14.1 Many operations, such as machining, grinding, electroplating, and casehardening, may induce residual stresses in a material. Discuss, in general terms, the effect of such residual stresses on the fatigue life of the material.

In general, compressive residual stresses, no matter what the source, are beneficial for fatigue. That is why compressive stresses are intentionally introduced in surface layers of structures that will experience fatigue loading by techniques such as shot peening, etc. Surface treatments such as carburizing and nitriding of steels also result in compressive residual stresses in the surface layers. However, Ni or Cr plating of steels results in tensile stresses in the surface layer, and are, therefore, harmful for fatigue. Welding or joining can result in pores or other flaws, which reduce the fatigue strength.

14.2 A steel has the following properties:

Young's modulus E = 210 GPa,
Monotonic fracture stress σ_f = 2.0 GPa,
Monotonic strain at fracture ε_f = 0.6,
Exponent b (cyclic) = 0.09; c (cyclic) = 0.06.

Compute the total strain that a bar of this steel will be subjected to under cyclic straining before failing at 1,500 cycles.

The total strain ($\Delta\varepsilon_t$) is given by the following equation (see p. 718 in the text):

$$\frac{\Delta\varepsilon_t}{2} = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f\,(2N_f)^c$$

where $b = 0.09$

and $c = 0.06$

Therefore,

$$\frac{\Delta\varepsilon_t}{2} = \frac{2}{210}(2\times1500)^{0.09} + (0.6)(2\times1500)^{0.06}$$

$$= 0.019 + 0.124 = 0.143$$

$$\Delta\varepsilon_t = 0.286$$

14.3 The low-cycle fatigue behavior of a material can be represented by

$$\sigma_{\mathrm{L}} = \sigma_{\mathrm{UTS}}\ N_f^{-0.5}$$

where $\sigma_{\mathrm{L}}$is the endurance limit, σ_{UTS} is the ultimate tensile strength, and N_f is the number of cycles to failure. If σ_{UTS} for this material is 500 MPa, find its endurance limit. A sample of the material is subjected to block loading consisting of 40, 30, 20, and 10 % of fatigue life at σ_{L}, $1.10\sigma_{\mathrm{L}}$, $1.2\sigma_{\mathrm{L}}$, and $1.3\sigma_{\mathrm{L}}$, respectively. Use the Palmgren--Miner relationship to estimate the fatigue life of the sample under this block loading.

We can write the Palmgren-Miner relationship for the given block loading as

$$0.4\ \sigma_{\mathrm{L}} + 0.3 \text{ x } 1.1\ \sigma_{\mathrm{L}} + 0.2 \text{ x } 1.2\ \sigma_{\mathrm{L}} + 0.1 \text{ x } 1.3\ \sigma_{\mathrm{L}} = 1$$

$$\sigma_{\mathrm{L}}\ (0.4 + 0.33 + 0.24 + 0.13) = 1$$

$$1.1\ \sigma_{\mathrm{L}} = 1$$

$$\sigma_{\mathrm{L}} = \sigma_{\mathrm{UTS}} N_f^{-0.5} = 1/1.1$$

Or, $500 \text{ x } 1.1 = 550 = N_f^{0.5}$

Fatigue life, $N_f = 3\ 10^5$ cycles

Endurance limit, $\sigma_{\mathrm{L}} = 500\ (3\ 10^5)^{-0.5} = 288$ MPa

14.4 A microalloyed steel was subjected to two fatigue tests at ± 400 MPa and ± 250 MPa. Failure occurred after 2 x 10^4 and 1.2 x 10^6 cycles, respectively, at these two stress levels. Making appropriate assumptions, estimate the fatigue life at ± 300 MPa of a part made from this steel that has already suffered 2.5 x 10^4 cycles at ± 350 MPa.

