5.1 Calculate the dislocation spacing in a symmetrical tilt boundary ($\theta = 0.5°$) in a copper crystal.

Copper is FCC with

$$a_o = 36\ nm$$
$$\therefore b = \frac{\sqrt{2}}{2} a_o = 23.4\ nm$$
$$\theta = 0.5° = 0.0087\ rad$$

Dislocation spacing in the symmetrical tilt boundary,

$$D = \frac{b}{\theta}$$
$$= \frac{2.34 \times 10^{-10}}{0.0087}$$
$$D = 29.1\ nm$$

5.2 Starting from the equation $E = E_O \theta \left(A - \ell n\theta\right)$ for a low-angle boundary, show how one can obtain graphically the values of E_O and A.

$$E = E_O \theta \left(A - \ell n\theta\right)$$
$$\frac{E}{\theta} = E_O A - E_O \ell n\theta$$

A plot of E/θ vs. $\ln\theta$ will be a straight line of slope $-E_o$.

5.3 Taking A = 0.3, compute the value of θ_{max}.

$$E = E_O \theta (A - \ell n \theta)$$

$$At \quad \theta = \theta_{\max}, \frac{dE}{d\theta} = 0$$

$$Thus, \quad E_O (A - \ell n \theta_{\max} - 1) = 0$$

$$\ell n \theta_{\max} = A - 1$$

$$\theta_{\max} = \exp (A - 1) = \exp (0.3 - 1)$$

$$= \exp (-0.7)$$

$$\theta_{\max} = 0.496\, rad = 28.30°$$

5.4 Show that, for a low-angle boundary, we have

$$\frac{E}{E_{\max}} = \frac{\theta}{\theta_{\max}} \left(1 - \ell n \frac{\theta}{\theta_{\max}}\right)$$

where E_{max} and θ_{max} correspond to the maximum in the E-versus-θ curve.

$$E = E_O \theta \, (A - \ell n \theta)$$

$$At \quad \theta = \theta_{\max}, \frac{dE}{d\theta} = 0$$

$$Thus, \quad E_O (A - \ell n \theta_{\max} - 1) = 0$$

$$\ell n \theta_{\max} = A - 1$$

$$\theta_{\max} = \exp \, (A - 1)$$

$$Therefore, \frac{E}{E_{\max}} = \frac{E_o (A - \ell n \theta)}{E_o \theta_{\max} (A - \ell n \theta_{\max})}$$

$$= \frac{\theta}{\theta_{\max}} \times \frac{A - \ell n}{A - A + 1}$$

$$= \frac{\theta}{\theta_{\max}} \{1 + \ell n \theta_{\max} - \ell n \theta\}$$

$$or, \quad \frac{E}{E_{\max}} = \frac{\theta}{\theta_{\max}} \quad \{1 - \ell n \frac{\theta}{\theta_{\max}}\}$$

5.5 Consider two parallel tilt boundaries with misorientations θ_1 and θ_2. Show that, thermodynamically, we would expect the two boundaries to join and form one boundary with misorientation $\theta_1 + \theta_2$.

$$E_1 = E_0\theta(A - \ell n\theta_1)$$
$$E_2 = E_0\theta_2(A - \ell n\theta_2)$$

Adding the two, we get

$$E = E_1 + E_2 = E_0A(\theta_1 + \theta_2) + E_0\ell n\left(\frac{1}{\theta_1}\right)^{\theta_1} + E_0\ell n\left(\frac{1}{\theta_2}\right)^{\theta_2} \qquad \text{(A)}$$

$$Let \quad \theta_1 + \theta_2 = \theta$$

Then, for the new tilt boundary, we can write

$$E = E_0A(\theta_1 + \theta_2) + E_0\,\ell n\left(\frac{1}{\theta_1 + \theta_2}\right)^{\theta_1 + \theta_2} \qquad \text{(B)}$$

Comparing (A) and (B) (put $\theta_1 = \theta_2$, for simplicity), we note that $E_1 + E_2$ is greater than E, the energy of the new boundary obtained by joining the two individual boudnaries. Thus, the thermodynamically we would expect the two boundaries with energies E_1 and E_2 to join to form a lower energy (E) boundary.

