CE 598 – Peridynamics
Solution to HW #4

4.5 A vertical-axis cylindrical water tower is 120 ft high and 50 ft in diameter. It is constructed of 0.5-inch-thick 4340 high-strength steel (F_y = 214 KSI, plane strain K_{Ic} = 90 K-in$^{-3/2}$). Water weighs 62.4 lb/ft^3.

(a) Is it valid to use the plane strain (rather than plane stress) fracture toughness?

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$$2.5\left(\frac{K_{Ic}}{F_y}\right)^2 = 2.5\left(\frac{90^{\text{ksi-in}^{-3/2}}}{214^{\text{ksi}}}\right)^2 = 0.442 \text{ in.}$$

Since this is smaller than the plate thickness of 0.5", we expect plain-strain conditions at the crack tip! According to Eq. 2.89 in Anderson,

$$K_{crit} = K_{Ic}\left(1 + 1.4\beta_{Ic}^2\right)^{1/2}$$

$$\text{where } \beta_{Ic} = \frac{1}{B}\left(\frac{K_{Ic}}{\sigma_{ys}}\right)^2 = \frac{1}{.5"}\left(\frac{90}{214}\sqrt{\text{in}}\right)^2 = 0.353$$

$$\text{so } K_{crit} = 90^{\text{ksi}\sqrt{\text{in}}}\left(1 + 1.4(.353)^2\right)^{1/2} = 97.6^{\text{ksi}\sqrt{\text{in}}} \text{ for a } 1/2" \text{ PL}$$

It is conservative to use $K_{Ic} = 90^{\text{ksi}\sqrt{\text{in}}}$

(b) How large a vertical crack, a_{cr}, at the base of the tower can be tolerated if crack growth is to be prevented?

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$$K_{Ic} = \sigma_H\sqrt{\pi a_{cr}} \;;\quad \sigma_H = \frac{PR}{t}$$

$2PR = \sigma 2t$

$\sigma = \frac{PR}{t}$

$$P = 62.4^{\text{pcf}} \times 120' = 7,488^{\text{psf}} = 52 \text{ psi}$$

$$\sigma_H = \frac{52\text{psi} \times 25' \times 12"/'}{0.5"} = 31.2^{\text{ksi}}$$

$$a_{cr} = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_H}\right)^2 = \frac{1}{\pi}\left(\frac{90^{\text{ksi}\sqrt{\text{in}}}}{31.2^{\text{ksi}}}\right)^2 = 2.64 \text{ in}$$

The crack will become unstable when its length is $2a = 5.3"$.
(if LEFM is applicable)

(c) Once the crack reaches this critical size, a_{cr}, will failure be slow or will it be catastrophic?

c) Once the crack reaches this critical size, will failure be slow or catastrophic?

Because G_H will remain approximately constant (if little leakage), and K_{Ic} will be constant, the crack will grow catastrophically

(d) Does LEFM apply to this problem?

d.) Does LEFM apply? $r_p \cong \frac{1}{\pi}\left(\frac{K_{Ic}}{F_y}\right)^2 = \frac{1}{\pi}\left(\frac{90\,\text{ksi}\sqrt{\text{in}}}{214\,\text{ksi}}\right)^2 = 0.056'' \ll a_{cr}$

$\therefore$ LEFM applies

4.6 The principal stress trajectories in Fig. 4.11(a) show that a crack in an infinite plate causes stress relief in the shaded triangular regions next to the crack. As an approximation, assume the stress relief region to be limited by lines of constant slope, k, as shown in Fig. 4.11(b), and assume the stresses inside the stress relief region are zero while remaining unchanged outside.

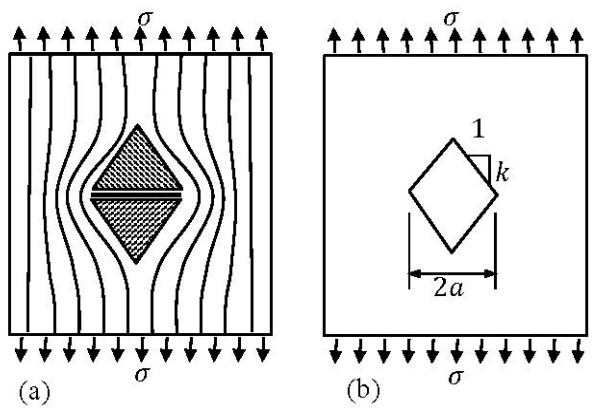

Figure 0.1. Infinite plate with crack for Problem 4.6.

