

3.1.1 Viscoelastic rod subjected to constant axial load

Product: Abaqus/Standard

This example, taken from Collingwood et al. (1985), is intended to verify the coding of the time domain linear viscoelastic material model.

Problem description

The problem is a rod of length of 254 mm (10 in) and diameter of 25.4 mm (1 in). The rod is fixed in the axial direction on one end and a constant axial load of 0.689 MPa (100 psi) is applied suddenly to the other end. The rod is modeled using one quadratic, axisymmetric, hybrid continuum element (CAX8H).

Material

The linear viscoelastic material model used in this example can be represented by a combination of linear springs and a dashpot, as shown in [Figure 3.1.1–1](#). The extensional relaxation function is

$$E_R(t) = k_1 + k_2 e^{-t/\tau_R},$$

where $\tau_R = \eta/k_2$, η is the damping coefficient and k_1 and k_2 are constants. In this case k_1 is 6.89 MPa (1000 psi); k_2 is 62.01 MPa (9000 psi); τ_R is 1.0 sec; and the bulk modulus, K , is 689 MPa (100,000 psi) and is independent of time.

Short-term material properties are specified using the [*ELASTIC](#) option ([“Linear elastic behavior,” Section 17.2.1 of the Abaqus Analysis User's Manual](#)), which requires the instantaneous Young's modulus, E_0 , and Poisson's ratio, ν_0 . The time-dependent behavior is specified using the [*VISCOELASTIC](#) option, in which the shear relaxation modulus and the bulk modulus are defined by a Prony series (see [“Time domain viscoelasticity,” Section 17.7.1 of the Abaqus Analysis User's Manual](#)). For the Abaqus analysis of this problem, it is assumed that no volumetric relaxation occurs.

E_0 is immediately available as $E_0 = k_1 + k_2 = 68.9$ MPa (10000 psi), and ν_0 is

$$\nu_0 = \frac{3K - k_1 - k_2}{6K} = 0.4833.$$

The time-dependent material behavior is approximated with a single-term Prony series for the shear relaxation modulus:

$$G_R(t) = G_0 (1 - \bar{g}_1^P (1 - e^{-t/\tau_1^G})).$$

We need to compute G_0 , \bar{g}_1^P , and τ_1^G from the extensional relaxation function. The limiting cases of both the shear and extensional relaxation functions are used for this purpose. The long-term ($t \rightarrow \infty$) properties of the material approach that of a linear elastic solid, with $E = k_1$. The long-term shear modulus, G , can be calculated using the relationship between the bulk, shear, and extensional moduli:

$$G = \frac{3Kk_1}{9K - k_1}.$$

Likewise, the “instantaneous” or “glassy” shear modulus, G_0 , is

$$G_0 = \frac{3K(k_1 + k_2)}{9K - (k_1 + k_2)}.$$

Then $\bar{g}_1^P = 1 - G/G_0 = 0.901001$.

The shear relaxation time, τ_1^G , is obtained by writing the rate of change of the shear modulus in terms of the rate of change of the extensional modulus at time $t = 0$:

$$\left. \frac{dG_R}{dt} \right|_{t=0} = \left(\frac{dG_R}{dE_R} \frac{dE_R}{dt} \right) \Big|_{t=0}.$$

After some algebra we obtain

$$\tau_1^G = \frac{9K - k_1 - k_2}{9K - k_1} \tau_R = 0.9899 \text{ sec.}$$

The same problem is also treated as a large-strain example. The relaxation behavior is defined in the same way, but the short-term elastic properties are given with the [*HYPERELASTIC](#) option. The polynomial formulation with $N=1$ is used, and the constants are $C_{10} = 6.89 \text{ MPa}$ (1000 psi), $C_{01} = 4.59 \text{ MPa}$ (666.67 psi), and $D_1 = 1.378 \times 10^{-7} \text{ MPa}^{-1}$ (0.00002 psi⁻¹). These constants are such that the initial Young's modulus and initial Poisson's ratio are equal to E_0 and ν_0 , respectively, and produce a close fit to a linear material. (See [“Hyperelastic behavior of rubberlike materials,” Section 17.5.1 of the Abaqus Analysis User's Manual](#), for further discussion of the choice of constants when $N=1$.)

