Suppose X and Y are independent random variables that are uniform on the intervals $[2, 10]$ and $[1, 12]$, respectively. What is the probability that $\max(X, Y)$ is less than 5 ?

(a) 3/22

(b) 35/88

(c) 21/88

(d) 7/22

(e) 3/8

(f) 5/8

(g) 7/11

(h) 0

(i) 1

(j) None of these

Suppose X and Y are independent random variables that are uniform on the intervals $[1, 10]$ and $[3, 11]$, respectively. What is the probability that $\max(X, Y)$ is less than 5 ?

(a) 1/9

(b) 5/12

(c) 1/3

(d) 1/6

(e) 4/9

(f) 5/9

(g) 3/4

(h) 0

(i) 1

(j) None of these

Suppose X and Y are independent random variables that are uniform on the intervals $[1, 11]$ and $[3, 9]$, respectively. What is the probability that $\max(X, Y)$ is less than 4 ?

(a) 1/20

(b) 7/12

(c) 1/4

(d) 1/12

(e) 3/10

(f) 7/10

(g) 5/6

(h) 0

(i) 1

(j) None of these

Solution:
If X is uniform on $[A, B]$ and Y is uniform on $[C, D]$, then

$$P(\max X, Y < t) = P(X < t, Y < t) = F_{X,Y}(t,t) = F_X(t)F_Y(t) = \frac{t-A}{B-A} \cdot \frac{t-C}{D-C}.$$