7.1 In a polyvinyl chloride (PVC) plate, there is an elliptical, through-the-thickness cavity. The dimensions of the cavity are:

major axis = 1mm,
minor axis = 0.1mm.

Compute the stress concentration factor K_t, at the extremities of the cavity.

$$K_t = 2\sqrt{\frac{a}{\rho}}$$

$$\rho = \frac{b^2}{a} = \frac{(0.1/2\,mm)^2}{1/2\,mm} = 0.005\,mm$$

$$\therefore K_t = 2\sqrt{\frac{1/2}{0.005\,mm}} = 20$$

7.2 Calculate the maximum tensile stress at the surfaces of a circular hole (in the case of a thin sheet) and of a spherical hole (in the case of a thick specimen) subjected to a tensile stress of 200 MPa? The material is Al_2O_3 with v = 0.2.

Circular hole: Using Inglis's formula for elliptical hole and setting a = b, we obtain:

$$\sigma_{\max} = 3 \cdot \sigma = 600\,MPa$$

$$\sigma_{\max} = 600\,MPa$$

Spherical hole: Using the formula by Goodier for the equatorial plane ($\theta = \pi/2$) and setting r = a in order to obtain the stress at the surface of the hole.

$$\sigma_{\theta\theta} = \left[1 + \frac{4-5v}{2(7-5v)} \cdot \frac{a^3}{r^3} + \frac{9}{2(7-5v)} \cdot \frac{a^5}{r^5}\right]\sigma$$

$$= \left[1 + \frac{3}{12} + \frac{9}{12}\right]\sigma$$

$$= 2 \cdot \sigma = 400\,MPa$$

$$\sigma_{\max} = 400\,MPa$$

Thus, the stress concentration factor is smaller for a spherical hole than for a circular hole.

7.3 Calculate the maximum tensile stress if the applied stress is compressive for a circular hole for which $\sigma_c = 200\,MPa\ and\ v = 0.2$.

$$\sigma_c = 200\ MPa \quad v = 0.2$$

$$(\sigma_\theta)_{\theta=o} = -\frac{3+15v}{2(7-5v)}\sigma$$

$$\Rightarrow an\ applied\ compresive\ force\ generates\ a\ tensile\ stress$$

$$= -\frac{3+15(0.2)}{2(7-5(0.2))}(-200\ MPa) = 100\ MPa$$

7.6 An Al_2O_3 specimen is being pulled in tension. The specimen contains flaws having a size of $100\,\mu m$. If the surface energy of Al_2O_3 is 0.8 J/m^2, what is the fracture stress? Use Griffith's criterion. E = 380 GPa.

According to Griffith's criterion, the critical stress required for the crack to propagate in the plane-stress situation.

$$\sigma_c = \sqrt{\frac{2E\gamma_s}{\pi a}}$$

where γ_s *is the surface energy* $= 0.8\, J/m^2$

a is half the crack length $= 50\,\mu m$

$$\sigma_c = \sqrt{\frac{2 \times 380 \times 10^9 \times 0.8}{50 \times 10^{-6}}}$$

$$\sigma_c = 62.2\ MPa$$

7.7 A thin plate is rigidly fixed at its edges (see Figure Ex. 7.7). The plate has a height L and thickness t (normal to the plane of the figure). A crack moves from left to right through the plate. Every time the crack moves a distance Δs, two things happen:

1. Two new surfaces (with specific surface energy) are created.
2. The stress falls to zero behind the advancing crack front in a certain volume of the material.

Obtain an expression for the critical stress necessary for crack propagation in this case. Explain the physical significance of this expression.

Strain energy per unit volume $= \dfrac{\sigma^2}{2E}$

Total strain energy released when the crack moves a distance Δx = $\frac{\sigma^2}{2E}$. L.t.Δx.

At the same time as the strain energy is released due to the crack propagation, two new surfaces are created which result in an increase in surface energy equal to $\gamma.2.\Delta x$ .t.

If U represents the change in energy, then for crack propagation to occur, we must have

$$\frac{\partial U}{\partial \Delta x} = 0$$

$$U = -\,(L.\Delta x.t)\,\frac{\sigma^2}{2E} + 2\gamma.\Delta x.t$$

Therefore,

$$\frac{\partial U}{\partial \Delta x} = Lt\frac{\sigma^2}{2E} + 2\gamma t = 0$$

Or, the critical stress for crack propagation is

$$\sigma = \sqrt{\frac{4E\gamma}{L}} = 2\sqrt{\frac{E\gamma}{L}}$$

Significance: The thinner is the plate (i.e., the smaller is the L), the larger is the stress necessary for crack propagation. The situation is akin to that obtained in an adhesive joining to two parts. The thinnest possible adhesive layer will lead to the strongest possible bond.

