12.4 An intermetallic compound of Al and Mg has a stable range of 52Mg−48Al to 56Mg−44Al (on a weight basis). What atomic ratios do these compositions correspond to? The atomic weight of Al is 27 and that of Mg is 24.31.

Weight percentage: 52% Mg-48% Al
56% Mg-44% Al

Atomic Weight: Al= 27 amu
Mg= 24.31 amu

$\boxed{\text{First Ratio}}$: 48% Al, 52% Mg

Total weight= 27x + 24.31y

$$\frac{27x}{27x + 24.31y} = \text{Percentage of Al} = 0.48$$

27x- 12.96x = 11.6688y

14.04x = 11.6688y

$$\frac{351x}{25} = \frac{7293y}{625}$$

$$\frac{8775x}{625} = \frac{7293y}{625}$$

$$\frac{6}{5}x \approx 1.203x$$

1.203x = y$\rightarrow$ $6x = 5y$ $\approx Al_6Mg_5$

$\boxed{\text{Second Ratio}}$: 44% Al, 56% Mg

Total Weight = 27x + 24.31y

$$\frac{27x}{27x + 24.31y} = \text{ Percentage Al} = 0.44$$

27x – 11.88x = 10.6964y

15.12x = 10.6964y

1.414x = y

707x = 500y

X= 7 Y= 5

$$\approx Al_7 Mg_5$$

12.5 A metallic laminate consists of FeAl as matrix and Ti as reinforcement.
If the temperature rises from 300 K to 325 K, estimate the expansion of laminated composite. What kinds of problems do you think will be caused by this? Explain.

ΔT = 25K : temperature increase from 300K- 325K

$$\alpha_c = \frac{\alpha_{FeAl} E_{FeAl} V_{FeAl} + \alpha_{Ti} E_{Ti} V_{Ti}}{E_{FeAl} V_{FeAl} + E_{Ti} V_{Ti}}$$

Assume: volumetric ratio of FeAl 80% Ti 20%

	α $10^{-6} K^{-1}$	E [GPa]	Vol %
FeAl	21.5	160 −250	.8
Ti	8.6	110 −130	.2

Coefficient of thermal expansion of composite (using average values of moduli, E_{FeAl} = 205 GPa and E_{Ti} = 120 GPa)

$$\alpha_c = 19.85 \times 10^{-6} K^{-1}$$

Expected expansion strain in composite:
$\alpha_c * \Delta T = 496 \text{ x } 10^{-6}$
This is the overall strain of the laminate. Individual components will have tensile stress in FeAl and compressive stress in Ti.

12.6 Find the relationship between pressure and relative density of powder from the data shown in Figure 12.22 for Fe, Ni, Cu, and W. See section 12.4.5

The overall behavior can be represented by:

$$P = \frac{1}{K}\left[\ln\frac{1}{1-D} + B\right]$$

$D =$ relative density = $\dfrac{\rho}{\rho_s}$

P = pressure

$$P = \frac{1}{K}\ln\left(\frac{1}{1-D}\right) + \frac{B}{K}$$

B, K are experimental parameters

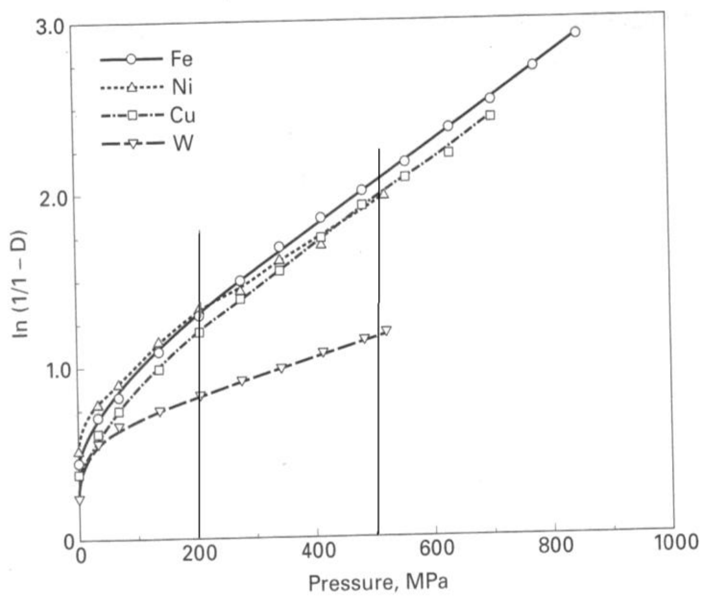

Fig. 12.22 Relationship between pressure and relative green density for several powders. (Adapted from R. M. German, *Powder Metallurgy Science* (Princeton, NJ: Powder Industries Federation), 1984.)

