ME 562 - Assignment 5: Plasticity

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Abstract

The objective of this assignment was to add additional subroutines to the Driver program allowing for the analysis of elastic-plastic constitutive equations. These additional subroutines allow for the simulation of an elasatic-plastic constitutive equation with a von Mises Yield Function (F) and various hardening functions (H).

1 Add Plasticity Capability to Driver Program

1.0.1 Summary of Elastic-Plastic Constitutive Equations

During plastic deformation, the following equations describe the constitutive relationships:

- Elasticity:
  - $\dot{\underline{\underline{\sigma}}} = \underline{\underline{\underline{\underline{E}}}} \cdot\cdot (\underline{\underline{e}} - \underline{\underline{e}}^p)$
  - $\underline{\underline{e}}^e = \underline{\underline{e}} - \underline{\underline{e}}^p$
- Evolution (Flow Rule):
  - $\dot{\underline{\underline{e}}}^p = \dot{\lambda} \underline{\underline{M}}$
  - $\dot{\overline{e}}^p = \dot{\lambda} \overline{m}$, or a more general form of $\{\dot{\overline{\epsilon}}\} = \dot{\lambda} \{\overline{m}\}$
- Consistency:
  - $\dot{F} = 0$

1.0.2 Summary of Subroutines

Four different subroutines were written to enable the calculation of a general elastic-plastic constitutive equation. Those subroutines were:

- Strs_Meas(strsv): input is a vector of the six stress components and the output is a 27 vector of various stress measures. The output is:
  - strs_measv[0:2] = principal components of the stress tensor (σ_1,σ_2, σ_3) ;
  - strs_measv[3:5] = principal components of the deviatoric stress tensor, which are the principal stress components minus the mean stress ($\sigma_1^{dv}, \sigma_2^{dv}, \sigma_3^{dv}$);
  - strs_measv[6] = maximum shear stress ($\frac{\sigma_1 - \sigma_3}{2}$);
  - strs_measv[7] = first invariant of the stress tensor (I_1);
  - strs_measv[8:9] = second and third invariants of the deviatoric stress tensor (J_2, J_3);
  - strs_measv[10] = mean stress or pressure (p);
  - strs_measv[11:13] = repeated values, principal components of the deviatoric stress tensor, which are the principal stress components minus the mean stress ($\sigma_1^{dv}, \sigma_2^{dv}, \sigma_3^{dv}$);

- strs_measv[14:15] = horizontal (x) and vertical directions (y) directions on the deviatoric stress plane (q_1, q_2);
- strs_measv[16:17] = both values are the same (use alternate calculation methods) Mises stress, which is the magnitude from the origin of the deviatoric stress plane (q);
- strs_measv[18:20] = Lode coordinates (r, z, θ);
- strs_measv[21] = stress triaxiality, which is the ratio of the mean stress to mises stress ($\frac{p}{q}$);
- strs_measv[22:27] = six components of the deviatoric stress tensor ($\sigma_{11}^{dv}, \sigma_{22}^{dv}, \sigma_{33}^{dv}, \sigma_{12}^{dv}, \sigma_{23}^{dv}, \sigma_{31}^{dv}$);

- Hard_Func(yield_0, eff_pstrn, harden_model, harden_exp, harden_type, harden_assoc, lambda_0, eff_pstrn_ref): input are the initial yield stress (yield_0) and the current set of plastic parameter values describing the yield condition (eff_pstrn, lambda_0) and specific hardening rule (hardent_model, harden_exp, harden_type, harden_assoc, eff_pstrn_ref). The output is:
  - H = value of the hardening parameter which defines the current yield stress;

- Yield_Func_02(strs_measv, model, yield_0, H, theta, c, path): input for this function are the stress measures (strs_measv), identification of the yield model (model), initial yield stress (yield_0), current yield stress (H), along with model and path specific parameters (theta, c, path). The output is:
  - F = value of the current yield function. Where:
    - F < 0: deformation is elastic;
    - F = 0: deformation is plastic;
    - F > 0: not admissible;

- Elastic_Plastic(matpropv, strn_incv, strnv, estrnv, pstrnv, strsv, term_type, p_max, p_min, SM, irow, path): input consists of the material parameters (matpropv), prescribed strain increment (strn_incv), along with current states of strain and stress (strnv, estrn, pstrnv, strsv). Additionally, various other parameters defining specific paths (path), how a leg is terminated (term_type, p_max, p_min) and previous values (SM) are also called. The output is:
  - pstrnv = six vector containing the current values of plastic strain;
  - estrnv = six vector containing the current values of elastic strain;
  - strsv = six vector containing the current values of stress;
  - plastic_storage = five vector containing plasticity specific parameters to be stored in SM;

1.1 Module for von Mises Plasticity

The subroutines Hard_Func and Yield_Func_02 were modified to allow for the appropriate calculation of von Mises plasticity with an associated flow rule and isotropic-linear hardening:

- Hard_Func was modified to allow for linear-isotropic hardening: $H = \sigma_0(1 + \frac{\overline{e}^p}{\overline{e}^p_{ref}})$

- Yield_Func_02 was to allow for Mises plasticity with an associated flow rule via:
  - Mises plasticity: $F = \frac{q-H}{\sigma_0}$, whch has been normalized by the initial yield stress (σ_0);
  - associated flow:
    - $\underline{\underline{M}} = \sqrt{\frac{3}{2}}\left(\frac{\underline{\underline{\sigma}}^{dv}}{q}\right)$
    - $\overline{m} = \sqrt{3}$, note: this is a different value that presented in class on April 23, 2015, lecture 28, page 2.

