

**8.4** Alumina specimens contain flaws introduced during processing; these flaws are, approximately, the grain size. Plot the fracture stress vs. grain size (for grains below 200  $\mu\text{m}$ ), knowing that the fracture toughness for alumina is equal to 4  $\text{MPa m}^{1/2}$ . Assume  $Y = 1$ .

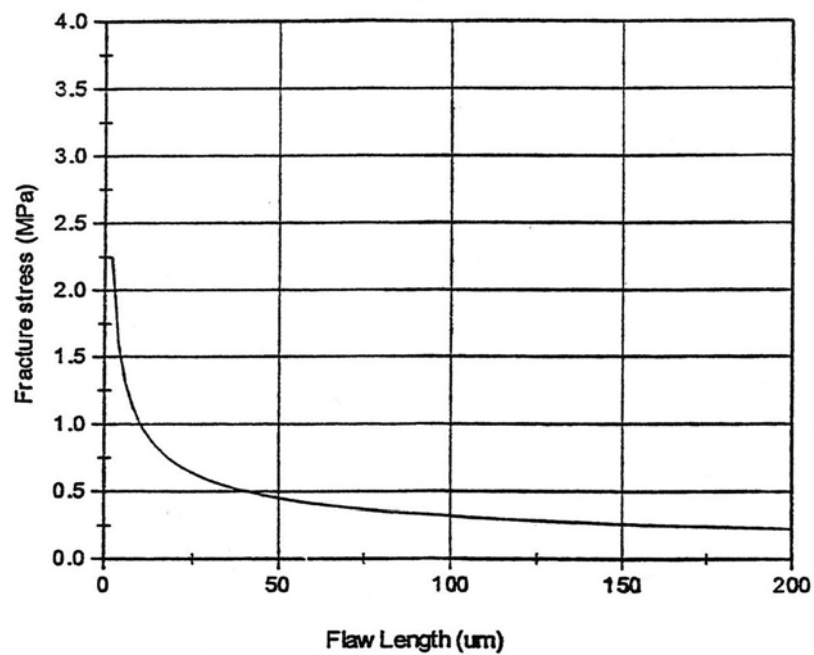
$$K_{Ic} = 4 \text{ MPa m}^{1/2}$$

$$K_{Ic} = \sigma_c \sqrt{\pi a}$$

$$D = 2a$$

$$\sigma_c = K_{Ic} / (\sqrt{\pi D/2}) = 4 / (\sqrt{\pi D/2}) = 3.19 / \sqrt{D}$$

The figure below shows the plot between the fracture stress and the grain size or the flaw size.



**8.11**  $Si_3N_4$  has a surface energy equal to  $30 J / m^2$  and in an atomic spacing  $a_o \approx 0.2 nm$ . Calculate the theoretical strength of this material (see Chapter 1), and compare the value you get with the one experimentally observed in tension testing ( $\sigma = 550 MPa$ ). Calculate the flaw size that would cause this failure stress.

From Table 2.8 (p. 116 of the textbook), the modulus of the silicon nitride is 320-365 GPa. The average value of the modulus is 342.5 GPa.

$$\begin{aligned}\sigma_{\max} &= \sqrt{\frac{E\gamma}{a_o}} \\ &= \sqrt{\frac{342.5 GPa \times 30 J / m^2}{0.2 nm}} \\ &= 227 GPa >> 550 MPa\end{aligned}$$

From Table 8.4 (p. 447 of the textbook), the toughness value of silicon nitride is  $5.0 MPam^{1/2}$ .

$$\begin{aligned}K_{IC} &= \sigma \sqrt{\pi a} \\ a &= \left( \frac{K_{IC}}{\sigma} \right)^2 \frac{1}{\pi} \\ &= \frac{(5 MPam^{1/2})^2}{(550 MPa)^2 \times 3.14} \\ &= 26.3 \mu m\end{aligned}$$

**8.12** The theoretical density of a polymer is  $1.21 \text{ g cm}^{-3}$ . By an optical technique, it was determined that the crazed region in this polymer had 40% porosity. What is the density of the crazed region? Can you estimate the elastic modulus of the crazed region as a percentage of the modulus of the normal polymer?

The density of the crazed region is given by a rule of mixtures:

$$\rho_c = \rho_m V_m + \rho_p V_p,$$

where  $\rho$  is the density,  $V$  is the volume fraction, and the subscripts  $m$  and  $p$  represent polymer and porosity, respectively. The density of porosity is zero. Thus,

$$\rho_c = \rho_m V_m + \rho_p V_p = 1.21 \times 0.4 = 0.48 \text{ g cm}^{-3}$$

The Young's modulus of crazed region can be written as (see p. 113 of the text)

$$E = E_0(1 - 1.9 p + 0.9 p^2),$$

where  $E_0$  is the modulus of the uncrazed region,  $E$  is the modulus of the craze, and  $p$  is volume fraction of porosity. We can recast this as fractional modulus

$$E/E_0 = (1 - 1.9 p + 0.9 p^2) = 1 - 1.9 \times 0.4 + 0.9 \times 0.4^2 = 1 - 0.76 + 0.144 = 0.384$$

**8.13** A polycarbonate sample showed a craze growth length and time relationship of

$$l = k \log(t/t_0),$$

where  $l$  is the craze length at time  $t$ ,  $t_0$  is the time crazing is initiated after the application of the load, and  $k$  is a constant. For a given temperature and stress, find the rate of craze growth. Comment on the implications of the relationship that you obtain.

We differentiate  $l = k \log(t/t_0)$  to obtain the craze growth rate

$$dl/dt = k/t.$$

Implications of this expression are that the craze growth varies inversely with time, i.e., craze rate decreases as the time increases.

**8.14** Craze formation is a plastic deformation mechanism that occurs without lateral contraction. What can you say about the Poisson's ratio of the crazed material?

Poisson's ratio,

$$\nu = -\epsilon_2/\epsilon_1,$$

where  $\epsilon_2$  and  $\epsilon_1$  are the strains in the transverse and longitudinal directions, respectively. The lateral contraction being zero means the transverse strain is zero. Thus, the Poisson's ratio must be zero for the crazed material.

**8.16** The famous accident of the NASA Challenger space shuttle that occurred on a cold night was caused by a faulty O-ring. Explain the accident.

This disaster resulted from the embrittlement of the elastomeric material used for making O-rings. An O-ring is a mechanical gasket made of rubber or elastomer in the shape of a ring, hence the name. It can provide sealing that can withstand pressure in the MPa range. O-rings are very commonly used to seal various mechanical designs.

On the morning of the shuttle lift-off, the temperature at the Kennedy space center dropped very low. The temperature was below the glass transition temperature of the elastomer used to make the O-ring (the material used was a fluoroelastomer), making it very brittle, resulting in failure to seal the joints of solid rocket boosters (SRBs), which in turn led to the tragic explosion of the Challenger shuttle. During his investigation of the Challenger failure, Richard Feynman pointed out out-gassing from the SRB at the joint between two segments, moments before the explosion. This was caused by the failure of the O-ring seal. The escaping high temperature gas impinged upon the external tank, causing the entire vehicle to be destroyed.

Since the Challenger accident, new procedures have been instituted to inspect the O-rings. For tracking purposes, the O-rings have with coding information about the batch and date, etc. There is more stringent quality assurance of the material. NASA has installed onboard heaters if the ambient temperature drops too low.