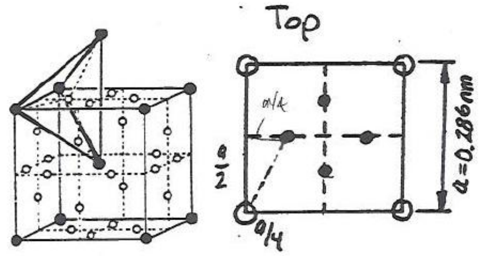
Problem 4.1

Calculate the radii of the tetrahedral and octahedral holes in BCC and FCC iron, assume lattice parameters of 0.286 nm and 0.357 nm, respectively.

Atomic radius of iron, Fe (see backcover of the book): $R_{Fe} = 0.124$ nm

$\boxed{\text{BCC}}$ ○ Fe   ● Interstitial atom

Tetrahedral



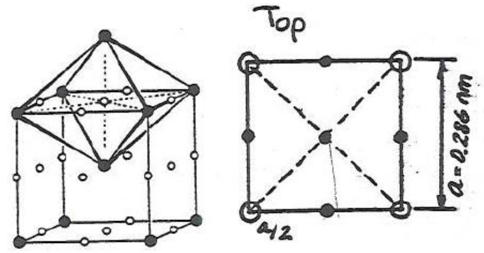
$$(R_{Fe}+r)^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{4}\right)^2$$

$$(R_{Fe}+r)^2 = \frac{a^2}{4} + \frac{a^2}{16} = \frac{5}{16}a^2$$

$$R_{Fe}+r = \frac{\sqrt{5}}{4}a$$

$$r = \frac{\sqrt{5}}{4}a - R_{Fe} = 36\text{ pm} \quad \boxed{r = 36\text{ pm}}$$

Octahedral



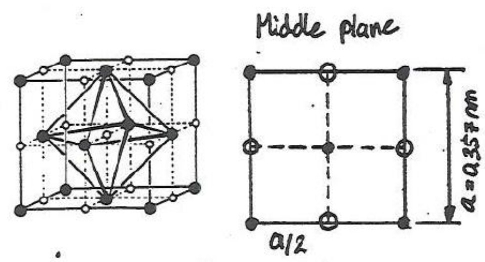
$$(R_{Fe}+r) = \frac{a}{2}$$

$$r = \frac{a}{2} - R_{Fe} = 19\text{ pm}$$

$$\boxed{r = 19\text{ pm}}$$

$\boxed{\text{FCC}}$

Octahedral

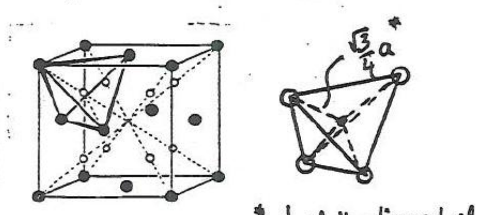


$$(R_{Fe}+r) = \frac{a}{2}$$

$$r = \frac{a}{2} - R_{Fe} = 55\text{pm}$$

$$\boxed{r = 55\text{ pm}}$$

Tetrahedral



* $\frac{1}{4}$ of the diagonal of the cube

$$(R_{Fe}+r) = \frac{\sqrt{3}}{4}a \quad \text{where } a = 0.357\text{ nm}$$

$$r = \frac{\sqrt{3}}{4}a - R_{Fe} = 31\text{ pm}$$

$$\boxed{r = 31\text{ pm}}$$

4.2 Calculate the concentration of monovacancies in gold at 1,000 K, knowing that $H_f = 1.4 \times 10^{-19}$ J. If the gold is suddenly quenched to ambient temperature, what will be the excess vacancy concentration?

$$C_v = \exp\left(\frac{-Ef}{kT}\right)$$

Take $E_f \cong H_f$ for metals

<u>At 1000 K</u>

$$Cv_{1000} = \exp\left(-\frac{1.4 \times 10^{-19}}{1000 \times 1.38 \times 10^{-23}}\right)$$

$$= 3.9 \times 10^{-5}$$

<u>At 298 K</u>

$$Cv_{298} = \exp\left(-\frac{1.4 \times 10^{-19}}{1.38 \times 10^{-23} \times 298}\right)$$

$$= 1.64 \times 10^{-15}$$

Excess Vacancy Conc.

$$Cv_{1000} - Cv_{298} \cong 3.9 \times 10^{-5}$$

4.3 How many vacancies per cubic centimeter are there in gold, at ambient temperature, assuming a lattice parameter of 0.408 nm?

$$Cv_{298} = \frac{n}{N} = \exp\left(-\frac{1.4\times10^{-19}}{1.38\times10^{-23}\times298}\right)$$

$= 1.64 \text{ x } 10^{-15}$ vacant sites/total number of atomic sites

Au is FCC $\rightarrow$ 4 atoms/unit cell

Vol. of unit cell = $a^3 = (0.408 \text{ x } 10^{-9} \text{ x } 10^2)^3 \text{ cm}^2$

