ME 562 Constitutive Equations and Num. Algorithms 8 March 2012

50 Min. Closed Book Exam

The weights (square brackets) associated with the problems indicate the relative degree of thoroughness (and time) that should govern your answers.

1. [10] Give alternative forms for the following expressions in which $(\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{u})$ is a set of vectors. In each case categorize the result as a scalar, vector or tensor of a specified order. Express your results in both direct and indicial notation.
(i) $\boldsymbol{u}\cdot(\boldsymbol{b}\otimes\boldsymbol{c})$ (ii) $(\boldsymbol{a}\otimes\boldsymbol{b}\otimes\boldsymbol{c})\cdot\boldsymbol{u}$

2. (a) [5] Define what is meant by a gradient with respect to a vector.
(b) [15] Find the gradient with respect to the vector $\boldsymbol{v}$ of the following:
   (i) $sin[(\boldsymbol{v}\cdot\boldsymbol{v})^{1.5}]$,
   (ii) $\boldsymbol{v}(\boldsymbol{v}\cdot\boldsymbol{\sigma}\cdot\boldsymbol{v})$ in which $\boldsymbol{\sigma}$ is a second-order tensor that is symmetric and constant.

3. Suppose $\boldsymbol{A}$ *and* $\boldsymbol{B}$ are second-order tensors with $\boldsymbol{A}=\boldsymbol{A}^T$ and $\boldsymbol{B}=-\boldsymbol{B}^T$.
(a) [5] For each of the following is the result a scalar, a vector or a tensor of what order.
   (i) $\boldsymbol{A}\otimes\boldsymbol{B}$ (ii) $\boldsymbol{A}\cdot\boldsymbol{B}$ (iii) $\boldsymbol{A}\cdot\cdot\boldsymbol{B}$
(b) [5] Expand and give the result for the product (iii).

4. [17] Consider the stress tensor $\boldsymbol{\sigma}$.
(i) What are the spherical and deviatoric parts of the stress tensor?
(ii) Suppose it is claimed that **e** is an eigenvector of $\boldsymbol{\sigma}$ with eigenvalue λ. How would you verify the claim?
(iii) Suppose **e** is an eigenvector of $\boldsymbol{\sigma}$. Is **e** also an eigenvector of the stress deviator? If so, what is the corresponding eigenvalue?
(iv) Define what is meant by an invariant of $\boldsymbol{\sigma}$. Are λ and $\boldsymbol{e}$ invariants?

5. [5] What is the definition of a projector? Give a couple of examples of projectors.

6. [10] What are the basic assumptions associated with the first and second laws of thermodynamics. Give a resulting implication of each law.

7. [8] Define each of the following loading paths:
(i) uniaxial stress, (ii) uniaxial strain, (iii) triaxial compression, and (iv) proportional loading.

8. [20] Outline the basic ideas associated with linear elasticity. In your discussion provide definitions for elasticity, elastic stiffness tensor, major and minor symmetries, flexibility, isotropy, anisotropy and orthotropy.

ME 562 Inelastic Continuum Mechanics (2:00 – 2:50 AM) 26 April 2012

Closed Book Exam

Write a "summary" article providing an overview of the basic concepts associated with the theory of plasticity and material stability. Credit will be allocated as follows:

[25] Item 1. General flow and logical structure of your article, completeness and the introduction of terms as necessary that are not listed below.

[75] Item 2. Technical definitions and arguments. This credit is broken down as listed below and is based on the accuracy of your descriptions and definitions that are included in your article. Both word descriptions and compact mathematical equations are expected.

1. [15] What is meant by the paths of uniaxial stress, uniaxial strain and triaxial compression? Sketch the paths in the pi (deviatoric) plane and the Mises stress – pressure plane. What would you expect to be the difference between experimental data for metals and data for concrete or rock? What features of experimental data warrant the use of a plasticity model?

2. [24] What is meant by (i) Mises stress, (ii) plastic strain, (iii) effective plastic strain, (iv) yield function, (v) yield surface, (vi) evolution equations, (vii) evolution functions, and (viii) consistency?

3. [6] Give a complete set of the elastic-plastic constitutive equations

4. [6] What is the difference between isotopic and kinematic hardening? What features of experimental data or problem type would suggest you choose one form over the other?

5. [6] What is meant by rate-dependence and rate independence? If your formulation is rate-independent, how would you add in a rate dependence?

6. [9] What is meant by tangent tensor, acoustic tensor, and material instability.
Outline the numerical algorithm necessary to check for material instability.

7. [9] With regard to the tangent tensor, what is meant by minor symmetries? What is meant by major symmetry? Under what condition, if any, does the tangent tensor satisfy major symmetry? What is the implication of major symmetry on material stability?

ME 562 Inelastic Continuum Mechanics (10:00 – 12:00 AM) 8 May 2012
Final Exam - Closed Book

Credit is shown in square brackets. This indicates roughly the relative weighting you should place on your effort. Be sure to answer your easy problems first.
Total points: 120

1. [20] What is meant by each of the following: (i) invariant of a tensor, (ii) deviatoric projection, (iii) elasticity, (iv) isotropic elasticity, (v) plasticity, (vi) viscoelasticity, (vii) ductile material, (viii) brittle material , (ix) shear enhanced compaction, and (x) dilatation.

2. [25] Start with linear elasticity as defined by $\boldsymbol{\sigma} = \boldsymbol{E} \cdot\cdot \boldsymbol{e}$ *or* $\sigma_{ij} = E_{ijkl} e_{kl}$. What is meant by major and minor symmetry of $\boldsymbol{E}$? What is the Voigt-Mandel notation? Why is it used? Relate the components of the vectors and matrices in the Voigt_mandel formulation to the components of $\boldsymbol{\sigma}, \boldsymbol{e}$ *and* $\boldsymbol{E}$. Suppose you have a material governed by orthotropic elasticity. What is meant by orthotropy? How would you handle the siruration where the global axes chosen for obtaining numerical solutions to the equations of motion are different from the "material axes" of your orthotropic material.

3. [40] For a generic plasticity model, give a rather complete mathematical description of the constitutive equations. Be sure to define all terms. Include definitions for yield surface, yield function, perfect plasticity, kinematic hardening, associated flow law and the consistency condition. Outline a numerical algorithm that might be used to solve these equations for prescribed strain increments. How can you handle a prescribed stress path? How would you incorporate a rate effect? What is material instability? When might you expect to predict material instability with a plasticity model?

4. This problem consists of two parts related to decohesive modeling of material failure.
4.1: [20] Provide definitions or descriptions for each of the following items: (i) Material failure, (ii) displacement discontinuity, (iii) decohesion function, (iv) failure surface, (v) evolution equations, (vi) evolution functions, (vii) a set of discrete constitutive equations, (viii) axial splitting, (ix) mode of failure, and (x) orientation of failure surface.
4.2: [15] What is the essential idea that motivated each of the following failure criteria, what is the essential feature of each criterion, what are the limitations, if any, and for what material or class of materials might each criterion be useful: (i) Tresca, (ii) Mohr-Coulomb, and (iii) Rankine.