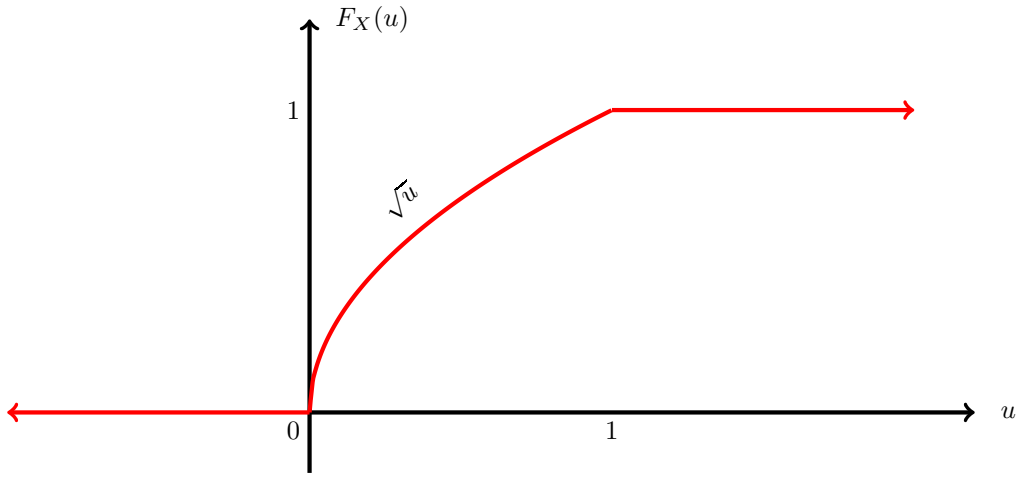
Let X be a random variable, whose cummulative distribution function equals $\sqrt{u}$ in the interval $[0, 1]$, as shown below. What is the variance of X ?



(a) 4/45

(b) 2/15

(c) 1/5

(d) 1/3

(e) 1/9

(f) 1/4

(g) 3/4

(h) 0

(i) 1

(j) 1/2

(k) None of these

Solution:

$$
\begin{aligned}
f_X(u) &= \frac{d}{du} u^{1/2} = \frac{1}{2\sqrt{u}} \\
E[X^n] &= \int_0^1 \frac{u^n du}{2\sqrt{u}} \\
&= \frac{1}{2} \int_0^1 u^{n-(1/2)} du \\
&= \frac{1}{2n+1} u^{n+(1/2)} \Big|_0^1 \\
&= \frac{1}{2n+1} \\
\sigma_X^2 &= E[X^2] - (E[X])^2 = (1/5) - (1/3)^2 \\
&= 4/45
\end{aligned}
$$