$$\Delta\sigma(N_f)^a = c$$

$$800(2x10^4)^a = 500(1.2x10^6)^a$$

$$\frac{800}{500} = 1.6 = \left(\frac{1.2\times10^6}{2\times10^4)}\right)^a = (60)^a$$

$$a = 0.115$$

$$c = 800(2\times10^4)^{0.115} MPa$$

$$= 2498 \ MPa$$

$$At \quad \pm 350 \ MPa$$

$$N_{f1} = \left(\frac{c}{\Delta\sigma}\right)^{1/a} = \left(\frac{2498}{700}\right)^{1/0.115}$$

$$= (3.57)^{8.7} = 6.4\times10^4 \ cycles$$

$$For \quad 2.4\times10^4 \, cycles, \quad \frac{N_1}{N_{f1}} = \frac{2.5\times10^4}{6.4\times10^4} = 0.39$$

$$At \pm 300\, MPa$$

$$N_{f2} = \left(\frac{c}{\Delta\sigma}\right)^{1/a} = \left(\frac{2498}{600}\right)^{1/0.115}$$

$$= (4.16)^{8.7} = 2.45 \times 10^5\ cycles$$

$$From\ Palmgren - Miner's\ Rule$$

$$\frac{N_1}{N_{f1}} + \frac{N_2}{N_{f2}} = 1$$

$$\frac{N_2}{N_{f2}} = 1 - \frac{N_1}{N_{f1}} = 1 - 0.39 = 0.61$$

$$Therefore,\ N_2 = 0.61\ N_{f2}$$

$$= 0.61 \times 2.45 \times 10^5$$

$$= 1.49 \times 10^5\ cycles$$

14.5 The curve of crack growth rate da/dN vs. cyclic stress intensity ΔK for a material in the Paris regime is shown in Figure Ex 14.5 (p. 759 in the text). Determine the parameters C and m for this material.

We first calculate the slope of the like, m by taking two points on the curve:

Point 1: ΔK = 30 $MPa\sqrt{m}$, da/dN = 0.1 mm/cycle

Point 2: ΔK = 20 MPa $\sqrt{m}$, da/dN= 10^{-2} mm/cycle

$$m = \frac{\log\ 10^{-1} - \log 10^{-2}}{\log\ 30 - \log\ 20} = 5.68$$ m = 5.68

Then, find C by applying the Paris relation to one of the points:

$$C = \frac{da/dN}{(\Delta K)^m} = \frac{0.1}{(30)^{5.68}} = 4.07 \times 10^{-10}$$ C = 4.07×-10-10

14.6 A steel has the following properties:

Yield Stress σ_y= 700 MPa,
Fracture Toughness, K_{Ic} = 165 MPa $\sqrt{m}$

A plate of this steel containing a single edge crack was tested in fatigue under $\Delta\sigma$ = 140 MPa, R = 0.5, and a_o = 2mm. It was observed experimentally that fatigue crack propagation in the steel could be described by the Paris-type relationship.

$$\frac{da}{dN}(m/cycle) = 0.66 \cdot 10^{-8}(\Delta K)^{2.25}$$
$$where\, \Delta K\ is\ measured\ in\, MPa\sqrt{m}.$$

(a) What is the critical crack size a_c at σ_{max}?
(b) Compute the fatigue life of the steel.

$$R = 0.5 = \frac{\sigma_{\min}}{\sigma_{\max}} \to \sigma_{\min} = 0.5.\sigma_{\max}$$

$$\Delta\sigma = 140 MPa = \sigma_{\max} - \sigma_{\min} = 0.5\sigma_{\max} = o\,\sigma_{\max} = 280\, MPa\, and\, \sigma_{\min} = 140\, MPa$$

For a single edge crack

$$K_{Ic} = Y\sigma_{\max}\sqrt{\pi a_c} \qquad where\, Y = 1.12\ for\ small\ crack$$

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{Y \cdot \sigma_{\max}}\right)2 = \frac{1}{\pi} \cdot \left(\frac{165}{1.12 \times 280}\right)2 = 8.81 \times 10^{-2} m \qquad a_c = 88.1\ mm$$

$$\frac{da}{dN} = 0.66 \cdot 10 - 5(\Delta K)2.25 \quad where\ \Delta K = Y\Delta\sigma\sqrt{\pi a}$$

$$\frac{da}{dN} = 0.66 \cdot 10^{-8} \cdot (1.12)^{2.25} \cdot (140)^{2.25} \cdot \pi^{1.125} \cdot a^{1.125}$$

$$a^{-125} \cdot da = 2.0815 \cdot 10^{-3}\ dn$$

$$\int_{a}^{a} a^{-1.125} da = 2.0815 \cdot 10^{-3} \int_{D}^{Nf} dn$$

$$-\left[\frac{a^{-0.125}}{0.125}\right] \int_{a_o}^{a_c} = 2.0818 \cdot 10^{-3} \cdot N_f \ \ with\ a_o = 2\,mm\, and\ a_c = 88.1 mm$$

$$N_f = -\frac{\left[(88 \cdot 1 \cdot 10^{-3})^{-0.125} - (2 - 10^{-3})^{-0.125}\right]}{0.125 \cdot 2.0815 \cdot 10^{-3}} = 3 \cdot 150\, cycles \qquad N_f = 3150\, cycles$$