1.2 Match Uniaxial Stress-Strain Curve

Parameters were chosen such that the initial (pure elastic) stress-strain curve had a slope of 18, and the stress-strain curve in the elastic-plastic regime had a slope of approximately 4.5 (Figure 1). Parameter values used for fitting were:

- Effective plastic strain reference value ($\bar{e}^p_{ref}$) = 3.5
- Initial increment size in the plastic parameter lambda ($\dot{\lambda_0}$) = 10^{-10}
- Initial yield stress (σ_0) = 10
- Tolerance of "F" to equal zero (ϵ) = 10^{-3}
- Constant strain increment size in the "11" direction ($\dot{e}_{11}$) = 0.05

Additionally, Figures 2 and 3 verify that a uniaxial state of stress was maintained during all strain increments and that the plastic strains in the 22 and 33 directions were each half the magnitude in the opposoite direction of the strain in the 11 (loading) direction.

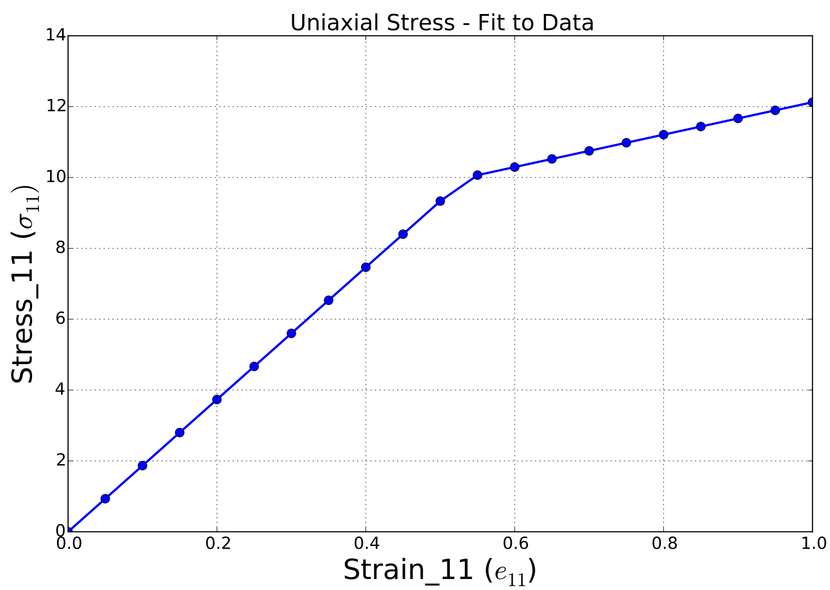

Figure 1: σ_{11} versus e_{11} data for the theoretical "fit" to data.

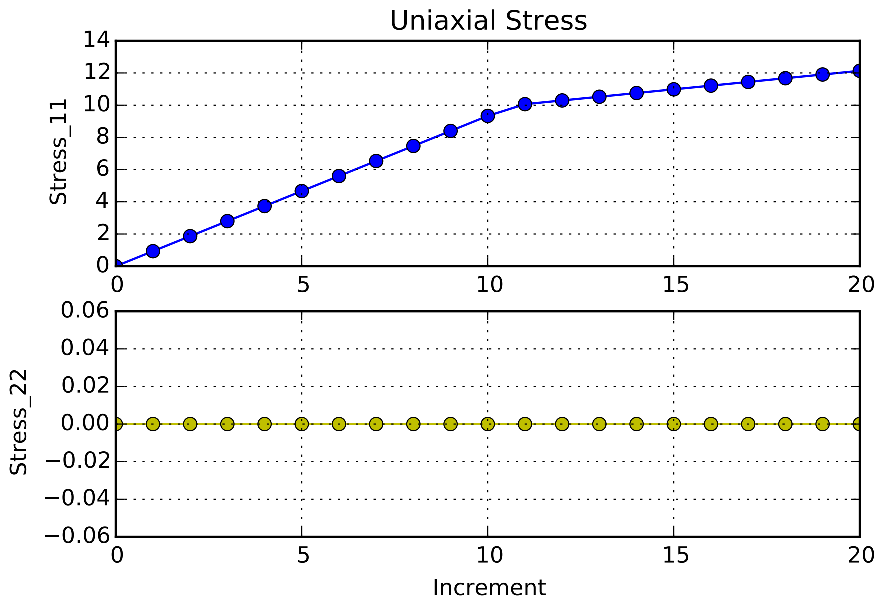

Figure 2: σ_{11} and σ_{22} versus strain increment.

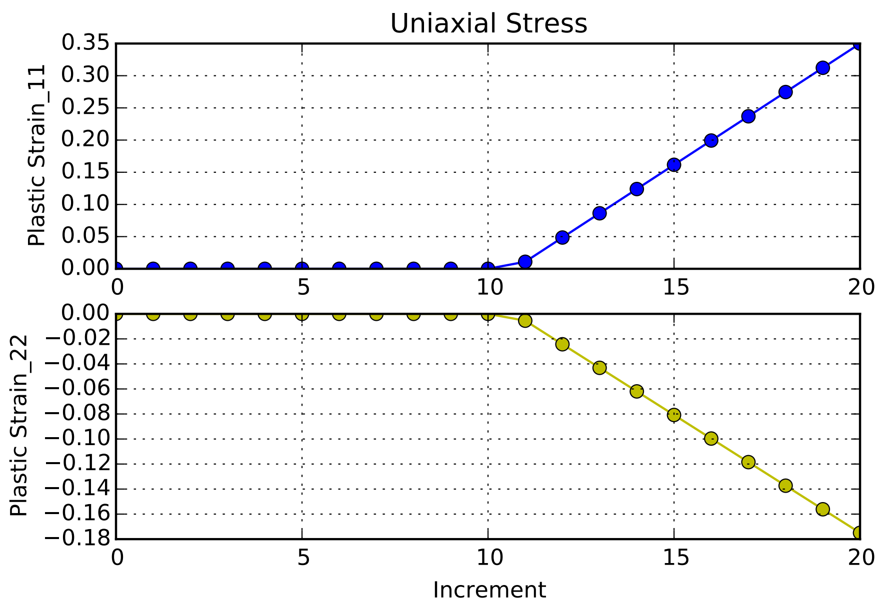

Figure 3: e_{11}^{p} and e_{22}^{p} versus strain increment.

2 Evaluation of Algorithm Robustness

The influence of toleranace (ϵ) and strain increment ($\dot{e}_{11}$) parameters were evalauated to see how they influenced the convergence of my algorithm to a numerically "exact" solution, i.e., a solution obtained using a very small tolerance and increment.

2.1 Calculation of a Numerically "Exact" Solution

Below are a listing for the parameters used in the calculation of the "exact" solution:

- Effective plastic strain reference value ($\bar{e}^p_{ref}$) = 3.5
- Initial increment size in the plastic parameter lambda ($\dot{\lambda}_0$) = 10^{-10}
- Initial yield stress (σ_0) = 10
- Tolerance of "F" to equal zero (ϵ) = 10^{-4}
- Constant strain increment size in the "11" direction ($\dot{e}_{11}$) = 0.05

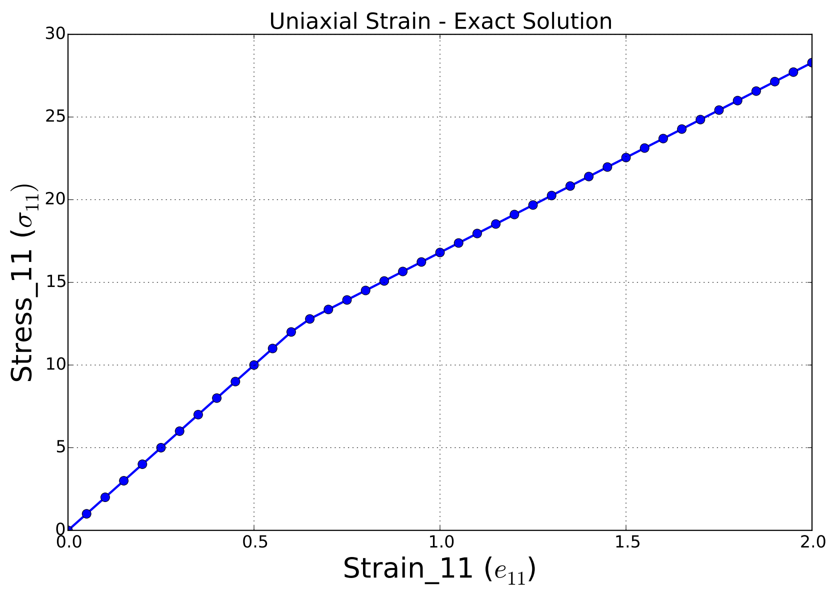

Figure 4: σ_{11} versus e_{11} data for the numerically "exact" solution.

2.2 Evaluation of Tolerance Restriction for F = 0

Theoretically, during plastic deformation the consistency conditions implies that F = 0. However, when this is numerically implemented, F is rarely equal to to zero and is actually equal to zero plus or minus a tolerance. I have written my algorithm such that F will never be less than zero during plastic deformation; therefore my defined tolerance (ϵ) controls the potential magnitude of F greater than zero.

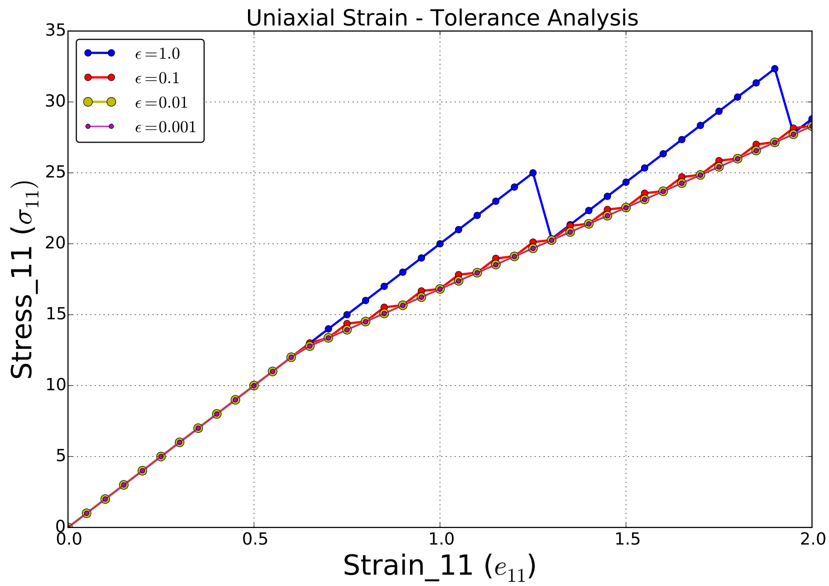

Figure 5: σ_{11} versus e_{11} data over a range of $\epsilon' s$.

2.3 Evaluation of Strain Increment Size

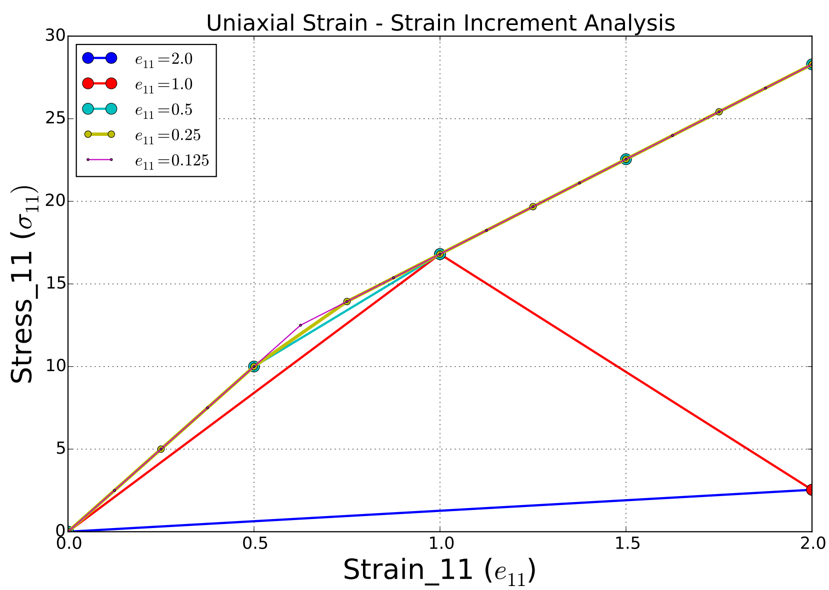

Figure 6: σ_{11} versus e_{11} data over a range of $\dot{e}_{11}$.