5.6 Can you suggest a quick technique to check weather lines observed in an optical microscope on the surface of a polished sample after deformation are slip lines or twin markings?

Slip lines are just surface markings while twin markings or boundaries extend through the volume of material. Thus, a simple repolishing and etching of the surface will let us know weather the markings seen before repolishing were slip lines or twin boundaries. Slip marking will disappear on repolishing and etching while twin markings will reappear on etching.

5.7 A twin boundary separates two crystals of different orientations; however, we do not necessarily need dislocations to form a twin. Why?

Basically, twinning represents a homogeneous deformation process while slip represents an inhomogeneous deformation process. The twinned region is a mirror image of the parent material.

In slip, we have s shear displacement of an entire block of the crystal. Of course, the entire crystal block is not moved in one step but the same effect is achieved by the passage of a dislocation, which leaves a step at the crystal surface equal to the Burgers vector of the dislocation.

In twinning, on the other hand, we have a uniform lattice shear. Macroscopically, the twinned region is formed by shear on the twinning plane in the twinning direction. The shear is homogeneous throughout the entire twinned volume and is determined by the crystal structure. On an atomic scale, the displacement of a lattice point in a twinned material is proportional to the distance from and parallel to the twin boundary. Thus, theoretically, in twinning one does not need the kind of leverage provided by dislocation in the slip process.

5.8 Let m be the total length of dislocations per unit area of a grain boundary. Assume that at yield all the dislocations in the grain interior (ρ) are the ones emitted by the boundaries. Assume also that the grains are spherical (with diameter d). Derive the Hall-Petch relation ($\theta = \theta_0 + kd^{-1/2}$) for this case, and give the expression for k.

Total length of dislocation lines per unit area of grain boundary = m. Yielding results in emission of dislocation from the grain boundary (dislocation density = ρ).

We assume that the grains are spherical (diameter = d). Then, surface area of each grain is

$$4\pi\left(\frac{d}{2}\right)^2 = \pi\, d.$$

The total length of dislocation lines per unit area of grain boundary being m, a total of ($m\,\pi d^2$) of dislocation line length will be emitted. However, this total length will be shared by two grains as each boundary belongs to two grains. Thus, ½ ($m\pi d^2$) of dislocations will go into each grain. Thus the dislocation density

$$\rho = \frac{length \quad of \quad dislocations}{volume} = \frac{1/2\,(m\,\pi\,d^2)}{4/3\,\pi\left(\frac{d}{2}\right)^3}$$

$$or \quad \rho = \frac{3m}{d}$$

Assuming that the flow stress $\tau = \tau_o + \alpha\, Gb\sqrt{\rho}$, we have

$$\tau = \tau_o + \alpha\, Gb\sqrt{\frac{3m}{d}}$$

This is a Hall- Patch type relation with

$$k = \alpha Gb\sqrt{3m}$$

5.9 Consider a piano wire that has a 100% pearlitic structure. When this wire undergoes a reduction in diameter from D_o to D_e, the interlamellar spacing normal to the wire axis is reduced from d_o to d_e, that is,

$$\frac{d_o}{d_e} = \frac{D_o}{D_e},$$

where the subscript O refers to the original dimensions, while the subscript e refers tp the dimensions after a true plastic strain of ε. If the wire obeys a Hall-Petch type of relationship between the flow stress and the pearlite interlamellar spacing, show that the flow stress of the piano wire can be expressed as

$$\sigma = \sigma_1 + \frac{k'}{\sqrt{d_o}} \exp\left(\frac{\varepsilon}{4}\right).$$

We have the following relationship between true strain, ε and engineering strain, e

$$\varepsilon = \ell n(1 + e)$$

Now

$$\varepsilon = \ell n \frac{\ell_\varepsilon}{\ell_o}$$

and

$$\frac{\ell_\varepsilon}{\ell_o} = \frac{A_o}{A_\varepsilon} = \left(\frac{D_o}{D_\varepsilon}\right)^2$$

$$\varepsilon = \ell n\left(\frac{D_o}{D_\varepsilon}\right)^2 = 2\ell n\left(\frac{D_o}{D_\varepsilon}\right)$$

$$\textit{Since} \quad \frac{D_o}{D_\varepsilon} = \frac{d_o}{d_\varepsilon} \quad (\textit{given}), \textit{we can write}$$

$$\varepsilon = 2\ \ell n\left(\frac{d_o}{d_\varepsilon}\right) \quad or \quad d_\varepsilon = d_o \exp\left(-\frac{\varepsilon}{2}\right)$$