14.11 (Design Problem) Estimate the life of a hip implant of 304L stainless steel if it contains initial flaws with length 2a = 200 nm and a height 2a = 100 μm. Assume that the force applied on the artificial hip is,

walking: 3W and running: 7W

where W is the weight of the person. The fatigue response of 304L can be represented by

$$\frac{da}{dN}(mm/cycle) = 5.5\times10^{-9}\ \left(\Delta K\left(MPa\sqrt{m}\right)\right)^3$$

Make and list all necessary assumptions.

We make the following assumptions

1) The mass of the person is m = 80 kg.
2) The flaw is oriented most unfavorably.
3) The flaw is located where the normal stress due to bending is maximum.
4) The flaw is located close to the surface of the hip implant.
5) The load P is vertical.
6) Critical stress intensity factor $K_{Ic} = 110 MPa\sqrt{m}$

(a) The person is assumed to walk 3 hour per day.

Elliptical flaw most unfavorably oriented, K_I (p. 347)

$K_I = Y\sigma\sqrt{\pi,a}$ where Y is given by

$$\frac{\left(\sin^2\theta + \frac{a^2}{c^2}\cos^2\theta\right)^{1/4}}{\frac{3\pi}{8}+\frac{\pi}{8}\frac{a^2}{c^2}}$$

K_I is maximum when Y is maximum ⇒θ = 90°⇒Y= 0.784

Let us calculate the critical crack size, a_c

$$K_{Ic} = Y\sigma_{\max}\sqrt{\pi a_c} \qquad a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{Y\sigma_{\max}}\right)^2$$

Compute the maximum normal stress due to bending.

$$\Rightarrow \qquad I = \frac{\pi\, r^4}{4}$$

$$\sigma_b = \frac{12mgx}{\pi^3} = \frac{12 \times 80 \times 9 \times 81 \times \left(20 \times 10^{-3}\right)}{\pi\left(8 \cdot 10^{-3}\right)^3} = 117MPa$$

$\sigma_b = \frac{M \cdot r}{I}$ where $M = P \cdot x = 3 \cdot mgx$ and

Compression stress due to load P,

$$\sigma_c = \frac{P \cdot \cos 20°}{A} = \frac{P \cdot \cos 20°}{r\pi^2} = \frac{3 \cdot 80 \cdot 9 \cdot 81 \cdot \cos 20°}{\pi\left(8 \cdot 10^{-3}\right)^2} = 11MPa$$

Resultant stress,
$\sigma_r = \sigma_b - \sigma_c = 117 – 11 = 106$ MPa

The parameters of the fatigue cycle are,
$\sigma_{max} = \sigma_r = 106$ MPa
$\sigma_{min} = 0$
R = 0
$\Delta\sigma = 106$ MPa

We can calculate now the critical flow size numerically,

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{Y \cdot \sigma_{\max}}\right)^2 = \frac{1}{\pi}\left(\frac{110}{0.784 \cdot 106}\right)^2 \cdot 0.557m$$

The critical size is larger than the dimensions of the hip implant!

The hip will fail when the crack reaches the surface → a_c = 16mm

Let us compute the fatigue life,

$$\frac{da}{dN}\left(m/cycle\right)=5.5\times10^{-12}\left(\Delta K\left(MPa\sqrt{m}\right)\right)^{3} \; with \quad \Delta K = Y\Delta\sigma\sqrt{a\pi}$$

$$\frac{da}{dN}=5.5\times10^{-12}\times\left(Y\Delta\sigma\sqrt{a\pi}\right)^{3}$$

$$\frac{da}{dN}=5.5\times10^{-12}\times Y^{3}\times\Delta\sigma^{3}\times\pi^{1.5}\times a^{1.5}$$

$$a^{-1.5}da=5.5\times10^{-12}\times\left(0.784\right)^{3}\times\left(106\right)^{3}\cdot\pi^{1.5}\cdot dN$$

