6.1 Discuss the merits and demerits of the use of transmission electron microscopy techniques to study the dislocation behavior in crystalline materials.

Transmission electron microscopy (TEM) is technique that allows one to make direct observation of dislocations. That is the main attraction. It does have some problems, though. The main negative points are:

- The specimen must be transparent to the electron beam, i.e., it must be very thin, which does not allow one to study the behavior of dislocations in bulk.
- The specimen preparation is very time consuming and involves destruction of the surface features. There is always the possibility of introducing structural artifacts during specimen preparation.

6.2 Explain why a metal like lead does not work-harden when deformed at room temperature, whereas a metal such as iron does.

Work-hardening occurs when a metal is cold-worked, i.e., when it is deformed below its recrystallization temperature. In general, room temperature is above the recrystallization temperature of lead, i.e., when lead is deformed at room temperature, it is undergoing hot deformation. With iron, it is the opposite case. Room temperature deformation of iron is below its recrystallization temperature, so it is cold working and iron work hardens.

6.3 What is the effect of cold work and annealing on the Young's modulus of metal?

There is no significant effect of cold rolling on the Young's modulus of a metal because the modulus is determined by the bonding which does not change with cold work or annealing. A small contribution can come from any preferential alignment of grains on cold rolling.

6.6 Consider dislocations blocked with an average spacing of ℓ in a copper crystal. If the flow stress is controlled by the stress necessary to operate a Frank-Read source, compute the dislocation density ρ in this crystal when it is deformed to a point where the resolved shear stress in the slip plane is 42 MPa. Take G = 50 GPa.

$$\ell = \rho^{-1/2}$$

Flow Stress = Stress necessary to operate a Frank-Read source

$$\tau = \frac{Gb}{\ell} = Gb\sqrt{\rho}$$

For copper, $b = \dfrac{3.6 \times 10^{-10}\sqrt{2}}{2} = 0.255\ nm$

where we have taken the Cu lattice parameter, $a_0 = 3.6 \times 10^{-10} m$. Thus,

$$\rho = \frac{\tau^2}{G^2 b^2} = \frac{\left(42 \times 10^6\right)^2}{\left(50 \times 10^9\right)^2 \times \left(2.55 \times 10^{-10}\right)^2}$$

or $\rho = 1.09 \times 10^{13} m^{-2}$

6.8 The stress axis in an FCC crystal makes angles of 31° and 62° with the normal to the slip plane and with the slip direction, respectively. The applied stress is 10 MN/m^2.

(a) Determine the resolved stress in the shear plane.

(b) Is the resolved stress larger when the angles are 45° and 32°, receptively?

(a) $\theta = 31°$

$\lambda = 62°$

Schmid factor, $M = \cos\theta \cos\lambda = 0.402$

Resolved shear stress, $\tau = \sigma \cos\theta \cos\lambda = \sigma \times M$

where σ is the applied normal stress

$\tau = \sigma \times M = 10 \times 0.402 = 4.02$ MPa

$\tau = 4.02$ MPa

(b) $\theta = 45°$

$\lambda = 32°$

$M = \cos\theta \cos\lambda = 0.600$

$\tau = \sigma \times M = 10 \times \text{MPa} \times 0.600 = 6.00$ MPa

$\tau = 6.00$ MPa

Yes, the resolved shear stress is ≈ 33% larger.

6.9

Magnesium oxide is cubic (having the same structure as NaCl). The slip planes and directions are [110] and <110>, respectively. Along which directions, if any, can a tensile (or compressive) stress be applied without producing slip?

For deformation condition, Schmid factor = 0

[001] direction is perpendicular to (110) plane.

M = $\cos\phi \cos\lambda$

$$\cos\phi = \frac{(110)\bullet(001)}{\sqrt{1^2+1^2+0^2}+\sqrt{0^2+0^2}+1^2} = 0$$

$M = 0 \cdot \cos\lambda = 0$

Thus, no deformation.

6.17 For an FCC polycrystalline metal, TEM analysis showed that the dislocation density after cold working was 5 x 10^{10} m^{-2}. If the friction stress is 100 MPa, G = 40 GPa, and b = 0.3 nm, compute the flow stress of this metal.

$$\rho = 5 \times 10^{10} m^{-2}$$
$$\sigma = 100 MPa$$
$$G = 40 GPa$$
$$b = 0.3 nm$$
$$\alpha = 0.3 - 0.6$$

$$Take\, \alpha = 0.5$$
$$\sigma = \sigma_o + \alpha G b \sqrt{\rho}$$
$$= 100 + 0.5 \times (40 \times 10^3) \times (0.3 \times 10^{-9}) \sqrt{5 \times 10^{10}}\ MPa$$
$$= 101.33 MPa$$

6.18 The stress--strain curve of a polycrystalline aluminum sample can be represented by
$\sigma = 25 + 200\, \varepsilon^{0.5}$.

Calculate the energy of deformation per unit volume corresponding to uniform strain (i.e., just prior to the onset of necking) in this material.

$$U = \int_0^{\varepsilon} \sigma \, d\varepsilon$$

$$= \int_0^{\varepsilon} \left(25 + 200\varepsilon^{.5}\right) d\varepsilon$$

$$= 25\varepsilon + \frac{200\varepsilon^{.5+1}}{.5+1}$$

$$\boxed{U = 25\varepsilon + \frac{400}{3}\varepsilon^{1.5} \left[J/m^3\right]}$$

6.19 An FCC crystal is pulled in tension along the [100] direction.

(a) Determine the Schmid factor for all slip systems.

(b) Identify the slip system(s) that will be activated first.

(c) What is the tensile stress at which this crystal will flow plastically? (τ = 50 MPa)

(a) FCC $\rightarrow$ 12 independent slip systems.

Slip plane	Slip direction	cosθ	cosλ	M=cosλcosθ
(111)	$[0\bar{1}1]$	$\frac{1}{\sqrt{3}} = 0.577$	0	0
*(111)	$[\bar{1}01]$	0.577	$-\frac{1}{\sqrt{2}} = -0.707$	0.408
*(111)	$[\bar{1}10]$	0.577	$-\frac{1}{\sqrt{2}} = -0.707$	0.408
$(\bar{1}11)$	$[0\bar{1}1]$	$-\frac{1}{\sqrt{3}} = -0.577$	0	0
* $(\bar{1}11)$	[101]	–0.577	$\frac{1}{\sqrt{2}} = 0.707$	0.408
* $(\bar{1}11)$	[110]	–0.577	$\frac{1}{\sqrt{2}} = 0.707$	0.408
$(1\bar{1}1)$	[011]	$\frac{1}{\sqrt{3}} = 0.577$	0	0
* $(1\bar{1}1)$	$[\bar{1}01]$	0.577	–0.707	0.408

* $(\bar{1}11)$	[110]	0.577	$\frac{1}{\sqrt{2}} = 0.707$	0.408
$(11\bar{1})$	[011]	$\frac{1}{\sqrt{3}} = 0.577$	0	0
* $(11\bar{1})$	[101]	0.577	0.707	0.408
* $(11\bar{1})$	$[\bar{1}10]$	0.577	–0.707	0.408

(b) The slip systems that will be activated first are the ones with the highest Schmid factor. These are marked with an * and the primary slip systems.

(c) Schmid law: $\tau_c = \sigma \cos\theta \cos\lambda = M\ \sigma$
$\Rightarrow \sigma_o = \tau_c/M = 50/0.408 = 122.5$ MPa
Friction stress, $\sigma_o = 122.5$ MPa

6.20 Calculate the total energy due to dislocations for copper that underwent 20% plastic deformation, resulting in dislocation density of 10^{14} m $^{-2}$. Assume that b = 0.3 nm.

$$\ell = 10^{14} m^{-2}$$
$$b = 0.3 nm$$
$$20\% \ plastic\ deformation$$

$$G = 48.3\ GPa$$
$$U = \frac{Gb^2}{2}$$
$$U = 48.3 \times 10^9\ Pa\ (0.3 \times 10^{-9}\ m)^2$$
$$U = 4.34 \times 10^{-9}\ N$$
$$U_T = U\rho$$
$$U_T = 4.34 \times 10^{-9}\ (10^{14}\ m^{-2})$$
$$U_T = 434.7\ kJ/m^3$$

6.24 The response of copper to plastic deformation can be describes by Holloman's equation, $\sigma = K\varepsilon^{0.7}$. It is known that for $\varepsilon = 0.25$, $\sigma = 120$ MPa. The dislocation density varies with flaw stress according to the well-known relationship, $\sigma = K^{'}\rho^{1/2}$.

(a) If the dislocation density at a plastic strain of 0.4 is equal to 10^{11}cm $^{-2}$, plot the dislocation density versus strain.
(b) Calculate the work performed to deform the specimen.
(c) Calculate the total energy stored in the metal as dislocations after deformation of 0.4, and compare this value with the one obtained in part (b).
(d) Explain the difference.

(a) Dislocation density vs. plastic strain.

Holloman's equation : $\sigma = K\varepsilon^{0.7}$
For $\varepsilon = 0.25 \Rightarrow \sigma = 120$ MPa
Thus, $K = \sigma/\varepsilon^{0.7} = 120 \times 10^{6} / 0.25^{0.7} = 316.7$ MPa

But we also have:σ $K^{'}\rho^{1/2}$ where ρ is the dislocation density
We know, for $\varepsilon = 0.4 \Rightarrow \rho = 10''\text{cm}^{-2}$
$\sigma = K\,\varepsilon^{0.7} = K' \times \rho^{1/2} \Rightarrow \rho = (K/K')^2\varepsilon^{1.4}$

Thus, $K' = K\varepsilon^{0.7}\rho^{-1/2} = 316 \times 10^{6} \times 0.4^{0.7} \times (10^{15})^{-1/2} = 5.27$ Pa m

Finally, we can plot the dislocation density vs. strain.

(b) Work performed to deform the specimen.

$$U_{tot} = \int_{o}^{0.4} \sigma\, dE = \int_{o}^{0.4} K \cdot E^{0.9} dE = \frac{K}{1.7} E^{1.7} \int_{o}^{0.4}$$
$$= \frac{316.7 \times 10^{6}}{1.7} 0.4^{1.7} = 39.2 \times 10^{6}\, J/m^{3}$$
$$U_{tot} = 39.2 \times 10^{6}\, J/m^{3}$$

c) Total energy stored in the metal as dislocations after deformation of 0.4, and compare this value with the one obtained in part (b). Explain the difference.

$$U_{disl.} = \rho \frac{Gb^2}{2} \quad \textit{where } \rho \ldots \textit{dislocation density } \rho = 10'' cm^{-2} = 10^{15} m^{-2}$$

$$b \ldots \textit{Burgers vector} \quad b = 2r_{Cu}$$

Copper ,Cu:

G = 48.3 GPa (chap.2,table 2.5, p. 114)

R_{Cu} = 0.128 nm (Appendix, p. 846)

$$U_{disl.} = 10^{15} \times \frac{48.3 \times 10^9 (0.256 \times 10^{-9})^2}{2} = 1.58 \times 10^6 \, J/m^3$$

$$U_{disl.} = 1.58 \times 10^6 \, J/m^3$$

The energy stored into the dislocation represents only 4 % of the energy expended. The rest is used to move the dislocation (overcome the obstacles) and the movement of the dislocation is essentially a frictional effect which leads to a rise in the temperature of the specimen.

6.25 A single crystal of silver is pulled in tension along the [100] direction. Determine the Schmid factor for all slip systems. What is the tensile stress atwhich this crystal will flow plastically? (τ = 100 MPa.)

Cu is FCC, i.e., it has 12 slipstreams

$$\ell = [100] \qquad Cos\phi = \frac{n \cdot \ell}{|n| \cdot |\ell|} \qquad Cos\lambda = \frac{s \cdot \ell}{|s| \cdot |\ell|}$$

Schmid Law : $\tau = \sigma \cos(\theta) \cos(\lambda) = \sigma M$

$$\sigma = \frac{\tau}{M}, \quad \tau = 100 \text{ MPa}$$

Slip plane (n)	Slip direction (s)	$Cos\phi$	$Cos\lambda$	$M = Cos\phi Cos\lambda$	σ (MPa)
(111)	$[0\bar{1}1]$	$\frac{1}{\sqrt{3}}$	0	0	Not deformed
	$[\bar{1}01]$	$\frac{1}{\sqrt{3}}$	$\frac{-1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	245
	$[\bar{1}10]$	$\frac{1}{\sqrt{3}}$	$\frac{-1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	245
$(\bar{1}11)$	$[0\bar{1}1]$	$\frac{-1}{\sqrt{3}}$	0	0	Not deformed
	$[101]$	$\frac{-1}{\sqrt{3}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	245
	$[110]$	$\frac{-1}{\sqrt{3}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	245
$(1\bar{1}1)$	$[011]$	$\frac{1}{\sqrt{3}}$	0	0	Not deformed
	$[\bar{1}01]$	$\frac{1}{\sqrt{3}}$	$\frac{-1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	245
	$[110]$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	245
$(\bar{1}\bar{1}1)$	$[011]$	$\frac{1}{\sqrt{3}}$	0	0	Not deformed
	$[101]$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	245
	$[\bar{1}10]$	$\frac{1}{\sqrt{3}}$	$\frac{-1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	245

$\boxed{6.28}$

Cu $\rightarrow$ FCC has 12 slipstreams

$$\ell = [\bar{1}21] \qquad Cos\phi = \frac{n \cdot \ell}{|n| \cdot |\ell|} \qquad Cos\lambda = \frac{s \cdot \ell}{|s| \cdot |\ell|}$$

Slip plane (n)	Slip direction (s)	$Cos\phi$	$Cos\lambda$	$M = Cos\phi Cos\lambda$
(111)	$[\bar{1}10]$	$\frac{5\sqrt{3}}{9}$	0	0
	$[\bar{1}01]$	$\frac{5\sqrt{3}}{9}$	$\frac{\sqrt{2}}{6}$	$\frac{5\sqrt{6}}{54}$
	$[0\bar{1}1]$	$\frac{5\sqrt{3}}{9}$	$\frac{\sqrt{2}}{6}$	$\frac{5\sqrt{6}}{54}$
$(\bar{1}11)$	$[110]$	$\frac{\sqrt{3}}{9}$	$\frac{4\sqrt{2}}{6}$	$\frac{4\sqrt{6}}{54}$
	$[101]$	$\frac{\sqrt{3}}{9}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{6}}{18}$
	$[0\bar{1}1]$	$\frac{\sqrt{3}}{9}$	$\frac{\sqrt{2}}{6}$	$\frac{\sqrt{6}}{18}$
$(1\bar{1}1)$	$[110]$	$\frac{\sqrt{3}}{9}$	$\frac{4\sqrt{2}}{3}$	$\frac{4\sqrt{6}}{27}$
	$[\bar{1}01]$	$\frac{\sqrt{3}}{9}$	$\frac{\sqrt{2}}{6}$	$\frac{\sqrt{6}}{54}$
	$[011]$	$\frac{\sqrt{3}}{9}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{6}}{18}$
$(1\bar{1}1)$	$[\bar{1}10]$	$\frac{5\sqrt{3}}{9}$	0	0
	$[101]$	$\frac{5\sqrt{3}}{9}$	$\frac{\sqrt{2}}{2}$	$\frac{4\sqrt{6}}{18}$
	$[011]$	$\frac{5\sqrt{3}}{9}$	$\frac{\sqrt{2}}{2}$	$\frac{4\sqrt{6}}{18}$

The first systems to be activated are the ones with the largest Schmid values. For this system they are located in slip plane $(1\bar{1}1)$ in slip directions $[101]$, $[011]$.

6.26 Determine the area of the slip plane Ni deformed parallel to [100] and under load $P = 150 \times 10^{3}$ N. The shear stress is 600 MPa.

Ni is FCC.

τ = 600 MPa
$P = 150 \times 10^3$ N
Slip plane [111]
Slip direction [110]

$$\cos\theta = \frac{\vec{n}\times\vec{\ell}}{|\vec{n}|\,|\vec{\ell}|} = \frac{1}{\sqrt{3}},\ \cos\lambda = \frac{\vec{s}\times\vec{\ell}}{|\vec{s}|\,|\vec{\ell}|} = \frac{1}{\sqrt{2}}$$

$\tau = P/A \cos\theta \cos\lambda$

$$600\times10^{6} = \frac{150\times10^{3}}{A}\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$A = \frac{150\times10^{3}}{600\times10^{6}}\frac{1}{\sqrt{6}}$$

$$A = 1.02\times10^{-4}\,m^{2}$$

6.27 Compute the dislocation density in tungsten if the stress is controlled by the stress necessary to operate a Frank-Read source. The stress in the plane is 50 MPa. Take G = 166 GPa.