Two unknowns, we need at least 3 points from the graph to solve, since it is a nonlinear plot.

	Pt 1 P = 0 MPa	Pt 2 P = 200 MPa	Pt 3 P = 500 MPa
Fe, $\ln\frac{1}{1-D}$	$\propto$.45	1.25	2.1
Ni, $\ln\frac{1}{1-D}$	$\propto$ .25	1.30	1.95
Cu, $\ln\frac{1}{1-D}$	$\propto$ .40	1.2	1.95
W, $\ln\frac{1}{1-D}$	$\propto$ .50	.80	1.2

* Points are interpolated from graph

K = slope of linear region B = theoretical intercept

$$(1)\quad K = \frac{\Delta\left(\ln\frac{1}{1-D}\right)}{\Delta P} = \frac{\left(\ln\frac{1}{1-D}\right)_2 - \ln\left(\frac{1}{1-D}\right)_1}{P_2 - P_1}$$

$$(2)\quad P = \frac{1}{K}\left[\ln\frac{1}{1-D} + B\right] \Rightarrow PK - \ln\frac{1}{1-D} = B$$

$\boxed{\text{For all materials: } P_2 = 500 \text{ MPa},\ P_1 = 200 \text{ MPa},\ \Delta P = 300 \text{ MPa}}$

Fe:

$$K = \frac{2.1 - 1.25}{300} = 0.0028$$

Pt. 3

$B = 500\ (0.0028) - 2.1 = -0.6833$

$$P = 352.9\left[\ln\frac{1}{1-D} - 0.6833\right] \text{ in MPa}$$

Ni:

$$K = \frac{1.95 - 1.30}{300} = 0.0021$$

Pt. 3

$B = 500\ (.0022) - 1.95 = -0.85$

$$P = 454.5\left[\ln\frac{1}{1-D} - 0.85\right] \text{ in MPa}$$

Cu:

$$K = \frac{1.95 - 1.20}{300} = 0.0025$$

Pt. 3

$B = 500\ (0.0025) - 1.95 = -0.70$

$$P = 400\left[\ln\frac{1}{1-D} - 0.70\right] \text{ in MPa}$$

W:

$$K = \frac{1.20 - 0.80}{300} = 0.00133$$

Pt. 3

$B = 500\ (0.0013) - 1.2 = -0.55$

$$P = 750\left[\ln\frac{1}{1-D} - 0.55\right] \text{ in MPa}$$

12.8 Calculate C_1 in the equation:

$$\frac{\rho^*}{\rho_s} = C_1\left(\frac{t}{l}\right)^2$$

Hint: Assume fully dense material.

$\rho_{s=}$ density of solid material

$\rho^* =$ density of cellular material

Assuming the material is fully dense, i.e., $\rho_s = \rho^*$ and t = $\frac{l}{2}$

$$\frac{\rho^*}{\rho_s} = 1$$

$$C_1 = \left(\frac{l}{t}\right)^2$$

Plug t into the previous equation

$$C_1 = \left(\frac{l}{\frac{l}{2}}\right)^2 = 4$$

$$C_1 = 4$$

12.9 Determine the pressure required to densify a copper powder (σ_0 = 100 MPa) to 90% of the theoretical density using:
(a) The Fischmeister and Arzt equation:
(b) The Carroll--Holt--Torre equation.