7.8 A central through-the-thickness crack, 50 mm long, propagates in a thermoset polymer in an unstable manner at an applied stress of 5 MPa. Find K_c.

For a central through-the-thickness crack, we take Y = 1. Thus

$$K = \sigma\sqrt{\pi\, a}$$

$K_c = 5\sqrt{\pi(0.05)}$

$= 19.8 \text{ MPa}\sqrt{m}$

7.10 An AISI 4340 steel plate has a width W of 30 cm and has a central crack 2a of 3 mm. The plate is under a uniform stress, σ. This steel has a K_{Ic} value of 50 $MPa\sqrt{m}$ and a services stress of 1500 MPa. Compute the maximum crack size that the steel may have without failure.

$$K_{Ic} = f\left(\frac{a}{w}\right) \quad \sigma\sqrt{\pi \quad a_c}$$

$$\frac{a}{w} = 0.005$$

$$\sigma = 1500\ MPa$$

$$K_{Ic} = 50\ MPa\sqrt{m}$$

$$For\ \frac{a}{w} \leq 0.074, \quad f\left(\frac{a}{w}\right) = 1$$

$$a_c = \left(\frac{50}{1500}\right)^2 \times \frac{1}{\pi}$$

$$= 0.00035\ m$$

$$2a_c = 0.0007m = 0.7\ mm$$

7.11 A microalloyed steel, quenched and tempered at 250°C, has a yield strength (σ_y) of 1750 MPa and a plane-strain fracture toughness K_{Ic} of 43.50 $MPa\sqrt{m}$. What is the largest disk-type inclusion, oriented most unfavorably, that can be touched in this steel at an applied stress f 0.5 σ_y?

$$\sigma_y = 1750\ MPa$$

$$K_{Ic} = 43.5\ MPa\sqrt{m}$$

A disk type inclusion, oriented most unfavorably, can be likened to a penny-shaped internal crack. In the most general case, such a crack will have an elliptical form. The stress intensity factor for an elliptical crack is given by

$$K_I = \frac{\sigma\sqrt{\pi}}{\sqrt{a}E(k)} \quad \left(a^2 \sin^2\theta + b^2\cos^2\theta\right)^{1/4}$$

where a and b the semi-major and semi-minor axes of the ellipse, respectively, and E(k) is the complete elliptical integral of the second kind. For the present case, we can consider the inclusion to be a circular one of radius, a. Then, putting a = b, we have (see p. 426 in the text)

$$K_I = \frac{2}{\pi}\sigma\sqrt{\pi\, a}$$

$$For \quad \sigma = 0.5\,\sigma_{y,} \quad a_c = \pi\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$

$$= \pi\left(\frac{43.5}{875}\right)^2 = 0.0078\ m$$

$$= 7.8\ mm$$

The diameter of the largest inclusion tolerable is

d = 2a_c = 15.6 mm

7.12 A 25-mm^2 bar of cast iron contains a crack 5 mm long and normal to one face. What is the load required to break this bar if it is subjected to three- point bending with the crack toward the tensile side and the supports 250 mm apart?

A = 5 mm
B = W = 25 mm
L = 250 mm

For three point bending situation, we have

$$K = \frac{PL}{BW^{3/2}}\left[2.9\left(\frac{a}{W}\right)^{1/2} - 4.6\left(\frac{a}{W}\right)^{3/2} + 21.8\left(\frac{a}{W}\right)^{5/2} - 37.6\left(\frac{a}{W}\right)^{7/2} + 38.7\left(\frac{a}{W}\right)^{3/2}\right]$$

To find the breaking load, we put $K_I = K_{Ic} = 20$ MPa $\sqrt{m}$ for cast iron, substitute for L, B, W and $\frac{a}{W}$ and put P = P_c. Thus

$$P_c = \frac{K_I B W^{3/2}}{L,\ f\left(\frac{a}{W}\right)}$$

$$For \quad \frac{a}{W} = 0.2,\ f\left(\frac{a}{W}\right) = 1.13$$

$$P_c = \frac{20\times10^6 \times 25\times10^{-3} \times \left(25\times10^{-3}\right)^{3/2}}{250\times10^{-3}\times1.13} = 6.9\ kN$$

7.13 Consider a maraging steel plate of thickness (B) 3 mm. Two specimens of width (W) equal to 50 mm and 5 mm were taken out of this plate. What is the largest through-the-thickness crack that can be tolerated in the two cases at an applied stress of $\sigma = 0.6\sigma_y$ where σ_y (yield stress) = 2.5 GPa? The plane-strain fracture toughness K_{Ic} of the steel is 70 MPa $\sqrt{m}$. What are the critical dimensions in the case of a single-edge notch specimen?

$$\sigma_y = 2500\ MPa$$
$$\sigma_{ap} = 0.6\sigma_y = 1500\ MPa$$
$$K_{Ic} = 70\ MPa\sqrt{m}$$
$$B = 3\ mm$$
$$Compare\ B\ with\ r_y$$

$$B \geq 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2 = 2.5\left(\frac{70}{2500}\right)^2 = 1.96\ mm$$

So the plastic zone is small.

