Flows in thin Film Coating

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December 15, 2011

Outline

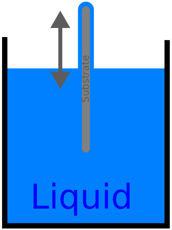
Why Coating?



- Bestow visual effects to material
- Bestow function to material
- Bestow protection/barrier to material

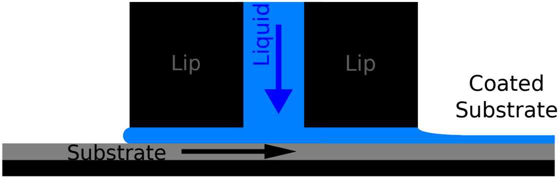
Different Coating Procedures

Self-Metered Coating



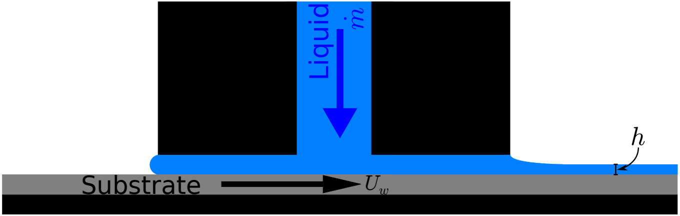
Dip Coating:
Coating thickness dependent on process

Pre-Metered Coating



Slot Coating in Bead Mode:
Coating thickness independent on process

Advantage of Pre-Metered Coating



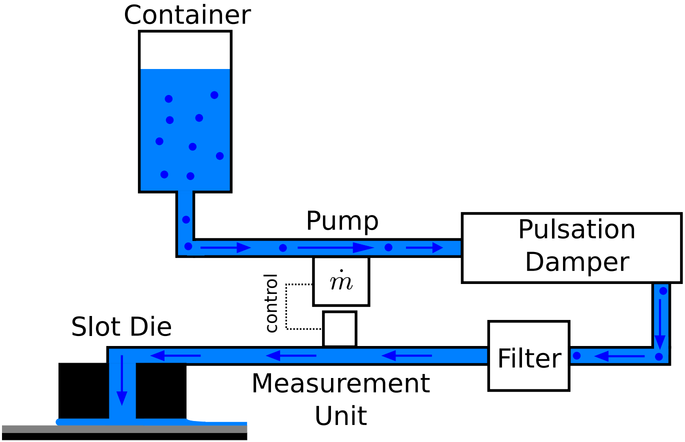
Mass Conservation

$$\dot{m} = h \cdot U_w \cdot \rho$$

Consequences for Slot Coating

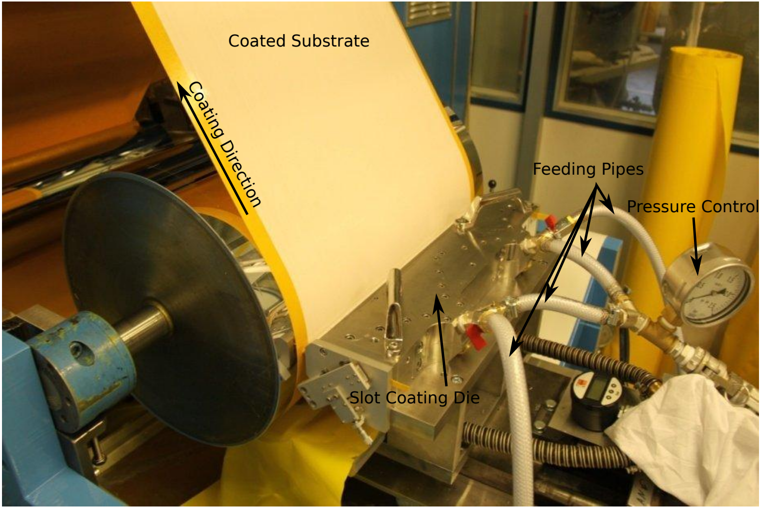
- Coating fluid directly applied to substrate
- No excess of coating fluid needed

Components of a Slot Coating System

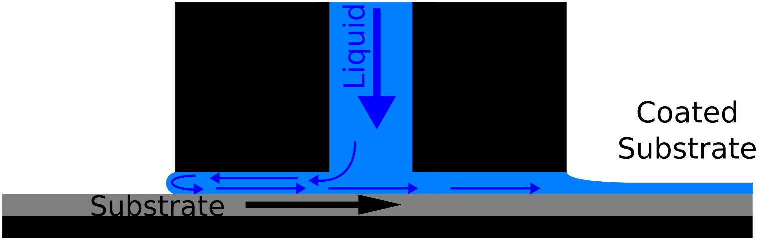


- Good feeding process is essential for perfect coating
- Slot coating is a closed system

Example: FMP Slot Coating Machinery



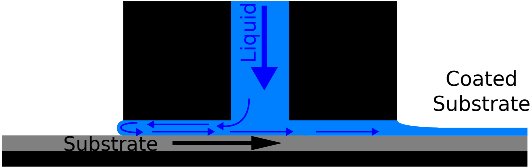
Eye on the Flow of the Fluid



We want to know . . .

