

Manual for the Calculation of Elastic-Plastic Materials Models Parameters

This Electronic Guide was produced as part of the Measurements for Materials System Programme on Design for Fatigue and Creep in Joined Systems

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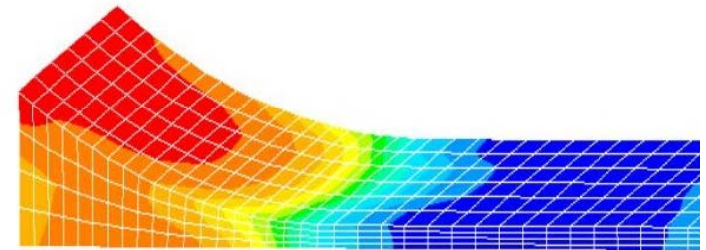
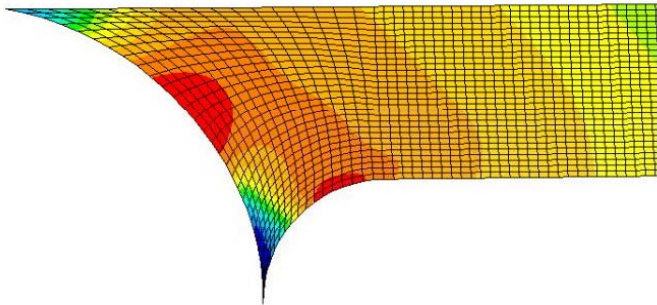
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INTRODUCTION

Finite element analysis is a computational tool that can be used for calculating forces, deformations, stresses and strains throughout a bonded structure. These predictions can be made at any point in the structure including within the adhesive layer. Furthermore, the element mesh can accurately describe the geometry of the bond line so the influence of geometrical features, such as the shape of fillets and boundaries with adherends, on joint performance can be accounted for in the analysis. This is particularly important in the design of adhesive joints because these features are usually associated with regions of stress and strain concentration likely to initiate joint failure.

Since adhesives are generally tough materials, they can sustain large strains before failure and, under these conditions, relationships between stress and strain are highly non-linear. The origin of non-linearity in a stress-strain curve for a polymeric adhesive can be explained in terms of enhanced creep brought about by an increase in molecular mobility caused by the application of stress. At moderate stresses this non-linear deformation is recoverable. At higher stresses, the molecular mobility is high enough for yielding to occur by plastic flow, which is not recoverable. Elastic-plastic materials are employed to describe deformation under large strain.

There are several models available for modelling the adhesive based on different criteria for plastic deformation. Predictions of joint performance at large strains close to joint failure depend on the model used. For the prediction of failure, stress and strain distributions in the adhesive need to be accurately calculated and a failure criterion for the adhesive needs to be established.

In its current form, this manual demonstrates the calculation of parameters for use with four different elastic-plastic material models. These four elastic plastic models are: von Mises, linear Drucker-Prager, exponent Drucker-Prager and Cavitation model.

Material Models	Data requirements
Von Mises	tensile only
Linear Drucker-Prager	tensile data and shear data at the same strain rate
Exponent Drucker-Prager	tensile data and shear data at the same strain rate
Cavitation model	tensile data, shear data and compressive data at the same strain rate

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Material models overview

Von Mises criterion is the most simple yield criterion. It interprets yielding as a purely shear deformation process which occurs when the effective shear stress reaches a critical value. Simple equations relate the tensile yield stress, shear yield stress and compressive yield stress to a material property. Tests on adhesives under additional stress states reveal that in many cases yielding is sensitive to the hydrostatic component of stress in addition to the shear component. Therefore the von Mises criterion is not realistic for many adhesives.

The **linear Drucker-Prager** model is a simple modification of the von Mises criterion that includes some hydrostatic stress sensitivity. A parameter (μ) is introduced which characterises the sensitivity of yielding to hydrostatic stress. Two different stress states e.g. tension and shear are required to determine this parameter. The linear Drucker-Prager model is not capable of accurately describing the non-linear behaviour of an important class of tough adhesives - the rubber toughened materials.

The **exponent Drucker-Prager** is a more complex elastic-plastic model, which is better able to describe behaviour under stress states in which there is a high component of hydrostatic tension. Two different stress states e.g. tension and shear are required to determine λ the exponent Drucker-Prager hydrostatic stress sensitivity parameter.

In rubber-toughened epoxy, nucleation of cavities occurs in the rubber phase. The **Cavitation model** includes the influence of void nucleation on ductility through adaptation of the linear Drucker-Prager model. This model has been developed at NPL and is currently being used to evaluate the accuracy of predictions of deformation in various joint geometries.

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TENSILE DATA INPUT

Tensile data are required for all four elastic-plastic materials models considered here. The data obtained from tensile test measurements of bulk adhesive are nominal (or engineering) values, where stresses and strains have been calculated using the initial specimen dimensions.

Measurements of the following data are required:

- Nominal Stress (MPa)
- Nominal Axial Strain
- Nominal Transverse Strain

For more information on tensile testing please click [here](#)

[Typical data](#) for a rubber-toughened epoxy

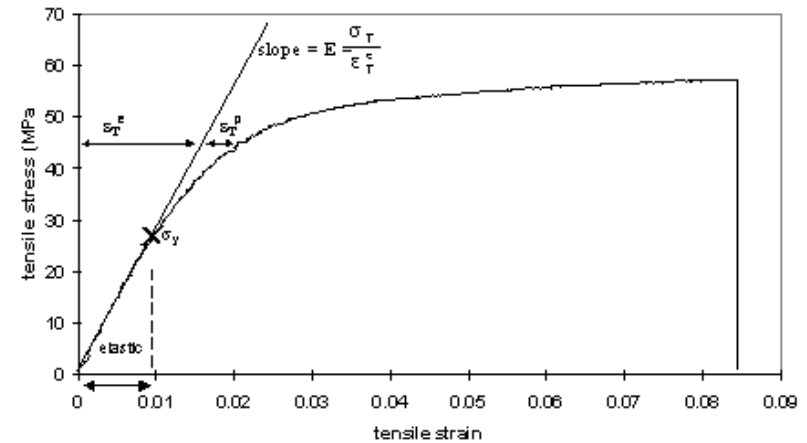
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Tensile Testing

Tensile tests [1] for the determination of Young's modulus (E) and Poisson's ratio (ν) are carried out on standard specimens [2, 3] under constant deformation rate in a tensile test machine at relatively low strain rates e.g. 10 mm/min. For best accuracy, contacting extensometers should be used for the measurement of axial and transverse strain, ϵ_T and ϵ_t . Two extensometers mounted on opposite faces of the specimen should preferably be used for the axial strain measurement to eliminate small non-uniformity in the strain through the thickness of the specimen caused by bending. The transverse strain measurement should be made close to the axial gauge section and, if possible, between the contact points of the extensometers. The contact pressure used to attach the extensometers to the specimen should be large enough to prevent slippage but insufficient to indent the specimen surface. Strain gauges are not recommended as they locally stiffen the specimen [1].



Values for Young's modulus and Poisson's ratio are calculated from the regression slopes in the linear region of the σ_T - ϵ_T and ϵ_t - ϵ_T curves. Use of regression slopes is preferable to single point values owing to the potential scatter in the data points (particularly the ϵ_t - ϵ_T data) that is mainly due to uncertainties in the small extensions measured. Whilst elastic values can be determined over any strain range where the data appear linear, the slight curvature due to viscoelastic effects will tend to reduce the value of E as the strain range widens.

The measurement of tensile hardening curves involves use of the same tests out to larger strains. Contacting extensometers, unless they have been modified, typically have an upper strain limit of around 0.05. They may also initiate premature failure in the specimen at a point of contact. For these tests, the use of a video extensometer is therefore preferable for the measurement of axial strain. The videoextensometer gauge markings are visible on the tensile test specimen shown above. These instruments are generally unsatisfactory for the measurement of small displacements and so a contacting device is best used to measure the lateral strain for the determination of true stresses and the plastic component of Poisson's ratio.



1. G.D. Dean and B.C. Duncan, Preparation and Testing of Bulk Specimens of Adhesives. NPL Measurement Good Practice Guide No 17, July 1998.
2. ISO 3167:1993, Plastics - Multipurpose test specimens.
3. ISO 527-2:1993, Plastics - Determination of tensile properties - Part 2: Test conditions for moulding and extrusion plastics.

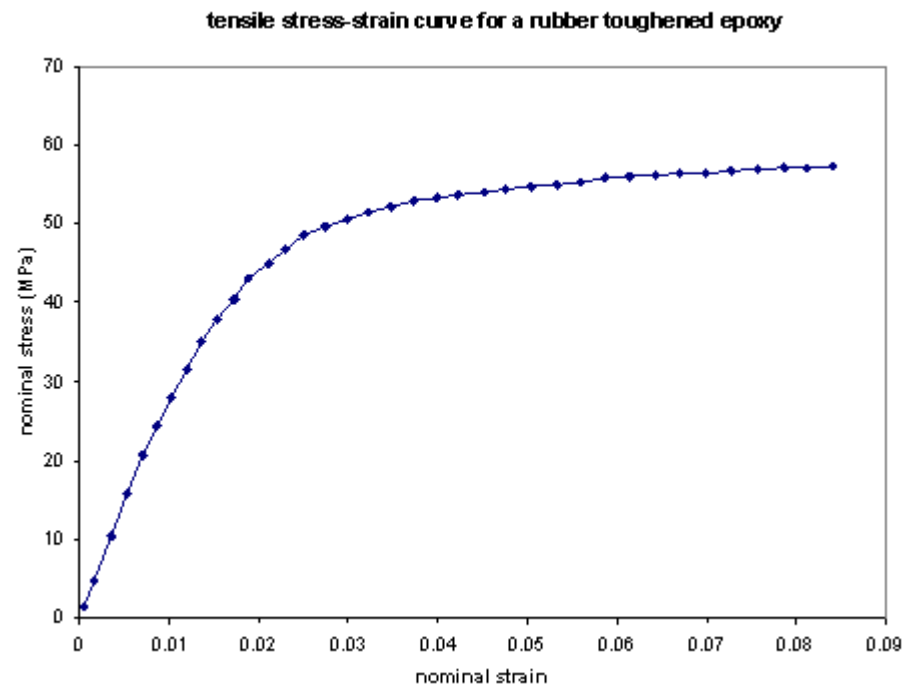
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Tensile Data

Here are typical tensile data for a rubber-toughened epoxy. The data are shown in tabular and graphical form.

Nominal Strain	Nominal Stress (MPa)
0.0004	1.44
0.0015	4.80
0.0035	10.34
0.0053	15.73
0.0070	20.71
0.0086	24.28
0.0103	28.02
0.0120	31.59
0.0135	34.93
0.0153	37.89
0.0172	40.46
0.0189	43.13
0.0211	45.03
0.0230	46.81
0.0251	48.57
0.0274	49.68
0.0298	50.65
0.0322	51.46
0.0348	52.25
0.0373	52.90
0.0399	53.36
0.0423	53.63
0.0452	54.07
0.0476	54.46
0.0504	54.80
0.0533	54.98
0.0560	55.33
0.0588	55.84
0.0615	55.97
0.0643	56.19
0.0670	56.32
0.0699	56.44
0.0728	56.79
0.0758	56.86
0.0786	57.09
0.0812	57.17
0.0842	57.26



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Tensile Data for Elastic-Plastic Models

Elastic-plastic models in FE systems require data in the form of elastic constants to describe elastic behaviour and parameters that describe the yield, hardening and flow behaviour to describe plastic (non-linear) behaviour. Further information on [elastic and plastic behaviour](#) is available. Data from tensile tests are required in the form of true stresses and strains for the calculation of the elastic-plastic model parameters. The change in cross-section of the specimen during tensile tests needs to be included to calculate true stress and strains. Under tensile loading, the specimen cross-section reduces. True values can be calculated from the nominal (engineering) values, which are based on the original specimen dimensions.

