The primary purpose of this assignment is to write a program to plot yield surfaces and to write a constitutive equation subroutine based on Mises plasticity. Your driver program is then used to illustrate predictions for specific paths.

Be sure to use a scaled dimensionless yield function. A simple way to define a dimensionless yield function is to divide *F* by a constant such that the normalized yield function assumes the value minus one when the intial stress and plastic strain are zero.

1. Add a plasticity capability to your constitutive equation subroutine. Modularize the subroutine to allow for an arbitrary yield function and arbitrary evolution functions for plastic strain rate and other internal plastic variables. Show that your algorithm is working by doing the following:
(i) Write a module for von Mises plasticity, $F = \bar{\sigma} - H(\bar{e}^{\,p})$, with an associated flow rule and isotropic, linear hardening.
(ii) Assume a bilinear uniaxial stress-strain curve. Consider this to be a fit to experimental data where the stress-strain curve has slope, Y, in the elastic regime and a slope of Y/4 beyond the initial yield stress. Obtain the corresponding hardening function. Exercise your plasticity subroutine through the driver for uniaxial stress. You should reproduce the original uniaxial stress-strain curve.

2. Evaluate the robustness and efficiency of your algorithm for the linear hardening function and uniaxial strain. You have two parameters at your disposal: (i) the value of a norm of your strain increments (the size of your strain step), and (ii) the error ε (with $F < \varepsilon$) you are willing to tolerate on satisfying the yield condition. Be sure to use a dimensionless form of *F* for this analysis.
(i) First obtain numerically an “exact” solution by using a small value for ε ($\varepsilon = 0.0001$ say) and then sequentially decreasing strain increments by ½ from initially large values until the stress-strain $(\sigma_{11}\ vs.\ e_{11})$ plots look the same.
(ii) For small and fixed strain increments, show how accuracy is affected by a change in the convergence criterion on the consistency condition $F < \varepsilon$, i.e., use three significantly different values for ε (for example, 0.1, 0.01, 0.001) for a given choice in strain increments. Here demonstrate accuracy by simply overlaying plots of your solutions with the “exact” solution obtained in (i).
(iii) Show how accuracy is affected when larger increments in strain are used holding the convergence criterion small and fixed? Do this study by doing a series of runs where the strain increments are doubled from one run to the next.

(iv) Suppose cost is proportional to the total number of iterations required to arrive at a certain level of stress (or strain). Is it more efficient to prescribe small steps in strain with a "loose" criterion, or a few large steps with a tight criterion, or a balance of the two? What value of ε do you recommend?

As time permits do either Problem 3 or 4.

3. Suppose the hardening function is

$$H = H_0 + (H_L - H_0)\sin[\frac{\pi}{2}(\hat{\bar{e}})^n] \qquad \hat{\bar{e}} \le 1$$

$$H = H_a + (H_L - H_a)[1 + s\bar{e}^*]e^{-s\bar{e}^*} \qquad \hat{\bar{e}} > 1$$

where

$$\hat{\bar{e}} = \frac{\bar{e}}{\bar{e}_L} \qquad \bar{e}^* = \frac{\bar{e} - \bar{e}_L}{\bar{e}_L} = \hat{\bar{e}} - 1$$

Choose what you think would be suitable values that represent features of material behavior (dimensional or dimensionless) for the material parameters in the set $(H_0, H_L, H_a, \bar{e}_L, s)$.

(i) Plot H as a function of $\hat{\bar{e}}$ for various choices of the material parameter n. Choose what you think is a reasonable value for n (the range might be 1/4,1/2,3/4,1,2,4).

(ii) Incorporate this form for H into your Mises formulation and show that the constitutive equation algorithm works equally well in the hardening and softening regimes for uniaxial strain.

(iii) For each of the following paths, apply this plasticity algorithm, plot the path in the q_1 - q_2 and $(\bar{\sigma} - P)$ planes, and plot the yield surface for a couple of points along the path:

(a) uniaxial strain

(b) triaxial compression,

(c) triaxial extension.

4. Add a purely kinematic plasticity model to your to your program based on the shifted Mises stress and an associated flow rule, i.e.,

$$F = \bar{s} - H_0 \qquad \bar{s} = \sqrt{\tfrac{3}{2}}\left(\mathbf{s}^{dev} \cdot\cdot\, \mathbf{s}^{dev}\right)$$

$$\mathbf{s}^{dev} = \boldsymbol{\sigma}^{dev} - \boldsymbol{\sigma}_c^{dev} \qquad \boldsymbol{\sigma}_c^{dev} = C\mathbf{e}^p$$

You may have to explore to find suitable values for the constants C and H_0. By plotting stress versus strain results for a suitable cyclic loading path such as uniaxial strain, argue that your program has satisfactorily incorporated the kinematic feature.