1) $S_P = \int_R tr(\underline{\underline{\sigma}} \cdot \underline{\underline{d}})\, dV$, where $\underline{\underline{\sigma}} \equiv$ Cauchy stress (symm.)

$\underline{\underline{d}} = \underline{\underline{L}}_{sym} \rightarrow \underline{\underline{d}} = \underline{\underline{d}}^T$

a) show : $S_P = \int_R tr(\underline{\underline{\Sigma}} \cdot \underline{\underline{D}}^*)\, dV$

where: $\underline{\underline{\Sigma}} = \underline{\underline{R}}^T \cdot \underline{\underline{\sigma}} \cdot \underline{\underline{R}}$ → Rotated Cauchy stress

$D^* = \underline{\underline{R}}^T \cdot \underline{\underline{d}} \cdot \underline{\underline{R}}$

$$S_p = \int_R tr(\underline{\underline{\sigma}} \cdot \underline{\underline{d}})\, dV$$

$$= \int_R tr(\underline{\underline{R}} \cdot \underline{\underline{\Sigma}} \cdot \underline{\underline{R}}^T \cdot \underline{\underline{R}} \cdot \underline{\underline{D}}^* \cdot \underline{\underline{R}}^T)\, dV \rightarrow \underline{\underline{R}} \text{ is orthogonal } \therefore \underline{\underline{R}}\,\underline{\underline{R}}^T = \underline{\underline{I}} = \underline{\underline{R}}\,\underline{\underline{R}}^{-1}$$

$$= \int_R tr(\underline{\underline{R}} \cdot \underline{\underline{\Sigma}} \cdot \underline{\underline{I}} \cdot \underline{\underline{D}}^* \cdot \underline{\underline{R}}^T)\, dV$$

*note: because $\underline{\underline{d}}$ + $\underline{\underline{\sigma}}$ are both symm. + $\underline{\underline{R}}$ is orthogonal $\underline{\underline{\Sigma}}$ + $\underline{\underline{D}}^*$ must bothe be diagonal Tensors. That is:

$$\underline{\underline{\sigma}} = \underline{\underline{R}} \cdot \underline{\underline{\Sigma}} \cdot \underline{\underline{R}}^T \rightarrow \underline{\underline{\sigma}} \cdot \underline{\underline{R}} = \underline{\underline{R}} \cdot \underline{\underline{\Sigma}}$$

$$\underline{\underline{d}} = \underline{\underline{R}} \cdot \underline{\underline{D}}^* \cdot \underline{\underline{R}}^T \rightarrow \underline{\underline{d}} \cdot \underline{\underline{R}} = \underline{\underline{R}} \cdot \underline{\underline{D}}^*$$

additionally, the diagonal elements of $\underline{\underline{\Sigma}}$ + $\underline{\underline{D}}^*$ are eigen values + colums of $\underline{\underline{R}}$ are eigen vectors. e.g., $\underline{\underline{\sigma}} \cdot \underline{R}_1 = \lambda_1 \underline{R}_1$; where $\underline{R}_1$ represents the first colum of $\underline{\underline{R}}$ + λ_1 represent the first diag. element of $\underline{\underline{\Sigma}}$.

$$= \int_R tr(\underline{\underline{R}} \cdot \underline{\underline{\Sigma}} \cdot \underline{\underline{D}}^* \cdot \underline{\underline{R}}^T)\, dV \rightarrow tr(\underline{\underline{R}} \cdot \underline{\underline{\Sigma}} \cdot \underline{\underline{D}}^* \cdot \underline{\underline{R}}^{-1}) = tr(\underline{\underline{R}} \cdot \underline{\underline{R}}^{-1} \cdot \underline{\underline{\Sigma}} \cdot \underline{\underline{D}}^*)$$

$$S_P = \int_R tr(\underline{\underline{\Sigma}} \cdot \underline{\underline{D}}^*)\, dV$$

↓

Show: $s_p = \int_{R_0} tr(\hat{P} \cdot \dot{F}^T)\, dV_0$

previously: $s_p = \int_R tr(\Sigma \cdot D^*)\, dV$

$$= \int_R tr(R^T \cdot \sigma \cdot \underbrace{R \cdot R^T}_{I} \cdot d \cdot R)\, dV \qquad * \; R \text{ is orth}: \; R^T = R^{-1}$$

$$= \int_R tr(\underbrace{R^T \cdot R}_{I} \cdot \sigma \cdot d)\, dV$$

$\rightarrow \hat{P} = J \sigma \cdot F^{-T} \rightarrow \sigma = \frac{1}{J} \hat{P} \cdot F^T$