2.4 Analysis of Results

Based on these observed results with linear hardening, my algorithm appears to require a reasonable small tolerance ($\epsilon = 0.01$) for continuous plastic stress increments. Large strain increments resulted in erronious results because the elastic-plastic transition (intial yield stress) value was overshot when large strains were used. Therefore, the upper limit of the strain increment size will be a function of the intial yield stress and bulk modulus of the modeled material. Because

3 Implementation of a Nonlinear Hardening Function

The hardening function as provided in problem three was implemented in my Driver program.

3.1 Evaluation of n on the Yield Stress (H) as a Function of the Normalized Effective Plastic Strain $(\hat{\bar{e}})$ Curve

Parameter used for this analysis were:

- Initial Yield Stress: $H_0 = 5$;
- Limit Yield Stress: $H_L = 8$;
- Final Minimum Yield Stress: $H_a = 2$;
- Limit Effective Plastic Strain (defines softening boundary): $\bar{e}^p_L = 5$;
- Softening Shape Factor: $s = 1$;

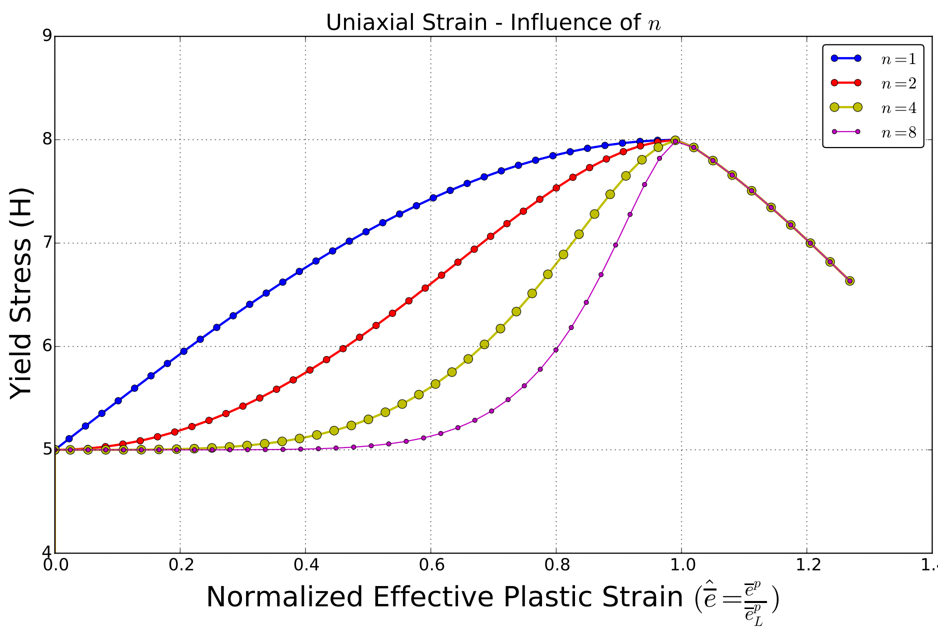

Figure 7: Evaluation of n on the Hardening Function.

3.2 Incorporation of Nonlinear Hardening Function into Mises Plasticity

The nonlinear hardening function was implemented into my Mises plasticity algorithm, and resutls are shown below for a strain prescribed path.

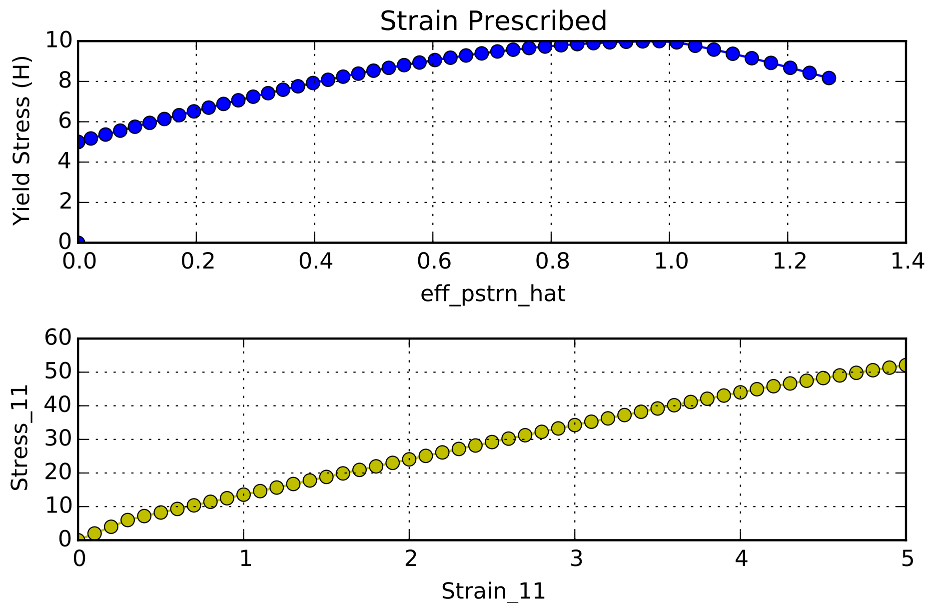

Figure 8: Results of Implementatin of Nonlinear Hardening with Mises Plasticity.

