10.1 Compute the hydrostatic stress, in terms of an (r, θ) coordinate, associated with an edge and a screw dislocation in an aluminum lattice. Take G = 26 GPa and b = 0.3 nm. For a given r, what is the maximum value of this stress?

For an edge dislocation, the hydrostatic pressure is:

$$\sigma_p = \frac{1+v}{1-v} \cdot \frac{Gb}{3\pi} \cdot \frac{\sin \theta}{\pi}$$

The maximum value is obtained when $0 = 90°$

$$\sigma_p \max = \frac{1+v}{1-v} \cdot \frac{Gb}{3\pi} \cdot \frac{1}{r}$$

For aluminum, G = GPa, b = 0.3 nm, v = 0.345

$$\sigma_{p \, \max} = \frac{1+0.345}{1-0345} \times \frac{26 \, x \, 10^9}{3 \, x \, 3.14} \times \frac{1}{r} = \frac{1.7}{r} GPa$$

For a screw dislocation, all the normal stress components are zero. Therefore,
$$\sigma_p = 0$$

10. 3   The interaction energy between an edge dislocation (at the origin) and a solute atom (at r, θ) is given by

$$U = \frac{A}{r} \sin \theta,$$

where A is constant. Transforming into Cartesian coordinates, plot lines of constant energy of interaction for different values (positive and negative) of A/2U. On the same graph, plot the curves for the interaction force. Indicate by arrows the direction in which the solute atoms, with ΔV positive, will migrate.

$$U = \frac{A}{r} \sin \theta$$

In Cartesian coordinates, we have

$$r^2 = x^2 + y^2$$

add

$$\sin \theta = \frac{y}{r}$$

Thus, we can write the above equation as follows

$$U = \frac{Ay}{r^2} = \frac{Ay}{x^2 + y^2}$$

$$or \quad x^2 + y^2 - \frac{A}{U} y = 0$$

This equation, for a given energy U, is nothing but the equation of a circle that is tangent to the line y = 0 and centered on x = 0 (see Fig. on the next page). The interaction force acting on the solute forcing it to migrate is given by the derivative of the energy equation. The interaction force (shown as broken line circles in the Fig.) will thus act perpendicular to the equipotential lines (full line circles in the Fig.). The interaction force will tend to move the solute atoms along the trajectory indicated by the broken circles until they reach the point of maximum binding energy at θ = 3π/2 and r = r$_o$.
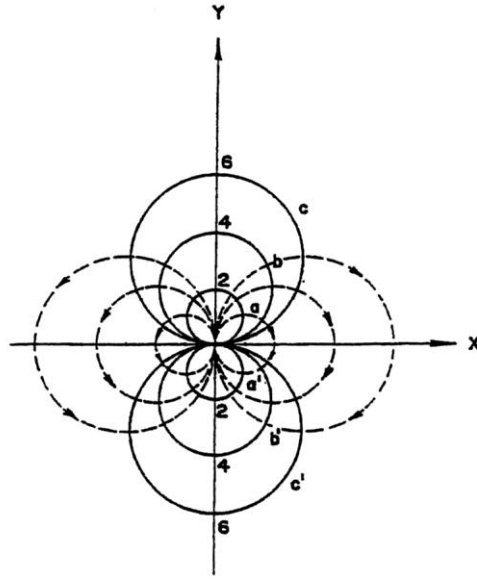
10. 4   Consider a metal with shear modulus G = 40 GPa and atomic radius r_o = 0.15 nm. Suppose the metal has a solute that results in a misfit of $\varepsilon = (R–r_0)/r = 0.14$. Compute the elastic misfit energy per mole of solute.

$$G = 40\,GPa$$

$$r_o = 0.15$$

$$\varepsilon = \frac{R - r_o}{r_o} = 0.14$$

$$U(r,\theta) = \frac{1+v}{1-v} \times \frac{Gb}{3} \times \frac{\sin\theta}{r} 4\varepsilon\, r_o^3$$

