CBE-521, Fall 2015

Homework No. 1

Due Date: Tuesday, November 17th, 2015 in class

1. Calculate the average linear velocity and the bulk flow rate of water at 293°K for a cylindrical nanocapillary with diameter 500 nm and length 1 cm. The applied pressure is 5 atm. (The viscosity of water is 9.93×10^{-4} Pa s).

2. Washburn equation for a horizontal capillary can be written in the form

$$\langle v \rangle = \frac{dL}{dt} = \frac{\gamma R}{4\mu L}$$

Derive expression for the time dependencies of the length of travel $L(T)$ and the average velocity of capillary driven fluid motion $\langle v\ t \rangle$.

3. The surface tension of pure water at room temperature is equal to 72 mN/m. Calculate the pressure drop at the water surface in a capillary with radius 0.5 mm. Assume perfect wetting of the walls.

4. Using the correct expression for the potential distributions (and low potential approximations), derive relationships for the surface charges at the solid liquid interface for a

   (a) single double layer
   (b) spherical double layer
   (c) single cylindrical double layer (*Hint:* $\frac{dK_0\ x}{dx} = -K_1\ x$ -- Modified *K* Bessel function of first order)
   (d) slit shaped channel
   (e) cylindrical capillary (*Hint:* $\frac{dI_0\ x}{dx} = -I_1\ x$ Modified *I* Bessel function of first order)

5. A particle is suspended in KCl solution with ionic strength equal to 0.001 M. When subjected to electric field with strength of 2000 V/m the particle moves with a velocity of 130 μm/s. Calculate the ζ-potential at room temperature (T = 298° K) if the particle radius is
   a. 500 nm
   b. 1 nm
   c. 10 nm

(ε = 78.25, $\varepsilon_0 = 8.854\times10^{-12}$ J^{-1} C^2 m^{-1}, η = 0.001 Pa s)

6. A cylindrical capillary filled with 0.01 M NaCl solution and has ζ-potential equal to -80 mV. The length of the capillary is 1m and its diameter is 1 mm.
    a. Check the validity of the Smoluchowski model for this dimensions and ionic strength.
    b. Calculate the electroosmotic linear and volumetric flow rates if a potential difference of 1000 V is applied at both ends.

($T = 298^{o}$ K, see above for the constants)