1. Provide a written summary of material related to the eigenproblem that is relevant to the following numerical problems. Be sure to include definitions.

2. Two matrices are given as follows:

$$[A] = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \qquad\qquad [A] = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Do the following by hand for each matrix [A]:

(a) Obtain the eigenpairs.
(b) Construct the modal matrix.
(c) Determine $[M^\circ]^T[A][M^\circ]$, $[M^\circ]^T[M^\circ]$ and $\sum_i \lambda_i \{e^i\}\{e^i\}^T$.
(d) Determine the rank, the range and the null space.

3. Consider the following matrix:

$$[A] = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

(a) Find a subroutine that provides eigenvalues and eigenvectors. Use the subroutine to obtain the eigensystem for [A]. Construct the matrices

(i) $[M^\circ]$ & $[\lambda]$, (ii) $[M^\circ]^T[M^\circ]$, (iii) $[A^*] = [M^0][\lambda][M^\circ]^T$,

(iv) $[A^*]^{-1} = [M^0][\lambda]^{-1}[M^\circ]^T$, and (v) $[A^*]^{-1}[A]$

(b) Use Gershgorin's theorem to obtain bounds on the eigenvalues of [A].

(c) Obtain the Rayleigh quotient R({v}) for <v> = <1, 2, 3, 2, 1>. Is the inequality $\lambda_1 \le R \le \lambda_n$ satisfied?

(d) Determine $\|[A]\|_1$, $\|[A]\|_2$ and $\|[A]\|_\infty$ Do these norms form an upper bound to the maximum eigenvalue?

(e) What is the condition number of [A]?

(f) Pick a solution $\{x\}$ for the linear algebraic problem and obtain the vector $\{b\}=[A]\{x\}$. Now assume $\{x\}$ is unknown and obtain one-mode, three-mode and five-mode solutions to the linear algebraic problem $[A]\{x\}=\{b\}$. Use the exact solution and an error norm of your choice to provide a measure of error for each case.

4. Apply the results of this chapter to Hilbert matrixes, $[H]_{nxn}$, defined in Problem 6, pg. 141, of the notes. Consider a range for n, (3 to 10 or higher, say). For representative values of n do the following:

(a) Obtain the eigenvalues.
(b) Determine the condition number.
(c) Using a direct solver, obtain a solution to the linear algebraic problem with {b} chosen such that you know the answer. Determine the error in your solution.
(d) Can you see any correlation of error to condition number?