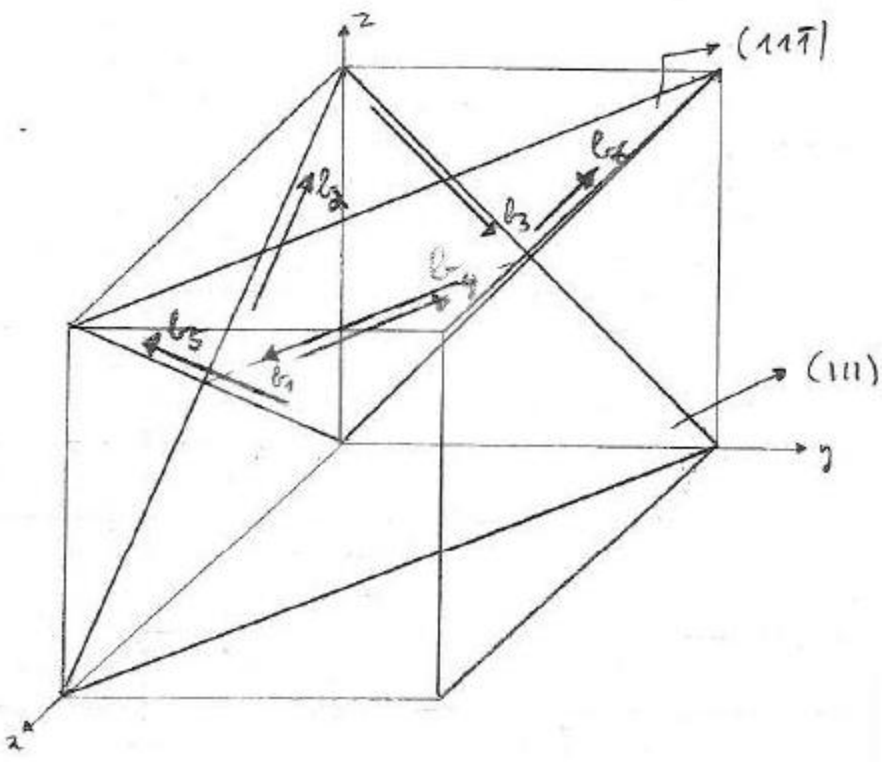
$= 6.79 \text{ x } 10^{-23} \text{ cm}^3$

$$1 \text{ cm}^3 \text{ contains } \frac{4}{6.79\ x\,10^{-23}} = 5.89\ x\,10^{22}\ atomic\ sites$$

$$\text{Number of vacancies/cm}^3 = 1.64 \text{ x } 10^{-15}\left(\frac{vacs}{sites}\right)\cdot 5.89\times10^{22}\left(\frac{sites}{cm^3}\right)$$

$= 9.7 \text{ x } 10^7 \text{ vacs/cm}^3$

4.8 Consider all possible reactions between partial Shockley dislocation (only the front dislocation, from the pair) in (111) and (111) in an FCC crystal. Among them, which ones will form a stair-rod dislocation?



For (111) plane

$$b_1 = \frac{a}{2}[\bar{1}10]$$
$$b_2 = \frac{a}{2}[\bar{1}01]$$
$$b_3 = \frac{a}{2}[0\bar{1}1]$$

$$b_4 = \frac{a}{2}[1\bar{1}0]$$
$$b_5 = \frac{a}{2}[10\bar{1}]$$
$$b_6 = \frac{a}{2}[01\bar{1}]$$

We need to check for each reaction if it results in a decrease in energy.

$b_1 - b_4 \Rightarrow$ Cancel each other

$b_1 - b_5 \Rightarrow \frac{a}{2}[1\bar{1}0] + \frac{a}{2}[101] = \frac{a}{2}[2\bar{1}1]$

$\Rightarrow \frac{a^2}{4}(1+1) + \frac{a^2}{4}(1+1) = a^2 < \frac{6a^2}{4}$

$b_1 - b_6 \Rightarrow \frac{a}{2}[1\bar{1}0] + \frac{a}{2}[011] = \frac{a}{2}[101]$

$\Rightarrow \frac{a^2}{4}(1+1) + \frac{a^2}{4}(1+1) = a^2 > \frac{a^2}{4}\cdot 2$

$\Rightarrow$ Forms Stair Rod

$b_2 - b_4 \Rightarrow \frac{a}{2}[\bar{1}01] + \frac{a}{2}[\bar{1}10] = \frac{a}{2}[-211]$

$\Rightarrow \frac{a^2}{4}(1+1) + \frac{a^2}{4}(1+1) = a^2 < \frac{6a^2}{4}$

$b_2 - b_5 \Rightarrow \frac{a}{2}[\bar{1}01] + \frac{a}{2}[101] = \frac{a}{2}[002]$

$\Rightarrow \frac{a^2}{4}(1+1) + \frac{a^2}{4}(1+1) = a^2 \cdot \frac{4}{4} = a^2$

$b_2 - b_6 \Rightarrow \frac{a}{2}[\bar{1}01] + \frac{a}{2}[011] = \frac{a}{2}[\bar{1}12]$

$\Rightarrow \frac{a^2}{4}(1+1) + \frac{a^2}{4}(1+1) = a^2 < \frac{6}{4}a^2$

$b_3 - b_4 \Rightarrow \frac{a}{2}[01\bar{1}] + \frac{a}{2}[\bar{1}10] = \frac{a}{2}[\bar{1}2\bar{1}]$

$\Rightarrow \frac{a^2}{4}(1+1) + \frac{a^2}{4}(1+1) = a^2 < \frac{6}{4}a^2$

$b_3 - b_5 \Rightarrow \frac{a}{2}[01\bar{1}] + \frac{a}{2}[101] = \frac{a}{2}[110]$

$\Rightarrow \frac{a^2}{4}(1+1) + \frac{a^2}{4}(1+1) = a^2 > \frac{a^2}{4}\cdot 2$

$\Rightarrow$ Forms stair Rod

$b_3 - b_6 \Rightarrow \frac{a}{2}[01\bar{1}] + \frac{a}{2}[011] = \frac{a}{2}[020]$

$\Rightarrow \frac{a^2}{4}(1+1) + \frac{a^2}{4}(1+1) = a^2 = a^2$

4.9 **(a)** Show that the reaction

$$\frac{a}{6}[10\bar{1}] \rightarrow \frac{a}{6}[21\bar{1}] + \frac{a}{6}[1\bar{1}\bar{2}].$$

is either vectorially correct or incorrect?

