1. Suppose **I** denotes the identity tensor and the components of the vectors **u** and **v** and the second-order tensor, **T**, are given as follows:

$$u_i \Rightarrow (1,\ -2,\ 2) \qquad v_i \Rightarrow (-2,\ 1, -3) \qquad T_{pq} \Rightarrow \begin{bmatrix} -1 & 2 & 3 \\ 2 & -2 & 2 \\ 4 & 3 & 4 \end{bmatrix}$$

Consider the following equations:

$$(a)\,(\phi?) = \boldsymbol{u}\cdot\boldsymbol{v} \qquad (b)\,(\phi?) = \boldsymbol{T}\cdot\boldsymbol{u} \qquad (c)\,(\phi?) = \boldsymbol{u}\cdot\boldsymbol{T}^T$$
$$(d)(\phi?) = \boldsymbol{v}\cdot\boldsymbol{T}\cdot\boldsymbol{u} \qquad (e)\,(\phi?) = \boldsymbol{u}\otimes\boldsymbol{v} \qquad \text{(f)}\ (\phi?) = \boldsymbol{I}\cdot\cdot\boldsymbol{T}$$

(i) For each equation, replace ϕ with an appropriate symbol to indicate that the quantity is a scalar, vector or second-order tensor.
(ii) Write each equation in direct notation, indicial notation and matrix notation.
(iii) Find the components of the resulting scalars, vectors, and tensors.
(iv) Determine the components of $\mathbf{T}^{sym}$ and $\mathbf{T}^{sk}$, the symmetric and skew-symmetric parts of **T**.
(v) Determine the components b_i, c_i and d_i where

$$b_i = \frac{1}{2}\varepsilon_{ijk}T_{jk} \qquad c_i = \frac{1}{2}\varepsilon_{ijk}T_{jk}^{sym} \qquad d_i = \frac{1}{2}\varepsilon_{ijk}T_{jk}^{sk}$$

2. Show that the relation $\mathbf{v} = (\mathbf{v}\cdot\mathbf{n})\mathbf{n} + \mathbf{n}\times(\mathbf{v}\times\mathbf{n})$ holds $\forall\ \mathbf{n}$ and that this represents a resolution of **v** into vectors parallel and perpendicular to **n** (a unit vector).

3. Suppose **T** and **U** are second-order tensors.
   (a) Show that $tr(\boldsymbol{T}\cdot\boldsymbol{U}) = tr(\boldsymbol{T}^T\cdot\boldsymbol{U})$ if either **T** or **U** is symmetric.
   (b) Show that $\mathrm{tr}(\mathbf{T}\cdot\mathbf{U}) = 0$ if one of the tensors is skew-symmetric and the other is symmetric.

4. If $\boldsymbol{A}$ is a second-order tensor, and $\boldsymbol{u}, \boldsymbol{v}$ and $\boldsymbol{w}$ are arbitrary vectors, use indicial notation to prove that

$$(\boldsymbol{A}\cdot\boldsymbol{u})\cdot(\boldsymbol{v}\times\boldsymbol{w}) + \boldsymbol{u}\cdot[(\boldsymbol{A}\cdot\boldsymbol{v})\times\boldsymbol{w}] + \boldsymbol{u}\cdot[\boldsymbol{v}\times(\boldsymbol{A}\cdot\boldsymbol{w})] = (tr\boldsymbol{A})\boldsymbol{u}\cdot(\boldsymbol{v}\times\boldsymbol{w}) \quad \forall\ \boldsymbol{u},\boldsymbol{v},\boldsymbol{w}$$

5. The angles between the respective base vectors in two systems are given in the table to the left. The components of a vector **v** and a tensor **T** in the $\mathbf{e}_i$ system are given to the right.

	$\mathbf{E}_1$	$\mathbf{E}_2$	$\mathbf{E}_3$
$\mathbf{e}_1$	90°	45°	135°
$\mathbf{e}_2$	45°	60°	60°
$\mathbf{e}_3$	45°	120°	120°

$$\{v\}^{e} = \begin{Bmatrix} 1 \\ -2 \\ 3 \end{Bmatrix} \qquad \overset{e-e}{[T]} = \begin{bmatrix} 2 & 0 & -3 \\ 0 & 6 & 0 \\ -3 & 0 & 4 \end{bmatrix}$$

(a) Express $\mathbf{e}_i$ in terms of $\mathbf{E}_A$; Express $\mathbf{E}_A$ in terms of $\mathbf{e}_i$. Verify that the basis $\mathbf{E}_A$ is a right-handed orthonormal system.

(b) Obtain the components of the transformation matrix. Leave answers in terms of the square roots of integers. Verify that the matrix is orthonormal.

(c) Find the components of **v** in the $\mathbf{E}_A$ system. Denote these components as $\{v\}^{\mathbf{E}}$. Apply the reverse transformation to show that you revert back to $\{v\}^{\mathbf{e}}$.

(d) Find the components of **T** in the $\mathbf{E}_A$ system, i.e., find $\overset{E-E}{[T]}$.

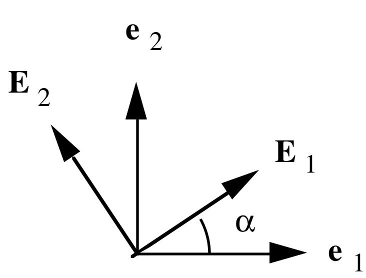
(e) Find the mixed components $\overset{e-E}{[T]}$ and $\overset{E-e}{[T]}$.

6. $\mathbf{E}_A$ is related to $\mathbf{e}_i$ and $\mathbf{g}_p$ is related to $\mathbf{E}_A$ as shown. Obtain the transformation matrices for transforming components from:

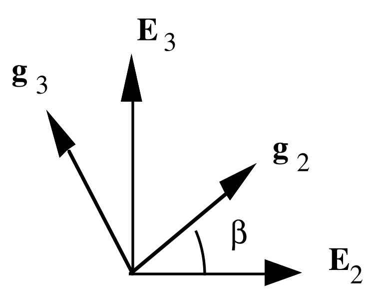
(a) the $\mathbf{e}_i$ basis to the $\mathbf{E}_A$ basis,

(b) the $\mathbf{E}_A$ basis to the $\mathbf{g}_p$ basis, and

(c) the $\mathbf{e}_i$ basis to the $\mathbf{g}_p$ basis.



$\mathbf{E}_3 = \mathbf{e}_3$



$\mathbf{E}_1 = \mathbf{g}_1$