15.1 Describe some composite materials that occur in nature. Describe their structure and properties.

Bone is a composite material consisting of a matrix, mineral hydroxy apatite and an organic fiber, collagen. The organization of the two constituents varies according to the functional needs. Cortical (dense) bone has a concentric ring like structure extending along the shaft of the bone. Cancellous (spongy) bone consists of a space-filling network of small beams of bone resulting in a porous, compliant material. Many long bones consist of hollow tubes of cortical bone with enlarged extremities filled with cancellous bone fibers oriented along the principal stress directions resulting from the major loading. As expected, cortical bone is stronger in the axial direction than in the transverse direction.

Wood is another versatile natural composite. It consists of crystalline cellulose fibers embedded in an amorphous matrix of lignin and hemicellulose. The matrix contributes to the stiffness of the wood as well as serves to protect the crystalline fibrous cellulose from moisture. Generally, the fiber volume fraction in wood is about 50%; and it has a high strength to weight ratio and toughness.

15.2 To promote wettability and avoid interfacial reactions, protective coatings are sometimes applied to fibers. Any improvement in the behavior of a composite will depend on the stability of the layer of coating. The maximum time t for the dissolution of this layer can be estimated by the diffusion distance

$$x = \sqrt{Dt},$$

where D is diffusivity of the matrix in the protective layer. Making an approximation that the matrix diffusion in the protective layer can be represented by self-diffusion, compute the time required for a 0.1-μm-thick protective on the fiber to be dissolved at T_m and $0.75T_m$, where T_m is the matrix melting point in kelvin. Assume a reasonable value D for self-diffusion in metals, taking into account the variation in D with temperature.

$$\text{Diffusion distance, } x = \sqrt{Dt},$$

where D is the matrix self-diffusion coefficient. For self-diffusion in metals, we can take

$$D = 10^{-13} m^2 s^{-1} \ \textit{near the m.p.,}\ T_m$$

$$\textit{and} \qquad D = 10^{-16} m^2 s^{-1} \quad \textit{near}\ 0.75\, T_m$$

Coating thickness = diffusion distance = 0.1 μm = 10^{-7} m

Then, near T_m, we have

$$t = \frac{x^2}{D} = \frac{\left(10^{-7}\right)^2}{10^{-13}}$$
$$t = 0.1\, s$$

At 0.75 T_m, we have

$$t = \frac{x^2}{D} = \frac{\left(10^{-7}\right)^2}{10^{-16}}$$
$$t = 100\, s$$

15.4 One can obtain two-dimensional isotropy in a fiber composite plate by having randomly oriented fibers in the plane of the plate. Show that the average in-plane modulus is

$$\overline{E}_\theta = \frac{\int_0^{\pi/2} E_\theta \, d\theta}{\int_0^{\pi/2} d\theta}.$$

Plot E_θ / E_{11} versus V_f for fiber reinforced composites with E_f / E_m = 1, 10, and 100.

From the discussion of anisotropy in fiber reinforced composites, see Sec. 15.5.4, after converting the compliances into engineering moduli, we can obtain the following expression for modulus at an angle θ:

$$\frac{1}{E_\theta} = \frac{\cos^4\theta}{E_{11}} + \frac{\sin^4\theta}{E_{22}} + \left(\frac{1}{G_{12}} - \frac{2\,\nu_{12}}{E_{11}}\right)\cos^2\theta\ \sin^2\theta$$

The average in-plane modulus for randomly oriented fibers in the plane of the plate is found by averaging the modulus value E_θ given in the above equation over all values of the angle θ. Thus

$$\overline{E_\theta} = \frac{\int_0^{\pi/2} E\theta \, d\theta}{\int_0^{\pi/2} d\theta}$$

We are given E_f / E_m = 1,10, 100. We need E_{11}, E_{22}, ν_{12} and G_{12} of the composite. We can safely take $\nu_f = \nu_m = \nu_{12} = 0.3$. Small variations in ν will not be significant for the final result. For the other three moduli we can write

$$E_{11} = E_f V_f + E_m V_m$$

$$\frac{1}{E_{22}} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

$$\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m}$$

Take E_m = 70 GPa (Al metal matrix). Then $G_m = E_m/2\,(1+\nu)$. Similarly for fiber (E_f/E_m = 1, 10, 100). Putting in these values in the above expressions for different V_f, one can get curves $\overline{E}_\theta/E_{11}$ vs. V_f for E_f/E_m = 1, 10, 100.