- how to obtain perfect coating quality
- how to configure the machinery *before* coating

Parameters in Slot Coating in Bead Mode



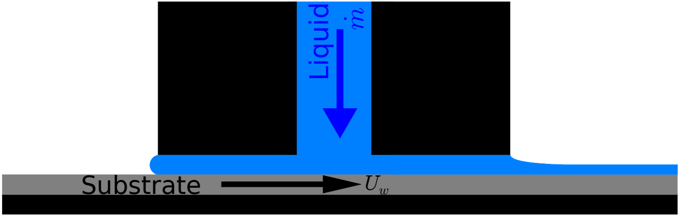
Fixed Parameters

- Properties of the coating liquid (density, viscosity, surface tension)
- Coating height h
- Length of lip of slot die l

Variable Parameters

- Distance between substrate and slot die D
- (Substrate velocity U_w)
- (Mass flow rate $\dot{m}$)

Roadmap



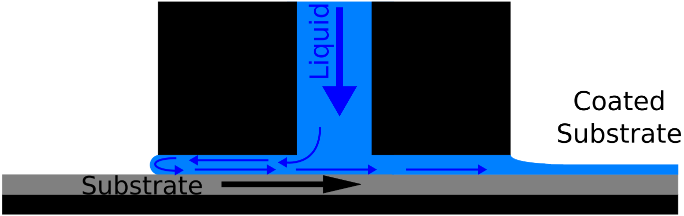
Goal

Express coating with respect to variable parameters

Way to the Goal

Investigate coating flow analytically

Characterization of the Coating Flow



Coating Flow consists of ...

- Only plane flows
- Flows between two plates
- Fluid transported by substrate
- Fluid influenced by pressure

Fundamental Formulas

Starting Formulas

- Continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho U_i)}{\partial x_i} = 0$$

- Navier-Stokes equations ($j = 1, 2$):

$$\rho \left[\frac{\partial U_j}{\partial t} + U_i \frac{\partial U_j}{\partial x_i} \right] = -\frac{\partial P}{\partial x_j} + \frac{\partial}{\partial x_i} \left[\mu \left(\frac{\partial U_j}{\partial x_i} - \frac{\partial U_i}{\partial x_j} \right) + \frac{2}{3} \mu \delta_{ij} \frac{\partial U_k}{\partial x_k} \right] + \rho g_j$$

Assumptions

- Restrictions to Newtonian fluids
- Constant density ρ and viscosity μ
- Consider 2D Navier-Stokes equations of stationary, fully developed flows

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- Navier-Stokes equations ($j = 1, 2$):

$$0 = -\frac{\partial P}{\partial x_j} + \mu \frac{\partial^2 U_j}{\partial x_i^2} + \rho g_j$$

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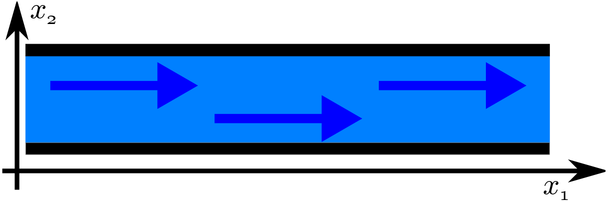
Plane Fluid Flows between Plates



- Continuity equation: $\dfrac{\partial U_1}{\partial x_1} = \dfrac{\partial U_2}{\partial x_2} = 0 \quad \Rightarrow U_2 = 0$
- Navier-Stokes equation in x_2 direction: $-\dfrac{\partial P}{\partial x_2} + \rho g_2 = 0$
- Pressure field has the form: $P = \rho g_2 x_2 + \Pi(x_1)$
- $\Pi(x_1)$ is externally imposed pressure (e.g. by a pump)
- Consequently: $\dfrac{\partial P}{\partial x_1} = \dfrac{\partial \Pi}{\partial x_1}$

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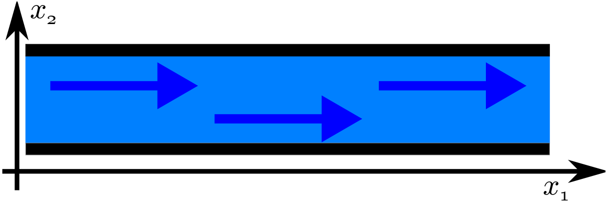

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Intermezzo

Solution of the velocity field ...