Solution:

(a) Use the Griffith criterion to evaluate G_I and K_I as functions of σ, E, k, and a.

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$$G = -\frac{\partial \Pi}{\partial A} = \frac{\partial U}{2B\partial a} = \frac{\partial}{2B\partial a}\left(\int_{Vol} \frac{\sigma \cdot \epsilon}{2}\, dVol\right)$$

$$G = \frac{\partial}{2B\partial a}\left[\frac{\sigma^2}{2E}(2a \cdot (ka)B)\right] = \frac{\partial}{2\partial a}\left[\frac{\sigma^2 ka^2}{E}\right]$$

$$G = \frac{a\sigma^2 k}{E} \cong \frac{a\sigma^2 k}{E} \quad \text{(plane stress)}$$

$$K_I = \sqrt{EG} \cong \sqrt{a\sigma^2 k} = \sigma\sqrt{ka} \quad \text{(plane stress)}$$

plane stress.

(b) Show that the energy release rate depends upon whether the specimen is in plane stress or plane strain, while the stress-intensity factor is the same for both cases.

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for plane strain, $\epsilon = \frac{\sigma}{E(1-\nu^2)}$ $\quad E' = \frac{E}{1-\nu^2}$

thus $G = \frac{a\sigma^2 k(1-\nu^2)}{E}$, and $K_I = \sqrt{E'G} = \sigma\sqrt{ka}$

plane strain

thus $G_{pl.\,stress} \neq G_{plane\,strain}$ but $K_{I\,pl.\,stress} = K_{I\,pl.\,strain}$

(c) What value of k gives the exact solution to this problem?

C) How far is the solution to (a) from the exact solution to this problem?

$K_{I\,approx} = \sigma\sqrt{ka}$ with $k=1$

$K_{I\,exact} = \sigma\sqrt{\pi a}$

error in K_I.

$$\text{so error} = e = \frac{\sigma\sqrt{a} - \sigma\sqrt{\pi a}}{\sigma\sqrt{\pi a}} = \frac{1-\sqrt{\pi}}{\sqrt{\pi}} = \frac{1-1.772}{1.772} = -43.5\%$$

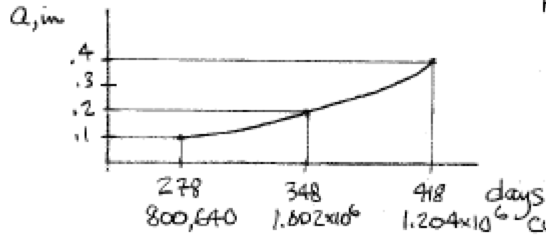
4.8 An offshore oilrig is subjected to a complete cycle of wave loading twice per minute. One of its components is a sheet of ¼” thick steel plate. During each cycle of wave loading, the plate is subject to a maximum average uniaxial tensile stress of 10 KSI and a minimum average uniaxial tensile stress of 2 KSI. In the first inspection, after 278 days of service, a pair of cracks, each of length 0.10 inch is discovered emanating in opposite directions (co-linearly) from a pre-existing ¼ inch – long slot. After another 70 days of service, a second inspection reveals that each of the cracks has grown to a length of 0.2 inch. Finally, after another 70 days of service, a third inspection shows that each of the cracks has grown to a length of 0.40 inch. The plate is known to have a plane strain fracture toughness of 50 ksi-in$^{1/2}$ and yield strength of 100 KSI.

a. Determine the material parameters, C and m, in the Paris Law from the available data.

