Analysis of an **orthotropic lamina**

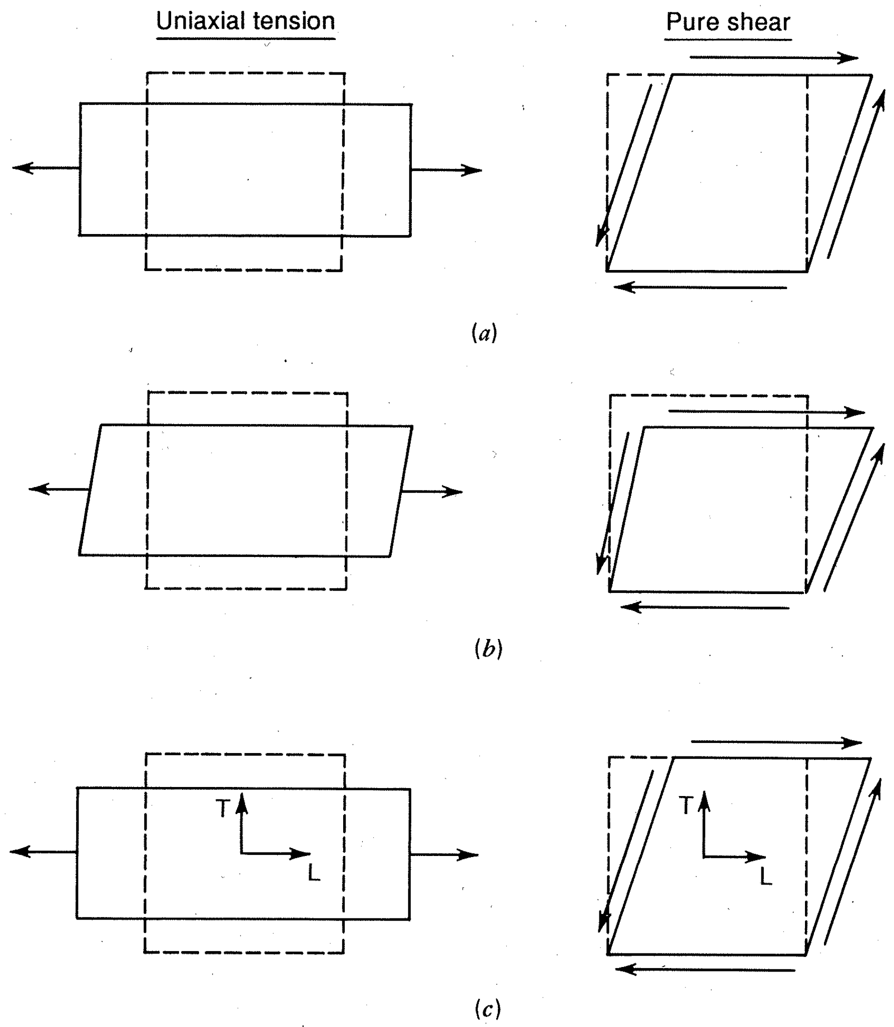
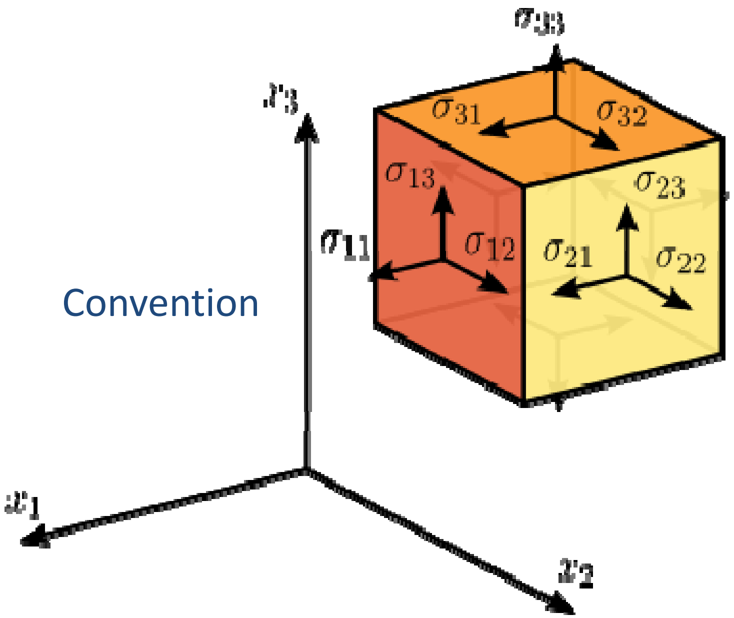


Figure 5.1. Deformation behavior of materials—response to uniaxial tension and pure shear: (*a*) isotropic material; (*b*) anisotropic and generally orthotropic material; (*c*) specially orthotropic material.