15.5 Consider a carbon fiber reinforced epoxy composite. The fibers are continuous, unidirectional aligned and 60% by volume. The tensile strength of carbon strength of carbon fibers is 3 GPa, and the Young's modulus is 250 GPa. The tensile strength of the epoxy matrix is 50 MPa, and its Young's modulus is 3 GPa. Compute the Young's modulus and the tensile strength of the composite in the longitudinal direction.

$$E_c = V_f E_f + (1 - V_f) E_m$$

$$E_c = 0.6 \text{ x } 250 + (1 - 0.6) \text{ x } 3$$

$$E_c = 151.2 \text{ GPa}$$

$$\sigma_c = \sigma_f V_f + \sigma_m (1 - V_f)$$
$$\sigma_c = 3\,(0.6) + 50\ \ 10^{-3}(1 - 0.6)$$
$$\sigma_c = 1.82 \quad GPa$$

15.6 A steel wire of diameter 1.25 mm has an aluminum coating such that the composite wire has a diameter of 2. 50 mm. Some other pertinent data are as follows:

Property	Steel	Aluminum
Elastic modulus E	210 GPa	70 GPa
Yield Stress σ_y	200 MPa	70 MPa
Poisson ratio ν	0.3	0.3
Coefficient of thermal expansion (linear)	$11 \times 10^{-6} K^{-1}$	$23 \times 10^{-6} K^{-1}$

(a) If the composite wire is loaded in tension, which of the two components will yield first? Why?

(b) What tensile load can the composite wire support without undergoing plastic strain?

(c) What is the elastic modulus of the composite wire?

(d) What is the coefficient of thermal expansion of the composite wire?

(a) For tensile loading of the wire, we have

$$e_c = e_{steel} = e_{A\ell}$$

Thus, the component that presents a lower yield strain will yield first. The yield strains of he two components can be found by using the relation, $e_y = \frac{\sigma_y}{E}$. Thus,

$$e_{y\ steel} = \frac{200}{210 \times 10^3} = 9.5 \times 10^{-4}$$
$$\sigma_{yA\ell} = \frac{70}{70 \times 10^3} = 10.0 \times 10^{-4}$$

Thus, <u>steel will yield first.</u>

(b) $V_{steel} = \frac{(1.25)^2}{(2.50)^2} = 0.25$ $\qquad V_{A\ell} = 1 - V_{steel} = 0.75$

Using the rule of mixtures, we can obtain the yield strength of the composite as the strength corresponding to a strain of 9.5 x 10^{-4} (see part (a) above).

$$\sigma_{yc} = 200 \times 0.25 + 70 \times 10^3 \times 0.95 \times 10^{-3} \times 0.75\ MPa$$
$$= 50 + 49.87$$
$$= 99.87\ MPa$$

The corresponding load before the onset of plastic deformation is given by

$$P_{cy} = \sigma_{yc} \times A = 99.87 \times \frac{\pi}{4} \times (2.50)^2 \times 10^{-6}$$

$$= 490.3\ MN$$

$$(c) \quad E_c = E_{A\ell} V_{A\ell} + E_{steel} V_{steel}$$

$$= 70 \times 0.75 + 210 \times 0.25$$

$$E_c = 105 \quad GPa$$

$$(d) \quad \alpha_{cL} = \frac{E_{A\ell}\alpha_{A\ell}V_{A\ell} + E_{steel}\alpha_{steel}V_{steel}}{E_{A\ell}V_{A\ell} + E_{steel}V_{steel}}$$

$$= \frac{210 \times 11 \times 10^{-6} \times 0.25 + 70 \times 23 \times 10^{-6} \times 0.75}{105}$$

$$\alpha_{cL} = 17 \times 10^{-6} K^{-1}$$

15.7 A boron/aluminum composite has the following characteristics:

Unidirectional reinforcement

Fiber volume fraction V_f =50%

Fiber length l = 0.1 m

Fiber diameter d = 100 μm

Fiber ultimate stress σ_{fu} = 3 GPa

Fiber strain corresponding to σ_{fu}, e_{fu} = 0.75% (uniform elongation)

Fiber Young's modulus E_f = 415 GPA

Matrix shear yield stress τ_{ym} = 75 MPa

Matrix stress at e = e_{fu}, $\sigma'_m |_{efu}$ = 93 MPa

Matrix ultimate stress $\sigma_{m\mu}$ = 200 MPa

Compute:

(a) The critical fiber length ℓ_c for the load transfer.

