ANSYS - Plate with a Hole (Results-Interpretation)

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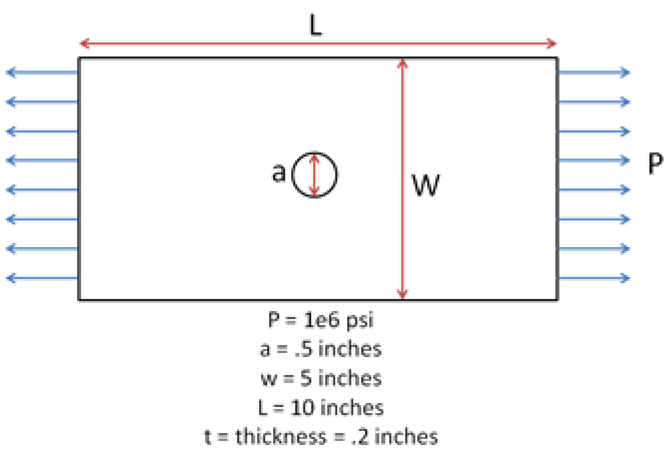
Plate with a Hole (Results-Interpretation)

Created using ANSYS 13.0

Problem Specification

Consider the classic example of a small circular hole in a rectangular plate of constant thickness subjected to an in-plane tensile load. The material is structural steel with a Young's Modulus of 29E6 psi and a Poisson ratio of 0.3. The geometric dimensions and applied tensile load are shown below.



Click here for an enlarged image

In this exercise, you are presented with the numerical solution to the above problem obtained from finite-element analysis (FEA) using ANSYS software. Compare FEA results for the stress distribution presented to you with the corresponding analytical solution for an infinite plate (there is no analytical solution for a *finite* plate). Justify agreements and discrepancies between the two approaches (FEA vs. Analytical).

Note that you will be using the ANSYS solution presented to you to explore the physics of the problem. You will be downloading the ANSYS solution prepared for you. The objective is to help you learn important fundamentals of mechanics through the interactive, visual interface provided by ANSYS. You will *not* be obtaining the FEA solution using ANSYS; there are other tutorials to help you learn this.

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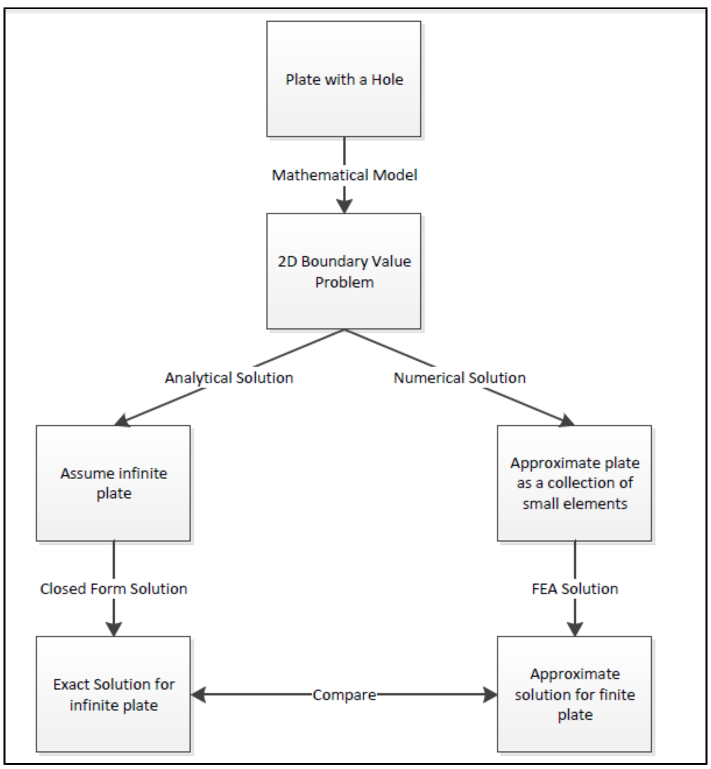
Pre-Analysis and Start-Up

Analytical vs. Numerical Approaches

We can either assume the geometry as an infinite plate and solve the problem analytically, or approximate the geometry as a collection of "finite elements", and solve the problem numerically. The following flow chart compares the two approaches.



