The purpose of this assignment is to write a program for numerical quadrature based on the general trapezoidal rule using subdomains, evaluate its convergence properties and to compare results with a quadrature scheme that provides a higher rate of convergence.

1. Provide a written summary of the relevant theory. Be sure to include definition of terms and what you propose to do in your own words. At the end, discuss your numerical results in connection with what you expected from the theory.

2. Write a program based on the general trapezoidal rule $I_\eta = \alpha f(1) + (1-\alpha) f(0)$. Demonstrate that your algorithm works for some value of $\alpha \neq 1/2$ and for $\alpha = 1/2$.

3. Select a nontrivial function and a domain not of unit length. Obtain the exact integral and the results from your numerical quadrature for at least 1, 2, and 4 subdomains. Plot ln(error) as a function of ln(h), where h is the subdomain length, and determine numerically the "rate of convergence." Do this for the two values of α chosen in problem 2. How do your rates compare with the theoretical values?

4. Now write an algorithm, or use an available code, for either Newton Cotes or Gauss quadrature that provide exact quadrature for at least third-order polynomials. Demonstrate that "exact" answers are obtained for third-order polynomials, and provide a convergence study analogous to that performed for Problem 3.