Near the core of an edge dislocation, we can take b ≈ r. Then,

$$U = \frac{1+v}{1-v} \times \frac{G}{3} \sin\theta\, 4\varepsilon\, r_o^3$$

Taking $v = 0.3$, we can further simplify this expression for U to:

$$U = \frac{1+0.3}{1-0.3} \times \frac{4}{3} G \sin\theta \, \varepsilon \, r_o^3$$

$$= \frac{5.2}{2.1} G \sin\theta \, \varepsilon \, r_o^3 \quad per \; atom$$

Putting in the values given for G, ε, and r_o , we get

$$U = \frac{5.2}{2.1} \times 40 \times 10^9 \times \sin\theta \times 0.14 \times \left(0.15 \times 10^{-9}\right)^3$$

$$= 4.68 \times 10^{-20} \sin\theta \; J \, / \, atom$$

$$1 \, mole = 6.02 \times 10^{23} atoms$$

$$U \; per \; mole = 4.68 \times 10^{-20} \times 6.02 \times 10^{23} \sin\theta \; J$$

$$= 28.17 \times 10^3 \sin\theta \quad J$$

$$or \quad U = 28.17 \sin\theta \; kJ \, / \, mole$$

$$U_{max} = 28.17 \; kJ \, / \, mole$$

10.5 Estimate the amount of solute (atomic percent) necessary to put one solute atom at each site along all the dislocations in a metal. Assume that 1 mm ^{3} of a metal contains about 10 ^{6} mm of dislocation lines.

There are 10^6 mm of dislocation lines in 1 mm^3 of metal. Let us take Fe as the host metal. Then, we know that 1 mole of Fe has 6 x 10 ^{23} atoms. Now, 1 mole of Fe has 56 g Fe (at. wt. of Fe is 55 .84) and density is 7.8 10^{-3} g/mm^3. Thus, 1 mm^3 of Fe has 7.8 x 10^{-3} x 6 x 10^{23}/56 atoms or about 8 x 10 ^{19} atoms. This gives about 4 x 10 ^{6} atoms per mm. There being 10^6 mm of dislocation lines in 1 mm ^{3}, we shall have 4 x 10 ^{6} x 10 ^{6} = 4 x 10 ^{12} atoms lying on dislocations. Therefore, the amount of solute required to put one solute atom at each site along all the dislocations is given by

$$\frac{4 \times 10^{12}}{8 \times 10^{19}} \times 100 = 0.5 \times 10^{-5} \; at. \; percent$$

10.6 Compute the condensation temperature T_c for the following cases:

    (a)   Carbon in iron with C_o (average concentration) = 0.01 % and U_i (interaction energy) = 0.08 aJ (0.5 eV).

    (b)   Zinc in copper with C_o = 0.01% and U_i = 0.019 aJ (0.12eV).

    (a)   Carbon in iron

$$C_o = 0.01\% = 10^{-4}$$

$$U_i = 0.08 \ aJ = 8 \times 10^{-20} J$$

$$T_c = \frac{U_i}{\left(k\ell n \dfrac{1}{C_o} \right)} = \frac{8 \times 10^{-20}(J)}{1.38 \times 10^{-23}(JK^{-1})\ell n 10^4}$$

$$T_c = 629 K$$

    (b)   Zinc in copper

$$C_o = 0.01\% = 10^{-4}$$

$$U_i = 0.019 \ aJ = 1.9 \times 10^{-20} J$$

$$T_c = \frac{1.9 \times 10^{-20}}{1.38 \times 10^{-23} \times \ell n \ 10^4}$$

$$T_c = 151 \ K$$

10.7 One of the Hume-Rothery rules for solid solutions is that the solubility of solute B in solvent A becomes negligible when the atomic radii of A and B differ by more than 15 %. Plot the maximum solubility (atomic percent) of Ni, Pt, Au, Al, Ag, and Pb as a function of the ratio of the solute and solvent (Cu) radii, and verify the solid solubility in Cu drops precipitously at a size of about 1.15. Use the following data:

r_{Ni} = 0.1246nm r_{Al} = 0.143 nm
r_{Pt} = 0.139 nm r_{Ag} = 0.1444 nm
r_{Cu} = 0.1278 nm r_{Pb} = 0.1750 nm
                              r_{Au} = 0.1441 nm

Copper solvent has an atomic radius, r_{Cu} = 12.78 nm while the atomic radii, the solubility in Cu and the ratio of atomic radii with respect to Cu of various solutes is given in the table below.