Loading

A distributed load of 0.689 MPa (100 psi) is applied instantaneously and held constant throughout the analysis. To model this, we use the [*VISCO](#) procedure ([“Quasi-static analysis,” Section 6.2.5 of the Abaqus Analysis User's Manual](#)) in two steps. The load is applied in the first step, which has a time period of 0.001 seconds, so that the instantaneous (glassy) behavior dominates. Since this step uses only one increment, CETOL is not specified on the [*VISCO](#) option. The second step has a time period of 50 seconds, during which the load is held constant and the rod is allowed to relax toward its long-term behavior. Automatic time incrementation is chosen by giving a value for CETOL, the maximum difference in the creep strain increment over a time increment. CETOL is selected so that its value is of the same order of magnitude as the maximum elastic strain. Therefore, for this example CETOL is set to 5×10^{-3} . The [*SECTION FILE](#) option is used to output the total force and the total moment on the loaded face of the model.

Results and discussion

The instantaneous and long-term behaviors provide a check on the Abaqus results. The instantaneous and long-term axial displacements of the rod tip can be calculated as follows:

$$\begin{aligned} \frac{u_2}{L}(t=0) &= \frac{\sigma_{22}}{k_1 + k_2} = 0.01 \\ \frac{u_2}{L}(t \rightarrow \infty) &= \frac{\sigma_{22}}{k_1} = 0.1. \end{aligned}$$

These values agree well with the Abaqus results. Similarly, the instantaneous and long-term values of the Poisson's ratio can also be calculated exactly:

$$\nu(t = 0) = \frac{3K - k_1 - k_2}{6K} = 0.4833$$

$$\nu(t \rightarrow \infty) = \frac{3K - k_1}{6K} = 0.4983.$$

The Poisson's ratio can be extracted from the Abaqus results by taking the ratio of the lateral strain to the axial strain at $t = 0.001$ and $t = 50$:

$$\nu(t = 0.001) = -\frac{\varepsilon_{11}}{\varepsilon_{22}} = -\frac{-4.8353 \times 10^{-3}}{1.0005 \times 10^{-2}} = 0.4833$$

$$\nu(t = 50) = -\frac{\varepsilon_{11}}{\varepsilon_{22}} = -\frac{-4.9478 \times 10^{-2}}{9.9291 \times 10^{-2}} = 0.4983.$$

Since this is an applied stress problem, obtaining the exact solution for the entire time period of the analysis requires inverting the original constitutive integral equation defining uniaxial stress in terms of uniaxial strain. To perform this inversion, we use the following relation (Pipkin, 1972) between the time-dependent relaxation modulus, $E_R(t)$, and the time-dependent creep compliance, $J_C(t)$:

$$\int_0^t J_C(\tau) E_R(t - \tau) d\tau = t.$$

Differentiation of this relation with respect to t yields

$$J_C(t) E_R(0) + \int_0^t J_C(\tau) \dot{E}_R(t - \tau) d\tau = 1.$$

With the previously used expression for $E_R(t)$ this takes the form

$$(k_1 + k_2) J_C(t) - \frac{k_2}{\tau_R} \int_0^t J_C(\tau) e^{(\tau-t)/\tau_R} d\tau = 1.$$

Differentiating this expression once more provides

$$(k_1 + k_2) \dot{J}_C(t) - \frac{k_2}{\tau_R} J_C(t) + \frac{k_2}{\tau_R^2} \int_0^t J_C(\tau) e^{(\tau-t)/\tau_R} d\tau = 0.$$

