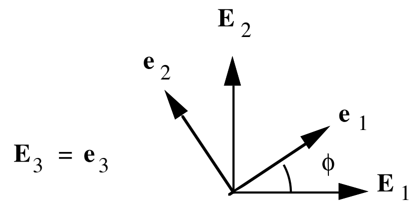
ME 512 CONTINUUM MECHANICS Fall 2014 ASSIGNMENT 9

The bases $\mathbf{e}_i$ and $\mathbf{E}_A$ are related as follows:



The parameters ϕ, α, β and γ are constants with $\alpha \neq 1, \beta \neq 1$ and $\gamma \neq 0$. Assume the origins of the two position vectors are identical.

The deformation is defined by

$$\phi \neq 0 \quad \text{and} \quad x_1 = \alpha X_1 + \gamma X_2 \qquad x_2 = \beta X_2 \qquad x_3 = X_3$$

1. Express $\boldsymbol{r}$ as a function of $\boldsymbol{R}$ for your choice of basis.
2. Express $\boldsymbol{R}$ as a function of $\boldsymbol{r}$ for your choice of basis.
3. Express $\boldsymbol{u}$ as a function of $\boldsymbol{R}$ for your choice of basis.
4. Express $\boldsymbol{u}$ as a function of $\boldsymbol{r}$ for your choice of basis.
5. Obtain the components of the two strain tensors using

$$\boldsymbol{E} = E_{ij}\boldsymbol{E}_i \otimes \boldsymbol{E}_j \qquad E_{ij} = \frac{1}{2}\left[\frac{\partial x_k}{\partial X_i}\frac{\partial x_k}{\partial X_j} - \delta_{ij}\right]$$

$$\boldsymbol{e} = e_{ij}\boldsymbol{e}_i \otimes \boldsymbol{e}_j \qquad e_{ij} = \frac{1}{2}\left[\delta_{ij} - \frac{\partial X_k}{\partial x_i}\frac{\partial X_k}{\partial x_j}\right]$$

6. Obtain the components of the two strain tensors using

$$\boldsymbol{E} = \frac{1}{2}\left[\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i} + \frac{\partial u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}\right]\boldsymbol{E}_i \otimes \boldsymbol{E}_j$$

$$\boldsymbol{e} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right]\boldsymbol{e}_i \otimes \boldsymbol{e}_j$$

7. Sketch the principal 11-components of $\boldsymbol{E}, \boldsymbol{e}$ and $ln(\boldsymbol{U})$ as functions of the principal stretch Λ_1.
8. Suppose a strain-gage rosette is attached to the surface of a body and the surface lies in the x-y plane. The directions x and y are marked on the surface. The individual gages are oriented at angles of α, β and γ with respect to the x-axis. Describe how you would infer the planar components of the strain tensor given readings from the strain gages.