ME 562 - Assignment 4

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Abstract

The objectve of this assignment was to implement a viscoelastic subroutine for use with the driver program. Multiple loading paths, such as creep, relaxation, and cyclic loading were simulated in an attempt to verify the numerical algorithms. Additionally, subroutines were written to provide values of defined yield surfaces when a material subjected to a prescribed state of stress.

1 Write Viscoelastic Subroutine and Verification

1.1 Add Viscoelastic Subroutine to Driver Program

A viscoelastic subroutine that numerically calculates results of the Standard Viscoelastic Model (both linear and nonlinear) was created. The current formulation of subroutine is in one dimension only. The subroutine was titled "Visco_1d" and is called by the driver.

- Visco_1d(matpropv, strn_inc, strnv, estrnv, bstrnv, strsv, path, SM, irow, t_inc, alpha)

A brief description of the calling arguments for Visco_1d is provided below:

1. matpropv: vector of material properties.
2. strn_inc: prescribed strain increment.
3. strnv: vector of current total strain values.
4. estrnv: vector of current elastic strain values.
5. bstrnv: vector of current back strain values (i.e. viscoelastic or time-dependent strain values).
6. path: specific loading path.
7. creep and relaxation paths require specific designation, while cyclic loading requires the description of a strain function).
8. SM: matrix of stored values.
9. irow: identificaiton of current step, used for storing values in the correct row of SM.
10. t_inc: size of time increment, which is used for numerical integration. Typical value is $t_{inc} = \frac{\tau_1^b}{10}$.
11. alpha: prescribes the type of numerical integration.
    (a) $\alpha = 0 \rightarrow$ explicit integration method that is conditionally stable (Euler-Forward method).
    (b) $\alpha = 1 \rightarrow$ implicit integration method that unconditionally stable (Euler-Backward method).
    (c) $0 < \alpha < 1 \rightarrow$ semi-implicit Euler methods, typically $\alpha = 0.5$ provides an increased rate of convergence with reasonable stability.

1.2 General Formulation of One-Dimensional Viscoelasticity

The following set of equations outline the Standard Viscoelastic Model. Letting c^* equal zero results in the Standard Linear Viscoelastic Model. In addition to the elastic material parameters (contained in E), an additional set of material parameters ($\tau$and τ^b) have been added to describe the viscous (time dependent) strain, referred to here as backstrain (e^b).

$$\begin{aligned}
\text{Stress:} \quad & \sigma = E(e - e^b) \\
\text{Backstrain Rate:} \quad & \dot{e}^b = (1 + \bar{\sigma}_{ve})\left(\frac{1}{\tau}e - \frac{1}{\tau^b}e^b\right) \\
\text{Viscoelastic Effective Stress:} \quad & \bar{\sigma}_{ve} = \frac{\left((\sigma - \sigma_r)^2\right)^{0.5}}{\sigma^*} = c^*\left((\sigma - \sigma_r)^2\right)^{0.5} \\
\text{Relaxation Stress:} \quad & \sigma_r = E\left(1 - \frac{\tau^b}{\tau}\right)e
\end{aligned}$$

1.3 Verification of Relaxation Problem

The analytical solution to the relaxation (constant strain) problem was utilized in the verification of the Visco_1d subroutine. The problem prescribed a constant total strain of $e_0 = 0.1$, and the associated stress was calculated. Material parameters Y, τ and τ^b were assigned values of 5, 101 and 100, respectively. Results of this verification are shown below in Figure 1. The analytical solution to the to the SLVM relaxation problem is:

$$\sigma(t) = \tfrac{\sigma_0}{\tau}\left(\tau - \tau^b + \tau^b exp\left(\tfrac{-t}{\tau^b}\right)\right)$$

