ASSIGNMENT 2

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ME 512 - Continuum Mechanics

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1 Construct a new basis:

(i) Pick three distinct nonzero numbers as v_i of a vector $\boldsymbol{v}$.

- $v_i \Rightarrow \{v\} = \begin{Bmatrix} 3 \\ 2 \\ 1 \end{Bmatrix}$
- $\boldsymbol{v} = v_i \boldsymbol{e_i} = 3\boldsymbol{e_1} + 2\boldsymbol{e_2} + 1\boldsymbol{e_3}$

(ii) Construct a second vector, $\boldsymbol{u}$, with two arbitrary components and the calculated third component so that $\boldsymbol{u} \perp \boldsymbol{v}$.

- Two vectors are perpendicular if the dot product between the two equals zero: $\boldsymbol{u} \bullet \boldsymbol{v} = 0$
- $u_i \Rightarrow \{u\} = \begin{Bmatrix} 2 \\ 2 \\ -10 \end{Bmatrix}$
- $\boldsymbol{u} = u_i \boldsymbol{e_i} = 2\boldsymbol{e_1} + 2\boldsymbol{e_2} - 10\boldsymbol{e_3}$

(iii) Construct a new orthonormal basis $\boldsymbol{E}$ as follows:

- $\boldsymbol{E_1} = \frac{\boldsymbol{v}}{||\boldsymbol{v}||}, \qquad \boldsymbol{E_2} = \frac{\boldsymbol{u}}{||\boldsymbol{u}||}, \qquad \boldsymbol{E_3} = \boldsymbol{E_1} \times \boldsymbol{E_2}$
- $\boldsymbol{E_1} = 0.802\boldsymbol{e_1} + 0.535\boldsymbol{e_2} + 0.267\boldsymbol{e_3}$
- $\boldsymbol{E_2} = 0.192\boldsymbol{e_1} + 0.192\boldsymbol{e_2} - 0.962\boldsymbol{e_3}$
- $\boldsymbol{E_2} = -0.566\boldsymbol{e_1} + 0.823\boldsymbol{e_2} + 0.051\boldsymbol{e_3}$

(iv) Construct a transformation matrix relating $\boldsymbol{E}$ to $\boldsymbol{e}$:

- $\left[{}^{E}a^{e}\right] = \begin{bmatrix} 0.802 & 0.535 & 0.267 \\ 0.192 & 0.192 & -0.962 \\ -0.566 & 0.823 & 0.051 \end{bmatrix}$

2 Show that the transformation matrix is orthogonal and the determinant equals +1

- The matrix is orthogonal because: $[{}^{E}a^{e}][{}^{E}a^{e}]^T = [I]$,
  where $[I]$ is the identity matrix: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- The determinant of the transformation matrix (i.e., $|[{}^{E}a^{e}]|$) equals 1.

3 Choose three nonzero distinct numbers as the components, w_i^E, of a vector $\boldsymbol{w}$, with respect to the basis $\boldsymbol{E_i}$. Find the components w_i^e.

- $w_i^E \Rightarrow \{w^E\} = \begin{Bmatrix} 3 \\ 4 \\ 5 \end{Bmatrix}, \qquad \boldsymbol{w} = w_i^E \boldsymbol{E_i}$
- $w_i^e = [{}^e a^E] w_i^E,$

  $[{}^e a^E] = [{}^E a^e]^{-1} = [{}^E a^e]^T = \begin{bmatrix} 0.802 & 0.192 & -0.566 \\ 0.535 & 0.192 & 0.823 \\ 0.267 & -0.962 & 0.051 \end{bmatrix}$

  $w_i^e \Rightarrow \{w^e\} = \begin{Bmatrix} 0.346 \\ 6.49 \\ -2.79 \end{Bmatrix}$

4 Show that the magnitude of $\boldsymbol{w}$ is the same with respect to both basis ($w_i^E w_i^E = w_A^e w_A^e$)

- The magnitude squared is an invariant, i.e., doesn't change between bases:
  $w_i^E w_i^E = 50.0$
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5 Pick components of Z_i^e, with respect to basis $\boldsymbol{e_i}$ and evaluate $\boldsymbol{w} \bullet \boldsymbol{Z}$

- $Z_i^e \Rightarrow \{Z^e\} = \begin{Bmatrix} 7 \\ 8 \\ 9 \end{Bmatrix}, \qquad \boldsymbol{Z} = Z_i \boldsymbol{e_i}$
- $w_i^e \Rightarrow \{w^e\} = \begin{Bmatrix} 0.346 \\ 6.49 \\ -2.79 \end{Bmatrix}, \qquad \boldsymbol{w} = w_i \boldsymbol{e_i}$
- $\boldsymbol{w} \bullet \boldsymbol{Z} = 29.2$