(b) Is the reaction energetically favorable?

a)

$$\left(\frac{a}{3}i + \frac{a}{6}j - \frac{a}{6}k\right) + \left(\frac{a}{6}i + \frac{a}{6}j - \frac{a}{3}k\right)$$
$$= \frac{a}{2}i + \frac{a}{3}j + \left(-\frac{a}{2}k\right)$$
$$= \frac{a}{2}\left[1\frac{2}{3}\bar{1}\right]$$

It is vectorially incorrect.

(b)

Dislocation line energy is $Gb^2/2$.

$a^2/2 \ > \ a^2/6 + a^2/6$

It is energetically favorable.

4.10 10^{7} and 10^{11} cm $^{-2}$ are typical values for the dislocation of annealed and deformed nickel, respectively. Calculate the average space between dislocation lines (assuming a random dislocation distribution) as well as the line energy for edge and screw dislocations, in both cases. In nickel, E = 210 GPa, ν = 0.3, and the lowest distance between atom centers is 0.25 nm.

Assuming a two-dimensional array of dislocations:

$$\rho = \frac{1}{L^2} \therefore L = \frac{1}{\sqrt{\rho}}$$

The average space between dislocation lines
in the annealed condition, L = 3.16 μm
in the deformed condition, L = 0.316 nm

Line Energy:

$$G = \frac{E}{2(1+\nu)} = 80.77\ GPa,\ b = 0.25 \times 10^{-9}\ m$$

Screw dislocation

$$U = \frac{Gb^2}{4\pi(1-\nu)}(1-\nu)\ell n\frac{\rho^{-1/2}}{5b}$$

In the annealed condition, U = 3.14 x 10 $^{-9}$ J
In the deformed condition, U = 1.29 x 10 $^{-9}$ J

Edge Dislocation:

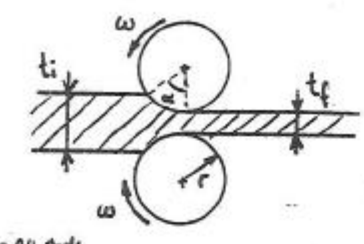
$$U = \frac{Gb^2}{4\pi(1-\nu)}\ell n\frac{\rho^{-1/2}}{5b}$$

In the annealed condition, U = 4.5 x 10 $^{-9}$ J
In the deformed condition, U = 1.85 x 10 $^{-9}$ J/m

Problem 4.18

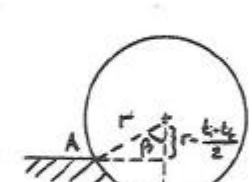
Nickel sheet is being rolled at ambient temperature in a rolling mill (roll diameter 50 cm, velocity 200 rpm). The initial thickness is 20 mm and the final thickness is 10 mm (one pass).

$t_i = 20\,mm$
$t_f = 10\,mm$
$r = 25\,cm$
$\omega = 200\,rpm$
$= \frac{2\pi \cdot 200}{60} = 20.94\,rad/s$



Nickel, Ni (see backcover)
$r_{Ni} = 0.125\,nm$, $\rho_{Ni} = 8.9\,g/cm^3$
Table 4.1, p. 216 $G = 76\,GPa$
Table 2.5, p. 92 $\nu = 0.312$

(a) Calculate the average strain rate



$$\beta = \arccos\left(\frac{r-\frac{t_i-t_f}{2}}{r}\right) = 11.48° = 0.200\,rad$$

$$t_{AB} = \frac{\beta}{\omega} = \frac{0.200\,rad}{20.94\,rad/s} = 9.57 \cdot 10^{-3}\,s = 9.57\,ms$$

$$\varepsilon_e = \frac{t_f-t_i}{t_i} = \frac{10-20}{20} = -0.5$$

$$\dot{\varepsilon} = \frac{\varepsilon_e}{t} = \frac{0.5}{9.57\cdot 10^{-3}} = 52.3\ s^{-1} \qquad \boxed{\dot{\varepsilon} = 52.3\,s^{-1}}$$

(b) Calculate the energy that will be stored in the material, assuming that the final density is $10^{11}\,cm^{-2}$.

Energy of dislocations: $U_r = \frac{Gb^2}{10} + \frac{Gb^2}{4\pi(1-\nu)}(1-\nu\cos^2\alpha)\ln\frac{\rho^{-1/2}}{5b}$

where G... shear modulus
b... Burgers vector $b = 2 \cdot r_{Ni} = 0.250\,nm$
ρ... dislocation density
ν... Poisson's ratio
α... dislocation parameter (mixed ⇒ $\alpha = \frac{\pi}{4}$)

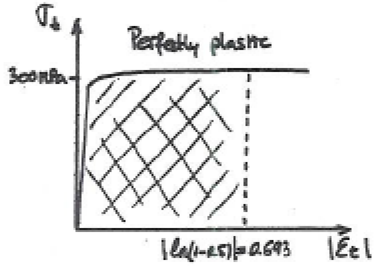
The approximation $U_r \approx \frac{Gb^2}{2}$ can also be used.

Problem 4.18 (continued)

(b) $$U_r = \frac{76\cdot10^9(0.250\cdot10^{-9})^2}{10} + \frac{76\cdot10^9(0.250\cdot10^{-9})^2}{4\pi\,(1-0.312)}\left(1-0.312\cos^2\frac{\pi}{4}\right)\ln\frac{(10^4\cdot10^4)^{-1/2}}{5\cdot0.250\cdot10^{-9}}$$

$$= 1.973\cdot10^{-9}\ \mathrm{J/m} \qquad \boxed{U_r = 1.973\ \frac{\mathrm{nJ}}{\mathrm{m}}}$$

(c) Determine the total energy expenditure per unit volume, assuming a flow stress equal to 300 MPa.