$$\sigma_0 = Y e_0$$

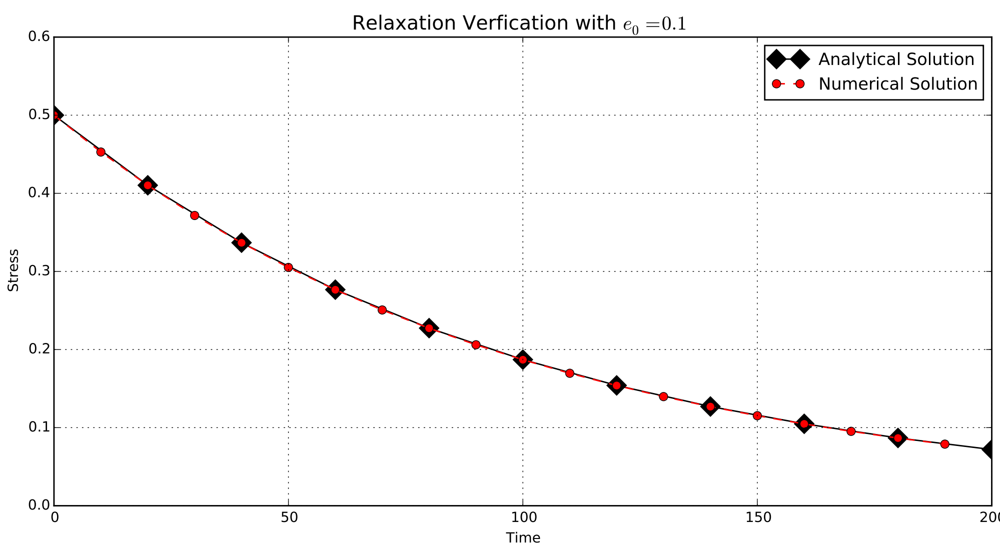

Figure 1: Analytical and Numerical Results to a Relaxation Problem.

1.4 Verification of Creep Problem

The analytical solution to the creep (constant stress) problem was also used for verification. This problem prescribed a constant stress of $\sigma_0 = 0.5$, and the associated strains were calculated. Material parameters Y,

τ, and τ^b were again assigned values of 5, 10.1 and 10, respectively. Results of this verification are shown below in Figure 2.

$$e(t) = \frac{e_0}{\tau - \tau^b}\left(\tau - \tau^b exp\left(\frac{-t}{\tau^*}\right)\right)$$
$$e_0 = \sigma_0 / Y$$
$$\tau^* = \frac{\tau\tau^b}{\tau - \tau^b}$$

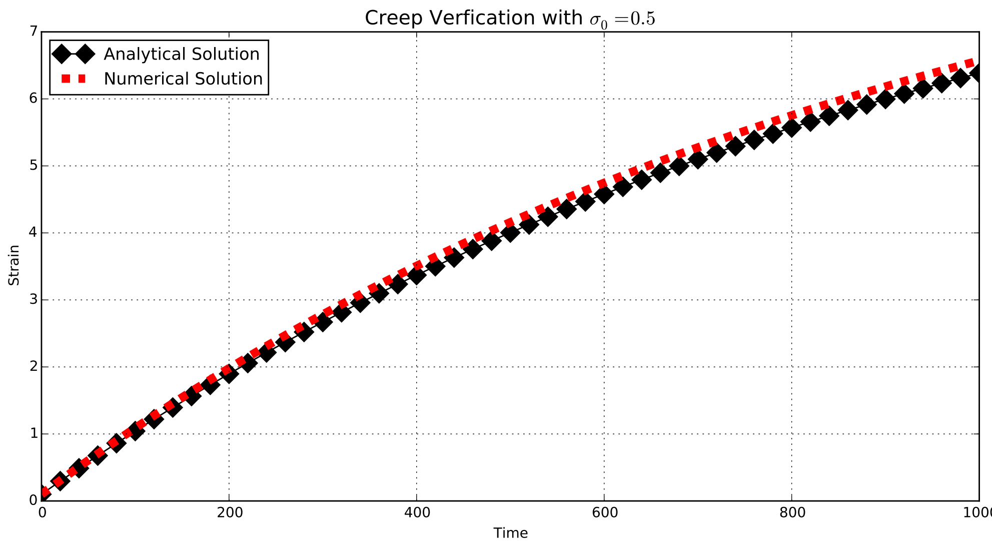

Figure 2: Analytical and Numerical Results to a Creep Problem.

