ME 512 CONTINUUM MECHANICS Fall 2014 ASSIGNMENT 5

1. In the notes the following expressions for gradients were obtained by using total differentials. Show that you get the same answers by taking derivatives with respect to components of the independent variable with the bases assumed constant.

(i) $\phi = (\boldsymbol{v}\cdot\boldsymbol{v})^{3/2} \qquad (\phi)\bar{\nabla}_{\boldsymbol{v}} = 3(\boldsymbol{v}\cdot\boldsymbol{v})^{1/2}\boldsymbol{v}$

(ii) $\phi = tr(\boldsymbol{T}) + tr(\boldsymbol{T}^2) + (\boldsymbol{T}\cdot\cdot\boldsymbol{T})^{1/2} \qquad (\phi)\bar{\nabla}_{\boldsymbol{T}} = \boldsymbol{I} + 2\boldsymbol{T}^T + \dfrac{\boldsymbol{T}}{(\boldsymbol{T}\cdot\cdot\boldsymbol{T})^{1/2}}$

(iii) $\boldsymbol{\phi} = \boldsymbol{v}(\boldsymbol{v}\cdot\boldsymbol{v})^{3/2} \qquad (\boldsymbol{\phi})\bar{\nabla}_{\boldsymbol{v}} = \left[\boldsymbol{I}(\boldsymbol{v}\cdot\boldsymbol{v})^{3/2} + 3(\boldsymbol{v}\cdot\boldsymbol{v})^{1/2}(\boldsymbol{v}\otimes\boldsymbol{v})\right]$

2. If F and ϕ are scalars, use indicial notation to show that

$$
\begin{aligned}
&(i)\,(F\boldsymbol{v})\cdot\bar{\nabla} = \boldsymbol{v}\cdot\left(F\bar{\nabla}\right) + F\left(\boldsymbol{v}\cdot\bar{\nabla}\right)\\
&(ii)\,\left(F\bar{\nabla}\right)\times\bar{\nabla} = 0\\
&(iii)\,\left(\boldsymbol{v}\times\bar{\nabla}\right)\cdot\bar{\nabla} = 0\\
&(iv)\,\left(\boldsymbol{v}\times\bar{\nabla}\right)\times\bar{\nabla} = \left(\boldsymbol{v}\cdot\bar{\nabla}\right)\bar{\nabla} - \boldsymbol{v}\bar{\nabla}^2\\
&(v)\,(\boldsymbol{u}\otimes\boldsymbol{v})\cdot\bar{\nabla} = (\boldsymbol{u}\bar{\nabla})\cdot\boldsymbol{v} + \boldsymbol{u}(\boldsymbol{v}\cdot\bar{\nabla})\\
&(vi)\,curl\left(grad\ \phi\right) = 0
\end{aligned}
$$

3, If $\boldsymbol{u} = x_1x_2x_3\boldsymbol{e}_1 + x_1x_2\boldsymbol{e}_2 + x_1\boldsymbol{e}_3$, determine the gradient of $\boldsymbol{u}$, the divergence of $\boldsymbol{u}$, and the curl of $\boldsymbol{u}$. Verify that (iv) of Problem 2 is satisfied.

4. If $\boldsymbol{T}$ is a second-order tensor, express $(i)\,\boldsymbol{T}\times\bar{\nabla}\ and\ (ii)\,C_{13}\left(\boldsymbol{I}\times\boldsymbol{T}\right)$ in terms of components and base vectors.

5. If $\boldsymbol{T}$ is a second-order tensor and $\boldsymbol{f}$ is a vector, express $\boldsymbol{T}\cdot\bar{\nabla} + \boldsymbol{f} = \boldsymbol{0}$ in indicial form and then expand to give the "long-hand" form.