The following data can be calculated:

- True stress, σ_T
- True strain, ϵ_T
- True transverse stress, ϵ_t
- Nominal Poisson's ratio, ν'
- Young's Modulus, E'
- True Elastic modulus, E
- True Poisson's ratio, ν
- True plastic strain, ϵ_T^P
- True transverse plastic strain, ϵ_t^P
- True plastic Poisson's ratio, ν^P
- [Hardening curve](#)

For more information on the calculation of these parameters please click [here](#)

Of the above data, the Young's modulus, Poisson's ratio, true stress, true plastic strain, and true plastic Poisson's ratio are required for the elastic-plastic models parameter calculations. The hardening curve is also required. This is a plot of true stress versus true plastic strain, which is sampled to reduce the number of data points and then used in tabular form within elastic-plastic materials models. More information on the hardening curve can be obtained by clicking on the link above.

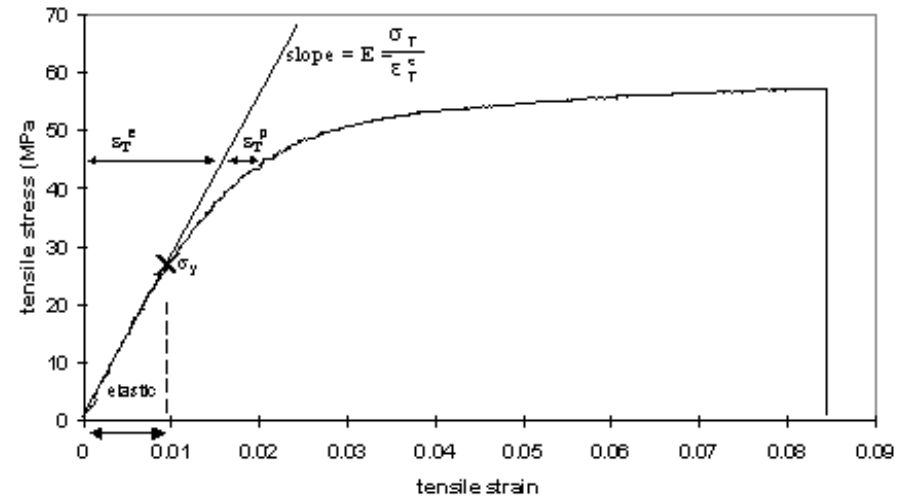
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Elastic-Plastic Behaviour

With elastic-plastic models, calculations of stress and strain distributions at low strains are based on linear elasticity. The onset of non-linearity is attributed to plastic deformation and occurs at a stress level regarded as the first yield stress. The subsequent increase in stress with strain is associated with the effects of strain hardening, and increases to a maximum corresponding to the flow region. In this non-linear region, the total strain is considered to be the sum of a recoverable elastic component and a plastic component, which is non-recoverable. Stress analysis calculations then involve the use of multiaxial yield criteria and a flow law. The yield criterion relates components of applied stress field to material parameters after the onset of yielding. The material parameters will depend upon the plastic strain for a strain hardening material.



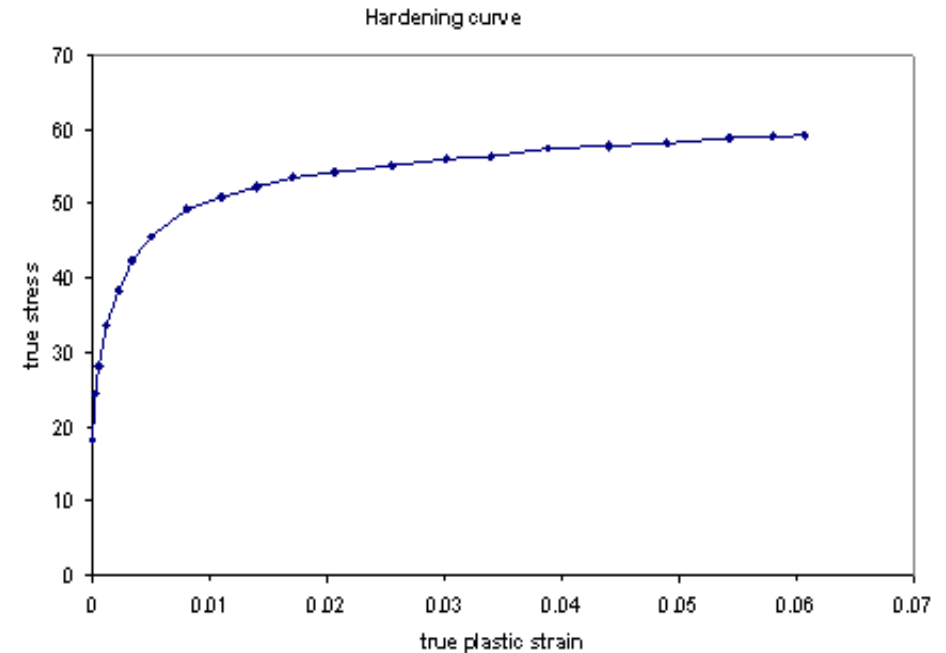
The calculation of plastic strain components is achieved in plasticity theory using a flow rule, which relates increments of plastic strain to a plastic flow potential. If the flow behaviour for a particular material is such that the flow potential can be identified with the yield function then this is termed associated flow. In general, this will be an approximation and extra information is needed to characterise non-associated flow. In order to calculate some of the parameters in elastic-plastic models, it is necessary to select stress values from different tests under the same state of yielding. This requires the definition of an effective plastic strain, and equivalent stresses are then a set of stresses that characterise stress states having the same effective plastic strain.

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Hardening Curve

Strain hardening data can be given in either single or multiple rate form. If a single strain rate hardening curve is used, this strain rate is assumed to be the average strain rate in the adhesive layer. The use of multiple rate hardening curves allows for regions of varying strain rate within the adhesive layer. Generally four strain hardening curves separated by a factor of 10 in strain rate are used. One is chosen with a strain rate approximately the same as the average strain rate in the joint, one higher rate curve and one lower rate curve. An extremely low rate curve is also required and is designated the zero rate curve. Analysis difficulties can occur if more hardening curves are used or if too many data points are used to characterise each curve. The plastic strain value must always increase otherwise the analysis will not run. The best approach is to sample the hardening data until a representative curve is achieved. The sampling density of the hardening curves shown in the figure has been used successfully.



The hardening curve data are required in the tabular form of yield stress with plastic strain where the first pair of numbers must correspond to the initial yield stress at zero plastic strain. If strains in the analysis exceed the maximum effective strain supplied in the hardening curve then the analysis will assume that additional extension occurs with no hardening. One means of getting around this is to extrapolate a point on the hardening curve at a significantly higher strain.

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Tensile Data Calculations

Data from tensile tests are required in the form of true stresses and strains for the calculation of the elastic-plastic model parameters. These are related to nominal (engineering) values and based on the original specimen dimensions by the equations shown below.

Click [here](#) for a definition of the symbols

True Stress	$\sigma_T = \frac{\sigma'_T}{(1 - \nu' \epsilon'_T)^2}$
True Strain	$\epsilon_T = \ln(1 + \epsilon'_T)$
True Transverse Strain	$\epsilon_t = \ln(1 + \epsilon'_t)$
Nominal Poisson's Ratio	$\nu' = -\frac{\epsilon'_t}{\epsilon'_T}$
Young's Modulus	$E = \frac{\sigma_T}{\epsilon_T^e}$
True Poisson's Ratio	$\nu = -\frac{\epsilon_t}{\epsilon_T}$
True Plastic Strain	$\epsilon_T^p = \epsilon_T - \ln\left(1 + \frac{\sigma_T}{E}\right)$
True Transverse Plastic Strain	$\epsilon_t^p = \epsilon_t - \ln\left(1 - \nu' \frac{\sigma_T}{E}\right)$
True Plastic Poisson's Ratio	$\nu^p = -\frac{\epsilon_t^p}{\epsilon_T^p}$

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The definition of each symbol is given below

$\sigma'_T, \epsilon'_T, \epsilon'_t$	nominal (engineering) values of tensile stress, axial strain and transverse strain respectively, calculated using original specimen dimensions
$\sigma_T, \epsilon_T, \epsilon_t$	true values of tensile stress, axial strain and transverse strain respectively, these take into account the instantaneous specimen dimensions.
E, ν'	Young's modulus and nominal Poisson's ratio respectively. Modulus and Poisson's ratio values are calculated at a point or by regression over a strain range e.g. 0.0005 to 0.0025 strain range. The Young's modulus is calculated from stress divided by strain, and at small strains the true values are equivalent to the nominal values.
ν, ϵ_T^e	true Poisson's ratio and true elastic axial strain respectively
$\sigma_T^p, \epsilon_T^p, \nu^p$	true axial plastic strain, true transverse plastic strain and true plastic Poisson's ratio respectively

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VON MISES PARAMETERS

Von Mises parameters can be output after tensile testing as these parameters are calculated from tensile data only.
The parameters required are Young's modulus, Poisson's ratio and a hardening curve (true tensile stress vs true tensile plastic strain)

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VON MISES MATERIAL MODEL

The most simple yield criterion interprets yielding as a purely shear deformation process which occurs when the effective shear stress σ_e reaches a critical value. This effective stress is defined in terms of principal stress components σ_i ($i = 1, 2$ or 3) by

$$\sigma_e = \left\{ \frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \right\}^{1/2}$$

The von Mises criterion then relates σ_e to the yield stress in tension σ_T by

$$\sigma_e = \sigma_T$$

The tensile yield stress σ_T is now a material parameter and has a minimum value, which denotes the limit of elastic behaviour and the start of plastic deformation and will increase with tensile plastic strain. The variation of yield stress with plastic strain is called the tensile strain hardening function.

The von Mises criterion predicts that the tensile yield stress, effective shear yield stress and compressive yield stress are related by

$$\sigma_T = \sigma_C = \sqrt{3}\sigma_S$$

Tests on adhesives under additional stress states such as shear and compression reveal that yielding is sensitive to the hydrostatic component of stress in addition to the shear component. The von Mises criterion is therefore not realistic for some materials.

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VON MISES PARAMETERS

The von Mises materials model parameters are shown below in both a general format and also the format required by the ABAQUS finite element software

Young's modulus, $E = 2970 \text{ MPa}$
Poisson's ratio, $\nu = 0.35$

ABAQUS FORMAT

*ELASTIC, TYPE=ISO
2970., 0.35

true tensile plastic strain	true tensile stress (MPa)
0.0000	18.197
0.0002	24.425
0.0006	28.210
0.0012	33.578
0.0022	38.299
0.0034	42.436
0.0050	45.664
0.0080	49.321
0.0110	50.979
0.0139	52.377
0.0170	53.557
0.0206	54.412
0.0254	55.243
0.0301	56.128
0.0340	56.527
0.0389	57.455
0.0440	57.920
0.0490	58.287
0.0543	58.989
0.0580	59.213
0.0607	59.376

*PLASTIC
18.197, 0
24.425, 0.0002
28.21, 0.0006
33.578, 0.0012
38.299, 0.0022
42.436, 0.0034
45.664, 0.005
49.321, 0.008
50.979, 0.011
52.377, 0.0139
53.557, 0.017
54.412, 0.0206
55.243, 0.0254
56.128, 0.0301
56.527, 0.034
57.455, 0.0389
57.92, 0.044
58.287, 0.049
58.989, 0.0543
59.213, 0.058
59.376, 0.0607

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SHEAR DATA INPUT

Shear data are required for the calculation of Drucker-Prager and cavitation model parameters

The following data are required from shear measurements on bulk adhesive specimens:

- Shear Stress (MPa)
- Shear Strain

For more information on shear testing please click [here](#)

[Typical data](#) for a rubber-toughened epoxy

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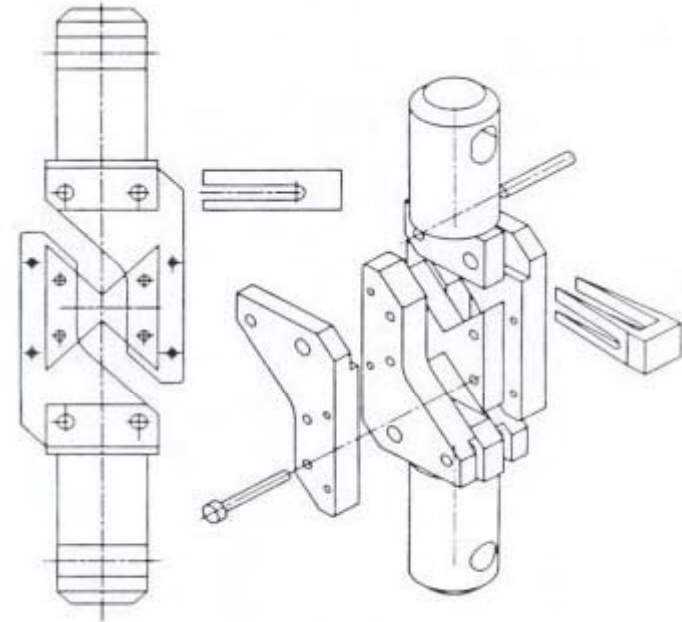
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Shear Testing

Shear tests on bulk specimens can be carried out using the notched-plate shear (Arcan) [1], [2] or notched-beam shear (Iosipescu) [2], [3] methods. These methods are similar in that they use a double-notched specimen to achieve a region of predominantly pure shear in the centre of the specimen between the notches. Of these methods, the notched-plate shear test is probably the preferred one because the loading stage is simpler to construct, extensometers can be more conveniently used for strain measurement [4] and thinner test specimens can be employed. A schematic diagram of the specimen and loading arrangement is shown.