Substituting this value of d_ε in the Hall-Petch relation $\sigma = \sigma_i + k\, d_\varepsilon^{-1/2}$, we get

$$\sigma = \sigma_i + k\left(d_o \exp(-\frac{\varepsilon}{2})\right)^{-1/2}$$

$$or \quad \sigma = \sigma_i + \frac{k}{d_o^{\ 1/2}} \exp\left(\frac{\varepsilon}{4}\right)$$

5.10 **(a)** Determine the mean lineal intercept, the surface area per unit volume, and the estimated grain diameter for the specimen shown in Figure Ex 5.10.
(b) Estimate the yield stress of the specimen (AISI 304 stainless steel).
(c) Estimate the parameters of part (a), excluding the annealing twins. By what percentage is the yield stress going to differ?

$$(a) \quad L = 8.5\ cm$$

$$N = 7$$

$$Magnification, \quad M = \frac{0.5\ cm}{20\ \mu m} = 250$$

$$\Rightarrow \ell = \frac{8.5}{250 \times 7}$$

$$\ell = 4.86 \times 10^{-3}\ cm$$

$$\ell = 4.86\ \mu m$$

$$(b) \quad \sigma_y = \sigma_o + kd^{-1/2}$$

$$\sigma_o = 265\ MN/m^2$$

$$k = 0.74\ MN/m^{3/2}$$

$$\sigma_y = 265 \times 10^6 + (0.74 \times 10^6)(4.86 \times 10^{-6})^{-1/2}$$

$$\sigma_y = 600\ MPa$$

5.13 If the grain size of a metal is doubled by an appropriate annealing, by what percentage is the surface area per unit volume of the metal changed?

Assume grains are spherical:

Surface area = $4\pi r^2$

Volume = $\frac{4}{3}\pi r^3$

$S_v =$ Grain boundary surface area per unit volume

$$S_v = \left(\frac{1}{2}\right)\frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{2r} = \frac{3}{2D}$$

Initial diameter $D_1 = D$ (1) $\qquad S_{v1} = \frac{3}{2D_1}$

Final diameter $D_2 = 2D$ (2) $\qquad S_{v2} = \frac{3}{2D_2}$

Percentage charge in $S_v = 100 \times \frac{S_{v1} - S_{v2}}{S_{v1}} = 100 \times \left(\frac{\frac{3}{2D_1} - \frac{3}{2D_2}}{\frac{3}{2D_1}}\right) = 100 \times \left(1 - \frac{D_1}{D_2}\right)$ (3)

Substitute (1) and (2) into (3)

$$= 100 \times \left(1 - \frac{D}{2D}\right) = 50\%$$

A 50% decrease in surface area per unit volume can be expected if grain size is doubled.

5.16 Estimate the average grain diameter and the grain-boundary area per unit volume for a material that has isotropic grains (the same dimension in all directions) and ASTM grain size 6.

Since the ASTM grain size is 6, the number of grains in an area of 1 in^2 at 100 X is

$N = 2^n = 2^6 = 32\ in^{-2}$

Assume all the grains are equiaxial.

$$N \frac{1}{4}\pi (D \times Mag)^2 = 1 in^2$$

$$D = \sqrt{4/\pi N}\ / Mag$$

$$= \left(\sqrt{4/(\pi \times 32)}\right) 25.4/100$$

$$= 5.07 \times 10^{-2} mm$$

$$= 50.7 \mu m$$

$$Sv = \frac{3}{D} = 59.2\, mm^2 / mm^3$$

5.17 How many grains in an area of 5 x 5 cm would be counted, in a photomicrograph taken at a magnification of 500X, for a metal with ASTM grain size 3?

n = 3
N = 2^{n-1}
N = 2^{3-1}
N = 4

N = $\frac{number\, of\ grains}{area} = 4$

$$Number\, of\ grains = 4\, area = 4 \times 25 = 100$$

5.18 A graduate student (undergraduates are much brighter!) measured the grain size of metallic specimen and found that it was equal to ASTM #2. However, he had the wrong magnification in his picture (400X instead of 100X). (a) What is the correct ASTM grain size? (b) What is the approximate grain diameter?