3.3 Evaluation of Different Stress Paths

3.3.1 Uniaxial Strain

Parameter used for this analysis were:

- Initial Yield Stress: $H_0 = 5$;
- Limit Yield Stress: $H_L = 10$;
- Final Minimum Yield Stress: $H_a = 2$;
- Limit Effective Plastic Strain (defines softening boundary): $\bar{e}^p_L = 4$;
- Softening Shape Factor: $s = 1$;
- Hardening Shape Factor: $n = 1$;
- Strain Increment Size: $\dot{e}_{11} = 0.1$;
- Matrix of Elasticity Coefficients: $E = \begin{bmatrix} 20 & 4 & 4 & 0 & 0 & 0 \\ 4 & 20 & 4 & 0 & 0 & 0 \\ 4 & 4 & 20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

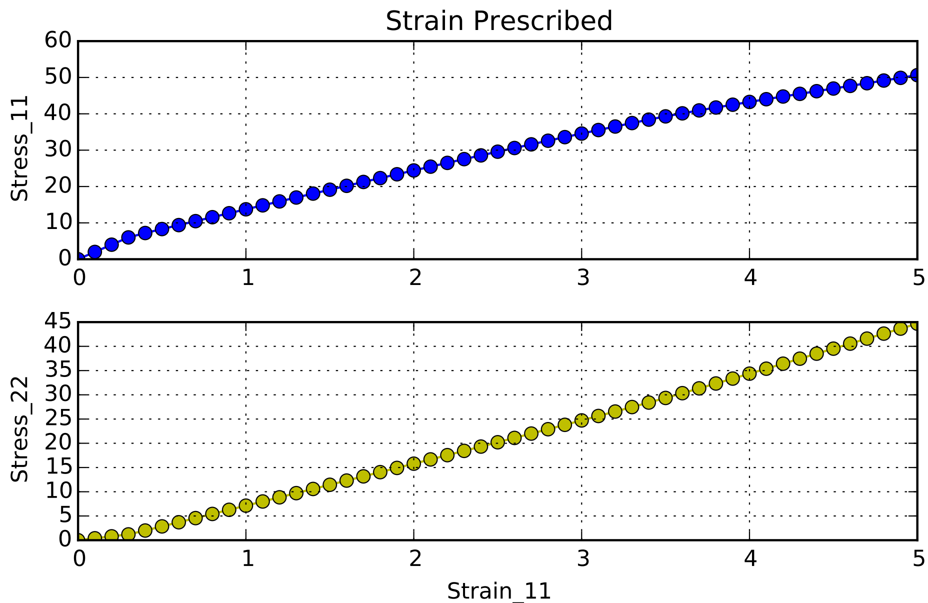

Figure 9: Stress (σ_{11} and σ_{22}) Versus Strain (e_{11}).

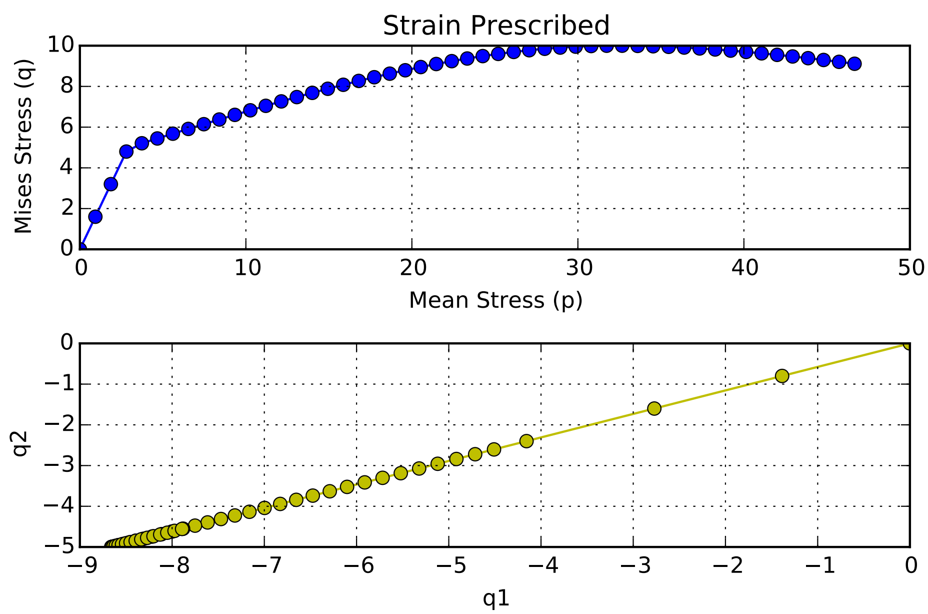

Figure 10: Mean Stress (p) Versus Mises Stress (q) and q_1 Versus q_2 Stress Paths.

