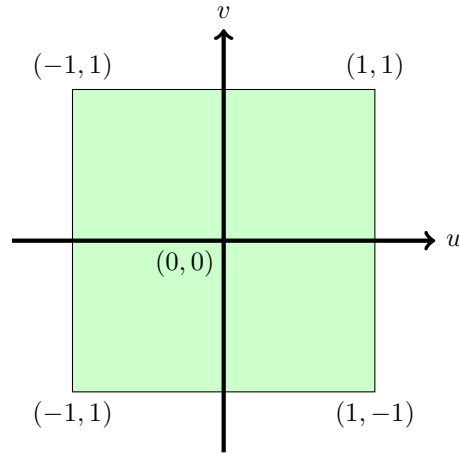Suppose X and Y are random variables whose joint probability density function is constant in the set $\{(u,v) : u^2 + v^2 < 2\}$ and is zero elsewhere. What is the probability that the point (X,Y) lies inside the green square shown below?

(a) $2/\pi$

(b) $2/\pi^2$

(c) $\pi/6$

(d) $\pi/4$

(e) $1/(2\pi)$

(f) $1/(4\pi)$

(g) $1/2$

(h) $1/3$

(i) $1/4$

(j) 1

(k) None of these

Solution:

The square is inscribed in the circle and $f_{X,Y}(u,v) = 1/2\pi$. The probability that (X,Y) lies in the square is $\int_{-1}^{1} \int_{-1}^{1} (1/2\pi) du dv = 4/2\pi = 2/\pi$.