Practical Peridynamics - HW 1

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1 Chapter 1: Excercises 1.1 through 1.6

1.1 Show why mass density is undefined on the boudnary of a homogeneous material body.

Density is the ratio of an amount of matter having mass (Δm) to the volume that matter occupies (ΔV). Let P represent the center of mass (centroid) of the volume ΔV. When density is defined as $lim_{\Delta V \to 0}(\Delta m/\Delta V)$, this definition is physically limited (in an engineering sense) to a minimium ΔV with a characteristic length that is greater than the mean free path of the molecules which compose the homogenous body. Therefore, when P is chosen on the boundary of a homogenous body ΔV will encompass a volume that is not a part of that homogenous body. This will then provide a value of ρ that is not representative of the homogenous body. Therefore, ρ is only meaningful when $\Delta V$is of an order greater than that of the mean free path and greater than the characteristic length of ΔV from the boundary.

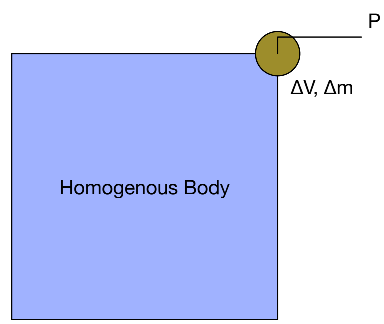

Figure 1: Point on the boundary of a homogenous body where $\rho$is undefined.

1.2 Describe and sketch three physical situations in which the traction vector is undefined at a point.

Tractions are typically undefined as a result of the ΔA being undefined, the following lists some situations in which this is so and Figure 2 illustrates these:

- A) At a corner: the area which the traction is applied is said to be uniquely described by the unit normal to that area; however, at an edge a unique normal vector does not exist and therefore the traction is not defined.
- B) At a crack front: the force applied by the traction encompasses a very small area, which may result in an unrealistically high magnitude of the traction. Additionally, the assumption of continuity is no longer valid as new faces are formed.
- C) Point of material seperation: again, the force applied by the traction encompasses a very small area, which may result in an unrealistically high magnitude of the traction. Additionally, the assumption of continuity is no longer valid as new faces are formed.

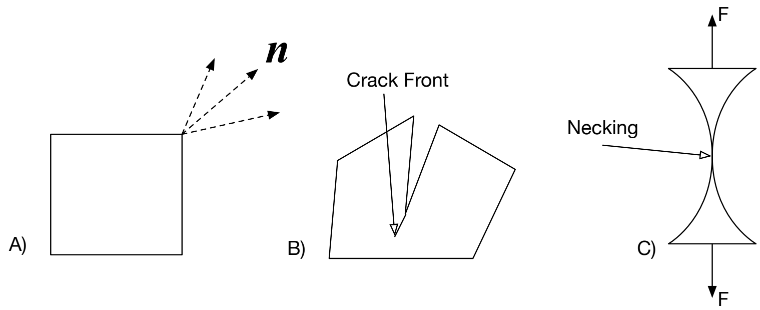

Figure 2: Examples of undefined tracstions.

1.3 Describe and sketch three physical situations in which the strain component at a point ϵ_{xx} is undefined.

Below are situations where strain components are undefined and (at least a partial) reason why (shown in Figure 2):

- A) At a corner: ΔX is undefined at the boundary of a body.
- B) At a crack front: the area ΔX is discontinuous and therefore not differentiable. This results in a strong formulation of strain being undefined and thus requiring a weak formulation.
- C) Point of material seperation: again, strains are discontinuous and the area ΔX is vanishingly small, which result in strain being undefined.

1.4 Determine the maximum permitted load for the beam shown in Fig. 1.2 when the maximum shear stress is 15,000 psi.

- a) Work for this problem is shown on the following pages. Calculated solution for the maximum value of P was 90,000 lb.
- b) Results were calculated using Abaqus. Results (in Figure 3) show the maximum allowable shear stress (J_2) was obtained when the applied load $P = 7,250$ psi. Maximum stress was observed at the edges near the two reaction forces on the lower boundary of the beam with the following material parameters:
  - Young's modulus $= 30 * 10^6 psi$
  - Poisson's ratio $= 0.3$

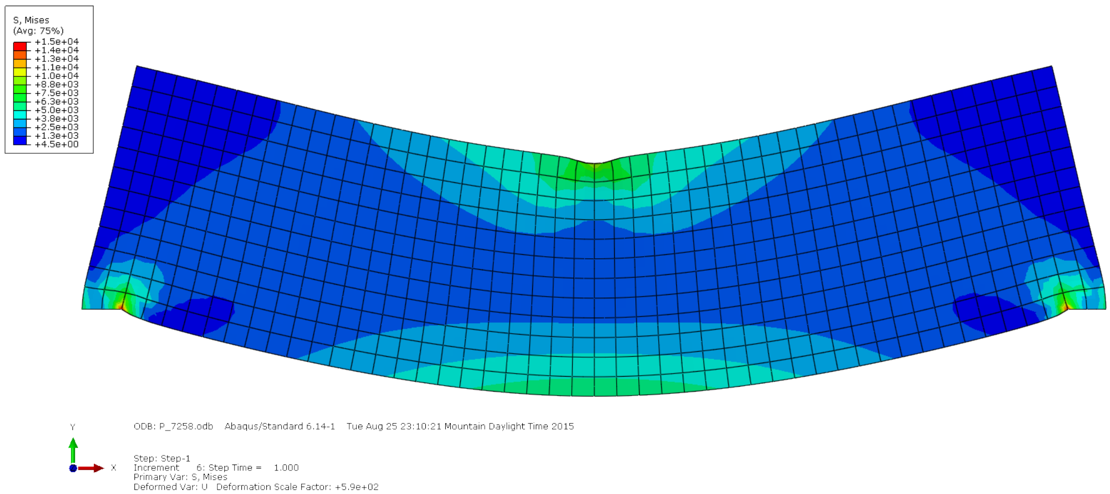

Figure 3: Contours of the maximum shear stress results on the deformed beam.

- c) The problem statement defines the load and reaction to be "concentrated". A concentrated force implies the force will be applied over a negligible area. Because of this, the magnitude of the resulting traction will be infinite (undefined) for any nonzero force vector. Therefore, the permitted load (P) in this situation is zero.

1.5 What is the load at which the glass plate with a hole in it will break?

A solution was obtained from literature where the stress concentration factor (K) at the boundary of the whole was defined as:

$$K = 3.000 - 3.140(d/D) + 3.667(d/D)^2 - 1.527(d/D)^3$$

Where d is the thickness of the plate and D is the height of the plate. This definition of K yielded a value of 2.875 for this problem. Therefore, the load at which the glass plate will break was calculated as:

$$P_{max} = \sigma_{ult} A/K = 10,000 * (6 * 0.25)/2.875 = 5.217 \text{ kip}$$

1.6 When the two cracks are each 0.5 inches in length, what is the maximum stress in the plate?

A FEA was not completed for this problem.