The definition of a creep problem is that of a prescribed constant stress; however, the Driver program has been written in terms of a prescribed strain increment. Therefore, path specific modifications were made in order to evaluate a creep path. Those modifications include:

- prescribe a constant stress value
- assume all strains resulting from the prescribed stress were elastic
- calculate a stress value based on this value of elastic strain, which is the typical method for a strain prescribed problem
- define an an error tolerance between the prescribed and calculate stress values and compare
- if the difference between the two stress values is greater than the tolerance, then allocate a portion of the elastic strain to the back strain while keeping the total strain constant
  - Stress is increased by the addition of elastic strain; therefore, by decreasing the elastic strain the calculated stress value will also decrease
- continue this process until the difference between the two stress values is within the desired tolerance.

Figure 3 shows the variation of the actual stress, where the prescribed stress value was 0.5. The calculated stress starts at a value of 4.999996 + 0.5e-7 (tolerance limit) and gradually approaches the prescribed value.

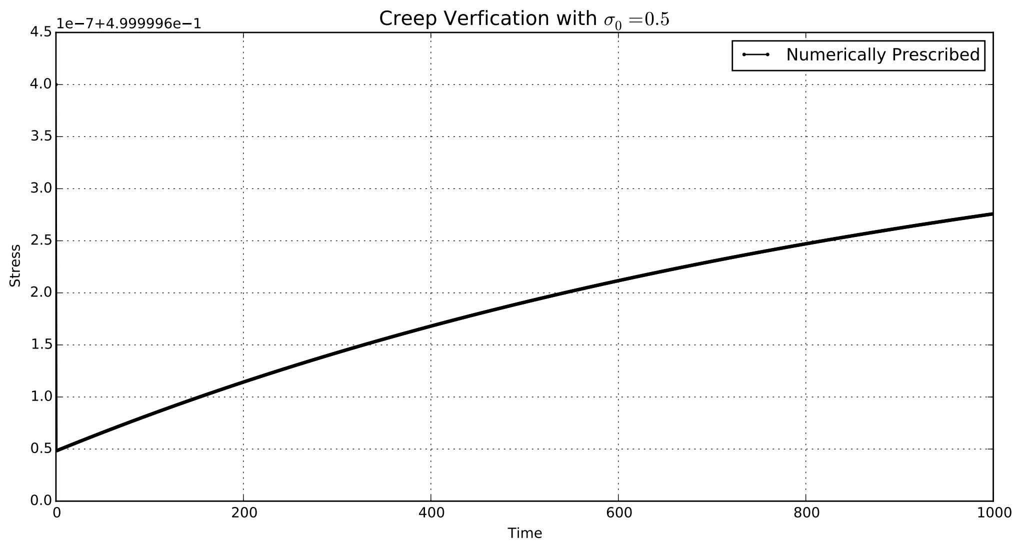

Figure 3: Variation of Calculated Stress During Creep Problem.

2 Show the Effects of Choosing Different Values of Backstrain Terms $\tau$and τ^b

Influence of the independent backstrain parameters τ and τ^b on creep and relaxation simulations for the linear viscoelastic model were analyzed. The choice of τhas a large effect on the predicted strain and stress rates for both creep and relaxation simulations, respectively (Figures 4 and 5). This is because τ^b must be chosen such that is has a value that is less than the value of τ. That is, for stability to be maintained, the restriction of $\tau^b < \tau$ has been applied, the reason for this restriction is evident in the definition of τ^* above.

