Recursive Procedure Calls

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CSC342 - Instructor: Prof. Izidor Gertner 3/30/2015

Objective:

Recursion refers to the occurrence of allowing a function to call itself. Meaning the solution to a larger procedure depends on the smaller cases of the same procedure. Inside memory, a new stack frame must be created for each instance of the recursive procedure during execution. This is what we will primarily analyze though debugging and disassembling over three major operating systems; Windows, Linux, and MIPS/MARS.

Windows OS

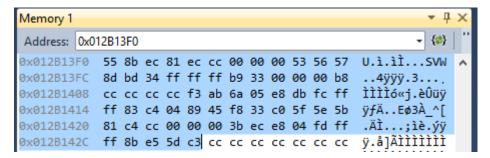
For Windows, we will be using the MS Debugger from Microsoft Visual C++ 2010 Express on a 64-bit Intel Core i5 CPU. Let's debug and analyze the following code segment written in C++:

We have two functions -- the factorial function, which utilizes recursion to calculate the factorial of a value n, and the main function, which calls the factorial function on the value 5. The yellow arrow indicates the location where we will be entering the program (*i.e.* the main function).

The instruction code for our program is compiled and stored into memory.

Memory 1 ▼ Д X														
Address:	0x012B	1380											- {¢}	"
0x012B13	80 55	8b	ec	81	ec	с0	00	00	00	53	56	57	U.ì.ìÀSVW	٨
0x012B13	8C 8d	bd	40	ff	ff	ff	b9	30	00	00	00	b8	@ÿÿÿ.0,	
0x012B13	98 cc	сс	СС	cc	f3	ab	83	7d	08	01	75	07	ÌÌÌÌó«ƒ}u.	
0x012B13	A4 b8	01	00	00	00	eb	13	8b	45	08	83	e8	,ëE.fè	
0x012B13	B0 01	. 50	e8	39	fd	ff	ff	83	с4	04	0f	af	.Pè9ýÿÿƒÄ⁻	
													E^[.ÄÀ;	
0x012B13	C8 ec	e8	63	fd	ff	ff	8b	e5	5d	с3	cc	cc	ìècýÿÿ.å]ÃÌÌ	

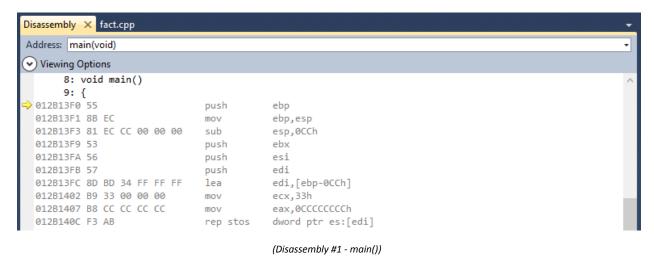
(Instruction Code - Factorial Function)



(Instruction Code - Main Function)

Instance #1: Before First Instruction

Now let's begin debugging. The disassembly of the main function at the instance before any instruction is executed is shown below:



(Registers #1)

Let's bring our attention to three specific registers which are highlighted above:

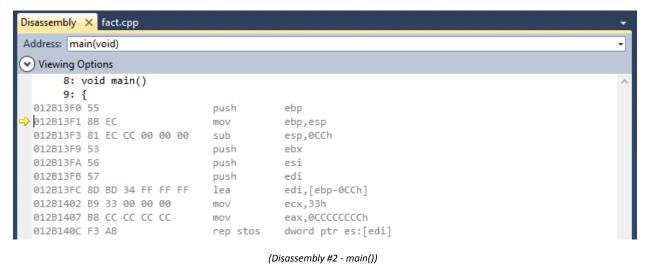
- The EIP stores the address of the instruction which will be executed next. This is also
 indicated by the yellow arrow displayed next to the same address in the disassembly
 window.
- The ESP typically stores the address at the top of the stack. In this case, it contains the address which stores the return address that the main procedure must return to after execution is completed.
- The EBP typically stores the address at the base of the stack. In this case, it contains the address which stores the base pointer of the previous calling procedure.



** NOTE -- Intel Processors store data in Little Endian Notation

Instance #2: Creating the Main() Stack Frame

When the main function begins, a stack frame must be created in memory. This can be seen at the first instruction where a base pointer (ebp) is pushed into memory. This marks the beginning of the main stack frame, and does so by first setting up the stack pointer (esp).



```
Registers

EAX = 00902110 EBX = 7F5AF000 ECX = 00903D20 EDX = 000000001 ESI = 012B1109

EDI = 012B1109 EIP = 012B13F1 ESP = 0044FA10 EBP = 0044FA60 EFL = 00000206
```

(Registers #2)

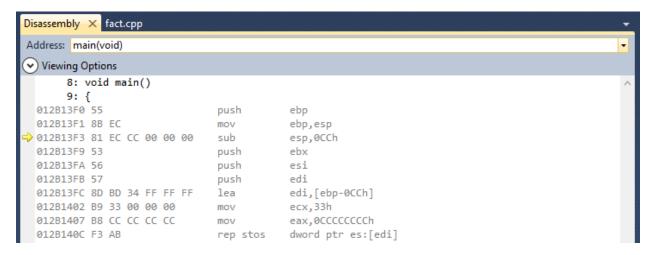
```
Memory 1

0x0044FA10 60 fa 44 00
0x0044FA14 af 19 2b 01
0x0044FA18 01 00 00 00
0x0044FA1C 20 3d 90 00
0x0044FA20 10 21 90 00
(Memory #2 - ESP)
```

This instruction has set ESP equal to the address 0x0044FA10. It contains the address of the base pointer of the previous calling procedure, 0x0044FA60.