Solute	Atomic radius $r_{solute,}$nm	$r_{soulute}/r_{Cu}$	Solubility in Cu (at.%)
Ni	12.46	0.98	100
Pd	13.67	1.07	100
Pt	13.80	1.08	100
Au	14.31	1.12	100
Al	14.31	1.12	19
Ag	14.44	1.14	6
Pb	17.50	1.37	0

The plot between solubility and r_{solute} /r_{Cu} (see Fig. on the next page) shows a precipitous drop in solid solubility occurring at around a size ratio of 1.15.
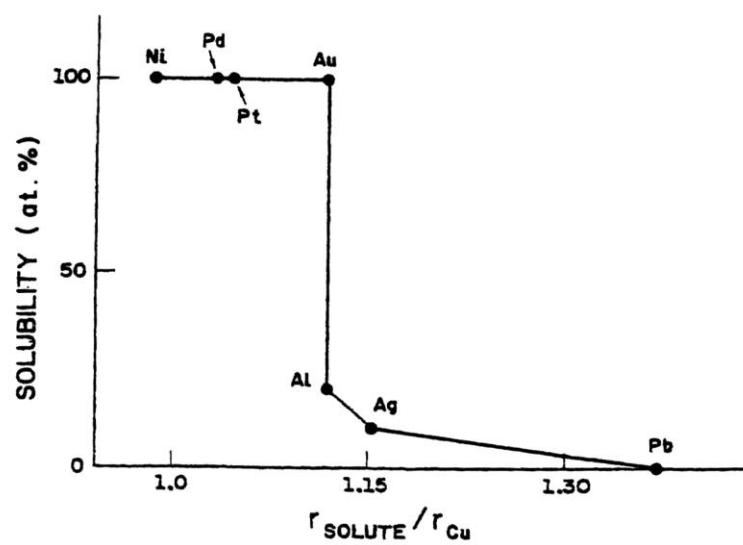
10.8 A steel specimen is being tested at a strain rate of 3 x 10^{-3} s^{-1}. The cross-sectional length is equal to 0.1 m. A Lüders band forms at the section with an instantaneous strain of 0.2. What is the velocity of propagation of the two Lüders fronts?

$$\dot{\varepsilon} = 3 \; x \; 10^{-3} \; s^{-1}$$

$$\dot{\varepsilon} = \frac{2v_1}{L.}$$

$$\therefore \; v_1 = \frac{\left(3x10^{-3} \; s^{-1}\right)\left(0.1m\right)}{2} = 1.5 \, x \, 10^{-4} \; m/s$$

By conservation of mass:

$$A_1 \, v_1 = A_2 \, v_2 \quad \therefore \; v_2 = \frac{A_1}{A_2} v_1$$

By conservation of volume:

$$A_1 dL_1 = A_2 dL_2$$

Instantaneous strain at Lüders band = 0.2

$$0.2 = \frac{dL_2 - dL_1}{dL_1}, \quad or \quad \frac{dL_2}{dL_1} = 1.2$$

$$\frac{A_1}{A_2} = \frac{dL_2}{dL_1} = 1.2$$

$$\therefore \; v_2 = 1.2 \, v_1 = 1.2 \left(1.5 \times 10^{-4} \; m/s\right) = 1.8 \times 10^{-4} \; m/s$$

10.10 Consider a unit cube of a matrix containing uniform spherical particles (with radius r) of a dispersed second phase.

     a)   Show that the average distance between the particles is given by

$$\Lambda = \frac{1}{(N)^{1/3}} - 2R$$

     where N is the number of particles per unit volume.

     b)   Compute Λ for a volume fraction f of particles equal to 0.001 and $r = 10^{-6}$ cm.

The second phase is uniformly dispersed. We can image that there is a cube of unit volume, which is composed of N elemental cubes. In every elemental cube there is a particle. Thus the side of such an elemental cube is $(1/N)^{1/3}$. Then, the inter-particle spring is $\lambda = \left(\dfrac{1}{N}\right)^{1/3} - 2r$.