Let's first review the analytical results for the infinite plate. We'll then use these results to check the numerical solution from ANSYS.

Analytical Results

Displacement

Let's *estimate* the expected displacement of the right edge relative to the center of the hole. We can get a reasonable estimate by neglecting the hole and approximating the entire plate as being in uniaxial tension. Dividing the applied tensile stress by the Young's modulus gives the uniform strain in the x direction.

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E}$$

$$\epsilon_{xx} = \frac{\Delta L}{L}$$

$$\Delta L = \frac{\sigma_{xx} \times L}{E}$$

$$\sigma = 1 \times 10^6 \text{ psi}, \ L = 5 \text{ inches}, \ \text{ and } E = 29 \times 10^6 \text{ psi}$$

$$\Delta L = .1724 \text{ inches}$$

Multiplying this by the half-width (5 in) gives the expected displacement of the right edge as ~ 0.1724 in. We'll check this against ANSYS.

Sigma-r

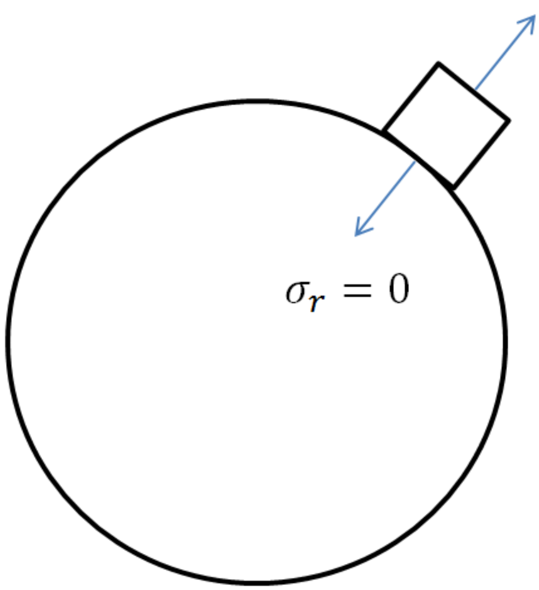
Let's consider the expected trends for Sigma-r, the radial stress, in the vicinity of the hole and far from the hole. The analytical solution for Sigma-r in an infinite plate is:

$$\sigma_r(r,\theta) = \frac{1}{2}\sigma_o[(1 - \frac{a^2}{r^2}) + (1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2})cos(2\theta)]$$

where *a* is the hole radius and Sigma-o is the applied uniform stress (denoted *P* in the problem specification). At the hole (r=a), this reduces to

$$\sigma_r = 0$$

This result can be understood by looking at a vanishingly small element at the hole as shown schematically below.



We see that Sigma-r at the hole is the normal stress at the hole. Since the hole is a free surface, this has to be zero.

For r>>a,

$$\sigma_r(r,\theta) = \sigma_r(\theta) = \frac{1}{2}\sigma_o[1 + cos(2\theta)]$$

Far from the hole, Sigma-r is a function of theta only. At theta = 0, Sigma-r ~ Sigma-o. This makes sense since r is aligned with x when theta = 0. At theta = 90 deg., Sigma-r ~ 0 which also makes sense since r is now aligned with y. We'll check these trends in the ANSYS results.

Sigma-theta

Let's next consider the expected trends for Sigma-theta, the circumferential stress, in the vicinity of the hole and far from the hole. The analytical solution for Sigma-theta in an infinite plate is:

$$\sigma_\theta(r,\theta) = \frac{1}{2}\sigma_o[(1 + \frac{a^2}{r^2}) - (1 + 3\frac{a^4}{r^4})cos(2\theta)]$$

At r = a, this reduces to

$$\sigma_\theta = \frac{1}{2}\sigma_o(2 - 4cos(2\theta))$$

At theta = 90 deg., Sigma-theta = 3*Sigma-o for an infinite plate. This leads to a stress concentration factor of **3** for an infinite plate.

