ME562 - Constitutive Modeling Course Notes (Feb. 17, 2015)

March 6, 2015

1 Experimental Loading Paths and Additional Stress Parameters

Buck is basing the premise of this course on the principle that we are analyzing a material with somewhat unique properties that are not represented in an existing constitutive equation. For example, no satisfactory constitutive equations exist in Sierra, Abaqus, etc.

1.1 How to determine if a constitutive equation is suitable?

You must compare experimental data to the results predicted from the constitutive model. How many different stress paths are needed to validate a particular constitutive equation... an infinite amount. However, we typically only have one or two if we are lucky! You must check the constitutive equation against data to validate that it is of use. If an existing version of a constitutive equation is not satisfactory, you must develop your own version and then validate it! This assumes your constitutive equation represents certain features that are not currently represented by existing models. For example, if you only have access to an isotropic model but experimental results show that the material of interest is anisotropic, then you may need to develop a new constitutive equation.

With these ideas in mind, we must be aware of how experimental data is obtained, and that is why we are reviewing stress paths.

1.1.1 How constitutive equations are written in "big" codes

Typically the constitutive equation subroutine is called with strain increments, from which stress increments are computed. For complicated constitutive equations, the calculation of stress increments is very computationally expensive. Once the stress increments are computed, the stresses are updated with this increments. On the contrary, experimental data often has prescribed stress increments, not strain; therefore, we must adjust our calculations to accommodate this fact. Also, a stress or strain increment does not necessarily imply transient behavior (i.e. varying with time). Even in a static analysis, stresses will be modeled as increasing monotonically over a defined number of increments and with each stress increment is an associated strain increment... nothing happens instantaneously.

1.2 Underlying assumptions

Almost all constitutive equations implicitly assume that stress and strain are homogenous (e.g., values do not vary throughout the material). This assumption means that stress components and all strain components are the same! This assumption is violated typically near the limit point, which is the maximum stress that a material can maintain, and this assumption is definitely violated when a material begins to soften (e.g., stress decreases and the strain rate increases). The homogeneity assumption is violated near the limit point because localization of stresses and strains occurs e.g., necking, bulging, or shear banding.

Experimentalists can get around violated this assumption by measuring displacements. This way, strains may be defined in such a way that averaged over the entire body e.g., by calculating strains via LVDTs at the end of the specimen rather than using strain rosettes over small regions. If LVDTs are used, then the experimentalists reports AVERAGE strains (displacement over the length)! Another alternative would be to present displacement (not strain) versus force (no stress).

1.3 Common experimental paths and features of experimental data

1.3.1 Uniaxial stress in tension (see hand notes page 11-2)

Commonly applied to metals (cohesive materials), that have strength derived from metallic bonds (i.e., an electron cloud) that have isotropic strength (i.e., not directional dependent). For this test, $\sigma_{11} \neq 0$ (where the σ_{11} is in the axial direction) and all other stress components are zero. Additionally, σ_{11} is assumed to be homogenous (uniform) across the center of the specimen.

Note, the calculation of stress in the region of a loading or confining device (grips on end caps) is nearly impossible without the use if the FEM. But that's okay, because are are not concerned with the ends of specimen but rather the middle where stresses and strains are assumed uniform. Because the surface traction in the region of concern is zero (a boundary condition), $\tau = \underline{\mathbf{0}}$, then $\sigma_{22} = \sigma_{33} = \sigma_{12} = \sigma_{13} = \sigma_{23} = 0$ at the surface of the material (note, the $\sigma_{23} = 0$ argument is made based on symmetry of the horizontal plane). The argument is then that because the cross-sectional region of concern is so small, the internal stresses will not have sufficient area to grow. Therefore, we want the diameter of width of the specimen to be small over a large length ($\frac{D}{L} << 1$) . If the diameter is not small, stresses not aligned with σ_{11}may develop and violate assumptions.

1.3.2 Uniaxial stress in compression

Typically a length to diameter ration $>> 2$ is used. Effort should be made to minimize end effects, i.e., minimize the friction between loading platens and specimen. Shear tractions are inevitable

1.3.3 Hydrostatic Compression

1.3.4 Triaxial Compression

1.3.5 Triaxial Extension

1.3.6 Torsion - Pure Shear

1.3.7 Uniaxial Strain

1.3.8 One and Two Point Bending Tests

1.3.9 Creep and Relaxation Tests

1.3.10 Hopkinson Bar and Flyer Plate Tests

2 Terminology

Enhanced Ductility: occurs when the deformation becomes more ductile (i.e. greater plastic deformation). This occurs in geologic materials when the mean pressure is increased. This implies that the amount of strain energy stored up to the limit point in greater with greater mean pressures.

Hydrostat: loading path during hydrostatic compression