Midterm

Graded

Student

BRANDON LO

Total Points

49.25 / 50 pts

Question 1

Copying Instances when Training Decision Tree

2 / 2 pts

✔ **– 0 pts Correct**: False

– 2 pts Incorrect: True

Question 2

Perceptron Maximum Iteration

2 / 2 pts

✔ **– 0 pts Correct**: False

– 2 pts Incorrect: True

Question 3

Pruned Decision Tree

2 / 2 pts

✔ **– 0 pts Correct**: True

– 2 pts Incorrect: False

Question 4

1-Nearest Neighbors

2 / 2 pts

✔ **– 0 pts Correct**: False

– 2 pts Incorrect: True

Question 5

Gradient Descent on Function

2 / 2 pts

✔ **– 0 pts Correct**: False

– 2 pts Incorrect: True

– 1 pt Incorrect: Correct work shown

Question 6

1-NN and Linear Separability

2 / 2 pts

✔ **− 0 pts Correct**: True

− 2 pts Incorrect: False

− 0 pts Correct: Valid explanation

Question 7

Training Error of Perceptron

2 / 2 pts

✔ **− 0 pts Correct**: False

− 2 pts Incorrect: True

Question 8

Perceptron vs. Logistic Regression

2 / 2 pts

✔ **− 0 pts Correct**: False

− 2 pts Incorrect: True

− 1 pt Incorrect: Has comparison of cost functions

Question 9

Permuting Instances

2 / 2 pts

✔ **− 0 pts Correct**: True

− 2 pts Incorrect: False

− 1 pt Incorrect: References Convergence Theorem/properties

Question 10

Log Likelihood

2 / 2 pts

✔ **− 0 pts Correct**: True

− 2 pts Incorrect: False

− 1 pt Incorrect: Comparison of derivatives

Question 11

Nearest Neighbors

3 / 3 pts

✔ **+ 0.75 pts Correct**: Option A ***not*** selected

✔ **+ 0.75 pts Correct**: Option B selected

✔ **+ 0.75 pts Correct**: Option C selected

✔ **+ 0.75 pts Correct**: Option D ***not*** selected

Question 12

Comparing Accuracies

3 / 3 pts

✔ **+ 0.75 pts Correct**: Option A ***not*** selected

✔ **+ 0.75 pts Correct**: Option B selected

✔ **+ 0.75 pts Correct**: Option C ***not*** selected

✔ **+ 0.75 pts Correct**: Option D ***not*** selected

Question 13

Reducing Overfitting in Decision Trees

3 / 3 pts

+ 0.75 pts Correct: Option A selected

+ 0.75 pts Correct: Option B ***not*** selected

+ 0.75 pts Correct: Option C selected

+ 0.75 pts Correct: Option D selected

✔ **+ 3 pts All Correct**: A,C,D selected, B not selected

+ 0 pts incorrect

Question 14

XOR Training Error

2.25 / 3 pts

✔ **+ 0.75 pts Correct**: Option A selected

✔ **+ 0.75 pts Correct**: Option B ***not*** selected

✔ **+ 0.75 pts Correct**: Option C ***not*** selected

+ 0.75 pts Correct: Option D selected

Question 15

Logistic Regression for Yelp Reviews

3 / 3 pts

- ✔ **+ 0.75 pts Correct**: Option A selected
- ✔ **+ 0.75 pts Correct**: Option B selected
- ✔ **+ 0.75 pts Correct**: Option C ***not*** selected
- ✔ **+ 0.75 pts Correct**: Option D ***not*** selected

Question 16

Decision Tree: Entropy of Y

3 / 3 pts

- ✔ **− 0 pts Correct**: Option C
- **− 1 pt Partial**: Most work correct, but selected wrong option
- **− 2 pts Partial**: Work shows general understanding, but with a major flaw
- **− 3 pts Incorrect**: Work shows little to no understanding; potentially missing

Question 17

Decision Tree: Conditional Entropy rel. X1

3 / 3 pts

- ✔ **− 0 pts Correct**: Option C
- **− 1 pt Partial**: Most work correct, but selected wrong option
- **− 2 pts Partial**: Work shows general understanding, but with a major flaw
- **− 3 pts Incorrect**: Work shows little to no understanding; potentially missing

Question 18

Decision Tree: Conditional Entropy rel. X2

3 / 3 pts

- ✔ **− 0 pts Correct**: Option D
- **− 1 pt Partial**: Most work correct, but selected wrong option
- **− 2 pts Partial**: Work shows general understanding, but with a major flaw
- **− 3 pts Incorrect**: Work shows little to no understanding; potentially missing