Stress tensor

Convention



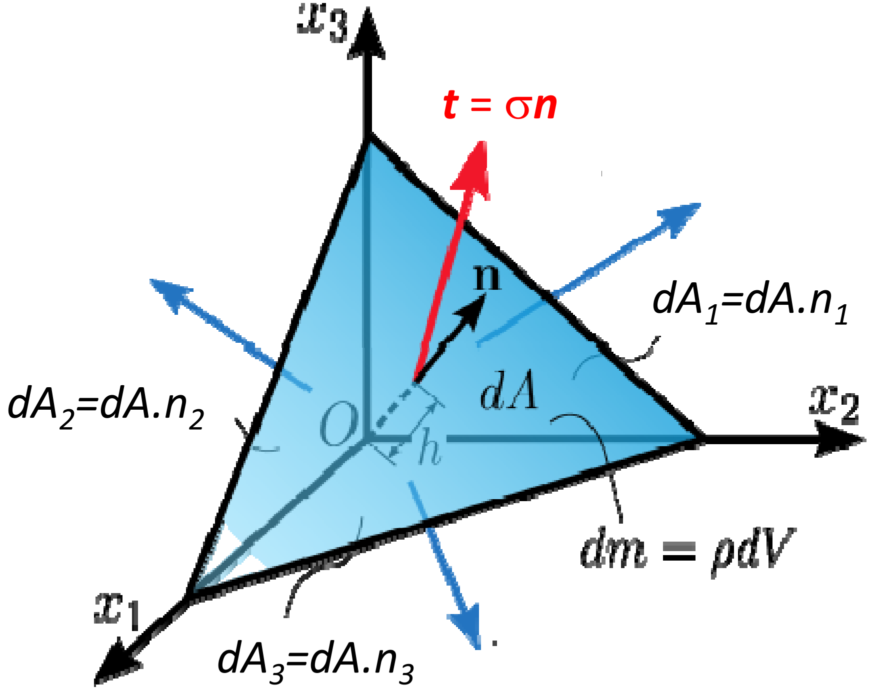
Equilibrium:

$$t_1 dA = (\sigma_{11}n_1 + \sigma_{21}n_2 + \sigma_{31}n_3)dA$$

$$t_2 dA = (\sigma_{12}n_1 + \sigma_{22}n_2 + \sigma_{32}n_3)dA$$

$$t_3 dA = (\sigma_{13}n_1 + \sigma_{23}n_2 + \sigma_{33}n_3)dA$$

$$\boxed{t_i = \sigma_{ij}n_j}$$



Principal stresses

The principal directions are solutions of $$\boxed{\sigma_{ij}n_j = \lambda n_j}$$

$$|\sigma_{ij} - \lambda\delta_{ij}| = \begin{vmatrix} \sigma_{11}-\lambda & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\lambda & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\lambda \end{vmatrix} = -\lambda^3 + I_1\lambda^2 - I_2\lambda + I_3 = 0$$

Stress invariants:

$$\left[\begin{aligned} I_1 &= \sigma_{11} + \sigma_{22} + \sigma_{33} \\ &= \sigma_{kk} \\ I_2 &= \begin{vmatrix} \sigma_{22} & \sigma_{23} \\ \sigma_{32} & \sigma_{33} \end{vmatrix} + \begin{vmatrix} \sigma_{11} & \sigma_{13} \\ \sigma_{31} & \sigma_{33} \end{vmatrix} + \begin{vmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{vmatrix} \\ &= \sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{11}\sigma_{33} - \sigma_{12}^2 - \sigma_{23}^2 - \sigma_{31}^2 \\ &= \frac{1}{2}(\sigma_{ii}\sigma_{jj} - \sigma_{ij}\sigma_{ji}) \\ I_3 &= \det(\sigma_{ij}) \\ &= \sigma_{11}\sigma_{22}\sigma_{33} + 2\sigma_{12}\sigma_{23}\sigma_{31} - \sigma_{12}^2\sigma_{33} - \sigma_{23}^2\sigma_{11} - \sigma_{31}^2\sigma_{22} \end{aligned}\right.$$

In a coordinate system oriented along the principal directions, the stress tensor is diagonal:

$$\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

$$\boxed{\begin{aligned} I_1 &= \sigma_1 + \sigma_2 + \sigma_3 \\ I_2 &= \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \\ I_3 &= \sigma_1\sigma_2\sigma_3 \end{aligned}}$$

Stress invariants in principal coordinates

Generalized Hooke's law

$$\boxed{\sigma_{ij} = E_{ijkl}\epsilon_{kl}}$$

4^{th} order tensor of elastic constants

$$E_{ijkl} = E_{ijlk}$$

Because of the symmetry of the strain tensor

$$E_{ijkl} = E_{jikl}$$

Because of the symmetry of the stress tensor

$$E_{ijkl} = E_{klij}$$

Constitutive Equation: $\dfrac{\partial U}{\partial \epsilon_{ij}} = \sigma_{ij}$

$$\frac{\partial U}{\partial \epsilon_{ij}} = E_{ijkl}\epsilon_{kl}$$

$$\frac{\partial}{\partial \epsilon_{kl}}\left(\frac{\partial U}{\partial \epsilon_{ij}}\right) = E_{ijkl}$$

The order of partial differentiation May be changed

$$\frac{\partial}{\partial \epsilon_{ij}}\left(\frac{\partial U}{\partial \epsilon_{kl}}\right) = \frac{\partial}{\partial \epsilon_{kl}}\left(\frac{\partial U}{\partial \epsilon_{ij}}\right)$$

Because of the 3 symmetry relationships, the number of independent elastic constants is reduced from **3^4=81** to **21** in the most general anisotropic material

Change of coordinates in the elastic constants

Let two coordinate systems *x* and *x'* related by the rotation matrix $A = a_{ij}$

$$x = Ax' \quad x' = A^T x \quad x_i = a_{ij}\, x'_j \quad x'_i = a_{ji}\, x_j$$

Change of coordinates of the **stress tensor**:

$$t = \sigma n$$

$$t' = A^T t \; = \; A^T \sigma n \; = A^T \sigma A \, n'$$

$$\boxed{\begin{gathered} \sigma' = A^T \sigma A \\ \sigma'_{ij} = a_{mi} a_{nj} \sigma_{mn} \end{gathered}}$$

