3.1 A polycrystalline metal has a plastic stress-strain curve that obeys Hollomon's equation.

$$\sigma = K\varepsilon^n$$

Determine n, knowing that the flow stresses of this material at 2% and 10% plastic deformation (offset) are equal to 175 and 185 MPa, respectively.

$\sigma = K\varepsilon^n$

At 2% strain (0.02), σ = 175 MPa
At 10% strain (0.1) σ = 185 MPa

$\log(\sigma) = \log(K) + n \log(\varepsilon)$

$\log(175) = \log(K) + n \log(0.02)$
$\log(185) = \log(K) + n \log(0.1)$

$\log(175) / \log(185) = n \log(0.02) / \log(0.1)$

$0.9459 = 0.2n$
$\log(0.9459) = n \log(0.2)$
$n = 0.0345$

3.2 You are traveling in an airplane. The engineer who designed it is, casually, on your side. He tells you that the wings were designed using von Mises' criterion. Would you feel safer if he had told you that Tresca's criterion had been used? Why?

Yes. The Tresca criterion always predicts yielding at or before the von Mises criterion, so it is more conservative. One would feel safer with a more conservative wing design.

3.3 A material is under a state of stress such that $\sigma_1 = 3\sigma_2 = 2\sigma_3$. It starts to flow when $\sigma_2 = 140$ MPa.

(a) What is the flow stress in uniaxial tension?
(b) If the material is used under conditions in which $\sigma_1 = -\sigma_3$ *and* $\sigma_2 = 0$, which value of σ_3 will it flow, according to Tresca's and von Mises' criteria?

$$\sigma_1 = 3\sigma_2 = 2\sigma_3 \; and \; \sigma_2 = 140 \; MPa \;\; so \; \sigma_1 = 420; \sigma_2 = 140; \sigma_3 = 210$$

a) The problem does not say whether to use Tresca or von Mises to get σ_o. We shall try both.

<u>Tresca</u> $\sigma_{max} - \sigma_{min} = 420 - 140 = 280 MPa = \sigma_o$

<u>von Mises</u> $\frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2) + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}$

$$= \frac{1}{\sqrt{2}}\sqrt{(280)^2 + (210)^2 + (70)^2} = 252 MPa = \sigma_o$$

b) We need to pick a value of σ_o from part a. Let us use σ_o= 280 MPa (We could also choose 252 MPa)

$$\sigma_1 = \sigma_3; \sigma_2 = 0 \; and \; assume \; \sigma_3 > 0$$

<u>Tresca:</u> $\sigma_{max} - \sigma_{min} = \sigma_3 - (-\sigma_3) = \sigma_0 = 280 \; so \; \sigma_3 = 140 \; MPa$

<u>von Mises</u> $\frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}$

$$= \frac{1}{\sqrt{2}}\sqrt{\sigma 3^2 + 4\sigma 3^2 + \sigma 3^2} = \sqrt{3}\sigma_3 = 280$$

so $\sigma_3 = 280/3 = 162 \; MPa$

3.4 A steel with a yield stress of 300 MPa is tested under a state of stress where $\sigma_2 = \sigma_1 / 2 \; and \; \sigma_3 = 0$. What is the stress at which yielding occurs if it is assumed that:

(a) The maximum-normal-stress criterion holds?
(b) The maximum-shear-stress criterion holds?
(c) The distortion-energy criterion holds?

According to the maximum-normal-stress criterion

$$\sigma_1 = \sigma_y = 300 \; MPa$$
$$\sigma_2 = \sigma_1 / 2 = 150 \; MPa$$
$$and \; \sigma_3 = 0$$

According to the maximum-shear-stress criterion

$$\sigma_1 - \sigma_3 = \sigma_y$$
$$\sigma_1 = \sigma_3 + \sigma_y = 300 \; MPa$$
$$\sigma_2 = \sigma_1 / 2 = 150 \; MPa$$
$$and \; \sigma_3 = 0$$

According to the distortion-energy criterion

$$\frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2} = \sigma_y$$
$$\frac{1}{\sqrt{2}} \sqrt{(\sigma_1 / 2)^2 + (\sigma_1 / 2)^2 + (\sigma_1)^2} = \sigma_y$$

$$\sigma_1 = \frac{2}{\sqrt{3}} \sigma_y = 346.4 \; MPa$$
$$\sigma_2 = \sigma_1 / 2 = 173.2 \, MPa$$
$$and \; \sigma_3 = 0$$

3.5 Determine the maximum pressure that a cylindrical gas reservoir can withstand, using the three flow criteria. Use the following information:

Material: AISI 304 stainless steel- hot finished and annealed, $\sigma_y = 205\ MPa$
Thickness: 25 mm
Diameter: 500 mm
Length: 1 m
Hint: Determine the longitudinal and circumferential (hoop) stresses by the method of sections.

<u>Use only Tresca and von Mises criteria</u>

For a cylindrical pressure vessel with internal pressure,

Hoop Stress

$$\sigma_1 = \frac{pD}{2t}$$

Longitudinal Stress,

$$\sigma_2 = \frac{pD}{4t} = \frac{1}{2}\sigma_1$$

$$\sigma_3 = 0\ (plane\ stress)$$

$$So \quad \sigma_1 = \frac{500}{2(25)}p = 10p$$

$$\sigma_2 = 5p$$

$$and\ \sigma_3 = 0$$

Tresca

$$\sigma_{\max} - \sigma_{\min} = 10p - 0 = \sigma_o = 205 MPa$$

$$and \quad p = 20.5\ MPa$$

von Mises

$$\frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}$$

$$=\frac{1}{\sqrt{2}}\sqrt{25p^2+100p^2+25p^2}=8.66p=205\ MPa$$

Therefore, $p = 23.7\ MPa$

3.6 Determine the value of Poisson's ratio for an isotropic cube being plastically compressed between two parallel plates.

Initial Volume = $V_i = 1$

Final Volume = $V_f = (1+\varepsilon_2)(1+\varepsilon_1)(1+\varepsilon_3) = 1+\varepsilon_1+\varepsilon_2+\varepsilon_3$

Isotropic material: $\varepsilon_1 = \varepsilon_2$ (transverse strains are equal)

$$\Delta V = V_f - V_i = 0$$

$$1+\varepsilon_1+\varepsilon_2+\varepsilon_3-1=0$$

$$\varepsilon_1+\varepsilon_2=-\varepsilon_3$$

$$2\varepsilon_1=-\varepsilon_3$$

$$\boxed{\nu=\frac{\varepsilon_1}{\varepsilon_3}=\frac{1}{2}}$$

3.7 A low - carbon -steel cylinder, having a height of 50 mm and a diameter of 100 mm, is being forged (upset) at 1,200 °C and a velocity of 1 m/s, until its height is equal to 15 mm. Assuming an efficiency of 60%, and assuming that the flow stress at the specified strain rate is 80 MPa, determine the power required to forge the specimen.

