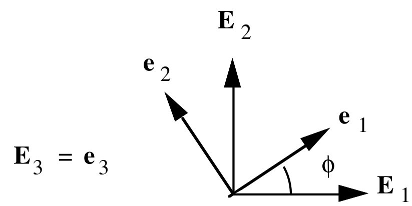
ME 512 CONTINUUM MECHANICS Fall 2014 ASSIGNMENT 7
Due: Th. Oct. 16.

The deformation of a continuous medium is defined by the equations

$$\begin{aligned} x_1 &= \frac{1}{2}(X_1 + X_2)\alpha + \frac{1}{2}(X_1 - X_2)\beta \\ x_2 &= \frac{1}{2}(X_1 + X_2)\alpha - \frac{1}{2}(X_1 - X_2)\beta \\ x_3 &= X_3 \end{aligned} \tag{1}$$

in which α and β are constants and the bases $\mathbf{e}_i$ and $\mathbf{E}_A$ are related as follows:



1. Obtain the transformation matrices for the two bases.

2. Obtain $\boldsymbol{F} = \dfrac{\partial x_i}{\partial X_j} \boldsymbol{e}_i \otimes \boldsymbol{E}_j$; i.e., obtain $\dfrac{\partial x_i}{\partial X_j}$.

3. Invert this expression to obtain $\boldsymbol{F}^{-1} = \overset{E-e}{F_{ij}^{-1}} \boldsymbol{E}_i \otimes \boldsymbol{e}_j$, i.e., obtain $\overset{E-e}{F_{ij}^{-1}}$.

4. Use your transformation matrices to obtain $\overset{E-E}{F}_{ij}$ and $\overset{E-E}{F_{ij}^{-1}}$. Then obtain $\overset{E-E}{F}_{ij} \overset{E-E}{F_{jk}^{-1}}$.

5. Invert the deformation of (1) to obtain $X_i(x_j)$. What condition (if any) must be met for such an inversion to exist? Use this result to obtain $\boldsymbol{F}^{-1} = \dfrac{\partial X_i}{\partial x_j} \boldsymbol{E}_i \otimes \boldsymbol{e}_j$, i.e., find $\dfrac{\partial X_i}{\partial x_j}$. Does this agree with your result for problem 3?

6. How would you obtain components of $\boldsymbol{F}$ and $\boldsymbol{F}^{-1}$ with respect to the $\boldsymbol{e}_i \otimes \boldsymbol{e}_j$ basis?

7. Suppose a line element in the original configuration is $d\boldsymbol{R} = \boldsymbol{E}_1 + \boldsymbol{E}_2$. Find (i) the unit tangent vector $\boldsymbol{t}_0$, (ii) the corresponding element length, dS_0 , (iii) the deformed line element $d\boldsymbol{r}$, (iv) the “deformed unit tangent vector $\boldsymbol{t}$, and (v) the length of the deformed line element ds . Do your element lengths ds and dS_0 satisfy the relation

   $ds = \{\boldsymbol{t}_0 \cdot \boldsymbol{F}^T \cdot \boldsymbol{F} \cdot \boldsymbol{t}_0\}^{1/2} dS_0$?

8. Use Nanson’s relation to relate area elements between the two configurations for an area element originally in the $X_1 - X_2$ plane.

9. What is the relation between volume elements for the two configurations.