Applies **to any second order tensor**
(same rule for the strain tensor)

Tensor of elastic constant: $\sigma_{ij} = E_{ijkl}\,\varepsilon_{kl}$

$$\sigma'_{mn} = a_{im}\,a_{jn}\,\sigma_{ij}$$

$$= a_{im}\,a_{jn}\,E_{ijkl}\,\varepsilon_{kl}$$

$$\varepsilon_{kl} = a_{ku}\,a_{lv}\,\varepsilon'_{uv}$$

$$\sigma'_{mn} = a_{im}\,a_{jn}\,a_{ku}\,a_{lv}\,E_{ijkl}\,\varepsilon'_{uv}$$

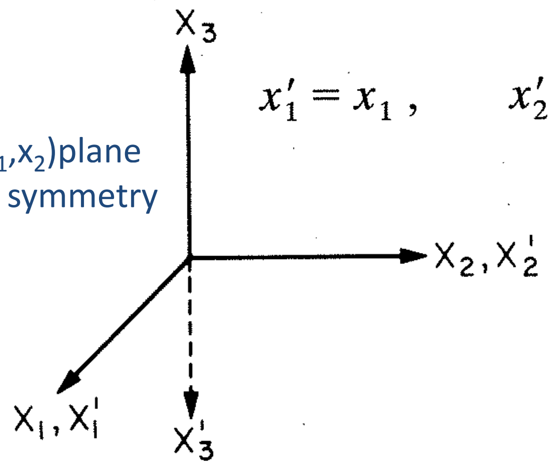
$$\boxed{E'_{mnuv} = a_{im}\,a_{jn}\,a_{ku}\,a_{lv}\,E_{ijkl}}$$

Orthotropic composite: 3 axes of symmetry

The elastic constants do not change under coordinate transformations that preserve symmetry

$$\boxed{E'_{ijkl} = E_{ijkl}}$$

(x_1,x_2)plane of symmetry



$$x_1' = x_1 \,, \qquad x_2' = x_2 \,, \qquad x_3' = -x_3$$

Direction cosines:

	x_1'	x_2'	x_3'
x_1	$a_{11} = 1$	$a_{12} = 0$	$a_{13} = 0$
x_2	$a_{21} = 0$	$a_{22} = 1$	$a_{23} = 0$
x_3	$a_{31} = 0$	$a_{32} = 0$	$a_{33} = -1$

One finds:

$$E'_{1111} = E_{ijkl} a_{i1} a_{j1} a_{k1} a_{l1} = E_{1111}$$

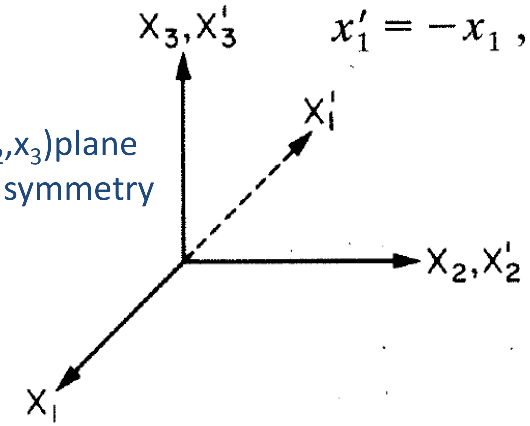
$$E'_{1112} = E_{ijkl} a_{i1} a_{j1} a_{k1} a_{l2} = E_{1112}$$

$$E'_{1113} = E_{ijkl} a_{i1} a_{j1} a_{k1} a_{l3} = -E_{1113}$$

Must be = 0

Similarly, 8 constants must be equal to 0:

$$\boxed{E_{1113},\ E_{2223},\ E_{1123},\ E_{2213},\ E_{1213},\ E_{1223},\ E_{1333},\ E_{2333}}$$



(x_2,x_3)plane
of symmetry

$$x'_1 = -x_1\,, \qquad x'_2 = x_2\,, \qquad x'_3 = x_3$$

Direction cosines:

	x'_1	x'_2	x'_3
x_1	$a_{11} = -1$	$a_{12} = 0$	$a_{13} = 0$
x_2	$a_{21} = 0$	$a_{22} = 1$	$a_{23} = 0$
x_3	$a_{31} = 0$	$a_{32} = 0$	$a_{33} = 1$

The following contants
Must also be equal to 0: $\boxed{E_{1233},\ E_{1323},\ E_{1222},\ E_{1112}}$

There is no additional condition coming from the third plane of symmetry (x_1,x_3).
Overall, there are 21-12= **9 independent elastic constants for an orthotropic material**.

Orthotropic materials: 9 independent elastic constants

$$(E_{ijkl}) = \begin{bmatrix} E_{1111} & E_{1122} & E_{1133} & 0 & 0 & 0 \\ E_{1122} & E_{2222} & E_{2233} & 0 & 0 & 0 \\ E_{1133} & E_{2233} & E_{3333} & 0 & 0 & 0 \\ 0 & 0 & 0 & E_{2323} & 0 & 0 \\ 0 & 0 & 0 & 0 & E_{1313} & 0 \\ 0 & 0 & 0 & 0 & 0 & E_{1212} \end{bmatrix}$$

Hooke's law may be written in *matrix* form

$$\sigma_i = Q_{ij}\epsilon_j \qquad i, j = 1, 2, 3, 4, 5, 6$$

Vector of engineering stress components

Engineering strain components

Stiffness matrix

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix}$$

$$\gamma_{ij} = 2\varepsilon_{ij}$$

[the axes 1,2,3 coincide with the natural (orthotropy) axes of the material]

