Solve $\frac{d^2 y(x)}{dx^2} - c\, y(x) = 0$:

Assume a solution will be proportional to $e^{\lambda x}$ for some constant λ.

Substitute $y(x) = e^{\lambda x}$ into the differential equation:

$$\frac{d^2}{d x^2}\left(e^{\lambda x}\right) - c\, e^{\lambda x} = 0$$

Substitute $\frac{d^2}{dx^2}\left(e^{\lambda x}\right) = \lambda^2 e^{\lambda x}$:

$$\lambda^2 e^{\lambda x} - c\, e^{\lambda x} = 0$$

Factor out $e^{\lambda x}$:

$$\left(-c + \lambda^2\right) e^{\lambda x} = 0$$

Since $e^{\lambda x} \neq 0$ for any finite λ, the zeros must come from the polynomial:

$$-c + \lambda^2 = 0$$

Solve for λ:

$\lambda = \sqrt{c}$ or $\lambda = -\sqrt{c}$

The root $\lambda = -\sqrt{c}$ gives $y_1(x) = k_1\, e^{-\sqrt{c}\, x}$ as a solution, where k_1 is an arbitrary constant.

The root $\lambda = \sqrt{c}$ gives $y_2(x) = k_2\, e^{\sqrt{c}\, x}$ as a solution, where k_2 is an arbitrary constant.

The general solution is the sum of the above solutions:

Answer:

$$y(x) = y_1(x) + y_2(x) = \frac{k_1}{e^{\sqrt{c}\, x}} + k_2\, e^{\sqrt{c}\, x}$$

Solve $\frac{d^2 y(x)}{dx^2} + c\, y(x) = 0$:

Assume a solution will be proportional to $e^{\lambda x}$ for some constant λ.

Substitute $y(x) = e^{\lambda x}$ into the differential equation:

$$\frac{d^2}{dx^2}\left(e^{\lambda x}\right) + c\, e^{\lambda x} = 0$$

Substitute $\frac{d^2}{dx^2}\left(e^{\lambda x}\right) = \lambda^2 e^{\lambda x}$:

$$\lambda^2 e^{\lambda x} + c\, e^{\lambda x} = 0$$

Factor out $e^{\lambda x}$:

$$\left(c + \lambda^2\right) e^{\lambda x} = 0$$

Since $e^{\lambda x} \neq 0$ for any finite λ, the zeros must come from the polynomial:

$$c + \lambda^2 = 0$$

Solve for λ:

$\lambda = i\sqrt{c}$ or $\lambda = -i\sqrt{c}$

The root $\lambda = -i\sqrt{c}$ gives $y_1(x) = k_1\, e^{-i\sqrt{c}\, x}$ as a solution, where k_1 is an arbitrary constant.

The root $\lambda = i\sqrt{c}$ gives $y_2(x) = k_2\, e^{i\sqrt{c}\, x}$ as a solution, where k_2 is an arbitrary constant.

The general solution is the sum of the above solutions:

$$y(x) = y_1(x) + y_2(x) = \frac{k_1}{e^{i\sqrt{c}\, x}} + k_2\, e^{i\sqrt{c}\, x}$$

Apply Euler's identity $e^{\alpha + i\beta} = e^{\alpha}\cos(\beta) + i\, e^{\alpha}\sin(\beta)$:

$$y(x) = k_1\left(\cos\left(\sqrt{c}\, x\right) - i\sin\left(\sqrt{c}\, x\right)\right) + k_2\left(\cos\left(\sqrt{c}\, x\right) + i\sin\left(\sqrt{c}\, x\right)\right)$$

Regroup terms:

$$y(x) = (k_1 + k_2)\cos\left(\sqrt{c}\, x\right) + i\,(-k_1 + k_2)\sin\left(\sqrt{c}\, x\right)$$

Redefine $k_1 + k_2$ as k_1 and $i\,(k_2 - k_1)$ as k_2, since these are arbitrary constants:

Answer:

$$y(x) = k_1\cos\left(\sqrt{c}\, x\right) + k_2\sin\left(\sqrt{c}\, x\right)$$

Solve $\frac{d^2y(x)}{dx^2} - c\,y(x) = 1$:

The general solution will be the sum of the complementary solution and particular solution.

Find the complementary solution by solving $\frac{d^2y(x)}{dx^2} - c\,y(x) = 0$:

Assume a solution will be proportional to $e^{\lambda x}$ for some constant λ.

Substitute $y(x) = e^{\lambda x}$ into the differential equation:

$$\frac{d^2}{dx^2}\left(e^{\lambda x}\right) - c\,e^{\lambda x} = 0$$

Substitute $\frac{d^2}{dx^2}\left(e^{\lambda x}\right) = \lambda^2 e^{\lambda x}$:

$$\lambda^2 e^{\lambda x} - c\,e^{\lambda x} = 0$$

Factor out $e^{\lambda x}$:

$$\left(-c + \lambda^2\right) e^{\lambda x} = 0$$

Since $e^{\lambda x} \neq 0$ for any finite λ, the zeros must come from the polynomial:

$$-c + \lambda^2 = 0$$

Solve for λ:

$\lambda = \sqrt{c}$ or $\lambda = -\sqrt{c}$

The root $\lambda = -\sqrt{c}$ gives $y_1(x) = k_1 e^{-\sqrt{c}\,x}$ as a solution, where k_1 is an arbitrary constant.