(b) The ultimate tensile strength of the composite.

(c) V_{min} and V_{crit} for the composite system.

(a) $$\ell_c = \frac{\sigma_{fu} \times d}{2\,\tau_{ym}}$$

$$\sigma_{fu} = 3\,GPa,\ \tau_{ym} = 75\,MPa, \quad d = 100\ \mu\text{m}$$

$$\ell_c = \frac{3\times10^{9}\times100\times10^{-6}}{2\times75\times10^{6}}\,m$$
$$\ell_c = 2\times10^{-3}$$

(b) $$\sigma_{cu} = \sigma_{fu}V_f + \sigma_m(1 - V_f)$$

σ'_m =93 MPa

$$\sigma_{cu} = 3\times10^{9}\times0.5 + 93\times10^{6}\times0.5$$
$$\sigma_{cu} = 1.546\,GPa$$

(c)

$$V_{\min} = \frac{\sigma_m - \sigma'_m}{\sigma_{fu} + \sigma_m - \sigma'_m}$$
$$= \frac{200-93}{3000+200-93} = \frac{107}{3107} = 3.4\%$$

$$V_{crit} = \frac{\sigma_{mn} - \sigma'_m}{\sigma_{fu} - \sigma'_m}$$
$$= \frac{200-93}{3000-93} = \frac{107}{2907} = 3.6\%$$

15.8 Determine Young's modulus for a steel fiber/aluminum matrix composite material, parallel and perpendicular to the fiber direction. The reinforcement steel has E 210 and $V_f = 0.3$, and the aluminum matrix has E = 70 GPa and $V_m = 0.7$.

Parallel

$E_c = E_m V_m + E_f V_f$

$E_c = 210 \times 0.3 + 70 \times 0.7 = 112$ GPa

Perpendicular

$$\frac{1}{E_c} = \frac{V_f}{E_f} + \frac{V_m}{E_f}$$

$$\frac{1}{E_c} = \frac{0.3}{210} + \frac{0.7}{70}$$

$$E_c = 87.5\,GPa$$

15.10 A glass fiber reinforced polymer matrix composite has the following characteristics:

fiber maximum strength = 2 GPa
Interfacial shear strength = 50 MPa
Fiber radius = 10 μm

Compute the critical fiber length for this system.

Fiber diameter, d = 2 r = 20 μm

$$\frac{l_c}{d} = \frac{\sigma_b}{2\tau_i}$$

$$l_c = \frac{2\times10^9}{2\times50\times10^6}\cdot 20\times10^{-6}\,m$$

$$= 0.4\;10^{-3}\,m = 0.4mm$$

15.13 List some nonstructural applications of composite materials.

The following is a list of nonstructural applications of fiber composites:

i. Carbon-carbon composites are used for ablative surface applications in rockets and other space vehicles.

ii. Carbon and glass fiber reinforced polymer composites are used for corrosion resistance applications, e.g., storage and transport of corrosive chemicals. Of course, there is a structural requirement to these composites as well.

iii. Carbon fibers combine excellent mechanical properties with very low x-ray absorption characteristics. This has resulted in their use in X-ray analysis and treatment devices.

iv. Carbon fibers are good electrical conductors. Thus, carbon fibers (short) in a polymer matrix are used in places where static charge removal is required, e.g., in high-speed printers.

v. Carbon-carbon composites are being used as brake-liners in aircraft wheels where temperatures can get very high for very short duration. This application also requires structured characteristics in addition to the efficient heat dissipation.

vi. Metal matrix composites consisting of Nb_3Sn of Nb-Ta filaments distributed in a Cu matrix are used commercially in superconducting magnets.

vii. Unidirectionally solidified eutectic composites show very interesting electrical, optical, thermal, and magnetic properties. For example, InSb – NiSb eutectic composite is used as a magneto resistive device in the following applications: a) measurement of magnetic fields and field dependent quantities, b) contactless variable resistor and potentiometer, c) modulation of small dc currents and voltages, d) contactless control.

