^{*}} \end{aligned}\right\} \qquad \text{Nonlinear}$$

1. First you have to add additional capability to your driver program. Think in terms of uniaxial strain first (or just a shear component if you prefer). Perform the following steps: (i) Add in a subroutine that provides values of strain and strain rate for a prescribed function. (ii) For a prescribed (by you) time step, add the subroutine for a system integrator that provides back strain and stress for the standard linear visco-elastic model.
(iii) Demonstrate your program works by comparing your numerical solution to the analytical solution for the relaxation problem.
(iv) Add in the “overide” segment on strain increment for a prescribed stress problem and demonstrate your program works by comparing your numerical solution to the analytical solution for the creep problem.

2. Show the effects of choosing various values for two of the three parameters, τ, τ^{b} and τ^{*} for the creep and relaxation curves.

3. Add in a second back strain and for fixed values of τ, τ^{b} and τ^{*}, and show the effects of changes in values of the two additional parameters on the creep and relaxation curves.

4. Add in the nonlinear term for the back strain. For the relaxation path, demonstrate the effects of different choices of values for σ_0 and σ^{*}. Plot your stress results as a function of σ/σ_0.

5. Now we demonstrate the feature of the model for high strain rate. Use a strain-prescribed function of time, t, as follows:

$$e = \frac{e_{max}}{2}[1 + \cos 2\pi(\frac{t}{T} - \frac{1}{2})] \qquad 0 \le t \le T$$

The duration, or period, is T. The function starts and ends with both a value and a slope that is zero and is assumed to be representative of the motion in a half-space that is loaded impulsively. The maximum value of the strain over the cycle is e_{max}. The maximum and minimum strain rates are $\dot{e}_{max} = \pm(\pi / T)e_{max}$ and these two values occur when t = T/4 and t = 3T/4, respectively. If the maximum strain and maximum strain rate are specified, then the period must satisfy $T = \pi e_{max} / \dot{e}_{max}$. As representative values, suppose the maximum strain is 0.002 and maximum strain rate is 1000/s. Choose viscous elastic parameters so that for this case, the stress increases by approximately a factor of 2 at a strain of 0.001. Show plots of one cycle of stress versus strain for maximum strain rates of 0, 250/s, 500/s and 1000/s.

6. The purpose of this problem is to write a program to plot yield surfaces in terms of principal stresses.

Write a program to plot the yield surface for an arbitrary yield function, F in terms of principal stresses in the deviatoric $(q_1 - q_2)$ space, and in the space of $\sigma_1 - \sigma_2$ for plane stress $(\sigma_3 = 0)$.

As an example of how one might proceed we note that any yield function, F, is defined to be negative when all components of stress are zero, and $F = 0$ defines the yield surface. Consider the case of plane stress. Let $\sigma_1 = r\cos\alpha$ and $\sigma_2 = r\sin\alpha$. For a given value of α start with $r = 0$ and let r increase with small increments. For each value of r evalueate the pricipal stresses and, consequently, F. Once the value of the F becomes negative, note the value of r and compute the corresponding values of σ_1 and σ_2. This is one point on the yield surface. Repeat the process for values of α from 0 to 2π.

Apply your program to obtain yield surfaces for a Mises, a Tresca and a Mohr-Coulomb yield function. Note that for these yield functions it is elementary to plot yield surfaces directly from the analytical expressions. The purpose here is to have an algorithm for tracking the evolution of yield functions that are much more complicated such as those you may develop as part of your research.