If random variables X and Y have joint probability density function $f_{X,Y}(u,v) = Ce^{2v(2-v)-u^2}$, where C is a constant, then what is the sum of their variances?

(a) 3/4

(b) 3/2

(c) 4/3

(d) 6

(e) 3

(f) 2

(g) $\sqrt{3}/2$

(h) 5

(i) 4

(j) 1

(k) None of these

Solution:

$$f_{X,Y}(u,v) = Ce^{2v(2-v)-u^2}$$

$$= Ce^2 e^{-u^2} e^{-2(v-1)^2}$$

$$= Ce^2 \cdot e^{-\frac{1}{2}\left(\frac{u^2}{1/2}\right)} \cdot e^{-\frac{1}{2}\left(\frac{(v-1)^2}{1/4}\right)}$$

$$\therefore \ \sigma_X^2 + \sigma_Y^2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}.$$