$\rightarrow L = \underset{\text{sym.}}{d} + \underset{\text{skw. sym}}{W} \rightarrow tr(L) = tr(d) + \cancel{tr(W)}^{\,0}$

$\therefore tr(L) = tr(d)$

$$= \int_R tr\left(\frac{1}{J} \hat{P} \cdot F^T \cdot L\right) dV$$

$\rightarrow J = \frac{dV}{dV_0} \rightarrow \frac{1}{J} = \frac{dV_0}{dV}$

$\rightarrow \dot{F}^T = F^T \cdot L$

$$= \int_{R_0} tr(\hat{P} \cdot \dot{F}^T)\, dV_0 \rightarrow \text{in undeformed config.}$$

Show: $s_p = \int_{R_0} tr(P \cdot \dot{E})\, dV_0$

Previously: $s_p = \int_{R_0} tr(\hat{P} \cdot \dot{F}^T)\, dV_0$

$\rightarrow \hat{P} = F \cdot P$

$\rightarrow tr(\dot{F}^T) = tr(\dot{F}^T \cdot L) = tr(\dot{F}^T \cdot d)$

$$= \int_{R_0} tr(F \cdot P \cdot \dot{F}^T \cdot d)\, dV_0$$

$\rightarrow \dot{E} = \dot{F}^T \cdot d \cdot F$

$$= \int_{R_0} tr(P \cdot \dot{F}^T \cdot d \cdot F)\, dV_0 = \int_{R_0} tr(P \cdot \dot{E})\, dV_0$$

2) given: $\underline{\underline{\sigma}} \Rightarrow \overset{s-s}{[\sigma]}$ + $\underline{\underline{d}} \Rightarrow \overset{s-s}{[d]}$

$$\underline{\underline{F}} = \underline{r}\overleftarrow{\nabla} = \frac{\partial \underline{r}}{\partial \underline{R}} = \frac{\partial x_i}{\partial X_j}\, \underline{e}_i \otimes \underline{E}_j \Rightarrow \overset{e-E}{[F]} \text{ ; spatial - material}$$

$$\underline{\underline{U}}^2 = \underline{\underline{F}}^T \cdot \underline{\underline{F}} \Rightarrow \overset{E-e}{[F^T]}\,\overset{e-E}{[F]} = \overset{E-E}{[U^2]} \text{ ; material - material}$$

$$\underline{\underline{R}} = \underline{\underline{F}} \cdot \underline{\underline{U}}^{-1} \Rightarrow \overset{e-E}{[F]}\,\overset{E-E}{[U]} = \overset{e-E}{[R]} \text{ , spatial - material}$$

$$\underline{\underline{\Sigma}} = \underline{\underline{R}}^T \cdot \underline{\underline{\sigma}} \cdot \underline{\underline{R}} \Rightarrow \overset{E-e}{[R^T]}\,\overset{e-e}{[\sigma]} \cdot \overset{e-E}{[R]} = \overset{E-E}{[\Sigma]} \text{ ; material - material}$$

$$\hat{\underline{\underline{P}}} = J\,\underline{\underline{\sigma}} \cdot \underline{\underline{F}}^{-T} \Rightarrow J\,\overset{e-e}{[\sigma]}\,\overset{e-E}{[F^{-T}]} = \overset{e-E}{[\hat{P}]} \text{ ; spatial - material}$$

$$\underline{\underline{P}} = \underline{\underline{F}}^{-1} \cdot \hat{\underline{\underline{P}}} \Rightarrow \overset{E-e}{[F^{-1}]}\,\overset{e-E}{[\hat{P}]} = \overset{E-E}{[P]} \text{ ; material - material}$$

$$\underline{\underline{D}} = \underline{\underline{F}}^T \cdot \underline{\underline{d}} \cdot \underline{\underline{F}} \Rightarrow \overset{E-e}{[F^T]}\,\overset{e-e}{[d]}\,\overset{e-E}{[F]} = \overset{E-E}{[D]} \text{ ; material - material}$$

$$\underline{\underline{D}}^* = \underline{\underline{R}}^T \cdot \underline{\underline{d}} \cdot \underline{\underline{R}} \Rightarrow \overset{E-e}{[R^T]}\,\overset{e-e}{[d]}\,\overset{e-E}{[R]} = \overset{E-E}{[D^*]} \text{ ; material - material}$$

→ mixed tensors $\underline{\underline{a}}, \underline{\underline{R}}, \underline{\underline{Q}}, \underline{\underline{F}}$, along with the transposes, inverses, + transpose of the inverse, may be used to transform tensors.

$\underline{\underline{a}}, \underline{\underline{R}}$, + $\underline{\underline{Q}}$ are orthogonal

→ m-m tensors remain unchanged during rigid body rotation.

3. i

Let $\underline{e}_i = \underline{E}_i$

→ because for all other parts of this class 'i' has represented a set of 3 numbers, I will continue with this practice.

$\underline{r} = \underline{r}(\underline{B}, t) = x_i \underline{e}_i$

$= X_1(1+\varepsilon t)\cos\theta - X_2 \sin\theta \, \underline{e}_1 +$

$X_1(1+\varepsilon t)\sin\theta + X_2 \cos\theta \, \underline{e}_2 + X_3 \underline{e}_3$

→ Assumed this was $\cos(\theta)$

note: $\theta = \omega \cdot t$

$\underline{v} = \left.\dfrac{\partial \underline{r}}{\partial t}\right|_{\underline{B}} = \left(X_1 \varepsilon \cos\theta - X_1(1+\varepsilon t)\sin\theta\cdot\omega - X_2\cos\theta\cdot\omega\right)\underline{e}_1 +$

$\left(X_1 \varepsilon \sin\theta + X_1(1+\varepsilon t)\cos\theta\cdot\omega - X_2\sin\theta\cdot\omega\right)\underline{e}_2 +$

$0\,\underline{e}_3$

3.1. i

$\underline{\underline{F}} = \dfrac{\partial \underline{r}}{\partial \underline{B}}$

$\overset{e-E}{[F]} = \begin{bmatrix} (1+\varepsilon t)\cos\theta & -\sin\theta & 0 \\ (1+\varepsilon t)\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\underline{\underline{F}}^T \Rightarrow \overset{e-E}{[F]}^T = \overset{E-e}{[F^T]} = \begin{bmatrix} (1+\varepsilon t)\cos\theta & (1+\varepsilon t)\sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\underline{\underline{U}}^2 = \underline{\underline{F}}^T \cdot \underline{\underline{F}}$

$\overset{E-E}{[U^2]} = \overset{E-e}{[F^T]}\,\overset{e-E}{[F]} = \begin{bmatrix} [(1+\varepsilon t)\cos\theta]^2 + [(1+\varepsilon t)\sin\theta]^2 & -(1+\varepsilon t)\cos\theta\sin\theta + (1+\varepsilon t)\sin\theta\cos\theta & 0 \\ -\sin\theta\cos\theta(1+\varepsilon t) + \cos\theta\sin\theta(1+\varepsilon t) & \sin^2\theta + \cos^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} (1+\varepsilon t)^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

because $\underline{\underline{U}}$ in the E-E basis has only diag. comp.,
→ E-E is a principal basis for $\underline{\underline{U}}$;
this is a spectral representation of $\underline{\underline{U}}$

$\overset{E-E}{[U]} = \begin{bmatrix} (1+\varepsilon t) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, stretch only in the $\underline{E}_1$ direction

3.1.i cont.

$$\dot{\underline{\underline{F}}} = \underline{V}\,\overleftarrow{\nabla}_0 = \frac{\partial \underline{V}}{\partial \underline{R}}$$

$$[\dot{F}]^{e\cdot E} = \begin{bmatrix} \mathcal{E}\cos\theta - (1+\mathcal{E}t)\sin\theta\cdot\omega & -\cos\theta\cdot\omega & 0 \\ \mathcal{E}\sin\theta + (1+\mathcal{E}t)\cos\theta\cdot\omega & -\sin\theta\cdot\omega & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\dot{\underline{\underline{U}}} = \frac{\partial \underline{\underline{U}}}{\partial t}$$

$$[\dot{U}]^{E\cdot E} = \begin{bmatrix} \mathcal{E} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$, stretch only in the $\underline{E}_1$ dir.

3.1 cont.

$$\underline{\underline{R}} = \underline{\underline{F}} \cdot \underline{\underline{U}}^{-1}$$

$$\overset{e-E}{[R]} = \overset{e-E}{[F]}\ \overset{E-E}{[U^{-1}]} = \begin{bmatrix} \cos\theta(1+\varepsilon t) & -\sin\theta & 0 \\ \sin\theta(1+\varepsilon t & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (1+\varepsilon t)^{-1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\dot{\underline{\underline{R}}} = \frac{d\underline{\underline{R}}}{dt} \Rightarrow \begin{bmatrix} -\sin\theta\cdot\omega & -\cos\theta\cdot\omega & 0 \\ \cos\theta\cdot\omega & -\sin\theta\cdot\omega & 0 \\ 0 & 0 & 0 \end{bmatrix} = \overset{e-E}{[\dot{R}]}$$

* Recall: $\theta = \omega t$

$$\underline{\underline{R}}^T \Rightarrow \overset{E-e}{[R^T]} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{\Omega}} = \dot{\underline{\underline{R}}} \cdot \underline{\underline{R}}^T \Rightarrow \overset{e-E}{[\dot{R}]}\ \overset{E-e}{[R^T]} = \overset{e-e}{[\Omega]}$$

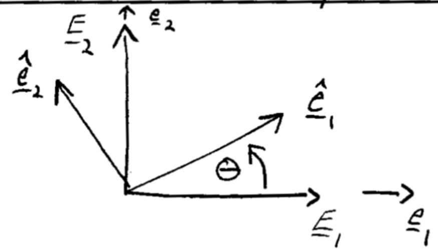
$$\overset{e-e}{[\Omega]} = \begin{bmatrix} -\sin\theta\cdot\omega & -\cos\theta\cdot\omega & 0 \\ \cos\theta\cdot\omega & -\sin\theta\cdot\omega & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\sin\theta\cos\theta\cdot\omega + \sin\theta\cos\theta\cdot\omega & -\sin^2\theta\cdot\omega - \cos^2\theta\cdot\omega & 0 \\ \cos^2\theta\,\omega + \sin^2\theta\cdot\omega & \sin\theta\cos\theta\cdot\omega - \sin\theta\cos\theta\cdot\omega & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \text{Skw. Symm}$$

3. ii

$$\hat{\underline{e}}_1 = \cos\Theta \, \underline{e}_1 + \sin\Theta \, \underline{e}_2$$
$$\hat{\underline{e}}_2 = -\sin\Theta \, \underline{e}_1 + \cos\Theta \, \underline{e}_2$$
$$\hat{\underline{e}}_3 = \underline{e}_3$$



$$\overset{\hat{e}-e}{[a]} = \begin{bmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Theta = \omega t$

Let: $\underline{E}_3 = \underline{e}_3 = \hat{\underline{e}}_3$

$\overset{\hat{e}-e}{[a]} = \overset{e-E}{[a]}$?

$$\hat{\underline{r}} = x_i \hat{\underline{e}}_i = X_1(1+\varepsilon t)\,\hat{\underline{e}}_1 + X_2\,\hat{\underline{e}}_2 + X_3\hat{\underline{e}}_3$$
$$\langle \overset{e}{r} \rangle = \langle \overset{\hat{e}}{r} \rangle \overset{\hat{e}-e}{[a]}$$
$$= \langle X_1(1+\varepsilon t)\cos\Theta - X_2\sin\Theta,\ X_1(1+\varepsilon t)\sin\Theta + X_2\cos\Theta,\ X_3 \rangle$$

3.1, ii

$$\underline{\underline{F}} = \frac{\partial \underline{r}}{\partial \underline{R}} = \frac{\partial x_i}{\partial X_j}\, \underline{e}_i \otimes \underline{E}_j$$

$$\overset{e-E}{[F]} = \begin{bmatrix} (1+\varepsilon t)\cos\Theta & -\sin\Theta & 0 \\ (1+\varepsilon t)\sin\Theta & +\cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{F}}^T \Rightarrow \overset{e-E}{[F]}^T = \overset{E-e}{[F^T]} = \begin{bmatrix} (1+\varepsilon t)\cos\Theta & (1+\varepsilon t)\sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{U}}^2 = \underline{\underline{F}}^T \cdot \underline{\underline{F}}$$

$$\overset{EE}{[U^2]} = \begin{bmatrix} [(1+\varepsilon t)\cos\Theta]^2 + [(1+\varepsilon t)\sin\Theta]^2 & -(1+\varepsilon t)\cos\Theta\sin\Theta + (1+\varepsilon t)\sin\Theta\cos\Theta & 0 \\ -(1+\varepsilon t)\cos\Theta\sin\Theta + (1+\varepsilon t)\sin\Theta\cos\Theta & \sin^2\Theta + \cos^2\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ again, E-E is a principal basis

$$\overset{E-E}{[U]} = \begin{bmatrix} (1+\varepsilon t) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\overset{E-E}{[U^{-1}]} = \begin{bmatrix} (1+\varepsilon t)^{-1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3.1 ii Cont.

$$\underline{\underline{R}} = \underline{\underline{F}} \cdot \underline{\underline{U}}^{-1}$$

$$\overset{e-E}{[R]} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\dot{\underline{\underline{F}}} = \underline{v} \overleftarrow{\nabla_0} = \frac{\partial \underline{v}}{\partial \underline{R}}$$

$$\rightarrow \underline{v} = \frac{\partial \underline{r}}{\partial t} = \frac{\partial x_i}{\partial t} \underline{e}_i \quad ; \quad \theta = \omega t$$

$$\langle \overset{e}{v} \rangle = \langle X_1 \varepsilon \cos\theta - X_1(1+\varepsilon t)\sin\theta\cdot\omega - X_2 \cos\theta\cdot\omega,$$
$$X_1 \varepsilon \sin\theta + X_1(1+\varepsilon t)\cos\theta\cdot\omega - X_2 \sin\theta\cdot\omega, \ 0 \rangle$$

$$\overset{e-E}{[\dot{\vec{F}}]} = \begin{bmatrix} \varepsilon\cos\theta - \omega(1+\varepsilon t)\sin\theta & -\omega\cos\theta & 0 \\ \varepsilon\sin\theta + \omega(1+\varepsilon t)\cos\theta & -\omega\sin\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\dot{\underline{\underline{R}}} = \frac{\partial \underline{\underline{R}}}{\partial t}$$

$$\overset{e\ E}{[\dot{R}]} = \begin{bmatrix} -\sin\theta\cdot\omega & -\cos\theta\cdot\omega & 0 \\ \cos\theta\cdot\omega & -\sin\theta\cdot\omega & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\dot{\underline{\underline{U}}} = \frac{\partial \underline{\underline{U}}}{\partial t}$$

$$\overset{E-E}{[\dot{U}]} = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad , \text{ deformation in 1D}$$

$$\underline{\underline{\Omega}} = \dot{\underline{\underline{R}}} \cdot \underline{\underline{R}}^T$$

$$\overset{e\ e}{[\Omega]} = \overset{e-E}{[\dot{R}]} \overset{E-e}{[R^T]} = \begin{bmatrix} -\sin\theta\cdot\omega & -\cos\theta\cdot\omega & 0 \\ \cos\theta\cdot\omega & -\sin\theta\cdot\omega & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\cos\theta\sin\theta\cdot\omega + \cos\theta\sin\theta\cdot\omega & -\sin^2\theta\cdot\omega - \cos^2\theta\cdot\omega & 0 \\ \cos^2\theta\cdot\omega + \sin^2\theta\cdot\omega & \sin\theta\cos\theta\cdot\omega - \cos\theta\sin\theta\cdot\omega & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\downarrow$

$$\overset{e-e}{[\Omega]} = \begin{bmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- yes, the same results were obtained.

3.2 $\rightarrow$ are $\dot{e}_i$ + E_i supposed to be $\dot{\underline{\underline{e}}}_i$, $\dot{\underline{e}}_i$, $\dot{\underline{\underline{E}}}_i$, $\dot{\underline{E}}_i$?

$$\dot{\underline{\underline{e}}} = -\frac{1}{2}\left[\dot{\underline{\underline{F}}}^{-T} \cdot \underline{\underline{F}}^{-1} + \underline{\underline{F}}^{-T} \cdot \dot{\underline{\underline{F}}}^{-1}\right]$$

$$\overset{e-e}{[\dot{e}]} = -\frac{1}{2}\left[\overset{e-E}{[\dot{F}^{-T}]}\,\overset{E-e}{[F^{-1}]} + \overset{e-E}{[F^{-T}]}\,\overset{E-e}{[\dot{F}^{-1}]}\right]$$

$$\underline{\underline{F}}^{-1} = \frac{\partial \underline{R}}{\partial \underline{r}} = \frac{\partial X_i}{\partial x_j}\, \underline{E}_i \otimes \underline{e}_j$$

$$\underline{R} = \underline{R}(\underline{r}(t), t) = X_i\, \underline{E}_i$$

$$X_1 = \frac{\begin{vmatrix} x_1 & -\sin\theta \\ x_2 & \cos\theta \end{vmatrix}}{1+\varepsilon t} = \frac{x_1\cos\theta + x_2\sin\theta}{(1+\varepsilon t)}$$

$$X_2 = \frac{\begin{vmatrix} (1+\varepsilon t)\cos\theta & x_1 \\ (1+\varepsilon t)\sin\theta & x_2 \end{vmatrix}}{1+\varepsilon t} = x_2\cos\theta - x_1\sin\theta$$

$$X_3 = x_3$$

$$\overset{E-e}{[F^{-1}]} = \begin{bmatrix} \frac{\cos\theta}{1+\varepsilon t} & \frac{\sin\theta}{1+\varepsilon t} & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{L}} = \dot{\underline{\underline{F}}} \cdot \underline{\underline{F}}^{-1}$$

$$\overset{ee}{[L]} = \overset{e-E}{[\dot{F}]}\,\overset{E-e}{[F^{-1}]} = \begin{bmatrix} \frac{\varepsilon\cos^2\theta}{1+\varepsilon t} & \frac{\sin\theta}{1+\varepsilon t}\left(\varepsilon\cos\theta - \omega(1+\varepsilon t)\sin\theta\right) - \omega\cos^2\theta \\ \frac{\cos\theta}{1+\varepsilon t}\left(\varepsilon\sin\theta + \omega(1+\varepsilon t)\cos\theta\right) + \sin^2\theta\,\omega & \frac{\sin\theta}{1+\varepsilon t}\left(\varepsilon\sin\theta + \omega(1+\varepsilon t)\cos\theta\right) - \omega\sin\theta\cos\theta \\ 0 & 0 \end{bmatrix}$$

3.2 Cont.

$$\overset{e\text{-}c}{[L]} = \begin{bmatrix} \frac{\varepsilon\cos^2\theta}{1+\varepsilon t} & \frac{\varepsilon\sin\theta\cos\theta}{1+\varepsilon t} \overbrace{- \omega\sin^2\theta - \omega\cos^2\theta}^{-\omega} & 0 \\ \frac{\varepsilon\sin\theta\cos\theta}{1+\varepsilon t} + \underbrace{\omega\cos^2\theta + \sin^2\theta\,\omega}_{\omega} & \frac{\varepsilon\sin^2\theta}{1+\varepsilon t} + \omega\sin\theta\cos\theta - \omega\sin\theta\cos\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\varepsilon\cos^2\theta}{1+\varepsilon t} & \frac{\varepsilon\sin\theta\cos\theta}{1+\varepsilon t} - \omega & 0 \\ \frac{\varepsilon\sin\theta\cos\theta}{1+\varepsilon t} + \omega & \frac{\varepsilon\sin^2\theta}{1+\varepsilon t} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\dot{\underline{\underline{F}}}^{-1} = -\underline{\underline{F}}^{-1} \cdot \underline{\underline{L}}$ → a mess

$$\Rightarrow -\begin{bmatrix} \left(\frac{\cos\theta}{1+\varepsilon t}\right)\left(\frac{\varepsilon\cos^2\theta}{1+\varepsilon t}\right) + \left(\frac{\sin\theta}{1+\varepsilon t}\right)\left(\frac{\varepsilon\sin\theta\cos\theta}{1+\varepsilon t} + \omega\right) & \left(\frac{\cos\theta}{1+\varepsilon t}\right)\left(\frac{\varepsilon\sin\theta\cos\theta}{1+\varepsilon t} - \omega\right) + \left(\frac{\sin\theta}{1+\varepsilon t}\right)\left(\frac{\varepsilon\sin^2\theta}{1+\varepsilon t}\right) & 0 \\ \cdots & & \end{bmatrix}$$

→ my wrist is failing

3.3 $d\underline{X} = dX_1 \underline{E}_i$

$$\underline{\underline{U}} \cdot d\underline{X} \Rightarrow \overset{E-E}{[U]} \{ \overset{E}{dX} \} = \begin{bmatrix} (1+\varepsilon t) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} dX_1 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} dX_1(1+\varepsilon t) \\ 0 \\ 0 \end{Bmatrix}$$

$$\underline{\underline{R}} \cdot (\underline{\underline{U}} \cdot d\underline{X}) \Rightarrow \begin{bmatrix} (1+\varepsilon t)^{-1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} dX_1(1+\varepsilon t) \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} dX_1 \\ 0 \\ 0 \end{Bmatrix}$$

-> results in the original element.

-> How can/should this be interpreted....

~~the element experiences stretch only~~

-here the rotatation of the element is the inverse of the stretch?