ME-504 Homework Assignment 5: Finite Element Theory

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Strong Form of the Problem:

$$((1 - x^2)u')' + 12u = 0, \quad 0 < x < 1$$

$$u(0) = 0$$

$$u(1) = 1$$

Weak Form of the Problem:

$$\int_0^1 [v'(1 - x^2)u' - 12(v)(u)]dx = 0$$

0.1 Examine Convergence:

Approximate solutions over the problem domain are shown in Figure 1 and first derivatives of the approximate solutions are shown in Figure 2. Convergence was analyzed at $x = 0.5$, and the solution appears to have converged when 16 or more elements were used, as shown in Figure 3.

0.2 Evaluate the Error:

Figure 4 shows the calculated values of the Energy and L2 error norms with respect to the inverse of the element width. The errors were calculated between elements of width h and width $\frac{h}{2}$, not with respect to the analytical solution. The error norms scale with the order of the basis function such that the energy error norm scales with h^{k+1} and the L2 error norm scales with h^k, where h is the element wid th and k is the order of the polynomial used for the basis function (order 1 polynomials were used for this assignment).

0.2.1 Energy Error Norm

The work function is:

$$\prod[u] = \frac{1}{2}\int_0^1 [(1 - x^2)u'^2 + 12u^2]dx - \int_0^1 [fu]dx = 0$$

Therefore, the relative energy norm is defined as:

$$||e||_{Energy} = \Big(\frac{1}{2}\int_0^1 [(1 - x^2)(u'_N - u'_{2N})^2 + 12(u_N - u_{2N})^2]dx\Big)^{\frac{1}{2}}$$

0.2.2 L2 Error Norm:

$$||e||_{L2} = \Big(\int_0^1 [(u_N - u_{2N})^2]dx\Big)^{\frac{1}{2}}$$

0.3 Discussion of Results

Convergence appears to have been achieved when 16 or more elements were used. The both error norms decreased when an increasing number of elements were used.