Specimen dimensions of 12 mm for the notch separation with a radius of the notches of 1.5 mm have been used successfully. A purpose-built extensometer [4] has been used for measuring the relative displacement of two points either side of a vertical line through the centre of the specimen. The separation of these points is 3 mm. Finite element analyses of the stress and strain distributions in the specimen reveal some non-uniformity in the shear stress between the notches and a small contribution to the measured displacements from bending. These give rise to typical errors of about 7% in shear modulus and 2% in the shear flow stress.



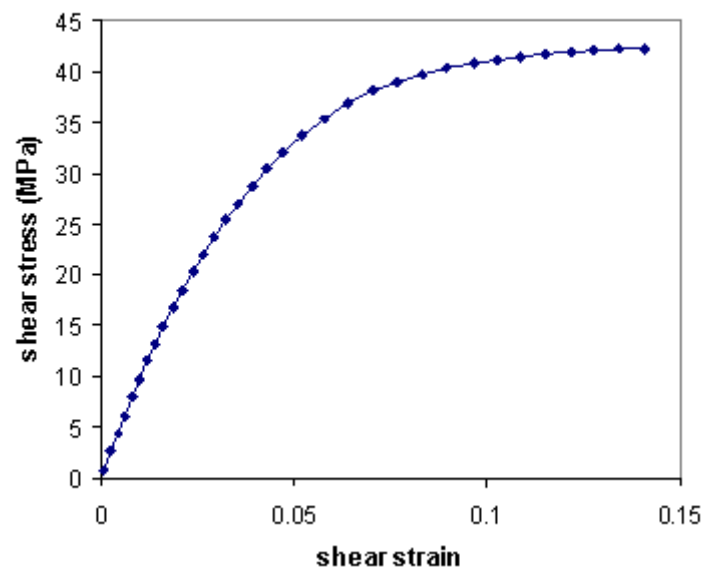
1. Arcan, Z. Hashin and A. Voloshin, Experimental Mechanics, vol 18 (1978) p141.
2. G.D. Dean and B.C. Duncan, Preparation and Testing of Bulk Specimens of Adhesives. NPL Measurement Good Practice Guide No 17, July 1998.
3. D.F. Adams and D.E. Walrath, J. Composite Materials, vol 21 (1987) p494.
- 4 G. Dean, B. Duncan, R. Adams, R. Thomas and L. Vaughn, Comparison of bulk and joint specimen tests for determining the shear properties of adhesives. NPL report CMMT(B)51, April 1996.

Shear Data

Here are typical tensile data for a rubber-toughened epoxy. The data are shown in tabular and graphical form.

Shear Strain	Nominal Stress (MPa)
0.0010	0.838
0.0026	2.670
0.0044	4.374
0.0063	6.186
0.0082	7.997
0.0101	9.742
0.0121	11.560
0.0141	13.237
0.0163	14.961
0.0188	16.813
0.0212	18.477
0.0239	20.309
0.0266	21.965
0.0294	23.648
0.0325	25.339
0.0356	26.995
0.0393	28.760
0.0432	30.409
0.0474	32.059
0.0524	33.735
0.0579	35.318
0.0640	36.771
0.0705	38.029
0.0765	38.934
0.0834	39.732
0.0895	40.300
0.0966	40.800
0.1026	41.145
0.1087	41.422
0.1154	41.673
0.1217	41.848
0.1277	41.963
0.1344	42.126
0.1409	42.227

Shear stress-strain curve for a rubber toughened epoxy



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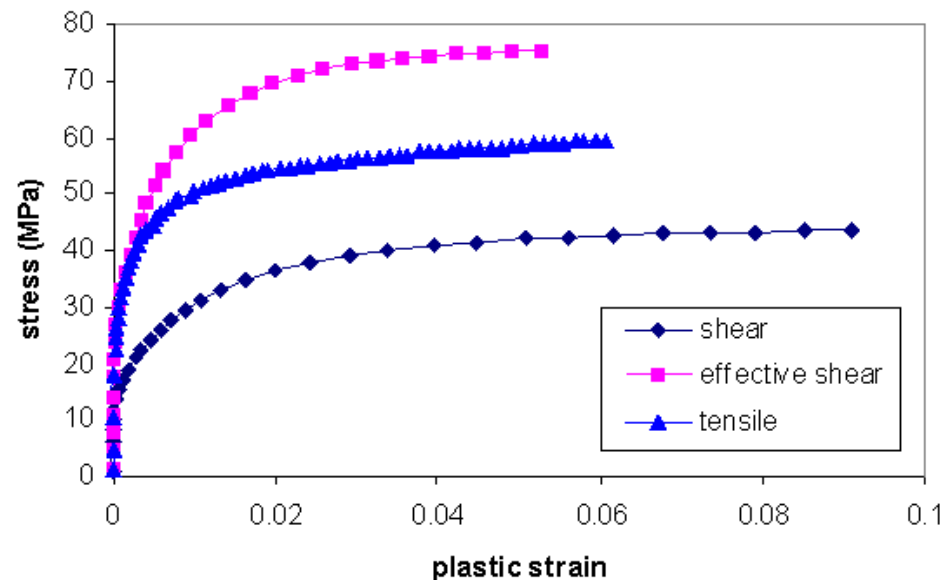
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The following data can be calculated from shear data:

- Shear Modulus, G
- Plastic strain, ϵ_s^p
- Effective Shear Stress, σ_{Seff}
- Effective Shear Plastic Strain, ϵ_{Seff}^p

For more information on the calculation of these parameters please click [here](#)

The experimental shear stress and plastic strain are required for the Drucker-Prager models parameter calculations. The effective stress and plastic strain data are useful for comparison with tensile results and are also used in the calculation of some of the Cavitation model parameters. For the Cavitation model a hardening curve is derived from the shear stress and plastic strain data.



Shear Data Calculations

The required shear data are obtained from the equations below.

Shear Modulus	$G = \frac{\sigma_s}{\epsilon_s}$
Plastic Strain	$\epsilon_s^p = \epsilon_s - \frac{\sigma_s}{G}$
Effective Shear Stress	$\sigma_{\text{Seff}} = \sqrt{3} \sigma_s$
Effective Shear Plastic Strain	$\epsilon_{\text{Seff}}^p = \frac{\epsilon_s^p}{\sqrt{3}}$

The definition of each symbol is given below:

σ_s, ϵ_s	measured values of shear stress and shear strain respectively
G, ϵ_s^p	shear modulus and plastic component of shear strain respectively
$\sigma_{\text{Seff}}, \epsilon_{\text{Seff}}^p$	effective shear stress and effective plastic strain respectively under a shear stress state

The measured shear stress and strain can be corrected to give a more accurate value of stress or strain in the centre of the arcan specimen [1]

1. [G.D. Dean, B.E. Read and B.C. Duncan, An Evaluation of Yield Criteria for Adhesive for Finite Element Analysis, NPL Report CMMT\(A\)117, January 1999](#)

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DRUCKER-PRAGER PARAMETERS

Tensile and shear testing and data analysis are required to obtain Drucker-Prager parameters.

The required parameters are:

Tan B	Hydrostatic stress sensitivity factor for linear Drucker-Prager
Ψ	Flow parameter for linear and exponent Drucker-Prager
a	Hydrostatic stress sensitivity factor for exponent Drucker-Prager
b	Exponent in exponent Drucker-Prager

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LINEAR DRUCKER-PRAGER MATERIAL MODEL

For the linear Drucker-Prager model the von Mises criterion is modified to include hydrostatic stress sensitivity as follows:

$$\sigma_e = \sigma_o - \mu \sigma_m \quad (1)$$

here σ_o is a material parameter that is related to the shear yield stress by:

$$\sigma_o = \sqrt{3} \sigma_s$$

and σ_m is the hydrostatic stress given in terms of principal stresses by:

$$\sigma_m = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3)$$

Equation (1) is identical to the linear Drucker-Prager model in the finite element system ABAQUS where the notation used is:

$$q = d + p \tan \beta$$

where $q = \sigma_e$, $p = -\sigma_m$, $\tan \beta = \mu$ and $d = \sigma_o$

The parameter μ depends on the adhesive material and characterises the sensitivity of yielding to hydrostatic stress. A value for μ is determined from tests under two different stress states. Using yield stresses from shear and tensile tests:

$$\tan \beta = \mu = 3 \left[\left(\sqrt{3} \sigma_s / \sigma_T \right) - 1 \right]$$

It should be noted that the above yield stresses σ_T and σ_s are associated with the same effective plastic strain.

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Associated and non-associated flow

The calculation of plastic strain components is achieved in plasticity theory using the flow rule in which increments of plastic strain are related to a plastic flow potential F by the equation:

$$d\varepsilon_{ij}^p = d\lambda \frac{\partial F}{\partial \sigma_{ij}}$$

where $d\lambda$ is a factor that depends on stress state and is determined by ensuring equivalence of the plastic work done under all stress states. For some materials, the flow potential F can be identified with the yield function, in which case the flow is said to be associated and:

$$F = \sigma_e + \mu \sigma_m - \sigma_0$$

This relationship is valid if the resultant of the strain increment during flow is directed normal to the yield surface. The validity of this assumption for adhesives needs to be verified and a more general expression for F is based on non-associated flow.

Under non-associated flow, a more general expression for F is: $F = \sigma_e + \mu' \sigma_m - \sigma_0$

The flow parameter μ' is then a material parameter that must be measured:

$$\mu' = \frac{3(1-2\nu^p)}{2(1+\nu^p)}$$

where ν^p is the plastic component of Poisson's ratio determined under uniaxial tension and is given by:

$$\nu^p = \frac{-\varepsilon_t^p}{\varepsilon_T^p}$$

In ABAQUS, the flow potential is:

$$F = q - p \tan \psi - d$$

where $\tan \psi$ has replaced μ' . If the calculated value of parameter μ' is not equal to μ then flow is termed non-associated. Associated flow is obtained by setting μ' equal to μ .

The flow behaviour of adhesives is generally non-associated. Non-associated flow results in a nonsymmetrical stiffness matrix and negative eigenvalues can occur even when the hardening data doesn't show softening. When an analysis reaches such a bifurcation point, an implicit solver may have difficulty converging. For simplicity, associated flow can be assumed by setting μ' equal to μ but the resulting loss in the accuracy of stress and strain calculations will be uncertain.

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EXPONENT DRUCKER-PRAGER MATERIAL MODEL

Although the linear Drucker-Prager yield criterion includes some sensitivity of yielding to the hydrostatic stress, it is not able to describe behaviour with any accuracy under stress states in which there is a high component of hydrostatic tension. Such stress states are common locally in adhesive bonds because of the high constraint imposed by the adherend under forces directed normal to the interface. An alternative criterion is significantly more accurate under these conditions and is often written in the form:

$$\sigma_e^2 = \lambda \sigma_T^2 - 3(\lambda - 1) \sigma_T \sigma_m$$

where λ is a hydrostatic stress sensitivity parameter that relates stresses σ_s and σ_T under shear and uniaxial tension by the equation:

$$\lambda = \frac{3\sigma_s^2}{\sigma_T^2}$$

This criterion is implemented in ABAQUS as the exponent Drucker-Prager model with the exponent parameter, b , equal to 2. The yield criterion equation is then expressed in the form:

$$aq^2 = p + p_1$$

where

$$a = \frac{1}{3\sigma_T(\lambda - 1)} \quad \text{and} \quad p_1 = a\lambda\sigma_T^2$$

A hyperbolic function has been chosen for the flow potential F . The asymptote of the hyperbola coincides with the flow potential for the linear Drucker-Prager model. The relevant material parameter in the flow law is therefore $\tan\psi = \mu'$ which can be calculated using:

$$\mu' = \frac{3(1 - 2\nu^p)}{2(1 + \nu^p)}$$

Because in ABAQUS, the flow potential assumed for this model is very similar to that used in the linear Drucker-Prager model, effective plastic strains and the associated equivalent stresses may be taken to be the same for the two models.