1) $\Delta\sigma = 10^{ksi} - 2^{ksi} = 8^{ksi}$

$\Delta K = \Delta\sigma\sqrt{\pi a}$

$f = 2\,\frac{\text{cycles}}{\text{minute}} \times 60\,\frac{\text{min}}{\text{hr}} \times 24\,\frac{\text{hrs}}{\text{day}} = 2880\,\frac{\text{cycles}}{\text{day}}$



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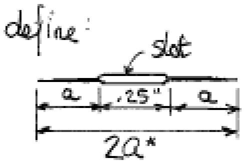

a) Paris Law: $\frac{da}{dN} = C\Delta K^n = C\left(\Delta\sigma\sqrt{\pi\left(a+\frac{1}{8}''\right)}\right)^n$

So $\displaystyle\int_{a_1}^{a_2} \frac{da}{C\left(\Delta\sigma\sqrt{\pi\left(a+\frac{1}{8}''\right)}\right)^n} = \int_{N_1}^{N_2} dN = N_2 - N_1$

So $\displaystyle\frac{1}{C(\Delta\sigma\sqrt{\pi})^n}\int_{a_1}^{a_2}\frac{da}{\left(a+\frac{1}{8}''\right)^{n/2}} = N_2 - N_1$

solving, for $n \neq +2$,

$$-\frac{2\left(a+\frac{1}{8}\right)^{\left(1-\frac{n}{2}\right)}}{(n-2)}\Bigg|_{a_1}^{a_2} = C(\Delta\sigma\pi)^n(N_2 - N_1)$$

setting $a_1 = 0,\ N_1 = 0$,

$$\frac{\left(a+\frac{1}{8}\right)^{(1-n/2)}}{(1-n/2)} - \frac{\left(\frac{1}{8}\right)^{(1-n/2)}}{(1-n/2)} = C(\Delta\sigma\pi)^n N$$

or

$$\left(a+\frac{1}{8}\right)^{(1-n/2)} - \left(\frac{1}{8}\right)^{(1-n/2)} = (1-n/2)\,C(\Delta\sigma\pi)^n N \qquad (*)$$

solving for C, n at 278 days and 418 days

278 days: $(0.1+.125)^{(1-n/2)} - .125^{(1-n/2)} = (1-n/2)\,C(8\pi)^n(800{,}640)$ (A)

418 days: $(0.4+.125)^{(1-n/2)} - .125^{(1-n/2)} = (1-n/2)\,C(8\pi)^n(1.204\times10^6)$ (B)

solving for C, n using Matlab:

$C = 7.518\times10^{-10}$, $n = 2.0$ (this is a trivial solution)

by trial and error,

$C = 1.1\times10^{-12}$, $n = 5$ approximately solves (A) & (B).

$$\boxed{\frac{da}{dN} = 1.1\times10^{-12}\Delta K^5 \qquad C \cong 1.1\times10^{-12},\ n \cong 5}$$

b. Predict the number of days (after the third inspection) to catastrophic failure of the plate.

b.) Predict the number of days (after the last inspection) to catastrophic failure of the plate.

$$K_{Ic} = 50^{ksi\sqrt{in}}$$

$$K_{Ic} = \sigma_{max}\sqrt{\pi a^*_{cr}} \; ; \; a^*_{cr} = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_{max}}\right)^2 = \frac{1}{\pi}\left(\frac{50^{ksi\sqrt{in}}}{10^{ksi}}\right)^2 = 7.96''$$

$$a_{cr} + .125'' = a^*_{cr} \; ; \; a_{cr} = a^*_{cr} - .125'' = 7.96'' - .125'' = 7.835''$$

from (*),

$$N = \frac{(a+\frac{1}{8})^{(1-n/2)} - (\frac{1}{8})^{(1-n/2)}}{(1-n/2)C(\Delta\sigma\pi)^n} = \frac{(7.96)^{-1.5} - .125^{-1.5}}{(-1.5)(1.1\times10^{-12})(8\pi)^5} = \frac{-22.58}{-1.655\times10^{-5}}$$

$$N = 1.365\times10^6 \text{ cycles.}$$

$$N_{after\,last\,inspection} = 1.365\times10^6 - 1.204\times10^6 = 1.607\times10^5 \text{ cycles.}$$

$$1.607\times10^5 \text{ cycles} \times \frac{1\text{ min}}{2\text{ cycles}} \times \frac{1\text{ hr.}}{60\text{ min}} \times \frac{1\text{ day}}{24\text{ hrs}} = 55.8 \text{ days.}$$

The plate will fail catastrophically 56 days after last inspection

c. How long will the cracks be just prior to catastrophic failure?

c.) The cracks will be 7.835" long just prior to catastrophic failure.