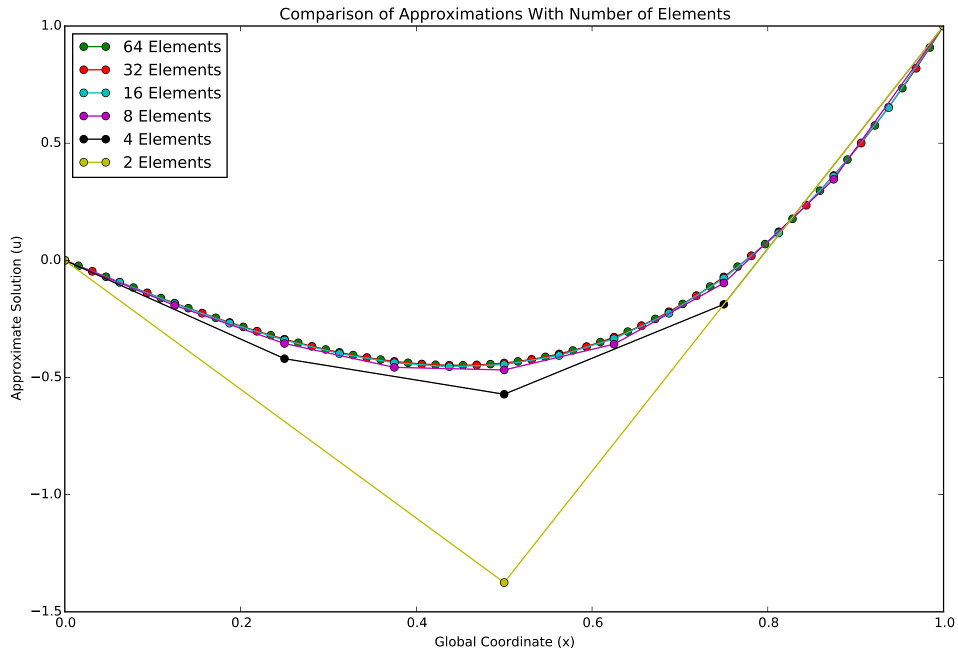

Figure 1: Approximate Solutions

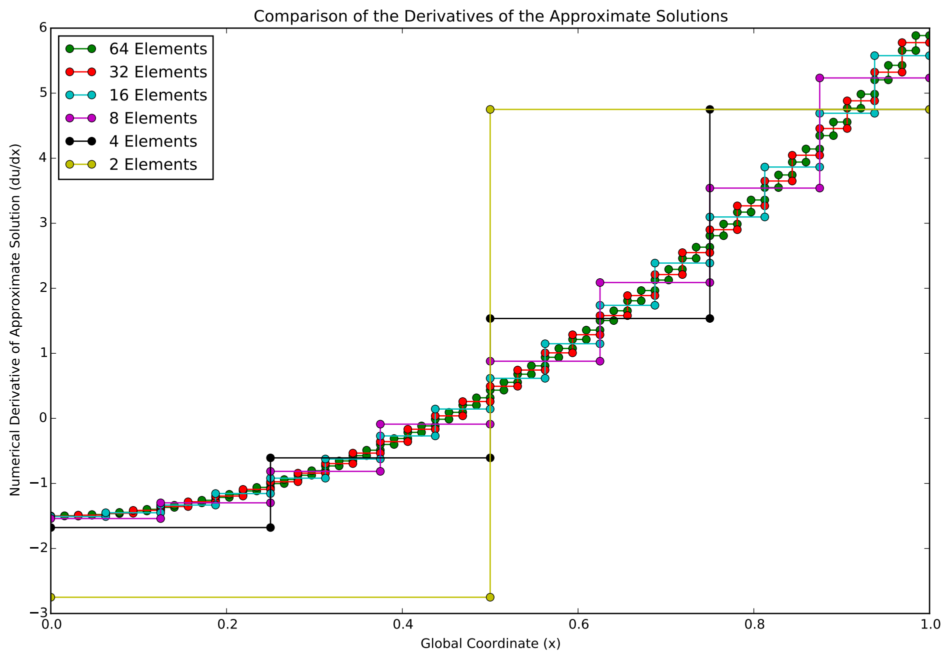

Figure 2: Derivatives of the Approximate Solutions

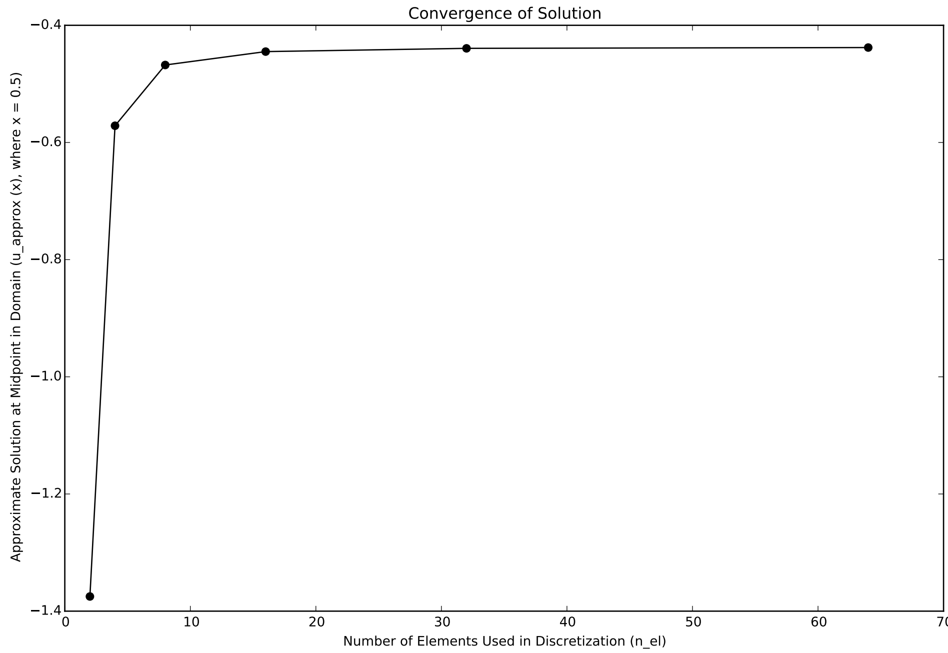

Figure 3: Approximate solutions at x = 0.5 for differing numbers of elements.

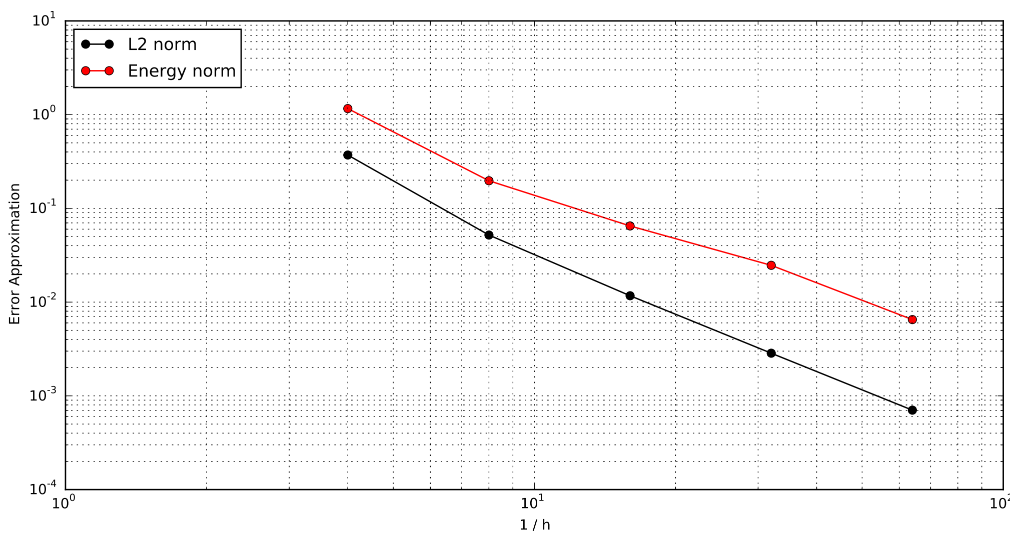

Figure 4: L2 and Energy error norms with respect to the inverse of element width.