- in x_2 direction: $U_2 = 0$
- in x_1 direction: $U_1 = ?$

Previous Formulas updated

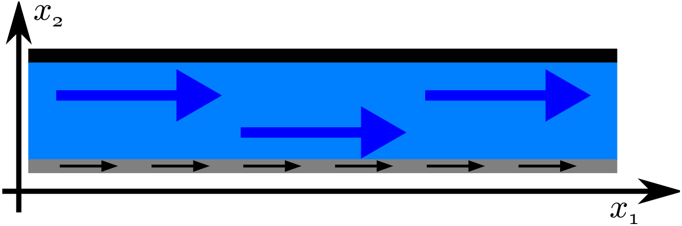
- Continuity Equation:

$$\frac{\partial U_i}{\partial x_i} = 0$$

- Navier-Stokes Equation in x_1 direction:

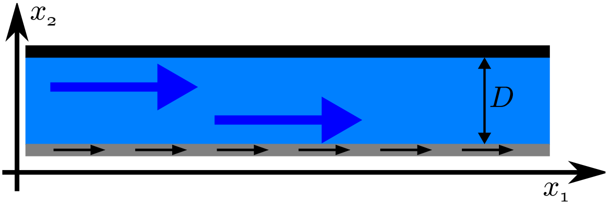
$$-\frac{\mathrm{d}\Pi}{\mathrm{d}x_1} + \mu \frac{\mathrm{d}^2 U_1}{\mathrm{d}x_2^2} + \rho g_1 = 0$$

Couette Flow



- Impose constant external pressure: $\dfrac{\mathrm{d}\Pi}{\mathrm{d}x_1} = 0$
- Gravitational forces only in x_2 direction $\Rightarrow g_1 = 0$
- Couette flow is characterized by $\mu \dfrac{\mathrm{d}^2 U_1}{\mathrm{d}x_2^2} = 0$

Velocity Profile of a Couette Flow



$$\mu \frac{\mathrm{d}^2 U_1}{\mathrm{d}x_2^2} = 0 \quad \rightsquigarrow \quad U_{1,\mathrm{c}} = C_1 x_2 + C_2$$

Solution of the Velocity Profile

- Boundary Conditions:
  - $x_2 = 0 : U_{1,\mathrm{c}} = U_w \quad \Rightarrow \quad C_2 = U_w$
  - $x_2 = D : U_{1,\mathrm{c}} = 0 \quad \Rightarrow \quad C_1 = -U_w/D$
- Solution: $U_{1,\mathrm{c}} = \frac{U_w}{D}(D - x_2)$ for $0 \leq x_2 \leq D$

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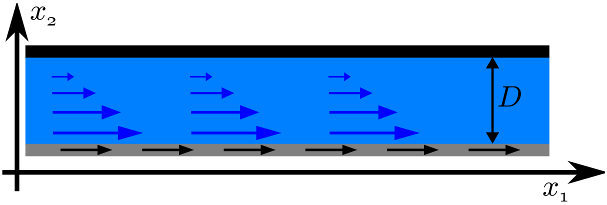

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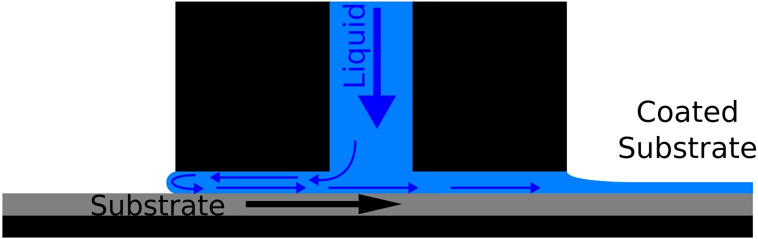
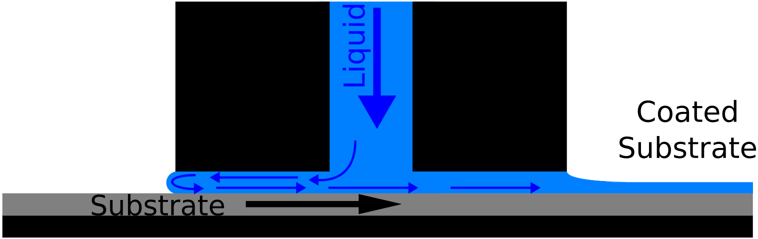
Stopover



So far we . . .

- incorporated plane flows
- discussed Couette flows
- did not consider pressure driven flows

Stopover



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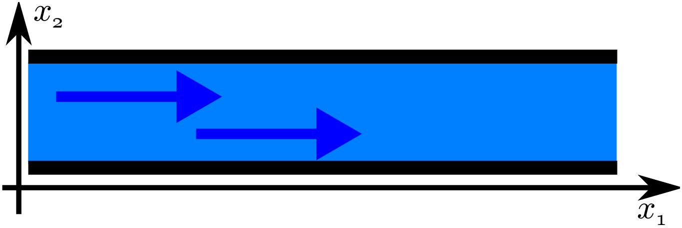
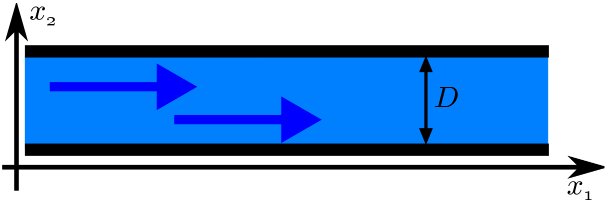
Hagen-Poiseuille Flow



- Navier-Stokes equation: $-\frac{\mathrm{d}\Pi}{\mathrm{d}x_1} + \mu\frac{\mathrm{d}^2 U_1}{\mathrm{d}x_2^2} + \underbrace{\rho g_1}_{=0} = 0$
- Determine $U_{1,\mathrm{hp}}$ by integration:

$$\mu\frac{\mathrm{d}^2 U_1}{\mathrm{d}x_2^2} = \frac{\mathrm{d}\Pi}{\mathrm{d}x_1} \quad \leadsto \quad U_{1,\mathrm{hp}} = \frac{1}{2\mu}\left(\frac{\mathrm{d}\Pi}{\mathrm{d}x_1}\right)x_2^2 + C_1 x_2 + C_2$$

Velocity Profile of a Hagen-Poiseuille Flow



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  - $x_2 = D : U_1 = 0 \quad \Rightarrow \quad \frac{1}{2\mu}\left(\frac{\mathrm{d}\Pi}{\mathrm{d}x_1}\right) D^2 - C_1 D = 0$
- Solution: $U_{1,\mathrm{hp}} = \frac{1}{2\mu}\left(\frac{\mathrm{d}\Pi}{\mathrm{d}x_1}\right) x_2 \left(x_2 - D\right)$ for $0 \leq x_2 \leq D$

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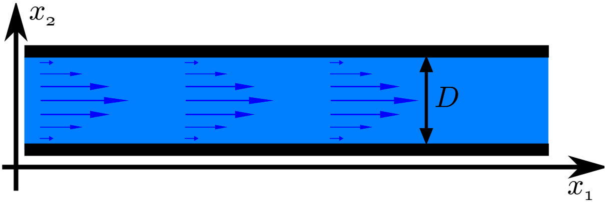

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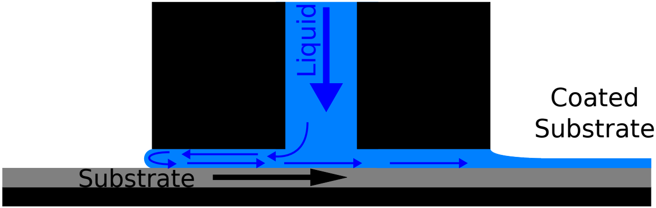
Assembling the Solution of the Coating Velocity Field



Obtaining the Coating Velocity Field

- Coating flow is superposed by Couette and Hagen-Poiseuille flows
- Summing up $U_{1,\mathrm{c}}$ and $U_{1,\mathrm{hp}}$ yields coating flow

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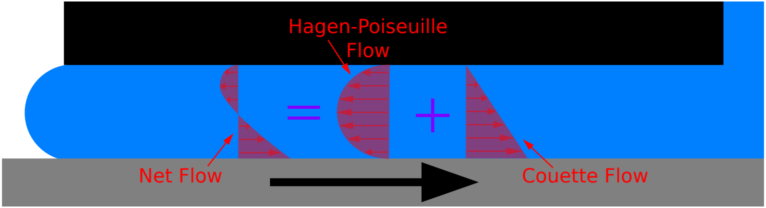

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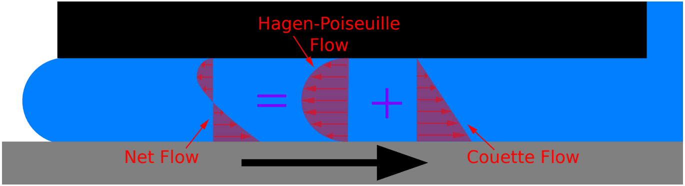
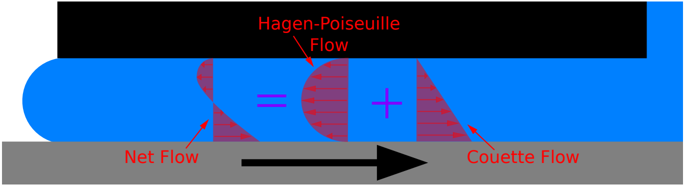


Superposition of $U_{1,\mathrm{c}}$ and $U_{1,\mathrm{hp}}$

- Couette flow: $U_{1,\mathrm{c}} = \frac{U_w}{D}\left(D - x_2\right)$
- Hagen-Poiseuille flow: $U_{1,\mathrm{hp}} = \frac{1}{2\mu}\left(\frac{\mathrm{d}\Pi}{\mathrm{d}x_1}\right) x_2\left(x_2 - D\right)$
- Coating flow: $U_1 = \left(x_2 - D\right)\cdot\left(\frac{x_2}{2\mu}\frac{\mathrm{d}\Pi}{\mathrm{d}x_1} - \frac{U_w}{D}\right)$

Velocity Profile of a Coating Flow



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Intermezzo

Solution of the velocity field ...