In two dimensions:

Stiffness matrix

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

Compliance matrix

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

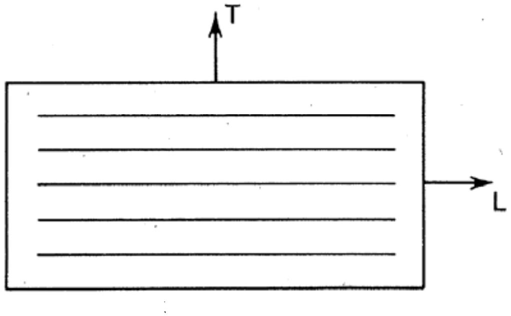
$$Q_{11} = \frac{S_{22}}{S_{11}S_{22} - S_{12}^2}$$

$$Q_{22} = \frac{S_{11}}{S_{11}S_{22} - S_{12}^2}$$

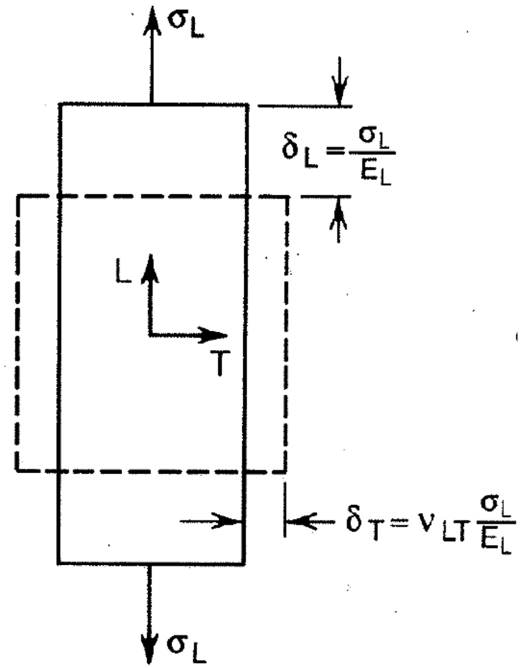
$$Q_{12} = -\frac{S_{12}}{S_{11}S_{22} - S_{12}^2}$$

$$Q_{66} = \frac{1}{S_{66}}$$

Stress-strain relations and engineering constants for orthotropic lamina



The compliance matrix may be constructed column by column by considering 3 load cases:



1. When σ_L is the only nonzero stress $(\sigma_T = \tau_{LT} = 0)$

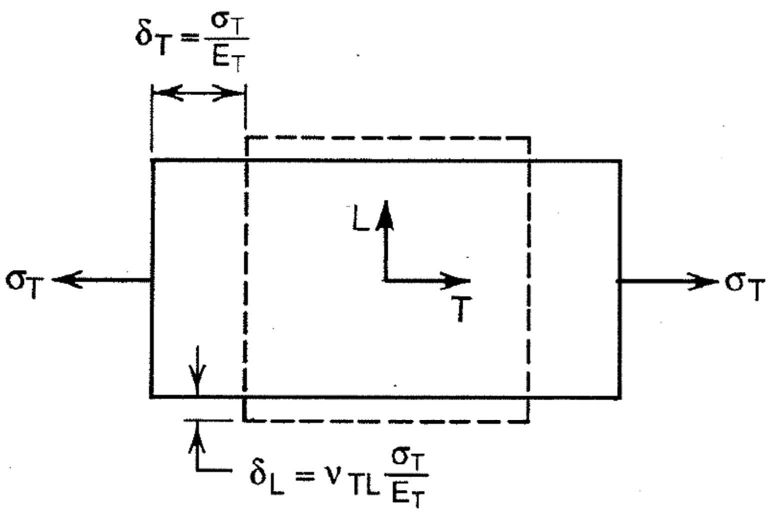
$$\varepsilon_L = \frac{\sigma_L}{E_L}$$

$$\varepsilon_T = -\nu_{LT}\varepsilon_L = -\nu_{LT}\frac{\sigma_L}{E_L}$$

$$\gamma_{LT} = 0$$

One gets the first column of the compliance matrix

2. When σ_T is the only nonzero stress $(\sigma_L = \tau_{LT} = 0)$:



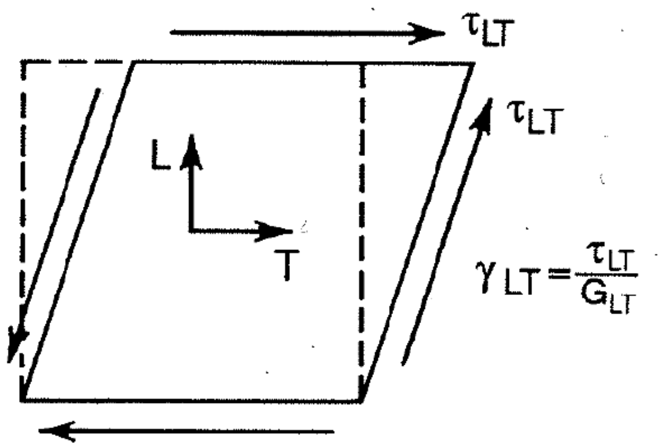
$$\varepsilon_T = \frac{\sigma_T}{E_T}$$