Question 19

Maximum Likelihood: Expression

3 / 3 pts

- ✔ **− 0 pts Correct**: Option A
- **− 1 pt Partial**: Most work correct, but selected wrong option
- **− 2 pts Partial**: Work shows general understanding, but with a major flaw
- **− 3 pts Incorrect**: Work shows little to no understanding; potentially missing

Question 20

Maximum Likelihood: Sample Mean

3 / 3 pts

- **− 0 pts** **Correct**: Option A
- **− 1 pt** **Partial**: Most work correct, but selected wrong option
- **− 2 pts** **Partial**: Work shows general understanding, but with a major flaw
- **− 3 pts** **Incorrect**: Work shows little to no understanding; potentially missing

CM146: Introduction to Machine Learning	Winter 2024

Midterm exam

Feb 13, 2024

- Please do not open the exam unless you are instructed to do so.
- This is an open book and open notes exam.
- Everything you need in order to solve the problems is supplied in the body of this exam OR in a cheatsheet at the end of the exam.
- Mark your answers ON THE EXAM ITSELF. If you make a mess, clearly indicate your final answer (box it).
- For true/false questions, CIRCLE True OR False. Justification for your choice is not needed but could be provided for partial credit.
- Unless otherwise instructed, for multiple-choice questions, CIRCLE ALL CORRECT CHOICES (in some cases, there may be more than one).
- You may use scratch paper if needed (provided at the end of the exam).
- You have 1 hour 45 minutes.

Good Luck! Legibly write your name and UID in the space provided below.

Name: Brandon Lo

UID: 105753560

True/False		/20
Multiple choice		/30
Total		/50

True/False (20 pts)

1. (2 pts) You are given a training dataset with attributes $A_1, \ldots, A_m$ and instances $x^{(1)}, \ldots, x^{(n)}$ and you use the ID3 algorithm to build a decision tree D_1. You then take one of the instances, add a copy of it to the training set (so your new training set will have $n + 1$ instances), and rerun the decision tree learning algorithm (with the same random seed) to create D_2. D_1 and D_2 are necessarily identical decision trees.

   True   False

   The addition of an instance changes the information gains of the attribute, which means a different attribute may become the ra

2. (2 pts) You run the PerceptronTrain algorithm with $maxIter = 100$. The algorithm terminates at the end of 100 iterations with a classifier that attains a training error of 1%. This means that the training data is not linearly separable.

   True   False

   It could be linearly separable but take over 100 iterations to converge.

3. (2 pts) You learn a decision tree with the $MaxDepth$ parameter set to infinity and then prune the resulting decision tree. The resulting pruned decision tree is less likely to overfit compared to the original decision tree.

   True   False

   Pruning would remove some of the leaves which would cause it to underfit more than the original decision tree.

4. (2 pts) We want to use 1-Nearest Neighbors (1-NN) to classify houses into one of two classes (cheap vs expensive) given a single feature that measures the area of the house. The predictions made by the 1-NN classifier data can change if the area of the house is measured in square metres instead of square feet. (You can neglect the effect of ties *i.e.*, two training instances that are both nearest neighbors to a test instance.)

   True   False

   The units should not change the classifier data as it should be proportional to the old units.

5. (2 pts) You run gradient descent to minimize the function $f(x) = (2x-3)^3$. Assume the step size has been chosen appropriately and you run gradient descent till convergence. Then gradient descent will return the global minimum of f.

   True   False

   $(2x-3)^3$ is not convex $f'(x) = 3 \cdot 2(2x-3)^2$

   $6(2x-3)^2$

   $f''(x) = 24(\ldots$ can be

$(x) = 4(2x-3)$ ← negative

6. (2 pts) On a dataset that is not linearly separable, the 1-nearest neighbors classifier obtains zero training error.

True   False

1-nearest neighbors always obtains 0 training error

7. (2 pts) The training error of the perceptron never increases with each iteration of the perceptron algorithm.

True   False

The next iteraton may cause points which were classified correctly to be classified incorrectly

8. (2 pts) On a linearly separable dataset, the perceptron and logistic regression learn the same separating hyperplane.

True   False

Logistic regression is a measure of probability while a perceptron will always find a linearly separable hyperplane, therefore they are not always th same.

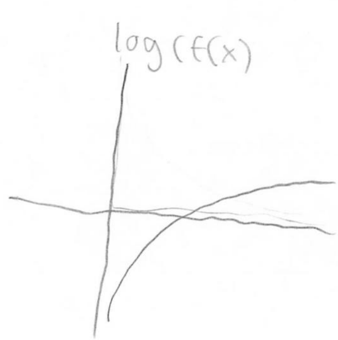
9. (2 pts) Permuting the order of instances in the training data can affect the number of iterations for convergence of the perceptron algorithm (assuming the data is linearly separable).