Instance #3: Set Base Pointer and Stack Pointer of Main()

Next, the stack pointer (esp) is moved to the location of the base pointer (ebp), which is achieved by setting the base pointer equal to the stack pointer. The base pointer will always point to the base of the stack, however, the stack pointer will mark the top of the stack, and will grow accordingly as data is pushed and popped into and from the stack.



(Disassembly #3 - main())

```
Registers

EAX = 00902110 EBX = 7F5AF000 ECX = 00903D20 EDX = 00000001 ESI = 012B1109

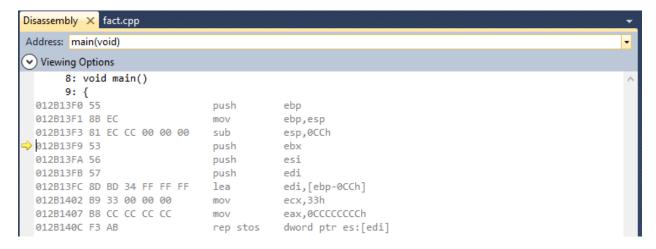
EDI = 012B1109 EIP = 012B13F3 ESP = 0044FA10 EBP = 0044FA10 EFL = 00000206
```

(Register #3)

```
Memory 1

0x0044FA10 60 fa 44 00
0x0044FA14 af 19 2b 01
0x0044FA18 01 00 00 00
0x0044FA1C 20 3d 90 00
0x0044FA20 10 21 90 00
(Memory #3 - EBP)
```

Instance #4: Stack Space Allocation



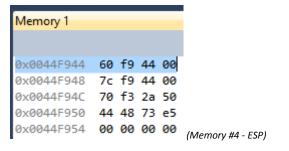
(Disassembly #4 - main())

```
Registers

EAX = 00902110 EBX = 7F5AF000 ECX = 00903D20 EDX = 00000001 ESI = 012B1109

EDI = 012B1109 EIP = 012B13F9 ESP = 0044F944 EBP = 0044FA10 EFL = 00000216
```

(Registers #4)



The instruction "81 EC CC 00 00 00 sub esp, OCCh" subtracts 204 bytes from the stack pointer, and stores the result by overwriting the stack pointer register. This can be seen from the instruction segment CC 00 00 00, or the segment OCCh, which represents the decimal number 204 in hexadecimal. What this is doing is moving the stack pointer and allocating 204 bytes between the stack pointer and the base pointer on stack. This brings the stack pointer to the new location 0x0044F944 and clears the allocated space over the next few instructions.

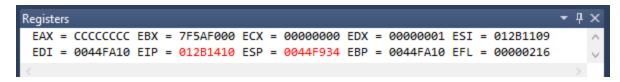
Memory 1																														
Address:	0x0044	F94	4																										-	{≉}
0 x0044F9	44 c	с с	с сс	cc	сс	СС	сс	сс	cc	cc	cc	сс	cc	cc	СС	сс	cc	cc	cc	сс	сс	cc	cc	сс	СС	сс	СС	СС	сс	cc
0x0044F9	62 c	c c	c co	cc																										
0x0044F9	80 c	cc	c co	cc																										
0x0044F9	9E c	c c	c co	cc																										
0x0044F9	BC c	c c	c co	cc																										
0x0044F9	DA c	c c	c co	cc																										
0x0044F9	F8 c	c c	c cc	cc	сс	cc	СС	cc	cc	cc	СС	60	fa	44	00	af	19													

(Memory #4 - ESP to EBP is cleared)

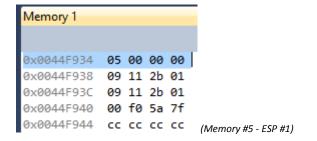
Instance #5: Call and Jump to Factorial(5)

```
Disassembly X fact.cpp
Address: main(void)
Viewing Options
     10:
            int n_fact = factorial(5);
 012B140E 6A 05
 012B1410 E8 DB FC FF FF
                                         factorial (12B10F0h)
                             call
 012B1415 83 C4 04
                              add
                                         esp,4
 012B1418 89 45 F8
                                         dword ptr [n_fact],eax
                             mov
     11: }
```

(Disassembly #5 - main())



(Registers #5)



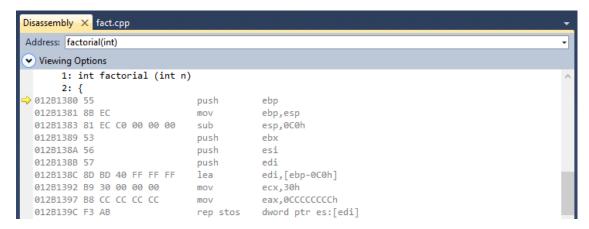
This instruction pushes the parameter n = 5 to the stack, just before calling the factorial (int n) function. The value can be seen at the location of the current stack pointer.