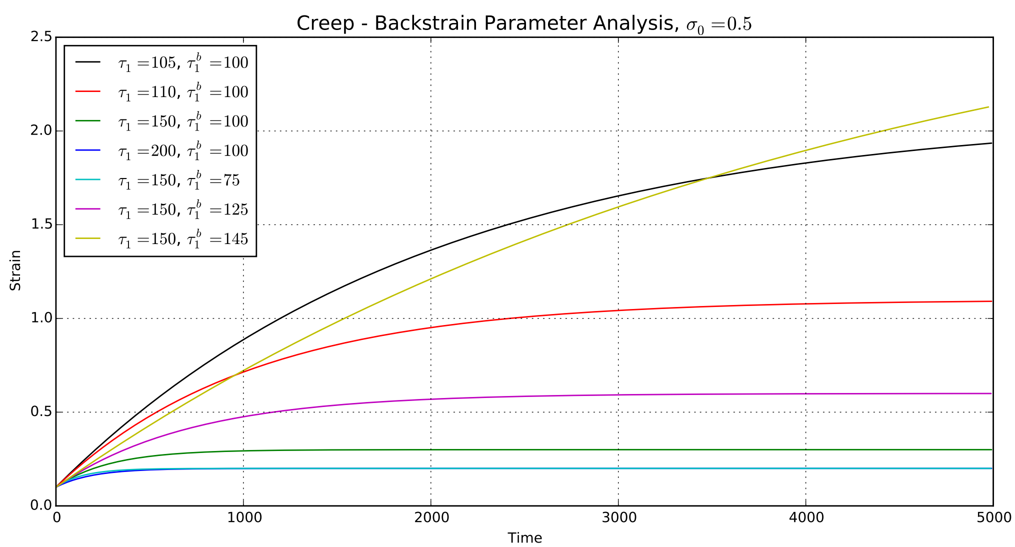

Figure 4: Results of Creep Analysis with Differing Values of τ and τ^b.

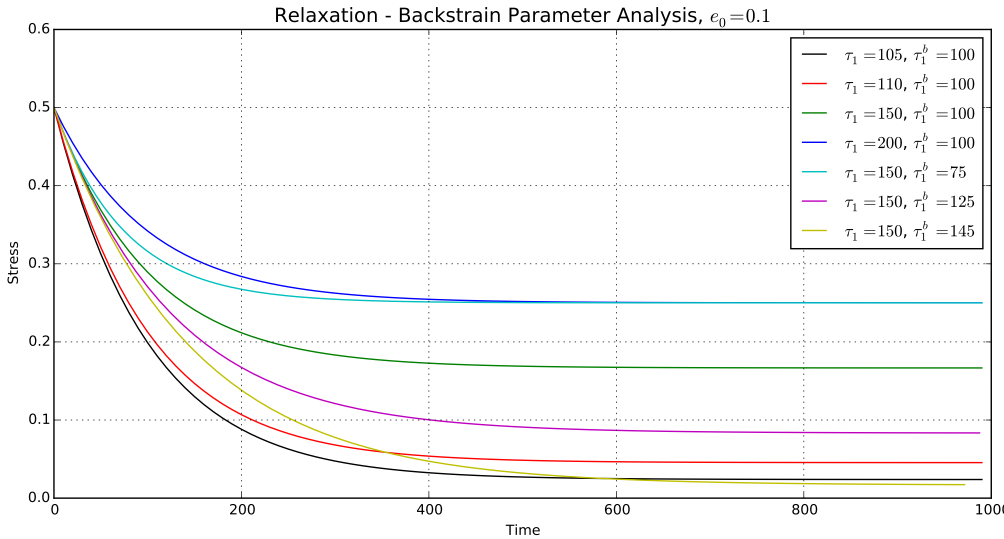

Figure 5: Results of Relaxation Analysis with Differing Values of $\tau$and τ^b.

3 Show the Effects of Adding a Second Set of Backstrain Terms

Influence of a second set of independent backstrain terms τ_2 and τ_2^b on the linear viscoelastic model for creep and relaxation were analyzed. These additional terms were incorporated into the viscoelastic model by simply summing the additional terms as:

$$\dot{e}^b = (1 + \bar{\sigma}_{ve})\left(\frac{1}{\tau_1}e - \frac{1}{\tau_1^b}e^b\right) + (1 + \bar{\sigma}_{ve})\left(\frac{1}{\tau_2}e - \frac{1}{\tau_2^b}e^b\right)$$

The results of these additional backstrain terms (τ_2 and τ_2^b) on the predicted creep and relaxation simulation results are shown below (Figures 6 and 7)