True   False

The perceptron will always converge if linearly separable. However, the order of instances can affect how many iterations it takes due to differing err

10. (2 pts) The value of x at which $f(x)$ attains its maximum is the same as the value of x at which $log(f(x))$ attains its maximum (assume that $f(x) > 0$ for all x).

True   False



log(x) is an increasing function, so this is true

Multiple choice (30 pts)

CIRCLE ALL CORRECT CHOICES (in some cases, there may be more than one)

11. (3 pts) In k-nearest neighbor classification, which of the following statements are true? (circle all that are correct)

    (a) The decision boundary is smoother with smaller values of k. converse is true

    (b) k-NN does not require any parameters to be learned in the training step (for a fixed value of k and a fixed distance function). Stores training set only

    (c) If we set k equal to the number of instances in the training data, k-NN will predict the same class for any input. It will predict whichever class is the majority

    (d) For larger values of k, it is more likely that the classifier will overfit than underfit.

12. (3 pts) Assume we are given a set of one-dimensional inputs and their corresponding output (that is, a set of $\{(x_i, y_i)\}, x_i \in \mathbb{R}, y_i \in \mathbb{R}$). We would like to compare the following two models where $\theta \in \mathbb{R}$:

$$\mathrm{A} : y = \theta^2 x$$
$$\mathrm{B} : y = \theta x$$

For each model, we split our data into training and testing data to evaluate the generalization accuracy of the learned model (assume that the number of instances in the training and the test data are large). Which of the following is correct?

(a) There are datasets for which A would be more *accurate* than B.

(b) There are datasets for which B would be more *accurate* than A.

(c) Both (a) and (b) are correct.

(d) They would perform equally well on all datasets.

θ^2 can only be positive and certain values while θ can be any value.

13. (3 pts) What strategy can help reduce over-fitting in decision trees.

(a) Pruning

(b) Make sure it achieves zero training error

(c) Adding more training data

(d) Enforce a maximum depth for the tree

14. (3 pts) Which of the following algorithms can achieve zero training error on the XOR problem?

(a) Decision tree

(b) Logistic regression

(c) Perceptron

(d) 1-Nearest Neighbors

} data is not linearly separable

} nearest neighbor would be of opposite class

15. (3 pts) Consider a logistic regression model to predict if a yelp review is positive or not ($y = 1$ means the review is positive) based on two features: x_1 and x_2. x_1 is the number of times the word "great" appears and x_2 is the number of times the word "not" appears. The logistic regression model $P(y = 1|x; \boldsymbol{\theta}) = \sigma(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$ with $\boldsymbol{\theta} = (\theta_0, \theta_1, \theta_2) = (-0.5, 1, -2)$. Which of the following is true ?

(a) The decision boundary is given by the line $x_1 - 2x_2 - 0.5 = 0$

(b) If the word "great" appears more often (assuming everything else about the review is the same), probability that the review is classified as positive becomes closer to 1.

(c) If the word "not" appears more often (assuming everything else about the review is the same), probability that the review is classified as positive becomes closer to 1.

(d) If the review contains neither the word "great" nor the word "not", it will be classified as positive.

If x_1 la
$P(y=1|x;\theta$
is large

If x_2 is la
$P(y=1|x;$
is smal
because
negati

If x_1 and x_2 are zero you end up with θ_0 which is -0.5, which is not positive.

$-2x_2 + x_1 - 0.5$

Decision Tree learning

Suppose you want to build a decision tree for a problem. In the dataset, there are two classes (*i.e.*, Y can take one of two possible values), with 60 examples in the + class and 30 examples in the − class. Recall that the information gain for target label Y and feature X is defined as $Gain = H[Y] - H[Y|X]$, where $H[Y] = -E[\log_2 P(Y)]$ is the entropy. See cheatsheet at the end of this exam for entropy values.

16. (3 pts) What is the entropy of the response variable Y?

(a) 0.73
(b) 0.81
(c) 0.92
(d) 0.97

$P(Y \text{ is } +) = \frac{2}{3}$ $\quad P(Y \text{ is } -) = \frac{1}{3}$

$H[Y] = 0.92$

17. (3 pts) For this data, we are interested in computing the information gain of a binary feature X_1. In the + class, the number of instances that have $X_1 = 0$ and $X_1 = 1$ respectively: $(30, 30)$. In the − class, these numbers are: $(0, 30)$. What is the conditional entropy of Y relative to X_1?

(a) 0
(b) 0.33
(c) 0.67
(d) 0.92

$-\frac{30}{90} \underbrace{\log_2\left(\frac{1}{2}\right)}_{1} - \frac{30}{90} \underbrace{\log_2\left(\frac{1}{2}\right)}_{1} = \frac{2}{3}$

18. (3 pts) We are interested in computing the information gain of a binary feature X_2. In the + class, the number of instances that have $X_2 = 0$ and $X_2 = 1$ respectively are: $(40, 20)$. In the − class, these numbers are: $(20, 10)$. What is the conditional entropy of Y relative to X_2?

