Appendix A: p. 819, Ex. 1

a. $[\boldsymbol{\tau}\cdot\mathbf{v}]_x = (3)(5)+(2)(3)+(-1)(-2) = 23$

$[\boldsymbol{\tau}\cdot\mathbf{v}]_y = (2)(5)+(2)(3)+(1)(-2) = 14$

$[\boldsymbol{\tau}\cdot\mathbf{v}]_z = (-1)(5)+(1)(3)+(4)(-2) = -10$

b $[\mathbf{v}\cdot\boldsymbol{\tau}]_x = (5)(3)+(3)(2)+(-2)(-1) = 23$

$[\mathbf{v}\cdot\boldsymbol{\tau}]_y = (5)(2)+(3)(2)+(-2)(1) = 14$

$[\mathbf{v}\cdot\boldsymbol{\tau}]_z = (5)(-1)+(3)(1)+(-2)(4) = -10$

Note that these results are the same as those in part (*a*). Normally $[\boldsymbol{\tau}\cdot\mathbf{v}] \neq [\mathbf{v}\cdot\boldsymbol{\tau}]$, but since $\boldsymbol{\tau}$ is symmetric, the two operations give identical results.

c. $(\boldsymbol{\tau}:\boldsymbol{\tau}) = \sum_i \sum_j \tau_{ij}\tau_{ji} = \sum_i \sum_j \tau_{ij}^2$

since $\boldsymbol{\tau}$ is symmetric. Then

$$(\boldsymbol{\tau}:\boldsymbol{\tau}) = (3)^2+(2)^2+(-1)^2+(2)^2+(2)^2+(1)^2 \\ +(-1)^2+(1)^2+(4)^2 = 41$$

d $(\mathbf{v}\cdot[\boldsymbol{\tau}\cdot\mathbf{v}]) = \sum_i v_i[\boldsymbol{\tau}\cdot\mathbf{v}]_i$

$= (5)(23)+(3)(14)+(-2)(-10) = 117$

e $$\mathbf{vv} = \boldsymbol{\delta}_x\boldsymbol{\delta}_x(25)+\boldsymbol{\delta}_x\boldsymbol{\delta}_y(15)+\boldsymbol{\delta}_x\boldsymbol{\delta}_z(-10) \\ +\boldsymbol{\delta}_y\boldsymbol{\delta}_x(15)+\boldsymbol{\delta}_y\boldsymbol{\delta}_y(9)+\boldsymbol{\delta}_y\boldsymbol{\delta}_z(-6) \\ +\boldsymbol{\delta}_z\boldsymbol{\delta}_x(-10)+\boldsymbol{\delta}_z\boldsymbol{\delta}_y(-6)+\boldsymbol{\delta}_z\boldsymbol{\delta}_z(4)$$

f. $[\boldsymbol{\tau}\cdot\boldsymbol{\delta}_x]_x = (3)(1)+(2)(0)+(-1)(0) = 3$

$[\boldsymbol{\tau}\cdot\boldsymbol{\delta}_x]_y = (2)(1)+(2)(0)+(1)(0) = 2$

$[\boldsymbol{\tau}\cdot\boldsymbol{\delta}_x]_z = (-1)(1)+(1)(0)+(4)(0) = -1$

Appendix A: p. 823, Ex. 3

$a.\quad (\nabla\cdot\mathbf{v})=0$

$(\nabla\mathbf{v})_{xy}=b$ and all other component are zero

$$[\nabla\cdot\mathbf{vv}]_x=\frac{\partial}{\partial x}v_xv_x+\frac{\partial}{\partial y}v_yv_x+\frac{\partial}{\partial z}v_zv_x=0$$
$$[\nabla\cdot\mathbf{vv}]_y=\frac{\partial}{\partial x}v_xv_y+\frac{\partial}{\partial y}v_yv_y+\frac{\partial}{\partial z}v_zv_y=0$$
$$[\nabla\cdot\mathbf{vv}]_z=\frac{\partial}{\partial x}v_xv_z+\frac{\partial}{\partial y}v_yv_z+\frac{\partial}{\partial z}v_zv_z=0$$

$b.\quad (\nabla\cdot\mathbf{v})=b$

$(\nabla\mathbf{v})_{xx}=b$ and all other components are zero

$$[\nabla\cdot\mathbf{vv}]_x=\frac{\partial}{\partial x}v_xv_x+\frac{\partial}{\partial y}v_yv_x+\frac{\partial}{\partial z}v_zv_x=2b^2x$$
$$[\nabla\cdot\mathbf{vv}]_y=\frac{\partial}{\partial x}v_xv_y+\frac{\partial}{\partial y}v_yv_y+\frac{\partial}{\partial z}v_zv_y=0$$
$$[\nabla\cdot\mathbf{vv}]_z=\frac{\partial}{\partial x}v_xv_z+\frac{\partial}{\partial y}v_yv_z+\frac{\partial}{\partial z}v_zv_z=0$$

$c.\quad (\nabla\cdot\mathbf{v})=0$

$(\nabla\mathbf{v})_{xy}=b,\ (\nabla\mathbf{v})_{yx}=b$ and all others are zero

$$[\nabla\cdot\mathbf{vv}]_x=\frac{\partial}{\partial x}v_xv_x+\frac{\partial}{\partial y}v_yv_x+\frac{\partial}{\partial z}v_zv_x=b^2x$$
$$[\nabla\cdot\mathbf{vv}]_y=\frac{\partial}{\partial x}v_xv_y+\frac{\partial}{\partial y}v_yv_y+\frac{\partial}{\partial z}v_zv_y=b^2y$$
$$[\nabla\cdot\mathbf{vv}]_z=\frac{\partial}{\partial x}v_xv_z+\frac{\partial}{\partial y}v_yv_z+\frac{\partial}{\partial z}v_zv_z=0$$

d. $(\nabla \cdot \mathbf{v}) = 0$

$(\nabla \mathbf{v})_{xy} = -b, \ (\nabla \mathbf{v})_{yx} = b$ and all others are zero

$$[\nabla \cdot \mathbf{vv}]_x = \frac{\partial}{\partial x} v_x v_x + \frac{\partial}{\partial y} v_y v_x + \frac{\partial}{\partial z} v_z v_x = -b^2 x$$
$$[\nabla \cdot \mathbf{vv}]_y = \frac{\partial}{\partial x} v_x v_y + \frac{\partial}{\partial y} v_y v_y + \frac{\partial}{\partial z} v_z v_y = -b^2 y$$
$$[\nabla \cdot \mathbf{vv}]_z = \frac{\partial}{\partial x} v_x v_z + \frac{\partial}{\partial y} v_y v_z + \frac{\partial}{\partial z} v_z v_z = 0$$