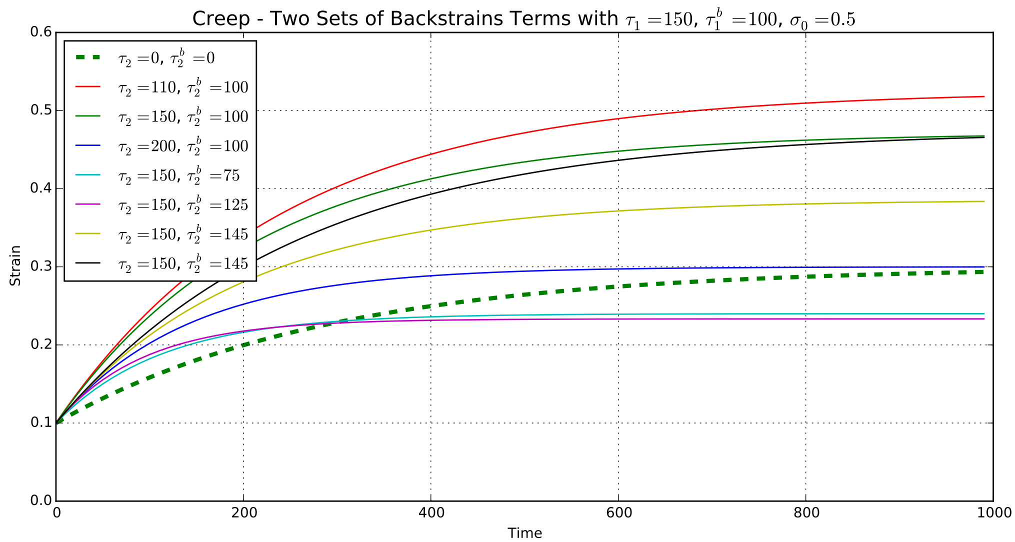

Figure 6: Results of Creep Analysis with Differing Values of τ_2 and τ_2^b.

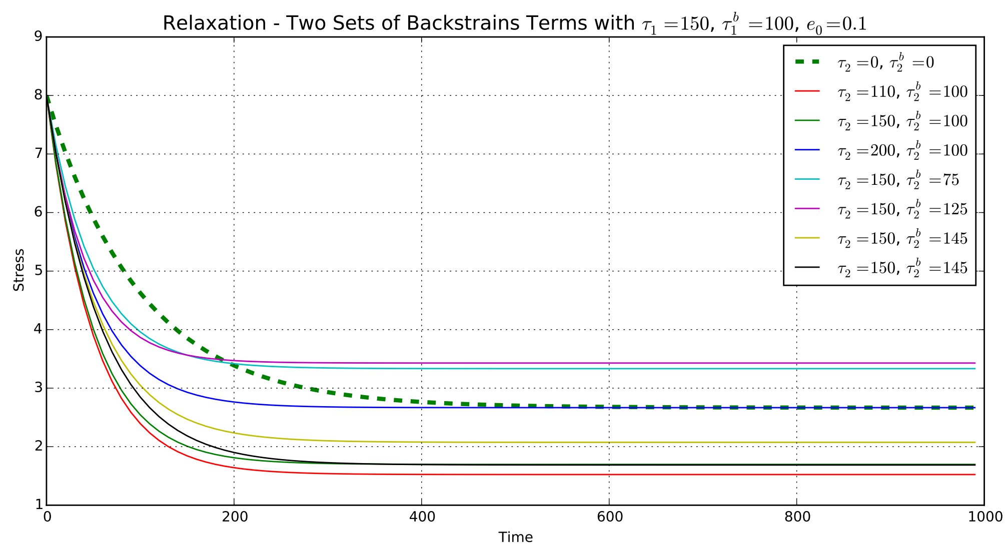

Figure 7: Results of Relaxation Analysis with Differing Values of τ_2 and τ_2^b.

4 Show the Effects of a Nonlinear Backstrain Term

As discussed above, by letting c^* have a value greater than zero results in a nonlinear viscoelastic model. The following figures show the influence of σ^* and σ_0 values on the predicted creep and relaxation results (Figures 8 and 9). Definitions of σ^* and σ_0 are provided below:

$$\text{Nonlinear Magnitude:} \qquad \sigma^* = \frac{1}{c^*}$$

Initial Stress: $\sigma_0 = E e_0^e$

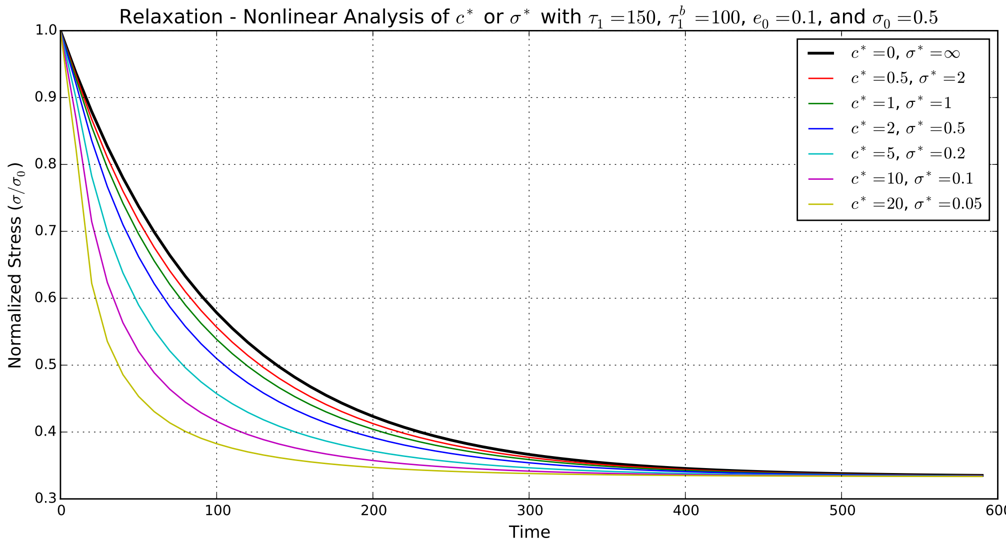

Figure 8: Influence of σ^* on the Results of a Nonlinear Relaxation Analysis.

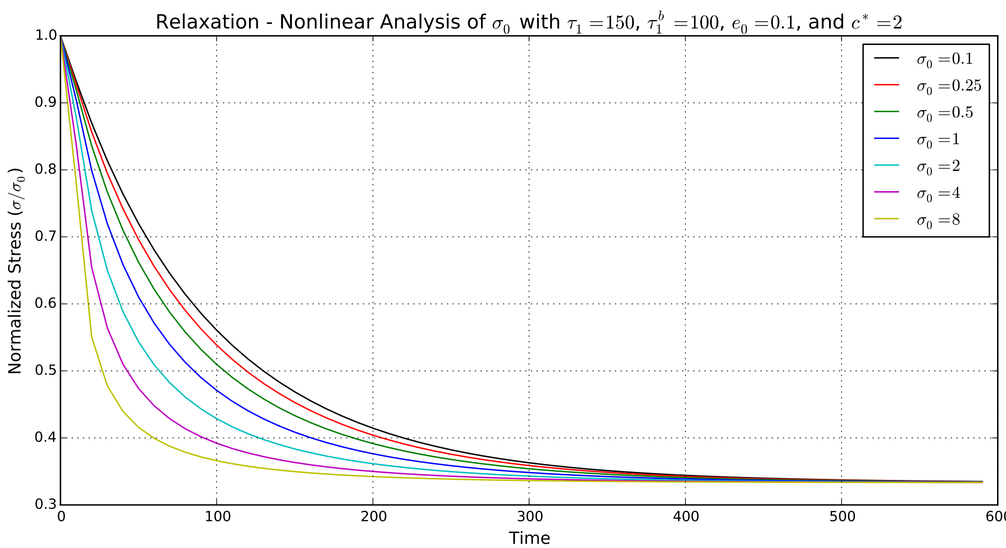

Figure 9: Influence of σ_0 on the Results of a Nonlinear Relaxation Analysis.

5 Cyclic Loading

The effects of a high strain rate on the were analyzed with the use of the prescribed strain as a function of time (t):

$$e = \frac{e_{max}}{2}\left[1 + cos\big(2\pi\big(\frac{t}{T} - \frac{1}{2}\big)\big)\right] \quad 0 \leq t \leq T$$

$$\dot{e}_{max} = \pm(\pi/T)e_{max}$$

$$T = \pi\frac{e_{max}}{\dot{e}_{max}}$$

Analysis of cyclic loading was completed with maximum strain rates ($\dot{e}_{max}$) of 0, 250/sec, 500/sec, and 1000/sec, and results from these analyses are shown in Figures 10 through 13. For ease of analysis, $\dot{e}_{max}$ was defined as having a value of $1e-8/sec$ instead of $0/sec$. This approximation results the defined strain cycle occuring over appoximately 10^6 seconds (11 days), which may be considered a quasi-static analysis. The backstrain parameters were held constant during these analyses, with $\tau_1 = 1e-6$ and $\tau_1^b = 8e-7$, where both τ_1 and τ_1^b have units of seconds. In order for stable integration, the size of time increment was modified based on $\dot{e}_{max}$ according to:

$$t_{inc} = \frac{\tau_1^b}{\big(\dot{e}_{max}/200\big)}$$

One and a half strain cycles were analyzed for each of the defined values of $\dot{e}_{max}$, this allowed for the observation of hysteretic responses at higher strain rates.