$$\varepsilon_L = -\nu_{TL}\varepsilon_T = -\nu_{TL}\frac{\sigma_T}{E_T}$$

$$\gamma_{LT} = 0$$

2nd column of the Compliance matrix

3. When τ_{LT} is the only nonzero stress $(\sigma_L = \sigma_T = 0)$



$$\varepsilon_L = 0$$

$$\varepsilon_T = 0$$

$$\gamma_{LT} = \frac{\tau_{LT}}{G_{LT}}$$

3rd column of the Compliance matrix

Orthotropic lamina in natural axes

1. Compliance matrix

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

$$S_{11} = \frac{1}{E_L}$$

$$S_{22} = \frac{1}{E_T}$$

$$S_{12} = -\frac{\nu_{LT}}{E_L} = -\frac{\nu_{TL}}{E_T}$$

$$S_{66} = \frac{1}{G_{LT}}$$

For an isotropic lamina,
$E_L = E_T$, $G = E/2(1+\nu)$

2. Stiffness matrix

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

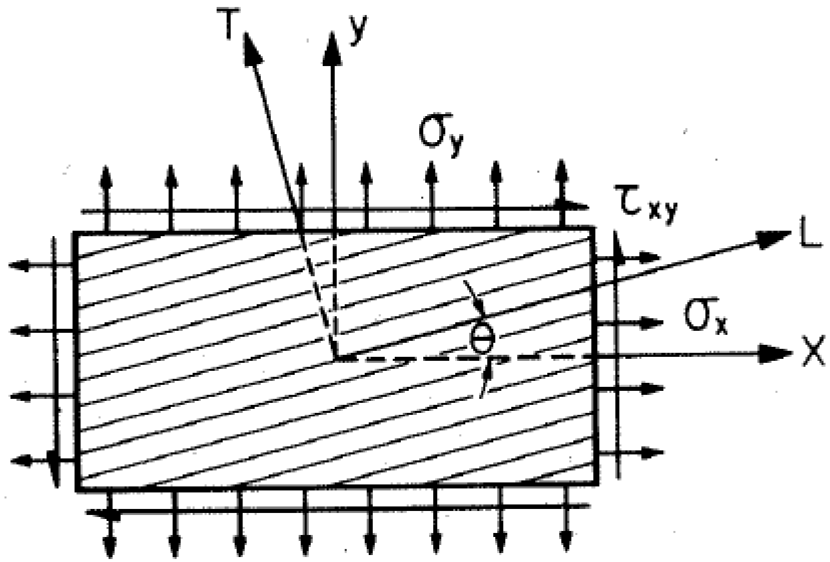
$$Q_{11} = \frac{E_L}{1 - \nu_{LT}\nu_{TL}}$$

$$Q_{22} = \frac{E_T}{1 - \nu_{LT}\nu_{TL}}$$

$$Q_{12} = \frac{\nu_{LT}E_T}{1 - \nu_{LT}\nu_{TL}} = \frac{\nu_{TL}E_L}{1 - \nu_{LT}\nu_{TL}}$$

$$Q_{66} = G_{LT}$$

Change of reference frame



We seek the transformation matrix [T]

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = [T] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \epsilon_L \\ \epsilon_T \\ \frac{1}{2}\gamma_{LT} \end{Bmatrix} = [T] \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \frac{1}{2}\gamma_{xy} \end{Bmatrix}$$

Change of coordinates
For a 2nd order tensor: $\boxed{\sigma' = A^T \sigma A}$

$$\begin{bmatrix} \sigma_L & \tau_{LT} \\ \tau_{LT} & \sigma_T \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

[T] is not a rotation matrix !!

$$[T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [\mathrm{T}]^{-1} \begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix}$$

$$[\mathrm{T}]^{-1} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & -2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & 2\sin\theta\cos\theta \\ \sin\theta\cos\theta & -\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

$$\boxed{[T(\theta)]^{-1} = [T(-\theta)]}$$

Stress-strain relationship

In L-T axes:

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & 2Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_L \\ \epsilon_T \\ \frac{1}{2}\gamma_{LT} \end{Bmatrix}$$

In arbitrary axes:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [\mathrm{T}]^{-1} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & 2Q_{66} \end{bmatrix} [\mathrm{T}] \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \frac{1}{2}\gamma_{xy} \end{Bmatrix}$$

Stiffness matrix in arbitrary axes

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{aligned}
\bar{Q}_{11} &= Q_{11}\cos^4\theta + Q_{22}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta \\
\bar{Q}_{22} &= Q_{11}\sin^4\theta + Q_{22}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta \\
\bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\cos^4\theta + \sin^4\theta) \\
\bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta + \cos^4\theta) \\
\bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})\cos^3\theta\sin\theta - (Q_{22} - Q_{12} - 2Q_{66})\cos\theta\sin^3\theta \\
\bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})\cos\theta\sin^3\theta - (Q_{22} - Q_{12} - 2Q_{66})\cos^3\theta\sin\theta
\end{aligned} \tag{5.61}$$

Compliance matrix in arbitrary axes:

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

$$S_{11} = \frac{1}{E_L}$$

$$S_{22} = \frac{1}{E_T}$$

$$S_{12} = -\frac{\nu_{LT}}{E_L} = -\frac{\nu_{TL}}{E_T}$$

$$S_{66} = \frac{1}{G_{LT}}$$

$$\begin{aligned}
\bar{S}_{11} &= S_{11}\cos^4\theta + S_{22}\sin^4\theta + (2S_{12} + S_{66})\sin^2\theta\cos^2\theta \\
\bar{S}_{22} &= S_{11}\sin^4\theta + S_{22}\cos^4\theta + (2S_{12} + S_{66})\sin^2\theta\cos^2\theta \\
\bar{S}_{12} &= (S_{11} + S_{22} - S_{66})\cos^2\theta\sin^2\theta + S_{12}(\cos^4\theta + \sin^4\theta) \\
\bar{S}_{66} &= 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})\cos^2\theta\sin^2\theta + S_{66}(\cos^4\theta + \sin^4\theta) \\
\bar{S}_{16} &= (2S_{11} - 2S_{12} - S_{66})\cos^3\theta\sin\theta - (2S_{22} - 2S_{12} - S_{66})\cos\theta\sin^3\theta \\
\bar{S}_{26} &= (2S_{11} - 2S_{12} - S_{66})\cos\theta\sin^3\theta - (2S_{22} - 2S_{12} - S_{66})\cos^3\theta\sin\theta
\end{aligned} \tag{5.63}$$

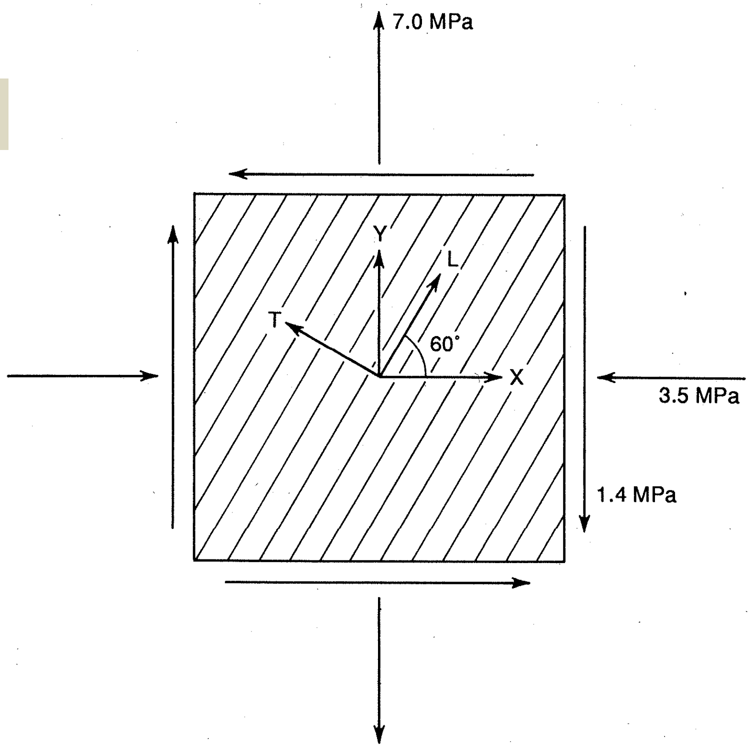
Example: find the strains in the lamina :θ=60°

Elastic constants:

$$E_L = 14 \text{ GPa}$$
$$E_T = 3.5 \text{ GPa}$$
$$G_{LT} = 4.2 \text{ GPa}$$
$$\nu_{LT} = 0.4$$
$$\nu_{TL} = 0.1$$

Stresses:

$$\sigma_x = -3.5 \text{ MPa}$$
$$\sigma_y = 7.0 \text{ MPa}$$
$$\tau_{xy} = -1.4 \text{ MPa}$$



Step 1: compute the stresses in orthotropy axes

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = [T] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

$$[T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

$$\begin{Bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{Bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & \frac{\sqrt{3}}{2} \\ \frac{3}{4} & \frac{1}{4} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{4} & \frac{\sqrt{3}}{4} & -\frac{1}{2} \end{bmatrix} \begin{Bmatrix} -3.5 \\ 7.0 \\ -1.4 \end{Bmatrix} = \begin{Bmatrix} 3.16 \\ 0.34 \\ 5.24 \end{Bmatrix}$$

Step 2: compute the strains in natural axes:

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

$$S_{11} = \frac{1}{E_L}$$

$$S_{22} = \frac{1}{E_T}$$

$$S_{12} = -\frac{\nu_{LT}}{E_L} = -\frac{\nu_{TL}}{E_T}$$

$$S_{66} = \frac{1}{G_{LT}}$$

$$\sigma_L = 3.16 \text{ MPa}$$
$$\sigma_T = 0.34 \text{ MPa}$$
$$\tau_{LT} = 5.24 \text{ MPa}$$

$$E_L = 14 \text{ GPa}$$
$$E_T = 3.5 \text{ GPa}$$
$$G_{LT} = 4.2 \text{ GPa}$$
$$\nu_{LT} = 0.4$$
$$\nu_{TL} = 0.1$$

$$\varepsilon_L = \frac{3.16 \times 10^6}{14 \times 10^9} - 0.1\left(\frac{0.34 \times 10^6}{3.5 \times 10^9}\right) = 216 \times 10^{-6}$$

$$\varepsilon_T = \frac{0.34 \times 10^6}{3.5 \times 10^9} - 0.4\left(\frac{3.16 \times 10^6}{14 \times 10^9}\right) = 6.9 \times 10^{-6}$$

$$\gamma_{LT} = \frac{5.24 \times 10^6}{4.2 \times 10^9} = 1248 \times 10^{-6}$$

Step 3: compute the strains in the axes *(x,y)*

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \frac{1}{2}\gamma_{xy} \end{Bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{4} & \frac{3}{4} & -\frac{\sqrt{3}}{2} \\ \frac{3}{4} & \frac{1}{4} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{4} & -\frac{\sqrt{3}}{4} & -\frac{1}{2} \end{bmatrix}}_{[T(-\theta)]} \begin{Bmatrix} 216 \times 10^{-6} \\ 6.9 \times 10^{-6} \\ \underbrace{624 \times 10^{-6}}_{\gamma_{LT}/2} \end{Bmatrix} = \begin{Bmatrix} -481 \times 10^{-6} \\ 704 \times 10^{-6} \\ -221 \times 10^{-6} \end{Bmatrix}$$