n = 2

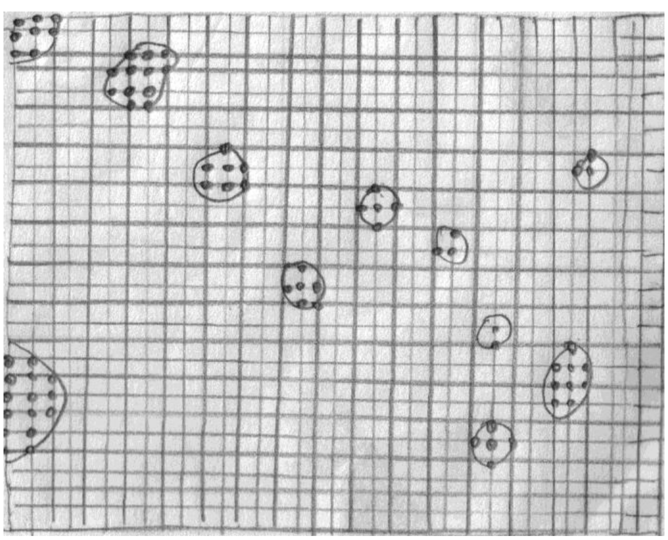
(a) The grain size should be $\left(\frac{1}{4}\right)\ell$

(b) The grain diameter is $D = \frac{3}{8}\ell$

5.20 Calculate the volume fraction of voids in the TiC specimen shown in Fig. 5.35(b).

(Reference Example 5.7)

We overlay a grid onto Fig 5.35(b) and count the intersections of lines falling within the voids. The ratio of intersections inside void to total number of intersections can be used to approximate porosity.



The total number of intersections in grid = $28 \times 35 = 980$

The total number of intersections in side the voids ≈ 75

Porosity approximation

$^{75}\!/_{980} \approx .0765$

Porosity $\approx \boxed{7.65\%}$

5.21 Examine the figure Ex. 5.21.

(a) Using the lineal intercept method, determine the mean lineal intercept and the grain size if the material is TiC.

(b) Determine the grain size using the ASTM method.

(a) The magnification of the picture in the textbook (p. 367) is

$$Mag = \frac{11.5mm}{100\mu m} = 115$$

Using the lineal intercept method, we draw two test lines, each 70 mm. The number of intercepts on the test line is 32. The mean lineal intercept is

$$\begin{aligned} l &= \frac{L}{N_1 M} \\ &= \frac{70 \times 2}{32 \times 115} \\ &= 38\mu m \end{aligned}$$

$$\begin{aligned} D &= \frac{3}{2} l \\ &= 57\mu m \end{aligned}$$

(b) Using the ASTM method, the number of the grains in 1 in^2 at 115X is 27. Then the number of the grains in 1 in^2 at 100X is:

$$N = \frac{27}{^{1}\!/_{1.15^2}} = 35.71$$

$$N = 2^{n-1}$$

$$\begin{aligned} n &= \frac{\ln N}{\ln 2} + 1 \\ &= \frac{\ln 35.71}{\ln 2} + 1 \\ &= 4.6 \end{aligned}$$

Assuming all grains are equiaxial, we get

$$N \frac{1}{4}\pi (D \times Mag)^2 = lin^2$$
$$D = \sqrt{4/\pi N}\ / Mag$$
$$= \left(\sqrt{4/(\pi \times 35.71)}\right) \times 25.4/100$$
$$= 48.0\mu m$$

5.22 (a) Using the mean lineal intercept, calculate the grain diameter for tantalum, given the micrograph in Figure 5.2(a). (Note: the figure is 5.2a, not 5.29a as indicated in the text)
(b) Calculate the ASTM grain size
(c) Estimate the yield stress for this specimen of tantalum, using values from Table 5.3