- in x_2 direction: $U_2 = 0$
- in x_1 direction: $U_1 = (x_2 - D) \cdot \left(\frac{x_2}{2\mu} \frac{\mathrm{d}\Pi}{\mathrm{d}x_1} - \frac{U_w}{D} \right)$

Connection to mass flow rate $\dot{m}$

- Initially defined: $\dot{m} = h \cdot U_w \cdot \rho$
- Alternative: $\dot{m} = \rho \int_0^D U_1(x_2)\,\mathrm{d}x_2$

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Computation of the Mass Flow Rate

Thin Film Velocity Profile

$$U_1 = (x_2 - D) \cdot \left(\frac{x_2}{2\mu} \frac{\mathrm{d}\Pi}{\mathrm{d}x_1} - \frac{U_w}{D} \right)$$

Mass Flow Rate

$$\begin{aligned} \dot{m} &= \rho \int_0^D U_1(x_2)\, \mathrm{d}x_2 \\ &= \rho \cdot \left[-\frac{D^3}{12\mu} \left(\frac{\mathrm{d}\Pi}{\mathrm{d}x_1} \right) + \frac{U_w D}{2} \right] \end{aligned}$$

How do we obtain $\frac{\mathrm{d}\Pi}{\mathrm{d}x_1}$?

Computation of the Mass Flow Rate

Thin Film Velocity Profile

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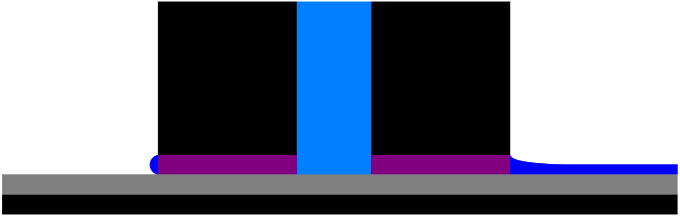
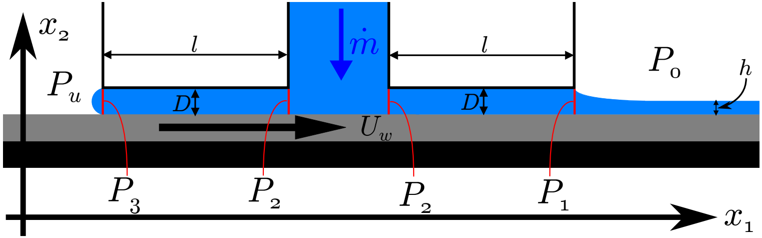
Properties of the Pressure



Assumptions

- Change in pressure is linear under the lips
- No change in pressure anywhere else

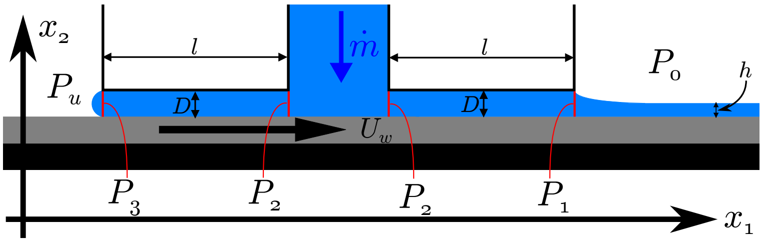
Approximation of the Pressure



Linear Approximation

- $$\frac{\mathrm{d}\Pi}{\mathrm{d}x_1} \approx \frac{\Delta P}{\Delta x_1} = \frac{P_a - P_b}{l}$$
- $$\dot{m} = \rho \cdot \left[-\frac{D^3}{12\mu} \left(\frac{\Delta P}{\Delta x_1} \right) + \frac{U_w D}{2} \right] \quad \Leftrightarrow \quad \frac{\Delta P}{\Delta x_1} = \left(\frac{\rho U_w D}{2} - \dot{m} \right) \frac{12\mu}{\rho D^3}$$

Approximation of the Pressure



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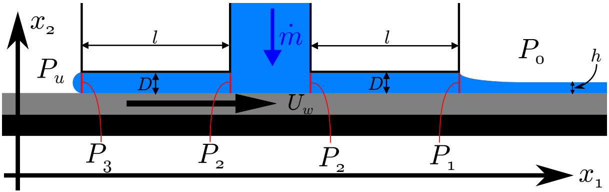
Approximation of the Pressure below the Lips



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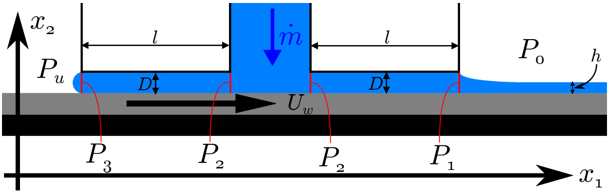
Pressure below front Lip

- $\dfrac{\Delta P}{\Delta x_1} l = P_{12} = P_1 - P_2 = \left(\dfrac{\rho U_w D}{2} - \dot{m}\right)\dfrac{12\mu}{\rho D^3} l$
- Recall: $\dot{m} = h \cdot U_w \cdot \rho$
- $P_{12} = \left(1 - \dfrac{2h}{D}\right)\left(\dfrac{6\mu U_w}{d^2} l\right)$

