Assignment 1 **ME 562** **Inelastic Continuum Mechanics** **Spring/15**
(25 points) Due Feb. 3

NOTE: There will be approximately 5 assignments each of which I consider to be fairly lengthy but not overly difficult. I have started out with 25 pts for this assignment. If I believe another assignment is longer or shorter, the number of points awarded will be raised or lowered accordingly. I expect you to summarize your results and conclusions with sufficient details appended to substantiate your claim (worth 1/5 of points for the assignment). It is good practice, and a nice way to help learn the theory, to summarize the theory and equations you are using. Do not waste your time on extravagant plots and programs. In the past, most type their notes. This is not necessary but your handwriting must be readable. If you write a new subroutine, annotate it as necessary with handwritten comments, and hand it in only the first time you use it. It is not necessary to hand the same subroutine in for subsequent assignments. If you use a commercial package, indicate which one you are using.

END OF NOTE.

The primary objectives of this assignment is to write a program for transforming components of second-order and fourth-order tensors from one orthonormal basis to another using conventional and Voigt-Mandel notation, and to demonstrate that you are comfortable with the eigenproblem.

1. First, consider an orthonormal basis, $\boldsymbol{E}_i$, as given. Construct a second orthonormal basis, $\boldsymbol{e}_i$, in three dimensions, that is related in a "nontrivial" manner to the first. Construct the resulting transformation matrix $\left[{}^{e}a^{E}\right]$ and its transpose $\left[{}^{E}a^{e}\right]$. Show numerically that $\left[{}^{\mathrm{e}}\mathrm{a}^{\mathrm{E}}\right]\left[{}^{\mathrm{E}}\mathrm{a}^{\mathrm{e}}\right]=[\mathrm{i}]$, i.e., show that the matrix is orthogonal, and obtain $det\left[{}^{e}a^{E}\right]$.

2. (i) Select components (non zero for all components), $\left[{}^{E}T^{E}\right]$, for a second-order symmetric tensor, $\mathbf{T}$, with respect to the $\boldsymbol{E}_i$ basis. Apply the transformation relation (again numerically)

$$[{}^{\mathrm{e}}\mathrm{T}^{\mathrm{e}}]=[{}^{\mathrm{e}}\mathrm{a}^{\mathrm{E}}][{}^{\mathrm{E}}\mathrm{T}^{\mathrm{E}}][{}^{\mathrm{E}}\mathrm{a}^{\mathrm{E}}]$$

to obtain components with respect to the other basis. Argue that your algorithm appears to be correct.

(ii) Obtain the spherical and deviatoric components of $\boldsymbol{T}$ in the $\boldsymbol{E}_i$ system.

3. (i) Use your favorite eigensystem program to find the eigenvalues and eigenvectors of **T** with respect to the $\boldsymbol{E}_i$ basis.
(ii) Construct an orthonormal basis, $\mathbf{P}_{\mathrm{A}}$, from the eigenvectors.
(iii) Construct the transformation matrix and transform components from your original basis to the principal system. Are the components in this principal system what you expected?
(iv) Obtain the components of the inverse of **T** in the principal basis (assuming your tensor is not singular). Transform these components back to the original basis. Is your result correct?
(v) Now use the components of **T** with respect to the $\boldsymbol{e}_i$ basis to find the eigenvalues and eigenvectors of **T**. Is the result the same as what you obtained in part (i); in particular, did you get the same result for the eigenvectors?

4. Write a subroutine that transforms components of a symmetric, second order tensor written as a 3x3 matrix to a Voigt-Mandel vector with six components, and a subroutine that transforms a six vector back to a 3x3 symmetric matrix.

5. (i) Suppose the components of the transformation matrix $\left[{}^{e}a^{E}\right]$ are given. Write a subroutine that constructs the Voigt-Mandel transformation matrix $\left[{}^{e}A^{E}\right]$ where the components of $\left[{}^{e}A^{E}\right]$ are expressed in terms of the components of $\left[{}^{e}a^{E}\right]$.
(ii) As an example to verify that your program is correct use the components of your matrix $\left[{}^{e}a^{E}\right]$ defined in Problem 1 to obtain a specific $\left[{}^{e}A^{E}\right]$. Repeat the examples of Problem 2 to show that you get the same results using the Voigt-Mandel notation of

$$\left\{\mathrm{T}^{\mathrm{e}}\right\}_{6}=\left[{}^{\mathrm{e}}\,\mathrm{A}^{\mathrm{E}}\right]_{6\mathrm{x}6}\left\{\mathrm{T}^{\mathrm{E}}\right\}_{6} \qquad \left\{\mathrm{T}^{\mathrm{E}}\right\}_{6}=\left[{}^{\mathrm{E}}\,\mathrm{A}^{\mathrm{e}}\right]_{6\mathrm{x}6}\left\{\mathrm{T}^{\mathrm{e}}\right\}_{6}$$

Demonstrate that $\left[{}^{\mathrm{e}}\,\mathrm{A}^{\mathrm{E}}\right]$ is orthogonal.

6. Write a program that provides the matrix components of the Voigt-Mandel versions of the fourth-order deviatoric projection and the fourth-order spherical projection denoted by $[\mathrm{P}^{\mathrm{d}}]$ and $[\mathrm{P}^{\mathrm{sp}}]$, respectively. Verify that when you numerically take the products

$$[\mathrm{P}]^{\mathrm{dev}}\{\mathrm{T}\}=\{\mathrm{T}\}^{\mathrm{dev}} \qquad [\mathrm{P}]^{\mathrm{sp}}\{\mathrm{T}\}=\{\mathrm{T}\}^{\mathrm{sp}}$$

the result is the same as your hand calculations from Problem 2(ii).

7. Suppose the matrices $[\mathrm{P}^{\mathrm{d}}]$ and $[\mathrm{P}^{\mathrm{sp}}]$ are considered to be components of their respective tensors with respect to the $\boldsymbol{e}_i$ basis. Use the Voigt-Mandel transformation matrix from Problem 5(ii) and determine the components of $[\mathrm{P}^{\mathrm{d}}]$ and $[\mathrm{P}^{\mathrm{sp}}]$ in the $\boldsymbol{e}_i$ basis. What tentative conclusion concerning anisotropy of the tensors $\mathbf{P}^{\mathrm{d}}$ and $\mathbf{P}^{\mathrm{sp}}$ can you make?