(a) grain diameter

$$D = \frac{3}{2}\ell$$
$$\sigma_o = 186.31\,MPa$$
$$k = 0.64\,MN/m^{3/2}$$

$$N_\ell = 25$$
$$L = 14.6cm$$
$$M = \frac{1.7cm}{100\mu m} = 170$$
$$\ell = \frac{L}{N_\ell M} = \frac{14.6}{25(170)} = 3.434 \times 10^{-3} cm$$
$$\therefore D = 5.153 \times 10^{-3} cm$$
$$= 51.53\,\mu m$$

(b) ASTM grain size = 34.35 μm

(c)

$$\sigma_y = \sigma + kd^{-1/2}$$
$$\sigma_y = 186.31\,MPa + 0.64\,MN/m^{3/2}\,(51.53\mu m)^{-1/2}$$
$$\sigma_y = 275.466\,MPa$$

5.23 A polycrystalline sample has 16 grains per square inch in a photomicrograph taken at magnification100X. What is the ASTM grain size number?

$$N = 2^{n-1}$$
$$16 - 2^{n-1}$$
$$\ell n16 = (n-1)\ell n2$$
$$n = \frac{\ell n16}{\ln 2} + 1$$
$$n = 4 + 1$$
$$n = 5$$

5.24 A 20-cm line gave seven intersections in a 100X micrograph. Using the lineal intercept method, determine the mean lineal intercept and the grain size.

Mean lineal intercept

$$\ell = \frac{20 \times 10^{-2}}{7(100)} = 285.71 \mu m$$

5.25 How many grains in an area of 5 × 5 cm would be counted in a photomicrograph taken at a magnification of 500× for a metal with an ASTM grain size 3?

ASTM n = 3

$N = 2^{n-1} = 2^{3-1} = 4\,\text{grains}/in^2$ at 100 x Magnification = 500 x

x = number of grains per square inch at 500 x Magnification

$$N = \left(\frac{500}{100}\right)^2 (x) = 2^{3-1}$$

$$x = \frac{4}{25} = .16 \text{ grains}/in^2$$

Total number of grains in 5 x 5 cm area

$$.16\, {grains}/{in^2} \bullet \frac{in^2}{2.54^2\, cm^2} \bullet 5 \times 5 cm^2 \; \boxed{= .62 \text{grains}}$$

5.26 The yield stress of AISI 1020 steel with a grain size of 200 μm is 200 MPa. Estimate the yield stress for a grain size of 10 μm if the Hall-Petch constant k = 0.8 $MN/m^{3/2}$.

$$\sigma_y = \sigma_o + kd^{-1/2}$$

$$200 \times 10^6 Pa = \sigma_o + 0.8 \times 10^6 \frac{N}{m^{3/2}} (200 \times 10^{-6} m)^{-1/2}$$

$$\sigma_o = 143.4 \times 10^6 Pa$$

$$For\ grain\ size = 10 \mu m$$

$$\sigma_y = 143.4 \times 10^6 Pa + 0.8 \times 10^6 \frac{N}{m^{3/2}} (10 \times 10^{-6} m)^{-1/2}$$

$$\sigma_y = 396.4\, MPa$$

5.27 A small-angle tilt boundary has a misorientation of 0.1°. What is the spacing between the dislocations in this boundary if the Burgers vector of the dislocation is 0.33 nm?

Spacing between the dislocations

$$D = \frac{b}{\theta}$$

$$b = 0.33\,nm$$

$$\theta = 0.1^{\circ}$$

$$\theta = 0.1\left(\frac{\pi}{180}\right) = 1.745 \times 10^{-3}\,rad$$

$$\therefore D = \frac{0.33}{1.745 \times 10^{-3}}\,nm$$

$$= 189.08\ nm$$

5.28 Calculate the dislocation spacing of low-angle tilt boundary in aluminum for $\theta = 0.5^{\circ}$. Take G = 26.1 GPa, v = 0.345, and r_{Al} = 0.143 nm.

