Final Exam

CBE 521, Fall 2014

Take-Home, Due on Tuesday December 8th at 11 am in FEC 235.

NOTE: Remember to state all of your assumptions and steps. Work as organized as possible! Good luck.

1. The mean square displacement $\langle x^2 \rangle$ of a Brownian particle in one dimension is defined by Einstein as

$$\left\langle x^2 \right\rangle = \int_{-\infty}^{\infty} x^2 f\left(x,t\right) dx$$

(a) Find $\langle x^2 \rangle$ if

$$f\left(x,t\right) = \frac{e^{\frac{x^2}{4Dt}}}{\sqrt{4\pi Dt}}$$

You may need $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$

(b) Show that the mean displacement

$$\left\langle x \right\rangle = \int_{-\infty}^{\infty} x f\left(x,t\right) dx = 0$$

(c) If the particle motion occurs in an infinite half-space then the above integral becomes

$$\left\langle x \right\rangle = \int_{0}^{\infty} x f\left(x,t\right) dx$$

Find $\langle x \rangle$.

2. A measurement of the time correlation function by Dynamic Light Scattering gave the data summarized in the table on the right.

Find the effective diffusion coefficient if the wave number is q = 1.87x10^7. Assume that the sample is mono disperse.

t	$g^{(1)}(t)$
0.000	1.000
0.001	0.498
0.002	0.248
0.003	0.124
0.004	0.061
0.005	0.030
0.006	0.015
0.007	0.008
0.008	0.004
0.009	0.002
0.001	0.001

3. A measurement of the time correlation function by Dynamic Light Scattering gave the data summarized in the table on the right.

   The wave number is $q = 1.87 \times 10^7$. The sample is bidisperse. Find the effective diffusion coefficients for the two particle species.

   Hint: assume that the sample can be represented by a half sum of tow exponential time correlation functions.

t	$g^{(1)}(t)$
0.000	1.000
0.001	0.373
0.002	0.155
0.003	0.069
0.004	0.033
0.005	0.016
0.006	0.008
0.007	0.004
0.008	0.002
0.009	0.001