(a) 0
(b) 0.33
(c) 0.67
(d) 0.92

$H[Y|X_2=1] = 0.92$

$H[Y|X_2=0] = 0.92$

$H[Y|X_2] = 0.92$

MLE

We observe a data set consisting of N samples: $x_1, \ldots, x_N$. $x_1, \ldots, x_N$ are i.i.d. random variables where each random variable is distribute as $Poisson(\lambda)$. The probability mass function for $X \sim Poisson(\lambda)$ is:

$$p(x; \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$$

19. (3 pts) What is the expression for the log-likelihood $l(\lambda)$ (all terms that do not depend on λ are refered to as $const$)?

(a) $l(\lambda) = -N\lambda + \log(\lambda)(\sum_n x_n) + const$
(b) $l(\lambda) = \lambda^N e^{-\lambda \sum_n x_n} + const$
(c) $l(\lambda) = -N\log(\lambda) + \log(\lambda)\sum_n x_n + const$
(d) $l(\lambda) = \sum_n \lambda e^{-\lambda x_n} + const$

$LL(\lambda) = \log(p(x_1, \ldots, x_N; \lambda))$
$= \log(p(x_1;\lambda)\, p(x_2;\lambda) \ldots p(x$
$= \log(p(x_1;\lambda)) + \log(p(x_2;\lambda)) \ldots$
$= \sum_{n=1}^{N} \log(p(x_n;\lambda))$
$= \sum_{n=1}^{N} \log\left(\frac{e^{-\lambda}\lambda^x}{x!}\right)$
$= \sum_{n=1}^{N} [\log(e^{-\lambda}\lambda^x) - \log(x!)]$

20. (3 pts) Let $\bar{x} = \frac{\sum_n x_n}{N}$ denote the sample mean of $(x_1, \ldots, x_N)$. What is the MLE, $\hat{\lambda}$, of λ?

(a) $\hat{\lambda} = \bar{x}$
(b) $\hat{\lambda} = \frac{1}{\bar{x}}$
(c) $\hat{\lambda} = e^{\bar{x}}$
(d) $\hat{\lambda} = \sum_n x_n$

$= \sum_{n=1}^{N} [\log(e^{-\lambda}) + \log(\lambda^x) - \log$
$= -N\lambda + \sum^{N} x_n \cdot \log(\lambda) - con$

a

$$\frac{d\,LL(\lambda)}{d\lambda} = \frac{d(-N\lambda + \log(\lambda)(\Sigma_n x_n) + const)}{d\lambda}$$
$$= \frac{d(-N\lambda)}{d\lambda} + \frac{d(\log(\lambda)(\Sigma_n x_n))}{d\lambda} + \frac{d(const)}{d\lambda}$$
$$= -N + \frac{1}{\lambda}\Sigma_n x_n = 0 \quad 0$$
$$\frac{1}{\lambda}\Sigma x_n = N$$
$$\frac{1}{\lambda} = \frac{N}{\Sigma_n x_n}$$
$$\lambda = \frac{\Sigma_n x_n}{N} = \bar{x} \quad \text{a}$$

Identities

Probability density/mass functions for some distributions

$$\text{Normal} \quad : \quad P(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$\text{Multinomial} \quad : \quad P(\boldsymbol{x};\boldsymbol{\pi}) = \prod_{k=1}^{K} {\pi_k}^{x_k}$$

$\boldsymbol{x}$ is a length K vector with exactly one entry equal to 1 and all other entries equal to 0

$$\text{Poisson} \quad : \quad P(x;\lambda) = \frac{\lambda^x \exp(-\lambda)}{x!}$$

Matrix calculus

Here $\boldsymbol{x} \in \mathbb{R}^n, \boldsymbol{b} \in \mathbb{R}^n, \boldsymbol{A} \in \mathbb{R}^{n \times n}$. $\boldsymbol{A}$ is symmetric.

$$\nabla \boldsymbol{x}^{\mathrm{T}} \boldsymbol{A} \boldsymbol{x} = 2\boldsymbol{A}\boldsymbol{x}, \qquad \nabla \boldsymbol{b}^{\mathrm{T}} \boldsymbol{x} = \boldsymbol{b}$$

Entropy

The entropy $H(X)$ of a Bernoulli random variable $X \sim \mathit{Bernoulli}(p)$ for different values of p:

p	$H(X)$
$\frac{1}{2}$	1
$\frac{1}{3}$	0.92
$\frac{1}{4}$	0.81
$\frac{1}{5}$	0.73
$\frac{2}{5}$	0.97