Multiplying this equation by τ_R and adding it to the previous equation yields the differential equation

$$(k_1 + k_2) \tau_R \dot{J}_C(t) + k_1 J_C(t) = 1.$$

With the introduction of the creep time constant $\tau_C = (1 + k_2/k_1) \tau_R$ this can be written as:

$$\tau_C \dot{J}_C(t) + J_C(t) = 1/k_1.$$

The general solution to this differential equation is

$$J_C(t) = 1/k_1 + C e^{-t/\tau_C},$$

where the coefficient C is defined by the initial condition $J_C(0) = 1/(k_1 + k_2)$, which yields

$$J_C(t) = \left(1 - \frac{k_2}{k_1 + k_2} e^{-t/\tau_C}\right) \frac{1}{k_1}.$$

For this problem the stress σ_{22} is a constant, so that

$$\varepsilon_{22} = J_C(t)\sigma_{22} = \left(1 - \frac{k_2}{k_1 + k_2} e^{-t/\tau_C}\right) \frac{\sigma_{22}}{k_1}.$$

From the values given above for k_1 , k_2 , and τ_R , as well as the fact that $\sigma_{22} = 0.689$ MPa (100 psi), ε_{22} becomes

$$\varepsilon_{22} = 0.1 \left(1 - 0.9e^{-t/10}\right).$$

From this equation, it is evident that the effective time constant for the problem is dramatically different (by a factor of 10 in this case), depending upon whether the loading is applied force or applied displacement. [Figure 3.1.1-2](#) is a time history plot of ε_{22} as predicted by the above equation and as calculated by Abaqus. The plot shows acceptable agreement between the Abaqus results and the exact solution. Closer agreement can be obtained by using a smaller value of CETOL.

The solution obtained with the large-strain formulation differs negligibly from the small-strain solution. Abaqus automatically converts frequency domain data into a time domain Prony series representation. The analysis results using Prony parameters calibrated from tabulated frequency-dependent moduli data are in good agreement with the analyses using time domain data directly.

Input files

[viscorod_smallstrain.inp](#)

Small-strain input data for this problem.

[viscorod_small_frq2tim.inp](#)

Small-strain input data for time domain analysis using Prony parameters calibrated from tabulated frequency-dependent moduli data.

[viscorod_largestrain.inp](#)

Large-strain input data for this problem.

[viscorod_large_frq2tim.inp](#)

Large-strain input data for this problem using Prony parameters calibrated from tabulated frequency-dependent moduli data.

[viscorod_c3d8.inp](#)

Model using the three-dimensional 8-node brick element, C3D8.

[viscorod_cps4.inp](#)

Model using the 4-node plane stress element, CPS4.

[viscorod_t3d2.inp](#)

Model using the 2-node truss element, T3D2.

[viscorod_postoutput.inp](#)

[*POST OUTPUT](#) job for the restart file generated in viscorod_largestrain.inp.

References

- Collingwood, G. A., E. B. Becker, and T. Miller, *User's Manual for the TEXVISC Computer Program*, Morton Thiokol, Inc., Document Numbers U-85-4550A and U-85-4550B, 1985.
- Pipkin, A. C., *Lectures on Viscoelasticity Theory*, Springer Verlag, New York, 1972.

Figures

Figure 3.1.1–1 Spring and dashpot model of viscoelastic rod.

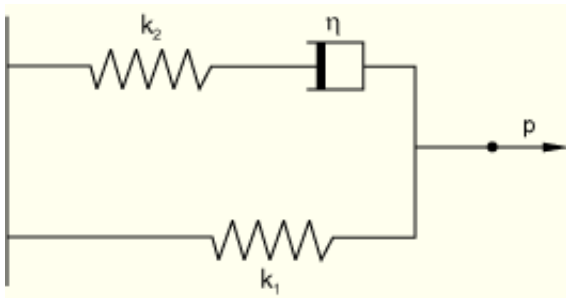


Figure 3.1.1–2 Time history of strain in the direction of load for viscoelastic rod.

—— ABAQUS
- - - PREDICTED VALUE

MMIN 0.000E+00
MMAX 5.000E+01
YMIN 1.000E-02
YMAX 9.939E-02