Approximation of the Pressure below the Lips



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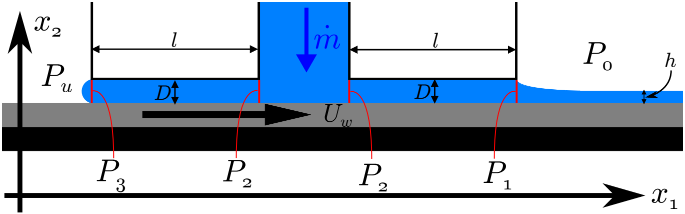
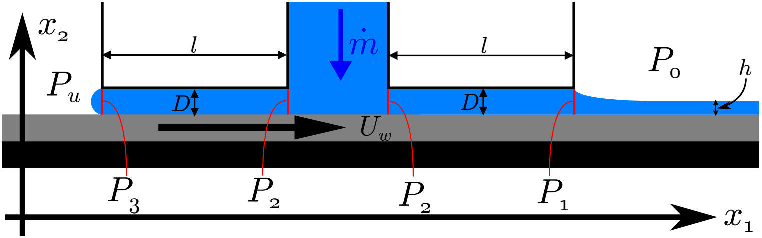


$$\frac{\Delta P}{\Delta x_1} = \left(\frac{\rho U_w D}{2} - \dot{m} \right) \frac{12\mu}{\rho D^3}$$

Pressure below back Lip

- Assumption: Net mass flow rate is zero $\Rightarrow \dot{m} = 0$
- $\dfrac{\Delta P}{\Delta x_1} l = P_{23} = P_2 - P_3 = \dfrac{6\mu U_w}{D^2} l$

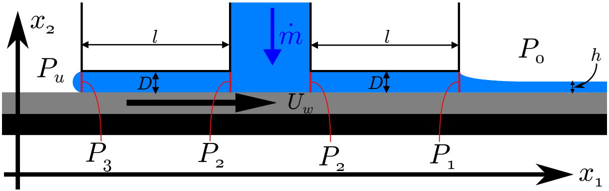
Approximation of the Pressure in the Menisci



Menisci Pressure Difference

- Front meniscus: $P_{01} = P_0 - P_1 = 1.34 \cdot (Ca)^{\frac{2}{3}} \cdot \frac{\sigma}{h}$ (by Rushack)
- Back meniscus: $P_{3u} = P_3 - P_u \leq \frac{2\sigma}{D}$ (free surface)

Pressure Distribution



Implications

- ΔP is obtained by superposition of the pressure differences:

$$\begin{aligned} \Delta P &= P_0 - P_u = P_{3u} + P_{23} + P_{12} + P_{01} \\ &\lesseqgtr 1.34 \cdot (Ca)^{\frac{2}{3}} \cdot \frac{\sigma}{h} + \left(2 - \frac{2h}{D}\right)\left(6\mu U_w \frac{l}{D^2}\right) \pm \frac{2\sigma}{D} \end{aligned}$$

- Above formula reflects both limiting cases of a convex and concave meniscus

Stopover

What have we achieved?

- $\Delta P = f(\underbrace{\mu, \sigma, \rho,}_{\text{fluid}} \underbrace{l,}_{\text{slot die}} \underbrace{D, h, U_w}_{\text{process}})$
- Stable coating is dependent on the back meniscus

Why do we need ΔP, if stable coating is dependent on the meniscus?

Stopover

What have we achieved?

- $\Delta P = f(\underbrace{\mu, \sigma, \rho,}_{\text{fluid}} \underbrace{l,}_{\text{slot die}} \underbrace{D, h, U_w}_{\text{process}})$
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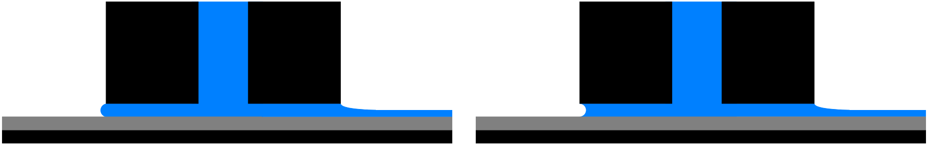
Why do we need ΔP, if stable coating is dependent on the meniscus?