Neglecting the slope of the elastic part of the curve:

$$U_{tot} = \sigma_f\cdot\varepsilon_t = 300\cdot10^6\cdot0.693$$
$$= 207.9\cdot10^6\ \frac{\mathrm{N}}{\mathrm{m}^2} \qquad [\mathrm{J}] = \mathrm{Nm}$$

$$\boxed{U_{tot} = 207.9\ \frac{\mathrm{MJ}}{\mathrm{m}^3}}$$

(d) Assuming that all energy not stored as dislocations is converted into heat, calculate the temperature rise if the process is adiabatic.

$$U_{tot} = U_{disl.} + U_{heat} \Rightarrow U_{heat} = U_{tot} - U_{disl.} = \overbrace{\rho_{Ni}}^{\text{density}} c_p\,\Delta T \quad \left(c_p = 0.49\ \frac{\mathrm{J}}{\mathrm{g}\,^\circ\mathrm{C}}\right)$$

$$U_{disl.} = U_r\cdot\overbrace{\rho}^{\text{disl. density}} = 1.973\cdot10^{-9}\cdot10^4\cdot10^4 = 1.97\cdot10^6\ \frac{\mathrm{J}}{\mathrm{m}^3} = 1.97\ \frac{\mathrm{MJ}}{\mathrm{m}^3}$$

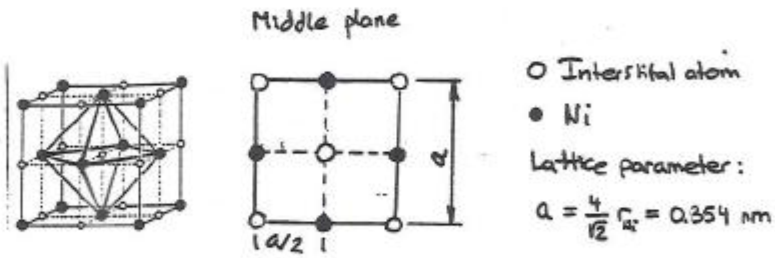
$$\Delta T = \frac{U_{tot} - U_{disl.}}{\rho_{Ni}\,c_p} = \frac{207.9\cdot10^6 - 1.97\cdot10^6}{8.9\cdot10^6\cdot0.49} = 47.2\ ^\circ\mathrm{C} \qquad \boxed{\Delta T = 47.2\ ^\circ\mathrm{C}}$$

Problem 4.19

Calculate the largest atom that would fit interstitially into

(a) nickel (FCC; atomic radius = 0.125 nm)

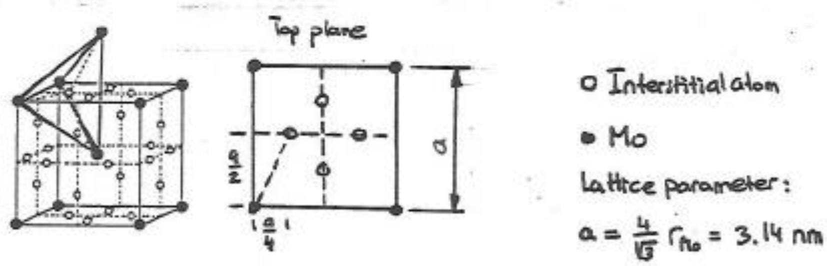
For FCC structures, the largest hole is the octahedral one.



$$r_{Ni} + R = \frac{a}{2} \Rightarrow R = \frac{a}{2} - r_{Ni} = 52 \text{ pm} \qquad \boxed{R = 52 \text{ pm}}$$

(b) molybdenum (BCC; atomic radius = 1.36 nm)

For BCC structures, the largest hole is the tetrahedral one.



$$(r_{Mo} + R)^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{4}\right)^2$$

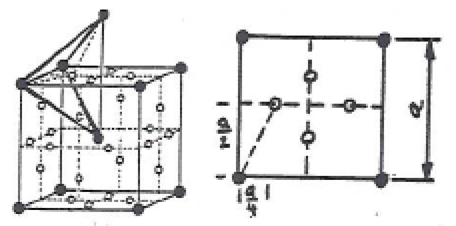
$$(r_{Mo} + R)^2 = \frac{5}{16} a^2$$

$$r_{Mo} + R = \frac{\sqrt{5}}{4} a \Rightarrow R = \frac{\sqrt{5}}{4} a - r_{Mo} = 396 \text{ pm} \qquad \boxed{R = 396 \text{ pm}}$$

Problem 4.20

Calculate, for tungsten (BCC; atomic radius = 0.1369 nm), the radii of the largest atoms that can fit into

(a) a tetrahedral interstitial site (at 0, 1/4, 1/2)



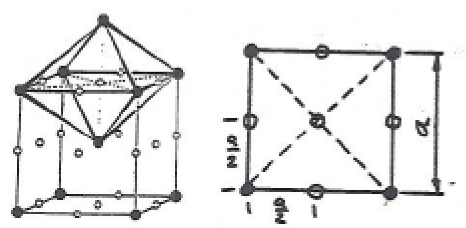
$a = \frac{4}{\sqrt{3}} r_W = 0.316$ nm

$$(r_W + R)^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{4}\right)^2$$

$$(r_W + R)^2 = \frac{5}{16} a^2$$

$$r_W + R = \frac{\sqrt{5}}{4} a \quad \Rightarrow R = \frac{\sqrt{5}}{4} a - r_W = 40 \text{ pm} \quad \boxed{R = 40 \text{ pm}}$$

(b) an octahedral interstitial site (at 0, 1/2, 1/2)



Lattice parameter: $a = 0.316$ nm

$$r_W + R = \frac{a}{2} \quad \Rightarrow R = \frac{a}{2} - r_W = 21 \text{ pm} \quad \boxed{R = 21 \text{ pm}}$$

