Suppose we flip a biased coin with $P(\text{Heads}) = 3/8$. If we get Heads, then we roll a fair 6-sided die and observe the output, and if we get Tails, then we just declare the output to be 2. What is the probability the output is either 1 or 2 ?

(a) 3/4

(b) 3/8

(c) 1/4

(d) 1/8

(e) 1/6

(f) 5/6

(g) 1/3

(h) 2/3

(i) 1

(j) 0

(k) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 5/6$. If we get Heads, then we roll a fair 6-sided die and observe the output, and if we get Tails, then we just declare the output to be 3. What is the probability the output is either 1 or 3 ?

(a) 4/9

(b) 2/9

(c) 5/9

(d) 5/18

(e) 1/6

(f) 5/6

(g) 1/3

(h) 2/3

(i) 1

(j) 0

(k) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 3/10$. If we get Heads, then we roll a fair 6-sided die and observe the output, and if we get Tails, then we just declare the output to be 4. What is the probability the output is either 1 or 4 ?

(a) 4/5

(b) 2/5

(c) 1/5

(d) 1/10

(e) 1/6

(f) 5/6

(g) 1/3

(h) 2/3

(i) 1

(j) 0

(k) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 5/8$. If we get Heads, then we roll a fair 6-sided die and observe the output, and if we get Tails, then we just declare the output to be 5. What is the probability the output is either 1 or 5 ?

(a) 7/12

(b) 7/24

(c) 5/12

(d) 5/24

(e) 1/6

(f) 5/6

(g) 1/3

(h) 2/3

(i) 1

(j) 0

(k) None of these

Solution: Let E and F be the events that the output is 1 and 5, respectively. Let G be the event the coin is Heads. Then

$$\begin{aligned}
P(E|G^c) &= 0 \\
P(E \cup F) &= P(E) + P(F) \\
&= \ P(E|G)P(G) + P(E|G^c)P(G^c) \\
&\ + P(F|G)P(G) + P(F|G^c)P(G^c) \\
&= (1/6)P(G) + 0 \\
&\ + (1/6) \cdot P(G) + 1 \cdot (1 - P(G)) \\
&= 1 - (2/3)P(G)
\end{aligned}$$