Assignment 1

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ME-500: Numerical Methods in Mechanical Engineering

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1 Summary of Relevant Theory

A numerical routine was written to approximate the definite integral of a function based on the Trapezoidal rule, and a convergence analysis was then performed. However; I am unsure of how to compare the theoretical rate of convergence to the numerical rate I computed. Additionally, I was unable to write a working Gauss quadrature routine; therefore, an available code was used to perform the Gauss quadrature of a third order polynomial.

2 Program General Trapezoidal Rule

The general trapezoidal rule (Equation 1) was utilized to calculate an numerical approximation to the definite integral (quadrature) of an arbitrarily chosen Function 2.

$$I_{num} = \alpha f(a) + (1-\alpha) f(b), \tag{1}$$

$$f(x) = sin(x) + x \tag{2}$$

The problem domain, $x \in [0, 10]$, was divided into 15 subdomains, where a and b in Equation 1 represent the upper and lower bounds of each subdomain, respectively. The analytical solution to the definite integral was calculated (Equation 3) and quadrature estimates with $\alpha = 0, 0.5, 1$ resulted in values of $I_{num} = 48.62, 51.77, 54.92$, respectively. Additionally, approximations for each subdomain are shown in Figure 1, where each point is centered at the horizontal midpoint of its respective subdomain.

$$\int_0^{10} sin(x) + x \, \mathrm{d}x = 51.84 \tag{3}$$

These results show that the trapezoidal rule most closely approximates the function when $\alpha = 0.5$, and quadrature results for $\alpha = 0$ and 1 proivde approximately the same error but on opposite sides of the analytical solution. The Python code for this algorithm is shown in the attached code listing.

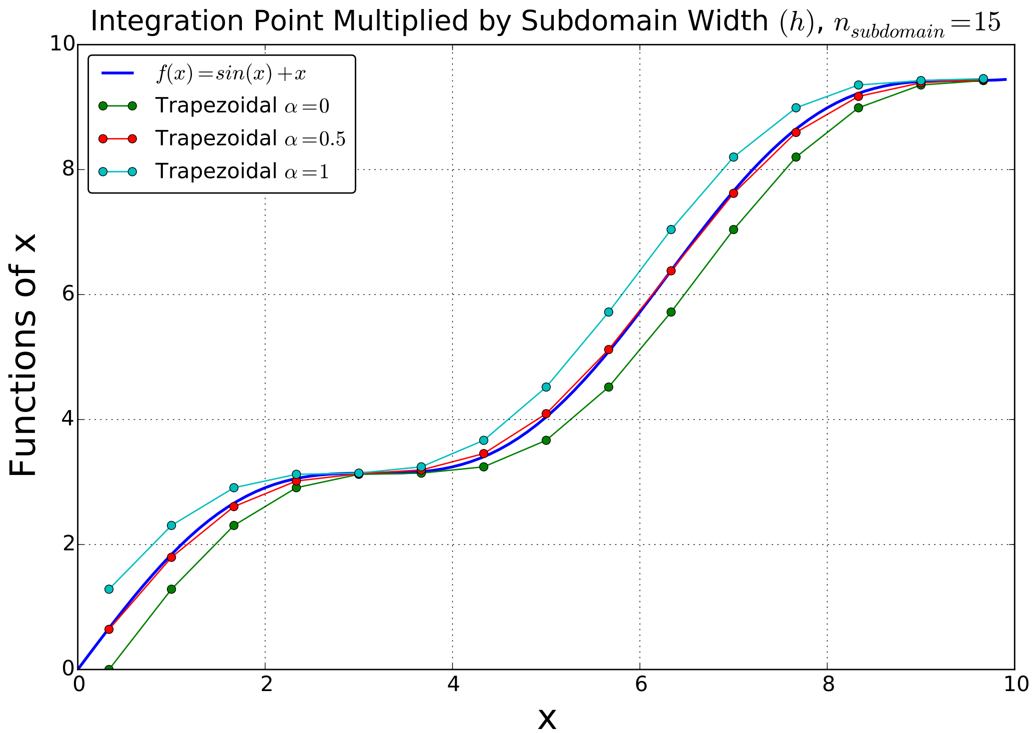

Figure 1: Results of the approximation to Equation 2.