Immediately after, the call factorial procedure instruction is made and the program is then taken to the address 0x012B10F0. This address contains the instruction code to jump to the start of the factorial function in memory, which is at address 0x012B1380. In addition, the return address is stored in the address 0x0044F930 by the stack pointer, which contains 0x012B1415 -- the instruction immediately after the call factorial procedure.

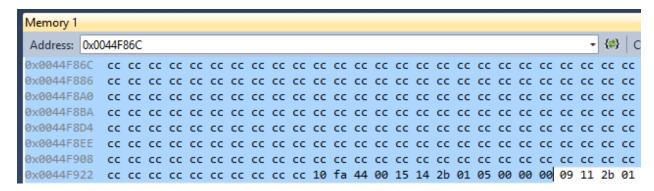
```
Memory 1

0x0044F930 15 14 2b 01
0x0044F934 05 00 00 00
0x0044F938 09 11 2b 01
0x0044F93C 09 11 2b 01
0x0044F940 00 f0 5a 7f
(Memory #5 - ESP #2)
```

Instance #6: Factorial (5)



(Disassembly #6 - factorial(5) #1)



(Memory #6 - factorial(5) stack frame)

Now a stack frame must be set up for the factorial(5) procedure call. Similar to how the main() stack frame was setup, we can see the base pointer is initialized to address 0x0044F92C, 0C0h (192 bytes) are allocated and cleared in memory, and the stack pointer is currently set to address 0x0044F86C.

```
Disassembly X fact.cpp
Address: factorial(int)
Viewing Options
           if (n==1)
     3:
 012B139E 83 7D 08 01
                                      dword ptr [n],1
 012B13A2 75 07
                                     factorial+2Bh (12B13ABh)
     4:
           return 1;
 012B13A4 B8 01 00 00 00
                           mov
                                     eax.1
 012B13A9 EB 13
                                      factorial+3Eh (12B13BEh)
                           jmp
     5: return (n*factorial(n-1));
 012B13AB 8B 45 08 mov
                                     eax,dword ptr [n]
sub
                                     eax,1
 012B13B1 50
                          push
                                    eax
 012B13B2 E8 39 FD FF FF
                          call
                                     factorial (12B10F0h)
 012B13B7 83 C4 04
                           add
                                     esp,4
 012B13BA 0F AF 45 08
                           imul
                                      eax, dword ptr [n]
     6: }
```

(Disassembly #6 - factorial(5) #2)

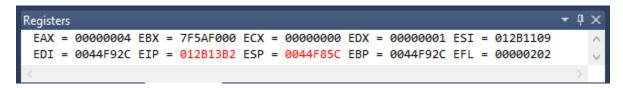
```
Registers

EAX = 00000005 EBX = 7F5AF000 ECX = 000000000 EDX = 00000001 ESI = 012B1109

EDI = 0044F92C EIP = 012B13AE ESP = 0044F860 EBP = 0044F92C EFL = 00000202
```

(Registers #6 - EAX stores value 5)

Here we can see the first conditional is ignored, and the program has jumped to the instruction at address 0x012B13AB. The value 5 is then stored into register EAX.



(Registers #6 - EAX and ESP store value 4)

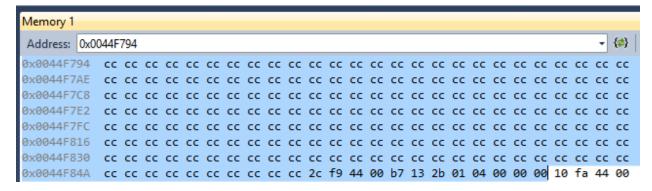
```
Memory 1
0x0044F85C 04 00 00 00
0x0044F860 10 fa 44 00
0x0044F864 09 11 2b 01
0x0044F868 00 f0 5a 7f
0x0044F86C cc cc cc cc
                            (Memory #6 - ESP store value 4)
```

In the following instructions, 1 is subtracted from the value stored in EAX, then EAX is pushed onto the stack by the stack pointer at address 0x0044F85C. The program then recalls the factorial() procedure, and jumps back to the address of the first factorial() procedure instruction, 0x012B1380. This time, however, n is equal to 4.

The return address is also stored at the address 0x0044F858, containing the address 0x012B13B7, which is the instruction immediately after the instruction that called the procedure.

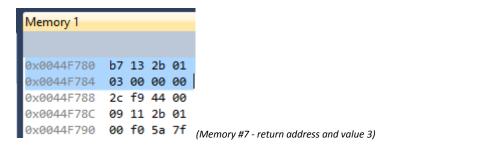
```
Memory 1
0x0044F858 b7 13 2b 01
0x0044F85C 04 00 00 00
0x0044F860 10 fa 44 00
0x0044F864 09 11 2b 01
0x0044F868 00 f0 5a 7f
            __ __ (Memory #6 - ESP stores return address)
```

Instance #7: Factorial(4)



(Memory #7 - factorial(4) stack frame)

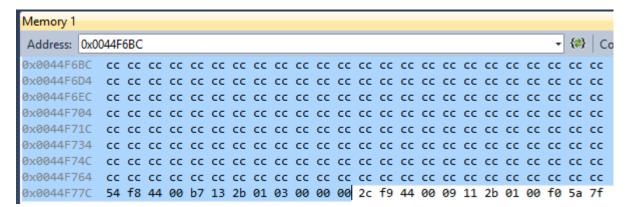
(Registers #7)



Similarly, a stack frame is created for the factorial(4) procedure call -- a base pointer is initialized to address 0x0044F854, n is decremented by 1, this value is then pushed onto the stack, the return address is stored, and the program recalls the factorial function once again.

One thing to notice is the return address for this frame stores the same instruction as the return address of the previous factorial(5) frame. This is because each instance of the recursive procedure is called by the same instruction, and will return to the instruction immediately afterwards.

Instance #8: Factorial(3)



(Memory #8 - factorial(3) stack frame)

```
Registers

EAX = 00000002 EBX = 7F5AF000 ECX = 00000000 EDX = 00000001 ESI = 012B1109

EDI = 0044F77C EIP = 012B1380 ESP = 0044F6A8 EBP = 0044F77C EFL = 00000202
```

(Registers #8)

```
Memory 1

0x0044F6A8 b7 13 2b 01
0x0044F6AC 02 00 00 00
0x0044F6B0 54 f8 44 00
0x0044F6B4 09 11 2b 01
0x0044F6B8 00 f0 5a 7f
(Memory #8 - return address and value 2)
```

Instance #9: Factorial(2)

(Memory #9 - factorial(2) stack frame)

(Registers #9)

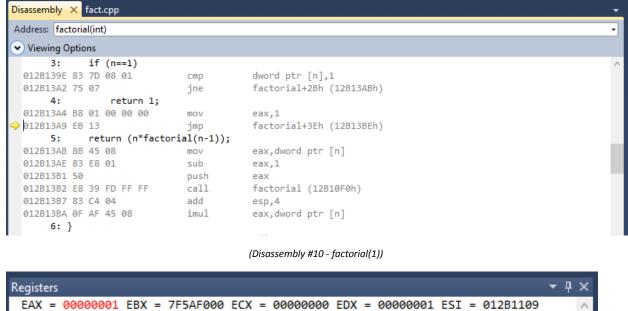
```
Memory 1

0x0044F5D0 b7 13 2b 01
0x0044F5D4 01 00 00 00
0x0044F5D8 7c f7 44 00
0x0044F5DC 09 11 2b 01
0x0044F5E0 00 f0 5a 7f
(Memory #9 - return address and value 1)
```

Instance #10: Factorial(1)

(Memory #10 - factorial(1) stack frame)

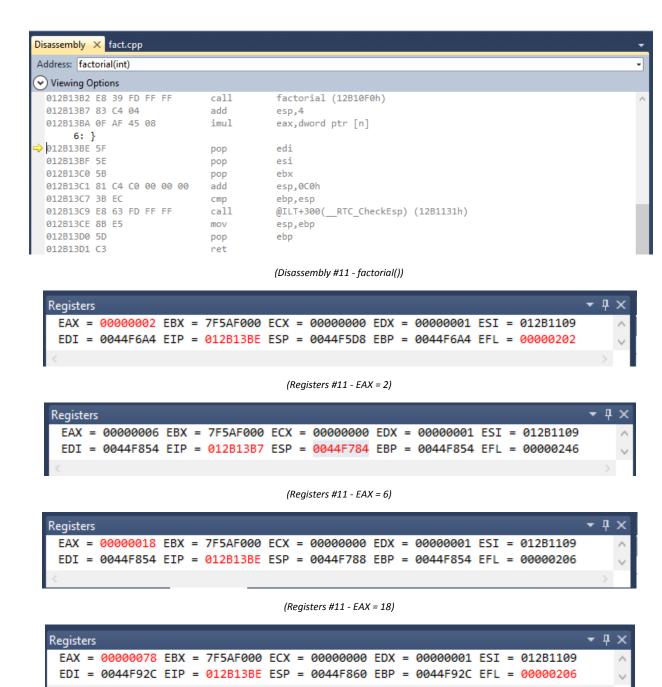
Now we have reached the last instance of the recursive procedure. This time, condition where n = 1 (the base case) will not be skipped, and is going to run. In this case, the condition simply returns the value 1. This value is then stored into register EAX.



(Registers #10 - EAX = 1)

Instance #11: Return and Evaluate n_fact

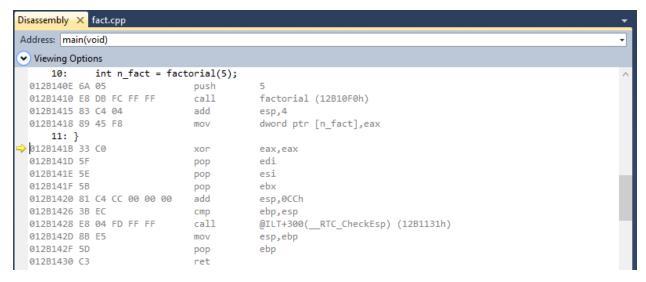
Next, the program jumps to the part of the code that will begin popping all of the factorial frames that are on stack. Each time, it multiplies our local variables cumulatively into register EAX. It will also continue to be called to the same return address (instruction 0x012B13B7) until all of the factorial stack frames are cleared.



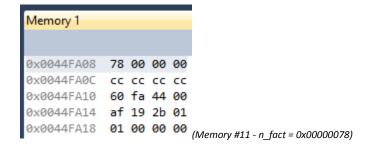
(Registers #11 - EAX = 78)

As we can see, EAX now equates to the hexadecimal value 0x00000078, or the decimal value 120, which is exactly what we expected. (5! = (5*4*3*2*1) = 120)

Lastly, we go back to main(), store the value of EAX into the variable n_fact (address 0x0044FA08), and proceed to popping the main stack frame, and returning to the address after the main call procedure.



(Disassembly #11 - main())



Stack Visual

Address	Content	
0044FA60	0044FA68	EBP of calling
0044FA14	012B19AF	Return Address
0044FA10	0044FA60	EBP of main() Main()
0044FA08	00000078	n_fact
0044F934	00000005	n = 5
0044F930	012B1415	Return Address Factorial(5)
0044F92C	0044FA10	EBP of fact(5)
		_
0044F85C	00000004	n = 4
0044F858	012B13B7	Return Address > Factorial(4)
0044F854	0044F92C	EBP of fact(4)
0044F784	00000003	n = 3
0044F780	012B13B7	Return Address Factorial(3)
0044F77C	0044F854	EBP of fact(3)
		·
0044F6AC	00000002	n = 2
0044F6A8	012B13B7	Return Address Factorial(2)
0044F6A4	0044F77C	EBP of fact(2)
0044F5D4	00000001	n = 1
0044F5D0	012B13B7	Return Address Factorial(1)
0044F5CC	0044F6A4	EBP of fact(1)
		_
		ESP

Stack Growth

Linux OS

For Linux, we will be using Ubuntu to run GCC and GDB on a 64 bit AMD Phenom II X4 CPU. Let's first look at the following code segment, written in C:

```
fact.cpp  x

int factorial(int n)

{
    if (n == 1)
        return 1;
    return (n * factorial(n - 1));

    int main()

    int n_fact = factorial(5);

    return 0;
}
```

(Source Code in C)

Instance 1: Main Frame

Now, if we begin debugging, we can set a break point at main(), and look at its disassembly:

```
(gdb) disassemble
Dump of assembler code for function main:
   0x0000000000400518 <+0>:
                                push
                                        %rbp
   0x0000000000400519 <+1>:
                                        %rsp,%rbp
                                MOV
   0x000000000040051c <+4>:
                                sub
                                        $0x10,%rsp
                                        $0x5,%edi
=> 0x0000000000400520 <+8>:
                                MOV
   0x0000000000400525 <+13>:
                                callq
                                        0x4004ed <factorial>
   0x000000000040052a <+18>:
                                mov
                                        %eax,-0x4(%rbp)
   0x000000000040052d <+21>:
                                leaveq
   0x000000000040052e <+22>:
                                retq
End of assembler dump.
```

(Disassembly - main())

Here, we can see the main stack frame is created by first pushing the base pointer (rbp) to stack, moving the stack pointer (rsp) to the same location as the base pointer, and allocating 0x10 (16 bytes) for the frame.

```
(gdb) info registers
гах
               0x400518 4195608
гЬх
               0x0
                        0
гсх
               0x0
                        0
гdх
               0x7fffffffde88
                                140737488346760
rsi
               0x7fffffffde78
                                140737488346744
rdi
               0x1
                        1
               0x7fffffffdd90
                                0x7fffffffdd90
гЬр
rsp
               0x7fffffffdd80
                                0x7fffffffdd80
г8
               0x7ffff7dd4e80 140737351863936
г9
               0x7ffff7dea560 140737351951712
г10
               0x7fffffffdc20
                                140737488346144
г11
               0x7fffff7a36dd0 140737348070864
г12
               0x400400 4195328
               0x7fffffffde70 140737488346736
г13
г14
               0x0
                        0
г15
               0x0
                        0
               0x400520 0x400520 <main+8>
гiр
eflags
                       [ IF ]
               0x202
cs
               0x33
                        51
SS
               0x2b
                        43
ds
               0x0
                        0
es
               0x0
                        0
fs
               0x0
                        0
---Type <return> to continue, or q <return> to quit---d
               0x0
                        0
gs
```

(Registers)

```
(gdb) info frame
Stack level 0, frame at 0x7fffffffdda0:
    rip = 0x400520 in main (fact.c:10); saved rip = 0x7ffff7a36ec5
    source language c.
    Arglist at 0x7ffffffdd90, args:
    Locals at 0x7ffffffdd90, Previous frame's sp is 0x7ffffffdda0
    Saved registers:
        rbp at 0x7ffffffdd90, rip at 0x7ffffffdd98
```

```
(gdb) x $rsp
0x7ffffffdd80: 0xffffde70
```

(Info Frame)

We can see the instruction pointer (rip) is at 0x400520, the base pointer is at 0x7FFFFFFDD90, and the stack pointer is at 0x7FFFFFFDD80.

Instance 2: Factorial(5) Frame

Next, the program stores the parameter n = 5 into the register edi, followed by calling the factorial procedure. The program is then taken to the location in memory where the factorial procedure has been compiled.

```
(gdb) disassemble
Dump of assembler code for function factorial:
   0x00000000004004ed <+0>:
                                push
   0x000000000004004ee <+1>:
                                       %rsp,%rbp
                                MOV
   0x00000000004004f1 <+4>:
                                sub
                                       $0x10,%rsp
                                       %edi,-0x4(%rbp)
   0x00000000004004f5 <+8>:
                                MOV
                                cmpl
   0x00000000004004f8 <+11>:
                                       $0x1,-0x4(%rbp)
   0x00000000004004fc <+15>:
                                jne
                                       0x400505 <factorial+24>
   0x00000000004004fe <+17>:
                                MOV
                                       $0x1,%eax
   0x0000000000400503 <+22>:
                                jmp
                                       0x400516 <factorial+41>
=> 0x0000000000400505 <+24>:
                                       -0x4(%rbp),%eax
                                MOV
   0x0000000000400508 <+27>:
                                sub
                                       $0x1,%eax
   0x000000000040050b <+30>:
                                       %eax,%edi
                                MOV
   0x000000000040050d <+32>:
                                       0x4004ed <factorial>
                                callq
   0x0000000000400512 <+37>:
                                imul
                                        -0x4(%rbp),%eax
   0x0000000000400516 <+41>:
                                leaveq
   0x0000000000400517 <+42>:
                                retq
End of assembler dump.
```

(Disassembly - factorial())

First, a stack frame is created for this procedure call, then, the value 5, stored in register edi, is moved into the memory location of the base pointer offset by 4 bytes (rbp - 4). The program then jumps to the else statement of our code, line 5. It will then move this value into register eax, subtract 1 from this value, and then move it back into register edi, just before recalling the factorial procedure on this new parameter.

```
(gdb) info frame
Stack level 0, frame at 0x7fffffffdd80:
  rip = 0x400505 in factorial (fact.c:5); saved rip = 0x40052a
  called by frame at 0x7ffffffdda0
  source language c.
  Arglist at 0x7fffffffdd70, args: n=5
  Locals at 0x7fffffffdd70, Previous frame's sp is 0x7fffffffdd80
  Saved registers:
  rbp at 0x7fffffffdd70, rip at 0x7fffffffdd78
```

```
(gdb) x $rbp-4
0x7ffffffdd6c: 0x00000005
```

Instance 3: Factorial(4) Frame

```
(gdb) info frame
Stack level 0, frame at 0x7fffffffdd60:
    rip = 0x400505 in factorial (fact.c:5); saved rip = 0x400512
    called by frame at 0x7fffffffdd80
    source language c.
    Arglist at 0x7fffffffdd50, args: n=4
    Locals at 0x7fffffffdd50, Previous frame's sp is 0x7ffffffdd60
    Saved registers:
        rbp at 0x7fffffffdd50, rip at 0x7ffffffdd58
(gdb) x $rbp-4
0x7fffffffdd4c: 0x00000004
```

(Info Frame)

Instance 4: Factorial(3) Frame

```
(gdb) info frame
Stack level 0, frame at 0x7fffffffdd40:
    rip = 0x400505 in factorial (fact.c:5); saved rip = 0x400512
    called by frame at 0x7fffffffdd60
    source language c.
    Arglist at 0x7fffffffdd30, args: n=3
    Locals at 0x7fffffffdd30, Previous frame's sp is 0x7fffffffdd40
    Saved registers:
    rbp at 0x7fffffffdd30, rip at 0x7ffffffdd38
(gdb) x $rbp-4
0x7fffffffdd2c: 0x00000003
```

(Info Frame)

Instance 5: Factorial(2) Frame

```
(gdb) info frame
Stack level 0, frame at 0x7fffffffdd20:
   rip = 0x400505 in factorial (fact.c:5); saved rip = 0x400512
   called by frame at 0x7ffffffdd40
   source language c.
   Arglist at 0x7ffffffdd10, args: n=2
   Locals at 0x7ffffffdd10, Previous frame's sp is 0x7ffffffdd20
   Saved registers:
   rbp at 0x7fffffffdd10, rip at 0x7ffffffdd18
(gdb) x $rbp-4
0x7ffffffdd0c: 0x00000002
```

(Info Frame)

Instance 6: Factorial(1) Frame

```
(gdb) info frame
Stack level 0, frame at 0x7fffffffdd00:
    rip = 0x4004f8 in factorial (fact.c:3); saved rip = 0x400512
    called by frame at 0x7fffffffdd20
    source language c.
    Arglist at 0x7fffffffdcf0, args: n=1
    Locals at 0x7fffffffdcf0, Previous frame's sp is 0x7ffffffdd00
    Saved registers:
        rbp at 0x7fffffffdcf0, rip at 0x7fffffffdcf8
(gdb) x $rbp-4
0x7fffffffdcec: 0x00000001
```

(Info Frame)

Instance 7: Return

The base case is finally met, and the factorial procedure returns 1, pops the frame from stack and jumps back to the previous frame. The program will continue to do this, at the same time begin to evaluate the product of all the stack arguments.

```
(gdb) info registers
гах
                        1
(gdb) info registers
               0x2
                        2
(gdb) info registers
                        б
(gdb) info registers
               0x18
                        24
гах
(gdb) info registers
                        120
               0x78
гах
```

We get the value for 5! to be 120, or 0x78, which is what we expected. Finally, the program will return to the instruction in main right after the instruction that called the first factorial function, and proceed to pop the main function off stack.

