ME 512 CONTINUUM MECHANICS Fall 2014 ASSIGNMENT 6

1. A force of magnitude F acts in a direction radially away from the origin at a point $\left(\frac{2a}{3}, \frac{b}{3}, \frac{2c}{3}\right)$ on the surface of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. Determine the component of the force in the direction of the normal to the surface.

2. (i) If $\mathbf{r}$ is the position vector, use the divergence theorem to express $\int_{\partial R} \mathbf{r} \cdot \mathbf{n}\, ds$ in terms of the volume of the region R.

(ii) Actually perform the surface integral for a unit cube with one corner at the reference point of a Euclidean point space.

3. A plane area in the x_1 - x_2 plane is bounded by the square with corners (0, 0), (b, 0), (b, b), (0, b). A vector $\mathbf{v}$ has components $v_1 = Ax_2, v_2 = Bx_2, v_3 = 0$ where A and B are constants. Verify that Stokes' theorem holds.

4. A vector $\mathbf{v}$ has components $v_1 = Ax_2, v_2 = Bx_2, v_3 = Cx_1x_3$ where A, B and C are constants.

(i) A regular hexahedron has one set of vertices defined by the coordinates $(x_1, x_2, x_3) \Rightarrow$ (0, 0, 0), (a, 0, 0), (a, b, 0), (0, b, 0) and a second set defined by $(x_1, x_2, x_3) \Rightarrow$ (0, 0, c), (a, 0, c), (a, b, c), (0, b, c).
Verify that the divergence theorem holds.

(ii) Now suppose the body is a tetrahedron with one set of corners located at $(x_1, x_2, x_3) \Rightarrow$ (0, 0, 0), (a, 0, 0), (a, b, 0), (0, b, 0) and a second set at $(x_1, x_2, x_3) \Rightarrow$ (0, 0, c), (0, b, c).
Verify that the divergence theorem holds.

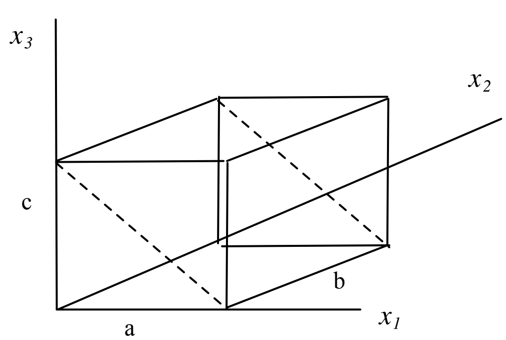

Fig. for Problem 4.