If the volume fraction of the second phase is f, then:

$$\frac{4}{3}\pi r^3 . N = f$$

$$\left(\frac{1}{N}\right)^{1/3} = \left(\frac{3f}{4\pi}\right)^{1/3} r$$

$$Thus\ \lambda = \left(\frac{1}{N}\right)^{1/3} - 2r$$

$$= r\left[\left(\frac{4\pi}{3f}\right)^{1/3} - 2\right]$$

If $f = 0.001,\ r = 10^{-6}$ cm, then

$$\Lambda = \left[\left(\frac{4 \times 3.14}{3 \times 0.001}\right)^{1/3} - 2\right] \times 10^{-8} = 141\ nm$$

10.13 Calculate the critical radius of precipitates for which an Al-Mg alloy containing 10% Mg will be strengthened by Orowan looping instead of particle shear. Use the following data:

$$\gamma_{Al_2Mg} = 1.4 \, J/m^2$$
$$G_{Al} = 26.1 \, GPa$$
$$r_{Al} = 0.142 \, nm$$

Using information provided in example 10.3 on p. 578 of the textbook:

Weight fraction of Al$_2$Mg $= (10\text{-}1)/(35\text{-}1) = 9/34 = 0.2647$

$\rho_{Al} = 2.7 \, \text{g/cm}^3$ $\qquad$ $\rho_{Al2Mg} = 2.3 \, \text{g/cm}^3$

∴ Volume fraction f can be obtained by dividing mass fraction by density:

$$f = \frac{0.2647/2.3}{0.7353/2.7 + 0.2647/2.3} = \frac{0.115}{0.2723 + 0.115} = 0.297$$

Assume the precipitates are arranged in a simple cubic arrangement. Therefore the spacing of precipitates, x, is equal to the length of the edge of the cube.

$$volume \ of \ Al_2Mg = \frac{4}{3}\pi \, r_{Al_2Mg}^3$$

$$volume \ fraction \ of \ Al_2Mg = f = \frac{\frac{4}{3}\pi \, r_{Al_2Mg}^3}{x^3} = 0.297$$

$$\therefore r_{Al_2Mg} = \left(\frac{3f}{4\pi}\right)^{1/3} x = 0.414x$$

$$For \ particle \ shear, \quad \tau_{shear} = \frac{\pi \, r_{Al_2Mg} \, \gamma_{Al_2Mg}}{2bx}$$

$$for \ bypass, \quad \tau_{orowan} = \frac{Gb}{x}$$

The critical condition is $\tau_{shear} = \tau_{orowan}$

$$\frac{\pi\, r_{Al_2Mg}\, \gamma_{Al_2Mg}}{2bx} = \frac{Gb}{x}$$

where $r_{Al_2Mg} = 0.414x \quad b = 2r_{Al}$

$$\frac{\pi(2.414x)(1.4J/m^2)}{(4)(r_{Al})(x)} = \frac{2(G)(r_{Al})}{x}$$

$$\therefore x = \frac{8\, r_{Al}^2\, G}{\pi(0.414)(1.4)} = \frac{8(0.143 \times 10^{-9})^2 (26.1 \times 10^9)}{\pi(0.414)(1.4)}$$

$$= 2.34\ nm$$

10.16 An aluminum alloy is strengthened by dispersed alumina particles. These particles are spherical and have a diameter of 15 μm. The weight percentage of alumina in aluminum is 3%. Estimate the dispersion strengthening.

Given:

$G_{Al} = 28$ GPa,
Density(Al) = 2.70 g/cm^3,
Density (Al$_2$O$_3$) = 3.96 g/cm^3.

Volume fraction of alumina, $f = \dfrac{0.3 \big/ 3.96}{0.7 \big/ 2.7 + 0.3 \big/ 3.96} = 0.23$

$$f = \frac{\dfrac{4}{3}\pi r^3}{x^3}$$

x = 0.197 μm

$$\tau_{orowan} = \frac{Gb}{x} = \frac{28\ 10^9\ 2x0.143\ 10^{-9}}{44.54\ 10^{-6}} = 0.4\ MPa$$