Suppose we flip a biased coin with $P(\text{Heads}) = 3/4$ twice. What is the probability the second flip is Heads, given that at least one of the flips is Tails?

(a) 3/7

(b) 4/7

(c) 1/7

(d) 3/4

(e) 1/4

(f) 9/16

(g) 7/16

(h) 1/3

(i) 1/2

(j) 1

(k) 0

(l) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 2/5$ twice. What is the probability the second flip is Heads, given that at least one of the flips is Tails?

(a) 2/7

(b) 5/7

(c) 1/7

(d) 2/5

(e) 3/5

(f) 4/25

(g) 21/25

(h) 1/3

(i) 1/2

(j) 1

(k) 0

(l) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 4/5$ twice. What is the probability the second flip is Heads, given that at least one of the flips is Tails?

(a) 4/9

(b) 5/9

(c) 1/9

(d) 4/5

(e) 1/5

(f) 16/25

(g) 9/25

(h) 1/3

(i) 1/2

(j) 1

(k) 0

(l) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 5/6$ twice. What is the probability the second flip is Heads, given that at least one of the flips is Tails?

(a) 5/11

(b) 6/11

(c) 1/11

(d) 5/6

(e) 1/6

(f) 25/36

(g) 11/36

(h) 1/3

(i) 1/2

(j) 1

(k) 0

(l) None of these

Solution: Let E be the event the first flip is heads and let F be the event the second flip is Heads. Then,

$$\begin{aligned} P(F|(EF)^c) &= \frac{P(F(EF)^c)}{P((EF)^c)} \\ &= \frac{P(F(E^c \cup F^c))}{1 - P(EF)} \\ &= \frac{P(FE^c)}{1 - q^2} \\ &= \frac{q(1-q)}{1-q^2} \\ &= \frac{q}{1+q} \end{aligned}$$

If $q = k/n$, then $P(F|(EF)^c) = \frac{k/n}{1+(k/n)} = \frac{k}{k+n}$.