

Solve $\frac{d^2 y(x)}{dx^2} - c y(x) = 0$:

Assume a solution will be proportional to $e^{\lambda x}$ for some constant λ .

Substitute $y(x) = e^{\lambda x}$ into the differential equation:

$$\frac{d^2}{dx^2}(e^{\lambda x}) - c e^{\lambda x} = 0$$

Substitute $\frac{d^2}{dx^2}(e^{\lambda x}) = \lambda^2 e^{\lambda x}$:

$$\lambda^2 e^{\lambda x} - c e^{\lambda x} = 0$$

Factor out $e^{\lambda x}$:

$$(-c + \lambda^2) e^{\lambda x} = 0$$

Since $e^{\lambda x} \neq 0$ for any finite λ , the zeros must come from the polynomial:

$$-c + \lambda^2 = 0$$

Solve for λ :

$$\lambda = \sqrt{c} \text{ or } \lambda = -\sqrt{c}$$

The root $\lambda = -\sqrt{c}$ gives $y_1(x) = k_1 e^{-\sqrt{c} x}$ as a solution, where k_1 is an arbitrary constant.

The root $\lambda = \sqrt{c}$ gives $y_2(x) = k_2 e^{\sqrt{c} x}$ as a solution, where k_2 is an arbitrary constant.

The general solution is the sum of the above solutions:

Answer:

$$y(x) = y_1(x) + y_2(x) = \frac{k_1}{e^{\sqrt{c} x}} + k_2 e^{\sqrt{c} x}$$

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