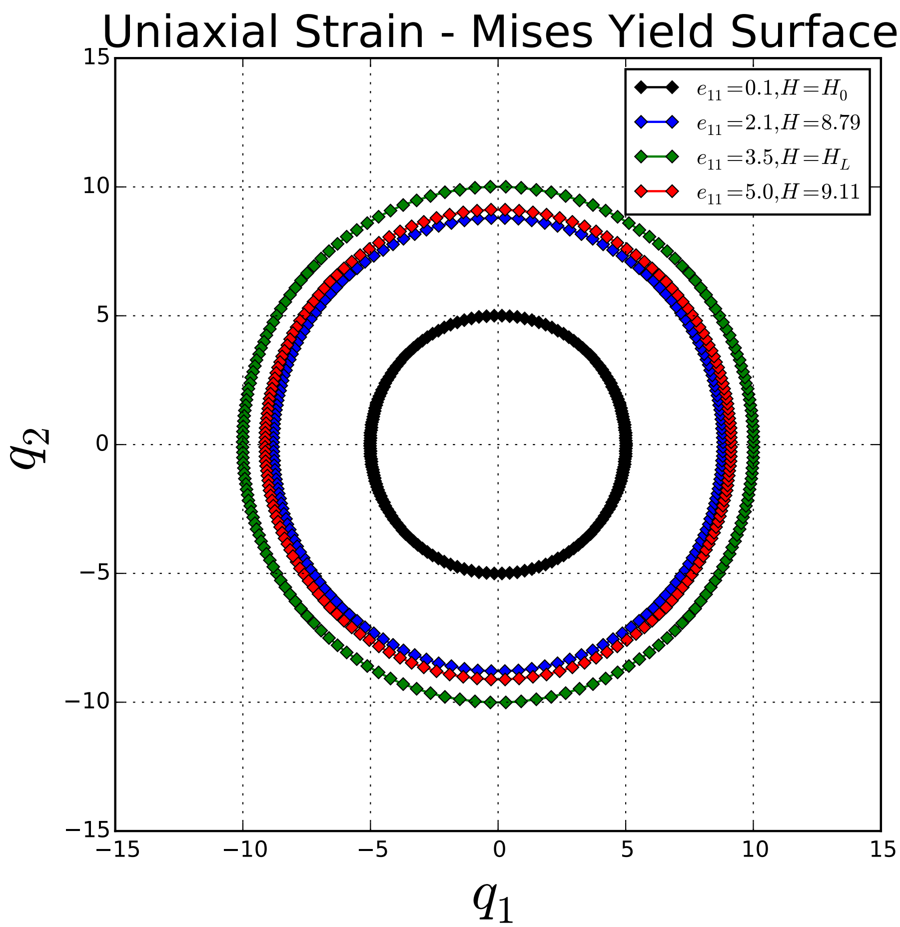

Figure 11: Yield Surface in the $q_1 - q_2$ Plane at Specific Points During Deformation.

3.3.2 Triaxial Compression

Parameter used for this analysis were:

- Initial Yield Stress: $H_0 = 5$;
- Limit Yield Stress: $H_L = 8$;
- Final Minimum Yield Stress: $H_a = 2$;
- Limit Effective Plastic Strain (defines softening boundary): $\bar{e}_L^p = 5$;
- Softening Shape Factor: 2;
- Hardening Shape Factor: $n = 1.5$;
- Strain Increment Size:
  - Hydrostatic Leg: $\dot{e}_{11} = -0.01$;
  - Triaxial Compression Leg:$\dot{e}_{11} = -0.1$;

- Matrix of Elasticity Coefficients: $E = \begin{bmatrix} 10 & 2 & 2 & 0 & 0 & 0 \\ 2 & 10 & 2 & 0 & 0 & 0 \\ 2 & 2 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 \end{bmatrix}$

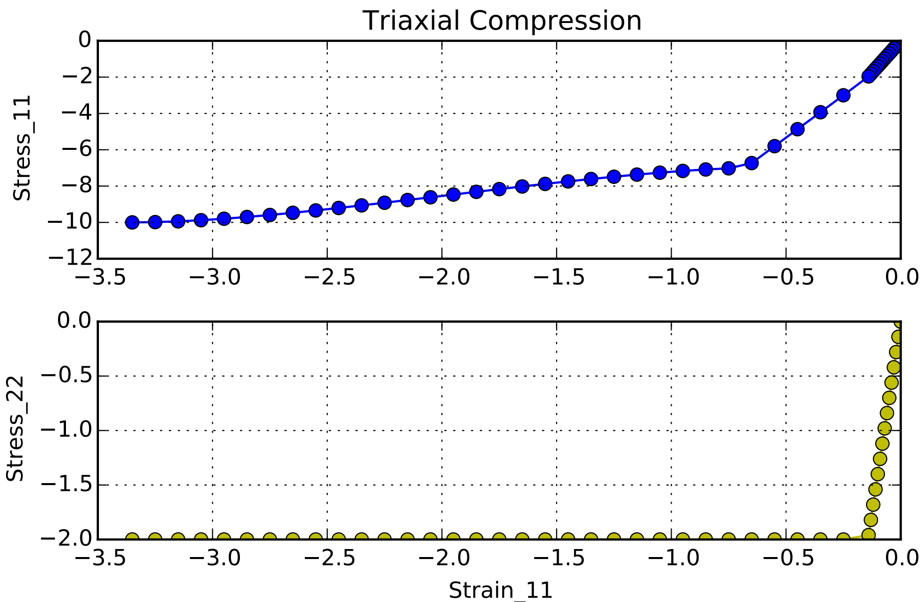

Figure 12: Stress (σ_{11} and σ_{22}) Versus Strain (e_{11}).