3 Determine the Rate of Convergence

The same function (Equation 2) was chosen with the domain of, $x \in [0, 10]$, and the exact integral to this function over the defined domain was calculated as $I_{ann} = 51.84$. Using this value, the error was defined as:

$$Error = |I_{ann} - I_{num}|$$

The error was calculated over a range of subdomains ranging from 1 to 64, with the corresponding suddomain legth (h) ranging from 10 down to 0.16. Figure 2 shows the results of the convergence analysis. The highest rate of convergence clearly occurs when $\alpha = 0.5$ with a value of approximately 2, the convergence rate for $\alpha = 0$ and 1 were approximately unity (values shown on Figure 2).

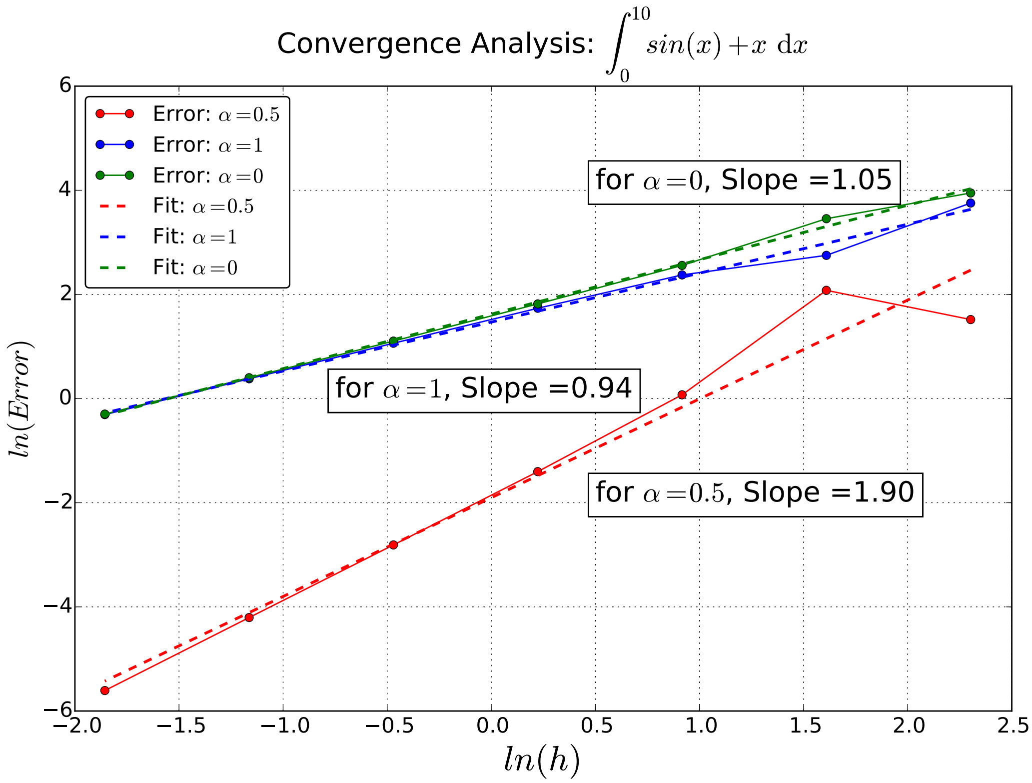

Figure 2: Convergence analysis.

I am unsure of how to compare the numerically calculated rate of convergence (slopes listed on plot) to the theoretical values. This is because the theoretical upper bound for the rate of convergence was said to be of the order of the polynomial function plus one ($n_p + 1$); however, the function I ingetrated was not a polynomial.

4 Gauss Quadrature

Third, fourth, fifth, and sixth-order taylor series expansions (TSE) of Equation 2 about $x = 5$ were performed to obtain the functions shown in Figure 3. The polynomial obtained from the third-order expansion is shown in Equation 4.

$$f(x) = -0.047x^3 + 1.189x^2 - 7.056x + 15.12 \tag{4}$$

I wrote a third-order Gauss quadrature routine ($GaussQuad\_3$) and evaluated the error associated each of the different TSE polynomials over a range of subdomain widths. Results of analysis hown in Figure4. These results show that a negligible amount of error was calculated when my third-order quadrature routine evaluated a third-order polynomail, hence the rate of convergence was zero. For higher order polynomials (fourth, fifth, and sixth), the rate of convergence was appears to be four, but when calculated these values were 3.6 and 2.4 for fifth and sixth-order polynomials, respectively.