4.21 If the enthalpy of formation for a vacancy is equal to 80 kJ/mol, what is the fraction of vacant sites at 1,500 K?

G_v = 80 kJ/mol = 80 kJ/6.02 10^{23} atoms = 1.33 10^{-19} J/atom

$$k = 1.38*10^{-23} J/atom*K$$
$$T = 1500K$$
$$\frac{n_v}{N} = \exp[\frac{G_v}{kT}] = \exp[\frac{1.33*10^{-19} J/atom}{(1.38*10^{-23} J/atom*K)(1500K)}] = 1.62*10^{-3} Vacancies/atom$$

4.22 The lattice parameter of a BCC crystal was measured at ambient temperature and at 1,000 ◦C. The parameter showed an increase of 0.5 percent due to thermal expansion. In the same interval of temperature, the density, measured by a separate method, showed a decrease of 2 percent.
(a) Assuming that, at room temperature, there is one vacancy per 1,000 atoms, what is the vacancy concentration at 1,000 ◦C?
(b) Calculate the activation energy necessary for the production of vacancies.

T_{293K}	T_{1273K}	
ρ_o	$\rightarrow \rho = \rho_o \times 0.98$	(decrease of 2%)
a_o	$\rightarrow 1.005\, a_o$	(increase of 0.5%)

$$\Rightarrow \rho_o = \frac{2M}{a_o^3 N_A}\left(1 - \frac{n_{r,o}}{N_o}\right)$$

M = molecular weight

N_A = avogadro's number.

BCC structure ⇒ 2 atoms in 1 unit cell

$$\frac{n_{r,o}}{N_o} = \frac{1}{1000}$$

$$\rho = \frac{2M}{(1.005\,a_0)^3 N_A}\left(1-\frac{n_v}{N}\right)$$

Now we use $0.98 \times \rho_0 = \rho$

So: $$\frac{2\cancel{M}}{(1,005\,\cancel{a_0})^3 \cancel{N_A}}\left(1-\frac{n_v}{N}\right) = \frac{2\cancel{M} \times 0.98}{\cancel{a_0^3}\ \cancel{N_A}}\left(1-\frac{n_{v,o}}{N_o}\right)$$

$$\Rightarrow \frac{\left(1-\frac{n_v}{N}\right)}{(1.005)^3} = 0.98\left(1-\frac{n_{v,o}}{N_o}\right)$$

$$\Rightarrow \left(1-\frac{n_v}{N}\right) = 0.98 \times \left(1-\frac{n_{v,o}}{N_o}\right) \times (1.005)^3$$

$$\Rightarrow \frac{n_v}{N} = 1 - \left\{0.98 \times \left(1-\frac{n_{v,o}}{N_o}\right) \times (1.005)^3\right\}$$

$$\Rightarrow \frac{n_v}{N} = 1 - \left\{0.98 \times (1-0.001) \times (1.005)^3\right\}$$

$$\Rightarrow \frac{n_v}{N} = 0.00622$$

$$\Rightarrow \boxed{\frac{n_v}{N} = \frac{1}{160} \text{ (= 1 vacancy on every 160 atoms)}}$$

b) To compute the formation energy G_v:

$$\frac{n_{v,o}}{N_o} = \exp\left(-\frac{G_v}{k T_o}\right) \qquad T_o = 293\,K$$

$$\ln\left(\frac{n_{v,o}}{N_o}\right) = -\frac{G_v}{k T_o}$$

$$\Rightarrow G_v = -k \cdot T_o \cdot \ln\left(\frac{n_{v,o}}{N_o}\right)$$

$$\Rightarrow G_v = -13.81 \times 10^{-24} \times 293 \cdot \ln(0.001) = 2.795 \times 10^{-20}\ J$$

$$\boxed{G_v = 0.175\ eV} \qquad (1eV = 1.6 \times 10^{-19} J)$$

4.23 The Burgers vector of a dislocation is 0.25 nm in a crystal. The shear modulus G = 40 GPa. Estimate the dislocation energy per unit length in this crystal.

$$U_r = \frac{Gb^2}{2}$$

$$= \frac{1}{2} \times 40 \times 10^9\, Pa \times (0.25 \times 10^{-9}\, m)^2$$

$$= 1.25 \times 10^{-9}\ J/m$$

4.24 A dislocation is anchored between two points 10 μm distant. For a metal with b = 0.35 nm and G = 30GPa, compute the shear stress necessary to bow the dislocation into a semicircle. Take the dislocation line tension T≈ (1/2)Gb^2.

The dislocation line tension T≈ (1/2) Gb^2.
The force on a dislocation per unit length is given by

$$F = \tau\, b$$

where τ is the shear stress and b is the Burgers vector, and F = T/r, T being the line tension and r is the radius to which the dislocation is bent.

The shear stress necessary to bow the dislocation into a semicircle of radius, r is

$$\tau = F/b = T/br$$

$$\text{Or, } \tau = \frac{Gb}{2r} = \frac{Gb}{2(\ell/2)} = \frac{Gb}{\ell}$$

$$= \frac{30 \times 10^9\ Pa \times 0.35 \times 10^{-9}\ m}{10 \times 10^{-6}\, m}$$

$$= 1.05 \times 10^6\ Pa$$

$$= 1.05\, MPa$$

4.25 Consider an aluminum polycrystal with a grain size of 10 μm. If a dislocation source at the center of a grain emits dislocations under an applied shear stress of 50 MPa that pile up at the grain boundaries, what is the stress experienced by a grain boundary? Take G = 26 GPa and b = 0.3 nm.

$$\tau^* = n\tau$$

$$L = \frac{nGb}{\pi\tau} = \frac{D}{2}$$

$$n = \frac{\pi L\tau}{Gb} = \frac{\pi D\tau}{2Gb} = \frac{\pi\ 10\ 10^{-6}\ 50\ 10^{6}}{2\ \ 26\ 10^{9}\ 0.3\ 10^{-9}} = 100.7$$

$$\tau^* = n\tau = 100.7\ 50 10^{6} = 5.03\ GPa$$

4.26 (a) Iron (r = 0.124 nm, G = 70 GPa) is deformed to a shear strain of 0.3. A dislocation density equal to 10^{10} cm^{-2} results. What is the average distance each dislocation had to move?
(b) Assuming that the strain rate is 10^{-2} s^{-1}, what is the average dislocation velocity?