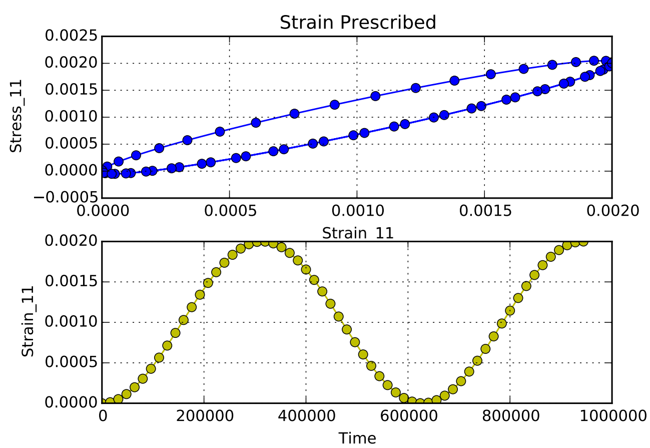

Figure 10: Predicted stress and strains for cyclic loads, $\dot{e}_{max} = 1e-8/sec$.

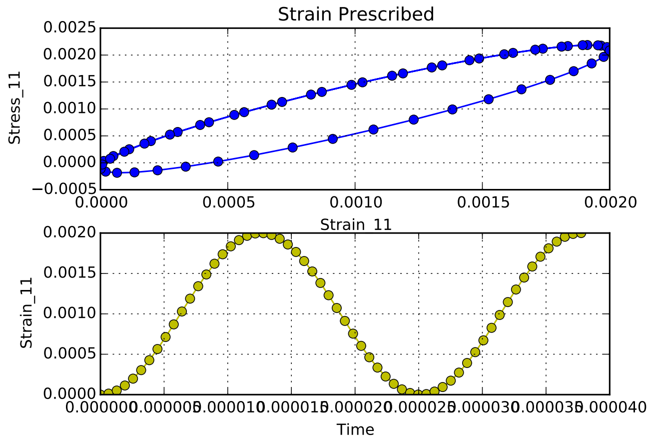

Figure 11: Predicted stress and strains for cyclic loads, $\dot{e}_{max} = 250/s$.

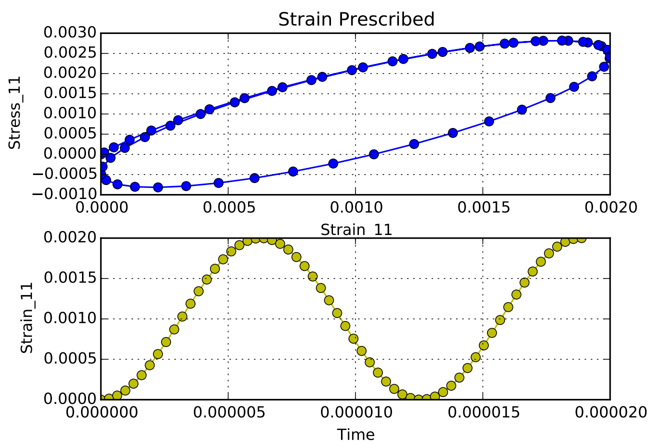

Figure 12: Predicted stress and strains for cyclic loads, $\dot{e}_{max} = 500/s$.

