3.1 A polycrystalline metal has a plastic stress-strain curve that obeys Hollomon's equation.

$$\sigma = K\varepsilon^n$$

Determine n, knowing that the flow stresses of this material at 2% and 10% plastic deformation (offset) are equal to 175 and 185 MPa, respectively.

$\sigma = K\varepsilon^n$

At 2% strain (0.02), σ = 175 MPa
At 10% strain (0.1) σ = 185 MPa

$\log(\sigma) = \log(K) + n \log(\varepsilon)$

$\log(175) = \log(K) + n \log(0.02)$
$\log(185) = \log(K) + n \log(0.1)$

$\log(175) / \log(185) = n \log(0.02) / \log(0.1)$

$0.9459 = 0.2n$
$\log(0.9459) = n \log(0.2)$
$n = 0.0345$

3.4 A steel with a yield stress of 300 MPa is tested under a state of stress where $\sigma_2 = \sigma_1 / 2$ *and* $\sigma_3 = 0$. What is the stress at which yielding occurs if it is assumed that:

(a) The maximum-normal-stress criterion holds?
(b) The maximum-shear-stress criterion holds?
(c) The distortion-energy criterion holds?

According to the maximum-normal-stress criterion

$$\sigma_1 = \sigma_y = 300\ MPa$$
$$\sigma_2 = \sigma_1 / 2 = 150\ MPa$$
$$and\ \sigma_3 = 0$$

According to the maximum-shear-stress criterion

$$\sigma_1 - \sigma_3 = \sigma_y$$
$$\sigma_1 = \sigma_3 + \sigma_y = 300\ MPa$$
$$\sigma_2 = \sigma_1 / 2 = 150\ MPa$$
$$and\ \sigma_3 = 0$$

According to the distortion-energy criterion

$$\frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2} = \sigma_y$$
$$\frac{1}{\sqrt{2}}\sqrt{(\sigma_1 / 2)^2 + (\sigma_1 / 2)^2 + (\sigma_1)^2} = \sigma_y$$

$$\sigma_1 = \frac{2}{\sqrt{3}}\sigma_y = 346.4\ MPa$$
$$\sigma_2 = \sigma_1 / 2 = 173.2\ MPa$$
$$and\ \sigma_3 = 0$$

3.16.

$$\sigma = 300 + 450\ e^{0.5}$$

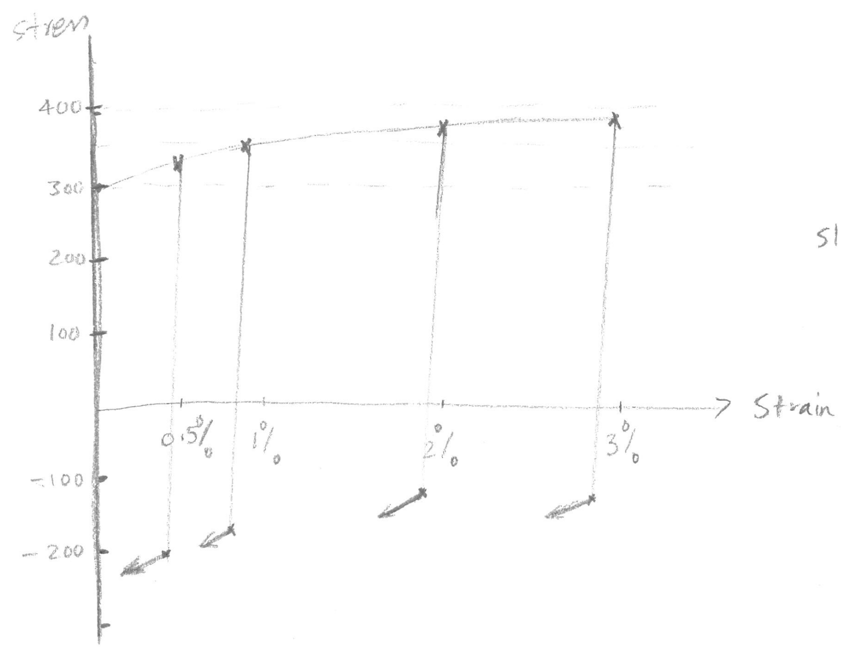
Pre-strain $e(\%)$	σ	σ_c / σ_t Fig. 3.17	σ_t
0.5	332	0.62	206
1	345	0.5	173
2	364	0.38	138
3	378	0.36	136