Engineering constants

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

The compliance matrix in arbitrary axes may be written:

$$\epsilon_x = \frac{\sigma_x}{E_x} - \nu_{yx}\frac{\sigma_y}{E_y} - m_x \frac{\tau_{xy}}{E_L}$$

$$\epsilon_y = \frac{\sigma_y}{E_y} - \nu_{xy}\frac{\sigma_x}{E_x} - m_y \frac{\tau_{xy}}{E_L}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G_{xy}} - m_x \frac{\sigma_x}{E_L} - m_y \frac{\sigma_y}{E_L}$$

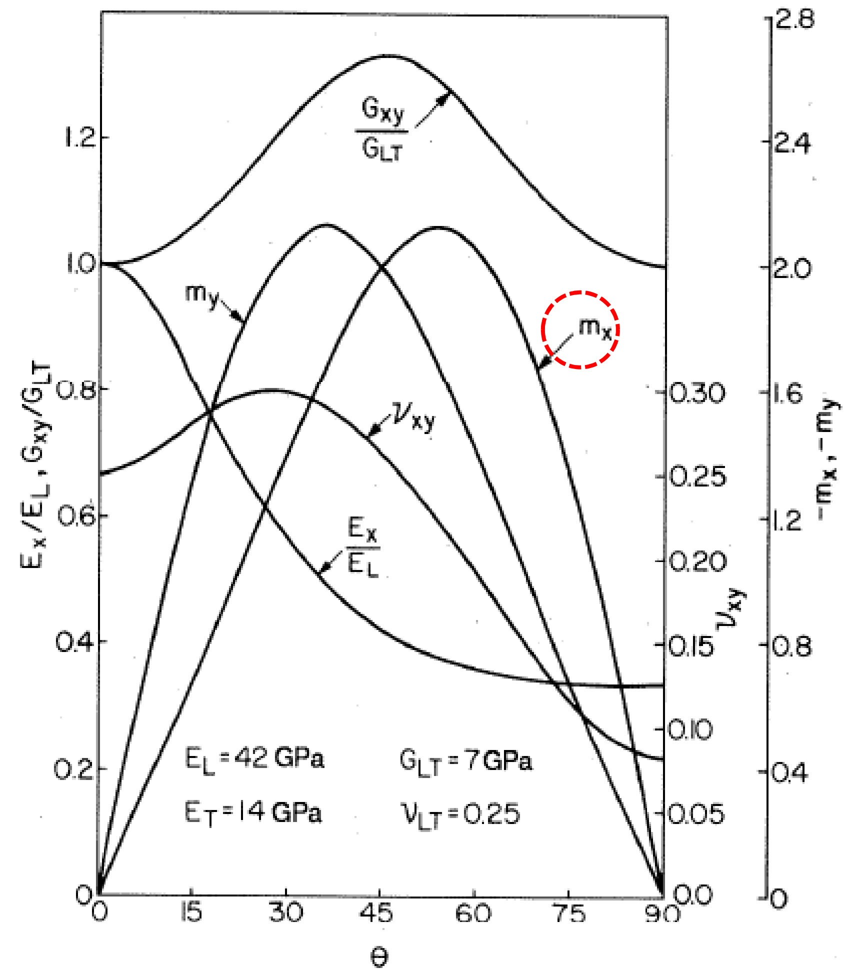
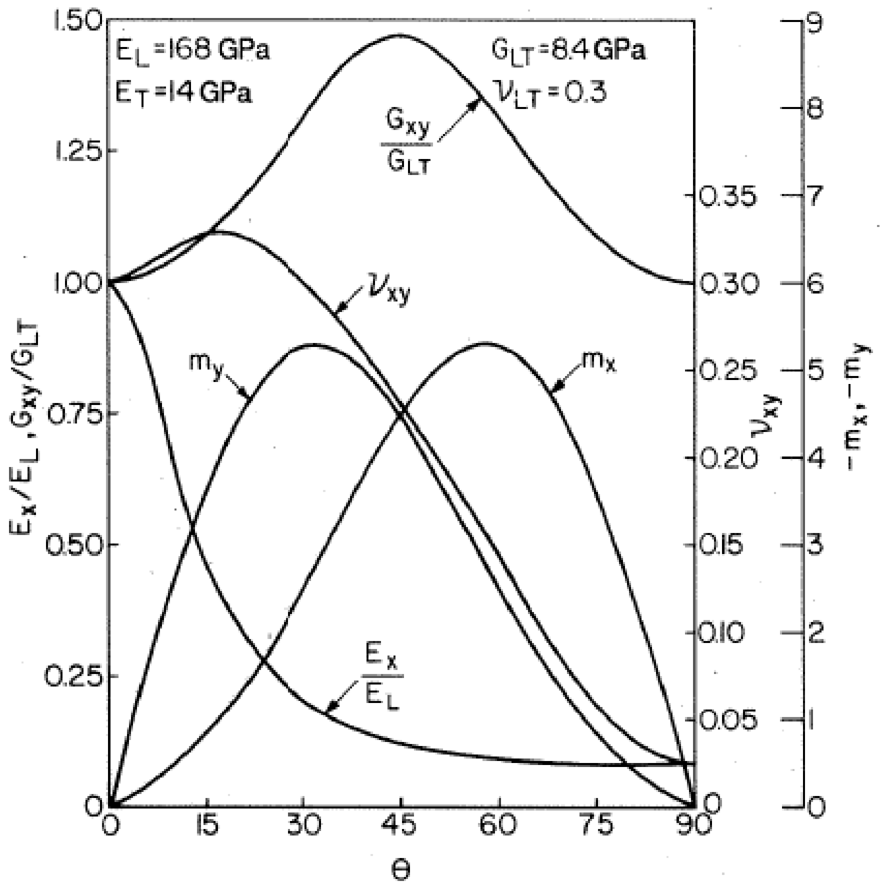