$$\int_{a_o}^{a_c} a^{-1.5}da=1.759\times10^{-5}\int_{\circ}^{n_f} dN \quad with. \quad a_o=50\mu m \quad and \quad a_c=16mm$$

$$\left[-\frac{a^{-0.5}}{0.5}\right]_{a_o}^{a_c}=1.759\times10^{-5}\cdot N_f$$

$$\Rightarrow N_f=\frac{a_o}{0.5\times1.759\times10^{-5}}=\frac{\left(50\times10^{-6}\right)^{-0.5}-\left(16\times10^{-8}\right)^{-0.5}}{0.5\times1.759\times10^{-5}}=15{,}178{,}346 \quad cycles$$

Let us assume that we have 1 cycle/s when the person is walking. The person walks 3 hours a day which is 3 x 3600 = 10, 800 cycles/day.

The lifetime will then be,

$$\frac{N_f}{10{,}800}=1405 \quad days \quad or \quad 3.85 \quad years$$

(b) The person is assumed to walk 3 hours per day and jog 20 minutes per day.
When the person jogs, the resultant stress is 7/3 times larger than when she walks.

Thus, ΔK will be 7/3 times larger and the life time $(7/3)^3$ times smaller.

The final effect is the same than for a person walking

3 h + $(7/3)^3$(20 min x 3) = 15.70 h per day (assuming we have 3 cycles/s when the person jogs)

The lifetime in this case,

$$\frac{N_f}{3600\times15\times70}=268 \quad days \quad or \quad 0.74 \quad years$$

The lifetime of the hip implant is decreased by a factor more than 5 if the person, in addition to walking 3 h per day, runs 20 min per day.

14.12 Assuming that fatigue failures are initiated at the "weakest link," we may use the Weibull frequency distribution function to represent the fatigue lives of a group of specimens tested under identical conditions. We have

$$f(N) = \frac{b}{N_a - N_o}\left(\frac{N - N_o}{N_a - N_o}\right)^{b-1} \exp\left[-\left(\frac{N - N_o}{N_a - N_o}\right)^{b}\right]$$

where N is the specimen's fatigue life, N_o is the minimum life ≥ 0, N_a is the characteristic life at 36.8 % survival of the population (36.8 % = 1/e , where e = 2.718) and b is the shape parameter of the Weibull distribution curve. Letting $x = (N - N_o)/(N_a - N_o)$, plot curves f(N) versus x for b = 1,2,and 3.

$$f(N) = \frac{b}{N_a - N_o}\left(\frac{N - N_o}{N_a - N_o}\right)^{b-1} \exp\left[-\left(\frac{N - N_o}{N_a - N_o}\right)^{b}\right]$$

Let $\quad x = (N - N_o)/(N_a - N_o)$, we have

$$f(N) = \frac{b}{N_a - N_o}(x)^{b-1} \exp\left[-x^{b}\right]$$

$$f(N) = \frac{b}{N_a - N_o}(x)^{b-1} \frac{1}{\exp\left[x^{b}\right]}$$

For b = 1

$$f(N) = \frac{1}{N_a - N_o}\frac{1}{\exp[x]}$$

-a simple exponential distribution function

For b = 2

$$f(N) = \frac{2}{N_a - N_o} \frac{x}{\exp[x^2]}$$

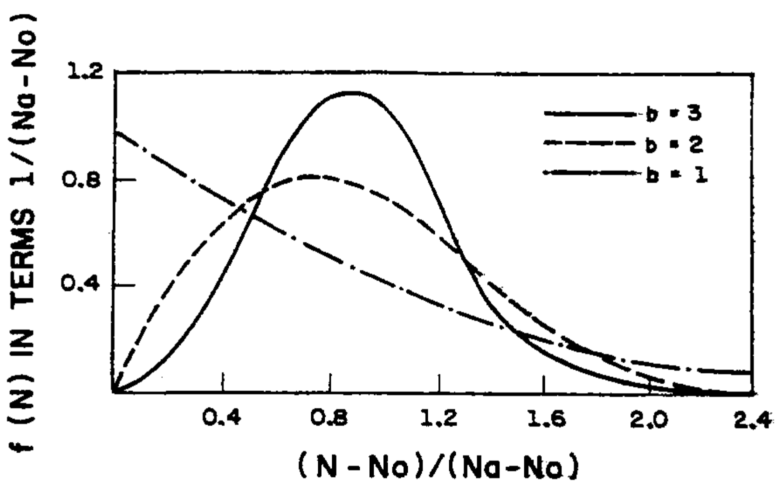
- this is called a Rayleigh distribution function and is used in acoustic fatigue analysis.

For b = 3

$$f(N) = \frac{3}{N_a - N_o} \frac{x^2}{\exp[x^3]}$$

- this is close to the normal distribution function. In fact b = 3.57 gives a better approximation to normal distribution.

Figure below shows the plots for b = 1, 2, and 3.



14.13 Fatigue data are, generally, analyzed cumulatively to determine the survival percentage. The Weibull cumulative function for the fraction of population failing at N is integration of the expression for f(N) in the preceding exercise. Show this function is

$$F(N) = 1 - \exp\left[-\left(\frac{N - N_o}{N_a - N_o}\right)^b\right]$$

Missing text of prob.???

$$F(N) = 1 - \exp\left[-\left(\frac{N - N_o}{N_a - N_o}\right)^b\right]$$

$$1 - F(N) = \exp\left[-\left(\frac{N - N_o}{N_a - N_o}\right)^b\right]$$

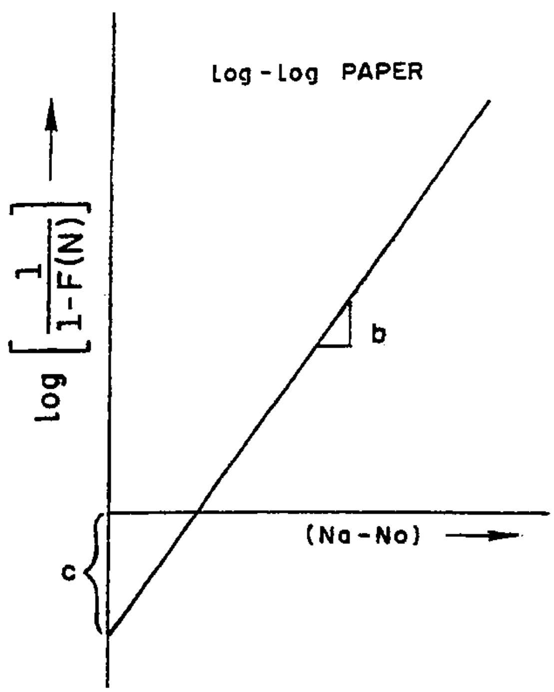
$$\frac{1}{1 - F(N)} = \exp\left[\left(\frac{N - N_o}{N_a - N_o}\right)^b\right]$$

Taking logarithms twice, we get

$$\log\log\frac{1}{1 - F(N)} = b\log\left[\left(\frac{N - N_o}{N_a - N_o}\right)^b\right] + \log\log e$$

$$\log\log\frac{1}{1-F(N)} = b\log(N-N_o)(N-N_o)$$
$$= b\log(N-N_o)+c$$
$$where \quad c = -b\log(N-N_o)-0.36$$

On a log-log plot, this equation will give a straight line (y = mx + c) as indicated in the Fig. below, from which we can find graphically various parameters as indicated.



14.14 (Design Problem) One if the worst single aircraft accidents in history resulted in the loss of 520 lives. It was produced by the growth of a fatigue crack in the back of the bulkhead of a Boeing 747 plane. (see Figure Ex14.14). The fatigue fracture was caused by a repair that replaced a double row of rivets by a single row in certain places. The atmospheric pressure decreases by 12 Pa for every meter increase in altitude.

(a) Calculate the stress cycle to which the pressurized cabin and bulkhead were subjected in
each takeoff-landing sequence of the plane.
(b) Establish the critical crack length for which catastrophic growth would occur.
(c) Assuming the fatigue failure started at one of the rivet holes (which had a diameter of 12 mm) and that it propagated through subsequent holes. Calculate the number of cycles necessary to bring down the "big bird," given the following data:

Paris relationship constants, $C = 5 \times 10^{-8}$ and $m = 3.6$.

$$\sigma_y = 310\ MPa$$

$$\sigma_{UTS} = 345\ MPa$$

(a) Treat the fuselage as thin-walled pressure vessel. The hoop stress is given by

$$\sigma = \frac{pr}{t}$$

$$\Delta\sigma = stress\ cycle = \sigma_{max} - \sigma_{min} = \frac{\Delta p \times r}{t} = \frac{(p_{max} - p_{\min}) \times r}{t}$$

$$r = inner\ radius = 2.316\ \text{m}$$

$$t = wall\ thickness = 0.009\ m$$

$$p_{min} = 1\ atm = 101{,}325\ Pa$$

$$p_{max} = (101{,}3215\ Pa) + \left(12\frac{Pa}{m} \times 7{,}200\ m\right) = 187{,}725\ Pa$$

$$\boxed{\Delta\sigma = \frac{(187{,}725\ Pa - 101{,}325\ Pa) \times (2.316\ m)}{(0.009\ m)} = 22.23\ MPa}$$

(b) Treat the problem to be single edge notch crack.

$$K_{Ic} = Y\sigma_{max}\sqrt{\pi a}$$

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{Y\sigma_{max}}\right)^2$$

$$K_{Ic} = 37\ MPa\sqrt{m} \qquad [1]$$

$$Y = 1.12\ \ (for\ a\ single\ edge\ crack)$$

$$\sigma_{max} = 48.31\ MPa$$

$$\boxed{a_c = \frac{1}{\pi}\left(\frac{37\ MPa\sqrt{m}}{(1.12)*(48.31\ MPA)}\right)^2 = 14.88\ cm}$$

(c)

$$\frac{da}{dN} = C\Delta K^m$$

$$C = 5 \times 10^{-8}$$

$$m = 3.6$$

$$\Delta K = 1.12\Delta\sigma\sqrt{\pi a}$$

$$\frac{da}{dN} = 5 * 10^{-8}\left[1.12\Delta\sigma\sqrt{\pi a}\right]^{3.6}$$

$$a^{-1.8}da = 5 * 10^{-8} * (1.12)^{3.6} * (22.23\ MPa)^{3.6} * (\pi)^{1.8} dN$$

$$\int_{a_0}^{a_c} a^{-1.8} da = 0.0415 \int_0^{N_f} dN$$

$$N_f * 0.0415 = -\left[\frac{a^{-0.8}}{0.8}\right]_{12}^{148.8}$$

$$N_f * 0.0415 = 37.307$$

$$\boxed{N_f = 899\ cycles}$$

14.17 The low cyclic fatigue of a metallic alloy can be described by the Coffin-Manson expression relating the number of cycles to failure, N_f, to plastic strain, $\Delta\epsilon_{pl}$ as follows:

$$N_f^{0.5}\ \Delta\epsilon_{pl} = 0.4$$

Compute the number of cycles to failure if the alloy fails at a plastic strain of 5×10^{-3}.

Coffin-Manson relationship

$$N_f^{0.5}\ \Delta\epsilon_{pl} = 0.4$$

$$N_f^{0.5} = 0.4/\Delta\epsilon_{pl} = 0.4/(5\text{x}10^{-3})$$

$$N_f = 0.4/(5\text{x}10^{-3})^{1/0.5} = 6.4 \times 10^3 \text{ cycles.}$$

14.19 A 2024-T6 aluminum alloy used for an aircraft was tested in Wöhler-type machine rotating at 400 rpm (sinusoidal stress variation, mean stress equal to zero). The following results were obtained:

Stress Range = 310 MPa; N = 10^4 cycles

Stress Range = 230 MPa; N = 10^7 cycles

Predict the life of the aircraft if it is in the air 16 h per day and if it is subjected to the stress range of 180 MPa at the same frequency. The material obeys Basquin law.

$\Delta \, Nf^a = c$

Apply 2 conditions, (310 MPa) $(10^4 \text{ cycles})^a$ = (230 MPa) $(10^4$ cycles)

$$\frac{310 \text{ MPa}}{230 \text{ MPa}} = \frac{(10^7 \text{ cycles})^a}{(10^4 \text{ cycles})^a}$$

$1.35 = 10^{3a}$

$\log(1.35) = 3a$

$a = 4.34 \times 10^{-2}$

$c = (310 \text{ MPa}) (10^4 \text{ cycles})^{4.34 \times 10\text{-}2}$

$c = 4.62 \times 10^{-2}$ MPa

If $\Delta\sigma = 180$ MPa » $Nf^a = \frac{C}{\Delta\sigma} = \frac{4.62 \times 10^{-2}}{180 \text{ MPa}} = 2.57$