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DATA ANALYSIS

The following data need to be calculated from tensile and shear data

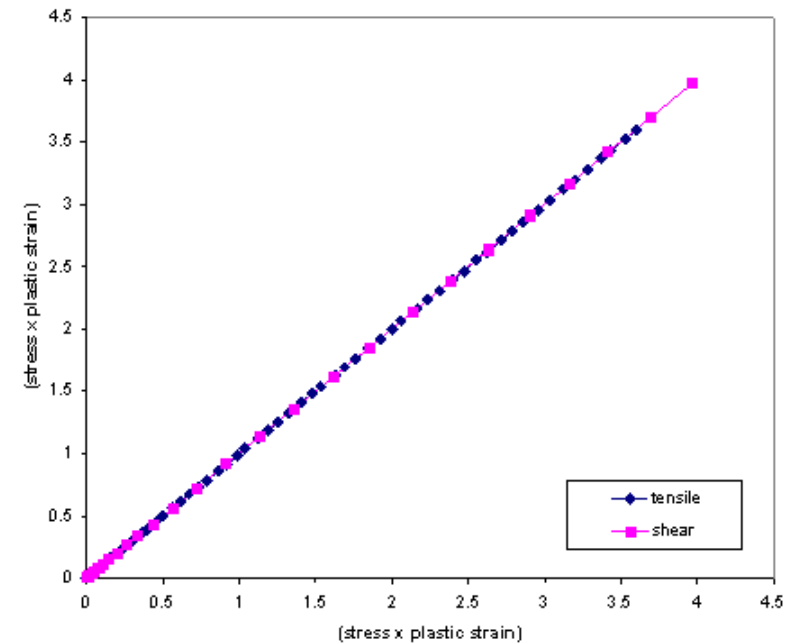
$$\sigma_T \epsilon_T^P$$

$$\sigma_S \epsilon_S^P$$

The above values obtained from tensile and shear data are used to obtain [equivalent data points](#) for calculation of Drucker-Prager parameters. When the resultant value from a tensile data point equals that from a shear data point, the plastic strain components from these two points are said to be equivalent (this is most easily seen graphically, as shown). The stresses corresponding to these two points are values that are used to calculate the model parameters.

These values are not unique, there may be other equivalent stress values for given sets of data. How these vary depends on the shape of the stress strain curves - if the yield stress data show a plateau (horizontal) region, then yield stresses do not vary much with effective plastic strain.

An example showing the effect of equivalent data point selection on parameters can be seen [here](#).



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Equivalent Data Points

The parameters μ and λ are calculated from yield stresses obtained from tests under different stress states. The yield stress values used in the calculations must correspond to the same effective plastic strain ϵ_{Seff}^p . Associated with an effective plastic strain is an effective stress σ_{Seff} which is defined with reference to a pure shear stress state by the relationship:

$$\sigma_{\text{eff}} = \sqrt{3} J_{2D}^{1/2}$$

so that

$$\sigma_{\text{eff}} = \sqrt{3} \sigma_s$$

Effective stresses and plastic strains σ_{ij} and ϵ_{ij}^p under an arbitrary stress state are then defined such that the plastic work done by this stress state is the same as the effective plastic work so that:

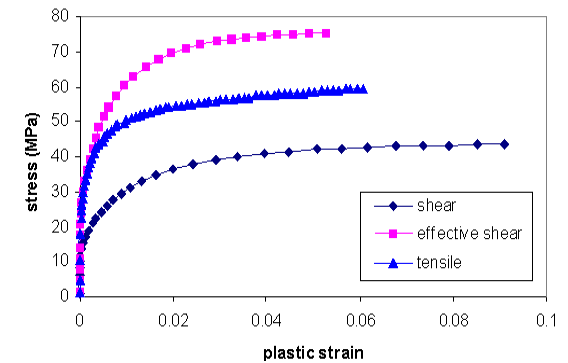
$$\sigma_{ij} \epsilon_{ij}^p = \sigma_{\text{eff}} \epsilon_{\text{eff}}^p$$

In the case of shear and tensile stresses:

$$\sigma_s \epsilon_s^p = \sigma_{\text{eff}} \epsilon_{\text{eff}}^p = \sigma_T \epsilon_T^p$$

so from the top equation the effective plastic strain is given by:

$$\epsilon_o^p = \frac{1}{\sqrt{3}} \epsilon_s^p$$



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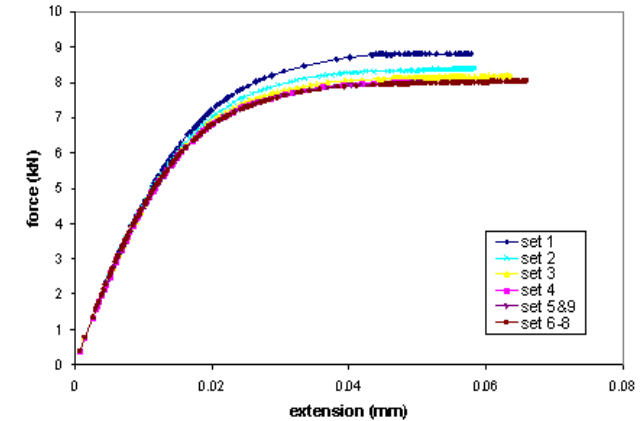
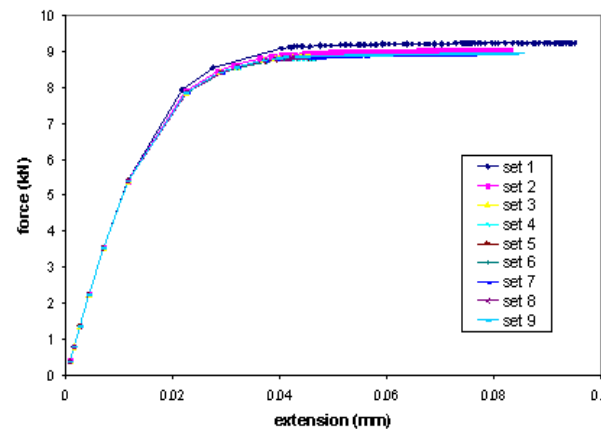
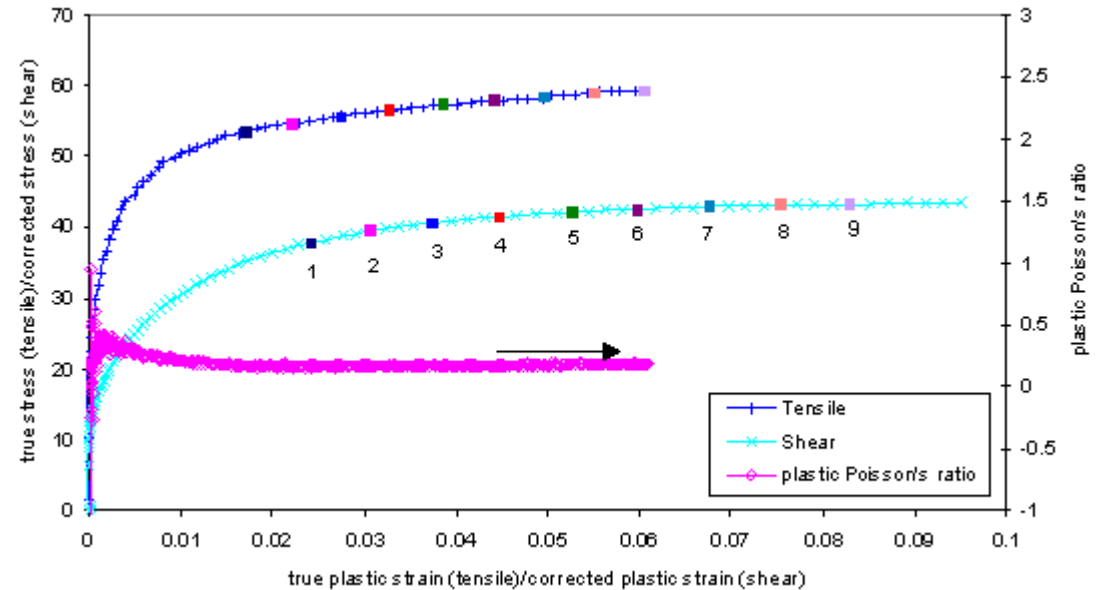
Effect of equivalent data point selection on parameters

This graph marks the location of 9 pairs of equivalent data points. Parameter values will vary depending on which pair of data points are selected. This can be seen in the table below, which shows parameters calculated for the linear and exponent forms of the Drucker-Prager model. In this case, the parameter ψ does not change as this is related to plastic component of the Poisson's ratio, which does not vary significantly over the strain range of interest for this material (see graph) and so is taken as a constant.

Data point set	Linear		Exponent	
	β	ψ	a	ψ
1	34.004	28.515	0.0124	28.515
2	37.171	28.515	0.0107	28.515
3	38.530	28.515	0.00996	28.515
4	39.204	28.515	0.00955	28.515
5	39.279	28.515	0.00938	28.515
6	39.330	28.515	0.00927	28.515
7	38.934	28.515	0.00931	28.515
8	38.958	28.515	0.00926	28.515
9	38.388	28.515	0.00939	28.515

Force-extension predictions obtained for a scarf joint using the above Drucker-Prager parameters are shown on the right. It can be seen that predictions obtained from parameter sets obtained at lower plastic strains (sets 1 and 2) are higher than those from the other data sets. Predictions from these other seven parameter sets are much more consistent. This is due to the fact that the parameters have been calculated from equivalent data points that are in the plateau regions of the tensile and shear stress strain curves.

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DRUCKER-PRAGER PARAMETERS

After careful data analysis the Drucker-Prager parameters can be obtained.

[Typical LINEAR Drucker-Prager parameters](#)
for a rubber-toughened epoxy

[Typical EXPONENT Drucker-Prager parameters](#)
for a rubber-toughened epoxy

Continue for calculation of Cavitation model parameters

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[TYPICAL PREDICTIONS](#)

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Linear Drucker-Prager Parameters

The linear Drucker-Prager materials model parameters shown below are in the format required by the ABAQUS finite element software

```
*ELASTIC, TYPE=ISO
2970., 0.35
**--Linear Drucker-Prager plasticity model
**--( Beta,K,Psi)
*DRUCKER PRAGER,SHEAR CRITERION=LINEAR
39.2, 1 ,28.5
*DRUCKER PRAGER HARDENING, TYPE=TENSION
18.197, 0
24.425, 0.0002
28.21, 0.0006
33.578, 0.0012
38.299, 0.0022
42.436, 0.0034
45.664, 0.005
49.321, 0.008
50.979, 0.011
52.377, 0.0139
53.557, 0.017
54.412, 0.0206
55.243, 0.0254
56.128, 0.0301
56.527, 0.034
57.455, 0.0389
57.92, 0.044
58.287, 0.049
58.989, 0.0543
59.213, 0.058
59.376, 0.0607
```

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Exponent Drucker-Prager Parameters

The exponent Drucker-Prager materials model parameters shown below are in the format required by the ABAQUS finite element software

```
*ELASTIC, TYPE=ISO
2970., 0.35
**--Exponent Drucker-Prager plasticity model
**--(a,b,"not used",Psi)
*DRUCKER PRAGER,SHEAR CRITERION=EXPONENT FORM
0.0093 , 2 , 0 , 28.5.
*DRUCKER PRAGER HARDENING, TYPE=TENSION
18.197, 0
24.425, 0.0002
28.21, 0.0006
33.578, 0.0012
38.299, 0.0022
42.436, 0.0034
45.664, 0.005
49.321, 0.008
50.979, 0.011
52.377, 0.0139
53.557, 0.017
54.412, 0.0206
55.243, 0.0254
56.128, 0.0301
56.527, 0.034
57.455, 0.0389
57.92, 0.044
58.287, 0.049
58.989, 0.0543
59.213, 0.058
59.376, 0.0607
```

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COMPRESSION DATA INPUT

To calculate the cavitation model parameters the following data are required from compression measurements on bulk adhesive specimens:

- Compressive Stress (MPa)
- Compressive Strain

For more information on compression testing please click [here](#)

[Typical data](#) for a rubber-toughened epoxy

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Compression Testing

Tests under uniaxial compression are difficult to perform because of the promotion of buckling of the specimen under high forces.