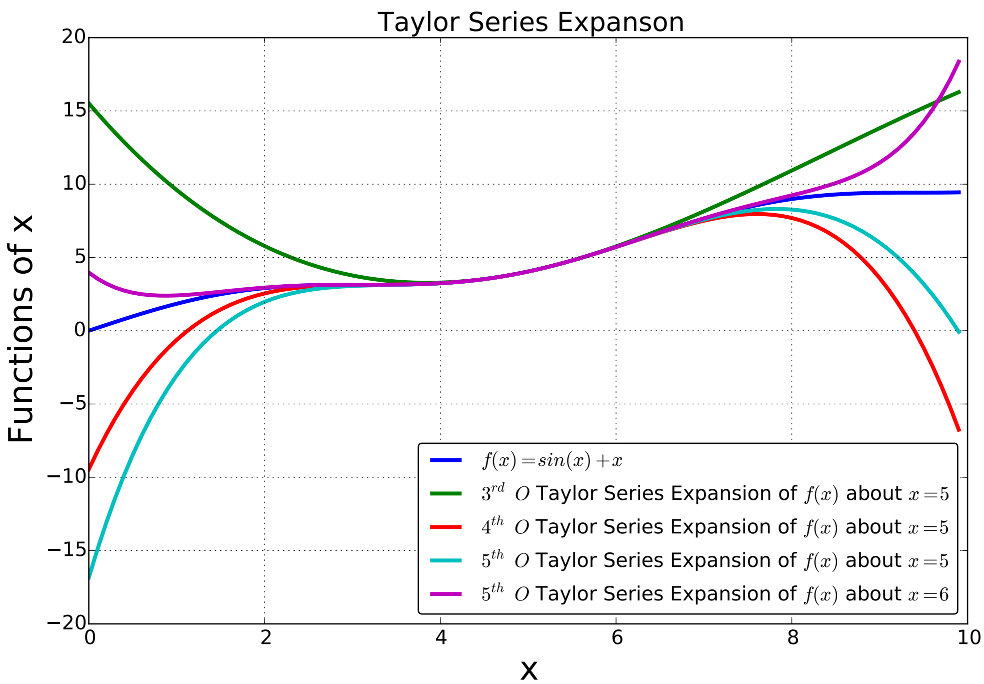

Figure 3: Taylor series expansions of Equation 2.

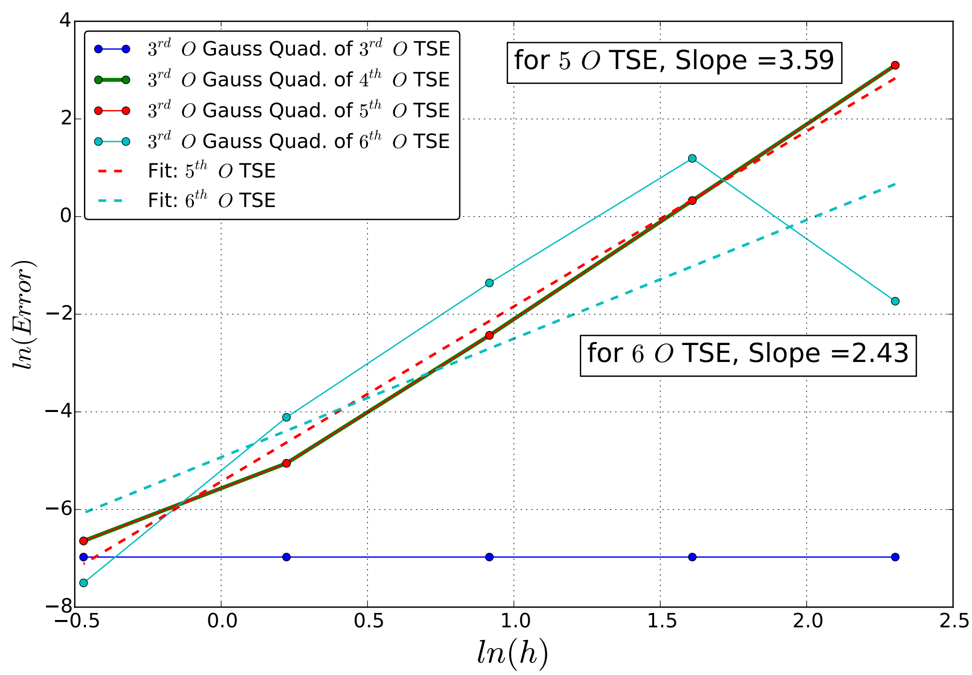

Figure 4: Analysis of Gauss quadrature of Equation 4.