Central through thickness crack

$$K_I = \sigma\left(W \tan\frac{\pi a}{W}\right)^{1/2} \; or \; K_I = \sigma\sqrt{\pi a}\left(\sec\frac{\pi a}{W}\right)^{1/2}$$

$$Assume \; \frac{a}{W} \leq\leq 1, \; \sec\frac{\pi a}{W} \to 1$$

$$K_{Ic} = \sigma\sqrt{\pi a_c}$$

$$or \quad a_c = \left(\frac{K_{Ic}}{\sigma}\right)^2 \frac{1}{\pi} = \left(\frac{70}{1500}\right)^2 \frac{1}{\pi} = 0.69 \; mm$$

$$W = 50 \; mm$$

$$\frac{a}{w} = \frac{0.69}{50} = 0.0138$$

$$\sec\frac{a}{w} = 1, \; i.e., \; our \; assumption \; above \; is \; right \; and \; a_c = 0.69 \; mm,$$

Thus, no iteration is required.

$W = 5$ mm

$$\frac{a}{W} = \frac{0.69}{6} = 0.138$$

$$\sec\left(\frac{\pi a}{W}\right) = 1.102$$

which is not equal 1.

In this case, we must use an iterative process to calculate the a_c value until successive values are close enough. Thus

(i) $\sec\left(\frac{\pi a}{W}\right) = 1.1$

(ii) $\frac{a}{W} = \frac{0.630}{5} = 0.126$

$$\sec\left(\pi \times 0.126\right) = 1.084$$

$$a_2 = \left(\frac{70}{1500}\right)^2 \times \frac{1}{\pi} \times \frac{1}{1.084} = 0.639 \; mm$$

(iii)

$$\frac{a}{W} = \frac{0.639}{5} = 0.1278$$

$$\sec\left(\pi \frac{a}{w}\right) = 1.086$$

$$a_3 = \left(\frac{70}{1500}\right)^2 \times \frac{1}{\pi} \times \frac{1}{1.086} = .0638\ mm$$

$$Therefore\ a_c = 0.638\ mm$$

Single Edge Notch

$$K_I = f\left(\frac{a}{W}\right)\sigma\sqrt{a}$$

$$where\ f\left(\frac{a}{W}\right) = 1.99 - 0.41\left(\frac{a}{W}\right) + 18.7\left(\frac{a}{W}\right)^2 - 38.48\left(\frac{a}{W}\right)^3 + 53.85\left(\frac{a}{W}\right)^4$$

$$for\ \ 0 \le \frac{a}{W} \le 0.6$$

$$a_c = \left(\frac{K_{Ic}}{\sigma}\right)^2 \times \frac{1}{\left[f\left(\frac{a}{W}\right)\right]^2}$$

$$Assume\left(\frac{a}{W}\right) to\ be\ small,\ then\ f\left(\frac{a}{W}\right) = 1.99$$

$$\left(Note\ that\ 1.12\sqrt{\pi} = 1.99\right)$$

$$Then \qquad a_c = \frac{1}{(1.99)^2} \times \left(\frac{70}{1500}\right)$$

$$= 0.00055\ mm$$

$$a_c = 0.55\ mm$$

$$W = 50\ mm$$

$$for\ a = 0.5\ mm,\ \frac{a}{W} = 0.011\ and\ f\left(\frac{a}{W}\right) = 1.99$$

Therefore, $a_c = 0.55$ mm

For W = 5 mm

Let a = 0.55 m, $\frac{a}{W} = \frac{0.55}{5} = 0.11,\ f\left(\frac{a}{W}\right) = 2.18$

$$a = \frac{1}{(2.18)^2}\left(\frac{70}{1500}\right)^2 = 0.459\ mm$$

Taking this value of a, we estimate the new value of $f\left(\frac{a}{W}\right)$ until two successive values are about the same. Thus,

a (mm)	$\frac{a}{W}$	$f\left(\frac{a}{W}\right)$	$a_c\ (mm)$
0.459	0.0918	2.08	0.503
0.503	0.1006	2.10	0.494
0.494	0.0988	2.10	0.494

$a_c = 0.494$ mm

7.14 An infinitely large plate containing a central crack of length 2a = 50/π mm is subjected to a nominal stress of 300 MPa. The material yields at 500 MPa. Compute:

(a) The stress intensity factor at the crack.

(b) The size of the plastic zone at the crack up.

Comment on the validity of Irwin's correction for the size of the plastic zone in this case.