The ISO standard test ISO 604, Plastics - Determination of compressive properties can be used to produce satisfactory results. This test employs short specimens with approximate dimensions 10 mm high x 10 mm wide. The thickness should be at least 3 mm and preferably higher. The top and bottom faces of the specimen must be machined smooth and accurately parallel. These are loaded in a testing machine between platens whose faces are smooth and accurately parallel. To minimise constraints by the platen surfaces to lateral expansion of the specimen during loading, the surfaces of the platens are lubricated with oil. Extensometers are used to measure changes in the platen separation, and nominal strain values are derived from the original specimen length. At small strains, errors in strain values arise through non-uniformity of the strain along the length of the specimen by this loading method. It is possible to apply a correction to these strains using a knowledge of Young's modulus from tensile tests. However, it is the properties under plastic deformation that are of most interest from this test. At small strains, uncertainties in plastic strains will be large but should become smaller at larger plastic strains.

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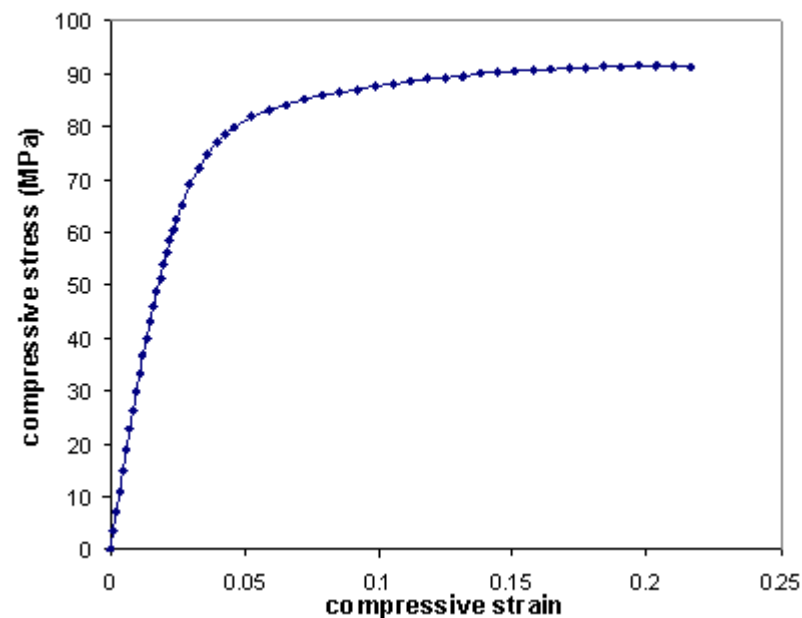
Compression Data

Here are typical tensile data for a rubber-toughened epoxy.
The data are shown in tabular and graphical form.

Compressive Strain	Compressive Stress (MPa)
0.0000	0.0317
0.0009	3.6501
0.0021	7.2030
0.0033	11.0594
0.0046	14.9195
0.0058	18.8314
0.0070	22.6558
0.0083	26.4015
0.0095	29.9514
0.0108	33.4151
0.0120	36.7533
0.0133	39.9197
0.0146	43.0115
0.0158	45.8811
0.0171	48.6312
0.0183	51.2853
0.0196	53.7570
0.0208	56.1216
0.0221	58.3276
0.0234	60.4476
0.0247	62.4127
0.0266	65.0851
0.0298	69.0216
0.0330	72.2359
0.0362	74.8374
0.0395	76.9510
0.0428	78.6097
0.0460	79.9529
0.0526	81.9421

Compressive Strain cont	Compressive Stress (MPa) cont
0.0592	83.2794
0.0657	84.2728
0.0723	85.1168
0.0788	85.8559
0.0854	86.4863
0.0920	87.0832
0.0985	87.6642
0.1051	88.1283
0.1117	88.5238
0.1182	88.9418
0.1248	89.3115
0.1313	89.6151
0.1379	89.9774
0.1445	90.2428
0.1510	90.4387
0.1576	90.6490
0.1642	90.8581
0.1707	91.0556
0.1772	91.1761
0.1837	91.2902
0.1902	91.3778
0.1968	91.4686
0.2033	91.5018
0.2098	91.4905
0.2163	91.3706

Compression stress-strain curve for a rubber-toughened epoxy



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Compression Data Calculations

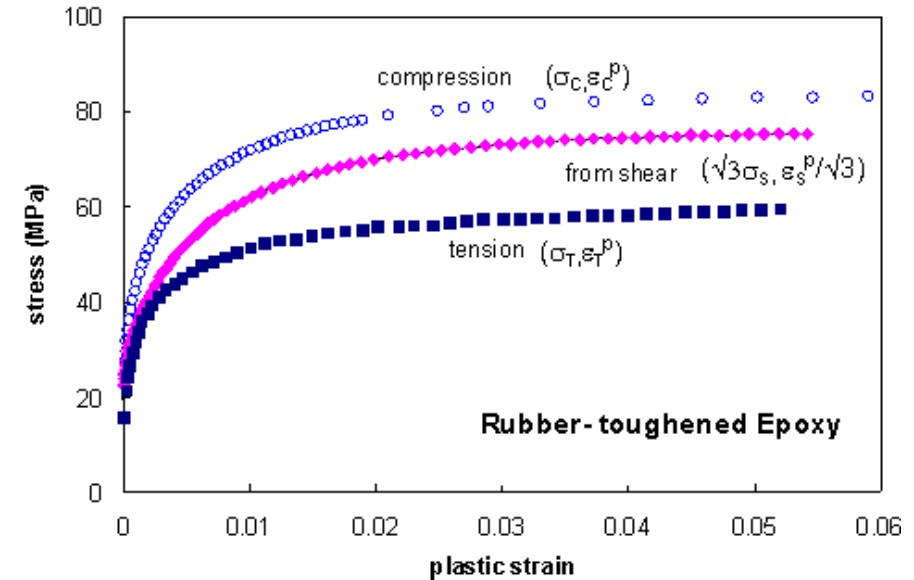
The required compression data are obtained from the equations below.

True Compressive Stress (MPa)	$\sigma_c = \frac{\sigma'_c}{(1 - \nu' \epsilon'_c)^2}$
True Compressive Strain	$\epsilon_c = \ln(1 + \epsilon'_c)$
True Compressive Plastic Strain	$\epsilon_c^p = \epsilon'_c - \frac{\sigma_c}{E}$

The definition of each symbol is given below:

$\sigma'_c, \epsilon'_c, \nu', E$ nominal (engineering) values of compressive stress, compressive strain, Poisson's ratio and Young's modulus respectively. The compressive stress and strain are negative values.

$\sigma_c, \epsilon_c, \epsilon_c^p$ true values of compressive stress, compressive strain and compressive plastic strain respectively. The compressive stress and strain are negative values.



Plot showing tensile, effective shear and compressive hardening data.

The above data are used along with tensile and effective shear data to calculate the Cavitation model parameters. A plot with data from all tests is shown above. To calculate the parameters, stress values from effective shear (σ_0), tensile (σ_T) and compression (σ_c) tests must be calculated at the same effective plastic strain. These are calculated as before e.g.

$$\sigma_s \epsilon_s^p = \sigma_c \epsilon_c^p$$

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CAVITATION MODEL PARAMETERS

The following Cavitation model parameters are calculated from tensile, shear and compression data. For more details about these parameters please click [here](#).

μ	hydrostatic stress sensitivity parameter
k	defines influence of rubber on yield stress
q	cavity interaction parameter
V_{R0}	volume fraction of rubber
μ'	flow parameter
ε_{1v}	critical volumetric strain
ε_{2v}	mean volumetric strain
β	defines the distribution of strains for cavitation
σ_{0u}	Parameters used to fit shear data to produce an effective shear hardening curve
σ_{0r}	
β_0	
g_0	

[CAVITATION MODEL INFORMATION](#)

[TYPICAL CAVITATION MODEL PARAMETERS](#)

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CAVITATION MODEL PARAMETERS

$\mu = 3 \left(1 - \frac{\sigma_0}{\sigma_c} \right)$	The hydrostatic stress sensitivity parameter μ is determined from measurements of yield stress under two different stress states for which no cavities are present (shear and compression). This is calculated from the equation shown, where σ_0 , from the shear stress, and σ_c , the compressive stress, are at the same effective plastic strains.
k	k is a parameter defining the influence of rubber on yield stress and q is the cavity interaction parameter. k and q are taken to have typical values which have been determined from additional tests on rubber toughened thermoplastics. For a rubber-toughened epoxy k=1.35 and q=1.6.
V_{R0}	Volume fraction of rubber. If possible obtain a value for the volume fraction of rubber from the adhesive supplier. Note that the effective value for this parameter may be higher than this if sources other than rubber, such as filler, act as cavity nucleation sites. Otherwise, the volume fraction of rubber can be determined by substituting equation (i) below into the yield equation (ii) (σ_{01} is the effective shear stress of the adhesive when all the rubber particles have cavitared, and σ_T is a tensile yield stress) and solving for V_{R0} <div style="display: flex; justify-content: space-around;"> <div>Eq (i) $\sigma_{01} = \sigma_0 / (1 - k V_{R0})$</div> <div>Eq (ii) $\frac{\sigma_e^2}{\sigma_M^2} - (q_1 f)^2 + 2q_1 f \cosh \frac{3\sigma_m}{2\sigma_M} = \left(1 - \frac{\mu \sigma_m}{\sigma_M} \right)^2$</div> </div>
μ'	A value for the flow parameter μ' is determined from Poisson's ratio measurements under tension out to strain levels for which cavity nucleation is complete. An experimental value for v^p is obtained and μ' is determined using the equation below. If μ' is not equal to μ then flow is non-associated. $v^p = \frac{\left[\frac{\sigma_T}{\sigma_{01}} - q V_{R0} \sinh \left(\frac{\sigma_T}{2\sigma_{01}} \right) - \frac{2\mu'}{3} \left(1 - \frac{\mu' \sigma_T}{3\sigma_{01}} \right) \right]}{\frac{2\sigma_T}{\sigma_{01}} + q V_{R0} \sinh \left(\frac{\sigma_T}{2\sigma_{01}} \right) + \frac{2\mu'}{3} \left(1 - \frac{\mu' \sigma_T}{3\sigma_{01}} \right)}$
ε_{1v} ε_{2v} β	These cavity nucleation parameters are obtained by an iterative process to achieve satisfactory predictions of the shape of the tensile stress-strain curve and the Poisson's ratio vs strain curve in the strain range associated with cavity nucleation. This requires a method for obtaining solutions for the case of uniaxial tension for comparison with experiment. A routine has been coded which allows the effects of changes to these parameters on curve shapes to be rapidly explored for the determination of optimum values. ε_{1v} and ε_{2v} are the critical volumetric strain and the mean volumetric strain respectively. β is a parameter defining the distribution of strains for cavitation
$\sigma_{0u}, \sigma_{0r}, \beta_0, g_0$	These parameters are used to fit the shear data to produce an effective shear hardening curve which characterises hardening behaviour in the Cavitation model. In the Cavitation model the hardening curve can be input as an equation rather than in tabular form. The hardening curve is modelled using the function below. Although this function can describe tension or shear, it is the shear curve that is modelled for the Cavitation model i.e. σ_s replaces σ_T $\sigma_T = \left[\sigma_{0u} + (\sigma_{0r} - \sigma_{0u}) \left(1 - \exp - \left(\varepsilon_T^p / \varepsilon_{0s} \right)^{\beta_0} \right) \right] (1 + g_0 \varepsilon_T^p)$

CAVITATION MODEL

The linear Drucker-Prager yield criterion, $\sigma_e = \sigma_o - \mu \sigma_m$, is modified as follows to give a yield function Φ that includes the effect of cavitation of the rubber on yield stresses for the adhesive:

$$\Phi = \frac{\sigma_e^2}{\sigma_M^2} - (q_1 f)^2 + 2q_1 f \cosh \frac{3\sigma_m}{2\sigma_M} - \left(1 - \frac{\mu\sigma_m}{\sigma_M}\right)^2 = 0$$