The root $\lambda = \sqrt{c}$ gives $y_2(x) = k_2 e^{\sqrt{c}\,x}$ as a solution, where k_2 is an arbitrary constant.

The general solution is the sum of the above solutions:

$$y(x) = y_1(x) + y_2(x) = \frac{k_1}{e^{\sqrt{c}\,x}} + k_2 e^{\sqrt{c}\,x}$$

Determine the particular solution to $\frac{d^2y(x)}{dx^2} - c\,y(x) = 1$ by the method of undetermined coefficients:

The particular solution to $\frac{d^2y(x)}{dx^2} - c\,y(x) = 1$ is of the form:

$$y_p(x) = a_1$$

Solve for the unknown constant a_1:

Compute $\frac{d^2y_p(x)}{dx^2}$:

$$\begin{aligned}\frac{d^2y_p(x)}{dx^2} &= \frac{d^2}{dx^2}(a_1)\\ &= 0\end{aligned}$$

Substitute the particular solution $y_p(x)$ into the differential equation:

$$\frac{d^2y_p(x)}{dx^2} - c\,y_p(x) = 1$$

$$-c\,a_1 = 1$$

Solve the equation:

$$a_1 = -\frac{1}{c}$$

Substitute a_1 into $y_p(x) = a_1$:

$$y_p(x) = -\frac{1}{c}$$

The general solution is:

Answer:

$$y(x) = y_c(x) + y_p(x) = -\frac{1}{c} + \frac{k_1}{e^{\sqrt{c}\,x}} + k_2 e^{\sqrt{c}\,x}$$

Solve $\frac{d^2 y(x)}{dx^2} + c\, y(x) = 1$:

The general solution will be the sum of the complementary solution and particular solution.

Find the complementary solution by solving $\frac{d^2 y(x)}{dx^2} + c\, y(x) = 0$:

Assume a solution will be proportional to $e^{\lambda x}$ for some constant λ.

Substitute $y(x) = e^{\lambda x}$ into the differential equation:

$$\frac{d^2}{dx^2}\left(e^{\lambda x}\right) + c\, e^{\lambda x} = 0$$

Substitute $\frac{d^2}{dx^2}\left(e^{\lambda x}\right) = \lambda^2 e^{\lambda x}$:

$$\lambda^2 e^{\lambda x} + c\, e^{\lambda x} = 0$$

Factor out $e^{\lambda x}$:

$$\left(c + \lambda^2\right) e^{\lambda x} = 0$$

Since $e^{\lambda x} \neq 0$ for any finite λ, the zeros must come from the polynomial:

$$c + \lambda^2 = 0$$

Solve for λ:

$\lambda = i\sqrt{c}$ or $\lambda = -i\sqrt{c}$

The root $\lambda = -i\sqrt{c}$ gives $y_1(x) = k_1 e^{-i\sqrt{c}\, x}$ as a solution, where k_1 is an arbitrary constant.

The root $\lambda = i\sqrt{c}$ gives $y_2(x) = k_2 e^{i\sqrt{c}\, x}$ as a solution, where k_2 is an arbitrary constant.

The general solution is the sum of the above solutions:

$$y(x) = y_1(x) + y_2(x) = \frac{k_1}{e^{i\sqrt{c}\, x}} + k_2 e^{i\sqrt{c}\, x}$$

Apply Euler's identity $e^{\alpha + i\beta} = e^{\alpha}\cos(\beta) + i\, e^{\alpha}\sin(\beta)$:

$$y(x) = k_1\left(\cos\left(\sqrt{c}\, x\right) - i\sin\left(\sqrt{c}\, x\right)\right) + k_2\left(\cos\left(\sqrt{c}\, x\right) + i\sin\left(\sqrt{c}\, x\right)\right)$$

Regroup terms:

$$y(x) = (k_1 + k_2)\cos\left(\sqrt{c}\, x\right) + i\,(-k_1 + k_2)\sin\left(\sqrt{c}\, x\right)$$

Redefine $k_1 + k_2$ as k_1 and $i\,(k_2 - k_1)$ as k_2, since these are arbitrary constants:

$$y(x) = k_1\cos\left(\sqrt{c}\, x\right) + k_2\sin\left(\sqrt{c}\, x\right)$$

Determine the particular solution to $c\, y(x) + \frac{d^2 y(x)}{dx^2} = 1$ by the method of undetermined coefficients:

The particular solution to $c\, y(x) + \frac{d^2 y(x)}{dx^2} = 1$ is of the form:

$$y_p(x) = a_1$$

Solve for the unknown constant a_1:

Compute $\frac{d^2 y_p(x)}{dx^2}$:

$$\frac{d^2 y_p(x)}{dx^2} = \frac{d^2}{dx^2}(a_1)$$
$$= 0$$

Substitute the particular solution $y_p(x)$ into the differential equation:

$$\frac{d^2 y_p(x)}{dx^2} + c\, y_p(x) = 1$$
$$c\, a_1 = 1$$

Solve the equation:

$$a_1 = \frac{1}{c}$$

Substitute a_1 into $y_p(x) = a_1$:

$$y_p(x) = \frac{1}{c}$$

The general solution is:

Answer:

$$y(x) = y_c(x) + y_p(x) = \frac{1}{c} + k_1\cos\left(\sqrt{c}\, x\right) + k_2\sin\left(\sqrt{c}\, x\right)$$