$$\Delta Power = \frac{\bar{p} \cdot A \cdot \Delta h}{\Delta t}$$

$$where \quad \bar{p} = \frac{\sigma_o}{2}^2 \left(\frac{h}{\mu\, a}\right)^2 \left(e^{2\mu\, a/h} - \frac{2\mu\, a}{h} - 1\right)\pi\, a^2 \cdot dh$$

$$\bar{p} = \frac{1}{\Delta t}\frac{\sigma_o}{2}\int_{0.05}^{0.015} \left(\frac{h}{\mu\, a}\right)^2 \left(e^{2\mu\, a/h} - \frac{2\mu\, a}{h} - 1\right)\pi\, a^2 \cdot dh$$

$$Volume\ of\ cylinder = \pi\, a^2 h = \pi\,(905)^2 \cdot 905$$

$$a = \sqrt{\frac{905}{h}}$$

By integration using Simpson's Rule

$$power = \frac{1}{\Delta t} \cdot \frac{\sigma_u}{2}\left(6.935 \times 10^{-3}\right)$$

$$\Delta t = \frac{0.05 - 0.015(m)}{1(m/s)} = 0.035\ s$$

$$\sigma_o = 80\ MPa = 80 \times 10^6\ N/m^2$$

$$\therefore\ power = 7.926 \times 10^6\ J/s$$

Assuming an efficiency of 60 %, the required power = 7.926 x 10^6/0.6 =13.2 x 10^6 J/s

3.8 Obtain the work-hardening exponent *n* using Considere's criterion for the curve of Example 3.4.

The work-hardening coefficient n, is numerically equal to the true uniform strain, ε_u

$$\varepsilon_u = \ln\frac{A_0}{A_u} = \ln(1+\varepsilon_e)$$

For uniform strain, make a vertical line from the UTS point parallel to the stress axis.

$$\Delta l_u \approx 1.5mm$$

$$\varepsilon_e = \frac{1.5}{20} = .075$$

$$\varepsilon_u = \ln(1+.075)$$

$$\boxed{\varepsilon_u = .0723}$$

3.9 The stress-strain curve of a 70 –30 brass is described by the equation,

$$\sigma = 600\varepsilon_p^{0.35} \ MPa$$

until the onset of plastic instability.

(a) Find the 0.2% offset yield stress
(b) Applying Considere's criterion, find the real and engineering stress at the onset of necking.

(a) $\varepsilon = 0.002$ $\sigma_{0.2} = 600(0.002)^{0.35}$ $\sigma_{0.2} = 68.2$ MPa

(b) Considere's criterion says that at the onset of necking, the true uniform strain is numerically equal to the work- hardening coefficient, i.e. $\varepsilon_u = n$
where n is the exponent in the Holloman's equation m, $\sigma = K\varepsilon^n$
In the present case, $\varepsilon_t = \varepsilon_u = n$

Real stress : $\sigma_t = 600(0.35)^{0.35} = 415.5$ MPa

Engineering stress: $\sigma_t = (1 + \varepsilon_e)\sigma_e$

$\varepsilon_t = \ell n(1 + \varepsilon_e)$ $\varepsilon_e = e^{\varepsilon x} - 1 = e^n - 1$

$$\sigma_e = \frac{\sigma_t}{1+\varepsilon_e} = \frac{\sigma_t}{e^n} = \frac{415.5}{e^{0.35}} = 292.8\ MPa$$

$$\sigma_t = 415.5\ MPa \quad and \quad \sigma_e = 292.8\ MPa$$

3.11 A tensile test on a steel specimen having a cross-sectional area of 2 cm^2 and length of 10 cm is conducted in an Instron universal testing machine with stiffness of 20 MN/m. If the initial strain rate is 10^{-3} s^{-1}, determine the slope of the load-extension curve in the elastic range ($E = 210$ GN/m^2).

Solution:

Tensile test on steel specimen,
A = 2 cm^2
L = 10 cm
Stiffness = 20 MN/m
Strain rate = 10^{-3} s^{-1}
Determine the slope of load-extension curve in elastic range E=210 GN/m^2

$$\sigma = E\varepsilon \Rightarrow E = \frac{\sigma}{\varepsilon} = \frac{\frac{P}{A_0}}{\frac{\Delta l}{l_0}} = \frac{P * l_0}{A_0 * \Delta l}$$

$$\frac{P}{\Delta l} = \frac{E * A_0}{l_0} = \frac{210GPa * 2cm^2}{10cm} * \frac{1m}{100cm} = 420MN/m$$

3.12 Determine all the parameters that can be obtained from the load-extension curve (for a cylindrical specimen) shown in Figure Ex. 3.12, knowing that the initial cross-section area in 4 cm^2, the crosshead velocity is 3 mm/s, the gage length is 10 cm, the final cross-sectional area is 2 cm, and the radius of curvature of the neck is 1 cm.

Engineering Stress-Strain Curve

a) Young's Modulus:
Slope of stress-strain curve

Point 1: F= 0 $\Delta\ell$ =0, $\sigma_e = 0$ $\varepsilon = 0$
Point 2: F= 240 N $\Delta\ell$ = 0.25 mm

$\sigma_e = F/A_o = 740 / 4 \times 10^{-4} = 1.85$ MPa

$\varepsilon_e = \Delta\ell/\ell_o = 0.025 / 10 = 0.0025$

$E = \sigma_c/\varepsilon_c = 0.74$ GPa

b) Ultimate tensile stress (UTS):
F_{max} = 1180 N = 1.18 kN
$\sigma_{uts} = F/A_o = 1180/4 \times 10^{-4} = 2.95$ MPa

c) Yield stress (0.2 % off set)
$\varepsilon_e = 0.002 = \Delta\ell/\ell$, therefore $\Delta\ell$ = (0.002)(10 cm) = 0.2 mm
F = 750 N
$\sigma_e = F/A_o = 750/4 \times 10^{-4}\ m^2 = 1.875$ MPa

d) Uniform Elongation:

$\Delta\ell$ @ UTS is 9.4 mm
$\varepsilon_e = 0.94/10 = 0.094$

e) Total Elongation:

$\Delta\ell$ @ Fracture is 16 mm
$\varepsilon_e = 1.6/10 = 0.16$

f) Stress @ fracture:

F = 800 N
$\sigma_e = F/A_o = 800/4 \times 10^{-4}$ m = 2.0 MPa

g) Reduction of area @ fracture:

$(A_o – A_f)/A_o = (4 – 2)/4 = 0.5 = 50\ \%$

h) Strain rate:

$\varepsilon_e = v/L_o = 3mm/s/100mm = 0.03\ mm^{-1}$

True Stress –Strain Curve

$\sigma_t = \sigma_e(1 + \varepsilon_e)$
$\varepsilon_t = \ell n\ (1 + \varepsilon_e)$

both are valid up to necking (UTS)

Beyond UTS, we use :
ε_t @ fracture = $\ell n\ (A_o/A_c)$
σ_t @ fracture = P_f/A_f

a) Young's Modulus:

Same as in Engineering Stress-strain case

b) UTS :

$\sigma_e = 2 .95$ MPa $\quad \varepsilon_e = 0.094$
$\sigma_t = \sigma\ (1 + \varepsilon_e) = 2.95\ (1 + 0.094) = 3.23$ MPa

c) 0.2 % off set yield stress:

$\sigma_e = 1.875$ MPa $\varepsilon_e = 0.002$
$\sigma_t = \sigma_e\ (1 + \varepsilon_e) = 1.88$ MPa

d) Uniform elongation:

$\varepsilon_e = 0.094$
$\varepsilon_t = \ell n\ (1 + 0.094) = 0.09$

e) Stress and strain @ fracture:
$\varepsilon_{tf} = \ell n\ A_f/A_f = \ell n\ 4/2 = 0.692$
$\sigma_{tf} = F/A_f = 800\ N/ 2x\ 10^{-4}\ m^2 = 4.0$ MPa

f) Bridgman Correction Factor:
@ fracture

$\sigma = \sigma_{av}/(1 + 2R/r_n)\ \ell n\ (1 + r_n/2R)$

R = radius of convecture of neck = 1 cm
r_n = radius of cross – reaction of thinnest part
= $\sqrt{(2/\pi)}$ = 0 .8 cm

Therefore, $\sigma = 4.0/(1 + 2(1)/0.8)\ \ell n\ (1 + 0.8/2) = 3.36$ MPa

3.13 Draw the engineering-stress-engineering-strain and true-stress-true-strain (with and without Bridgman correction) curves for the curve in Exercise 3.12.

Eng. Strain	Eng. Stress (MPa)	True Strain	True Stress (MPa)	True Stress with Bridgman Corr.
0	0	0	0	0
0.0025	1.85	0.0025	1.85	1.85
0.003	1.875	0.003	1.875	1.875
0.05	2.8	0.049	2.94	2.94
0.094	2.95	0.09	3.23	3.23
0.125	2.8	0.693	4	3.36
0.16	2			

3.14 What is the strain-rate sensitivity of AISI 1040 steel at a strain of 0.02 and a strain of 0.05. Obtain your data from Figure 3.12(a).

Strain rate sensitivity:

$$m = \left.\frac{\partial \ell n \sigma}{\partial \ell n \varepsilon}\right|_{\varepsilon.T}$$

Or it can also be written as $\sigma = K\varepsilon^m$

Applying this equation to two strain rates and eliminating K, we have:

$$m = \frac{\ell n(\sigma_2 / \sigma_1)}{\ell n(\varepsilon_2 / \varepsilon_1)}$$

$$At \varepsilon_e = 0.02:$$

$$m = \frac{\ell n(915/780)}{\ell n(10^{-1}/10^{-3})} = 0.035 \qquad m_{0.02} = 0.035$$

$$At\ \varepsilon_e = 0.05:$$

$$m = \frac{\ell n(935/870)}{\ell n(10^{-1}/10^{-3})} = 0.016 \qquad m_{0.05} = 0.016$$

3.17 The PMMA specimens shown in Figure Ex.3.17 were deformed in uniaxial tension. (a) Plot the total elongation, ultimate tensile stress, and Young's modulus as a function of temperature. (b) Discuss changes in these properties in terms of the internal structure of the specimen.

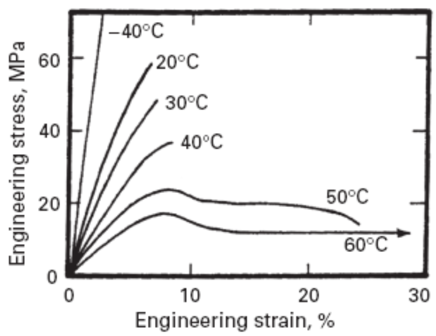
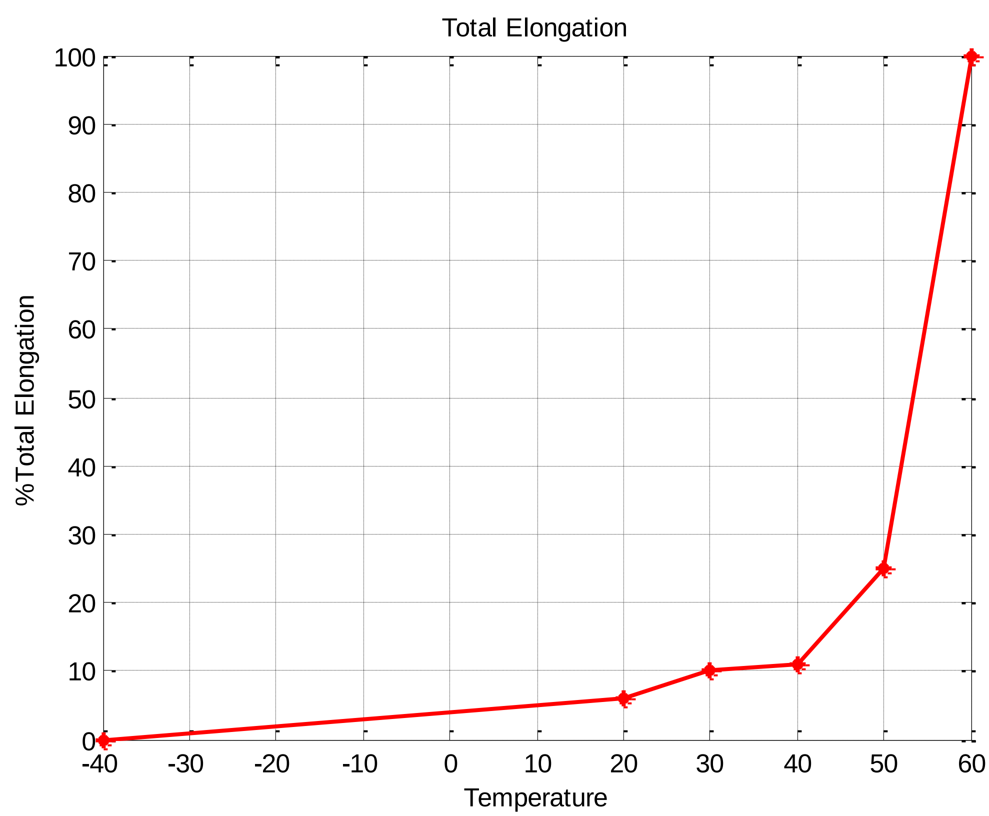
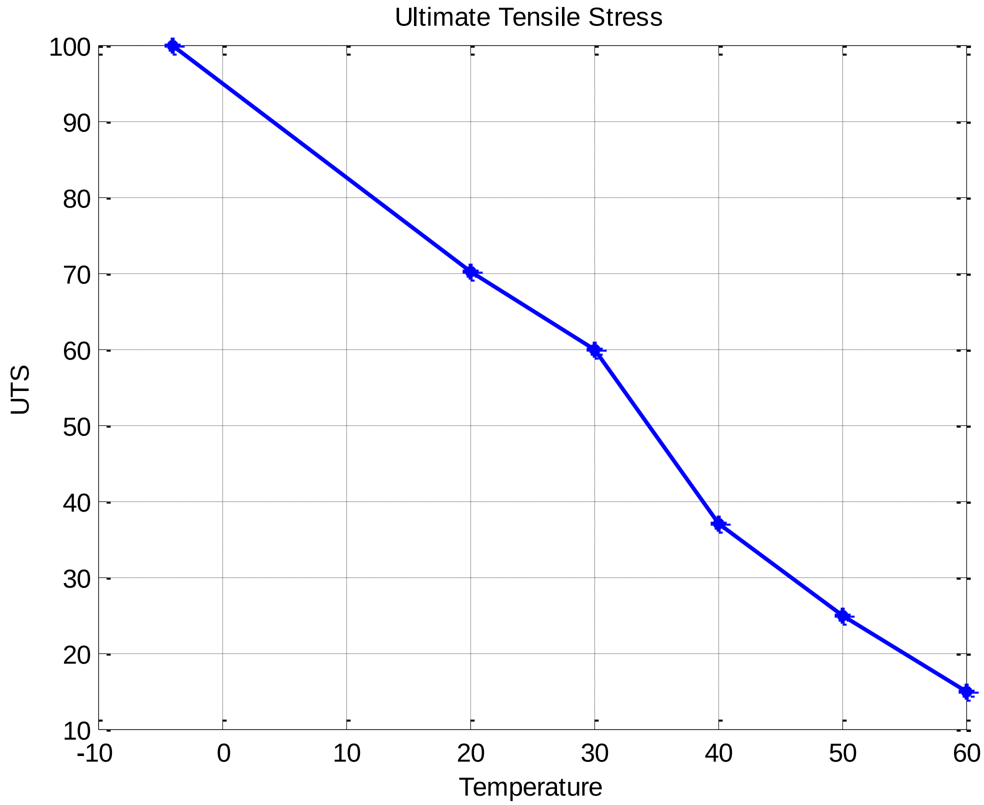


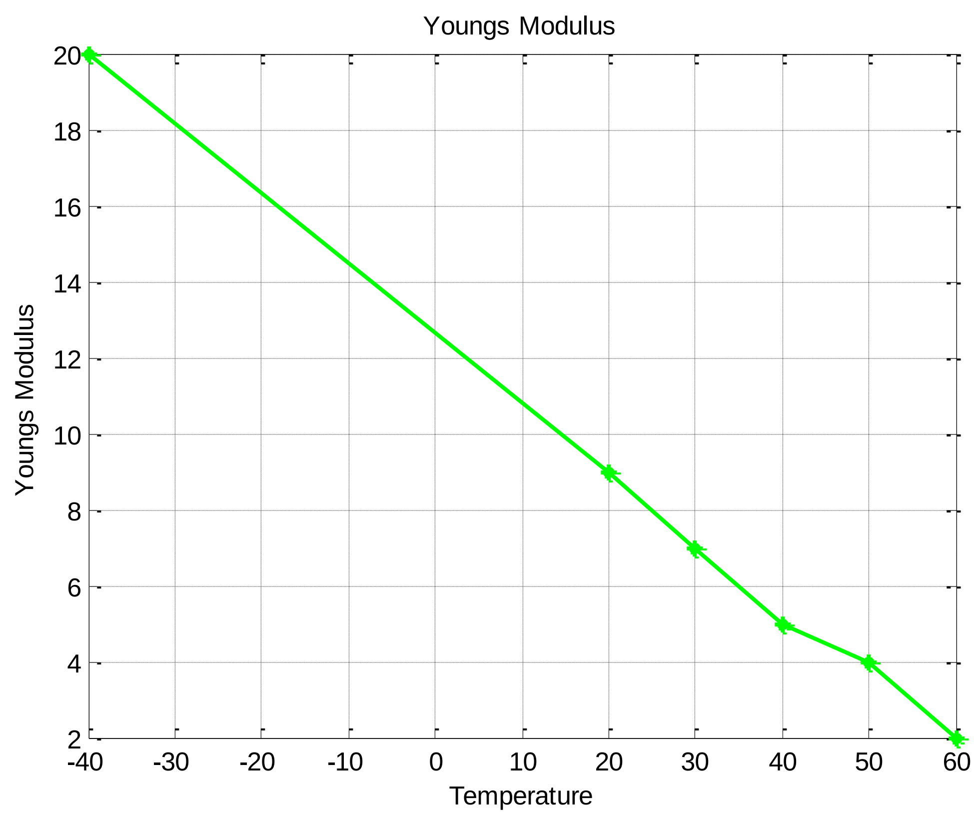
Fig. Ex.3.17

Solution:

(a) Plots,




Ultimate Tensile Stress
UTS
Temperature
100
90
80
70
60
50
40
30
20
10
-10
0
10
20
30
40
50
60


Youngs Modulus
Youngs Modulus
Temperature
20
18
16
14
12
10
8
6
4
2
-40
-30
-20
-10
0
10
20
30
40
50
60

(b) Discussion:

First plot:
The strain-to-failure increases as temperature increases. After about 50 ^{o}C, the strain seems to increase exponentially, probably because of start of necking.

Second plot:
The UTS decreases as the temperature increases due to break down of covalent bonds n at higher temperatures.

Third plot:
There appears to be an inverse relationship between the Young's modulus and temperature.

3.18 For the force-displacement curve of Figure Ex. 3.18, obtain the engineering and the stress-strain curves if the specimen was tested in compression.

ℓ_o = 0.59 mm d_o = 6.17 mm

Point 1:

$$\Delta\ell = 0.05mm \qquad F = 140\,N$$
$$\varepsilon_e = \frac{\Delta\ell}{\ell_o} = \frac{0.05}{6.59} = 0.00759 \Rightarrow \varepsilon_t = \ell n(1-\varepsilon_e) = 0.00762$$
$$\sigma_e = \frac{F}{A_o} = \frac{140}{\pi\,(6.17)^2 \times 10^{-4}} = 4.68\ MPa$$
$$\Rightarrow \sigma_t = \sigma_e(1-\varepsilon_e) = 4.64\ MPa$$

Point 2:

$$\Delta\ell = 0.1mm \quad F = 200\,N$$
$$\sigma_e = \frac{0.1}{6.59} = 0.015 \quad \Rightarrow \varepsilon_t = \ell n(1-\varepsilon_e) = 0.0151$$
$$\sigma_e = \frac{F}{A_o} = 6.69\ MPa \Rightarrow \sigma_t = \sigma_e(1-\varepsilon_e) = 6.59\ MPa$$

Point 3:

$$\Delta\ell = 0.5\,mm \qquad F = 270\ N$$
$$\varepsilon_c = \frac{0.5}{6.59} = 0.0759 \Rightarrow \varepsilon_t = 0.0789$$

3.19 Calculate the softening temperature for a soda--lime silica glass at which the viscosity is equal to 107 Pa · s if the activation energy for viscous flow is 250 kJ/mol and the viscosity at 1,000◦C is 103 Pa · s. Note: 1P = 0.1 Pa · s.

$$\eta = A\exp\left[\frac{Q}{RT}\right]$$

$$Q = \frac{250kJ}{mol} \qquad R = 8.314\frac{J}{mol \cdot K}$$

At T = 1000°C = 1273K, $n = 10^3$

$$A = \frac{n}{\exp\left[\frac{Q}{RT}\right]} = \frac{10^3}{\exp\left[\frac{250}{(8.314)(1273)}\right]} = 5.51 \times 10^{-8}$$

At the softening temperature: $\eta = 10^7\ Pa \cdot s$

$$\ln\left(\frac{\eta}{A}\right)\frac{R}{Q} = \frac{1}{T}$$

$$\ln\left(\frac{10^7}{5.51 \times 10^{-8}}\right)\frac{8.314}{250 \times 10^3} = \frac{1}{T} = .00109$$

$$\boxed{T_s = 915.8K}$$

3.20 The viscosity of a SiO2 glass is 1014 P at 1,000◦C and 1011 P at 1,300 ◦C. What is the activation energy for viscous flow in this glass? Note: 1 P = 0.1 Pa · s.

At point 1 $10^{14}\ P = 10^{13}\ Pa \cdot s$ T= 1000°C = 1273 K

At point 2 $10^{11}\ P = 10^{10}\ Pa \cdot s$ T= 1300°C = 1573 K

$$\eta = A\exp\left(\frac{Q}{RT}\right) \qquad R = 8.314\frac{J}{mol \cdot K}$$

For point 1

$$10^{13} = A\exp\left(\frac{Q}{8.314 \cdot 1273}\right) \qquad (1)$$

For point 2

$$10^{10} = A\exp\left(\frac{Q}{8.314 \cdot 1573}\right) \qquad (2)$$

Divide equation (1) by equation (2) to eliminate A

$$10^{3} = \exp\left[\frac{Q}{8.314}\left(\frac{1}{1273} - \frac{1}{1573}\right)\right]$$

$$Q = \frac{\ln\left(10^{3}\right)8.314}{\left(\frac{1}{1273} - \frac{1}{1573}\right)} = 383{,}338.8\frac{J}{mol}$$

$$\boxed{Q = 383.3\frac{kJ}{mol}}$$

3.21 When tested at room temperature, a thermoplastic material showed a yield of 51 MPa in uniaxial tension and 55 MPa in uniaxial address. Compute the yield strength of this polymer when tested in a pressure chamber with superimposed hydrostatic pressure of 300 MPa.

According to modified von Mises' Criterion for polymers, when a thermoplastic material showed yield strength of 51 MPa in uniaxial tension, we have:

$$(\sigma_1 - \sigma)^2 + (\sigma_2 - \sigma_3)^2 + [\sigma_3 - \sigma_1]^2 = 2\sigma_o{}^2 = 6(k_o + A_o\sigma_p)^2$$

$$where\ \sigma_1 = 51\ MPa\ ,\ \sigma_2 = \sigma_3 = 0,\ \sigma_p = \frac{\sigma_1}{3} = 17\ MPa$$

And when the thermoplastic material showed yield strength of 55 MPa in uniaxial compression, we get:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_o{}^2 = 6(k_o + A_o\sigma_p)^2$$

$$where\ \sigma_1 = -55 MPa,\ \sigma_2 = \sigma_3 = 0,\ \sigma_p = \frac{\sigma_1}{3} = -18.3\ MPa$$

From these two equations, we get

$$k_o = 30.5\ MPa,\ A_o = -0.065$$

Now this polymer is tested under a uniaxial stress in a pressure chamber with a superimposed hydrostatic pressure of 300 MPa. It is not specified if the uniaxial stress is tensile or compressive. We consider both the cases.

Tension

There will be a contribution to hydrostatic stress from the uniaxial stress equal to $\sigma_o/3$

$$2\sigma_o{}^2 = 6(k_o + A_o\sigma_p)^2$$

Inserting the values of $k_o = 30.5\ MPa,\ A_o = -0.065$ and $\sigma_p = -300 + \sigma_o/3$, we get

$$\sigma_o / \sqrt{3} = 30.55 - 300(-0.065) - 0.065\sigma_o / 3$$
$$0.599\sigma_o = 50.05$$
$$\sigma_o = 83.56MPa$$

Compression

This time the contribution to hydrostatic stress from the uniaxial stress is equal to $-\sigma_o/3$

$$2\sigma_o^2 = 6(k_o + A_o\sigma_p)^2$$

Inserting the values of $k_o = 30.5MPa,\ A_o = -0.065$ and $\sigma_p = -300 - \sigma_o/3$, we get

$$\sigma_o / \sqrt{3} = 30.55 - 300(-0.065) + 0.065\sigma_o / 3$$
$$0.555\sigma_o = 50.05$$
$$\sigma_o = 90.18MPa$$

3.27 You are given a 2.5 mm diameter cylindrical specimen 180 mm long. If the specimen is subjected to a torque of 50 N · m.
(**a)** Calculate the deflection of the specimen end, if one end is fixed.
(**b)** Will the specimen undergo plastic deformation?

$$d = 2.5mm$$
$$L = 180mm$$
$$\tau = 50N * m$$
$$c = \frac{2.5mm}{2} = 1.25mm$$
$$J = \frac{\pi c^4}{2} = \frac{\pi(1.25)^4}{2} = 3.83mm^4$$

(a) Calculate the deflection:

$$\tau_{\max} = \frac{\tau c}{J} = \frac{\tau c}{\frac{\pi c^4}{2}} = \frac{2\tau}{\pi c^3} = \frac{2(50N*m)}{\pi(0.00125m)^3}$$

$$\tau_{\max} = 1.62*10^{10} = 16.2GPa$$

$$\tau = G\gamma$$

$$G = \frac{E}{2(1+\nu)} = \frac{210}{2(1+0.3)} = 81GPa$$

$$\gamma = \frac{\tau}{G} = \frac{16.2}{81} = 0.2$$

Torsional deflection = angle of rotation

$$\theta = \frac{\gamma L}{c} = \frac{(0.2)(180)}{1.25} = 28.8rad$$

(b) Will the specimen undergo plastic deformation?

$$\tau_y = \frac{\sigma_y}{2}$$

$$\tau_y = \frac{300MPa}{2} = 150MPa$$

Yes, plastic deformation occurs.

3.28 Calculate the resulting rod diameter for 1040 carbon steel subjected to a 4000 N compressive load, with an initial diameter of 15 cm.

Given:

$F = 4000\,N$

$D_0 = 15cm$

Assume:

$E = 210GPa$

$\upsilon = .3$

A_o =.01767 m^2

$$\sigma = \frac{F}{A_0} = 226.37 \times 10^3\, Pa$$

$$\varepsilon_l = \frac{\sigma}{E} = 1.078 \times 10^{-6}$$ (compression)

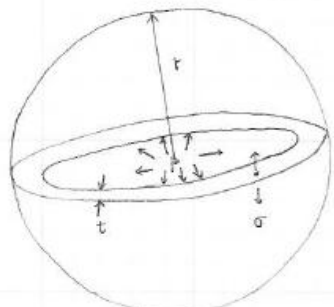
$$\varepsilon_t = -\varepsilon_l\, \nu = -(-1.078 \times 10^{-6})(.3) = 3.23 \times 10^{-7}$$

$$\varepsilon_t = \frac{D - D_0}{D_0}$$

$$D = (\varepsilon_t + 1) D_o = 15.0000049\ cm$$

3.29 You are asked to design a spherical pressure vessel for space application. The weight has to be minimized. Given that $\sigma = Pr/T$, choose, among materials below, which one you would select.

Alloy	Density (kg/m^3)	Y. S. (MPa)
304 SS	7.8	400
Ti6Al4V	4.46	850
2024 Al	2.7	400



$M = 4\pi r^2 t \rho$

$\sigma \le \frac{\sigma_y}{s}$ s - safety factor

$\frac{\sigma_y}{s} \geq \frac{Pr}{2} \cdot \frac{4\pi r^2 \rho}{M} = \frac{2\pi P r^3 \rho}{M}$

P - constant , r - fixed

① for 304 SS :

$\frac{400 \times 10^6}{s} \geq \frac{(2\pi P r^3)}{M} [7.8 \times 10^3 \ kg/m^3]$

$\Rightarrow M \geq 2\pi P r^3 s \left[\frac{7.8 \times 10^3 \ kg/m^3}{400 \times 10^6 \ Pa} \right] = 2\pi P r^3 s \times [1.95 \times 10^{-5}] \ kg$

② for Ti 6Al 4V :

$M \geq 2\pi P r^3 S \left[\frac{4.46 \times 10^3 \ kg/m^3}{850 \times 10^6 \ Pa} \right] = 2\pi P r^3 s \times [5.25 \times 10^{-6}] \ kg$

3.29 (cont'd)

③ for 2024 Al :

$$M \geq 2\pi P r^3 S \left[\frac{2.7 \times 10^3 \ kg/m^3}{400 \times 10^6 \ Pa} \right] = 2\pi P r^3 S \times [6.75 \times 10^{-6}] \ kg$$

∴ for a fixed r and const. P, Material Ti 6 Al 4 V seems to have the minimum mass.

3.30 You have a piece of steel, and you are able to measure its hardness: HV = 250 kg/mm^2. What is its estimated yield stress, in MPa?

Solution:

You have a piece of steel, and you are able to measure its hardness $HV = 250\frac{kg}{mm^2}$, what is its estimated yield stress, in MPa.

$$HV = 3\sigma_y = \frac{F}{A}$$

$$\sigma_y = \frac{HV}{3} = \frac{250 kg/mm^2}{3}$$

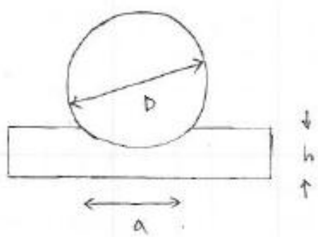
$$\sigma_y = 83.3 kg/mm^2 * \frac{9.8*10^6 Pa}{1 kg/mm^2}$$

$$\sigma_y = 816.63 MPa$$

3.31 You received a piece of cast iron, and you want to estimate its yield strength. You are able to make a hardness indentation using a 10 mm diameter tungsten carbide sphere. The diameter of the indentation is 4 mm. What is the estimated yield strength?

Solution:

Given: $D = 10$ mm (diameter of the sphere)
$a = 4$ mm (Indentation diameter)
$P = 3000$ kg·f



Standard Brinell test,

$$HB = 3\sigma_y = \frac{2P}{\pi D\left(D - \sqrt{D^2 - a^2}\right)}$$

$$3\sigma_y = \frac{2 \times 3000}{\pi \times 10\left(10 - \sqrt{100 - 16}\right)}$$

$$\sigma_y = 76.26\ \text{kg/mm} \times \frac{9.8\ \text{MPa}}{1\ \text{kgf/mm}^2}$$

$$\sigma_y = 747.35\ \text{MPa}$$

3.33 The shear yield strength of a polymer is 30% higher in compression than in tension. Determine the coefficient *A* that represents the dependence of yield stress on hydrostatic pressure.

Von Mises' Criterion for isotropic metals :

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 6k^2 = 2\sigma$$

$$k = k_0 + A\,\sigma_p$$

$$\sigma_y = 50 \text{ MPa}$$

Assume uniaxial stress state : $\sigma_2 = 0$, $\sigma_3 = 0$

① for tension

$$(\sigma_1 - \sigma_2) = \sigma_y$$

$$\sigma_2 = 0 \Rightarrow \sigma_1 = \sigma_y$$

$$\sigma_3 = 0$$

$$\sigma_p = \sigma_1/3$$

② for compression

$$(\sigma_1 - \sigma_2) = -1.3\,\sigma_y$$

$$\sigma_2 = 0 \Rightarrow \sigma_1 = -1.3\,\sigma_y$$

$$\sigma_3 = 0$$

$$\sigma_p = \sigma_1/3$$

for ① :

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 6k$$

$$\sigma_y^2 + (-\sigma_y)^2 = 6\,(k_0 + A\,\sigma_p)^2$$

$$\cancel{2}\,\sigma_y^2 = \cancel{6}^{\,3}\,(k_0 + A\,\sigma_p)^2$$

$$0.577\,\sigma_y = k_0 + A\,\sigma_p \quad \text{——} \quad ① \qquad \sigma_p = \sigma_1/3 = \sigma_y/3$$

for ② :

$$(-1.3\,\sigma_y)^2 + (1.3\,\sigma_y)^2 = 6\,(k_0 + A\,\sigma_p)^2$$

$$3.38\,\sigma_y^2 = 6\,(k_0 + A\,\sigma_p)^2$$

$$0.563\,\sigma_y^2 = (k_0 + A\,\sigma_p)^2$$

$$0.751\,\sigma_y = k_0 + A\,\sigma_p \quad \text{——} \quad ② \qquad \sigma_p = -1.3\,\sigma_y/3$$

3.33 (cont'd)

① $0.577\,(50\text{ MPa}) = k_0 + \frac{A}{3}\,(50\text{ MPa})$

$28.85 = k_0 + 16.67\,A$ —— ①

② $0.751\,(50\text{ MPa}) = k_0 - \frac{1.3A}{3}\,(50\text{ MPa})$

$37.55 = k_0 - 21.67\,A$ —— ②

② - ① $\Rightarrow$ $8.7 = -38.34\,A$

$A = -0.227$

from ① $\Rightarrow$ $28.85 = k_0 - 16.67 \times 0.227$

$k_0 = 32.63\text{ MPa}$

3.36 The following stresses were measured on a metal specimen:

$\sigma_{11} = 94$ MPa
$\sigma_{22} = 155$ MPa
$\sigma_{12} = 85$ MPa

Determine the yielding for both the Tresca and von Mises criteria, given that $\sigma_0 = 180$ MPa (yield stress). Which criterion is more conservative?

$\sigma_{11} = 94MPa$ $\qquad$ $\sigma_{22} = 155MPa$ $\qquad$ $\sigma_{12} = 85MPa$

Need to obtain principal stresses.

$$\sigma_{1,2} = \frac{\sigma_{11} + \sigma_{12}}{2} \pm \left[\left(\frac{\sigma_{11} - \sigma_{22}}{2}\right)^2 + \sigma_{12}{}^2\right]^{\frac{1}{2}}$$

$\sigma_1 = 214.81$ $\qquad$ $\sigma_2 = 34.19$ $\qquad$ $\sigma_3 = 0$

Tresca: $\tau_{\max} = \left(\dfrac{\sigma_1 - \sigma_3}{2}\right) =$ 107.4MPa

Tresca criterion: $\tau = \dfrac{\sigma_0}{2} = \dfrac{180}{2} =$ 90MPa

$\tau_{\max} > \tau$

von Mises:

$$J_2 = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2\right]$$

$J_2 =$ 13322.6

$J_2^M = \dfrac{1}{3}\sigma_0^2 = \dfrac{1}{3}(180)^2 =$ 10800

$J_2 > J_2^M$

Both criteria predict failure.

Tresca criterion is more conservative than von Mises.

3.37 A flat indenter strikes the surface of an iron block and sinks into the material by 0.4 cm. Assuming that the surface of a piece of iron (τ_0 = 6.6 GPa, σ_0 = 12.6 GPa, A = 0.5 cm2) can be modeled as triangular blocks as in Figure E2.10.2, determine the force with which the indenter hits the material.

Total work done by indenter = Total work done by blocks moving

u = .4 cm

$$Fu = 2 \times \underset{\Downarrow}{\frac{A\tau_0}{\sqrt{2}}} \times u\sqrt{2} + 2 \times \underset{\Downarrow}{A\tau_0} \times u + 4 \times \underset{\Downarrow}{\frac{A\tau_0}{\sqrt{2}}} \times \frac{u\sqrt{2}}{2}$$

Block 1 Blocks 2,3 Blocks 4,5

$$Fu = 2A\tau_0 + 2A\tau_0 + 2A\tau_0$$

$$Fu = 6A\tau_0$$

$$F = \frac{6A\tau_0}{u} = \frac{6(.5cm^2)(6.6GPa)}{.4cm}$$

$$F = \frac{6(.5\times 10^{-4}m^2)(6.6GPa)}{.4\times 10^{-2}} =$$

= 495 N

3.38 Determine the hardness of the copper specimen from the nanoindentation SEM image in Figure 3.42(b) knowing that the applied load is 2000 μN.

Load = $2000\times 10^{-6} N$

Berkovich tip was used

$$H = \frac{P_{\max}}{A}$$

A = $a + bhi^{\frac{1}{2}} + chi + dhi^{\frac{3}{2}} + 24.56hi^2$

Assume perfect tip a = b = c = d = 0

$$A = 24.56hi^2$$

$$L\sin\alpha = hi$$

$$\alpha = 65.3$$

L from figure $\approx 14\mu m = 14\times 10^{-6} m$

$$H = \frac{P_{max}}{24.56(L\sin\alpha)^2} = \frac{2000\times10^{-6}}{24.56(14\times10^{-6})^2(\sin 65.3)^2} = 503{,}370 \text{ Pa}$$

H = .503MPa

3.39 Calculate the projected area of an indentation made in keratin, the penetration depth *h* is 600 nm. Assume we used the Berkovich tip ($A = 24.5h^2$).

h = 600 nm

$$A = 24.5h^2 = 24.5(600\times10^{-9})^2$$

$$A = 8.82\times10^{-12}\,m^2$$

3.40 You are designing a kinetic energy penetrator for the M1 tank. This penetrator is made of depleted (non-radioactive but highly lethal!) uranium-0.75%Ti. Plot the stress–strain curve, from 0 to 1:

(a) At the following strain rates: 10^{-3} s^{-1}, 10^{3} s^{-1} (ambient temperature).
(b) At a strain rate of 10^{-3} s^{-1} and the following temperatures: 77 K, 100 K, 300 K.

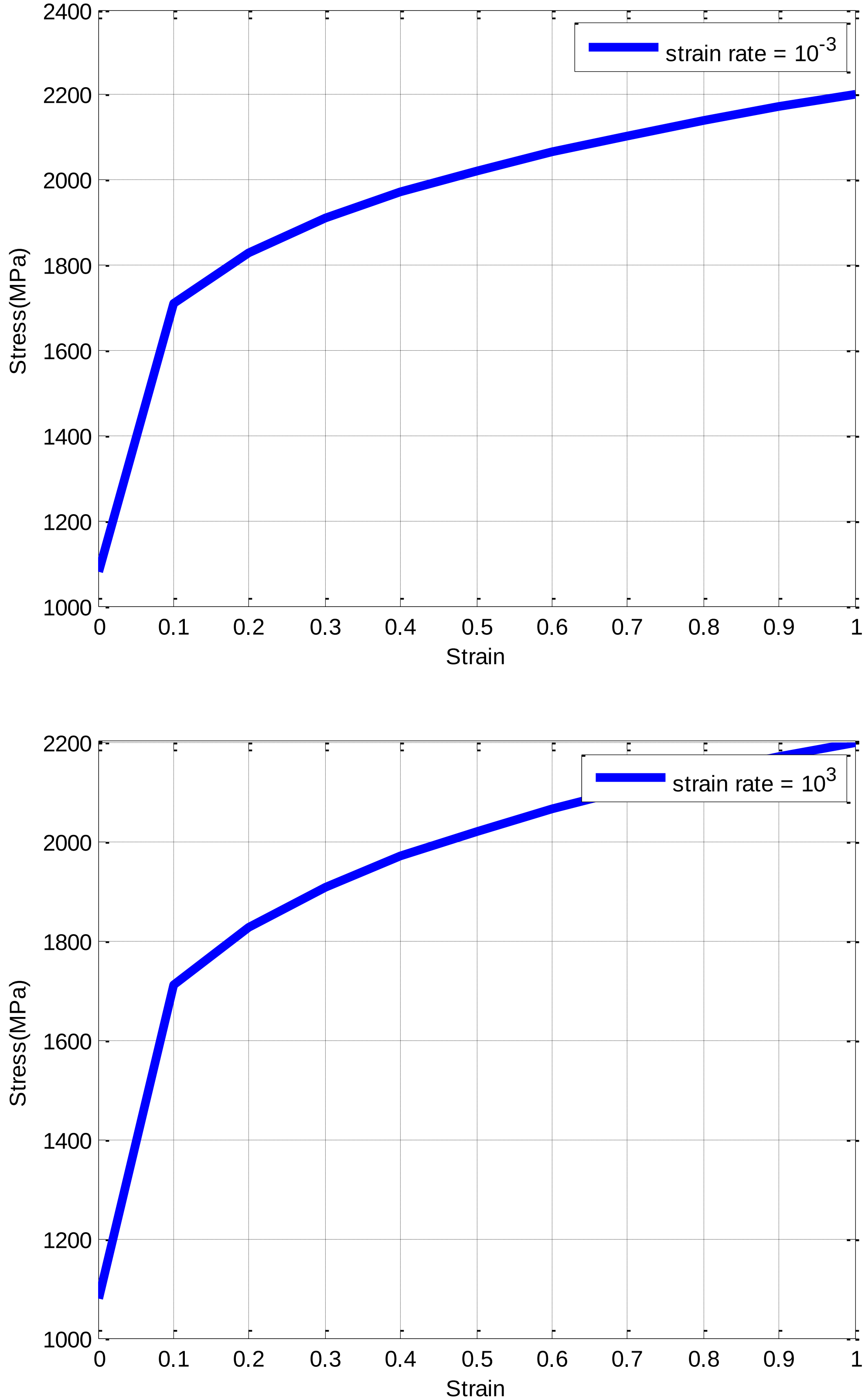
Given:

$T_m = 1473$ K
$\sigma_0 = 1079$ MPa
$K = 1120$ MPa
$N = 0.25$
$C = 0.007$
$m = 1$
$\dot{\varepsilon} = 10^{-4}$ s^{-1}

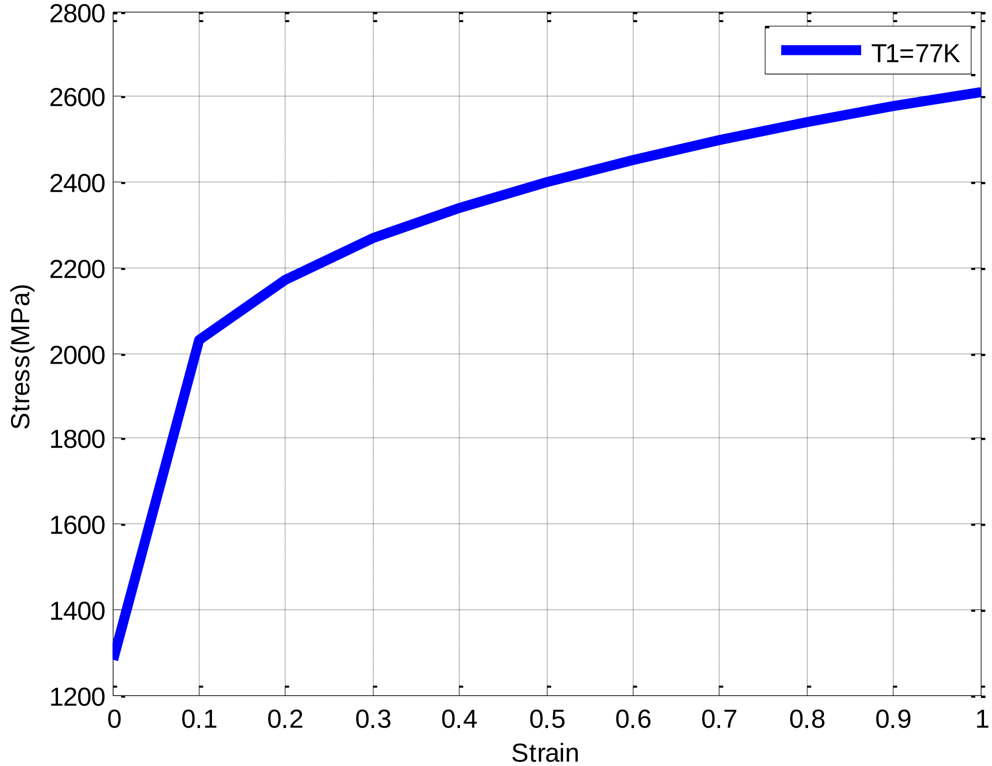
Solution:

$$\sigma = (\sigma_0 + K\,\varepsilon^n)\left(1 + C\ \ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)\left[1 - \left(\frac{T - T_r}{T_m - T_r}\right)^m\right]$$

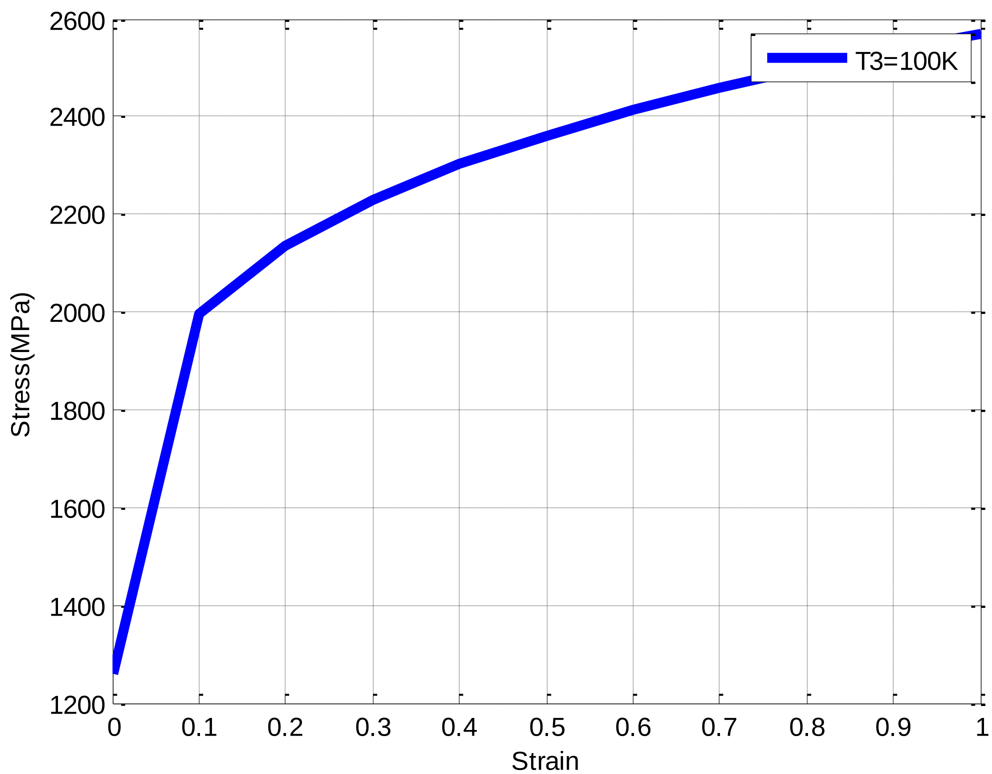
(a)


strain rate = 10^{-3}
Stress(MPa)
Strain
strain rate = 10^{3}
Stress(MPa)
Strain

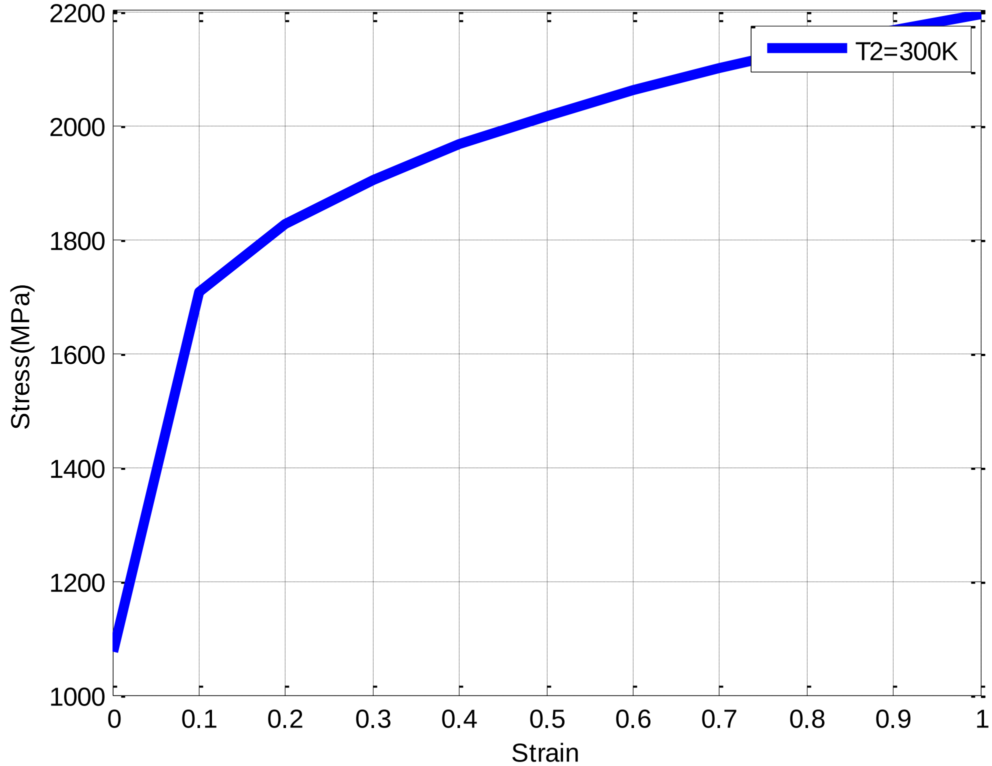
(b)



T1=77K
Stress(MPa)
Strain
2800
2600
2400
2200
2000
1800
1600
1400
1200
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



T3=100K
Stress(MPa)
Strain
2600
2400
2200
2000
1800
1600
1400
1200
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



2200
2000
1800
1600
1400
1200
1000
Stress(MPa)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Strain
T2=300K