Consider an elasticity (stiffness) tensor, $\mathbf{E}$, and flexibility tensor, $\mathbf{F}$, associated with orthotropic symmetry, i.e., the components of the flexibility tensor with respect to the basis, $\mathbf{E}_i$, that define the planes of symmetry in the Voigt-Mandel notation are $[F]_{6x6} = \begin{bmatrix} [F] & [0] \\ [0] & [F]_D \end{bmatrix}$. Each of the sub-matrices are 3x3 and

$$[F] = \begin{bmatrix} \frac{1}{Y_1} & -\frac{\nu_{12}}{Y_2} & -\frac{\nu_{13}}{Y_3} \\ -\frac{\nu_{21}}{Y_1} & \frac{1}{Y_2} & -\frac{\nu_{23}}{Y_3} \\ -\frac{\nu_{31}}{Y_1} & -\frac{\nu_{32}}{Y_2} & \frac{1}{Y_3} \end{bmatrix} \qquad [F]_D = \begin{bmatrix} \frac{1}{2G_{12}} & 0 & 0 \\ 0 & \frac{1}{2G_{23}} & 0 \\ 0 & 0 & \frac{1}{2G_{31}} \end{bmatrix}$$

where Y_i denote the Young's moduli, ν_{ij} the Poisson's ratios and G_{ij} the shear moduli with respect to the given basis. The Poisson's ratios are restricted so that [F] is symmetric. The elasticity matrix is

$$[E]_{6x6} = \begin{bmatrix} [E] & [0] \\ [0] & [E]_D \end{bmatrix} \qquad [E] = [F]^{-1} \qquad [E]_D = \begin{bmatrix} 2G_{12} & 0 & 0 \\ 0 & 2G_{23} & 0 \\ 0 & 0 & 2G_{31} \end{bmatrix}$$

1. Suppose the Young's moduli, the Poisson's ratios and the shear moduli are provided as input data. Write a program that provides the matrix of elasticity coefficients, [E], $[E]_D$ and $[E]_{6x6}$ with respect to the given basis. Show numerical results for material parameters selected by you to illustrate your program works. Make the necessary restriction on Young's moduli, Poisson's ratios and shear moduli for isotropy and show you obtain the correct stiffness matrix.

2. You cut two cubes of material out of the orthotropic material. One cube has its edges aligned with two of the planes of symmetry; the other cube not. Each cube is subjected to hydrostatic pressure $(\sigma_{11} = \sigma_{22} = \sigma_{33} = -P/3)$. For each cube answer the questions: (a) Are the normal components of strain the same? and (b) Are the shear components of strain equal to zero?

3. From a materials handbook, find a set of elasticity constants for an anisotropic material of interest to you that is orthotropic, or simpler, but not isotropic.

(i) Label the basis as $\mathbf{E}_i$. Construct the corresponding Voigt-Mandel matrices $\overset{E-E}{[E]}$ and $\overset{E-E}{[F]}$.

(ii) Use one of these matrices to obtain the best possible choices for the bulk and shear moduli if the material is to be approximated as an isotropic material. How good is the approximation?

4. In Assign. 1, you constructed the Voigt-Mandel transformation matrix $\overset{e-E}{[A]}$ that represents a transformation from one basis $\mathbf{e}_i$ to another basis, $\mathbf{E}_A$.

(i) Obtain the elasticity components of the material selected in Problem 3 in the second basis, $\mathbf{e}_i$. Denote these components as $\overset{e-e}{[E]}$. Use these components to obtain the best possible choices for the bulk and shear moduli if the material is to be approximated as an isotropic material. Compare the result with that obtained in Prob. 3. Is the result to be expected and why?

5. Suppose the matrix of stiffness parameters $\overset{e-e}{[E]}$ defined in Problem 4 is given to you with no knowledge of how the components were obtained. Find the "material axes" for your material by performing the following steps:

(i) Use an eigen-system package to obtain the eigenvectors of this elasticity matrix.

(ii) Based on the Voigt-Mandel notation (i.e., be careful with the root 2 factors) convert these vectors to components of second-order eigentensors.

(iii) Determine the eigenvectors for each of these second-order eigentensors. These eigenvectors are defined to be the "material vectors". The components of these vectors are associated with which basis?

(iv) From the eighteen material vectors obtained in (iii), select two different orthonormal bases. Obtain the V-M components of the elasticity matrix with respect to each of these bases.

(v) Obtain the components of the material vectors obtained in (iii) with respect to the basis $\mathbf{E}_i$. Sketch out the material vectors in the $\mathbf{E}_i$ space.