G = 166 GPa, τ = 50 MPa
$l = \rho^{-1/2}$
$\tau = Gb/l = Gb\sqrt{r}$

Tungsten is BCC
R_w = 0 .137 nm (see Appendix, p. 846)

$$4R_w = \sqrt{3}\,a$$

$$a = \frac{4}{\sqrt{3}} R_w = 0.322\ nm$$

$$b = \frac{a}{\sqrt{2}}$$

$$\therefore b = \frac{0.322}{\sqrt{2}} = 0.227\ nm$$

$$\rho = \frac{\tau^2}{G^2 b^2}$$

$$= \frac{(50\times10^6)^2}{(166\times10^9)^2(0.227\times10^{-9})^2}$$

$$= 1.813\times10^{12} m^{-2}$$

$$\therefore\ \rho = 1.813\times10^{12} m^{-2}$$

6.30 What is the dislocation density for iron with a shear strain of 0.4?
Given:

(a) $\tau = \tau_0 + k\gamma^n$, τ 0 = 50 MPa, k = 10^8 Pa, n = 0.5;

(b) $\tau = \tau_0 + \alpha G b\sqrt{\rho}$, G = 81.6 GPa, b = 0.25 nm, α = 0.5.

a) $\tau = \tau_0 + k\gamma^n$

$\tau_0 = 50\times10^6\,Pa$, $k = 10^8\,Pa$, $n = .5$ $\gamma = .4$

$$\tau = 50\times10^6 + 10^8(.4)^{.5} = 113.25\times10^6\,Pa$$

b) $\tau = \tau_0 + \alpha G b\sqrt{\rho}$

$G = 81.6 GPa \qquad b = .25 nm \qquad \alpha = .5$

$$\rho = \left(\frac{\tau - \tau_0}{\alpha G b}\right)^2 = \left(\frac{113.25\times10^6 - 50\times10^6}{(.5)(81.6\times10^9)(.25\times10^{-9})}\right)^2$$

$$\rho = 6.2\times10^6 m^{-2} = 6.2\times10^{10} cm^{-2}$$

6.31 The flow stress for an alloy is 100 MPa when its dislocation density is 10^6 cm^{-2}, and 150 MPa when its dislocation density is 10^8 cm^{-2}. If the flow stress is 190 MPa, what is the dislocation density?

Point 1 $\tau_1 = 100 MPa \quad \rho_1 = 10^6 cm^{-2}$

Point 2 $\tau_2 = 150 MPa \quad \rho_2 = 10^8 cm^{-2}$

Point 3 $\tau_3 = 190 MPa$

$$\tau = \tau_0 + \alpha G b\sqrt{\rho}$$
$$A = \alpha G b$$

For Point 1

$$\tau_1 = \tau_0 + A\sqrt{\rho_1}$$

$$100\times10^6 = \tau_0 + A\sqrt{10^6\times10^4}$$

$$\boxed{1\times10^8 - 1\times10^5 A = \tau_0} \qquad \textbf{(1)}$$

For Point 2

$\tau_2 = \tau_0 + A\sqrt{\rho_2}$

$$\boxed{\frac{1.50\times10^8 - \tau_0}{1\times10^6} = A}\ \textbf{(2)}$$

Two equations with two unknowns. Substitute (2) into (1) and solve for τ_0

$$1\times10^8 - \frac{1.0\times10^5\,(1.5\times10^8 - \tau_0)}{1\times10^6} = \tau_0$$

$$1\times10^8 - 1.5\times10^7 = \tau_0 - .1\tau_0$$

$$94.4\times10^6\,Pa = \tau_0$$

A = 55.5

$$\rho_3 = \left(\frac{\tau - \tau_0}{A}\right)^2 = \left(\frac{1.9\times10^8 - 94.4\times10^6}{55.5}\right)^2 = 2.96\times10^{12}\,m^{-2}$$

6.32 A copper sample exhibits work-hardening described by:

$\sigma = \sigma_0 + K\,\varepsilon^n$,

where σ_0 = 50 MPa, n = 0.5, K = 500 MPa.

Calculate the temperature rise when the sample is deformed up to a strain of 0.2. Assume that the conversion factor is 1.0, and given: density = 8.9 g/cm^3; heat capacity = 360 J/kg K.

$$\sigma = \sigma_0 + K\varepsilon^n = 50\,MPa + (500\,MPa)(0.2^{0.5}) = 273.6\,MPa$$

$$\Delta T = \frac{\beta}{\rho C_p}\sigma\Delta\varepsilon = \frac{1}{\left(8.9\times10^3\,\frac{kg}{m^3}\right)\left(360\,\frac{N\,m}{kg\,K}\right)}(273.6\times10^6 Pa)(0.2) = 17.08\text{ K}$$