ME 512 CONTINUUM MECHANICS Fall 2014 ASSIGNMENT 1

1. Given $T_{pq} \Rightarrow \begin{bmatrix} -4 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & 1 \end{bmatrix}$ $\quad u_i \Rightarrow (0, -1, 2) \quad v_i \Rightarrow (1, -2, -3)$

(a) What index (indices) should be associated with w in each of the following equations:

$(i)\ w_? = T_{ik}v_k \quad (ii)\ w_? = u_k v_p \quad (iii)\ w_? = T_{km}v_k u_m \quad (iv)\ w_? = T_{ps}u_p \quad (v)\ w_? = T_{rs}u_s v_r$

$(vi)\ w_? = T_{nn} \quad (vii)\ w_? = T_{pq}T_{qr} \quad (viii)\ w_? = T_{pq}T_{pr} \quad (ix)\ w_? = T_{AB}T_{AB} \quad (x)\ w_? = T_{pq}T_{qp}$

(b) For each case obtain the set of numbers associated with $w_?$.

(c) For each case construct the corresponding matrix expression.

2. What is wrong with each of the following indicial equations:

(a) $w_i = b_{ik}u_i v_k$ (b) $\phi = b_{ik}u_i$ (c) $\phi_{jp} = R_{ijkl}T_{kl}u_p$

Give forms that are correct.

3. Show that the ε - δ identity $\varepsilon_{ijk}\,\varepsilon_{irs} = \delta_{jr}\,\delta_{ks} - \delta_{js}\,\delta_{kr}$ holds when the free indices assume the following values:

$(j,k,r,s) = (1,1,1,1),(1,1,1,2),(1,1,1,3),(1,1,2,1),(1,1,2,2),(1,1,2,3)$ and $(1,2,2,3)$.

4. Show that the alternating symbol-determinant identity

$$\varepsilon_{ijk}a_{il}a_{jm}a_{kn} = \varepsilon_{lmn}\big\|[a]\big\|$$

holds when $(l,m,n) = (1,2,3)$.

5. Using the alternating symbol-determinant identity, prove that the determinant of the product of two matrices equals the product of the determinants of the matrices.

6. Use the cofactor matrix approach to find the inverse of $[T]$.