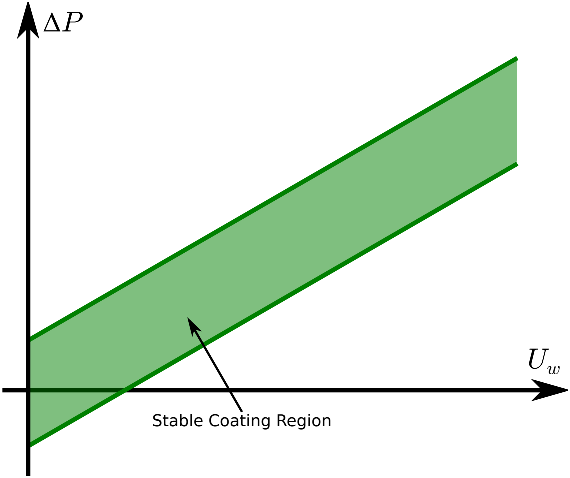
Limitations by the Surface Tension



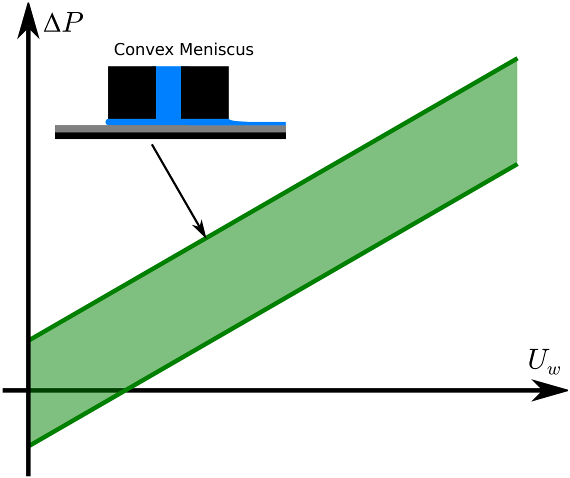
Stable Minisci

- $\Delta P = 1.34 \cdot (Ca)^{\frac{2}{3}} \cdot \frac{\sigma}{h} + \left(2 - \frac{2h}{D}\right)\left(6\mu U_w \frac{l}{D^2}\right) \pm \frac{2\sigma}{D}$
- Subtracting convex from concave limit: $\frac{4\sigma}{D}$
- We obtained stable coating 'width', i.e. coating window

