The primary tasks of this assignment, are to develop a working finite difference code for both steady-state and transient problems.

1. Provide a written summary of the theoretical formulation of the material related to your numerical investigations of finite difference solutions to the ODE $(k\phi,_x),_x + Q = 0$, numerical solutions to a one-degree-of-freedon ODE with time as the independent variable, and the partial differential equation $C\phi,_t = (k\phi,_x),_x + Q$.

2. For the equation $(k\phi,_x),_x + Q = 0$ do the following: (a) Choose data for a relatively simple problem for which you know the analytical solution with the following restrictions: (i) the coefficient function is constant, (ii) One boundary condition consists of a prescribed value for the primary variable; the other boundary condition consists of a prescribed value of the gradient of the primary variable, and (iii) the forcing function results in a solution of sufficient continuity that the part of your theoretical summary that provides the rate of converence is applicable.

(b) Write a finite difference algorithm based on a uniform mesh that provides approximate solutions.
(c) Provide plots that compare numerical solutions with the analytical solution for different mesh sizes.
(d) Provide a plot that shows the numerical rate of convergence and indicate how it compares with the theoretical rate.

3. (a) Consider the single-degree-of-freedom problem

$$\dot{T} + \Lambda T = 0 \qquad T(0) = 2; \qquad t \in [0, 8]$$

with $\Lambda = 1$. Overlay plots of the exact solution and approximate solutions obtained from the numerical integration algorithm based on the general trapezoidal rule for the following values of α and time step, s:

(a) $\alpha = 0$ $\quad$ s = 4, 2, 1, 0.5, 0.25, 0.125
(b) $\alpha = 1/2$ $\quad$ s = 4, 2, 1, 0.5, 0.25, 0.125
(c) $\alpha = 1$ $\quad$ s = 4, 2, 1, 0.5, 0.25, 0.125

(b) Now consider the transient problem when a smooth forcing term is present:

$$C\dot{T} + KT = F(t) \qquad C = 1 \qquad K = 2$$
$$F(t) = \sin \Omega t \qquad T(0) = 0 \qquad \Omega = 10$$

Let $t_p = 2\pi / \Omega$ be the period of the forcing function and let s_c be the critical time step for $\alpha = 0$. Obtain numerical solutions for three periods of time with time steps ranging from s_c up to $t_p/4$ for the trapezoidal rules defined by $\alpha = 0.5, \alpha = 0.75$ and $\alpha = 1$ and overlay plots of these solutions with the analytical solution.

4. Now combine the steady-state finite difference algorithm with time integration to obtain approximate solutions to the governing equation $C\phi,_t = (k\phi,_x),_x + Q$.

Common data for the two problems are:

Domains:	$0 \le x \le 1$	$t \ge 0$
Coefficient fns.	$C = 4$	$k = 2$
Forcing function	$F = 0$	
Initial Condition:	$T = 0$ for $0 \le x \le 1$	

Boundary conditions for the two problems are:

(i) $T(0) = 2 \quad T(1) = 0$ (ii) $T(0) = \sin 10t \quad T(1) = 0$

For problem (i) how do you resolve the contradiction beween the boundary condition and the initial condition at x = 0?