(a) The Fischmeister and Arzt equation. (Particle flattens)

$$D = Z\sigma_0 \frac{D(D - D_0)}{4(1 - D_0)}$$

$$D = 0.90$$
$$\sigma_0 = 100\,MPa$$
$$Z = 12D = 10.8$$
$$D_0 = 0.64$$

$$P = 10.8(100)\frac{0.9(0.9 - 0.64)}{4(1 - 0.64)} = 175.5\,MPa$$

(b) Carroll- Holt- Tore equation. (Hollow Sphere)

Equation 12.23

$$-P = -\frac{2}{3}\sigma_0 \ln\frac{b^3}{a^3}$$

D = 0.9

First solve for $\frac{b^3}{a^3}$

See p. 649 in the textbook

$$D = \frac{b^3 - a^3}{b^3} \Rightarrow b^3 - Db^3 = a^3$$

$$\boxed{\frac{b^3}{a^3} = \frac{1}{1 - D}}$$

$$-P = -\frac{2}{3}(100)\ln\left(\frac{1}{1 - 0.9}\right)$$

$$P = \frac{2}{3}(100)\left(\ln\frac{1}{1 - 0.9}\right)$$

$$P = 153.5\,MPa$$

12.10 From load vs. displacement for keratin shown in Figure Ex12.10, calculate hardness of keratin. Hardness is given by:

$$H = \frac{P_{\max}}{A_p}$$

where P = load, A = projected area. Assume Berkovich tip was used.

For standard Berkovich tip with face angles of 65.3° used for nanoindentation:

Section 3.8 (p. 227)

$$A = a + bh_i^{\frac{1}{2}} + chi + dh_i^{\frac{3}{2}} + 24.56h_i^2$$

For perfect tip :

$$a = b = c = d = 0$$

$$A = 24.56hi^2$$

h_i = maximum displacement

From figure : maximum load, P $\approx$ 961μN and displacement ≈ 225nm.

$$H = \frac{961 \times 10^{-6}}{24.56\left(225 \times 10^{-9}\right)^2} = 772.9 \times 10^6 \, Pa$$

$$H = 772 \, MPa$$

12.11 Figure Ex12.12 shows the compressive stress-strain curve from the foam of a toucan beak. Calculate the densification strain, Young's modulus, shear modulus, and plastic collapse stress of this foam. Assume this foam is open celled.

ρ^* = Density of foam ρ_s = Density of cell wall

E_s = Young's modulus of cell wall σ_y = Yield stress of cell wall

$$\rho^* = \frac{0.04g}{cm^3}$$

$$\rho_s = \frac{0.5g}{cm^3}$$

$$E_s = 12.7GPa$$

$$\sigma_y = 90MPa$$

Densification Strain: ε_d

$$\varepsilon_d = 1 - 1.4\left(\frac{\rho^*}{\rho_s}\right) = 1 - 1.4\left(\frac{0.04}{0.5}\right) = 0.888$$

From p. 643 in the textbook.

Experimental measurements indicate that $\frac{C_1}{24C_2} \approx 1$

Young's modulus (Equation 12.9)

$$\frac{E^*}{E_s} = \frac{C_1}{24C_2}\left(\frac{\rho^*}{\rho_s}\right)^2$$

$$\frac{E^*}{E_s} = \left(\frac{\rho^*}{\rho_s}\right)^2 = \left(\frac{0.04}{0.5}\right)^2 = 0.0064$$

$$E^* = 0.0064(E_s) = 81.28MPa$$

Shear modulus (Equation 12.11)

$$\frac{G}{E_s} = \frac{3}{8}\left(\frac{\rho^*}{\rho_s}\right)^2$$

$$G = E_s \cdot \frac{3}{8}\left(\frac{\rho^*}{\rho_s}\right)^2$$

$$G = \left(12.7 \times 10^9\right)\left(\frac{3}{8}\right)\left(\frac{.04}{.5}\right)^2 = 30.48 \times 10^6 = 30.48\,MPa$$

Plastic collapse stress

Equation 12.19

$$\frac{\sigma_p{}^*}{\sigma_y} = C_1^{-\frac{3}{2}}\left(\frac{\rho^*}{\rho_s}\right)^{\frac{3}{2}}$$

Section 12.4.3 – Best fit experimental results for data : $C_1^{\frac{-3}{2}} \approx 0.3$

$$\sigma_p{}^* = \sigma_y(0.3)\left(\frac{\rho^*}{\rho_s}\right)^{\frac{3}{2}} = \left(90 \times 10^6\right)(0.3)\left(\frac{0.04}{0.5}\right)^{\frac{3}{2}}$$

$$\sigma_p{}^* = 61.09 \times 10^4\ Pa$$