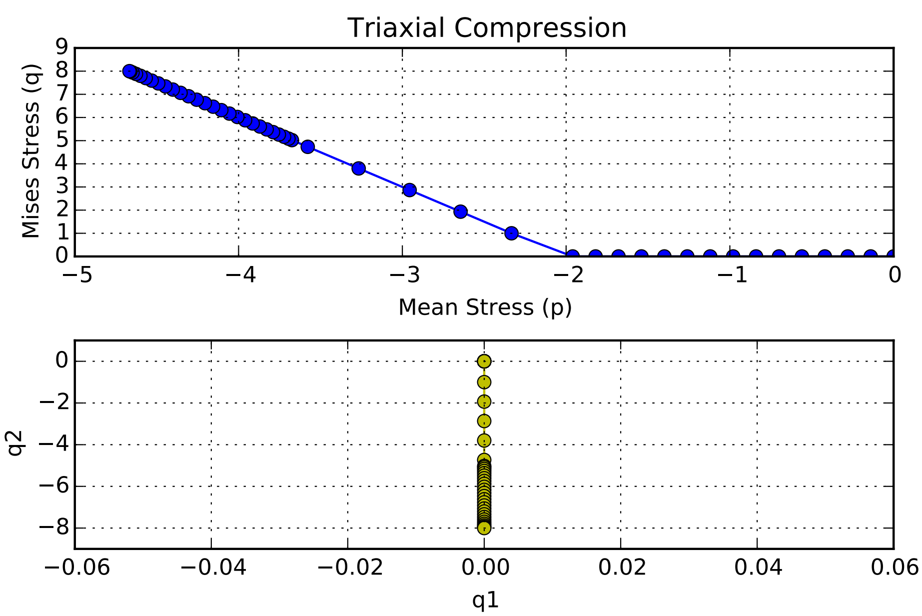

Figure 13: Mean Stress (p) Versus Mises Stress (q) and q_1 Versus q_2 Stress Paths.

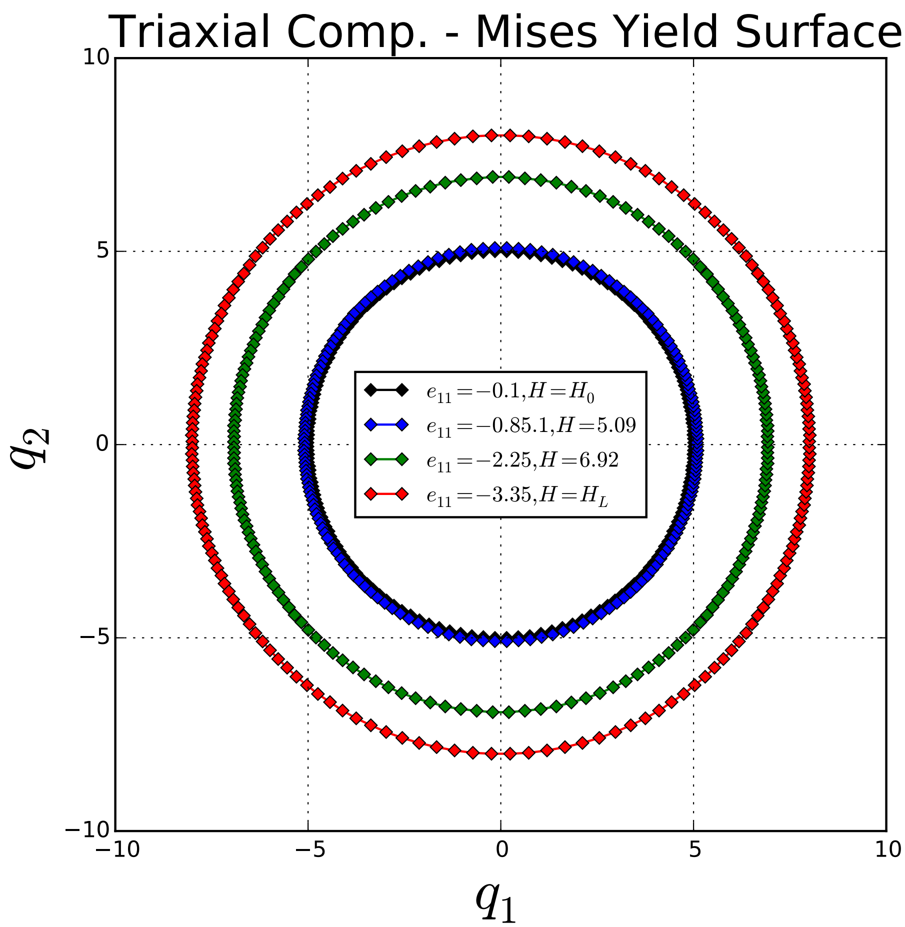

Figure 14: Yield Surface in the $q_1 - q_2$ Plane at Specific Points During Deformation.

3.3.3 Triaxial Extension

Parameter used for this analysis were:

- Initial Yield Stress: $H_0 = 5$;
- Limit Yield Stress: $H_L = 8$;
- Final Minimum Yield Stress: $H_a = 2$;
- Limit Effective Plastic Strain (defines softening boundary): $\bar{e}^p_L = 10$;
- Softening Shape Factor: 2;
- Hardening Shape Factor: $n = 1.5$;
- Strain Increment Size:
  - Hydrostatic Leg: $\dot{e}_{11} = -0.02$;
  - Triaxial Compression Leg:$\dot{e}_{11} = 0.01$;

- Matrix of Elasticity Coefficients: $E = \begin{bmatrix} 10 & 2 & 2 & 0 & 0 & 0 \\ 2 & 10 & 2 & 0 & 0 & 0 \\ 2 & 2 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 \end{bmatrix}$