(a) Since iron is BCC, b = 2r = 0.248 nm. Using Orowan's equation taking k = 1,

$$\ell = \frac{\gamma}{\rho b}$$

$$= \frac{0.3}{10^{10} \times 10^{4} m^{-2} \times 0.248 \times 10^{-9} m}$$

$$= 1.21 \times 10^{-5} m$$

(b) Average dislocation velocity

$$v = \frac{\dot{\gamma}}{\rho b}$$

$$= \frac{10^{-2}\ s^{-1}}{10^{10} \times 10^{4}\, m^{-2} \times 0.248 \times 10^{-9}\, m}$$

$$= 4.03 \times 10^{-7}\ m/s$$

4.27

(a) The average distance each dislocation had to move is:

$b = 2r = 0.248*10^{-9}m$

Using Orwan's equation k=1 and $\gamma = \rho b\bar{l}$

$$\bar{l} = \frac{\gamma}{\rho b} = 1.2*10^{-5}m$$

(b) $\bar{v} = \dfrac{\dot{\gamma}}{\rho b} = 4.0*10^{-7}m/s$

The average dislocation velocity is $4.0*10^{-7}m/s$

4.28 Consider the following dislocation reaction in a face-centered cubic material:

$$\frac{a}{2}[1\bar{1}0] \rightarrow \frac{a}{6}[2\bar{1}1] + \frac{a}{6}[1\bar{2}\bar{1}].$$

Show that the reaction will occur.

$$\frac{a}{2}[1\bar{1}0] \rightarrow \frac{a}{6}[2\bar{1}1] + \frac{a}{6}[1\bar{2}\bar{1}]$$

$$\left(\frac{a}{2}\right)^2 + \left(-\frac{a}{2}\right)^2 \rightarrow \left(\frac{2a}{6}\right)^2 + \left(\frac{-a}{6}\right)^2 + \left(\frac{a}{6}\right)^2 + \left(\frac{a}{6}\right)^2 + \left(\frac{-2a}{6}\right)^2 + \left(\frac{-a}{6}\right)^2$$

$$\frac{1}{2}a^2 \rightarrow \frac{1}{3}a^2$$

$$\Rightarrow \frac{1}{2}a^2 \textit{ is grater than } \frac{1}{3}a^2$$

Energy is lower after reaction; therefore, the reaction will occur.

4.29 Consider dislocations in gold. If the flow stress is controlled by the stress necessary to operate a Frank-Read source, compute the dislocation density ρ in the crystal when it is deformed to a point where the resolved shear stress on the slip plane is 45 MPa. Take G = 27 GPa.

$$\ell = \rho^{-1/2}$$

$$\tau = \frac{Gb}{\ell} = Gb\sqrt{\rho}$$

$$\textit{For gold (from table 4.1)}$$

$$b = 0.288 \times 10^{-9}\ m$$

$$\rho = \frac{\tau}{G^2b^2}$$

$$\rho = \frac{(45 \times 10^6)^2}{(27 \times 10^9)^2 (0.288 \times 10^{-9})^2 m^2}$$

$$\rho = 3.35 \times 10^{13} m^{-2}$$

4.31

Plot the energy of a single edge dislocation in copper as a function of dislocation density (in units of Gb^2). Start at a density of $10^6 cm^{-2}$ characteristic of well annealed material, and finish at $10^{11} cm^{-2}$, characteristic of work hardening material.

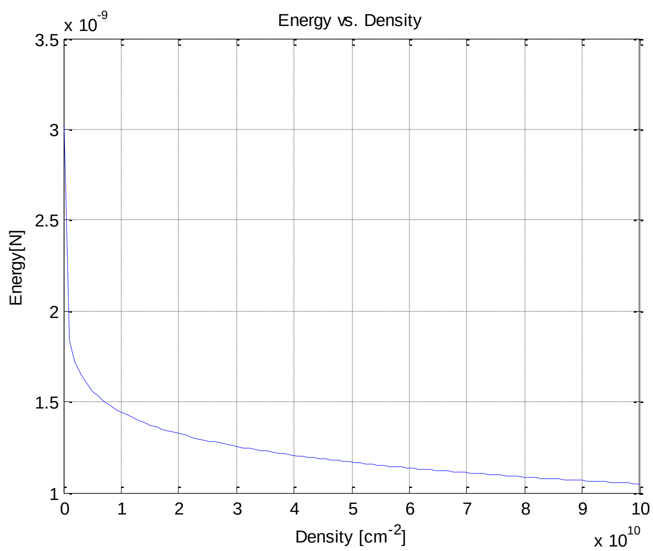
Given: b = 0.3 nm and G = 48 GPa

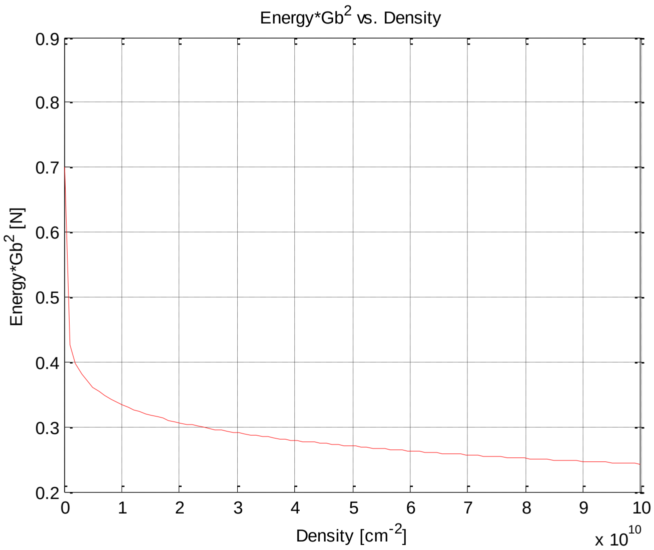
Eqn:

$$G = \frac{E}{2(1+\upsilon)}$$

$$U = \frac{Gb^2}{4\pi} \ln\left(\frac{\rho^{-1/2}}{5b}\right)$$

Plots of energy of dislocation per unit length (J/m or N) vs. dislocation density (cm/cm^3 or cm^{-2}) are shown below:





4.33

$$grainsize = 50\mu m = 50*10^{6}m = l$$

$$\gamma = 0.5$$

$$\dot{\gamma} = 10s^{-1}$$

$$G = 10GPa$$

$$b = 0.2nm = 2*10^{-10}m$$

$$k = 1$$

(a) Calculate the dislocation density required?

$$\gamma = \rho b l$$

$$\rho = \frac{\gamma}{bl} = \frac{0.5}{(2*10^{-10}m)(50*10^{-6}m)} = 5*10^{13}/m^{2}$$

(b) Calculate the velocity which each dislocation will move?

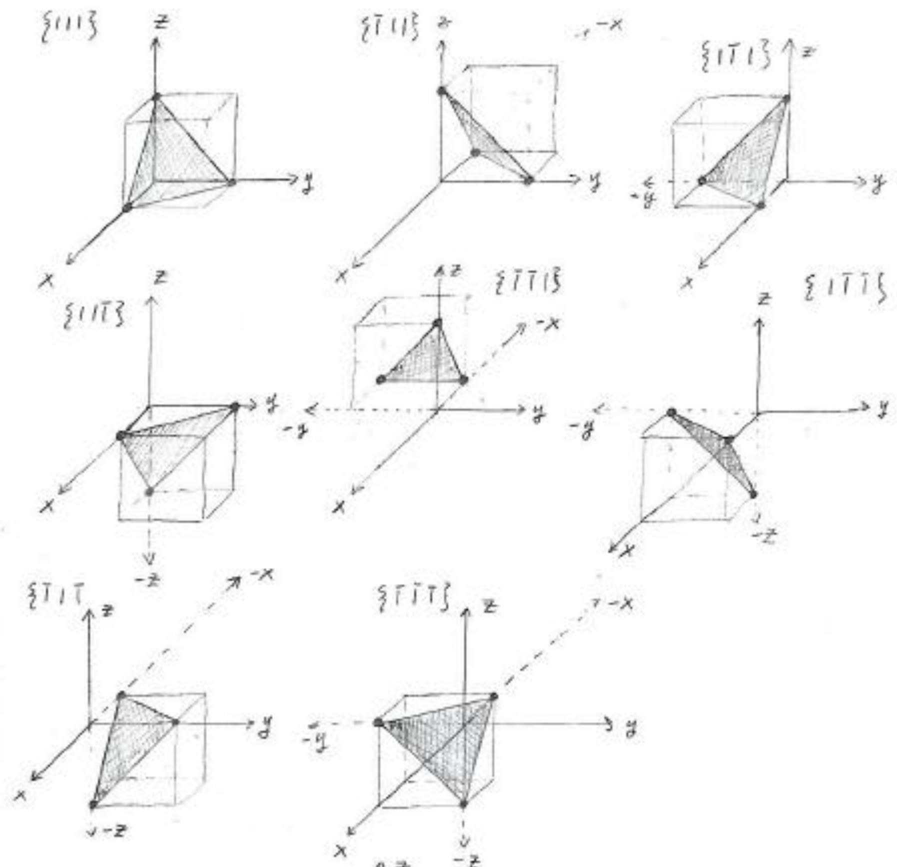
$$\dot{\gamma} = k\rho b\bar{v}$$

$$\bar{v} = \frac{\dot{\gamma}}{k\rho b} = \frac{10s^{-1}}{(1)(5*10^{13}m^{-2})(2*10^{-10}m)}$$

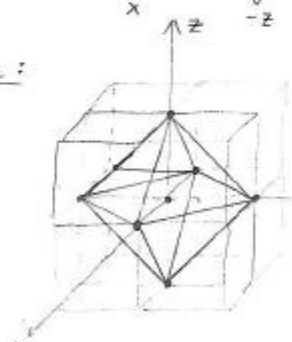
$$\bar{v} = 1*10^{-3}m/s$$

4.34

Soln: ✓$\{111\}$ ✓$\{\bar{1}\bar{1}1\}$
✓$\{\bar{1}11\}$ ✓$\{1\bar{1}\bar{1}\}$
✓$\{1\bar{1}1\}$ ✓$\{\bar{1}1\bar{1}\}$
✓$\{11\bar{1}\}$ ✓$\{\bar{1}\bar{1}\bar{1}\}$



