```
In [2]:  import sys
         from scipy import linalg as LA
         import math
         import numpy as np
         import blfunc as bl
         from IPython.display import display

         from sympy import *
         from sympy import symbols
         from sympy import init_printing
         init_printing()

         np.set_printoptions(precision= 4, suppress = True)
```

```
In [3]:  x = symbols('x')
         exact = sin(1.8 * pi * x)
         phi_1 = x*(1-0.5*x)
         phi_2 = x*(1-x)**2
         phi_3 = x
```

```
In [4]:  e_col = (exact - (-17.17*phi_1 + 14.67*phi_2))**2
         e_sub = (exact - (1.5463*phi_1 - 7.5762*phi_2))**2
         e_ls = (exact - (-11.002*phi_1 + 16.2186*phi_2))**2
         e_g1 = (exact - (7.9573*phi_1))**2
         e_g2 = (exact - (-9.2084*phi_1 + 13.1364*phi_2))**2
         e_g3 = (exact - (-10.6622*phi_1 + 14.3176*phi_2+ 0.8048*phi_3))**2
```

```
In [5]:  L2_col = float((integrate(e_col,(x, 0, 1))))
         L2_sub = float((integrate(e_sub,(x, 0, 1))))
         L2_ls = float((integrate(e_ls,(x, 0, 1))))
         L2_g1 = float((integrate(e_g1,(x, 0, 1))))
         L2_g2 = float((integrate(e_g2,(x, 0, 1))))
         L2_g3 = float((integrate(e_g3,(x, 0, 1))))
         print L2_col
         print L2_sub
         print L2_ls
         print L2_g1
         print L2_g2
         print L2_g3

         25.8724105193
         1.56272736429
         7.59415696183
         10.106136773
         5.23839423731
         5.19646172907
```

```
In [6]:  print L2_col**.5
         print L2_sub**.5
         print L2_ls**.5
         print L2_g1**.5
         print L2_g2**.5
         print L2_g3**.5

         5.08649294891
         1.25009094241
         2.7557498003
         3.17901506335
         2.28875386123
         2.27957490096
```

```
In [7]:  e_col = (exact - (-17.17*phi_1 + 14.67*phi_2))
         e_sub = (exact - (1.5463*phi_1 - 7.5762*phi_2))
         e_ls = (exact - (-11.002*phi_1 + 16.2186*phi_2))
         e_g1 = (exact - (7.9573*phi_1))**2
         e_g2 = (exact - (-9.2084*phi_1 + 13.1364*phi_2))
         e_g3 = (exact - (-10.6622*phi_1 + 14.3176*phi_2+ 0.8048*phi_3))
```

```
In [9]:  col_int = x * diff(e_col,x)**2
```

```
In [10]:  E = (0.5 * integrate(col_int,(x,0,1)))**(0.5)
```

```
In [13]:  col_int = x * diff(e_col,x)**2
          sub_int = x * diff(e_sub,x)**2
          ls_int = x * diff(e_ls,x)**2
          g1_int = x * diff(e_g1,x)**2
          g2_int = x * diff(e_g2,x)**2
          g3_int = x * diff(e_g3,x)**2
```

```
In [14]: col_E = (0.5 * integrate(col_int,(x,0,1)))**(0.5)
         sub_E = (0.5 * integrate(sub_int,(x,0,1)))**(0.5)
         ls_E = (0.5 * integrate(ls_int,(x,0,1)))**(0.5)
         g1_E = (0.5 * integrate(g1_int,(x,0,1)))**(0.5)
         g2_E = (0.5 * integrate(g2_int,(x,0,1)))**(0.5)
         g3_E = (0.5 * integrate(g3_int,(x,0,1)))**(0.5)
```

```
In [17]: print float(col_E)
         print float(sub_E)
         print float(ls_E)
         print float(g1_E)
         print float(g2_E)
         print float(g3_E)
```

```
         3.55634998598
         2.72936389766
         2.58787965101
         19.2735813686
         2.07909111026
         1.99487604937
```

```
In [ ]:
```