Here f is the effective volume fraction of cavities, which at small strains is zero but increases rapidly over some characteristic strain region responsible for cavity nucleation. The parameter q has been included to account for the effect of void interactions on the stress distribution in the matrix between cavities. The yield stress σ_M is $\sqrt{3}$ x the shear yield stress for the matrix material between voids. It is assumed that a cavity is nucleated in a rubber particle at some critical volumetric strain that decreases with increasing particle diameter. For a distribution of particle sizes, the void nucleation should then occur over a range of total volumetric strain ε_v related to the critical volumetric strain range for the rubber particles. Various mathematical functions relating f and ε_v have been considered and, based on tensile and shear results for the rubber-toughened adhesive and some rubber-toughened thermoplastics, the most suitable takes the form

$$f = 0 \quad \text{for } \varepsilon_v \leq \varepsilon_{1v} \quad \quad f = v_{Ro} \left\{ 1 - \exp \left[- \left(\frac{\varepsilon_v - \varepsilon_{1v}}{\varepsilon_{2v}} \right)^\beta \right] \right\} \quad \text{for } \varepsilon_v > \varepsilon_{1v}$$

These expressions imply that cavities start to nucleate when the volumetric strain exceeds a critical value ε_{1v} after which the volume fraction f rises from zero to the value v_{Ro} , for the volume fraction of rubber, over a range of volumetric strain determined by the parameters ε_{2v} and β . Furthermore, for stress states such as compression and shear that do not generate a significant volumetric strain, $f=0$ and the yield function above is identical to the linear Drucker-Prager yield criterion. $\sigma_o(\varepsilon_o^p)$ is the hardening function and is obtained from stress/strain data obtained in a shear test using the equation

$$\sigma_o = \sqrt{3} \sigma_s$$

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CAVITATION MODEL PARAMETERS

The materials model parameters shown below are those required by the NPL cavitation model user subroutine, which is read into the ABAQUS finite element software. The parameters are for a typical rubber-toughened epoxy.

```

***material
*integration theta_method_a 1.0 1e-8 1500

***behaviour prefd2ng explicit auto_step
**elasticity isotropic
young          2.970 GPa
poisson        0.35
**isotropic
npl_polymer
sig0u          0.0200 GPa
sig0r          0.0748 GPa
eps0s         0.0062
beta0          0.72
**model_coef
q              1.6
mu             0.300
mup0          0.1700
k              1.35
v_ro          0.16
eps1v         0.0030
eps2v         0.0040
betav         0.900
crit_str       0.005 GPa
***return

```

NOTE: In the cavitation model, the units used are GPa, this means predicted stresses are also output as GPa.

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TYPICAL PREDICTIONS

To investigate the effect of using different materials models on joint predictions please select a joint geometry. For each geometry the experimental curve is presented along with the corresponding predictions obtained using the von Mises, linear Drucker-Prager, exponent Drucker-Prager and Cavitation models. Contour plots of maximum principal stress in the adhesive are also available.

[Scarf joint](#)

[Lap joint](#)

[Tpeel joint](#)

[Butt joint](#)

NOTE: The predictions shown are only valid for these materials and joint geometries. The best choice of model will depend on the adhesive used.

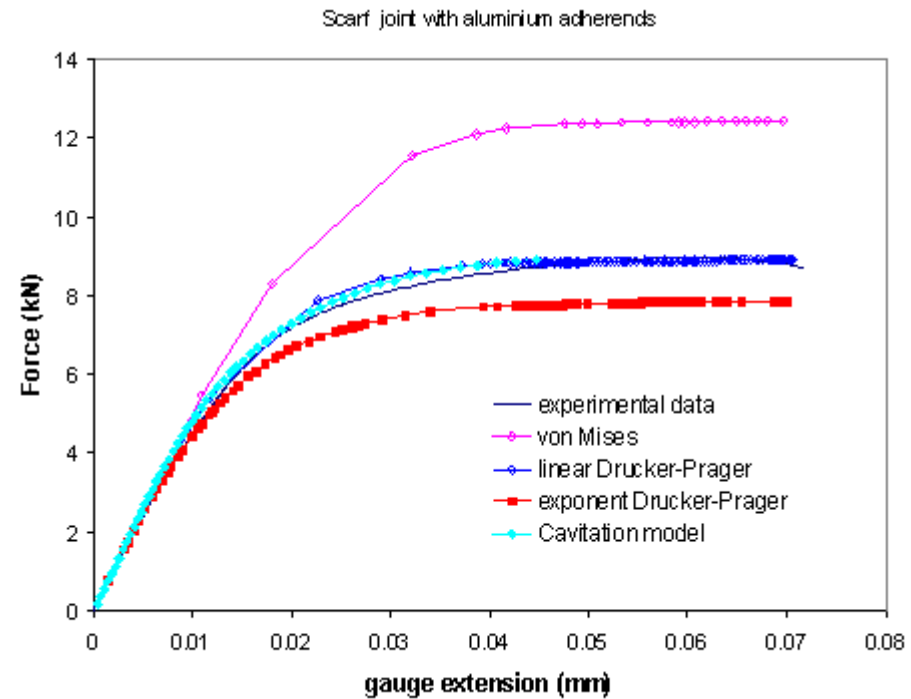
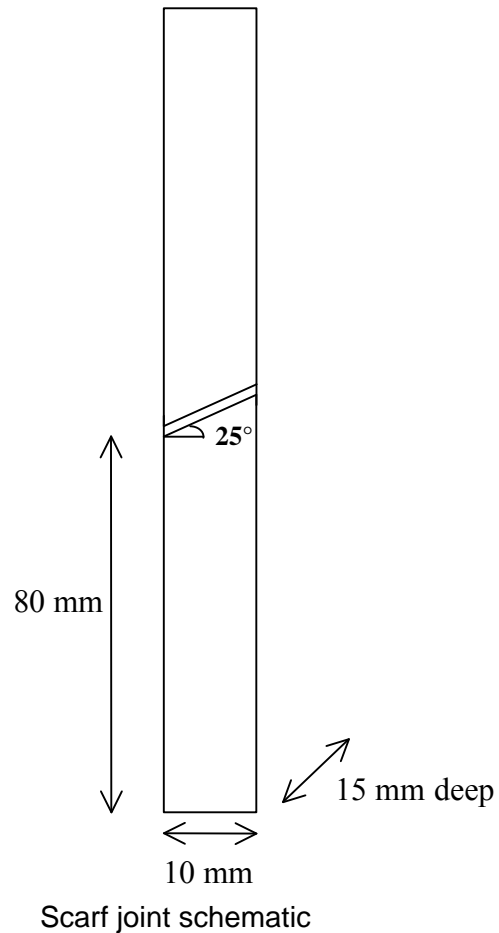
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SCARF JOINT

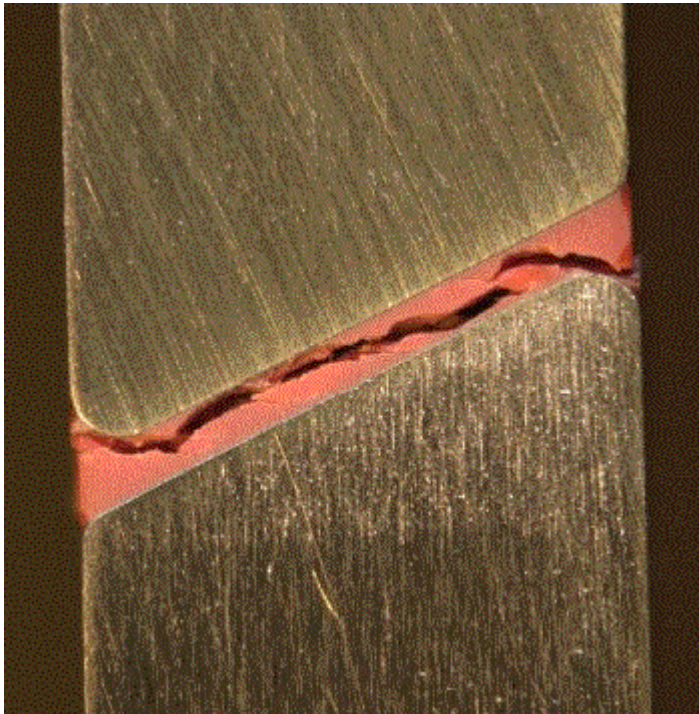
The scarf joint was bonded with a rubber-toughened epoxy and tested. FEA predictions were obtained from analyses using each of the four material models, von Mises, linear and exponent Drucker-Prager and cavitation model. The force-extension predictions are shown below. In this example, the linear Drucker-Prager and cavitation model produce similar predictions.



Scarf joint experimental curve with force-extension predictions from all four models

SCARF JOINT - Predictions of stress using different models

The values of stress and strain predicted in an analysis are very dependent on the material model used. To illustrate this, the maximum principal stress (SP3) is obtained for the scarf joint analyses at an extension of 0.0458 mm, the extension at which the scarf joint begins to fail. Contour plots were obtained for all four material models.



Photograph of failed scarf joint.
Crack initiated at an extension of 0.046 mm

Contour plots were obtained for all four models at an extension of 0.0458 mm (the point of crack initiation). The contour plots are available [here](#).

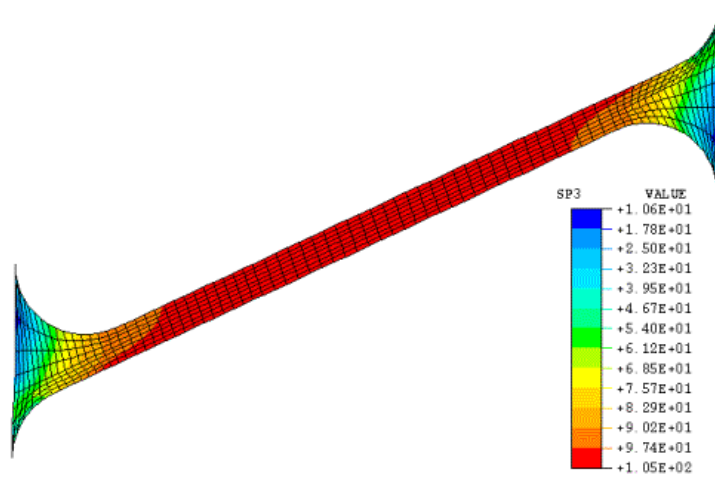
The predicted values of maximum principal stress are given in the table below

von Mises	105 MPa
linear Drucker-Prager	68.8 MPa
exponent Drucker-Prager	62.2 MPa
Cavitation model	67.6 MPa (6.76E-2GPa in contour plot)

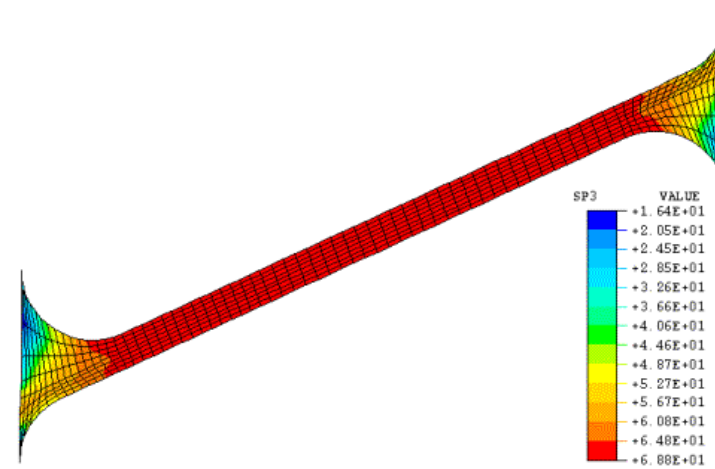
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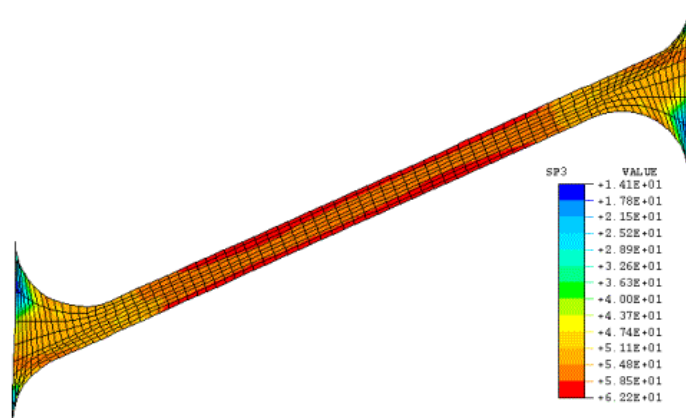
SCARF JOINT - Predictions of stress using different models



