ME 512 CONTINUUM MECHANICS Fall 2014 ASSIGNMENT 4

For the basis $\mathbf{e}_i$, the components of $\mathbf{T}$ are $\begin{bmatrix} 3 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

1. Find the components of $\mathbf{T}^2$ and $\mathbf{T}^3$ for the same basis.

2. Find $\mathrm{I}_T = \mathrm{tr}\,\mathbf{T}$, $\mathrm{II}_T = \mathrm{tr}\,\mathbf{T}^2$, and $\mathrm{III}_T = \mathrm{tr}\,\mathbf{T}^3$.

3. Find the eigenvalues of $\mathbf{T}$ and the eigenvectors. Construct a principal basis ($\boldsymbol{p}_A$ say) expressed in terms of $\mathbf{e}_i$.

4. What are the components of $\mathbf{T}$, $\mathbf{T}^2$, and $\mathbf{T}^3$ with respect to the principal basis. Determine I_T, II_T, and III_T using these components.

5. Set up the transformation matrix between $\boldsymbol{p}_A$ and $\mathbf{e}_i$. Start with the components of $\mathbf{T}$ in the principal basis obtained in Prob. 4 and use the transformation relation to obtain the components with respect to the basis $\mathbf{e}_i$.

6. (a) Obtain the values of the invariants $\hat{\mathrm{I}}_T$, $\hat{\mathrm{I}}\mathrm{I}_T$, and $\mathrm{I}\hat{\mathrm{I}}\mathrm{I}_T$.
   (b) Show that the Cayley-Hamilton theorem holds using components in the $\mathbf{e}_i$ system.
   (c) With the use of components in either system, show that

$$\mathrm{I}\hat{\mathrm{I}}\mathrm{I}_T \equiv \frac{1}{6}\left[\mathrm{I}_T^3 - 3\,\mathrm{I}_T\,\mathrm{II}_T + 2\,\mathrm{III}_T\right] = \det(\mathbf{T})$$

7. Find the components of the tensor $\mathbf{T}^{1/2}$ in the $\mathbf{e}_i$ system. Obtain $T_{ij}^{1/2}\,T_{jk}^{1/2}$.

8. In the $\mathbf{e}_i$ system find the components of $\mathbf{T}^{-1}$ from the equation

$$\mathbf{T}^{-1} = \left(\mathbf{T}^2 - \hat{\mathrm{I}}_T\,\mathbf{T} - \hat{\mathrm{I}}\mathrm{I}_T\,\mathbf{I}\right) / \left(\mathrm{I}\hat{\mathrm{I}}\mathrm{I}_T\right).$$

Transform these components to obtain the components of $\mathbf{T}^{-1}$ in the $\boldsymbol{p}_A$ system.