Problem 2.c ¶

Test the orthogonality of the system developed with the Lagrangian framework including calculating the mode shapes and natural frequencies

In [21]:
```
from sympy import *
from sympy import symbols
from scipy import linalg as LA
import scipy as sp
import numpy as np
import math
# from sympy.interactive.printing import init_printing
init_printing(use_unicode = True)

from sympy.matrices import Matrix, eye, zeros, ones, diag, GramSchmidt
```

In [21]:
```
M, L, g =symbols('M L g')
```

Define Mass Matrix

In [22]:
```
mass = Matrix(([3, 2], [1, 1]))
mass
```

Out[22]:
$$\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

Define Stiffness Matrix

In [23]:
```
stiff = Matrix(([-3, 0], [0, -1]))
stiff = g / L * stiff
stiff
```

Out[23]:
$$\begin{bmatrix} -\frac{3g}{L} & 0 \\ 0 & -\frac{g}{L} \end{bmatrix}$$

Numerical Analysis

In [43]:
```
M = sp.array([[3, 2],[1, 1]])
K = sp.array([[-3, 0], [0, -1]])
M_inv = LA.inv(M)
```

Implement a invert and shift pocedure to obtain the eigenproblem

In [44]:
```
A = M_inv.dot(K)
A
```

Out[44]:
```
array([[-3.,  2.],
       [ 3., -3.]])
```

In [45]:
```
lamda, p = LA.eig(A)
```

p => eigenvectors (modeshapes), normalized

In [46]:
```
p_t = np.transpose(p)
p_t
```

Out[46]:
```
array([[ 0.63245553,  0.77459667],
       [-0.63245553,  0.77459667]])
```

check that eigenvectors are orthonormal => [p][p]T = I; i.e., transpose = inverse

In [53]:
```
p.dot(p_t)
```

Out[53]:
```
array([[  8.00000000e-01,  -5.55111512e-17],
       [ -5.55111512e-17,   1.20000000e+00]])
```

lamda => eigenvalues (natural frequency - omega squared). Note that matrix is not positive definite.

In [54]:
```
lamda
```

Out[54]:
```
array([-0.55051026+0.j, -5.44948974+0.j])
```

Mode Shape associated with first Natural Frequency (omega squared)

In [58]:
```
phi_1 = p_t[0,:]
phi_1
```

Out[58]:
```
array([ 0.63245553,  0.77459667])
```

Mode Shape associated with second Natural Frequency (omega squared)

In [59]:
```
phi_2 = p_t[1,:]
phi_2
```

Out[59]:
```
array([-0.63245553,  0.77459667])
```