Stack Visual

Address	Content		
0x7FFFFFFDD90		RBP of main()	
			├ Main()
	0x78	n_fact	J
0x7FFFFFFDD70	0x7FFFFFFDD90	RBP of fact(5)	
0x7FFFFFFDD6C	0x5	n = 5	Factorial(5)
	•••		J
0x7FFFFFFDD50	0x7FFFFFFDD70	RBP of fact(4))
0x7FFFFFFDD4C	0x4	n = 4	Factorial(4)
			J
0x7FFFFFFDD30	0x7FFFFFFDD50	RBP of fact(3))
0x7FFFFFFDD2C	0x3	n = 3	Factorial(3)
	•••		J
0x7FFFFFFDD10	0x7FFFFFFDD30	RBP of fact(2)	
0x7FFFFFFDD0C	0x2	n = 2	Factorial(2)
	•••		J
0x7FFFFFFDCF0	0x7FFFFFFDD10	RBP of fact(1)	
0x7FFFFFFDCEC	0x1	n = 1	Factorial(1)
		RSP	J

Stack Growth

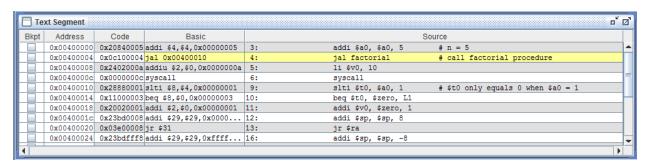
MIPS/MARS

Let's disassemble the following piece of code written in MIPS assembly:

```
fact.asm
            main:
2
                    addi $a0, $a0, 5
                                             \# n = 5
3
4
                    jal factorial
                                             # call factorial procedure
                    li $v0, 10
 6
                    syscall
8
                    slti $t0, $a0, 1
                                             # $t0 only equals 0 when $a0 = 1
9
                    beq $t0, $zero, L1
10
11
                    addi $v0, $zero, 1
12
                    addi $sp, $sp, 8
13
                    jr $ra
14
15
            L1:
                                             # the loop that handles decrementing n
                    addi $sp, $sp, -8
17
                    sw $ra, 4($sp)
                    sw $a0, 0($sp)
18
                    addi $a0, $a0, -1
19
                    jal factorial
20
                    lw $a0, 0($sp)
21
                    lw $ra, 4($sp)
22
23
                    addi $sp, $sp, 8
24
                    mul $v0, $a0, $v0
                    jr $ra
```

(MIPS Assembly Source Code)

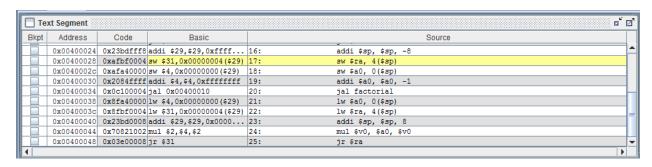
Instance 1:



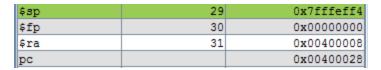
(Text Segment - main)



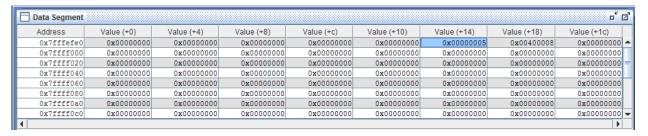
Instance 2:



(Text Segment - L1)



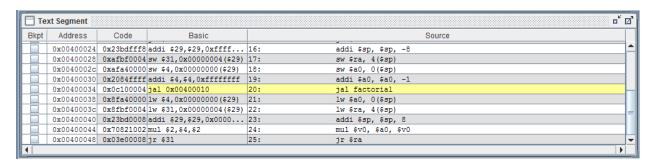
(Stack pointer (\$sp), return address (\$ra), program pointer (\$pc))



(Data Segment - return address and n = 5)

Here we can see the value 5 has been stored into memory address 0x7FFFEFF4, and the return address for this procedure call in 0x7FFFEFF8.

Instance 3:



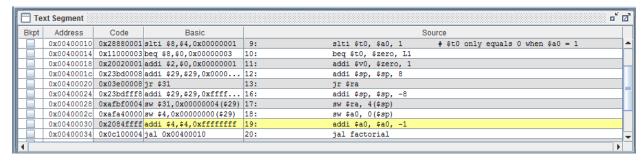
(Text Segment - L1)



(Register \$a0 stores parameter n = 4)

The value n, stored in register \$a0, has now been decremented by one.

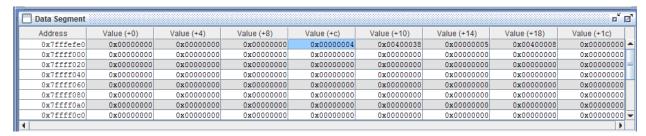
Instance 4:



(Text Segment - L1)

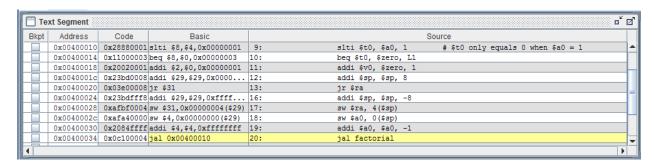
\$sp	29	0x7fffefec
\$fp	30	0x00000000
\$ra	31	0x00400038
pc		0x00400030

Stack pointer (\$sp), return address (\$ra), program pointer (\$pc))



(Data Segment - return address and n = 4)