2a = 50/π mm

σ_y = 500 MPa

σ = 300 MPa

(a) For an infinitely large plate containing a central crack,

$$K = \sigma\sqrt{\pi\, a}$$

$$= 300 \times \sqrt{\pi \times \frac{25}{\pi} \times 10^{-3}} \quad MPa\sqrt{m}$$

$$K = 47.4\ MPa\sqrt{m}$$

(b) Plastic zone size at crack tip,

$$r_y = \frac{1}{2\,\pi}\left(\frac{K}{\sigma_y}\right)^2$$

$$= \frac{1}{2\,\pi}\left(\frac{47.4}{500}\right)^2 = 1.43 \times 10^{-3}\, m$$

$$r_y = 1.43\ mm$$

<u>Comment:</u> It is valid to use Irwin's correction for the plastic zone when r_y is small. Specifically, in the present case, $\frac{r_y}{a} = \frac{1.43}{(25/\pi)} = 0.18$, i.e., the condition $r_y \le \frac{a}{50}$ is not satisfied. Thus, it would be valid to use Irwin's correction for plastic zone.

7.15 A steel plate containing a through-the-thickness central crack of length 15 mm is subjected to a stress of 350 MPa normal to the crack plane. The yield stress of the steel is 1500 MPa. Compute the size of the plastic zone of the plastic zone and the effective stress intensity factor.

$$\sigma_y = 1500\ MPa$$

$$\sigma = 350\ MPa$$

$$2a = 15\ mm$$

$$r_y = \frac{1}{2\pi}\left(\frac{K}{\sigma_y}\right)^2 = \frac{1}{2\pi}\frac{(350)^2\ \pi\ (0.0075)}{(1500)^2}$$

$$= 0.204\ mm$$

$$K_{ef} = 350\sqrt{\pi\ (0.0075 + 0.000204)}$$

$$= 54.44\ MPa\sqrt{m}$$

$$(2a)_{ef} = 2(a + r_y)$$

$$r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_y}\right)^2$$

$$K_I = \sigma\sqrt{\pi\ a}$$

Taking the given value of crack length as a_{ef}, we compute K_{ef} and r_y. This gives us a new value of a_{ef}. We recalculate K_{ef} and r_y and repeat the process until the successive values are close enough. Thus,

(i)

$$K_I = 350\sqrt{\pi\ (7.5 \times 10^{-3})}$$

$$= 53.7\ MPa\sqrt{m}$$

$$r_y = \frac{1}{2\pi}\left(\frac{53.7}{1500}\right)^2 = 0.204mm$$

(ii)

$$K_I = 350\sqrt{\pi\ (7.5\times10^{-3} + 0.204\times10^{-3})}$$
$$= 54.45\,MPa\sqrt{m}$$
$$r_y = \frac{1}{2\,\pi}\left(\frac{54.45}{1500}\right)^2 = 0.209mm$$

(iii)

$$K_I = 350\sqrt{\pi\ (7.5\times10^{-3} + 0.209\times10^{-3})}$$
$$= 54.46\,MPa\sqrt{m}$$
$$r_y = \frac{1}{2\,\pi}\left(\frac{54.46}{1500}\right)^2 = 0.209mm$$

Thus, r_y = 0.209 mm and K_{ef} = $54.46\ MPa\sqrt{m}$

7.16 The size of the plastic zone at the crack tip in the general plane stress case is given by

$$r_y = \frac{K_I^{\,2}}{2\,\pi\sigma_y^{\,2}} \cos^2\theta/2\left(4 - 3\cos^2\theta/2\right)^2$$

(a) Determine the radius if the plastic zone in the direction of the crack.

(b) Determine the angle θ at which the plastic zone is largest.

(a) Plastic zone in the crack direction

θ = 0 in the crack direction, therefore,

$$r_y = \frac{K_I^{\,2}}{2\,\pi\sigma_y^{\,2}}$$

(b) Angle at which the plastic zone is the largest.

In order to find this angle, we differentiate the expression for r_y with respect to θ and equate it to zero. Let A = $\frac{K_I^{\,2}}{2\,\pi\sigma_y^{\,2}}$, then

$$\begin{aligned} \mathrm{d}r_y/\mathrm{d}\theta &= -4A \quad \cos\theta/2 \quad \sin\theta/2 + 6A \quad \cos^3\theta/2 \quad \sin\theta/2 \\ &= A\sin\theta/2\,[3\cos^3\theta/2 \quad -2] = 0 \end{aligned}$$

Now, sinθ = 0 gives θ = 0, which corresponds to a minima in r_y . 3 $\cos^2$ θ/2 = 0 gives θ = 70.5°.