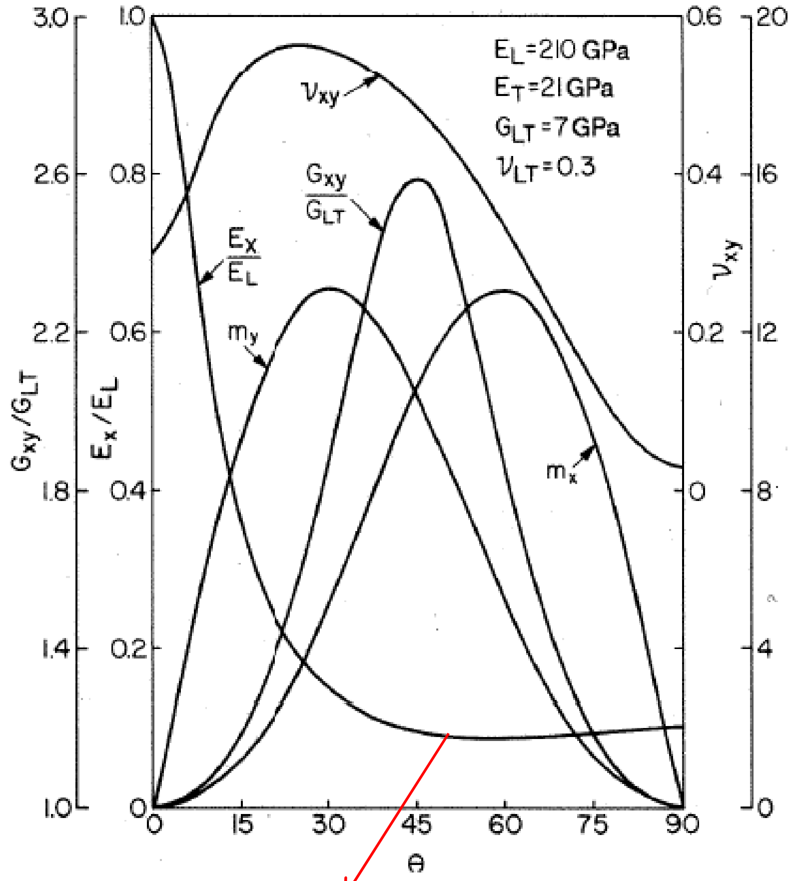
Figure 5.9. Variation in elastic constants of glass–epoxy systems.

All the elastic constants may be expressed in terms of the 4 constants: E_L, E_T, G_{LT}, ν_{LT}.

Graphite-epoxy system



Boron-epoxy system



$E_x < E_T$

Balanced lamina: $E_T=E_L, \nu_{LT}=\nu_{TL}$

Not isotropic !!

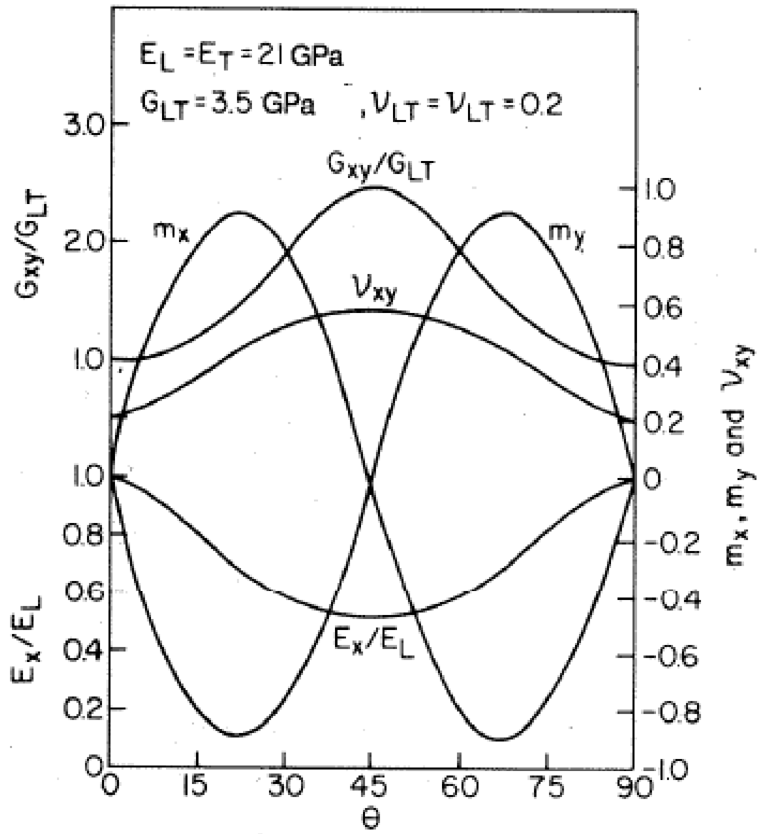


Figure 5.12. Variation in elastic constants of balanced lamina.