For r>>a,

$$\sigma_\theta(r,\theta) = \sigma_\theta(\theta) = \frac{1}{2}\sigma_o[1 - cos(2\theta)]$$

At theta = 0 and theta = 90 deg., we get

$$\sigma_\theta(0) = \frac{1}{2}\sigma_o[1 - cos(2(0))] = 0$$

$$\sigma_\theta(\frac{\pi}{2}) = \frac{1}{2}\sigma_o[1 - cos(2(\frac{\pi}{2}))] = \sigma_o$$

Far from the hole, Sigma-theta is a function of theta only but its variation is the opposite of Sigma-r (which is not surprising since r and theta are orthogonal coordinates; when r is aligned with x, theta is aligned with y and vice-versa). As one goes around the hole from theta = 0 to theta = 90 deg., Sigma-theta increases from 0 to Sigma-o. More trends to check in the ANSYS results!

Tau-r-theta

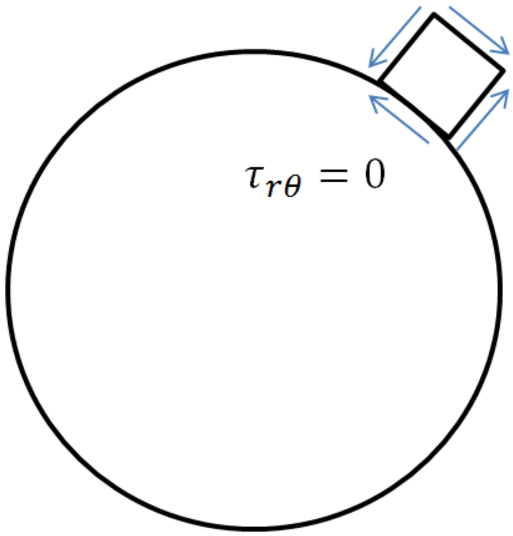
The analytical solution for the shear stress *Tau-r-theta* in an infinite plate is:

$$\tau_{r\theta} = -\frac{1}{2}\sigma_o(1 - 3\frac{a^4}{r^4} + 2\frac{a^2}{r^2})sin(2\theta)$$

At r=a,

$$\tau_{r\theta} = 0$$

By looking at a vanishingly small element at the hole , we see that Tau-r-theta at the hole is the shear stress at the hole. Since the hole is a free surface, this has to be zero.



For r>>a,

$$\tau_{r\theta}(r,\theta) = \tau_{r\theta}(\theta) = -\frac{1}{2}\sigma_o sin(2\theta)$$

We can deduce that, far from the hole, Tau-r-theta = 0 both at theta = 0 and theta = 90 deg. Even more trends to check in ANSYS!

Sigma-x

First, let's begin by finding the average stress, the nominal area stress, and the maximum stress with a concentration factor.

$$\sigma_o = \frac{F}{A} = \frac{1000000}{.2 \times 5}\frac{\text{lb}}{\text{in}^2} = 1 \times 10^6 \text{ psi}$$

$$\sigma_{nominal} = \frac{F}{A} = \frac{1000000}{.2 \times (5 - .5)}\frac{\text{lb}}{\text{in}^2} = 1.111 \times 10^6 \text{ psi}$$

The concentration factor for an infinite plate with a hole is K = 3. The maximum stress for an innite plate with a hole is

$$\sigma_{max} = K \times \sigma_o$$

$$\sigma_{max} = (3.0)(1.0 \times 10^6 \text{psi}) = 3.0 \times 10^6 \text{psi}$$

Although there is no analytical solution for a nite plate with a hole, there is empirical data available to find a concentration factor. Using a Concentration Factor Chart (3250 Students: See Figure 4.22 on page 158 in Deformable Bodies and Their Material Behavior), we find that d/w = 1 and thus K ~ 2:73 Now we can find the maximum stress using the nominal stress and the concentration factor

$$\sigma_{max} = K \times \sigma_{nominal} = (2.73)(1.111 \times 10^6 \text{ psi}) = 3.033 \times 10^6 \text{ psi}$$

Numerical Solution using ANSYS