7.17 For the plane-strain case, the expression for the size of the plastic zone is

$$r_y = \frac{K_I^2}{2\pi\, \sigma_\upsilon^2} \cos^2 \frac{\theta}{2}\left[4\left[1 - \nu(1-\nu) - 3\cos^2\frac{\theta}{2}\right]\right]$$

(a) Show that this expression reduces to the one for plane stress for ν = 0.
(b) Make plots of size of the plastic zone as a function of θ for = 0, ν = 1/3, and ν = ½. Comment on the size and form of the zone in the three cases.

Plane strain case

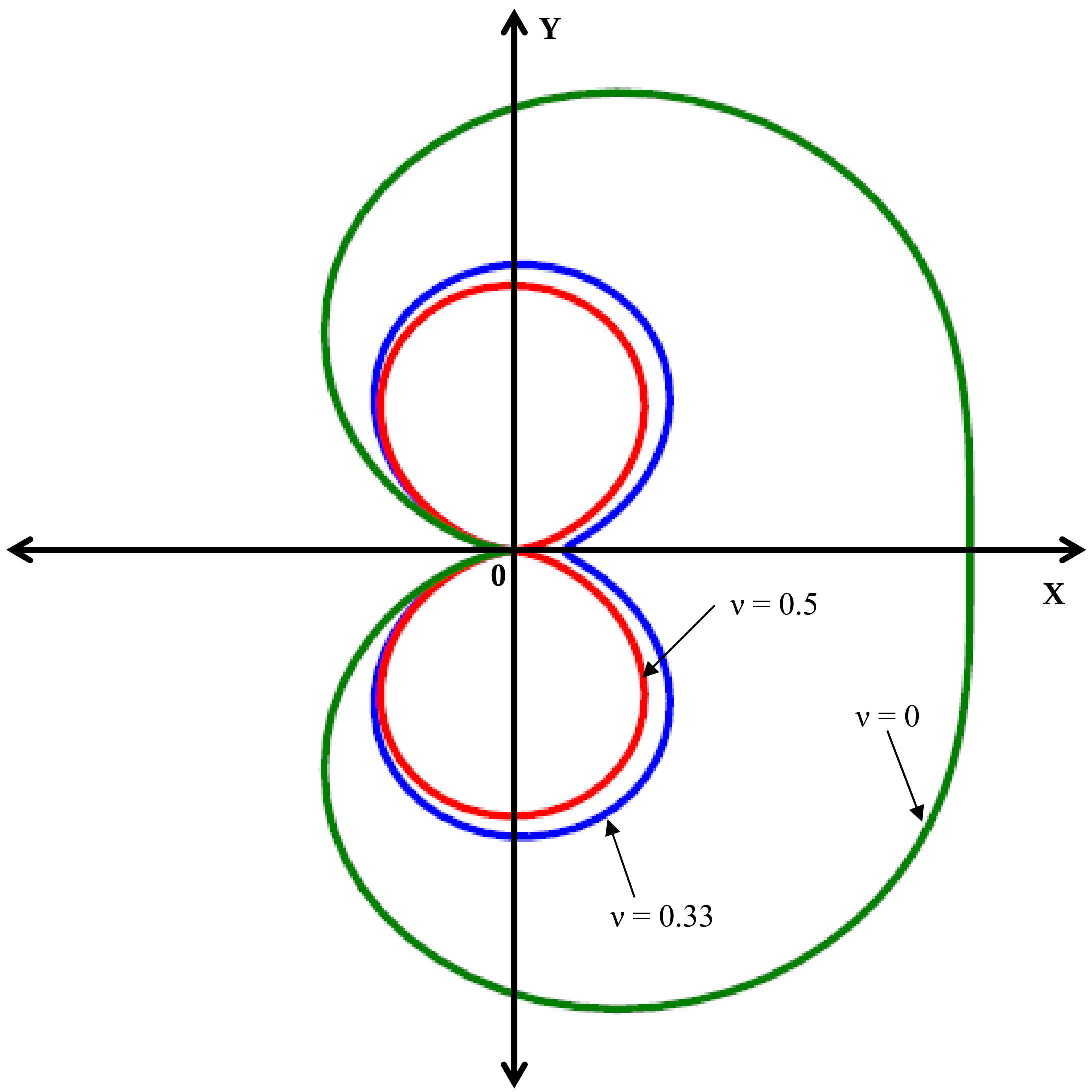
$$r_y = \frac{K_I^2}{2\pi\sigma y^2} \cos^2 \frac{\theta}{2}\left[4\left(1 - \nu(1-\nu) - 3\cos^2\frac{\theta}{2}\right)\right] \quad \textit{plane strain}$$

(a) For plane stress, ν = 0 and the above expression reduces to

$$r_y = \frac{K_I^2}{2\pi\, \sigma_y^{\ 2}} \cos^2 \frac{\theta}{2}\left[4 - 3\cos^2\frac{\theta}{2}\right]$$

which is the expression for the plane stress case.

(b) Plots of plastic zone sizes as function of θ for ν = 0, 1/3, and ½ are given in the figure below.