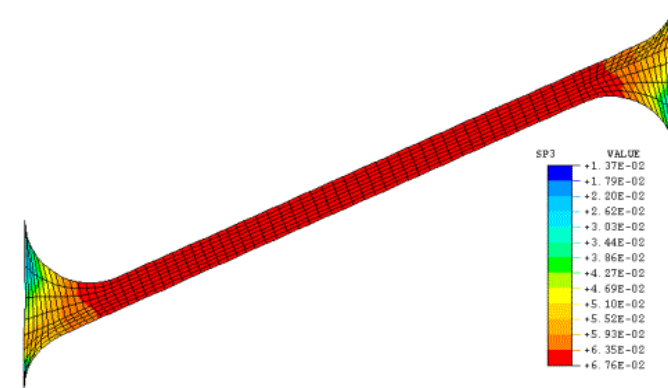
Von Mises



Linear Drucker-Prager



Exponent Drucker-Prager



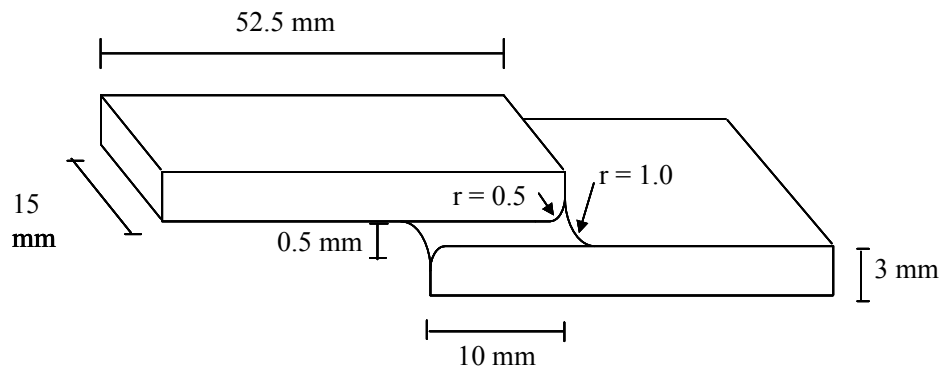
Cavitation

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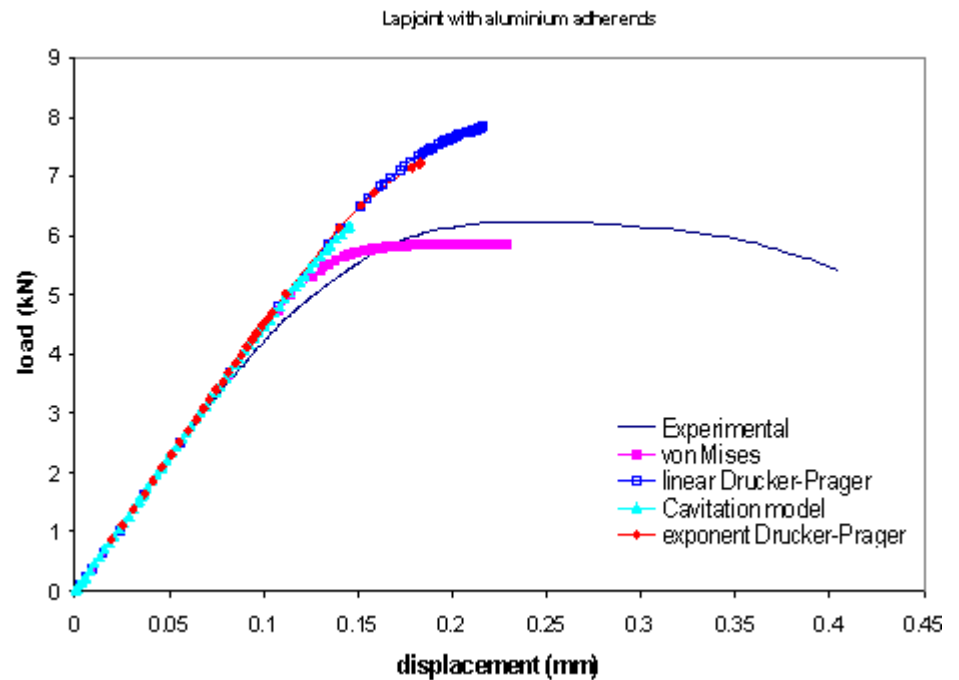
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LAP JOINT

The lap joint was bonded with a rubber-toughened epoxy and tested. FEA predictions were obtained from analyses using each of the four material models, von Mises, linear and exponent Drucker-Prager and cavitation model. The force-extension predictions are shown below. It should be noted that the degree to which the predicted force-extension curves fit the experimental behaviour is only of interest up to the point where failure initiates in the joint. After this point the experimental data will be lower than the predicted data due to the presence of a growing crack. In the lap joint, crack initiation is observed to occur at about 0.1 mm. At this extension the force-extension predictions are comparable for all four material models.



Lap joint schematic



Lap joint experimental curve with force-extension predictions from all four models

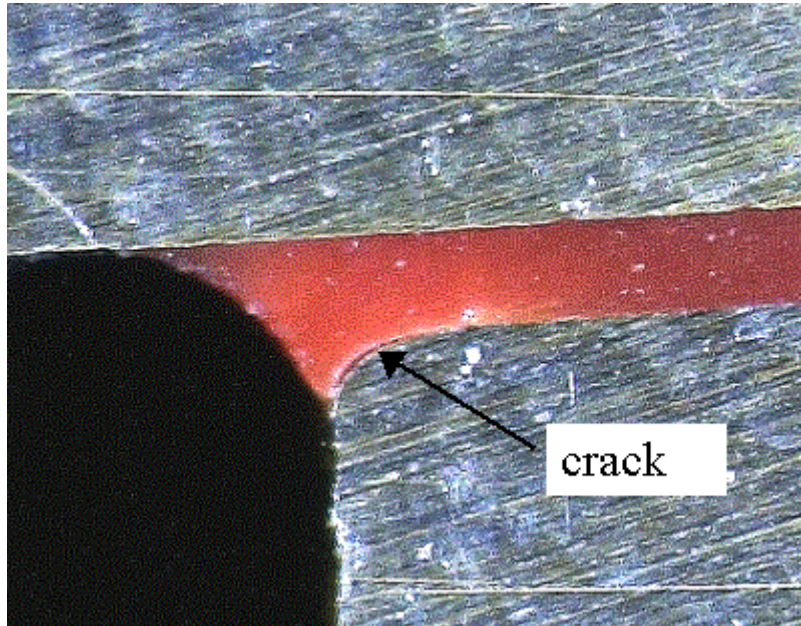
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[Effect of model on predicted stresses](#)

LAP JOINT - Predictions of stress using different models

The values of stress and strain predicted in an analysis are very dependent on the material model used. To illustrate this, the maximum principal stress (SP3) is obtained for the lap joint analyses at an extension of 0.09 mm. This is the extension at which a crack is observed to initiate in the adhesive layer of the joint. At this extension the force-extension predictions are comparable for all four material models.



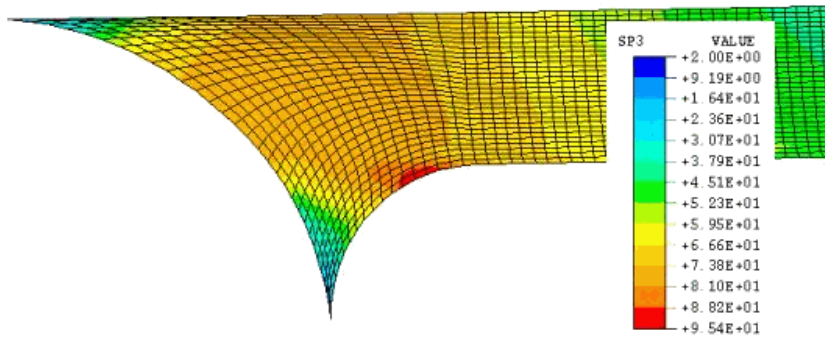
Photograph of failed lap joint.
Crack initiated at an extension of 0.09 mm

Contour plots were obtained for all four models at an extension of 0.09 mm (the point of crack initiation). The contour plots are available [here](#).

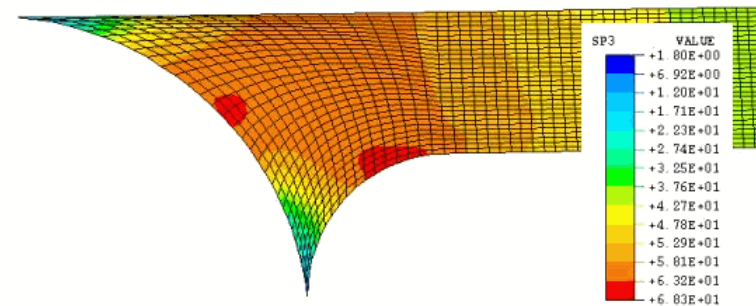
The predicted peak values of maximum principal stress are given in the table below. These peaks are found to occur in the region of crack initiation

von Mises	95.4 MPa
linear Drucker-Prager	68.3 MPa
exponent Drucker-Prager	60.7 MPa
Cavitation model	61.4 MPa (6.14E-2GPa in contour plot)

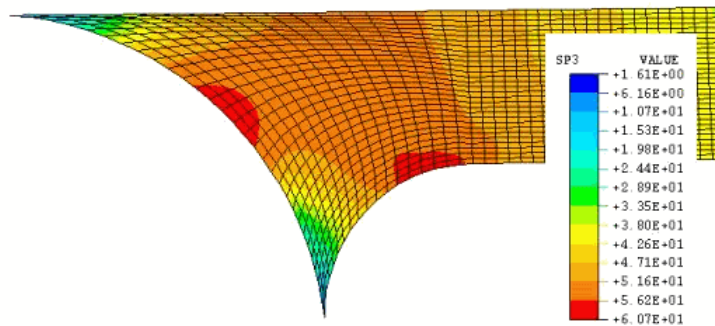
LAP JOINT - Predictions of stress using different models



