1. The stress power is defined to be $S_p = \int_R tr(\boldsymbol{\sigma} \cdot \boldsymbol{d})dV$ where $\boldsymbol{\sigma}$ is the Cauchy stress and $\boldsymbol{d} = \boldsymbol{L}_{sym}$.

Other measures of stress are

$\boldsymbol{\Sigma} = \boldsymbol{R}^T \cdot \boldsymbol{\sigma} \cdot \boldsymbol{R}$ Rotated Cauchy stress

$\hat{\boldsymbol{P}} = J\boldsymbol{\sigma} \cdot \boldsymbol{F}^{-T}$ Piola-Kirchoff stress of the first kind

$\boldsymbol{P} = J\boldsymbol{F}^{-1} \cdot \boldsymbol{\sigma} \cdot \boldsymbol{F}^{-T} = \boldsymbol{F}^{-1} \cdot \hat{\boldsymbol{P}}$ Piola-Kirchoff stress of the second kind

and other rates of deformation are

$$\boldsymbol{D} = \boldsymbol{F}^T \cdot \boldsymbol{d} \cdot \boldsymbol{F} = \dot{\boldsymbol{E}} \qquad \boldsymbol{D}^* = \boldsymbol{R}^T \cdot \boldsymbol{d} \cdot \boldsymbol{R}^T$$

Show that alternative expressions for the stress power are:

$$S_p = \int_R tr\,(S \cdot \boldsymbol{D}^*)dV = \int_{R_o} tr\,(\hat{\boldsymbol{P}} \cdot \dot{\boldsymbol{F}}^T)dV_o = \int_{R_o} tr\,(\boldsymbol{P} \cdot \dot{\boldsymbol{E}})dV_o$$

These combinations of stress and deformation rates are said to be "conjugate."

2. Recall that in connection with the study of a continuum, tensors could be defined as one of four possibilities: m-m, s-s, s-m, m-s where “m” denotes “material” and “s” denotes “spatial”. Recall the classifications of $\boldsymbol{F}$ and $\boldsymbol{R}$ from the notes. Assume $\boldsymbol{\sigma}$ and $\boldsymbol{d}$ are both s-s. Use the relations given in Prob. 1 to classify the tensors $\boldsymbol{\Sigma}, \hat{\boldsymbol{P}}, \boldsymbol{P}, \boldsymbol{D}$ and $\boldsymbol{D}^*$.

3. A bar of original length L that is initially horizontal (Fig. 1) deforms in a plane as the result of a simultaneous stretch and rotation as indicated in Fig. 2. The end O is fixed in space. The rotation is defined by $\theta = \omega t$ and the elongation of the end of the bar is $\delta_A = \varepsilon t$ with both ω and ε constant.

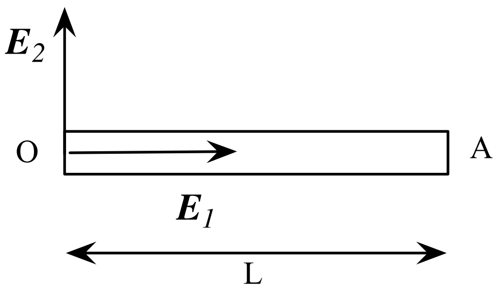

Fig. 1. Initial Position

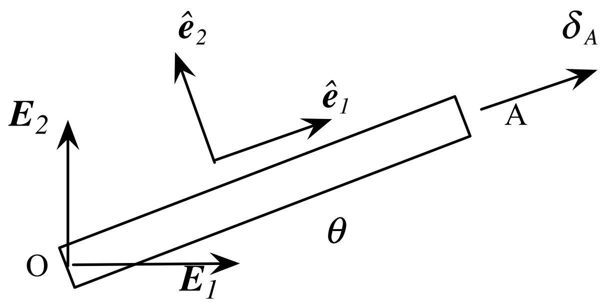

Fig. 2. Deformed position

Two approaches to handle the problem are
(i) Let $\boldsymbol{e}_i = \boldsymbol{E}_i$ for all time and define the deformation by

$$x_1 = X_1(1+\varepsilon t)\cos\theta - X_2 \sin\theta \qquad x_2 = X_1(1+\varepsilon t)\sin\theta + X_2 \sin\theta$$

or (ii) Let $\boldsymbol{e}_i = \hat{\boldsymbol{e}}_i$ and define the remainder of the deformation by $x_1 = X_1(1+\varepsilon t)$ $\qquad x_2 = X_2$

For approach 1, the basis $\mathbf{e}_I$ is fixed; for approach 2 these base vectors vary with time.

3.1. For each approach, determine $\boldsymbol{F}, \boldsymbol{R}, \boldsymbol{U}, \dot{\boldsymbol{F}}, \dot{\boldsymbol{R}}, \dot{\boldsymbol{U}}$ and $\boldsymbol{\Omega}$. Do you obtain the same results?

3.2 By using the transformation relations between the two bases for approach (ii) obtain $\dot{\boldsymbol{e}}_i$ in terms of $\boldsymbol{E}_i$ and then $\dot{\boldsymbol{e}}_i$ in terms of $\boldsymbol{e}_i$. Compare these latter results with $\boldsymbol{\Omega} \cdot \boldsymbol{e}_i$.

3.3 Consider an element $\mathrm{d}\mathbf{X} = \mathrm{d}X_1\mathbf{E}_1$. Determine the vector $\mathbf{U} \cdot \mathrm{d}\mathbf{X}$ and then the vector $\mathbf{R} \cdot (\mathbf{U} \cdot \mathrm{d}\mathbf{X})$.