Comments

For ν =0, we have the case of plane stress and the size of the plastic zone is the largest. For ν =1/3 and ν = ½, the constraint in the thickness direction increases and the plastic zone reduces in size accordingly.

For ν =1/2, we have the extreme case of an incompressible material which has r_y =0 at θ = 0°, as there will be equal triaxial tension in this case.

7.18 A sheet of polystyrene has a thin central crack with 2a = 50 mm. The crack propagates catastrophically at an applied stress of 10 MPa. Young's modulus polystyrene is 3.8 GPa, and the Poisson's ratio is 0.4. Find G_{Ic}.

$$2a = 50mm$$
$$a = 25mm$$
$$= 0.025m$$
$$\sigma = 10MPa$$
$$E = 3.8GPa$$
$$\nu = 0.4$$
$$\sigma = \sqrt{\frac{E\,G_{Ic}}{\pi\,a(1-\nu^2)}}$$
$$G_{Ic} = \frac{\sigma^2\pi\,a(1-\nu^2)}{E}$$
$$= \frac{(10\times10^6)^2\;\pi(0.025)(1-0.4^2)}{3.8\times10^9}$$
$$= 1736\;J\;m^{-2}$$

7.19 Compute the approximate size of the plastic zone, r_y for an alloy that has a Young's modulus E = 70 GPa, yield strength σ_y = 500 MPa and a toughness G_c = 20 kJ/m^2

$$E = 70GPa$$
$$\sigma_y = 500MPa$$
$$G_c = 20kJm^{-2} = \frac{K_{Ic}^2}{E}$$
$$r_y = \frac{1}{2\,\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2 = \frac{EG_c}{2\pi\,\sigma_y^{\,2}}\quad .$$
$$= \frac{70\times10^9\times20\times10^3}{2\,\pi\left(500\times10^6\right)^2}$$
$$r_y = 0.89\times10^{-3}\,m = 0.89\ mm$$

7.20 300-M steel, commonly used for airplane landing gears, has a G_c value of 10 kN/m. A nondestructive examination technique capable of detecting cracks that are 1 mm long is available. Compute the stress level that the landing gear can support without failure.

$$G_c = 10 kN/m$$

$$K_c^2 = E\,G_c = 210 \times 10^9 \times 10 \times 10^3 = 2100 \times 10^{12}$$

$$K_c = 45.8\ MPa\sqrt{m}$$

The nondestructive examination technique can detect 1 mm long cracks, i.e., a_c = 1 mm. In other words, we are assuming that when the cracks become detectable, the landing gear must be substituted. Assume that the situation in practice corresponds to a single edge notch, i.e.,

$$K_c = 1.12\sqrt{\pi\, a_c}$$

$$\sigma = \frac{K_c}{1.12\sqrt{\pi\, a_c}} = \frac{45.8}{1.12\sqrt{\pi \times 1 \times 10^{-3}}}$$

$$= 729.6\ MPa$$

7.25 An engineering ceramic has a flexure strength that obeys Weibull statistics with m = 10. If the flexure strength is equal to 200 MPa at 50 % survival probability, what is the flexure strength level at which the survival probability is 90%?

$$P(V) = 0.5 \quad \sigma = 200\ MPa$$

$$\ell n[1/P(V)] = [\sigma/\sigma_o]^m$$

$$\ell n \ell n[1/P(V)] = m\,[\ell n\, \sigma - \ell n \sigma_o]$$

$$\ell n \ell n[1/0.5] = 10\,[\ell n\, 200 - \ell n \sigma_o]$$

$$-0.37 = 10\,[\ell n\, 200 - \ell n \sigma_o]$$

$$\ell n \sigma_o = 5.33$$

$$P(V) = 0.9 \quad \sigma = ?$$

$$\ell n \ell n(1/0.9) = 10[\ell n \sigma - 5.33]$$

$$\sigma = 165\ MPa$$

7.26 What would be the flexure strength, at 40 % survival probability, if the ceramic in the preceding problem is subjected to a hot isostatic processing (HIP) treatment that greatly reduces the population of flaws and increases m to 60? Assume that the flexure strength at 50 % survival probability is unchanged.

Weibull modulus, m = 10

HIP treatment increases m to 60

σ = 200 MPa at 50 % survival probability

Weibull statistics:

$$P(v_o) = \exp\left(-\left(\frac{\sigma}{\sigma_o}\right)^m\right)$$

$$\ell n(P(v_o)) = -\left(\frac{\sigma}{\sigma_o}\right)^m$$

$$\sigma_o = \frac{\sigma}{\left[\ell n(1/P(v_o))\right]^{1/m}} = 201.2 MPa$$

$$\sigma \text{ at } 90\% \text{ survival probability} = ?$$

$$\sigma = \sigma_o\left[\ell n(1/P(v_o))\right]^{1/m} = 193.8 MPa \quad \sigma = 193.8 \quad MPa$$

The population of flaws is reduced after HIP treatment. Thus, the flexure strength is increased by about 17% in this case.