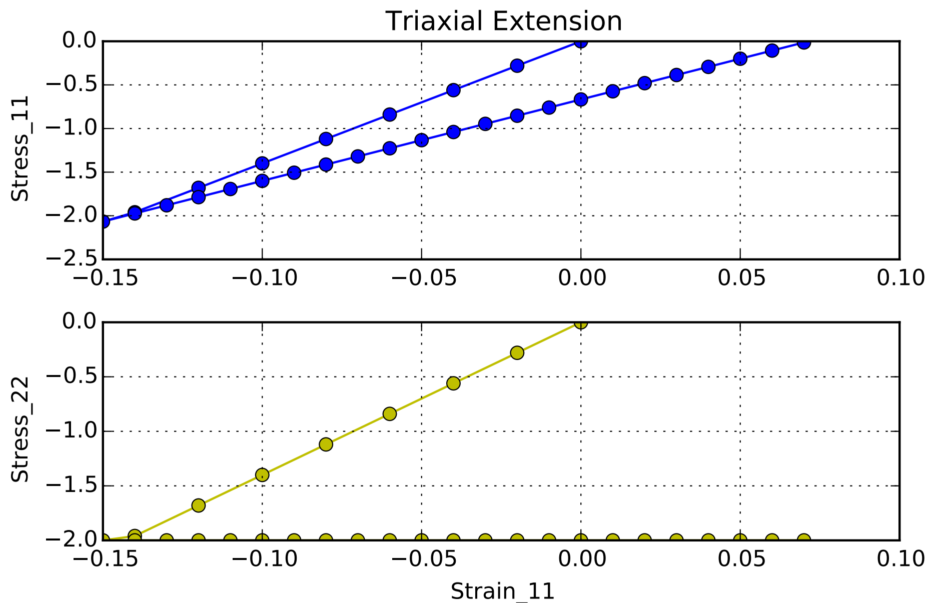

Figure 15: Stress (σ_{11} and σ_{22}) Versus Strain (e_{11}).

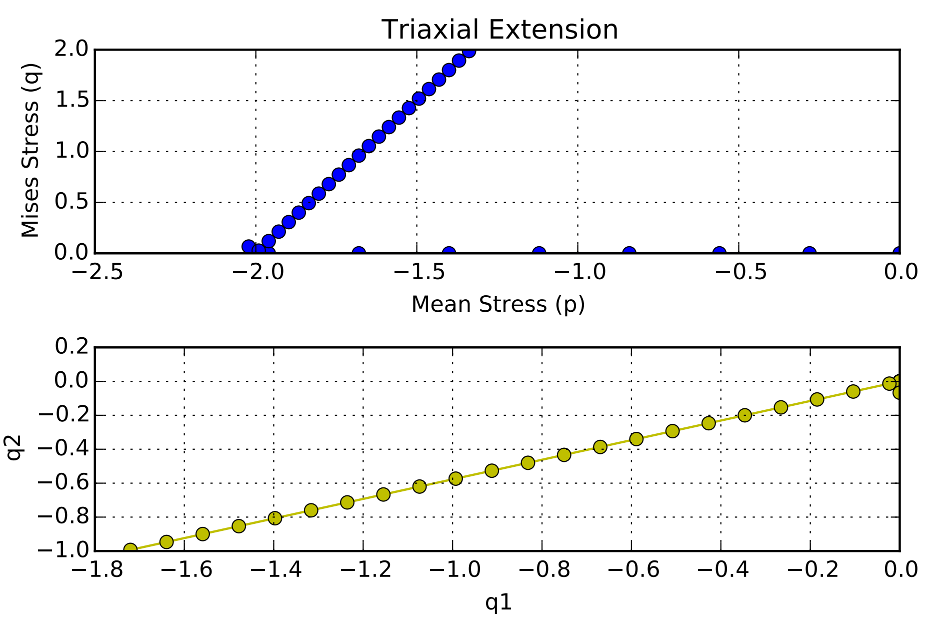

Figure 16: Mean Stress (p) Versus Mises Stress (q) and q_1 Versus q_2 Stress Paths.

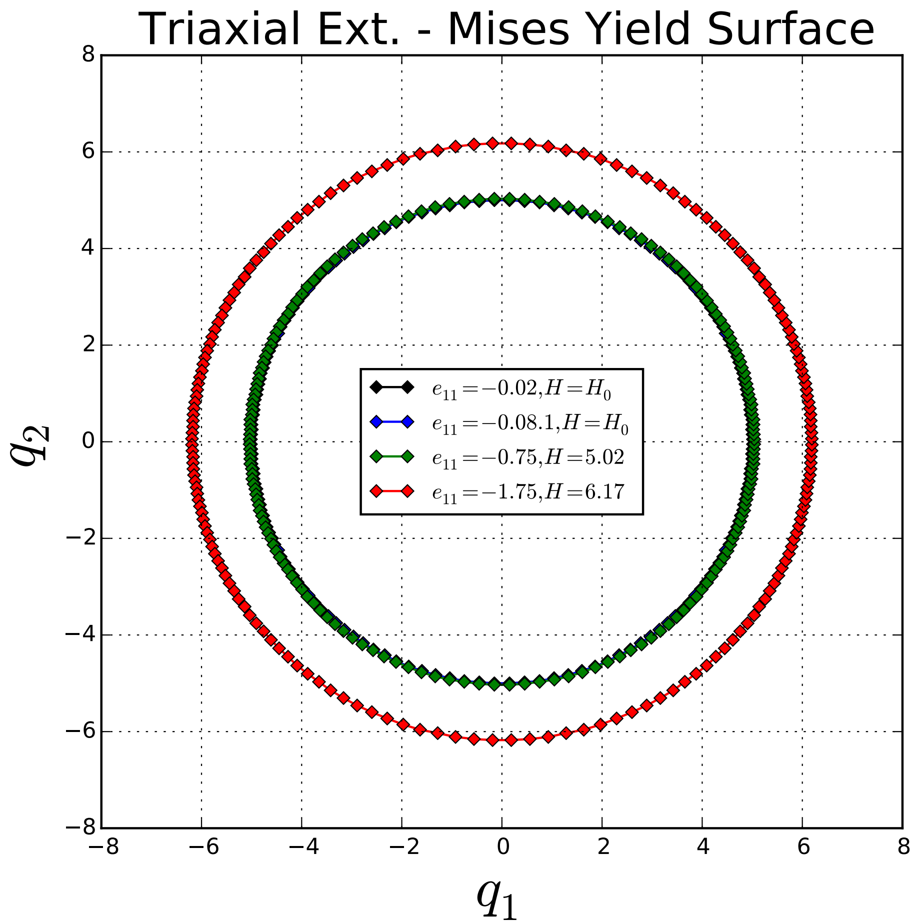

Figure 17: Yield Surface in the $q_1 - q_2$ Plane at Specific Points During Deformation.