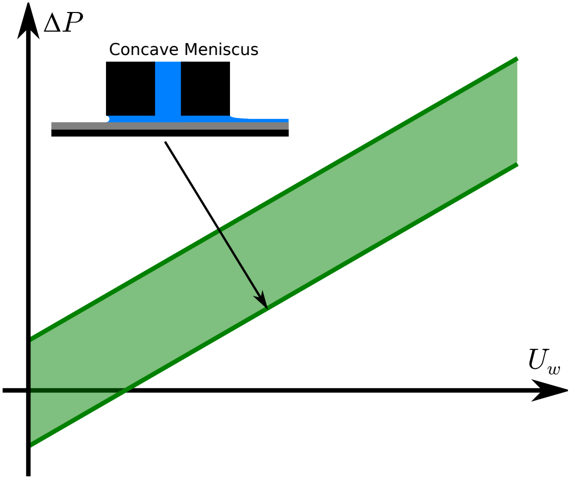
Coating Window



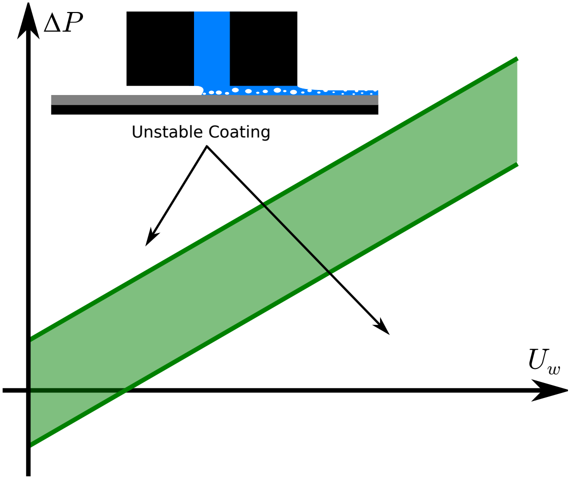
Coating Window



Coating Window

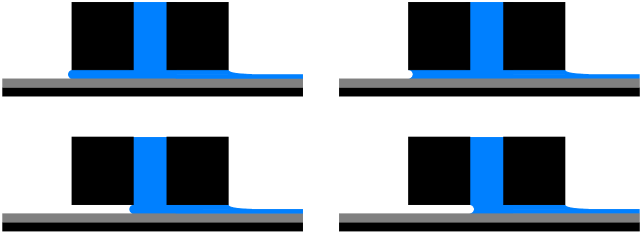


Coating Window

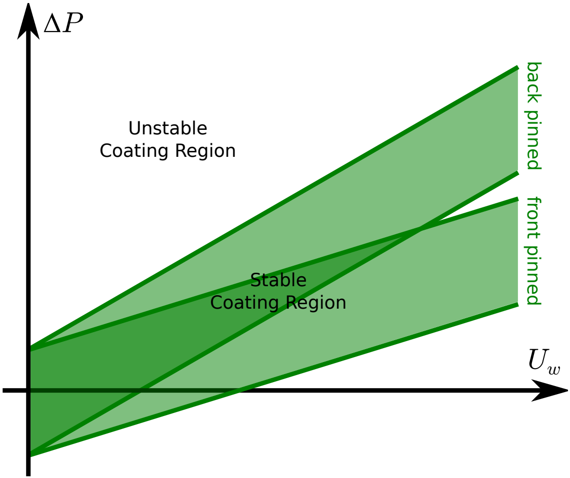


Stable back Menisci

	Convex	Concave
Back Pinned		
Front Pinned		

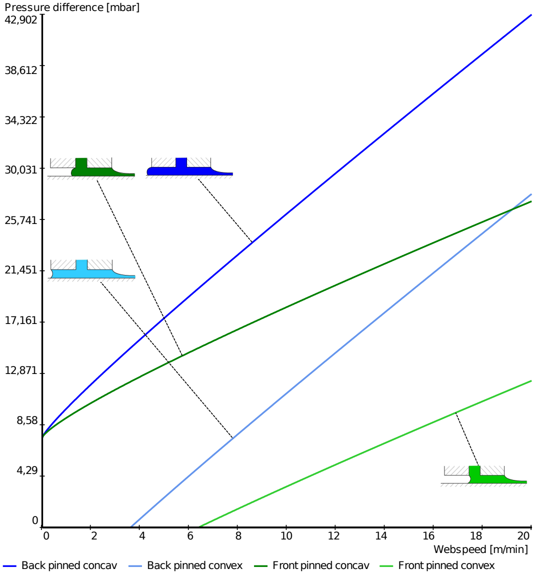


Stable Coating Regions



It is possible to determine U_w and ΔP such that coating is stable

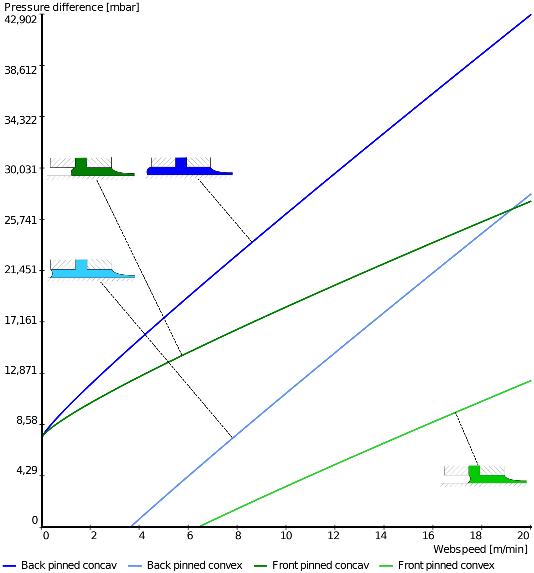
Practical Example: Nano Coating



Objective

Specs	Value
U_w	6 m/min
μ	10 mPas
σ	30 mN/m
h	10 μm
D	80 μm
l	500 μm

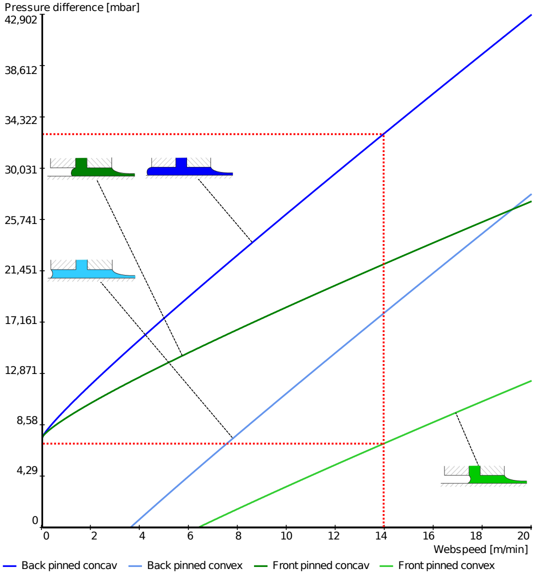
Practical Example: Nano Coating



Objective

Specs	Value
U_w	14 m/min
μ	10 mPas
σ	30 mN/m
h	10 μm
D	80 μm
l	500 μm

Practical Example: Nano Coating



Objective

Specs	Value
U_w	14 m/min
μ	10 mPas
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h	10 μm
D	80 μm
l	500 μm

Use sub-pressure chamber to achieve stable coating

Conclusions

Coating Methods

Pre-Metered Coating

- Coating thickness independent on process
- No excess of coating fluid required
- Coating in an integer multiple of the coating thickness

Self-Metered Coating

Coating thickness dependent on process

Slot Coating Fluid Properties

- Apply physical knowledge to describe coating process
  - Determine coating window
  - Draw connection between ΔP and U_w
- Efficient and controllable coating possible

Thank you for your Attention!

Sources

1 F. Durst: *Fluid Mechanics. An Introduction to the Theory of Fluid Flows*. Berlin 2008.

2 N. Dongari, R. Sambasivam, F. Durst: *Slot coaters operating in their bead mode*. Coating International 2007, pp. 10–15.

3 Images from Wikipedia.org and FMP Technology