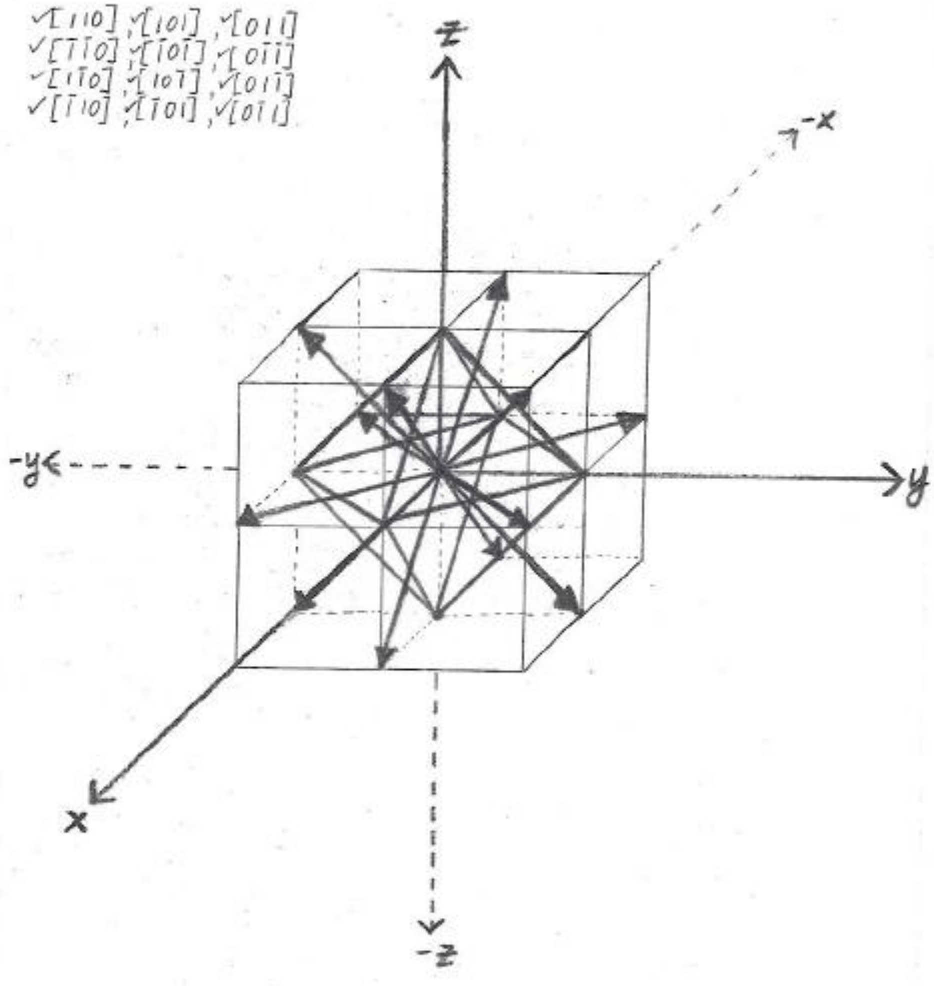
Overall Figure:



* See next pg for enlarged Overall Figure with all 110 directions.

110 directions:

✓$[110]$; ✓$[101]$; ✓$[011]$
✓$[\bar{1}\bar{1}0]$; ✓$[\bar{1}0\bar{1}]$; ✓$[0\bar{1}\bar{1}]$
✓$[1\bar{1}0]$; ✓$[10\bar{1}]$; ✓$[01\bar{1}]$
✓$[\bar{1}10]$; ✓$[\bar{1}01]$; ✓$[0\bar{1}1]$



4.35

The number of vacancies per cubic centimeter gold is:

$$\frac{n}{N} = \exp\left(\frac{-Gv}{kT}\right) = 9.18*10^{-16}$$

$$V = a^3 = 0.408^3 = 0.0679nm^3$$

$$N = \frac{4}{V} = 5.89*10^{22}$$

$$n = \exp\left(\frac{-Gv}{kT}\right)*N = 1.23*10^{8}$$

4.40

$$Copper(Cu)$$
$$G = 48.3GPa$$
$$b = 0.25nm = 2.5*10^{-10}m$$

(a) Find the force required to bend a dislocation into radius R=10um:

$$F = \frac{Gb^2}{2R} = \frac{(48.3*10^9 Pa)(2.5*10^{-10}m)^2}{2(10*10^{-6}m)} = 1.5*10^{-4} N/m$$

(b) Energy of the curved dislocation

$$U = \frac{Gb^2}{10} + \frac{Gb^2}{4\pi(1-\nu)}(1-\nu\cos^2\alpha)\ln(\frac{\rho^{-1/2}}{5b})$$

Screw dislocation, so α=0,

ν = 0.343 for copper

(c) Energy of the curved dislocation

$$U = \frac{Gb^2}{10} + \frac{Gb^2}{4\pi(1-\nu)}(1-\nu\cos^2\alpha)\ln(\frac{\rho^{-1/2}}{5b})$$

Screw Dislocation, so α=0,

ν = 0.343 for copper

$$\rho = \frac{1}{R^2} = \frac{1}{(10*10^{-6}m)^2} = 1*10^{10}m^{-2}$$

$$U = \frac{(48.3*10^9 Pa)(2.5*10^{-10}m)^2}{10} + \frac{(48.3*10^9 Pa)(2.5*10^{-10}m)^2}{4\pi(1-0.343)}(1-0.345\cos^2(0))*\ln[\frac{(1*10^{10}m^{-2})^{-1/2}}{5(2*10^{-10}m)}]$$

*U≈2.5077*10^{-9} Pa*m^{2}=2.5077*10^{-9} N*

or

$$U = \frac{Gb^2}{4\pi}\ln\left(\frac{R}{5b}\right) = \frac{(48.3*10^9 Pa)(2.5*10^{-10}m)^2}{4\pi}\ln\left(\frac{10*10^{-6}m}{5*2.5*10^{-10}m}\right) \text{ (eq 4.22a)}$$

$$= (2.402*10^{-10})(8.987) = 2.159*10^{-9} Pa*m^2 = 2.159*10^{-9} N$$