$Nf = (2.57)^{\frac{1}{4.34 \times 10 \text{-}2}}$

$Nf = 2.79 \times 10^9$ cycles.

$$Stress\ Range = 310\ MPa, N = 10^4$$
$$Stress\ range = 230\ MPa, N = 10^7$$
$$\Delta\sigma\left(N_f\right)^a = c$$

Equating to solve for a

$$310MPa\ (10^4)^a = 230MPa\ (10^7)^a$$
$$\frac{310MPa}{230MPa} = \left(\frac{10^7}{10^4}\right)^a$$
$$1.347 = 1000^a$$
$$\log(1.347) = a\ x \log(1000)$$
$$a = 0.043$$

Solving for c

$$310MPa(10^4)^{0.043} = c$$
$$c = 460.6MPa$$
$$180MPa\left(N_f\right)^{0.043} = 460.6MPa$$
$$N_f = 3.01X10^9\ cycles$$
$$400rpm\ x\ \frac{60min}{h}\ x\ \frac{16h}{1\ day} = 384000\frac{cycles}{day}$$
$$Predicted\ life\ to\ failure = \frac{3.01X10^9 cycles}{384000\ cycles/day} = 7838days = 21.47years$$

14.20 Explain the effect on fatigue life for the following design and environmental factors :

1) A high polish surface finish
2) A rivet hole
3) Increasing the mean stress, but keeping the range constant.
4) A corrosive atmosphere

1) A high polish surface finish-will reduce crack formation and crack propagation rate , in that sense, can minimize the risk of the fatigue failure , increase fatigue life.

2) Introducing a rivet hole will increase the stress concentration around the hole area; therefore lead to a decreased fatigue life.

3) Mean stress, $\sigma_m = (\sigma_{max} + \sigma_{min})/2$. Increasing the mean stress, but keeping the range constant, will make the material more vulnerable to fatigue, i.e., increasing the mean stress σ_m will decrease the fatigue life.

4) A corrosive atmosphere will increase the probability of causing nucleation and the resultant crack propagation, i.e., a decreased fatigue life.

14.15 Calculate the stress cycle to which the pressurized cabin and bulkhead were subjected in each takeoff-landing sequence.

$$\Delta\sigma = \frac{\Delta p \times r}{z \times t} = \frac{7200m \times 12Pu/m \times (4.6512)}{2 \times 9 \times 10^{-3}} = 44.2\ MPa$$

(a) Establish the critical crack length for which catastrophic growth would occur. Assume a single edge crack: $K_I = Y\sigma\sqrt{\pi a}$ $\quad$ *with* $Y = 1.12$

$$\sigma_{\max} = 44.2MPa, \quad \sigma_{\min} = 0 \quad R = 0$$

Let us assume (p. 367)**:** $K_{Ic} = 34MPa\sqrt{m}$

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{r \times \sigma_{\max}}\right)^2 = \frac{1}{\pi}\left(\frac{34}{1.12 \times 112}\right)^2 = 2.36\,m$$

(b) Assuming that fatigue failure started at one of the rivet holes (which had a diameter of 12 mm) and that it propagated through subsequent holes, calculate the number of cycles necessary to bring down the "big bird", given the following data:

$$\frac{da}{dn} = 5\times10^{-8}(\Delta K)^{3.6} \quad with\ \Delta K = Y\Delta\sigma\sqrt{\pi\ a}$$

$$\frac{da}{dn} = 5\times10^{=8}\times(1.12)^{3.6}\times(11.2)^{3.6}\times\pi^{1.8}\times a^{1.8}$$

$$a^{1.8}da = 3.53\times10^{-3}\,dN$$

$$\int_{a_o}^{a_c} a^{-1.5}da = 3.53\times10^{-3}\int_{\circ}^{N_f} dN$$

$$\left[-\frac{a^{-0.8}}{0.8}\right]_{a_o}^{a_c} = 3.53\times10^{-3}\times N_f \quad with\ a_c = 2.36\ m,\ a_o = 12\ mm\ (assumption)$$

$$N_f = -\frac{\left[(2.36)^{-0.8} - \left(12\times10^{-3}\right)^{-0.8}\right]}{0.8\times3.53\times10^{-3}} = 42000\ cycles$$