15.14 Bone is an excellent example of a natural composite. Describe the various components that make this composite.

Bone is a natural composite material consisting of a matrix, mineral hydroxyapatite and an organic fiber, collagen. The organization of the two constituents varies according to the functional needs of the bone. Typically, bone has an outer compact part and a soft or spongy part in the interior. In compact or cortical bone, soft collagen fibers are distributed in the hydroxyapatite mineral. The interior of the bone is made of spongy bone or cancellous bone which has a higher degree of porosity than the cortical bone. Compact bone is denser compared to spongy bone, but it still has a large amount of pores and pathways for blood vessels, lymphatic vessels, and nerves. Besides the compact and spongy bone types that make long bones such as the femur, there are flat bones in our skeletal system. For example, the skull bone has sandwich-like structure; it consists of dense cortical layers on the external surface with a thin cancellous structure between them.

15.15 Describe mechanical characteristics of bone and its ability to repair.

Bone is comprised of hydroxyapatite and mature bone cells called osteocytes. While the compact bone is the primary load carrier, the spongy bone helps to reinforce the compact bone. The direction of the collagen fibers is the main factor in the strength of bone. Reductions in the Young’s modulus can be as high as 60% when comparing the modulus along the fiber axis and the transverse axis. Microcracks can also play a role. In young healthy bone, microcracking can work as a method of stress relief during loading and the damage is eventually repaired by bone remodeling. However, as person ages, the collagen quality and the decreasing ability of the body to regenerate bone causes not only an increase in cracking but also sharper crack tips which can result in the decrease in fracture toughness.

15.16 A glass fiber reinforced polypropylene composite has 65% by volume of fibers unidirectionally aligned.
(**a)** Compute the weight fraction of glass fibers in this composite.
(**b)** What is the density of this composite?
(**c)** Compute the Young's modulus of the composite in a direction along the fiber and perpendicular to it.

15.17 A composite is made of unidirectional carbon fibers embedded in an epoxy matrix.
(**a)** Plot the Young's modulus as a function of the volume fraction of fibers parallel and perpendicular to the fiber direction.
(**b)** If the continuous fibers are replaced by chopped fibers with random orientation, where do you expect that the elastic properties would lie? Indicate the plot, given E_f = 390 GPa; E_m = 3 GPa.
(**c)** Name three applications for this composite

15.18 A carbon fiber/epoxy composite has 70% fibers. Determine the elastic modulus of composite along the perpendicular to fiber direction. Compute the density of this composite.

Given:
Density of carbon fibers = 1.3 g/cm^3,
Density of epoxy = 1.1 g/cm^3,
E_c = 270 GPa,
E_e = 4 GPa.

15.19 A composite is made with discontinuous alumina fibers in an aluminum matrix. The fibers have a diameter of 10 μm. If the volume fraction of fibers is 60%, what is the required length if we want the strength of composite to be equal to 50% of the same composite reinforced with continuous fibers?
Given:
Fiber E = 380 GPa,
Fiber strength = 1.7 GPa,
Matrix strength = 200 MPa.

15.20 A unidirectional reinforced composite has an aluminum matrix and steel fibers (40 vol.%). Determine its strength.

Given:
Aluminum: $\sigma = 100 + \varepsilon^{0.3}$ (in MPa);
Steel: σ = 2.5 GPa.

15.21 Name and describe five applications of composites in sports equipment.

Specify components of the composite.

15.22 Metals can be joined by welding, riveting, and bolting. Is it possible to apply these processes to polymer matrix composites? Explain why, and present alternative means of joining composites.

15.23 Give specific examples for the four different types of composites, and explain briefly the components involved (e.g. particle reinforced; short fiber reinforced, etc.).

15.24 Give an example of a composite. Compare its mechanical properties with those of the reinforcements and matrix materials, respectively, and explain its advantages.

15.25 Consider a steel and rayon-cord reinforced elastomer (rubber) with elastic moduli as shown in the table below.

Material	*E* (MPa)
Rubber	13
Rayon	6,000
Steel	210,000

Calculate the elastic modulus for the two different composites, if the volume fraction of fiber (rayon or steel) is 0.3.

15.26 Consider an elastomer matrix composite reinforced with steel cord (1 mm length and 0.5 mm diameter). What is the minimum fracture stress for the steel cord, if the interfacial shear strength is 20 MPa?

15.27 Consider an aluminum−titanium laminated composite. Calculate the longitudinal and transverse Young’s modulus of this composite if the volume fractions of the two metals are equal. Given E(Ti) = 116 GPa, E(Al) = 70 GPa.