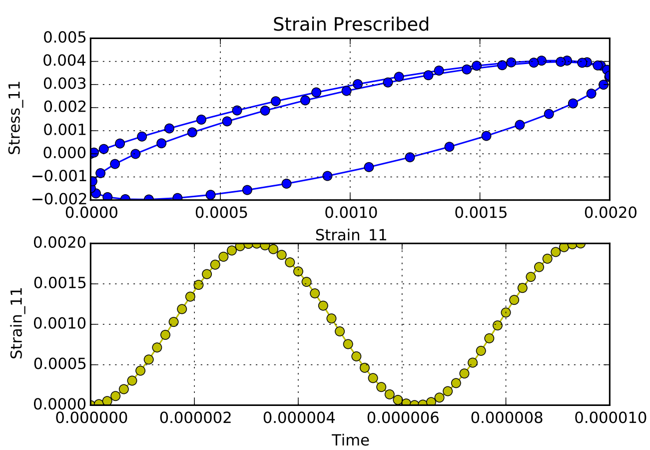

Figure 13: Predicted stress and strains for cyclic loads, $\dot{e}_{max} = 1000/s$.

6 Subroutine For Plotting Yield Surfaces

Two subroutines were developed for the plotting of yield surfaces, those are:

- Yield_Func(P1, P2, P3, q, func, sigma_test, theta, c)
- Yield_Surface(model, yield_stress, theta_cr, chohesion)
  - output from the Yield_Surface subroutine includes:
    - principal stress values (P_1, P_2, P_3)
    - coordinates in the deviatoric plane (q_1, q_2)
    - mean stress (p)
    - Lode coordinates (or Haigh-Westergaard coordinates) in the deviatoric plane (r, z, θ)
    - stress triaxiality $(p/\bar{\sigma}\ )$

The user defines the specific yield function (Mises, Tresca, Mohr-Coulomb, etc) in the Yield_Func subroutine. The Yield_Surface subroutine then allows the user to define which yield function (model), yield stress, and specific material parameters to use and then calls the Yield_Func subroutine. Figures 14 and 15 show three yield surfaces, where each surface was defined according to:

- Tresca: yield stress equal 2
- Mises: yield stress equal 2
- Mohr-Coulomb (M-C): ange of internal friction equal 15° and cohesion equal 0.8

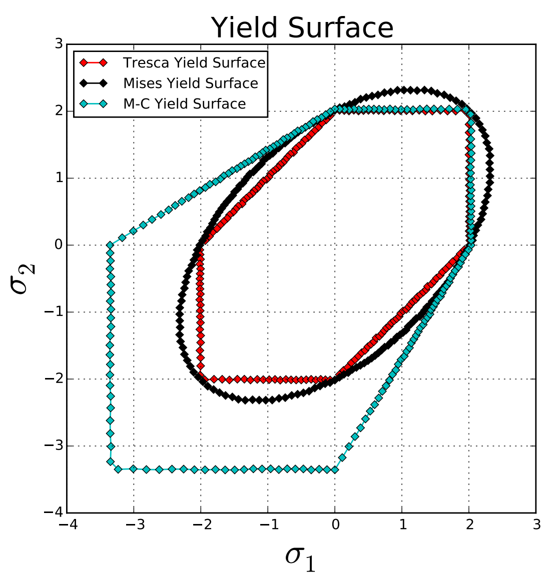

Figure 14: Yield surfaces in the $\sigma_1 - \sigma_2$ plane.

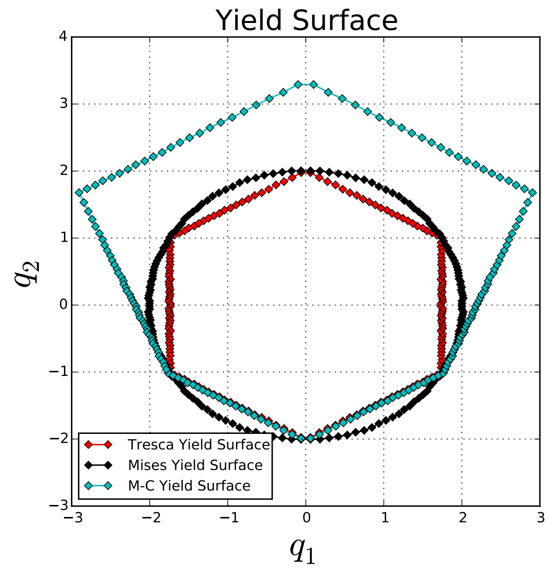

Figure 15: Yield surfaces in the deviatoric plane.