$$b = \frac{a}{\sqrt{2}}$$

$$4r_{Al} = \sqrt{2a}$$

$$a = \frac{4r_{Al}}{\sqrt{2}} = \frac{4(0.143nm)}{\sqrt{2}} = 0.4045\ nm$$

$$b = \frac{0.4045}{\sqrt{2}} = 0.286\ nm$$

$$\sigma = 0.5^{\circ}\left(\frac{\pi}{180}\right) = 8.727 \times 10^{-3}\,rad$$

$$D = \frac{b}{\theta} = \frac{0.286\ nm}{8.727 \times 10^{-3}}$$

$$\therefore D = 32.773\ nm$$

5.29 (**a)** Determine the grain size for the microstructure of zirconium shown in Figure Ex5.29, using the lineal intercept method. Use the scale given at the bottom.
(b) Use the Hall--Petch equation to determine the yield stress of this material, given σ_0 = 29 MPa, k = 0.25 MPa m$^{1/2}$.

a) $\bar{\ell} = \dfrac{L}{N_\ell M}$

L = Length of line drawn

N_ℓ = Number of intercepts

M = magnifications

From the scale on figure $M = \dfrac{.7\times10^{-2}\,m}{100\times10^{-6}\,m} = 70$

<u>Trial 1</u>

$L_1 = 4\times10^{-2}$

$N_{\ell 1} = 8$

$\bar{\ell}_1 = \dfrac{4\times10^{-2}}{8(70)} = 7.14\times10^{-5}m$

<u>Trial 2</u>

$L_2 = 3\times10^{-2}$

$N_{s2} = 9$

$\bar{\ell}_2 = \dfrac{3\times10^{-2}}{9(70)} = 4.76\times10^{-5}$

Average of the two:

Average $\bar{\ell} = \dfrac{\bar{\ell}_1 + \bar{\ell}_2}{2} = 5.95\times10^{-5}$

$d = \dfrac{3}{2}\bar{\ell} = 8.92\times10^{-5}m = \boxed{89.2\mu m}$

b) Use Hall-Petch equation to find yield stress

$$\sigma_0 = 29MPa \qquad k = .25MPa \ m^{\frac{1}{2}}$$

$$\sigma_y = \sigma_0 + kd^{\frac{-1}{2}} = (29\times10^6) + (.25\times10^6)(89.2\times10^{-6})^{\frac{-1}{2}}$$

$$\boxed{\sigma_y = 55.46MPa}$$

5.30 From Figure 5.2 find the grain diameter of a sample using the lineal intercept ASTM method, $N = 2^{n-1}$.

a) Tantalum

Refer to figure 5.2 (a)

$$\bar{\ell} = \frac{L}{N_\ell M}$$

L = Length of line drawn
N_ℓ =Number of intercepts
M = magnification

$$M = \frac{1.1\times10^{-2}}{100\times10^{-6}m} = 110$$

Following are the results of the two measurements and the average

<u>Trial 1</u>

$$L_1 = 7\times10^{-2}m$$

$$N_{\ell 1} = 18$$

$$\bar{\ell}_1 = \frac{7\times10^{-2}}{18(110)} = 3.53\times10^{-5}m$$

Trial 2

$$L_2 = 6\times10^{-2}$$

$$N_{\ell 2} = 15$$

$$\overline{\ell}_2 = \frac{6\times10^{-2}}{15(10)} = 3.63\times10^{-5}\,m$$

Average of the two:

$$\overline{\ell} = \frac{\overline{\ell}_1+\overline{\ell}_2}{2} = 3.58\times10^{-5}\,m$$

$$d = \frac{3}{2}\overline{\ell} = 5.37\times10^{-5}\,m = \boxed{53.7\,\mu m}$$

b) TiC from figure 5.2(b)

$$\overline{\ell} = \frac{L}{N_\ell M} \qquad M = \frac{1.3\times10^{-2}}{10\times10^{-6}} = 1300$$

Trial 1

$$L_1 = 6.5\times10^{-2}$$

$$N_{\ell 2} = 7$$

$$\overline{\ell}_1 = \frac{6.5\times10^{-2}}{7(300)} = 7.14\times10^{-6}\,m$$

Trial 2

$L_2 = 7$

$N_{\ell 2} = 8$

$$\bar{\ell}_2 = \frac{7\times 10^{-2}}{8(1300)} = 6.73\times 10^{-6}\, m$$

Average of the two:

$$\bar{\ell} = \frac{\bar{\ell}_1 + \bar{\ell}_2}{2} = 6.94\times 10^{-6}\, m$$

$$d = \frac{3}{2}\bar{\ell} = 1.04\times 10^{-5}\, m = \boxed{10.4\mu m}$$