7.27 Ten rectangular bars of Al_2O_3 (10 mm wide and 5 mm in height) were tested in three-point bending, the span being 50 mm. The failure loads were 1040, 1092, 1120, 1210, 1320, 1381, 1410, 1490, and 1540 N. Determine the characteristic flexure strength and Weibull's modulus for the specimens. (See Section 9.6.1 for the flexure formula.)

$$\text{Flexural strength in three-point bending} = \frac{3\,Fl}{2bh^2}$$

Probability of survival for N specimens, $P_i(v) = \dfrac{N+1-i}{N+1}$

L = 0.05 m, b= 0.01 m, h = 0.005 m

Load (N)	Sigma (Pa)	P(v)	ln(Sigma)	ln(ln(1/P(v))
1040	3.12E+08	0.909091	19.5585137	-2.35306187
1092	3.28E+08	0.818182	19.6073039	-1.60609
1120	3.36E+08	0.727273	19.6326217	-1.1442781
1210	3.63E+08	0.636364	19.7099134	-0.794106
1320	3.96E+08	0.545455	19.7969248	-0.5006512
1381	4.14E+08	0.454545	19.8421009	-0.237677
1410	4.23E+08	0.363636	19.8268827	0.01153414
1470	4.41E+08	0.272727	19.9045554	0.26181256
1490	4.47E+08	0.181818	19.9180692	0.53341735
1540	4.62E+08	0.090909	19.9510754	0.87459138

The Weibull plot has the slope, m given by

$m = y_2 - y_1/\ x_2 - x_1 = 0.9 - 0/19.95\text{-}19.86 = 10$

7.29 Aluminum has a surface energy of 0.5 Jm^{-2} and a Young's modulus of 70 GPa. Compute the stress at the crack tip for two different crack lengths: 1 mm and 1 cm.

$$\sigma_c = \sqrt{\frac{2E\gamma}{\pi\ a}}$$

$$\gamma = 0.5\ J/m^2$$
$$E = 70\ GPa$$

For 2a = 1 mm

$$\sigma_c = \sqrt{\frac{2\left(70\times10^9\right)(0.5)}{\pi\ \left(5\times10^{-4}\right)}}$$
$$\sigma_c = 6.68\ MPa$$

For 2a = 1 cm

$$\sigma_c = \sqrt{\frac{2\left(70\times10^9\right)(0.5)}{\pi\ \left(5\times10^{-3}\right)}}$$
$$\sigma_c = 2.11\ MPa$$

7.30 Determine the stress for crack propagation under plane strain for a crack length equal to 2 mm in aluminum. Take the surface energy equal to 0.018 J/m^2, Poisson's ratio to be 0.345, and the modulus of E = 70.3 GPa.

$$\gamma = 0.018\frac{J}{m^2}$$

$$E = 70.3\ GPa$$

$$\nu = 0.345$$

$$a = 2\times10^{-3}\times\frac{1}{2} = 1\times10^{-3}\,m$$

$$\sigma_c = \sqrt{\frac{2E\gamma}{\pi\ a\left(1-\nu^2\right)}}$$

$$\sigma_c = \sqrt{\frac{2\left(70.3\times10^9\right)\left(0.018\right)}{1\times10^{-3}\times\pi\times\left(1-0.345^2\right)}}$$

$$\sigma_c = 954.2\ kPa$$

7.31 Calculate the maximum load that a 2024-T851 aluminum alloy (10 cm x 2 cm) with a central through-the-thickness crack (length 0.1 mm) can withstand without yielding. Given: σ_y=500 MPa and K_{Ic}=30 MPa$\sqrt{m}$.

$$Y = 1 + 0.256\left(\frac{a}{W}\right) - 1.152\left(\frac{a}{W}\right)^2 + 13{,}200\left(\frac{a}{W}\right)^3$$

$$a = \frac{1}{100} \times \frac{1}{2} = 5 \times 10^{-3}\, m$$

$$W = 0.1\, m$$

$$Y = 1.011$$

$$\therefore \quad K_{Ic} = Y\sigma\sqrt{\pi\, a}$$

$$\sigma = \frac{30 MPa}{1.011\sqrt{\pi\left(5 \times 10^{-3}\right)}}$$

$$= 236.8\ MPa$$

$$\sigma = \frac{P}{A}$$

$$P = 236.8\ MPa\ (0.1)(0.02)$$

$$P = 473.5\ kN$$

7.32 An infinitely large sheet is subjected to a far-field stress of 300 MPa. The material has yield strength of 600 MPa, and there is a central crack $7/\pi$ cm long.

a) Calculate the stress intensity factor at the tip of the crack.
b) Estimate the size of the plastic zone size at the tip of the crack.

a) $K = \sigma\sqrt{\pi\, a}$

$$K = 300 \times 10^6 \sqrt{\pi \frac{7}{\pi} \cdot \frac{1}{2} \cdot \frac{1}{100}}$$

$$K = 56.12\ MPa\sqrt{m}$$

b) $\sigma_y = \dfrac{K}{\sqrt{2\pi\, r_y}}$ $-$ *for plane stress*

$$r_y = \frac{K^2}{2\pi\,\sigma_y}$$

$$r_y = \frac{(56.12\, MPa\sqrt{m})^2}{2\pi(600\, MPa)^2}$$

$$r_y = 1.39\, x10^{-3}\, m$$

$$r_y = \frac{1}{6\pi}\left(\frac{K}{\sigma y}\right)^2 - \textit{for plane strain}$$

$$r_y = \frac{1}{6\pi}\left(\frac{56.12\sqrt{m}}{600}\right)^2$$

$$r_y = 4.64\, x10^{-4}\, m$$

7.33 What is the maximum allowable crack size for a material that has $K_{Ic} = 55\ MPa\sqrt{m}\ \ and\ \sigma_v = 1{,}380\ MPa$? Assume a plane-strain condition and a central crack.

We take the design stress to be half the yield stress.

$$\sigma_{design} = \frac{1380}{2} = 690\ MPa$$

$$K_{Ic} = \sigma\sqrt{\pi a}$$

$$a = \left(\frac{K_{Ic}}{\sigma}\right)^2 \cdot \frac{1}{\pi}$$

$$a = \left(\frac{55}{690}\right)^2 \frac{1}{\pi}$$

$$a = 2.02\times10^{-3}\,m$$

$$\therefore 2a = 4\,mm$$

7.35 An Al_2O_3 specimen is being pulled in tension. The specimen contains flaws having a size of 100 μm.

a) If the surface energy of Al_2O_3 is 0.8 J/m^2, what is the fracture stress? Use Griffith criterion. E = 380 GPa

b) Estimate the fracture stress if the fracture toughness is 4 MPa m$^{0.5}$. Assume two positions for flaws :

1) in the center of an infinite body

2) in the edge of an infinite body.

(a) According to Griffith's criterion, the critical stress required for the crack to propagate in the plane-stress situation.

$$\sigma_c = \sqrt{\frac{2E\gamma_s}{\pi a}}$$

where γ_s *is the surface energy* $= 0.8\,J/m^2$

a is half the crack length $= 50\,\mu m$

$$\sigma_c = \sqrt{\frac{2 \times 380 \times 10^9 \times 0.8}{50 \times 10^{-6}}}$$

$$\sigma_c = 62.2\ MPa$$

(b)

1) For center cracked infinite body, Y = 1

$$K_{Ic} = Y\sigma\sqrt{\pi\, a}$$

$$\sigma = \frac{4MPa}{1.0\sqrt{\pi\left(50 \times 10^{-6}\right)}}$$
$$= 319.2\ MPa$$

2) For single edge cracked infinite body, Y = 1.12

$$K_{Ic} = Y\sigma\sqrt{\pi\, a}$$

$$\sigma = \frac{4MPa}{1..12\sqrt{\pi\left(100 \times 10^{-6}\right)}}$$
$$= 201\ MPa$$

7.37 A titanium alloy (Ti – 6Al – 4V) has a yield strength of 1050 MPa and a fracture toughness of 40 MPa $m^{0.5}$. If the applied stress is 0.3 yield strength, what would be the size of the surface crack that would lead to catastrophic failure?

$$\sigma_y = 1050\,MPa$$

$$\sigma = 0.3\,\sigma_y = 315\ MPa$$

$$K_{Ic} = Y\sigma\sqrt{\pi\, a}$$

$$Take\, Y = 1.$$

$$a = \left(\frac{K_{Ic}}{\sigma}\right)^2 \cdot \frac{1}{\pi}$$

$$a = \left(\frac{40}{315}\right)^2 \frac{1}{\pi}$$

$$a = 0.0041 \times 10^{-3}\, m$$

$$a = 4.1\, mm$$

7.40 What is the largest flaw size in a ceramic material that can support a strength of 280 MPa and K_{Ic} = 2.2 MPa $m^{0.5}$. Assume Y = 1.

$$K_{Ic} = Y\sigma\sqrt{\pi\, a}$$

$$a = \left(\frac{K_{Ic}}{Y\sigma}\right)^2 \cdot \frac{1}{\pi}$$

$$a = \left(\frac{2.2}{280}\right)^2 \frac{1}{\pi}$$

$$a = 0.0197 \times 10^{-3}\, m$$

$$2a = 0.039\ mm$$