Problem 2 reflects the primary task of this assignment, which is to develop a working finite difference code that includes the capability to analyze numerically the rate of convergence with mesh refinement. Then you are asked to explore a topic in some depth as reflected by either Problem 3 or Problem 4. In summary, do Problems 1, 2, and 3 **or** 4.

1. Provide a written summary of the theoretical formulation of the material related to your numerical investigations of finite difference solutions to the ODE $(k\phi,_x),_x + f = 0$.

2. (a) Choose data for a relatively simple problem for which you know the analytical solution and data with the following restrictions: (i) the coefficient function is constant, (ii) One boundary condition consists of a prescribed value for the primary variable; the other boundary condition consists of a prescribed value of the gradient of the primary variable, and (iii) the forcing function results in a solution of sufficient continuity that the part of your theoretical summary that provides the rate of converence is applicable.
(b) Write a finite difference algorithm based on a uniform mesh that provides approximate solutions.
(c) Provide plots that compare numerical solutions with the analytical solution for different mesh sizes.
(d) Provide a plot that shows the numerical rate of convergence and indicate how it compares with the theoretical rate.

3. Consider the modification to the force vector provided in class that has the potential of increasing the rate of convergence. Obtain the theoretical rate of convergence for a uniform mesh, and see if this rate is obtained numerically.

or

4. Extend the capability of your program and do numerical investigations, i.e., compare numerical solutions with analytical solutions as the mesh is refined, for at least two of the following three cases:
(i) a point forcing function.
(ii) a step change in the coefficient function.
(iii) a coefficient function that varies linearly with x.