Look up mechanical properties of AISI 1045 on Azom.com

σ_{ut} is 310 MPa So it is almost the same grade of steel as described by $\sigma = 300 + 450\ e^{0.5}$

Elastic modulus = 200 GPa (typical value for steel)

$$\left.\begin{aligned} \sigma &= E\varepsilon \\ 300\ \text{MPa} &= 200\ \text{GPa} \times \varepsilon \longrightarrow \varepsilon = 0.015 \end{aligned}\right\}$$ Neglected in (σ-ε) equation.



slope of unloading 200GPa

3.21 When tested at room temperature, a thermoplastic material showed a yield of 51 MPa in uniaxial tension and 55 MPa in uniaxial address. Compute the yield strength of this polymer when tested in a pressure chamber with superimposed hydrostatic pressure of 300 MPa.

According to modified von Mises' Criterion for polymers, when a thermoplastic material showed yield strength of 51 MPa in uniaxial tension, we have:

$$(\sigma_1 - \sigma)^2 + (\sigma_2 - \sigma_3)^2 + [\sigma_3 - \sigma_1]^2 = 2\sigma_o^{\ 2} = 6(k_o + A_o \sigma_p)^2$$

$$where\ \sigma_1 = 51\ MPa\ ,\ \sigma_2 = \sigma_3 = 0,\ \sigma_p = \frac{\sigma_1}{3} = 17\ MPa$$

And when the thermoplastic material showed yield strength of 55 MPa in uniaxial compression, we get:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_0^{\ 2} = 6(k_o + A_o\sigma_p)^2$$

$$where\ \sigma_1 = -55 MPa,\ \sigma_2 = \sigma_3 = 0,\ \sigma_p = \frac{\sigma_1}{3} = -18.3\ MPa$$

From these two equations, we get

$$k_o = 30.5\ MPa,\ A_o = -0.065$$

Now this polymer is tested under a uniaxial stress in a pressure chamber with a superimposed hydrostatic pressure of 300 MPa. It is not specified if the uniaxial stress is tensile or compressive. We consider both the cases.

Tension

There will be a contribution to hydrostatic stress from the uniaxial stress equal to $\sigma_o/3$

$$2\sigma_0^2 = 6(k_o + A_o\sigma_p)^2$$

Inserting the values of $k_o = 30.5\ MPa,\ A_o = -0.065$ and $\sigma_p = -300 + \sigma_o/3$, we get

$$\sigma_o / \sqrt{3} = 30.55 - 300(-0.065) - 0.065\sigma_o / 3$$
$$0.599\sigma_o = 50.05$$
$$\sigma_o = 83.56MPa$$

Compression

This time the contribution to hydrostatic stress from the uniaxial stress is equal to $-\sigma_o/3$

$$2\sigma_o^2 = 6(k_o + A_o\sigma_p)^2$$

Inserting the values of $k_o = 30.5MPa$, $A_o = -0.065$ and $\sigma_p = -300 - \sigma_o/3$, we get

$$\sigma_o / \sqrt{3} = 30.55 - 300(-0.065) + 0.065\sigma_o / 3$$
$$0.555\sigma_o = 50.05$$
$$\sigma_o = 90.18MPa$$

3.38 Determine the hardness of the copper specimen from the nanoindentation SEM image in Figure 3.42(b) knowing that the applied load is 2000 μN.

Load = $2000 \times 10^{-6}\, N$

Berkovich tip was used

$$H = \frac{P_{max}}{A}$$

$$A = a + bhi^{\frac{1}{2}} + chi + dhi^{\frac{3}{2}} + 24.56hi^{2}$$

Assume perfect tip a = b = c = d = 0

$$A = 24.56hi^{2}$$

$$L \sin\alpha = hi$$

$$\alpha = 65.3$$

L from figure $\approx 14\mu m = 14 \times 10^{-6} m$

$$H = \frac{P_{max}}{24.56(L\sin\alpha)^{2}} = \frac{2000 \times 10^{-6}}{24.56(14 \times 10^{-6})^{2}(\sin 65.3)^{2}} = 503{,}370$$ Pa

H = .503MPa