CS581 Assignment 04 -- Game Theory and Social Networks

I. Read the following in the text: chapter 6.1-6.9, 8.1-8.2, 9.1-9.6

II. Listen to the presentation for this week: Game_Theory_and_Social_Networks.mp4
The associated PPTx file is posted on the Canvas site.

III. Answer the questions below. *Use whole sentences and correct grammar for explanations.* Submit your answers in a .pdf or Word format document on Canvas, in a document labeled ***with your last name***, eg: Dugas_HW3.pdf. You may include legible hand-drawn diagrams in your document, but **all other text must be typed**. PowerPoint is a good tool for drawings.

Introduction:

Questions 1-4 concern 2-player strategic form games, where both players act simultaneously and without knowledge of the other player's actions. You are asked to include a matrix representation of the game as part of your answer. In the Standard Matrix Representation of a 2-player game:

- “row” player is player1; “column” player is player2
- rows correspond to actions player1's actions; columns correspond to player2's actions
- cells list utility or payoff values for each player: player1 first; then player2

		Player 2	
		H	T
Player 1	H	$-1,+1$	$+1,-1$
	T	$+1,-1$	$-1,+1$

The matrix to the right represents the Matching Pennies game.
Suggestion: For questions 1-3, a good approach might be to first identify what type of game is being described, and then proceed with the matrix and other questions.

QUESTIONS:

1. Consider a symmetric game where each player must decide whether to hunt or forage for food. If both players decide to hunt, they successfully capture an animal for meat, and they each get a payoff of 7. If both players forage, they will help each other find food faster and each will get payoff 4. If one player hunts and one forages, the hunter will be injured by the prey and get payoff 0, while the forager will gather some food and get payoff 3.

   a. Draw the matrix representation of this game.
   b. Does either (or both) player(s) have a strictly dominant strategy? Explain your answer.
   c. Identify any pure strategy Nash equilibria. Explain your answer.

2. You (player 1) are browsing the WWW on a computer and things are frustratingly slow. A popup appears that promises increased speed if you click on it. It will manage this by using an alternative protocol to standard TCP. A classmate (player 2) is having the same experience and has seen the same popup. If you both stay with TCP, you will each experience a 1 ms delay (a payoff of -1). If one player stays with TCP and the other goes with the alternative, the TCP player will experience a 4 ms delay and the alternative player will experience zero delay. If you both switch to the alternative protocol, you will each experience a 3 ms delay.

   a. Draw the matrix representation of this game.
   b. Does either (or both) player(s) have a strictly dominant strategy? Explain your answer.
   c. Identify any pure strategy Nash equilibria. Explain your answer.

3. Two drivers are playing chicken, the game where they drive straight toward each other and see if one or both will swerve to avoid a crash. If both drivers swerve, the game is a tie and each gets payoff 0. If one swerves and one stays straight, the driver who swerves is declared a "chicken" and gets payoff -10, while the driver who stays straight wins and gets payoff +10. If both drivers continue straight, they will crash and both get payoff -50.

   a. Draw the matrix representation of this game.
   b. Does either (or both) player(s) have a strictly dominant strategy? Explain your answer.
   c. Identify any pure strategy Nash equilibria. Explain your answer.

4. Draw the matrix representation of the game Rock Paper Scissors Lizard Spock.
   Use 0 for tie, 1 for win, -1 for lose.

   Note: Rock Paper Scissors Lizard Spock (from the Big Bang Theory show): As Sheldon explains:
   "Scissors cuts paper, paper covers rock, rock crushes lizard, lizard poisons Spock, Spock smashes scissors, scissors decapitates lizard, lizard eats paper, paper disproves Spock, Spock vaporizes rock, and as it always has, rock crushes scissors."

5. There are 80 cars which begin in city A and must travel to city B. There are two routes between city A and city B:

   Route I: begins with a local street leaving city A, which requires a travel time in minutes equal to 10 plus the number of cars which use this street, and ends with a highway into city B which requires one hour of travel time regardless of the number of cars which use this highway.

   Route II: begins with a highway leaving city A. This highway takes one hour of travel time regardless of how many cars use it, and ends with a local street leading into city B. This local street near city B requires a travel time in minutes equal to 10 plus the number of cars which use the street.

   Show your work for all calculations.

   a. Draw the network described above and label the edges with the travel time in minutes needed to move along the edge, in terms of x. **Let x be the number of travelers who use Route I.** The network should be a directed graph as all roads are one-way.

   b. What would be the travel time per car if all cars chose to use Route I?

   c. Assume that cars simultaneously chose which route to use.
      Find the Nash equilibrium value of x.

   d. Explain your answer to part c.

Now the government builds a new (two-way) road connecting the nodes where local streets and highways meet. **The new road is very short and takes no travel time.** This adds two new routes.

Route III: consists of the local street leaving city A (on Route I), the new road, and the local street into city B (on Route II).

Route IV: consists of the highway leaving city A (on Route II), the new road, and the highway leading into city B (on Route I).

e. What would the travel time be per car if all cars chose Route III?

f. What would the travel time be per car if all cars chose Route IV?

g. What happens to total travel time as a result of the availability of the new road?

h. If you can assign travelers to routes, it's possible to reduce total travel time relative to what it was before the new road was built. That is, the total travel time of the 80 cars can be reduced (below that in the original Nash equilibrium from part **c**) by assigning cars to routes. There are many assignments of routes that will accomplish this. Find one (where total travel time for all cars is lower than the Nash equilibrium total travel time). Explain why your reassignment reduces total travel time by giving the number of cars assigned to each of the Routes I, II, III, IV, the travel time per car on each route, and the total travel time of the 80 cars.

6. Consider an auction in which there is one seller who wants to sell one unit of a good and a group of bidders who are each interested in purchasing the good. The seller will run a sealed-bid, second-price auction. There will be around a dozen other bidders in addition to your firm. All bidders have independent, private values for the good. Your firm's value for the good is c.

   a. What bid should your firm submit? Explain your answer.

   b. How does your bid depend on the number of other bidders who show up?

7. A seller wants to sell one unit of a good to some bidders, by running a sealed-bid second-price auction. Assume that there are two bidders who have independent, private values **v** which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both 1/2. (If there is a tie at a bid of x for the highest bid the winner is selected at random from among the highest bidders and the price is x.)

   NOTES: (1) When you have multiple identical top bids (a multi-way tie), then the highest and second highest will be that same bid. That means the winner will pay the top bid, because that is both the first- and second-place bid value.
   (2) Do not use the formulas in section 9.7 of the text to calculate expected revenue. You should use the probabilities for the revenue for each of the bidding combinations: e.g. in 7a the probability that the two bidders each bid 3 (with a revenue of 3) is 1/4.

a. Show that the seller’s expected revenue is 6/4.

b. Suppose that there are three bidders who have independent, private values **v** which are either 1 or 3. For each bidder, the probabilities of 1 and 3 are both 1/2. What is the seller’s expected revenue in this case? Explain your answer.

c. Briefly explain why changing the number of bidders affects the seller’s expected revenue. Would the expected revenue increase or decrease? Explain your answer.