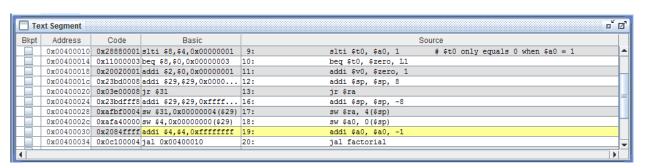
Instance 5:



(Text Segment - L1)



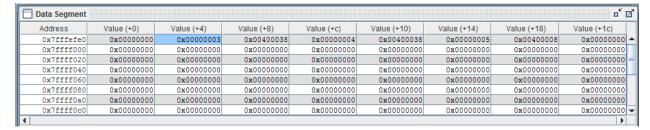
Instance 6:



(Text Segment - L1)

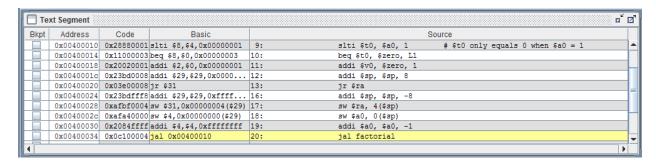
\$sp	29	0x7fffefe4
\$fp	30	0x00000000
\$ra	31	0x00400038
pc		0x00400030

(Stack pointer (\$sp), return address (\$ra), program pointer (\$pc))



(Data Segment - return address and n = 3)

Instance 7:

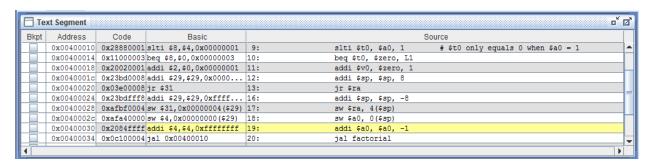


(Text Segment - L1)



(Register \$a0\$ stores parameter <math>n = 2)

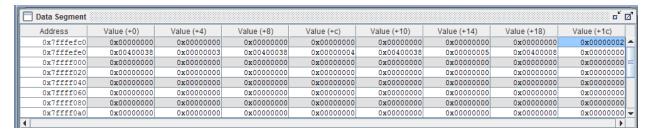
Instance 8:



(Text Segment - L1)

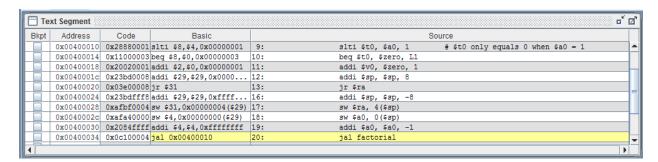
\$sp	29	0x7fffefdc
\$fp	30	0x00000000
\$ra	31	0x00400038
pc		0x00400030

(Stack pointer (\$sp), return address (\$ra), program pointer (\$pc))



(Data Segment - return address and n = 2)

Instance 9:

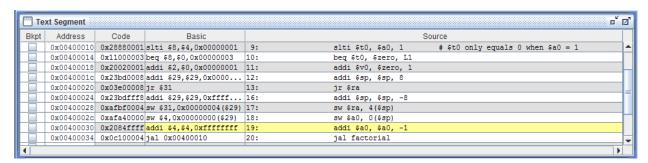


(Text Segment - L1)



(Register \$a0 stores parameter n = 1)

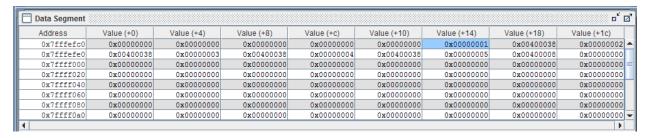
Instance 10:



(Text Segment - L1)

\$sp	29	0x7fffefd4
\$fp	30	0x00000000
\$ra	31	0x00400038
pc		0x00400030

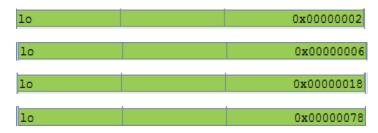
(Stack pointer (\$sp), return address (\$ra), program pointer (\$pc))



(Data Segment - return address and n = 1)

Instance 11:

Now the stack frames must be popped and the values must multiplied.



Once again, we get the end result of 0x78, or 120.

Stack Visual

Address	Content	
0x7FFFEFF8	0x00400008	return address \ \ Factorial(5)
0x7FFFEFF4	0x00000005	n = 5 ∫
0x7FFFEFF0	0x00400038	return address Factorial(4)
0x7FFFEFEC	0x00000004	n = 4
0x7FFFEFE8	0x00400038	return address
0x7FFFEFE4	0x00000003	n = 3 ∫
0x7FFFEFE0	0x00400038	return address \ \ \ Factorial(2)
0x7FFFEFDC	0x00000002	n = 2 ∫
0x7FFFEFD8	0x00400038	return address
0x7FFFEFD4	0x0000001	n = 1

Stack Growth

Conclusion:

We have used three different platforms to analyze how a recursive factorial procedure is interpreted by the CPU and stored in memory. We saw how stack frames must be created in memory for each instance the procedure is recalled. At each instance, the argument of n is decremented by one and stored in memory. Stack frames are then destroyed once the recursive base case is met, and the program links back to the previous frame via the return address. During this process, the arguments at each frame level are multiplied to give us the final result of n factorial. This summarizes how the CPU handles recursive procedure calls.