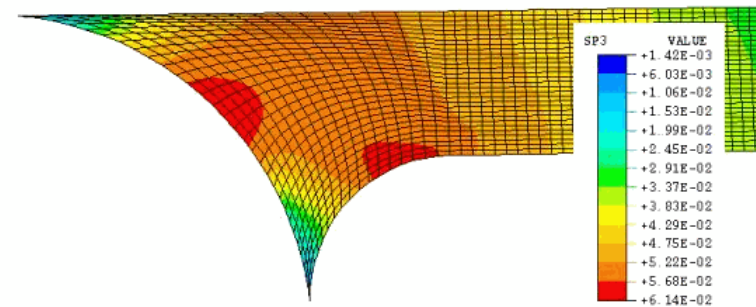
Von Mises



Linear Drucker-Prager



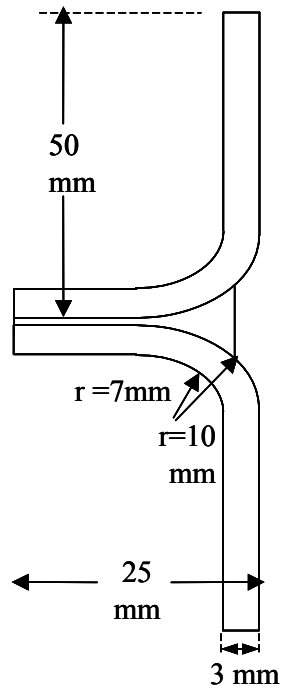
Exponent Drucker-Prager



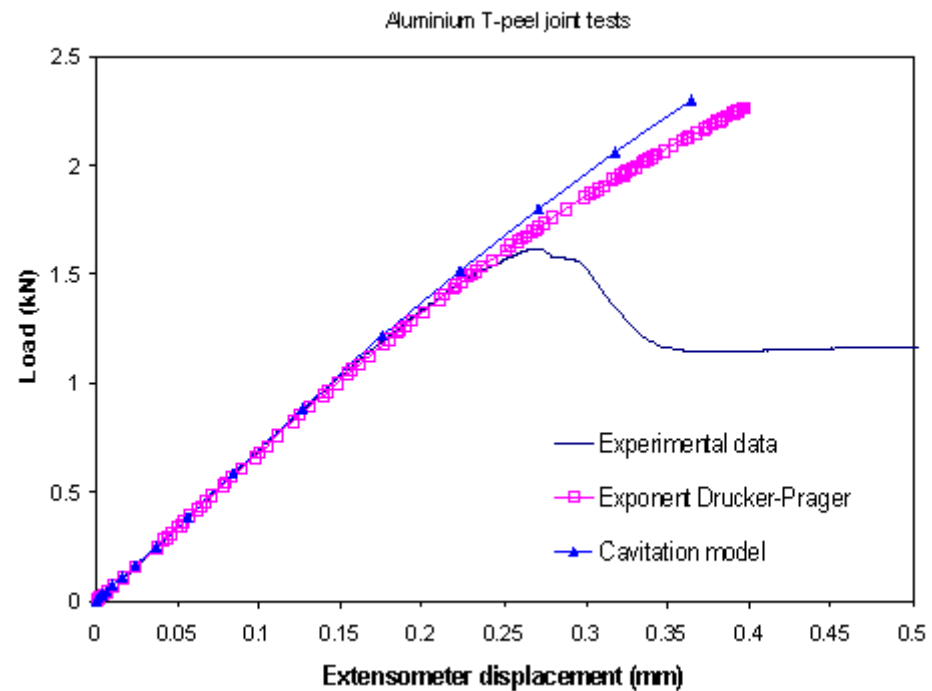
Cavitation

TPEEL JOINT

The tpeel joint was bonded with a rubber-toughened epoxy and tested. FEA predictions were obtained from analyses using two material models, exponent Drucker-Prager and cavitation model. The force-extension predictions are shown below.



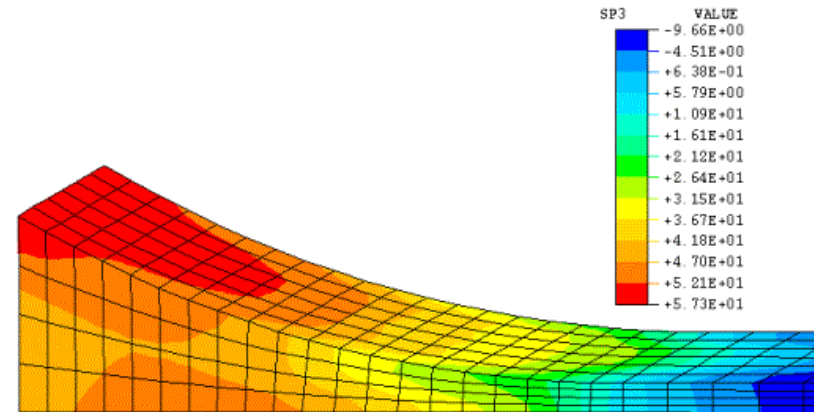
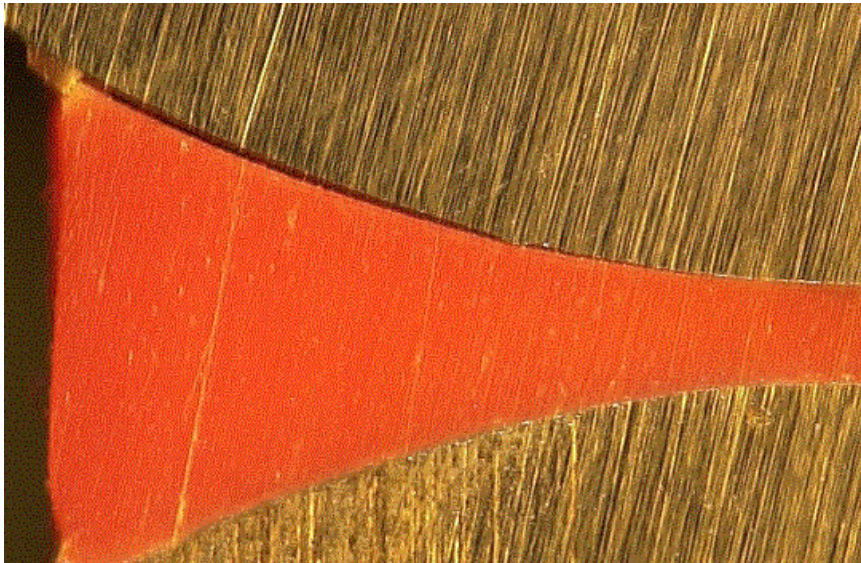
Tpeel joint schematic



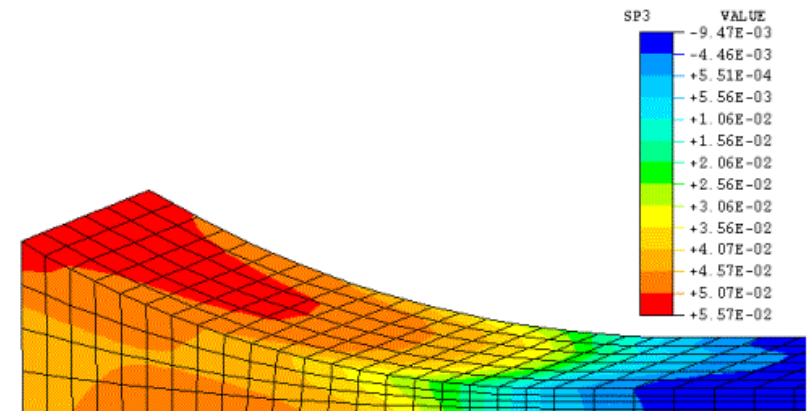
Tpeel joint experimental curve with force-extension predictions from two models

TPEEL JOINT - Predictions of stress using different models

Contour plots of maximum principal stress at an extension of 0.24 mm are shown below. This is the extension at which crack initiation is observed. The locations of the peak stresses in the contour plots tie-up with the experimentally observed crack initiation location.



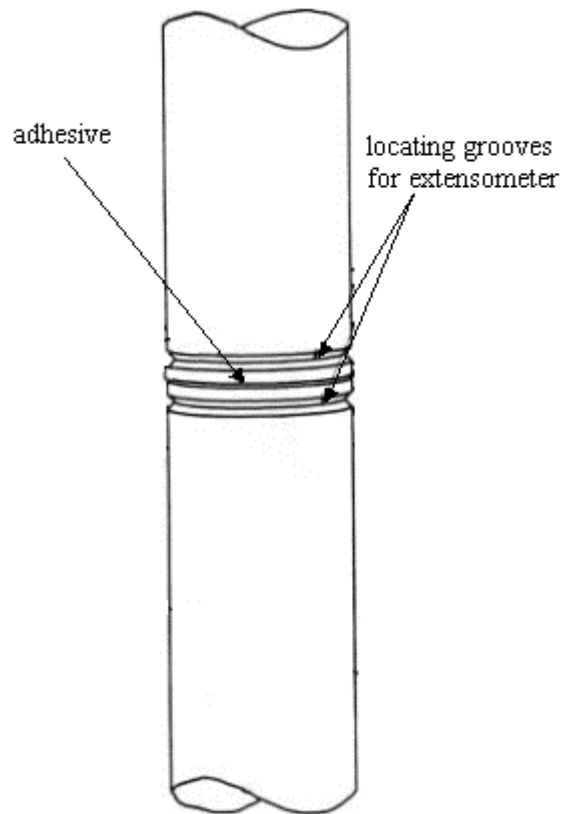
Exponent Drucker-Prager



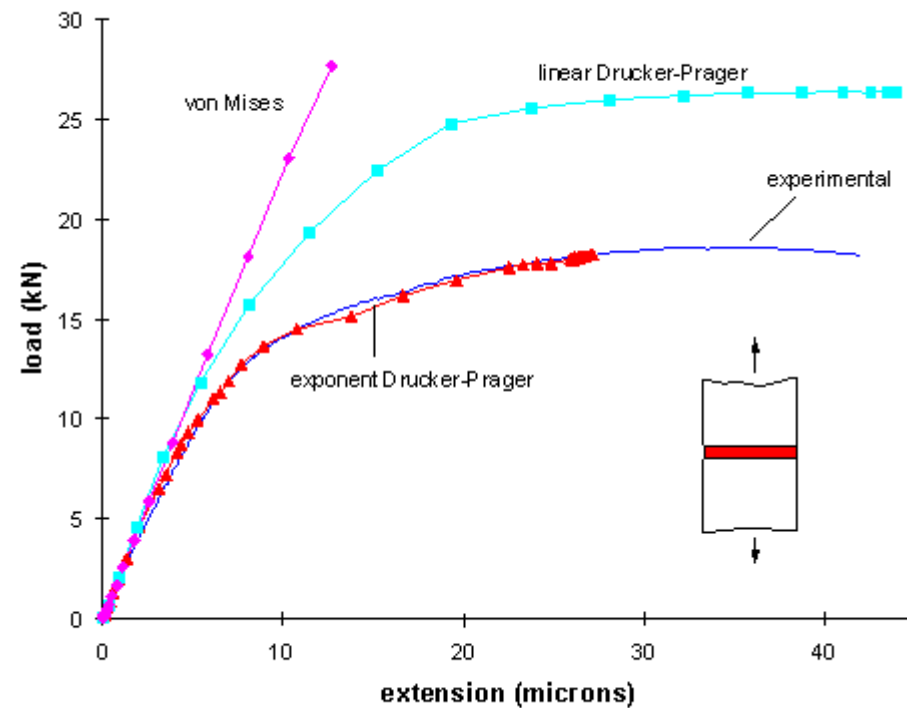
Cavitation

BUTT JOINT

The butt joint was bonded with a rubber-toughened epoxy and tested. FEA predictions were obtained from analyses using three material models, von Mises, linear Drucker-Prager and exponent Drucker-Prager. The elastic-plastic materials models differ in the way they account for the influence of hydrostatic stress on yield behaviour. The stress state in the adhesive in a butt-joint specimen loaded in tension contains a high component of hydrostatic stress. Hence experimental data from this test is useful for determining the applicability of different yield criteria for the adhesive to be assessed. The force-extension predictions are shown below.



Butt joint schematic



Butt joint experimental curve with force-extension predictions from three models

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THE PREDICTION OF FAILURE IN ADHESIVE JOINTS

A feasible approach to predicting when an adhesive joint will start to fail involves a finite element analysis to determine stress and strain distributions in the adhesive layer coupled with a criterion for failure based on achieving a critical level of some component of stress or strain sufficient to promote the initiation of a crack. In research studies on this subject, distributions of stress and strain components are calculated at loads or extensions where failure is believed to initiate in joint specimens of different geometry and dimensions. The aim is to explore whether there is a critical level of stress or strain that is common at failure initiation in these different types of joint. Stress and strain components that are commonly considered are :

- Maximum principal stress
- Hydrostatic stress
- Effective shear (Mises) stress
- Maximum principal strain
- Volumetric strain

The success of this work clearly relies on the ability to make accurate predictions of stress and strain distributions based on finite element analyses. Although, at present, no conclusions can be drawn regarding a generalised failure criterion, some relevant comments can be made.

With tough adhesives, there is a significant phase of plastic deformation in which the stress, for example in bulk specimens, changes little with strain. This would suggest that criteria based on a critical level of strain are likely to be more realistic than those based on a critical stress. However, in an adhesive joint, failure initiation often occurs at sites of high hydrostatic stress and at levels that are not generated in bulk specimens. It is possible therefore, that different criteria apply for failure in joints and bulk material.

It has been observed that failure strains can be high in tests on butt-joint specimens under torsion, even for adhesives that have relatively low tensile failure strains. The stress and strain states in this test are predominantly shear and dilatational components are minimal, which suggests that shear components are not major factors in determining failure initiation. In this context, it should be noted that high levels of maximum principal stress or strain can be generated under shear, and so criteria based on critical values of either of these components are unlikely to be valid unless the criterion includes an associated significant level of hydrostatic stress or strain.

Criteria for failure based on a critical level of hydrostatic stress or volumetric strain therefore deserve further investigation involving comparisons of measured and predicted deformation of joint specimens to failure.