In a numerical course I often ask my students to construct a “nontrivial” transformation matrix and, inevitably, they have a difficult time. Here I want you to construct one by using the steps I outline below. Each of you will have a different transformation matrix so you will have to perform certain steps to suggest your construction is valid but you may still not have a correct result, but this is how the real world operates. You will need a calculator but use absolutely no more than three significant figures.

Assume the basis $\boldsymbol{e}_i$ is known and components of vectors in Problem 1 are given with respect to this basis.

1. Construct a new basis $\boldsymbol{E}_i$, and the transformation matrix $\left[{}^{E}a^{e}\right]$ by performing the following steps:

(i) Pick three distinct nonzero numbers as components v_i of a vector $\boldsymbol{v}$.
(ii) Construct a second vector, $\boldsymbol{u}$, by choosing arbitrary numbers for the first two components u_1 and u_2. Obtain the third component so that the equation $\boldsymbol{u}\cdot\boldsymbol{v}=0$ is satisfied.
(iii)Construct a new orthonormal basis, $\boldsymbol{E}_i$, as follows:

$$\boldsymbol{E}_1=\frac{\boldsymbol{v}}{|\boldsymbol{v}|} \qquad \boldsymbol{E}_2=\frac{\boldsymbol{u}}{|\boldsymbol{u}|} \qquad \boldsymbol{E}_3=\boldsymbol{E}_1\times\boldsymbol{E}_2$$

(iv) Use the components of these vectors appropriately to construct the matrix $\left[{}^{E}a^{e}\right]$.

2. Show that your transformation matrix is orthogonal and the determinant is $+1$.

3. Choose three nonzero distinct numbers as the components, w_i^E, of a vector, $\boldsymbol{w}$, with respect to the basis $\boldsymbol{E}_i$. Find the components w_i^e.

4. Show that $w_i^e w_i^e = w_A^E w_A^E$.

5. Pick components, Z_i^e, with respect to the basis, $\boldsymbol{e}_i$, for another vector, $\boldsymbol{Z}$.

Evaluate $\boldsymbol{w}\cdot\boldsymbol{Z}$.