Assume a solution will be proportional to $e^{\lambda x}$ for some constant λ . Substitute $y(x) = e^{\lambda x}$ into the differential equation: $\frac{d^2}{dx^2}(e^{\lambda x}) - c e^{\lambda x} = 0$

Substitute
$$\frac{d^2}{dx^2}(e^{\lambda x}) = \lambda^2 e^{\lambda x}$$
:

Solve $\frac{d^2y(x)}{dx^2} - c y(x) = 0$:

$$\lambda^{2} e^{\lambda x} - c e^{\lambda x} = 0$$
Factor out $e^{\lambda x}$:

$$\left(-c + \lambda^2\right) e^{\lambda x} = 0$$

Since
$$e^{\lambda x} \neq 0$$
 for any finite λ , the zeros must come from the polynomial: $-c + \lambda^2 = 0$

Solve for
$$\lambda$$
:
 $\lambda = \sqrt{c}$ or $\lambda = -\sqrt{c}$

The root
$$\lambda = -\sqrt{c}$$
 gives $y_1(x) = k_1 \, e^{-\sqrt{c} \, x}$ as a solution, where k_1 is an arbitrary

The root
$$\lambda = -\sqrt{c}$$
 girconstant.

The root $\lambda = \sqrt{c}$ gives $y_2(x) = k_2 e^{\sqrt{c} x}$ as a solution, where k_2 is an arbitrary

constant.

The general solution is the sum of the above solutions:
 Answer:
$$y(x) = y_1(x) + y_2(x) = \frac{k_1}{\sqrt{c}} + k_2 e^{\sqrt{c} x}$$

Solve
$$\frac{d^2y(x)}{dx^2} + c y(x) = 0$$
:

Assume a solution will be proportional to $e^{\lambda x}$ for some constant λ . Substitute $y(x) = e^{\lambda x}$ into the differential equation:

$$\frac{d^2}{dx^2}(e^{\lambda x}) + c e^{\lambda x} = 0$$

Substitute $\frac{d^2}{dx^2}(e^{\lambda x}) = \lambda^2 e^{\lambda x}$:

$$\lambda^2 e^{\lambda x} + c e^{\lambda x} = 0$$

Factor out $e^{\lambda x}$:

$$(c + \lambda^2) e^{\lambda x} = 0$$

Since $e^{\lambda x} \neq 0$ for any finite λ , the zeros must come from the polynomial: $c + \lambda^2 = 0$

Solve for λ :

$$\lambda = i\sqrt{c}$$
 or $\lambda = -i\sqrt{c}$

The root $\lambda = -i\sqrt{c}$ gives $y_1(x) = k_1 e^{-i\sqrt{c} x}$ as a solution, where k_1 is an arbitrary constant.

The root $\lambda=i\sqrt{c}$ gives $y_2(x)=k_2\ e^{i\sqrt{c}\ x}$ as a solution, where k_2 is an arbitrary constant.

The general solution is the sum of the above solutions:

$$y(x) = y_1(x) + y_2(x) = \frac{k_1}{e^{i\sqrt{c}x}} + k_2 e^{i\sqrt{c}x}$$

Apply Euler's identity $e^{\alpha+i\beta} = e^{\alpha}\cos(\beta) + ie^{\alpha}\sin(\beta)$:

$$y(x) = k_1 \left(\cos(\sqrt{c} \ x) - i\sin(\sqrt{c} \ x)\right) + k_2 \left(\cos(\sqrt{c} \ x) + i\sin(\sqrt{c} \ x)\right)$$

Regroup terms:

$$y(x) = (k_1 + k_2)\cos(\sqrt{c} \ x) + i(-k_1 + k_2)\sin(\sqrt{c} \ x)$$

Redefine $k_1 + k_2$ as k_1 and i ($k_2 - k_1$) as k_2 , since these are arbitrary constants:

Answer:

$$y(x) = k_1 \cos(\sqrt{c} x) + k_2 \sin(\sqrt{c} x)$$

Substitute $y(x) = e^{\lambda x}$ into the differential equation: $\frac{d^2}{dx^2}(e^{\lambda x}) - c e^{\lambda x} = 0$ Substitute $\frac{d^2}{dx^2}(e^{\lambda x}) = \lambda^2 e^{\lambda x}$:

Find the complementary solution by solving $\frac{d^2y(x)}{dx^2} - c y(x) = 0$:

Assume a solution will be proportional to $e^{\lambda x}$ for some constant λ .

The general solution will be the sum of the complementary solution and

$$\lambda^{2} e^{\lambda x} - c e^{\lambda x} = 0$$
Factor out $e^{\lambda x}$:
$$(-c + \lambda^{2}) e^{\lambda x} = 0$$

Solve $\frac{d^2y(x)}{dx^2} - c y(x) = 1$:

particular solution.

Since $e^{\lambda x} \neq 0$ for any finite λ , the zeros must come from the polynomial:

$$-c + \lambda^2 = 0$$
Solve for λ :
$$\lambda = \sqrt{c} \text{ or } \lambda = -\sqrt{c}$$

The root $\lambda = -\sqrt{c}$ gives $y_1(x) = k_1 e^{-\sqrt{c} x}$ as a solution, where k_1 is an arbitrary constant. constant.

root
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 gives y_1 stant.
root $\lambda = \sqrt{c}$ gives $y_2(x)$ stant.
general solution is the $y_1(x) = y_1(x) + y_2(x) = \frac{k_1}{e^{\sqrt{c}}}$

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Determine the particular solution to
$$\frac{d^2y(x)}{dx^2} - c \ y(x) = 1$$
 by the method of undetermined coefficients:

The particular solution to $\frac{d^2y(x)}{dx^2} - c \ y(x) = 1$ is of the form:

The particular solution to $\frac{d^2y(x)}{dx^2} - c y(x) = 1$ is of the form:

particular solution to
$$\frac{d^2}{a}$$

$$= a_1$$
for the unknown cons

 $y_p(x) = a_1$ Solve for the unknown constant a_1 :

Solve for the unknown constant Compute
$$\frac{d^2y_p(x)}{dx^2}$$
:
$$\frac{d^2y_p(x)}{dx^2} = \frac{d^2}{dx^2}(a_1)$$

$$= 0$$

Substitute the particular solution $y_p(x)$ into the differential equation:

Substitute the partic
$$\frac{d^2y_p(x)}{dx^2} - c y_p(x) = 1$$
$$-c a_1 = 1$$

Solve the equation: $a_1 = -\frac{1}{a_1}$

Solve the equation:
$$a_1 = -\frac{1}{c}$$
 Substitute a_1 into $y_p(x) = a_1$:

 $y_p(x) = -\frac{1}{2}$

The general solution is:

Answer:

$$-c a_1 = 1$$

Solve the equation:
 $a_1 = -\frac{1}{2}$

 $y(x) = y_c(x) + y_p(x) = -\frac{1}{c} + \frac{k_1}{e^{\sqrt{c} x}} + k_2 e^{\sqrt{c} x}$

$$-c y(x) = 1$$
 is of the for a_1 :

 $\frac{d^2}{dx^2}(e^{\lambda x}) + c e^{\lambda x} = 0$ Substitute $\frac{d^2}{dx^2}(e^{\lambda x}) = \lambda^2 e^{\lambda x}$: $\lambda^2 e^{\lambda x} + c e^{\lambda x} = 0$

The general solution will be the sum of the complementary solution and

Find the complementary solution by solving $\frac{d^2y(x)}{dx^2} + c y(x) = 0$:

Assume a solution will be proportional to $e^{\lambda x}$ for some constant λ .

Substitute $y(x) = e^{\lambda x}$ into the differential equation:

Solve $\frac{d^2y(x)}{dx^2} + c y(x) = 1$:

particular solution.

Factor out
$$e^{\lambda x}$$
: $(c + \lambda^2) e^{\lambda x} = 0$
Since $e^{\lambda x} \neq 0$ for any finite λ , the zeros must come from the polynomial: $c + \lambda^2 = 0$

Solve for λ : $\lambda = i \sqrt{c} \text{ or } \lambda = -i \sqrt{c}$

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:
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 The root $\lambda = -i \sqrt{c}$ gives $y_1(x) = k_1 e^{-i \sqrt{c} x}$ as a solution, where k_1 is an arbitrary constant. The root $\lambda = i \sqrt{c}$ gives $y_2(x) = k_2 e^{i \sqrt{c} x}$ as a solution, where k_2 is an arbitra

The root $\lambda = i \sqrt{c}$ gives $y_2(x) = k_2 e^{i \sqrt{c} x}$ as a solution, where k_2 is an arbitrary constant. The general solution is the sum of the above solutions: $y(x) = y_1(x) + y_2(x) = \frac{k_1}{e^{i\sqrt{c}x}} + k_2 e^{i\sqrt{c}x}$

The general solution is the sum of the above solutions:
$$y(x) = y_1(x) + y_2(x) = \frac{k_1}{e^{i\sqrt{c} \ x}} + k_2 e^{i\sqrt{c} \ x}$$
Apply Euler's identity $e^{\alpha + i\beta} = e^{\alpha} \cos(\beta) + i e^{\alpha} \sin(\beta)$:
$$y(x) = k_1 \left(\cos(\sqrt{c} \ x) - i \sin(\sqrt{c} \ x)\right) + k_2 \left(\cos(\sqrt{c} \ x) + i \sin(\sqrt{c} \ x)\right)$$

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Regroup terms:
$$y(x) = (k_1 + k_2) \cos(\sqrt{c} \ x) + i (-k_1 + k_2) \sin(\sqrt{c} \ x)$$
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Redefine k_1 + k_2 as k_1 and i (k_2 – k_1) as k_2 , since these are arbitrary constants: $y(x) = k_1 \cos(\sqrt{c} x) + k_2 \sin(\sqrt{c} x)$

edefine
$$k_1+k_2$$
 as k_1 and i (k_2-k_1) as k_2 , since these are arbitrary constants x) = $k_1 \cos(\sqrt{c} \ x) + k_2 \sin(\sqrt{c} \ x)$ etermine the particular solution to c $y(x) + \frac{d^2y(x)}{dx^2} = 1$ by the method of

Determine the particular solution to $c y(x) + \frac{d^2y(x)}{dx^2} = 1$ by the method of

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ndetermined coefficients: the particular solution to
$$c \ y(x) + \frac{d^2 y(x)}{dx^2} = 1$$
 is of the form: $c(x) = a_1$

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the particular solution to
$$c y(x) + \frac{d^2 y(x)}{dx^2} = 1$$
 is of the form:
 $c(x) = a_1$

Substitute the particular solution $y_p(x)$ into the differential equation:

 $y(x) = y_c(x) + y_p(x) = \frac{1}{c} + k_1 \cos(\sqrt{c} x) + k_2 \sin(\sqrt{c} x)$

Solve for the unknown constant a_1 :

Compute $\frac{d^2y_p(x)}{dx^2}$:

 $\frac{d^2y_p(x)}{dx^2} = \frac{d^2}{dx^2}(a_1)$

 $\frac{d^2 y_p(x)}{dx^2} + c \, y_p(x) = 1$

Solve the equation:

Substitute a_1 into $y_p(x) = a_1$:

The general solution is:

 $c a_1 = 1$

 $a_1 = \frac{1}{2}$

 $y_p(x) = \frac{1}{2}$

Answer: